UBINET/SI5: Performance Evaluation of Networks

Correction of homework 2

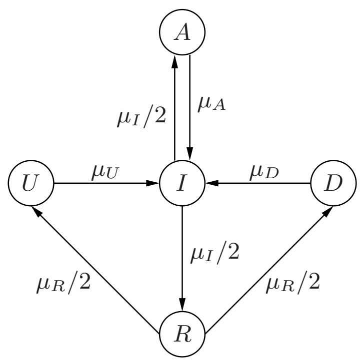
2.1 A functional database

1. According to the description of the process, we can see that it corresponds to construction rule number 1. The time spent in each state is exponentially distributed and once this time expires, the process changes state with some probability. These probabilities are 1 to go from states A, U or D to state I, and 1/2 to go from state I to R or A, and from state R to U or D. Therefore, according to construction rule 1 the stochastic process is a continuous-time Markov chain.

2. We have the following transition rates and infinitesimal generator (states are ordered according to the state-space $\mathcal{E} = \{I, R, A, U, D\}$):

$$\begin{array}{lcll} I & \to & R & \text{with rate } \mu_I/2 \\ I & \to & A & \text{with rate } \mu_I/2 \\ R & \to & U & \text{with rate } \mu_R/2 \\ R & \to & D & \text{with rate } \mu_R/2 \\ A & \to & I & \text{with rate } \mu_A \\ U & \to & I & \text{with rate } \mu_U \\ D & \to & I & \text{with rate } \mu_D \end{array} \qquad Q = \begin{bmatrix} -\mu_I & \mu_I/2 & \mu_I/2 & 0 & 0 \\ 0 & -\mu_R & 0 & \mu_R/2 & \mu_R/2 \\ \mu_A & 0 & -\mu_A & 0 & 0 \\ \mu_U & 0 & 0 & -\mu_U & 0 \\ \mu_D & 0 & 0 & 0 & -\mu_D \end{bmatrix} .$$

The transition diagram of the CTMC is



3. The chain is irreducible as we can find a path that visits all states: $I \to R \to U \to I \to R \to D \to I \to A \to I$. As the state-space is finite, the irreducibility implies the ergodicity. The CTMC is ergodic.

4. The vector $\pi = (\pi_I, \pi_R, \pi_A, \pi_U, \pi_D)$ is the solution of the steady-state equations (including the normalization equation)

$$\begin{cases} \mu_I \, \pi_I = \mu_A \, \pi_A + \mu_U \, \pi_U + \mu_D \, \pi_D \\ \mu_R \, \pi_R = \frac{1}{2} \mu_I \, \pi_I \\ \mu_A \, \pi_A = \frac{1}{2} \mu_I \, \pi_I \\ \mu_U \, \pi_U = \frac{1}{2} \mu_R \, \pi_R \\ \mu_D \, \pi_D = \frac{1}{2} \mu_R \, \pi_R \\ \pi_I + \pi_R + \pi_A + \pi_U + \pi_D = 1 \, . \end{cases}$$

Expressing all probabilities in terms of π_I yields

$$\pi_R = \frac{1}{2}\frac{\mu_I}{\mu_R}\pi_I \ , \quad \pi_A = \frac{1}{2}\frac{\mu_I}{\mu_A}\pi_I \ , \quad \pi_U = \frac{1}{4}\frac{\mu_I}{\mu_U}\pi_I \ , \quad \pi_D = \frac{1}{4}\frac{\mu_I}{\mu_D}\pi_I \ .$$

Using the notation suggested in the exercize statement, namely $\frac{1}{C} = \frac{4}{\mu_I} + \frac{2}{\mu_R} + \frac{2}{\mu_A} + \frac{1}{\mu_U} + \frac{1}{\mu_D}$, the normalization equation becomes

$$\frac{\mu_I}{4}\pi_I \frac{1}{C} = 1 \quad \Rightarrow \quad \pi_I = C\,\frac{4}{\mu_I} \ .$$

The steady-state distribution is then

$$\pi \ = \ C\left(\frac{4}{\mu_I}, \frac{2}{\mu_R}, \frac{2}{\mu_A}, \frac{1}{\mu_U}, \frac{1}{\mu_D}\right) \ .$$

Sanity check: each probability is in the interval $(0, 1)$ and the sum of the probabilities is equal to 1.

5. The database is utilized every time the process leaves state I. This occurs with rate $\mu_I\pi_I = 4C$. This is then the utilization rate.

6. The expected power consumption in steady-state is

$$P = 10\pi_I + 60\pi_R + 70(\pi_A + \pi_U + \pi_D) = C\left(\frac{40}{\mu_I} + \frac{120}{\mu_R} + \frac{140}{\mu_A} + \frac{70}{\mu_U} + \frac{70}{\mu_D}\right) \text{ W.}$$

7. The two operations that mostly impact the power consumption are the add operation followed by the read operation. To minimize the power consumption, it is important to reduce the expected duration of these operations, namely $1/\mu_A$ and $1/\mu_R$. In other words, one should try to increase μ_A and μ_R as much as possible. We observe that μ_I will be affected by changes in the duration of any operation time. Indeed, for the same number of requests by users (same utilization rate, C is constant), the idle time $1/\mu_I$ increases when the read/add/update/delete operations take less time, and the length of the change is the same. However, given the coefficients in the expression of the power consumption, the increase in $1/\mu_I$ is largely compensated by the decrease in the operation time, especially in $1/\mu_A$ or $1/\mu_R$. Therefore, the power consumption will decrease.