UBINET/SI5: Performance Evaluation of Networks

Homework 3

To be returned on 1 October 2024 at 9 am

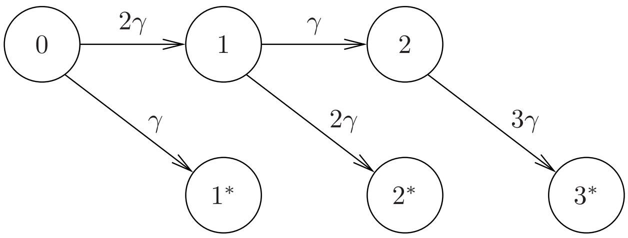
Homeworks are a personal effort. Copied solutions will get 0 for a grade.

3.1 A routing problem

Cars are getting equipped with wireless cards enabling direct wireless communications between them. The networks that are formed thanks to vehicle-to-vehicle communications are in the category of delay-tolerant networks. Routing messages in such networks can be achieved by using the so-called two-hop routing. In this protocol, the source of a message transmits a copy of its message to any node it meets in the network; such a node becomes a relay. Relays can only transmit messages to their respective destinations. As such a message can reach the destination after doing either one hop (case of source to destination transmission) or two hops (case of source to relay, then relay to destination transmissions). But there could be a potentially very large number of relays having a copy of the message when the destination receives the message.

Measurements collected from real-life encounters reveal that the inter-meeting times between *any two* nodes is roughly an exponential distribution. By assuming that inter-meeting times are independent of each other, the meeting process between any two nodes is Poisson with rate γ. We assume that when any two nodes meet they will exchange instantly all messages that they are allowed to transmit to each other, according to the two-hop routing protocol. Let $X(t)$ be the number of times a given message has been transmitted in the interval $[0, t[$ ($t = 0$ is seen as the message generation instant at the source). We have $X(0) = 0$.

We consider a network with only 4 nodes. Consequently, the state-space of the process $\{X(t), t > 0\}$ is $\mathcal{E} = \{0, 1, 2, 1^*, 2^*, 3^*\}$, where the asterisk denotes the fact that the message has reached its destination. For instance if the message has been transmitted a total of i times and has reached its destination by time t then $X(t) = i^*$. The transition diagram is



The performance of the two-hop routing is assessed through two metrics: (i) $T(0)$, the expected time to deliver a message to its destination given that $X(0) = 0$, and (ii) $\mathrm{E}[X]$, the expected number of transmissions until a message is delivered. (X is the stationary version of $X(t)$.) $T(0)$ is a quality metric while $\mathrm{E}[X]$ is a cost metric. Since b_{0,j^*} is the probability that j transmissions are needed for the message to be delivered given that $X(0) = 0$, we have $\mathrm{E}[X] := \sum_{j=1}^{3} j b_{0,j^*}$.

1. Say why $\{X(t), t > 0\}$ is an absorbing CTMC over $\mathcal{E}$. Mention specifically the transient/absorbing states.

2. Write the infinitesimal generator $\mathbf{Q}$.
3. Use appropriately $\mathbf{Q}$ to compute $T(0)$.
4. Derive the transition matrix $\mathbf{P}$ of the *embedded* Markov chain at *jump* times.
5. Use appropriately $\mathbf{P}$ to compute $\mathrm{E}[X]$.

We consider now a network with 5 nodes.

6. Write the new state-space $\mathcal{E}'$.
7. Explain the new transition rates between the possible states.
8. Draw the new transition diagram.
   Write the new infinitesimal generator $\mathbf{Q}'$.
9. Compute the expected time to deliver a message to its destination given that $X(0) = 0$.
   How does it compare with the same when there are four nodes in the network?
   Why could this be expected without performing the calculation?
10. Without making the computation, how does the expected number of transmissions until a message is delivered vary as the number of nodes in the network increases?
    Explain your reasoning.