Realisability of Global Types: Decidability and Verification

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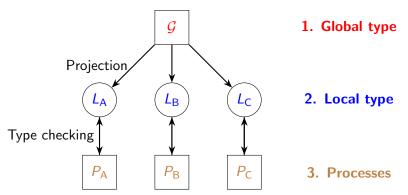
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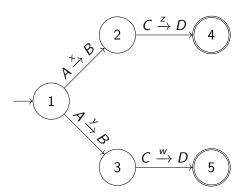
Multiparty Session Types

- ► Honda, K., Yoshida, N., and Carbone, M. (2008)
- Verification and design of communication protocols
- Avoid deadlocks, ensure progress, etc...



Global Types

- Description of a **global** behavior of a system.
- Defined as automata



Local type

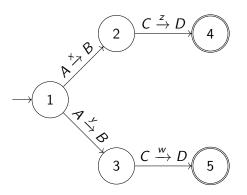
- ▶ Point of view of a participant
- Obtained via projection operation
- Behavior can differ for different communication semantics (p2p, sync)

Realisability problem: Does the implementation of a system **respects** the behavior described?

$$L(G) = L(\operatorname{proj}(G))$$

The example is not realisable

This example is **not** realisable because C doesn't know what A sent.



The trace $A \xrightarrow{\times} B$; $C \xrightarrow{w} D$ does not appear in L(G).

Reduction to sync [1]

A global type G is deadlock-free realisable in $\mathbf{p2p}$ iff:

- 1. $L_{p2p}(proj(G))$ is sync;
- 2. proj(G) is orphan-free in p2p;
- 3. $L_{p2p}(proj(G))$ is deadlock-free
- 4. *G* is weak realisable in sync
- 5. *G* is deadlock-free in sync

[1] Di Giusto, Cinzia, Etienne Lozes, and Pascal Urso. "Realisability and Complementability of Multiparty Session Types." (2025).

Reduction to sync [1]

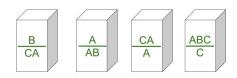
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First contribution

- Realisability for sync global types is undecidable. Proof: by reduction to the PCP problem.
- ▶ PCP: given a set of tiles, find an ordering such that the strings formed by the top and bottom halves are equal.
- Proof adaptated from Alur et al. [2]



[2] Alur, Rajeev, Kousha Etessami, and Mihalis Yannakakis. "Realizability and verification of MSC graphs." Theoretical Computer Science 331.1 (2005): 97-114.

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Second contribution

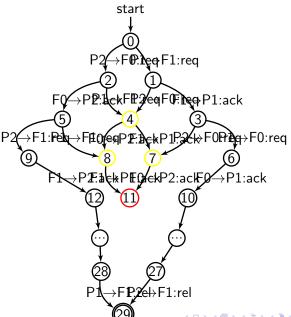
- ► RESCu: Model-checking TUI tool written in OCaml
- Added two verification if sync system:
 - ▶ *Deadlock*: a final state is always reachable
 - Progress: the system can always perform an action

RESCU Example - Dining Philosophers

Two Philosophers, two forks.

```
This system is RSC.
There are some sink states:
Sink: Id=11 Configuration={F0:4; F1:3; P1:2; P2:2}
There are some deadlock states:
Deadlock: Id=4 Configuration={F0:2; F1:1; P1:1; P2:1}
Deadlock: Id=11 Configuration={F0:4; F1:3; P1:2; P2:2}
...
```

RESCU Example - Dining Philosophers



Conclusion

Summary of contributions:

- Proof of undecidability for weak realisability in sync
- ► Enriched the tool RESCU

Future work:

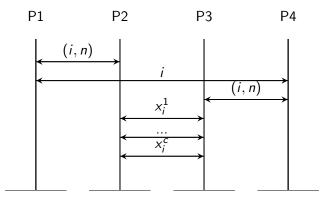
- Prove undecidability of deadlock-free realisability for sync global types
- ► Continue the development of RESCU

Thanks! Questions?

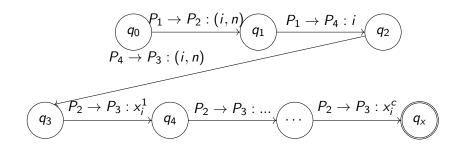
Weak and Safe realisability

- Weak realisability: the global type G is weak realisable in sync if there exist a CFSM system that can implement the global type.
- ▶ Weak realisability: the global type *G* is *safe* realisable in sync if it is *weak realisable* and the CFSM system is **deadlock free**.

The MSC M_i^n



The global type G_i^n



The global type L^*

