Discrete and Algorithmic Geometry

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Sheet 4

due on Monday, December 10, 2018

Important:

Please submit your solution to this exercise via a pull request so that we can discuss your work using the chat feature on github.

(1) Definition 9.2 in Ziegler's *Lectures on Polytopes* constructs the linear map

$$P \ \xrightarrow{\pi^c}\ Q^c := \left\{ \begin{pmatrix} \pi(x) \\ cx \end{pmatrix} : x \in P \right\} \ \subset \ \mathbb{R}^{q+1}$$

from a projection $\pi : P \subset \mathbb{R}^p \to Q \subset \mathbb{R}^q$ and a linear function $c \in (\mathbb{R}^q)^\star$. Is it possible to give an algorithm to determine the set of lower faces $\mathcal{L}^{\downarrow}(Q^c)$ of Q^c from just the set of facet normals of Q, the projection π, and the linear function c, without running a convex hull algorithm on Q^c?

(2) Show that

$$\int_P f(x)\,\mathrm{d}x \ = \ \mathrm{vol}(P) \cdot f(p_0)$$

for any polytope P, where $p_0 = \frac{1}{\mathrm{vol}(P)} \int_P x\,\mathrm{d}x$ denotes the barycenter of P.

(3) Complete the proof of Theorem 9.6 in Ziegler's *Lectures on Polytopes*, possibly referring to [1].

References

[1] Louis J. Billera and Bernd Sturmfels. Fiber polytopes. *Ann. Math. (2)*, 135(3):527–549, 1992.