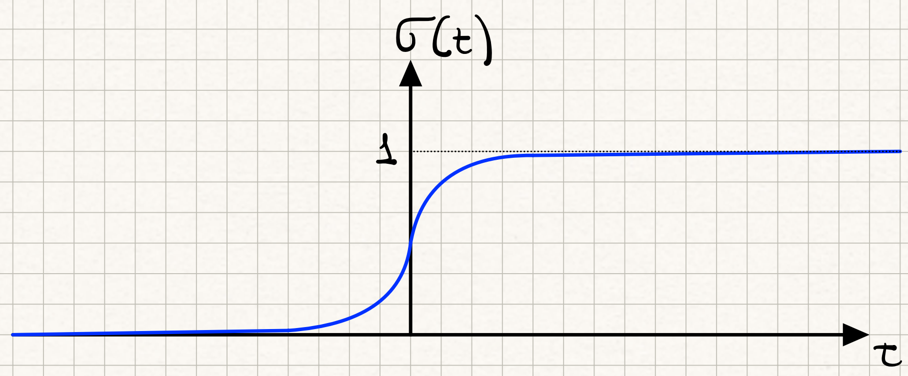
SIGMOIDE

È una funzione di attivazione non lineare:

$$\sigma(t) = \frac{1}{1+e^{-t}} \qquad \sigma : \mathbb{R} \to (0,1)$$



DERIVATA DELLA SIGMOIDE

$$\sigma(t) = (1+e^{-t})^{-1}$$

$$\frac{d\sigma(t)}{dt} = -1\,(1+e^{-t})^{-2} \cdot \frac{d\,(1+e^{-t})}{dt} =$$

$$= -1\,(1+e^{-t})^{-2} \cdot (-1\,e^{-t}) =$$

$$= (1+e^{-t})^{2}\, e^{-t}$$

$$= \frac{e^{-t}}{(1+e^{-t})^{2}}$$

Aggiungendo e togliendo $+1$ e -1 al numeratore:

$$= \frac{e^{-t}-1+1}{(1+e^{-t})^2} = \frac{(1+e^{-t})-1}{(1+e^{-t})^2} =$$

$$= \frac{1}{(1+e^{-t})} - \frac{1}{(1+e^{-t})^2} =$$

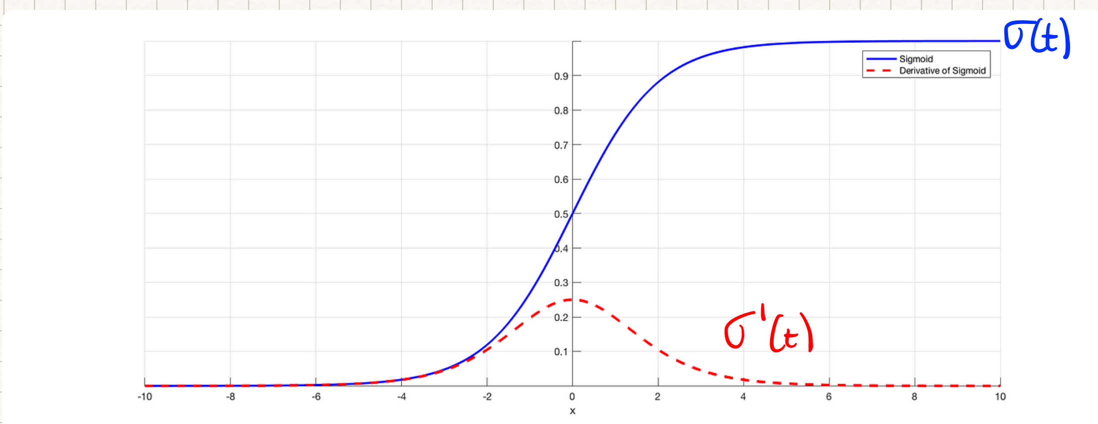
Mettendo in evidenza

$$= \underbrace{\frac{1}{(1+e^{-t})}}_{\sigma(t)} \underbrace{\left(1-\frac{1}{(1+e^{-t})}\right)}_{1-\sigma(t)}$$

Quindi la derivata della $\sigma(t)$:

$$\frac{d\sigma(t)}{dt} = \sigma(t)\left(1-\sigma(t)\right)$$

Graficamente:

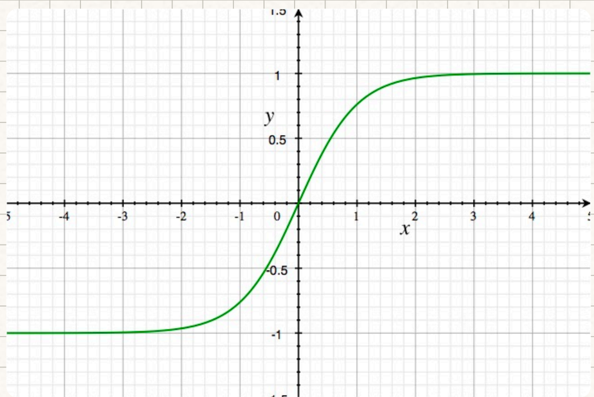


TANH

$$\tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

Derivata:

$$\tanh'(z) = \operatorname{sech}^2(z) = \frac{1}{\cosh(z)} = 1 - \tanh^2(z)$$

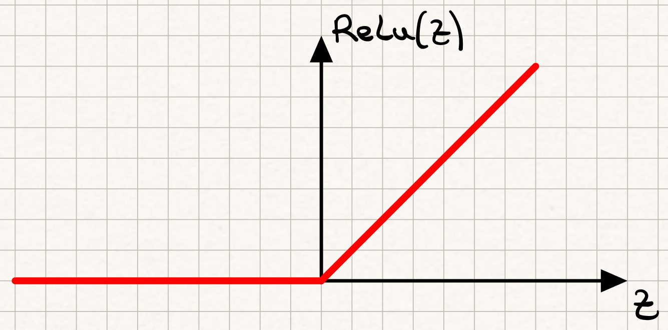


RELU

$$ReLu(z) = \begin{cases} z & \text{per } z > 0 \\ \varnothing & \text{altrimenti} \end{cases} = \max(\varnothing, z)$$

Derivata

$$ReLu'(z) = \begin{cases} 1 & \text{per } z > \varnothing \\ \varnothing.5 & \text{per } z = \varnothing \\ \varnothing & \text{per } z < \varnothing \end{cases}$$



LEAKY-ReLu

$$\text{Leaky-ReLu} = \max\left(\frac{z}{100},\ z\right)$$