UBINET: Performance Evaluation of Networks

Homework 1
To be returned on 17 September 2024 at 9 am

Homeworks are a personal effort. Copied solutions will get 0 for a grade.

1.1 A dysfunctional laptop

A laptop can be in one of four states: running CPU-intensive jobs, running memory-intensive jobs, swapping heavily, rebooting. A monitoring of the laptop's state on a minute basis reveals the following.

- If the laptop is running CPU-intensive jobs at a given minute, it will do the same in the following minute with a probability 19/20, and with the complementary probability it will be running memory-intensive jobs.
- If the laptop is running memory-intensive jobs at a given minute, it will do the same in the following minute with a probability 1/2, and with the complementary probability it will be swapping heavily.
- If the laptop is swapping heavily at a given minute, it will continue swapping with probability 2/3 or it will reboot with probability 1/3.
- If the laptop is rebooting, then it will continue rebooting with probability 3/4 or it will be running CPU-intensive jobs with probability 1/4.

For convenience, the four states of the laptop will be denoted A, B, C, and D in the order they were presented.

1. Say why the laptop's state can be described by a discrete-time Markov chain.
   Write the state-space $\mathcal{E}$.
   Draw the probability transition diagram.
   Write the transition matrix.
2. The laptop is running CPU-intensive jobs at minute 1. What is the probability that it is doing the same at minute 3?
   What is the probability that it is doing the same at minute 5?
3. Does the limiting distribution exist? If yes, say why.
   If yes, compute it.
4. The laptop's power consumption is 15 W when running CPU-intensive jobs, 20 W when running memory-intensive jobs, 25 W when swapping heavily, and 10 W when rebooting. What is the expected power consumption of the laptop in the stationary regime?

1.2 Exponential variables and Poisson processes

Let X and Y be two independent random variables having cumulative distribution function $F_X(\cdot)$ and $F_Y(\cdot)$. Let $Z = \min\{X, Y\}$ and $V = \max\{X, Y\}$. We define the following events:
A: "$X \leq t$"
B: "$Y \leq t$"
C: "$Z \leq t$"
D: "$V \leq t$"

1. Express events C and D in terms of events A and B or their complementary events.

2. Express the cumulative distribution function $F_Z(\cdot)$ of Z and $F_V(\cdot)$ of V in terms of $F_X(\cdot)$ and $F_Y(\cdot)$.

We assume now that the density function of X is $f_X(t) = \lambda e^{-\lambda t}$ for $t > 0$ and 0 otherwise, and that of Y is $f_Y(t) = \mu e^{-\mu t}$ for $t > 0$ and 0 otherwise, with $\lambda > 0$ and $\mu > 0$.

3. Compute the cumulative distribution functions $F_X(t)$ and $F_Y(t)$.

4. Compute the expectations $\mathrm{E}[X]$ et $\mathrm{E}[Y]$.

5. Compute $F_Z(t)$ and $\mathrm{E}[Z]$.
   What can you say about the distribution of the random variable Z?

Application: A data center is composed of two clusters and each time one of the clusters violates the server-level agreement (SLA), the data center itself violates the SLA which raises penalties to be paid by the provider. One of the clusters is more reliable than the other being more recent and better provisioned. The administrator of the data center estimates that the older cluster runs smoothly for a period of time that is exponentially distributed with a mean equal to 1 month. As for the newer cluster, the administrator estimates the period without SLA violation to be exponentially distributed with a mean equal to 2 months.

6. What is the nature of the *stochastic process* describing the number of occurrences of SLA violation over time of each of the clusters?

7. What is the nature of the process describing the number of occurrences of SLA violation over time of the data center?
   What is the rate of SLA violations of the data center?

8. The last SLA violation of the data center was in December 2020. What is the probability that a SLA violation will occur in March 2021?

9. We are in January 2021 and there has been no SLA violation since August 2020. What is the probability that no SLA violation will occur within 3 months?