Exercises Week 6

Ex. 1 — Let $f_1 : \mathbb{R}^d \to \mathbb{R}$ be a μ_1-strongly convex function and $f_2 : \mathbb{R}^d \to \mathbb{R}$ be a μ_2-strongly convex function. Prove that

1. For any $c > 0$, the function $g = cf_1$ is $(c\mu_1)$-strongly convex.
2. The function $g = f_1 + f_2$ is $(\mu_1 + \mu_2)$-strongly convex.

Ex. 2 — Let $X, W \subset \mathbb{R}^d$ and $Y \subset \mathbb{R}$. Consider the regression problem over the class of linear regressors $\mathcal{H} = \{h_w : X \to Y, w \in W\}$ where $h_w(x) = \langle w, x\rangle$, with the loss $\ell(w,(x,y)) = (\langle w, x\rangle - y)^2$.

1. Adapt the theory developed in class for RLM to demonstrate that, under appropriate additional conditions (to be determined), the ERM predictor is an agnostic PAC-learner for $\mathcal{H}$ and enjoys guarantees similar to those of RLM.
   Suggestions:
   - Identify conditions that ensure the empirical loss is strongly convex (without the need to add a regularization term).
   - Identify conditions under which the loss function is Lipschitz continuous.