1 2 ….. 6 ….. *k* *k*+1 *k*+2 ….. …*j…*

Horn main structure

*NJH* = 6

Contacting triplets

*k* = *NJH* + 1 + 3 × (*i* - 1)

*i* = 1,…,*NCP*

Tracked points

*j* = *NJH* + 3 × *NCP* + *i*

*i* = 1,…,*NTP*

Contacting points

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** | **….** | ***k*** | ***k*+1** | ***k*+2** | **….** | ***j*** |
| **BB** | 0 | 1 | 2 | 3 | 4 | 5 | …. | 6 | *k* | 6 | …. | 0 |
| **J\_type** | R\* | P | P | R | R | P | …. | R/R\* | R\* | R\* | …. | R\* |
| **S0** | 1 | 0 | 0 | 0 | 0 | 0 | …. | 0 | 0 | 0 | …. | 1 |
| **SE** | 0 | 0 | 0 | 0 | 0 | 0 | …. | 0 | 1 | 1 | …. | 1 |

( \* ) An asterisk close to a rotational joint will indicate that the joint is actually fixed.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **SS** | **1** | **2** | **3** | **4** | **5** | **6** | **….** | ***k*** | ***k*+1** | ***k*+2** | **….** | ***j*** |
| **1** | -1 | 1 | 0 | 0 | 0 | 0 | …. | 0 | 0 | 0 | …. | 0 |
| **2** | 0 | -1 | 1 | 0 | 0 | 0 | …. | 0 | 0 | 0 | …. | 0 |
| **3** | 0 | 0 | -1 | 1 | 0 | 0 | …. | 0 | 0 | 0 | …. | 0 |
| **4** | 0 | 0 | 0 | -1 | 1 | 0 | …. | 0 | 0 | 0 | …. | 0 |
| **5** | 0 | 0 | 0 | 0 | -1 | 1 | …. | 0 | 0 | 0 | …. | 0 |
| **6** | 0 | 0 | 0 | 0 | 0 | -1 | …. | 1 | 0 | 1 | …. | 0 |
| **…..** | ………………………………………………………………………… | | | | | | | | | | | |
| ***k*** | 0 | 0 | 0 | 0 | 0 | 0 | …. | -1 | 1 | 0 | …. | 0 |
| ***k*+1** | 0 | 0 | 0 | 0 | 0 | 0 | …. | 0 | -1 | 0 | …. | 0 |
| ***k*+2** | 0 | 0 | 0 | 0 | 0 | 0 | …. | 0 | 0 | -1 | …. | 0 |
| **…..** | ………………………………………………………………………… | | | | | | | | | | | |
| ***j*** | 0 | 0 | 0 | 0 | 0 | 0 | …. | 0 | 0 | 0 | …. | -1 |

Conical Horn (CH)

Fabric Cylinder (FC)

Metal Cylinder (MC)

Double Helical Spring (DHS)

Skirt Frame (SF)

*RDHS*

*RCH*

*LCH*

*LFC*

*LMC*

*RFC*

*RMC*

*TSF*

*LSF*

X

Z

Y

*α*0

*LDHS*

x

z

y

Inertial reference

system

**R**0

**R***H*

**R***iTP*, *i*=1..*NTP*

*i*th tracked point

Main body

reference system

*BSF*

*α* (*i*-1), *i*=1...*NCP*

*DSF*

x

y

1st triplet (*i*=1)

*i*th triplet

Horn lower tip (HLT)

**Figure 2: Geometric dimensions.**

6

5

2

3

1

8

9

7

x

z

1

3

6

7

0

x

z

y

y

z

x

y

x

z

y

x

z

y

z

x

y

4

z

x

y

z

x

y

*j*

x

y

z

*i*th tracked point, *j* = *NJH* + 3×*NCP* + *i*

z

x

y

x

y

z

8

9

7

6

7

x

y

*i*th triplet, *k* = 7 + 3×(*i*-1)

1st triplet, *k* = 7

*k*+1

*k*+2

*k*

*k*

z

z

*i* = 1..*NCP*

*i* = 1..*NTP*

Rotational joint

Rotational joint/end point

Prismatic joint

Red numbers are joints

Blue numbers are centroids

Link centroid

**Figure 3: Joints reference systems.**

6

5

2

3

1

8

9

7

x

z

1

3

6

7

0

x

z

y

x

z

y

4

z

x

y

z

x

y

*LCH*

*LFC*

*LMC*

RFC

RDHS

LSF

**R***iTP*

**R**H

y

*i* = 1..*NTP*

x

y

α (*i*-1)

x

z

*k*

*k* + 1

*k*

*k* + 2

*i* = 1..*NCP*

*j*

*j* = *NJH* + 3×*NCP* + *i*

*k* = 7 + 3×(*i*-1)

Rotational joint

Rotational joint/end point

Prismatic joint

Red numbers are joints

Blue numbers are centroids

Link centroid

**Figure 4: Centroids geometry and reference systems.**

**Equations**



(problem with using x as q, qdot)

Task space/joint space relationships:

 ↔ 

 ↔ 

 ↔ 

Manipulator dynamics in joint space:



Manipulator dynamics in task space: 

Sometime G term is written on the right with all the generalized forces since it is a force. In such case, the eqs. Looks like the usual ma = F.

Inertia matrices:



Centrifugal, gyroscopic, and Coriolis terms:

Gravity vector

Equations using the COM (generalize and use same symbols as previous, don’t do it specifically for contact):

**Kinetic and potential energy**

**Work**

The Newton equation (**L** part) makes it obvious that the robot needs external forces in order to move its COM in a direction other than that of gravity (typically the reaction force).

During the flight phase no external forces are present, so the COM moves as body in free falling under the effect of the gravity (). Linear and angular momentum are conserved but it is still possible to control and  to reconfigure the system.

**COM**

With: *mi* = link mass, **r***i* = link centroid position, *mt* = total mass, **r***COM* = COM position.

* Equation:

With: point *O* origin reference frame, **n** normal to the (flat) ground, **F***E* external forces, **R** ground reaction force, point *G* center of mass, **a**G body acceleration, .....etc ....

**COP**

If one single foot is in contact with the ground, the field of pressure forces (normal to the contact) is equivalent to a single resultant force exerted at the COP, where the resultant moment is zero.

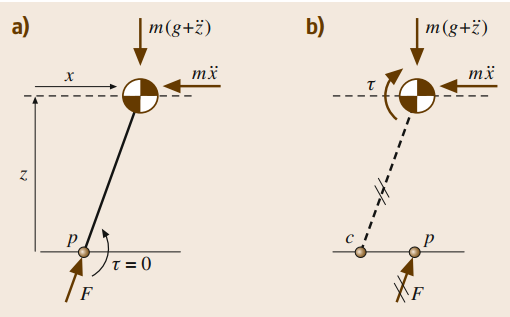


With: *FCi* = normal component of thecontact force, **r***Ci* = contact force position, **r***COP* = COP position. The COPs from multiple contacting elements can be combined to find the ZMP. For instance for two feet:



With: **r***COP1* = COP position first foot, **r***COP2* = COP position second foot, *FCOP1* = normal contact force first foot, *FCOP2* = normal contact force second foot.

**CMP**

If a net torque *τ* acts on the COM, then the GRF cannot not pass through the COM (point *c*) but is applied at point *p*, so to compensate for the COM torque. Point c, the CMP = Centroidal Moment Pivot, is defined as the point where the line of action of the current GRF would pass through the COM, and is given by:



With Fn the normal component of the GRF, p is the COP/ZMP point. Another component is along the other axis. In undisturbed human walking it was observed that the distance between ZMP and CMP is kept small, while the change of angular momentum plays a stronger role for push recovery

**Contact and friction model**

An end point *E* interested by a contact is defined by position  and velocity , both given wrt the inertial reference system (*X*,*Y*,*Z*). If *G*(*X*,*Y*) defines the surface (ground), i.e. the elevation as a function of the position in the plane (*X*,*Y*), a contact is present if

*zE* ≤ *G*(*xE*,*yE*)

(ref. Figure 5a). The actual contact point, CP, is assumed to be the point on the surface that is the closest to *E*. This point is determined as a minimization problem, using



Once the point CP is known, the normal **n***CP* is determined from the surface definition *G* (the normal is also directed as the line connecting points CP and E). The normal is defined as positive going outward wrt the surface, and this means that its component along *Z* axis is always positive.

The tangent direction is defined using the direction of motion of the end point, i.e.



(ref. Figure 5b). This direction will always lie on the plane defined by the normal **n***CP* and the end point velocity, and it will always point in the same direction as. In the particular case where the end point velocity is directed as the normal, the tangent direction is assumed as **t***CP* = {0,0,0} (since in this case the motion is along the normal direction, there is no friction force and thus the actual value of the tangent direction is not important). The normal contact force is defined as (ref. Figure 5c)



with



Constants *Kn* and *Dn* are, respectively, the stiffness and damping coefficients. *q* and *p* are also constant parameters. Because the way it has been defined, the indentation  is always negative. When  there is the compression phase, when  there is the restitution phase.

The tangential force can have two phases: sticking and sliding. A sticking phase begins if  and ends if . Term *FS* is the static friction force, and it has been defined, similarly to the normal contact force, as



where is the deformation along the tangential direction since the sticking phase begins, *Kt* and *Dt* are, respectively, the tangential stiffness and damping coefficients, *q* and *p* are constant parameters, and *μS*is the static coefficient of friction. The tangential deformation at time *tB* = *tA* + Δ*t* is determined as



(where Δ*t* is the integration time-step). If the sticking phase include more than one integration time-step, the total tangential deformation is assumed to be the sum of the single tangential deformations as determined with the previous equation. This is an approximation, because the tangent **t***CP* is not always the same (but if the surface is relatively smooth, the error should be acceptable). Finally, a sliding phase begins when a sticking phase ends and ends when a sticking phase begins. Note that for practical reasons, the beginning of the sticking phase is determined when  is smaller than a tiny number (defined as sticking velocity, *VS*), and not when it is exactly equal to zero. Summarizing, the tangential force is given by



where *μC*is the dynamic (Coulomb) coefficient of friction.

The total force dues to the contact wrt the inertial reference system is then given by (ref. Figure 5b)



No external torques are applied on the contact points, i.e. **T***CP* = {0,0,0}. In the table in the next page are listed the numerical values for the contact model parameters.

|  |  |  |
| --- | --- | --- |
| Name | Value | Element |
| *Kn* | 20000N/mq | Coefficient of stiffness (normal direction) |
| *Dn* | 250Ns/mp-1 | Coefficient of damping (normal direction) |
| *q* | 1.5 | Exponent for the elastic term (normal direction) |
| *p* | 1.5 | Exponent for the damping term (normal direction) |
| *Kt* | 20000N/mr | Coefficient of stiffness (tangential direction) |
| *Dt* | 100Ns/ms-1 | Coefficient of damping (tangential direction) |
| *r* | 1.5 | Exponent for the elastic term (tangential direction) |
| *s* | 1.5 | Exponent for the damping term (tangential direction) |
| *μs* | 0.30 | Coefficient of static friction |
| *μt* | 0.25 | Coefficient of dynamic (Coulomb’s) friction |
| *VS* | 10-7 m/s | Sticking velocity |

Damping terms *Dn* and *Dt* have been determined using the formula 

with damping coefficient *ζ* equal to, respectively, 0.707 and 0.30, and then rounding the results.

**Impacting surface (ground)**

The surface is represented by a plane that can assume any inclination. This plane is defined giving a reference point P*G*, belongings to the plane, and a vector **N**, normal to the plane (ref. Figure 6). Vector **N** is given using a spherical coordinates system, (*R*,*θ*,*φ*), with origin in P*G*. The components of **N** are

(setting *R* = 1 vector **N** becomes a unit vector). The limits on angle *φ* guarantee that *NZ*> 0, i.e. the normal points always toward the positive direction of axis *Z*. The equation of the plane wrt the inertial reference system (*X*,*Y*,*Z*) is



where *d*is determined using the coordinates (*XG*,*YG*,*ZG*) of the reference point PG. The elevation of a generic point A belonging to the plane is



and the corresponding normal is **N***A* **= N**. For the included simulations a flat plane has been used, thus:

**P***G* = {0, 0, 0} m, *θ* = 0 deg., *φ* = 0 deg.



**R***E*

*G*(*x*,*y*)

CP

*G*(*xE*,*yE*)

E

**R***CP*

**F***n*

**F***t*



CP

E

**n***CP*

**t***CP*

**δ***n*

*Fn*

*δn*



Compression

phase

Restitution

phase





**(a)**

**(b)**

**(c)**

X

Z

Y

x

z

y

**N**

φ

θ

PG