UBINET/SI5: Performance Evaluation of Networks

Homework 4
To be returned on 8 October 2024 at 9 am

Homeworks are a personal effort. Copied solutions will get 0 for a grade.

4.1 Fake news

A malicious person is hired to spread fake news over a social network. Each person on this social network that is exposed to the fake news will spread it in turn. We assume that each person—beside the source of the fake news—will eventually realize that the news is fake and will stop spreading it. However, the fake news will continue to spread over the social network as its membership is infinite.

It is assumed that when $i \geq 1$ persons are spreading the fake news

- it takes a random time for *each* spreader to convince a new person to start spreading the fake news; this convincing time is exponentially distributed with rate $\alpha > 0$.
- it takes a random time for *each* person (other than the source) spreading the fake news to awaken and realize that it is fake and stop then diffusing it; this awakening time is exponentially distributed with rate $\beta > 0$.

All convincing and awakening durations introduced above are assumed to be mutually independent. Denote $\rho := \alpha/\beta$. Let $X(t)$ be the number of persons spreading the fake news at time t and π_i be the stationary probability that i persons are spreading the fake news.

1. What is the nature of the stochastic process $\{X(t), t \geq 0\}$?
   What is the state-space (denoted by $\mathcal{E}$) on which this process is irreducible?
   Draw the transition rate diagram of this process.
2. Compute π_i for all $i \in \mathcal{E}$.
   What is the stability condition?
3. Compute $\overline{N}$, the expected number of fake news spreaders in steady-state.
4. Compute $\overline{\lambda}$, the average convincing rate (the rate of turning persons to become spreaders of fake news).
5. Compute $\overline{T}$, the average duration a person acts as a spreader of fake news.
   Any comment?
6. Assume now that the awakening time is exponentially distributed with rate $\beta/(i-1)$ when there are i spreaders of fake news. (This captures the idea that when more persons are spreading a fake news, it takes more time for a person to realize that it is fake.)
   Under what condition would this system be stable?

4.2 Dimensioning a server

A company's server consists of a single processor computer that serves a queue of jobs in a FIFO fashion. Jobs arrive according to a Poisson process with rate λ and require a service time that is exponentially distributed with rate μ. This server is stable because $\lambda < \mu$. Let $\overline{T}$ denote the mean response time of the server.

After a merger, the company starts offering a much wider range of services. The manager expects then the jobs arrival rate to double in the days following the merger. It is crucial that customers will not notice any degradation in their services. In a hurry, the manager orders a new server with twice the speed of the actual company's server.

1. What is the utilization ρ_{new} of the new server with the new arrival rate?
   Comment on the comparison with the old server before the merger.

2. What is the response time $\overline{T}_{\text{new}}$ of the new server with the new arrival rate?
   Comment on the comparison with the old server before the merger.

3. If the main objective is to maintain the same quality of service for customers, what would you have suggested to buy as new server for the company?