Final Project: Advanced Topics in Macro

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For The Last Session

1 Final Project

The objective of the final project is to expand the representative agent model to include two types of agents and then estimate this model using: i) SMM (Simulated Method of Moments) estimation and ii) a full Bayesian estimation.

Starting from the two-agent model described below:

1. Solve the model.
2. Write down the stationary equilibrium (i.e. de-trend the model).
3. Write the corresponding Dynare code.
4. Make sure the model run with the set of calibration provided in the text first.
5. For the SMM estimation: Estimate the following key structural parameters of the model: time preference β, the growth rate of the economy γ_y, the standard deviation of the TFP shock σ_A, the persistence of the TFP shock ρ_A, and the share of hand-to-mouth households ω. Use the SMM approach, as discussed in class, to match the following moments of aggregates and prices: the volatility of GDP and consumption growth rates ($Var(g_y)$ and $Var(g_c)$) with $g_y = \Delta log(y_t)$ and $g_c = \Delta log(c_t)$,[1] the mean interest rate $E(r_t)$, and the mean GDP growth rate $E(g_y)$ as well as the mean growth rate of aggregate consumption $E(g_c)$.
   - For the 5 moments, please target the moments of an economy of your choice. To do that you will need to download data on GDP, Aggregate Consumption, and Risk Free rate (e.g. a short term bond rate). Then create an Excel file with those data (remember they need to be of the same frequency). The rest can all be done in Dynare.

[1] Here all small letter variables denote stationary variables, while capital letter variables are level variables. For example, c_t and y_t represents the de-trended stationary variables.

6. For the Bayesian estimation: add a shock ϵ_t^G to public spending (such as now $G_t = 0.2 Y_t \epsilon_t^G)$[2] and then use the two data sets on GDP and aggregate consumption to estimate the two AR(1) shocks parameters, while you calibrate all the other parameters to the SMM values. Then show the shock decomposition for GDP and consumptions. Finally, run a counter factual analysis for different values of ω.

1.1 What You Need To Deliver

1. A short report with all the derivations, data used, results, and a short discussion.
2. A Dynare Code.
3. A Data file.

1.2 The Model

Consider an infinite time horizon problem. The economy is composed of a representative firm and two types of households.

Households

The representative hand-to-mouth household (SP) consumes their income $w_t l_t^{SP}$ minus taxes paid T_t^{SP}, where wages are w_t and labor supplied is l_t^{SP}. Labor is inelastic in this economy and set to 1/3 (i.e., $l_t^{SP} = 1/3$).

$$\max_{\{C_t^{SP}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t^{SP})$$

$$\text{s.t.} \quad C_t^{SP} = w_t l_t^{SP} - T_t^{SP}$$

The representative wealthy household (SA) decides how much to consume and save from their income $w_t l_t^{SA}$ and their asset holdings, including safe assets $r_t B_t$ and risky assets $r_t^K K_t$. Labor is also inelastic for savers (i.e., $l_t^{SA} = 1/3$).

$$\max_{\{C_t^{SA}, B_{t+1}, I_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t^{SA})$$

$$\text{s.t.} \quad C_t^{SA} + B_{t+1} + I_t = r_t B_t + r_t^K K_t + w_t l_t^{SA} - T_t^{SA}$$

$$\text{and} \quad K_{t+1} = (1-\delta) K_t + I_t$$

where δ is the depreciation rate of capital K_t, and I_t is investment in risky assets K_t that yield returns r_t^K. Finally, riskless bonds B_t yield returns r_t.

[2] Where ϵ_t^G follow an AR(1): $\log(\epsilon_t^G) = \rho_G \log(\epsilon_{t-1}^G) + \eta_G$ with $\eta_G \sim \mathcal{N}(0, \sigma_G^2)$.

Firms

The representative firm in this economy uses labor L_t (with $1-\alpha$, the elasticity of production with respect to labor, set to 2/3) and capital K_t to produce goods Y_t. Labor L_t is the weighted sum of labor from savers and spenders ($L_t = \omega L_t^{SP} + (1-\omega)L_t^{SA}$, with ω representing the share of spenders in the economy, set to 30%). Each labor is subject to the same labor augmenting technology with growth trend Γ_t such as $L_t^{SP} = \Gamma_t l_t^{SP}$ and $L_t^{SA} = \Gamma_t l_t^{SA}$. The economy trend $\Gamma_t = \gamma_y \Gamma_{t-1}$ where γ_y is the economy growth rate.

The firm maximizes lifetime discounted profits (with the discount rate $\beta \frac{\lambda_t}{\lambda_0}$, where λ_t is the marginal utility of savers). It decides: (i) how much labor L_t to employ and (ii) how much capital to use. The production is subject to TFP shocks, which introduce risk. The firm's problem is:

$$\max_{\{d_t, Y_t, L_t, K_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} d_t$$

$$\text{s.t.} \quad d_t = Y_t - w_t L_t - r_t^K K_t$$

$$\text{and} \quad Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$\text{with} \quad \log(A_t) = \rho_A \log(A_{t-1}) + \eta_A$$

where $\eta_A \sim \mathcal{N}(0, \sigma_A^2)$ and A_t represents the TFP. In the calibrated version ρ_A could be set at 0.9 while the standard deviation of the shock at 0.01.

Government

The government finances public spending using taxes:

$$G_t = T_t$$

Public spending is set as a fixed proportion of GDP (i.e., 20%):

$$G_t = 0.2Y_t$$

Aggregation and Market Clearing

Aggregate consumption is given by:

$$C_t = \omega C_t^{SP} + (1-\omega) C_t^{SA}$$

Taxes are levied uniformly across savers (wealthy) and spenders (hand-to-mouth):

$$T_t = \omega T_t^{SP} + (1-\omega) T_t^{SA}$$

and

$$T_t^{SP} = T_t^{SA}$$

The aggregate labor is:

$$L_t = \omega L_t^{SP} + (1-\omega) L_t^{SA}$$

Finally, the bonds are on net-zero supply (i.e. no debt):

$$B_t = 0$$