

# Regressione Lineare e Anova

## Progetto di Inferenza Statistica

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Luglio 2021

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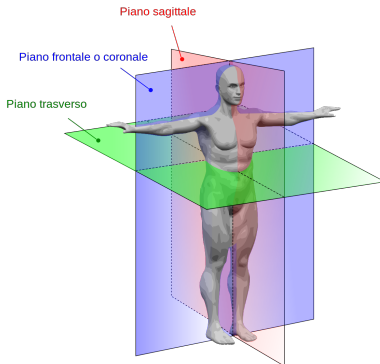


# Covariate presenti

- pelvic incidence (continua)
- pelvic tilt (continua)
- lumbar lordosis angle (continua)
- sacral slope (continua)
- pelvic radius (continua)
- grade of spondylolisthesis (continua)
- class (categorica)

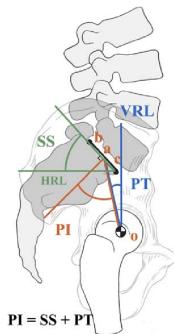
con 310 osservazioni.

Le classi sono *Hernia*, *Spondylolisthesis*, *Normal*.

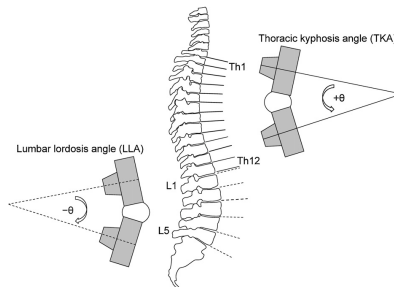




Vogliamo fare una regressione su misurazioni fisiche della zona pelvica...



... per poter stimare il **lumbar lordosis angle**, un parametro che riguarda la sezione medio-bassa della schiena che misura la concavità della zona lombare della schiena.



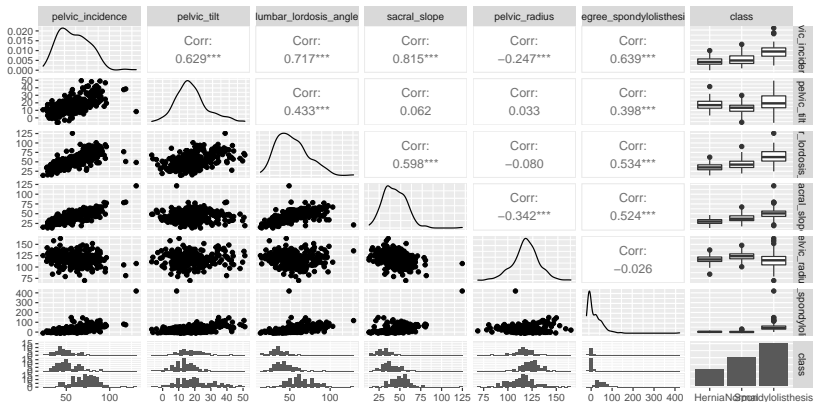




## Overview dei dati

	pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	sacral_slope	pelvic_radius
1	63.02782	22.552586	39.60912	40.47523	98.67292
2	39.05695	10.060991	25.01538	28.99596	114.40543
3	68.83202	22.218482	50.09219	46.61354	105.98514
4	69.29701	24.652878	44.31124	44.64413	101.86850
5	49.71286	9.652075	28.31741	40.06078	108.16872
6	40.25020	13.921907	25.12495	26.32829	130.32787
	degree_spondylolisthesis	class			
1	-0.254400	Hernia			
2	4.564259	Hernia			
3	-3.530317	Hernia			
4	11.211523	Hernia			
5	7.918501	Hernia			
6	2.230652	Hernia			

Non ci sono degli NA.







Prevediamo sacral\_slope in funzione di tutto il resto, tranne la risposta originale e la categorica.

Call:

```
lm(formula = sacral_slope ~ . - class - lumbar_lordosis_angle,  
    data = biomech)
```

Residuals:

Min	1Q	Median	3Q	Max
-1.091e-08	-5.072e-10	1.020e-10	3.703e-10	1.057e-08

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	6.414e-10	3.159e-09	2.030e-01	0.839
pelvic_incidence	1.000e+00	2.570e-11	3.891e+10	<2e-16 ***
pelvic_tilt	-1.000e+00	3.545e-11	-2.821e+10	<2e-16 ***
pelvic_radius	-8.878e-12	2.171e-11	-4.090e-01	0.683
degree_spondylolisthesis	4.338e-12	9.388e-12	4.620e-01	0.644

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.687e-09 on 305 degrees of freedom

Multiple R-squared: 1, Adjusted R-squared: 1

F-statistic: 6.337e+20 on 4 and 305 DF, p-value: < 2.2e-16

$R_{adj}^2$  vale 1, osservando i  $\beta$  scopriamo che

$$\text{sacral slope} + \text{pelvic tilt} = \text{pelvic incidence}$$

In ambito medico abbiamo conferma di questa cosa:

*Pelvic tilt and sacral slope are two angles directly correlated with the pelvic incidence angle. The angle of incidence is the algebraic sum of two angles: pelvic tilt (PT) and sacral slope (SS)*<sup>1</sup>

Escludiamo questa covariata.

---

<sup>1</sup><https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3175921/>

Procediamo con il nostro modello escludendo la `sacral_slope` e la categoria.

Call:

```
lm(formula = lumbar_lordosis_angle ~ . - class - sacral_slope,
    data = biomech)
```

Residuals:

Min	1Q	Median	3Q	Max
-76.720	-7.415	-1.261	6.878	70.183

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-10.55489	8.59928	-1.227	0.2206
pelvic_incidence	0.76884	0.06996	10.989	<2e-16 ***
pelvic_tilt	-0.11410	0.09650	-1.182	0.2380
pelvic_radius	0.14092	0.05911	2.384	0.0177 *
degree_spondylolisthesis	0.05166	0.02555	2.022	0.0441 *

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 12.76 on 305 degrees of freedom

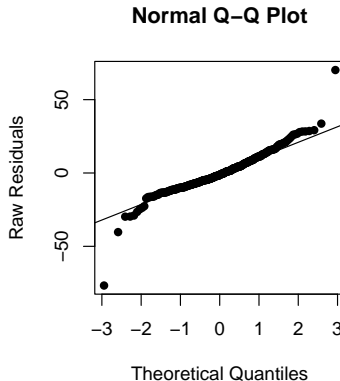
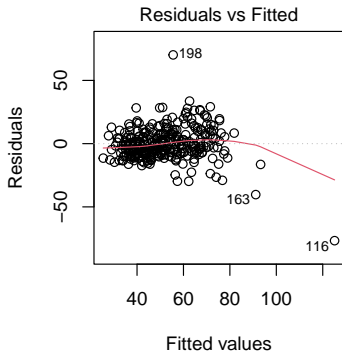
Multiple R-squared: 0.5334, Adjusted R-squared: 0.5272

F-statistic: 87.15 on 4 and 305 DF, p-value: < 2.2e-16



# Controlliamo le ipotesi

L'omoschedasticità non è fantastica e lo Shapiro test rifiuta la normalità con un p-value di 3.878e-11.



lm(lumbar lordosis\_angle ~ class - sacral)

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Procediamo dapprima con la pulizia del dataset, controllando se migliorano le ipotesi di lavoro, e nel caso procediamo con la trasformazione box cox.

I punti leva ( $h_{ii} > 2p/n$ ) risultano:

10	52	76	84	86	96	116	123	142	163
164	168	174	180	181	191	193	198	203	207

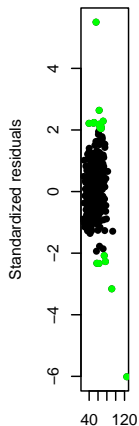
Vediamo anche residui studentizzati e standardizzati.

Gli standardizzati sono

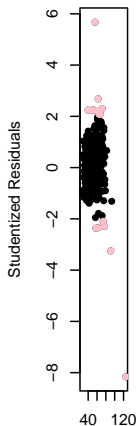
52	72	75	83	94	99	113	116
125	135	143	163	183	198	202	203

e coincidono con gli studentizzati.

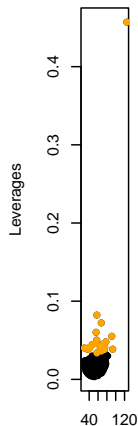
# Standardized residuals Studentized Resid Leverages



Fitted values

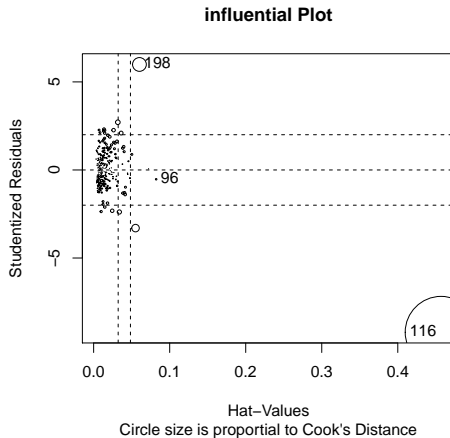


Fitted values



Fitted values

## Punti influenti



## Dopo aver tolto i leverage:

Call:

```
lm(formula = lumbar_lordosis_angle ~ . - class - sacral_slope,  
    data = biomech, subset = (lev < 2 * p/n))
```

Residuals:

Min	1Q	Median	3Q	Max
-38.740	-6.327	-0.724	5.792	26.955

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-14.65892	7.78281	-1.884	0.06065 .
pelvic_incidence	0.82717	0.06289	13.152	< 2e-16 ***
pelvic_tilt	-0.37281	0.09005	-4.140	4.58e-05 ***
pelvic_radius	0.15686	0.05499	2.853	0.00465 **
degree_spondylolisthesis	0.20055	0.03096	6.477	4.08e-10 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.12 on 285 degrees of freedom

Multiple R-squared: 0.6929, Adjusted R-squared: 0.6886

F-statistic: 160.7 on 4 and 285 DF, p-value: < 2.2e-16

## Dopo aver tolto gli studentizzati:

Call:

```
lm(formula = lumbar_lordosis_angle ~ . - class - sacral_slope,  
    data = biomech, subset = (abs(stud) < 2))
```

Residuals:

Min	1Q	Median	3Q	Max
-24.7494	-6.7045	-0.8731	6.1216	25.1839

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-7.11477	6.20910	-1.146	0.2528
pelvic_incidence	0.87282	0.05160	16.914	< 2e-16 ***
pelvic_tilt	-0.39784	0.07492	-5.310	2.19e-07 ***
pelvic_radius	0.08100	0.04293	1.887	0.0602 .
degree_spondylolisthesis	0.13697	0.02437	5.621	4.48e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9.021 on 289 degrees of freedom

Multiple R-squared: 0.7298, Adjusted R-squared: 0.7261

F-statistic: 195.2 on 4 and 289 DF, p-value: < 2.2e-16

## Dopo aver tolto i leverage e gli studentizzati:

Call:

```
lm(formula = lumbar_lordosis_angle ~ . - class - sacral_slope,  
    data = biomech, subset = (abs(stud) < 2 | lev < 2 * p/n))
```

Residuals:

Min	1Q	Median	3Q	Max
-34.290	-7.025	-0.731	5.986	31.766

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-6.11635	6.98922	-0.875	0.3822
pelvic_incidence	0.83386	0.05816	14.337	< 2e-16 ***
pelvic_tilt	-0.40375	0.08410	-4.801	2.50e-06 ***
pelvic_radius	0.09063	0.04823	1.879	0.0612 .
degree_spondylolisthesis	0.16506	0.02716	6.077	3.71e-09 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.32 on 300 degrees of freedom

Multiple R-squared: 0.6759, Adjusted R-squared: 0.6716

F-statistic: 156.4 on 4 and 300 DF, p-value: < 2.2e-16



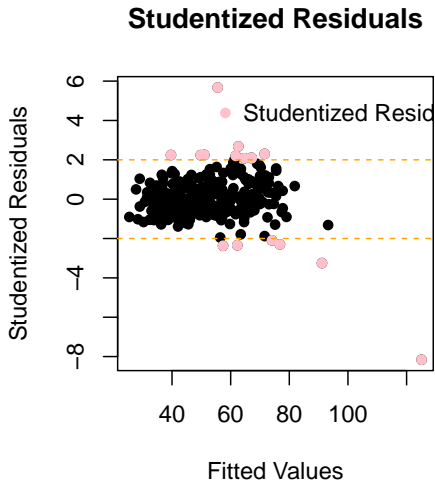
Aggiungiamo il calcolo dell'AIC.

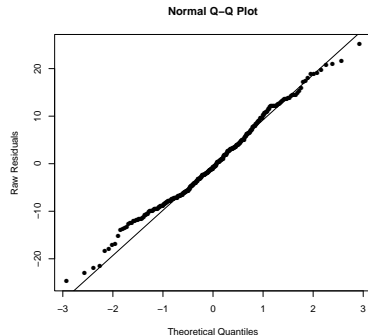
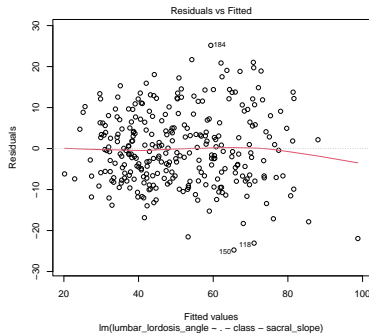
```
> AIC(g_post_lev)
[1] 2172.373
> AIC(g_post_rs)
[1] 2134.629
> AIC(g_post_both)
[1] 2296.307
```

Il modello migliore è quello senza i punti influenti trovati coi residui studentizzati, avendo  $R^2_{adj}$  maggiore e AIC minore.

L'  $R^2_{adj}$  aumenta notevolmente a 0.7261.

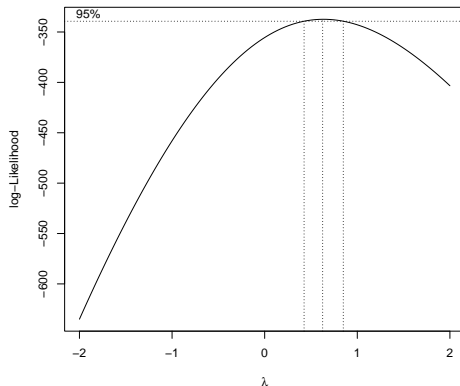
p-value è 2.2e-16, ci sono ancora covariate non significative, stavolta diverse da prima.





# Trasformazione box cox

Otteniamo  $\lambda = 0.6262626$



Generiamo il nuovo ML dove modelliamo  $\frac{Y^\lambda - 1}{\lambda}$

Call:

```
lm(formula = (lumbar_lordosis_angle~best_lambdagl - 1)/best_lambdagl ~  
  . - class - sacral_slope, data = biomech, subset = (abs(stud) <  
  2))
```

Residuals:

Min	1Q	Median	3Q	Max
-5.5750	-1.5905	-0.1978	1.4263	5.4229

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	3.517005	1.436634	2.448	0.0150 *
pelvic_incidence	0.204373	0.011940	17.117	< 2e-16 ***
pelvic_tilt	-0.096457	0.017334	-5.565	6.00e-08 ***
pelvic_radius	0.017717	0.009934	1.784	0.0755 .
degree_spondylolisthesis	0.030430	0.005638	5.397	1.41e-07 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.087 on 289 degrees of freedom

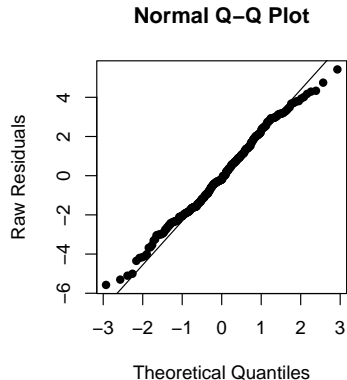
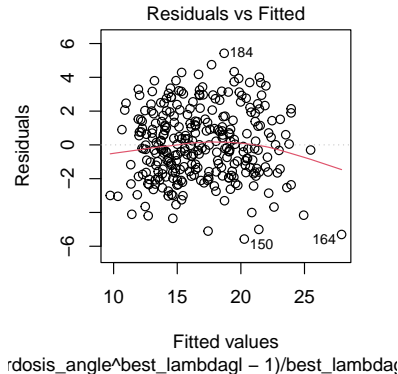
Multiple R-squared: 0.7288, Adjusted R-squared: 0.725

F-statistic: 194.1 on 4 and 289 DF, p-value: < 2.2e-16

$R^2_{adj}$  diminuisce da 0.7261 a 0.725.

Verifichiamo se ora le ipotesi di normalità sono soddisfatte.

L'omoschedasticità rimane la stessa, lo Shapiro test non rifiuta la normalità con un p-value di 0.196.



# Selezione delle covariate

Rimuoviamo `pelvic_radius` che ha un p-value *one-at-a-time* di 0.0755, c'è evidenza per dire che non è significativo.

Call:

```
lm(formula = (lumbar_lordosis_angle~best_lambdag1 - 1)/best_lambdag1 ~  
  . - class - sacral_slope - pelvic_radius, data = biomech,  
  subset = (abs(stud) < 2))
```

Residuals:

Min	1Q	Median	3Q	Max
-5.8320	-1.6205	-0.0449	1.4739	5.4439

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	5.902680	0.526115	11.219	< 2e-16 ***
pelvic_incidence	0.196926	0.011228	17.539	< 2e-16 ***
pelvic_tilt	-0.089619	0.016969	-5.282	2.52e-07 ***
degree_spondylolisthesis	0.031657	0.005617	5.635	4.13e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.095 on 290 degrees of freedom

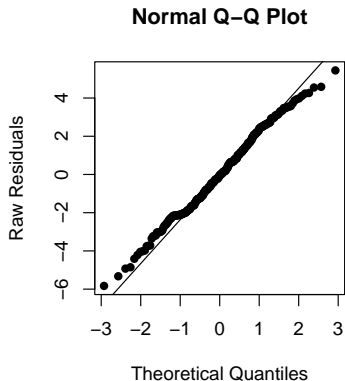
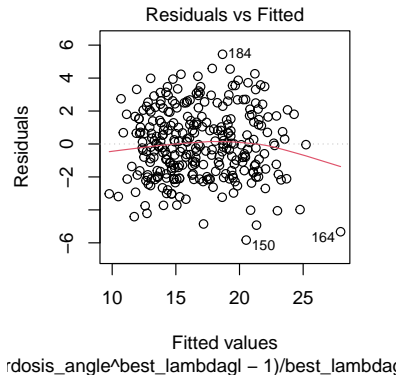
Multiple R-squared: 0.7258, Adjusted R-squared: 0.723

F-statistic: 255.9 on 3 and 290 DF, p-value: < 2.2e-16



$R_{adj}^2$  scende da 0.725 a 0.723, ma semplifica di molto il modello, quindi procediamo con questa modifica.  
Verifichiamo come cambiano le ipotesi di normalità e omoschedasticità.

L'omoschedasticità rimane la stessa, lo Shapiro test migliora con un p-value di 0.22.

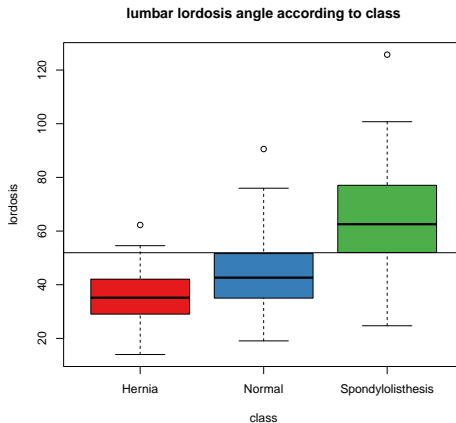


# Modello conclusivo

$$\frac{LLA^\lambda - 1}{\lambda} = 5.902680 + 0.196926 \cdot PI - 0.089619 \cdot PT + 0.031657 \cdot DS$$

dove  $\lambda = 0.6262626$ .

Procediamo ora con l'ANOVA, siamo interessati a capire se la media della nostra risposta LLA è diversa tra le classi.



Lo Shapiro test ci porta tuttavia a rifiutare la normalità nei gruppi.

Hernia	Normal	Spondylolisthesis
0.68493698	0.01650640	0.07089939

Sia il Levene test che il Bartlett test ci portano a rifiutare anche l'omoschedasticità tra i gruppi.

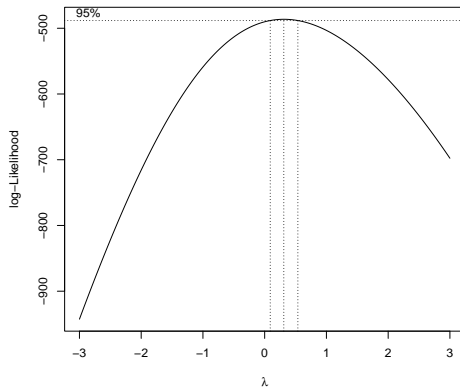
```
> leveneTest(biomech$lumbar_lordosis_angle, biomech$class)
Levene's Test for Homogeneity of Variance (center = median)
      Df F value    Pr(>F)
group  2  9.8185 7.357e-05 ***
      307
---
> bartlett.test(biomech$lumbar_lordosis_angle, biomech$class)

Bartlett test of homogeneity of variances

data: biomech$lumbar_lordosis_angle and biomech$class
Bartlett's K-squared = 23.183, df = 2, p-value = 9.242e-06
```

## Trasformazione box cox

Otteniamo  $\lambda = 0.31$





Generiamo ora il modello.

Call:

```
lm(formula = (biomech$lumbar_lordosis_angle^best_lambda - 1)/best_lambda ~
    class - sacral_slope, data = biomech)
```

Residuals:

Min	1Q	Median	3Q	Max
-2.91535	-0.59753	0.01302	0.66488	2.80340

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	6.4423	0.1179	54.623	< 2e-16	***
classNormal	0.6371	0.1492	4.271	2.6e-05	***
classSpondylolisthesis	1.9655	0.1395	14.084	< 2e-16	***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9136 on 307 degrees of freedom

Multiple R-squared: 0.4449. Adjusted R-squared: 0.4413

F-statistic: 123 on 2 and 307 DF, p-value:  $< 2.2e-16$



