**Jacobian**



Revolute

joint

Prismatic

joint

*  from ;
* , with  from , and  from ;
* Or , with  from , and  from ;

**NOTE.** If *N* is the number of joint variables of a manipulator, there is an important difference between the following two problems:

1) Find the Jacobian  based on a point on the end effector that is coincident with point *b*;

2) Find the Jacobian  of point *b*.

In the first case, the Jacobian is determined for *j* = 1,…, *N*. This is due to the fact that since point *b* is rigidly connected to the end effector, it must depend on all the joint variables.

In the second case, the Jacobian is determined for *j* = 1,…, *k*, where *k* is the number of joints before point *b*. This is because the position/orientation of point *b* can depend only on the previous joint variables.

**Generalized Inverse (Moore-Penrose Inverse)**

If the Jacobian is an *M* × *N* matrix, the forward velocity problem is defined as:



and the inverse velocity problem can be solved using the Moore-Penrose inverse:

 

This inverse does not give the exact solution, i.e.



but it minimizes the error .

**Lagrangian formulation**



  

 

  *j* = 1,…., *i*

  *i* = 1,…., *N*

**Newton-Euler formulation**

Outward iteration: *i* = 0,…,*N-1* (for a prismatic joint:, for a revolute joint:)

 

 



  

Inward iteration: *i* = *N*,…, *1*

  

Revolute joint:  Prismatic joint: 

**Manipulator dynamics in task space**

Task space/joint space relationships:

 ↔ 

 ↔ 

 ↔ 

Manipulator dynamics in joint space:



Premultiplying by  and substituting the joint accelerations :



Defining:

 

 

Manipulator dynamics in task space:



**Equations of motion**

 



   

Equations using the COM:

 

 

**Kinetic and potential energy**

 

**Work**

 