Bayesian mixture model for environmental application

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Data exploration

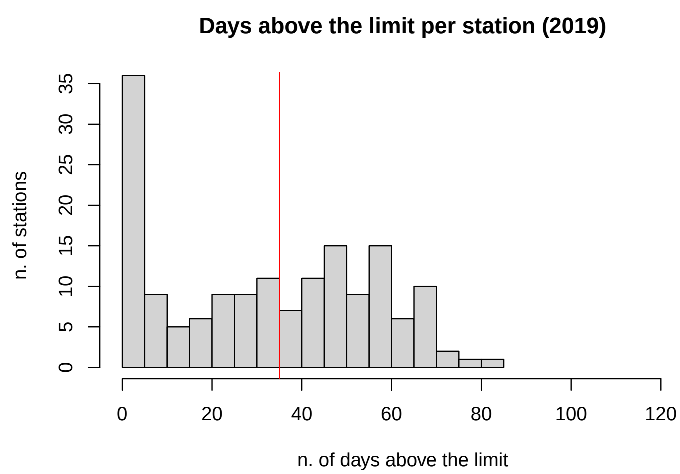
Data visualization

The data under consideration are collected from 162 monitoring stations between 2013 and 2019 in Northern Italy (European Environmental Agency) and for each station we have a daily measure of PM10 concentration.

The first analysis we've done was aimed to understand how many stations revealed levels of concentration above the limit threshold. To do so we plotted an histogram of the number of days in which a station was above such threshold in the year 2019:

```
day_above_limit <- function(year){
  poll <- loadData(year)
  max_daily_val = 50 #critic daily level
  max_dang_days = 35

  #stores for every station the num of "dangerous" days
  n_dang_days = colSums(poll>max_daily_val)

  #hist of the stations divided for the number of dangerous days
  hist(n_dang_days, breaks = 20, main=paste('Days above the limit
      per station (',year,')',sep=''), xlab='n. of days above the limit',
       ylab='n. of stations', xlim=c(0,125), ylim=c(0,35))
  abline(v=max_dang_days, col='red')
}
```

```
day_above_limit(2019)
```



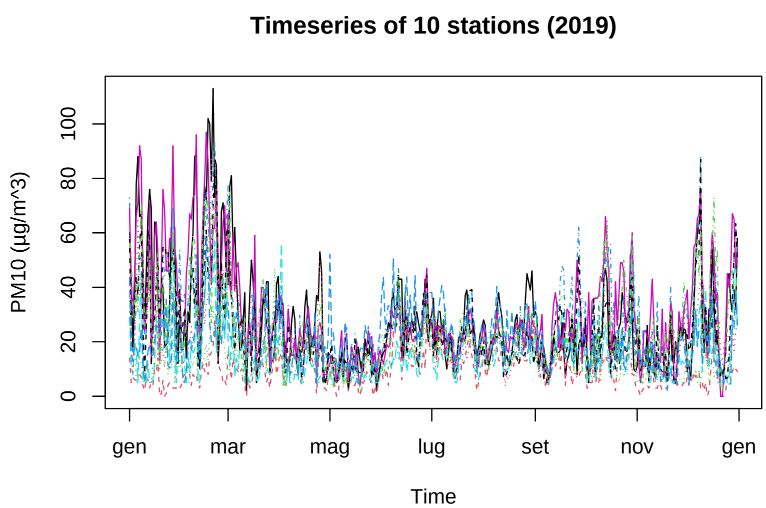
The European Environment Agency (EEA) has set two limit values for PM10: the PM10 daily mean value may not exceed 50 micrograms per cubic metre ($\mu m/m^3$) more than 35 times in a year and the PM10 annual mean value may not exceed 40 micrograms per cubic metre ($\mu m/m^3$).

From the previous plot, we can say that 77 stations out of 162 overcame the limit, which is fixed at 35 days per year. In particular, some of them reached the maximum concentration per day for 80 days.

Then we proceeded analizing the PM10 concentration of 10 stations just to have an idea of the level and seasonality of the time series.

```
matplot1 <- function(){
  poll <- loadData(2019)
  set.seed(5)
  rand_st <- sample(1:dim(poll)[2], 10)
  days= seq(as.Date("2019-01-01"), as.Date("2019-12-31"), by="days")
  matplot(days,poll[,rand_st],type='l',xlab='Time',ylab='PM10 (µg/m^3)',
          main="Timeseries of 10 stations (2019)")
}
```

```
matplot1()
```



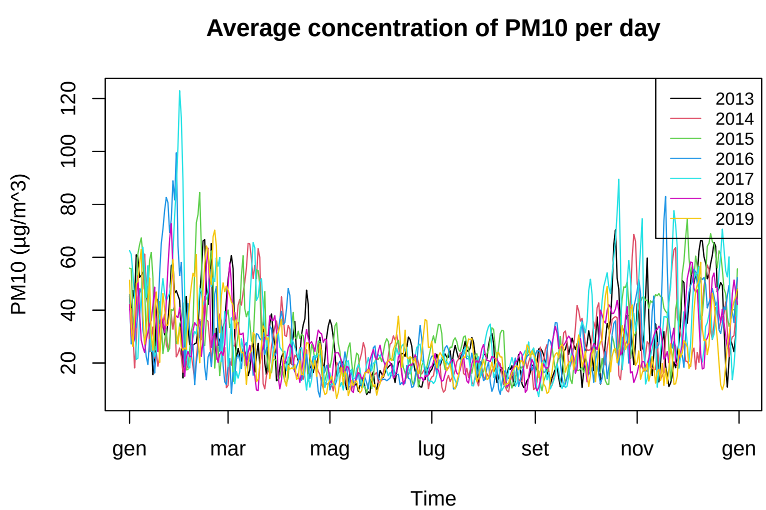
We can see how the time-series are pretty similar in level and seasonality.

Moreover, to give another insight on the seasonality trend we computed the mean of the stations in each year and compared:

```
matplot2 <- function(){

  pollutant <- read.csv('code/data/timeSeriesData.csv')
  N = dim(pollutant)[1] #time instants
  YEARS <- 7

  daily.avg = rep(0,N) #Mean in the day if there is at least one valid value
  for(i in 1:N){
    daily.avg[i] = mean(as.numeric(pollutant[i,]), na.rm=T)
  }
```

```r
  avg_per_y = matrix(NA, nrow = 365, ncol = YEARS)

  for (y in 1:YEARS){
    avg = rep(0,365)
    for(d in 1:365){
      i = (y-1)*365 + d
      avg_per_y[d,y] = daily.avg[i]
    }
  }

  days= seq(as.Date("2019-01-01"), as.Date("2019-12-31"), by="days")
  matplot(days,avg_per_y,type='l',xlab='Time',ylab='PM10 (µg/m^3)',
          main='Average concentration of PM10 per day',col=1:YEARS,lty=1)
  legend('topright',legend=2013:(2013+YEARS-1),col=1:YEARS,lty=1, cex=0.8)
}
```

```r
matplot2()
```



One can say that all the analysed years behave similarly, and the average concentration of PM10 is visibly lower in the summertime rather than in winter months.

Persistence analysis

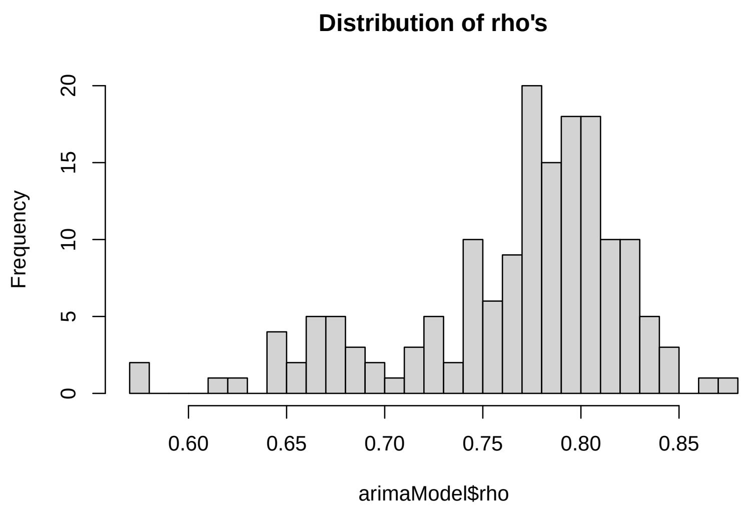
We then proceeded estimating the values of ρ and σ^2 through an autorgeressive process of order 1 using the R package `arima`,

```r
arima_model <- function(){
  pollutant <- read.csv('code/data/timeSeriesData.csv')
  rho <- c()
  sigma <- c()
  for(r in 1:dim(pollutant)[2]){
    a = arima(as.numeric(pollutant[,r]), order=c(1,0,0))
```

```
    rho[r] = as.numeric(a$coef[1])
    sigma[r] = sqrt(as.numeric(a$sigma2))

  }
  return(list(rho=rho,sigma=sigma))
}
```

and plotted its distribution both on an histogram:

```
arimaModel <- arima_model()
hist(arimaModel$rho, breaks = 25,main="Distribution of rho's")
```



and on the Northern Italy map:

```
plot_rho <- function(rho){
  stat_inf <- read.csv('code/data/stationsInfo.csv')
  sigma <- rep(NaN,length(rho))
  plotData <- data.frame(site=stat_inf$site,rho=rho,sigma=sigma,
                      latitude=stat_inf$latitude,longitude=stat_inf$longitude)

  #plot on the map
  pal <- colorNumeric(
    palette = colorRampPalette(c('green', 'yellow', 'red'))
    (length(plotData$rho)), domain = plotData$rho)


  map <- leaflet() %>% addProviderTiles(providers$Stamen.TonerLite,
                                  options = providerTileOptions(noWrap = TRUE)
  )
  map%>%
    addCircleMarkers(plotData$longitude,plotData$latitude,
                    color = pal(plotData$rho),
                    radius = 2,
                    label = plotData$rho,
```

```
                     labelOptions = labelOptions(textsize = "12px"),
                     popup = plotData$site_type
    )%>% addLegend('topright',
                   pal = pal,
                   values = plotData$rho)
}
```

```
plot_rho(arimaModel$rho)
```

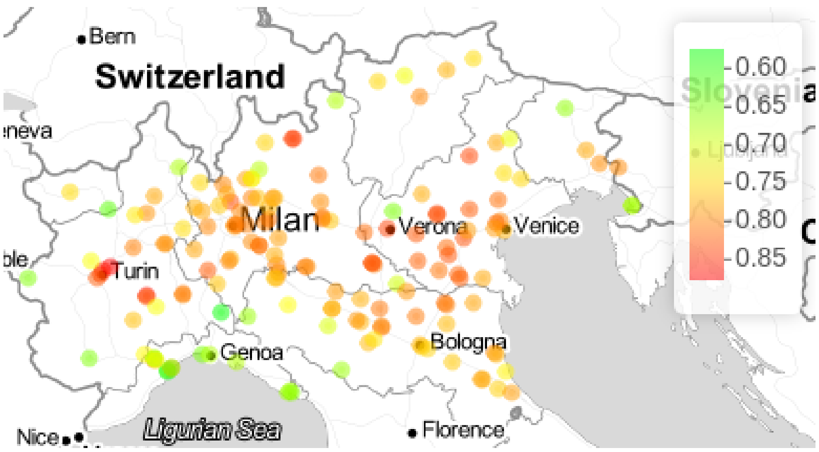


Figure 1: ρ's distribution in Northern Italy

Given the distribution of the ρ's shown in the histogram, we may be able to obtain a reasonable clustering.

In the last figure you can see the distribution of the ρ_i for each station considered. The green points are the stations with a low value of ρ, so a low persistence, while the red points are the ones with persistent time series. In particular, we can see that in the Po valley the persistence is high while in the zones near the sea or in a more mountaineer city the value of the persistence is smaller.