Homework 6 for Machine learning: Theory and Algorithms

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1 Exercise 1

1.1 Point one

If f_1 is μ_1-strongly convex, then it satisfies:

$$f_1(\lambda x + (1-\lambda)y) \leq \lambda f_1(x) + (1-\lambda) f_1(y) - \frac{\mu_1}{2}\lambda(1-\lambda)\|x-y\|^2$$

Now consider the function $g = cf_1$, where $c > 0$. We'll prove that g is $c\mu_1$-strongly convex.

For all $x, y \in R^d$ and $\lambda \in [0,1]$, we know that f_1 satisfies the strongly convex inequality. Multiply this inequality by c we got:

$$g(\lambda x + (1-\lambda)y) = cf_1(\lambda x + (1-\lambda)y) \leq c\Big(\lambda f_1(x) + (1-\lambda) f_1(y) - \frac{\mu_1}{2}\lambda(1-\lambda)\|x-y\|^2\Big)$$

This simplifies to:

$$g(\lambda x + (1-\lambda)y) \leq \lambda g(x) + (1-\lambda) g(y) - \frac{c\mu_1}{2}\lambda(1-\lambda)\|x-y\|^2$$

Therefore, g is $c\mu_1$-strongly convex.

1.2 Point two

For any $x, y \in R^d$ and $\lambda \in [0,1]$, since both f_1 and f_2 are strongly convex, they satisfy these two inequalities:

$$f_1(\lambda x + (1-\lambda)y) \leq \lambda f_1(x) + (1-\lambda) f_1(y) - \frac{\mu_1}{2}\lambda(1-\lambda)\|x-y\|^2$$

$$f_2(\lambda x + (1-\lambda)y) \leq \lambda f_2(x) + (1-\lambda) f_2(y) - \frac{\mu_2}{2}\lambda(1-\lambda)\|x-y\|^2$$

Now, add these two inequalities:

$$(f_1 + f_2)(\lambda x + (1-\lambda)y) \leq \lambda(f_1(x) + f_2(x)) + (1-\lambda)(f_1(y) + f_2(y)) - \frac{\mu_1 + \mu_2}{2}\lambda(1-\lambda)\|x-y\|^2$$

Therefore, the sum f_1+f_2 is $(\mu_1+\mu_2)$-strongly convex.

2 Exercise 2