FORWARD :

$Y_j^{\ell}(t) = \beta_j \delta_j^2 S_j(t) + \delta_j W(S_j(t))$

$\forall i$

$$\underbrace{F_j(t,t)}_{\text{MUST BE MARTINGALE}} = F_j(0,t) \exp\{f_j(t)\} \qquad f_j(t) = c_j t + Y_j^{\ell}(t) \overset{\mathcal{L}}{=} c_j t + \beta_j \delta_j^2 S_j(t) + \delta_j \sqrt{t}\sqrt{S_j(t)}\, g$$

$\Rightarrow \mathbb{E}_0[F_j(t,t)] = F_j(0,t)$ MUST BE IMPOSED

$\Rightarrow \mathbb{E}_0[\exp\{f_j(t)\}] = 1 \qquad \phi_{f_j(t)}(u) = \mathbb{E}[e^{iuf_j(t)}]$

$\phi_{f_j(t)}(-i) = \mathbb{E}[\exp\{f_j(t)\}]$

Thus at UNIT TIME

$\phi(-i) = 1 \qquad \phi(-i) = \mathbb{E}[\exp\{f_j\}] = \mathbb{E}[\exp\{c_j + Y_j^{\ell}(1)\}] =$

$= \exp\{c_j\}\, \mathbb{E}[\exp\{Y_j^{\ell}(1)\}] =$

$= \exp\{c_j\}\, \mathbb{E}[\exp\{\beta_j \delta_j^2 S_j(1) + \delta_j \sqrt{S_j(1)}\, g\}]$

$\Rightarrow \phi(u) = e^{iuc_j}\, \mathcal{L}\left(-iu\beta_j\delta_j^2 + \frac{1}{2}u^2\delta_j^2\right)$ By NMVM

$\Rightarrow e^{c_j}\, \mathcal{L}\left(\beta_j\delta_j^2 - \frac{1}{2}\delta_j^2\right) = 1$

$e^{c_j} = \dfrac{1}{\mathcal{L}(\beta_j\delta_j^2 - \frac{1}{2}\delta_j^2)} \qquad c_j = -\log\left(\mathcal{L}\left(\beta_j\delta_j^2 - \frac{1}{2}\delta_j^2\right)\right)$

Thus applying the MARTINGALE CONDITION :

$$f_i(t) = -\log\left(\mathcal{L}\left(\beta_j\delta_j^2 - \frac{1}{2}\delta_j^2\right)\right)t + \beta_j\delta_j^2 S_j(t) + \delta_j\sqrt{t}\sqrt{S_j(t)}\, g \qquad \forall t$$