Discrete and Algorithmic Geometry: Sheet 4

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1. Definition 9.2 in Ziegler's *Lectures on Polytopes* constructs the linear map

$$P \xrightarrow{\pi^c} Q^c := \left\{ \begin{pmatrix} \pi(x) \\ cx \end{pmatrix} : x \in P \right\} \subset \mathbb{R}^{q+1}$$

from a projection $\pi : P \subset \mathbb{R}^p \to Q \subset \mathbb{R}^q$ and a linear function $c \in (\mathbb{R}^p)^\star$. Is it possible to give an algorithm to determine the set of lower faces $\mathcal{L}^\downarrow(Q^c)$ of Q^c from just the set of facet normals of Q, the projection π, and the linear function c, without running a convex hull algorithm on Q^c?

2. Show that

$$\int_P f(x)\,\mathrm{d}x \;=\; \mathrm{vol}(P) \cdot f(p_0)$$

for any polytope P and linear function f, where $p_0 = \frac{1}{\mathrm{vol}(P)} \int_P x\,\mathrm{d}x$ denotes the barycenter of P.

3. Complete the proof of Theorem 9.6 in Ziegler's *Lectures on Polytopes*, possibly referring to [**?**].

1.

It is not possible to give such an algorithm.

This is because π, c and the set of facets of Q do not determine the lower faces of Q^c. Consider $\pi : \mathbb{R}^2 \to \mathbb{R}$ that deletes the last coordinate. Consdier then $c = (0,1)$. In this case, π^c is the identity in $\mathbb{R}^2$. Since in this case $q = 1$, the interval is the only polytope the set of facet normals of q is always the same, so $q = 1$, the only relevant information is π and c, but in this case, π^c is the identity. Therefore, it such algorithm existed, the set of lower faces would be the same for all polygons, which is not true.

2.

Using the fact that f is linear and linearity of the integral:

$$\mathrm{vol}(P) f(p_0) = \mathrm{vol}(P) f\left(\frac{1}{\mathrm{vol}(P)} \int_P x\,\mathrm{d}x\right) = f\left(\int_P x\,\mathrm{d}x\right) = \int_P f(x)\,\mathrm{d}x \tag{1}$$

3.

IN THE NEXT EPISODE.

References