Week 3 assignments

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Quantum Information and Computation

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Matrix Matrix multiplication

$$C_{ik} = \sum_{j=0}^{N} A_{ij} B_{jk}$$

Column-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

```
do ii = 1, size(mat_1,1)
    do kk = 1, size(mat_2,2)
        do jj = 1, size(mat_1,2)

            mat_3(ii,kk) = mat_1(ii,jj)*mat_2(jj,kk) + mat_3(ii, kk)
        end do
    end do
end do
```

Figure: Usual matrix multiplication, from outermost: ii, kk, jj

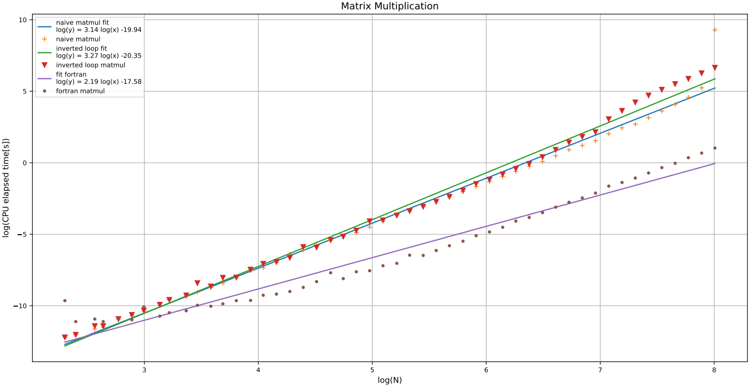
```
do ii = 1, size(mat_1,1)
    do jj = 1, size(mat_1,2)
        do kk = 1, size(mat_2,2)

            mat_3(ii,kk) = mat_1(ii,jj)*mat_2(jj,kk) + mat_3(ii, kk)
        end do
    end do
end do
```

Figure: "Cache" inefficient matrix multiplication, from outermost: ii, jj, kk

Matrix Matrix Multiplication: Results

With $y = CPUtime$, $x = N$ square matrix size, we have to fit a polynomial $y = ax^b$. This is equivalent to linear fitting

$$\log(y) = b\log(x) + b\log(a) \iff Y = bX + c$$



Random Matrix Theory

- A, D random hermitian and diagonal matrices $\sim U[-1,1]$
- call ZHEEV $\rightarrow \lambda_i$ stored in abscending order
- Check $\sum_i \lambda_i = \text{Tr}A$
- $s_i = \frac{\lambda_{i+1} - \lambda_i}{\bar{\Delta\lambda}}$
- Repeat 30 times for $N = 2001$ square matrices
- Compute $P(s)$

$$P(s) = \frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi} s^2} \quad \textit{Wigner Surmise}$$

$P(s) = a\, s^\alpha\, e^{-bs^\beta} \rightarrow P(s) = \frac{1}{\int_0^\infty ds\ P(s)} s^\alpha\, e^{-bs^\beta}$: normalization $\implies$ one less parameter (and removes minimization divergencies).

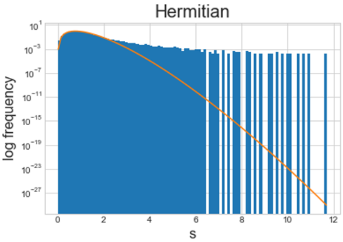
Entropy minimization: $S = -\sum P(s) \ln P(s) = -\langle \ln P(s) \rangle$
Functions: np.average, scipy.optimize.minimize, scipy.integrate.quad (for a)
Fitted logarithm of $P(s)$ for increasing numerical stability, nothing changed.

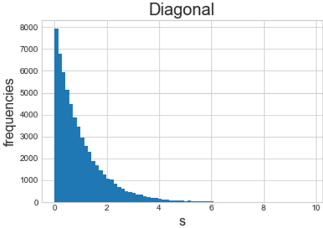
Hermitian Matrices

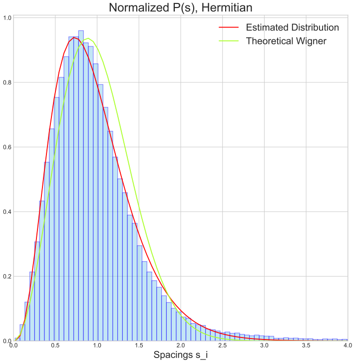
"Bad" fit: few spacings up to 11 conflict with exponential suppression $\rightarrow s < 3$. $\bar{\Delta}\lambda$ still problematic. $S_{exp} = 0.540$ vs $S_{th} = 0.558$

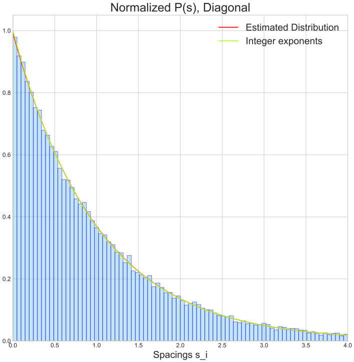
Diagonal Matrices

Histogram falls from the start: hypothesize $\alpha = 0$. Very good agreement.









Parameters	Wigner Surmise	Hermitian Exp	Diagonal Exp
a	$32/\pi^2 \approx 3.24$	10.0	0.99
b	$4/\pi \approx 1.27$	2.49	0.99
α	2	2.36	0
β	2	1.38	1.01