Exercises Week 2

Ex. 1 — Consider the following definition of learnability:
A hypothesis class $\mathcal{H}$ is learnable if there exist a learning algorithm A with the following property: For every distribution D over X , and for every labeling function $f : X \to \{0, 1\}$, if the realizable assumption holds with respect to $\mathcal{H}$, D, f, then when running the learning algorithm on m i.i.d. examples generated by D and labeled by f, it holds $\lim_{m\to\infty} \mathbb{E}_{S\sim D^m}[L_D(A(S))] = 0$.

1. Prove that this definition is equivalent to PAC-Learnability. Hint: you may want to use Markov's inequality.

Ex. 2 — Prove that if $\text{VCdim}(\mathcal{H}) = +\infty$, then $\mathcal{H}$ is not PAC-learnable. Hint: you may proceed by contradiction and rely on the no free lunch theorem.