Realisability of Global Types: Decidability and Verification

Presentata da: **Gabriele Genovese**

Relatore: Prof. Ivan Lanese
Correlatori: Prof. Cinzia Di Giusto
Chiar.mo Prof. Étienne Lozes
Controrelatore: Chiar.mo Prof. Luca Padovani

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i3S sophia antipolis

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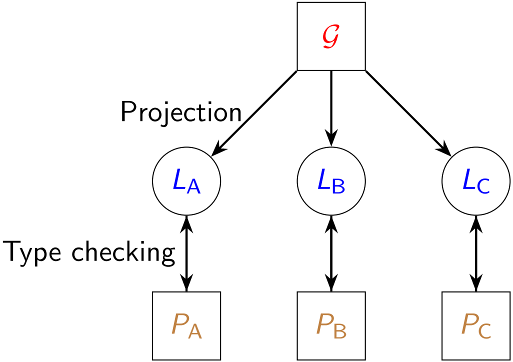
Multiparty Session Types

- Honda, K., Yoshida, N., and Carbone, M. (2008)
- Verification and design of *communication protocols*
- Avoid *deadlocks*, ensure *progress*, etc...



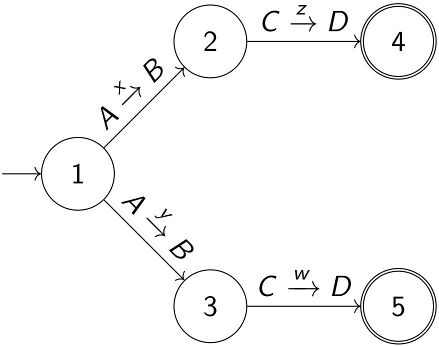
1. Global type

2. Local type

3. Processes

Global Types

- Description of a **global** behavior of a system.
- Defined as automata



Local type

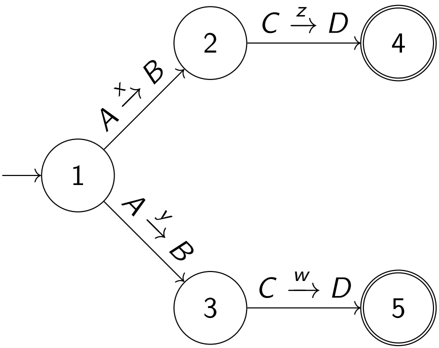
- Point of view of a participant
- Obtained via *projection* operation
- Behavior can differ for different communication semantics (FIFO, sync)

Realisability problem: Does the implementation of a system **respects** the behavior described?

$L(G) = L_{\text{com}}(\text{proj}(G))$ where com is FIFO or sync.

The example is not realisable

This example is **not** realisable because C doesn't know what A sent.



The trace $A \xrightarrow{x} B; C \xrightarrow{w} D$ does not appear in $L(G)$.

Reduction to sync [1]

A global type G is deadlock-free realisable in **FIFO** iff:

1. $L_{\mathrm{FIFO}}(\mathit{proj}(G))$ is sync;
2. $\mathit{proj}(G)$ is orphan-free in FIFO;
3. $L_{\mathrm{FIFO}}(\mathit{proj}(G))$ is deadlock-free
4. G is weak realisable in sync
5. G is deadlock-free in sync

[1] Di Giusto, Cinzia, Etienne Lozes, and Pascal Urso. "Realisability and Complementability of Multiparty Session Types." (2025).

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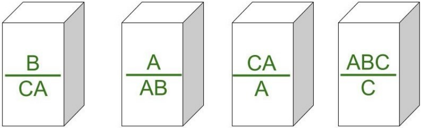
First contribution

- Realisability for sync global types is **undecidable**.
  Proof: by reduction to the PCP problem.
- PCP: given a set of tiles, find an ordering such that the strings formed by the top and bottom halves are equal.
- Proof adaptated from Alur et al. [2]



[2] Alur, Rajeev, Kousha Etessami, and Mihalis Yannakakis. "Realizability and verification of MSC graphs." Theoretical Computer Science 331.1 (2005): 97-114.

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Second contribution

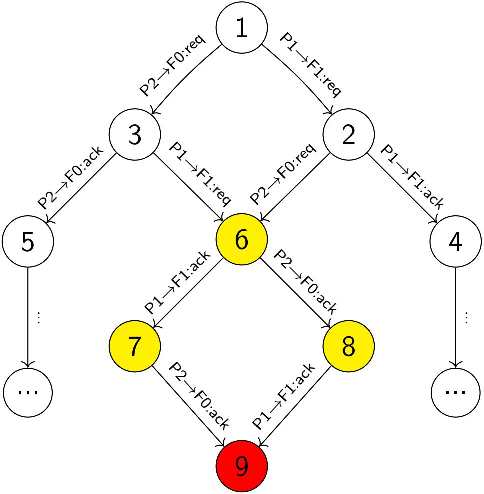
- ResCu: Model-checking TUI tool written in OCaml
- Added two verification if *sync* system:
  - *Deadlock-freedom*: a final state is always reachable
  - *Progress*: the system can always perform an action

ReSCu Example - Dining Philosophers

Two Philosophers, two forks.

```
This system is RSC.
There are some sink states:
Sink: Id=11 Configuration={F0:4; F1:3; P1:2; P2:2}
There are some deadlock states:
Deadlock: Id=4 Configuration={F0:2; F1:1; P1:1; P2:1}
Deadlock: Id=11 Configuration={F0:4; F1:3; P1:2; P2:2}
...
```

ReSCu Example - Dining Philosophers



Conclusion

Summary of contributions:

- Proof of undecidability for weak realisability in sync
- Enriched the tool ReSCu

Future work:

- Prove undecidability of deadlock-free realisability for sync global types
- Continue the development of ReSCu

Thanks! Questions?

Weak and Safe realisability

- Weak realisability: the global type G is *weak* realisable in sync if there exist a CFSM system that can implement the global type.
- Safe realisability: the global type G is *safe* realisable in sync if it is *weak realisable* and the CFSM system is **deadlock free**.

To prove:

$\Delta \in$ RPCP iff the global type L^* is not weak realisable.

Relaxed Post Correspondence Problem (RPCP)

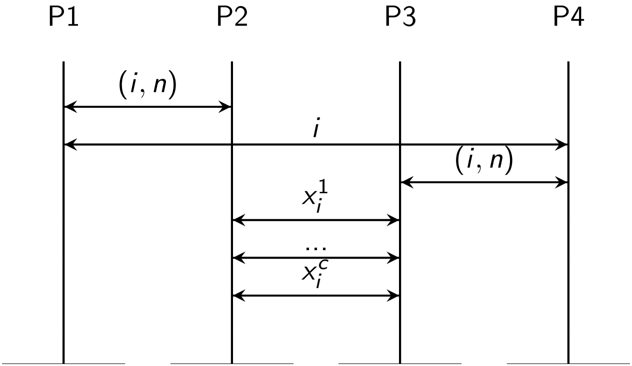
Given a set of tiles $\{(v_1, w_1), (v_2, w_2), \ldots, (v_r, w_r)\}$, determining whether there exist indices $i_1, \ldots, i_m$ such that

$$x_{i_1} \cdots x_{i_m} = y_{i_1} \cdots y_{i_m},$$

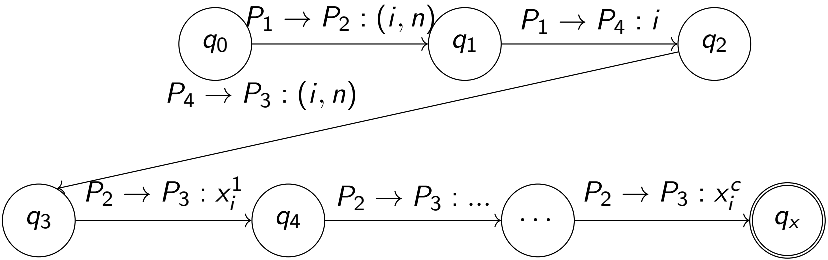
where $x_{i_j}, y_{i_j} \in \{v_{i_j}, w_{i_j}\}$, such that:

- there exists at least one index i_ℓ for which $x_{i_\ell} \neq y_{i_\ell}$, and
- for all $j \leq m$, $y_{i_1} \cdots y_{i_j}$ is a strict or not-strict prefix of $x_{i_1} \cdots x_{i_j}$.

The MSC M_i^n



The global type G_i^n



The global type L^*