Exercises Week 3

Ex. 1 — Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function. Prove that

1. If f is also concave, then f is affine.
2. If f is upper-bounded, then f is a constant.

Ex. 2 — Consider the hypothesis classes $\mathcal{H}_n = \{h_{a_1,a_2,\ldots,a_n} : [0,1] \to \{0,1\}, a_i \in \{0,1\}\}$, where

$$h_{a_1,a_2,\ldots,a_n}(x) = \begin{cases} a_i, & \text{if } x \in (\frac{1}{i+1}, \frac{1}{i}] \text{ for some } i \in \{1, 2, \ldots, n\}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Let $\mathbb{N}_{>0}$ denote the set of positive natural numbers. Consider that data is generated from a uniform distribution over $[0,1]$ ($X \sim \text{Uniform}(0,1)$) and labeled from the labeling function

$$f(x) = \begin{cases} 1, & \text{if } x \in (\frac{1}{2i+1}, \frac{1}{2i}] \text{ for some } i \in \mathbb{N}_{>0}, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

1. Compute the approximation error and the estimation error for the class $\mathcal{H}_n$.
2. Illustrate the bias-complexity tradeoff for $n \in \mathbb{N}_{>0}$.