Week 6 assignment

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Quantum Information and Computation

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Quantum State

Many-particle wavefunction

Single site: $|\alpha_1\rangle = \{|0\rangle, \ldots, |d-1\rangle\}$, $\mathcal{H}^d$

Many sites: $|\psi\rangle \in \mathcal{H}^{d^N}$, $|\psi\rangle = \sum_{\alpha_1,\ldots,\alpha_N} C_{\alpha_1,\ldots,\alpha_N} |\alpha_1\rangle |\alpha_2\rangle \ldots |\alpha_N\rangle$

Complex tensor can be reshaped into a vector $C_{\alpha_1 \ldots \alpha_N} \to C_i$, d^N entries.

Separable states: $|\psi\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_N\rangle \to d \cdot N$ coefficients

Density Matrix

$\rho = |\psi\rangle \langle\psi| = \sum |\alpha_1\alpha_2\rangle \langle\beta_1\beta_2| \psi^*_{\beta_1\beta_2} \psi_{\alpha_1\alpha_2}$ for 2 subsystems (bipartition)

Reduced density matrix: $\rho_A = Tr_B \rho = \sum_\gamma \langle\gamma|_B \rho |\gamma\rangle_B$

Reduction algorithm: $\rho = \sum_{ij} \rho_{ij} |i\rangle \langle j| = \sum_{\mu\nu kl} \rho_{\mu\nu kl} |\mu\rangle_A |\nu\rangle_B \langle k|_A \langle l|_B$

To get the reduced matrix insert $\delta^{\nu l}$:

$$(\rho_A)_{\mu k} = \sum_l \rho_{\mu lkl}$$

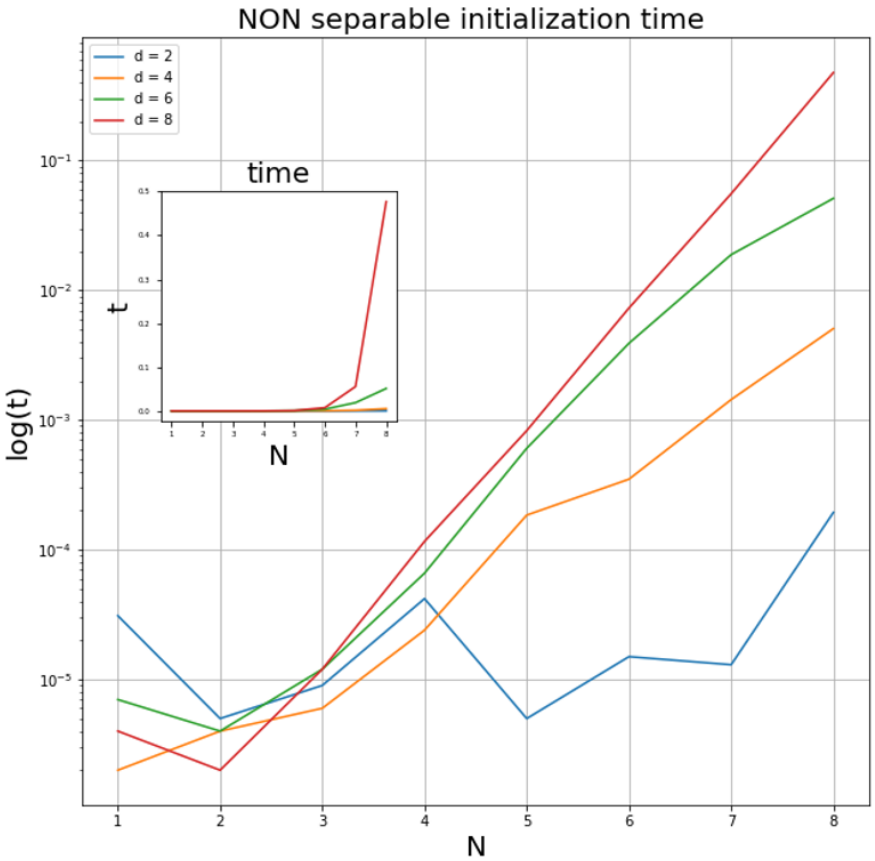
Compile: gfortran week6.f90 -o wk

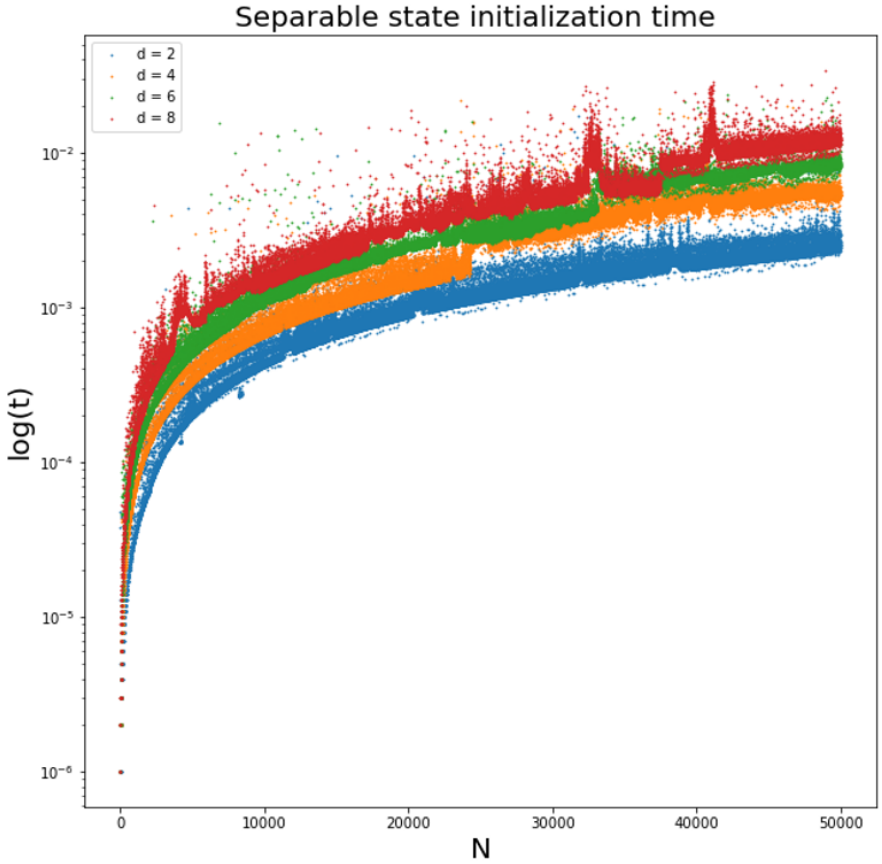
```
type STATE
                integer :: d
                integer :: N
                logical :: separable
                complex*16, dimension(:), allocatable :: psi
```

```
        SUBROUTINE init_random_state(sys)

                type(STATE) :: sys
                integer*4 :: total_dim
                real*8, dimension(:), allocatable    :: A, B
                real*8 :: psi_norm

                if (sys%separable .eqv. .TRUE.) then
                        total_dim = sys%d * sys%N
                else
                        total_dim = sys%d ** sys%N
                end if

                call Random_Number(A)
                call Random_Number(B)

                sys%psi = cmplx(A,B)
                psi_norm =  SQRT(dot_product(sys%psi, sys%psi))
                sys%psi = sys%psi/psi_norm
```

Efficiency Test

Compatible with exponential behaviour for non separable states, polynomial for separable ones. Issue: memory ($d = 8, N = 10 : 10^9$ complex to allocate, 16 bytes for each).





ρ from $|\psi\rangle$, check $\mathrm{Tr}\,\rho = 1$ and Hermiticity

```
NN = size(sys%psi)
!copy a vector ncopies times making it the rows (dim = 1) or the columns (dim = 2) of a matrix
density_matrix = spread(sys%psi(1:NN), dim = 2, ncopies = NN)  *
                 spread( CONJG(sys%psi(1:NN)), dim = 1, ncopies = NN)
```

```
function PARTIAL_TRACE(R, S, in_mat) result(out_mat)
                integer*4 :: R,S, ii, jj, uu
                complex*16    in_mat(R*S, R*S), reshaped_mat(R,S,R,S), out_mat(R,R)
                complex*16 :: trace
                !(0.d0,0.d0) is padding, double complex. Invert 1,2 and 3,4 (column major order)
                ! [1,2,3,4,5,6,7,8,9] ->          1 4 7     because columns are filled first.
                !                                 2 5 8
                !                                 3 6 9

                reshaped_mat = RESHAPE(in_mat, [R,S,R,S], [(0.d0,0.d0)], [2,1,4,3])

                !sum over indexes 2,4 to produce rho_a
                do ii = 1, R
                        do jj = 1, R

                                trace = 0
                                do uu = 1, S
                                        trace = trace + reshaped_mat(ii, uu, jj, uu )
                                end do

                                out_mat(ii, jj) = trace
                        end do
                end do
        end function
```

Test

Separable State

$$\frac{|0_A 0_B\rangle + |1_A 0_B\rangle}{\sqrt{2}} \; ; \; \rho = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
beefyboy@beefyboy:~/wk6$ ./wk
 Your state is
{0.707, 0.000} {0.000, 0.000} {0.707, 0.000} {0.000, 0.000}
 ************************************************************
 The density matrix of the system is
{0.500, 0.000} {0.000, 0.000} {0.500, 0.000} {0.000, 0.000}
{0.000, 0.000} {0.000, 0.000} {0.000, 0.000} {0.000, 0.000}
{0.500, 0.000} {0.000, 0.000} {0.500, 0.000} {0.000, 0.000}
{0.000, 0.000} {0.000, 0.000} {0.000, 0.000} {0.000, 0.000}
 ************************************************************
 The reduced density matrix for subsystem A is
{0.500, 0.000} {0.500, 0.000}
{0.500, 0.000} {0.500, 0.000}
 ************************************************************
 The reduced density matrix for subsystem B is
{1.000, 0.000} {0.000, 0.000}
{0.000, 0.000} {0.000, 0.000}
```

Entangled State

$$\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}} \; ; \; \rho = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

```
 Your state is
{0.707, 0.000} {0.000, 0.000} {0.000, 0.000} {0.707, 0.000}
 ************************************************************
 The density matrix of the system is
{0.500, 0.000} {0.000, 0.000} {0.000, 0.000} {0.500, 0.000}
{0.000, 0.000} {0.000, 0.000} {0.000, 0.000} {0.000, 0.000}
{0.000, 0.000} {0.000, 0.000} {0.000, 0.000} {0.000, 0.000}
{0.500, 0.000} {0.000, 0.000} {0.000, 0.000} {0.500, 0.000}
 ************************************************************
 The reduced density matrix for subsystem A is
{0.500, 0.000} {0.000, 0.000}
{0.000, 0.000} {0.500, 0.000}
 ************************************************************
 The reduced density matrix for subsystem B is
{0.500, 0.000} {0.000, 0.000}
{0.000, 0.000} {0.500, 0.000}
```