Week 7 assignment

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Quantum Information and Computation

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Ising 1D Chain in magnetic field (opposite)

Dimension $2^N \times 2^N$, opposite sign than usual

$$\mathcal{H} = \lambda \sum_i^N \sigma_z^i - \sum_i^{N-1} \sigma_x^{i+1} \sigma_x^i$$

Ground states

- $\lambda \to 0 \implies |\rightarrow\rightarrow \dots\rangle, |\leftarrow\leftarrow \dots\rangle$ with degeneracy 2.
- $\lambda \to \infty \implies |\downarrow\downarrow \cdots \downarrow\downarrow\rangle$ if $\lambda > 0$.

Mean field prediction

$$e = -1 - \frac{\lambda^2}{4} \qquad \lambda \in [-2, 2]$$
$$e = -|\lambda| \qquad \lambda \notin [-2, 2]$$

Theoretical result: $\lambda = 1$.

Compile: gfortran ising.f90 -o ising -llapack

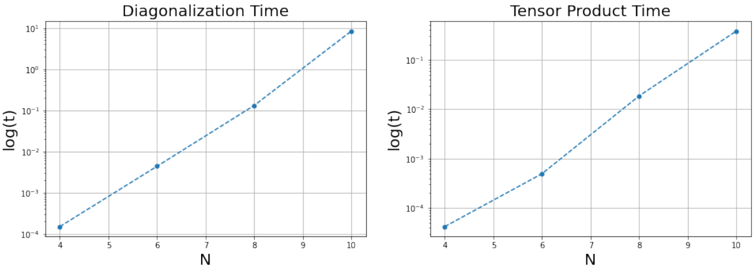
```fortran
!A_mxn, B_pxq
    function TPROD(A,B) result(C)
            allocate( C(m*p, n*q))
    !loop over A to get Aij that multiplies a whole B
            do ii = 1, m
                    do jj = 1, n

                        !start, finish: C[start:finish, ...]
                        srow = (jj - 1) * q + 1
                        frow =  jj * q
                        scol = (ii - 1) * p + 1
                        fcol =  ii * p

                        C(srow:frow, scol:fcol) = A(ii,jj) * B
```

```fortran
    function PAIR_SPIN_TERM(N, J) result(H_pair)
            id = cEYE(2)
            sigma_x = reshape([0, 1, 1, 0], shape(sigma_x), order = [2,1] )
            H_pair = 0
            do ii = 1, N-1
                    if (ii == 1) then
                            H_tmp = TPROD(sigma_x, sigma_x)
                            do jj = 3, N
                                    H_tmp = TPROD(H_tmp, id)
                            H_pair = H_pair + J * H_tmp
                    ELSE
                            H_tmp = id
                            do jj = 2, N - 1
                                    if (jj == ii) then
                                            H_tmp = TPROD(TPROD(H_tmp, sigma_x), sigma_x)
                                    else
                                            H_tmp = TPROD(H_tmp, id)
                            H_pair = H_pair + J * H_tmp
```

Efficiency

Diagonalization seems to take more time but

- For my limited VM, $N_{max} = 12$
- Expect $N_{max} = 14$ since $2^{14} \cdot 2^{14} \cdot 16 = 4Gb$ (complex*16)



$N = 2$ has exactly the 4 plotted eigenvalues, MF is bad until $\mathbb{I} \otimes \dots \sigma_z \dots \otimes \mathbb{I}$ dominates (no correlations).
Degeneracy breaks with $\lambda > 0$. For $\lambda \approx 1$, E_1 detaches from E_0 and reaches E_2, E_3.