UBINET/SI5: Performance Evaluation of Networks

Homework 5

To be returned on 15 October 2024 at 9 pm

Homeworks are a personal effort. Copied solutions will get 0 for a grade.

5.1 $M/G/1$ queue with priorities

Consider an $M/G/1$ queueing system with two customers classes where all service times and all inter-arrival times are independent. Customers in class k, or type k customers, for $k = 1, 2$, arrive according to a Poisson process with rate λ_k and their generic service time σ_k is exponentially distributed with mean $\overline{\sigma}_k = 1/\mu_k$. Denote $\rho_k = \lambda_k/\mu_k$, for $k = 1, 2$. Customers within each class are served according to the first-come-first-served discipline, however type 1 customers have non-preemptive priority over type 2 customers. In other words,

- if there are both a type 1 customer and a type 2 customer in the waiting room, the server serves the type 1 customer even if the type 2 customer entered the queue first;
- if a type 2 customer is being served and a type 1 customer arrives, the service of the type 2 customer will not be interrupted.

We assume the system is stable and at steady-state.

1. Draw over time the residual service time $R(t)$.
2. Compute $\overline{R}$, the time-average residual service time, using the curve $R(t)$.
3. Prove that the expected waiting time of customers in top priority class 1 is
$$\overline{W}_1 = \frac{\overline{R}}{1 - \rho_1} \ .$$
4. A type 2 customer entering the queue needs to wait for the service of all type 1 and type 2 clients in the queue, and all type 1 customers that arrive while waiting, before getting served. Prove that the expected waiting time of customers in low priority class 2 satisfies the relation
$$\overline{W}_2 = \overline{R} + \rho_1 \overline{W}_1 + (\rho_1 + \rho_2)\overline{W}_2 \ .$$
Hint: use the fact that the expected number of arrivals of a Poisson process with rate λ in an interval of duration d is equal to λd.
5. Find an expression for $\overline{W}_2$ as a function of $\overline{R}$, ρ_1 and ρ_2.
6. What is the stability condition of the system?

5.2 M/G/1 with different job types

Consider an M/G/1 queue that serves two types of jobs: red and blue. Red jobs arrive according to a Poisson process with rate $\lambda_R = \frac{1}{4}$ jobs/second, and blue jobs arrive according to a Poisson process with rate $\lambda_B = \frac{1}{2}$ jobs/second. Red job sizes have mean 1 and variance 1, whereas blue job sizes have mean 0.5 and variance 1. All jobs arrive to the same FCFS queue, so that, at any time, the server might be serving a red job or a blue one, and there might be jobs of one or both types in the queue.

1. What is the nature of the arrival process to this queue?
   Write its arrival rate λ.

2. What is the expected service time $\mathrm{E}[\sigma]$ of a job picked at random?

3. Compute the second moment $\mathrm{E}\left[\sigma^2\right]$ of a job picked at random?

4. Compute the load ρ of this M/G/1 queue.

5. What is the expected waiting time of red jobs? Of blue jobs?

6. What is the mean response time of red jobs $\overline{T}_R$?
   What is the mean response time of blue jobs $\overline{T}_B$?