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Three-dimensional Bone Models Generation for Numerical Biomechanical Simulation.

F. Ramírez[a,b], V. M. Calo[c],
G. A. Espinosa[a,b]

Uniandes Ph.D. Student:
Gabriel Andrés Espinosa Barrios
g-espino@uniandes.edu.co

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[a]Biomedical Engineering Group (GIB), Universidad de los Andes, Bogotá D.C., Colombia
[b]Department of Civil and Environmental Engineering, Universidad de los Andes, Bogotá D.C., Colombia
[c]Faculty of Science and Engineering, Curtin University, Bentley, Perth, Western Australia

It is defined that in the course of the First Semester a strong bibliographic review of NURBS (Non-uniform rational B-splines)was make to what it was and what it means. It was also study the basis functions of Splines, Bezier Curves, and the B-Splines. On the other hand, it was working to acquire the first Medical Images segmentation that works with the NURBS modeling of the bones of the hand. It was obtaining a first segmentation algorithm and the extraction of the cloud control points of the bones of the hand. The first limitation of this was that the segmentation is not perfect. There is a loss of information of the bone and there are additional data of other bones that it needs to be removed by hand.

The Second Semester was dedicated to the implementation. It was having more clear what are the NURBS and where it comes. Therefore, It was developed to implement own algorithms to generate not only Surface NURBS, that is like traditional, but also Solid Volumetric NURBS. Furthermore, it was revised the theory about geometrical meaning of the weights. It also was implemented what is known as Interpolation NURBS to force to pass the curves on the control points. Theories were reviewed about how to modify NURBS with least squares, but it wasn't possible to implement.

In the Third Semester, tutorial for Undergraduate B. S.(Bachelor of Science) students was made. Two tutorials for Medical Image Segmentation and for B-Splines and NURBS Geometry Model and IGA was make for this kind of students. On the other hand, a review of bone library was make to understand the bone to understand the pricipal topologies of the bones.

In the first internship, fourth semester an algorithm and methodology was generated to obtain the surface NURBS of the bone, and Solid Volumetric NURBS file for Stress - Strain simulations of the bones.

1 Introduction; Review of: "IGA(Isogeometric Analysis) Toward Integration of CAD and FEA". T. J.R. Hughes et al.[1]

1.1 Open Knot Vectors:

An indexed Vector "Knot Vectors" is open when it's first and last value are showed on $\mathbf{P}+1$ times. Where the $\mathbf{P}$ is the polynomial order. For instance, the polynomial cubic $\mathbf{P}=3$ is:

$$\Xi = [0,0,0,1,2,3,4,4,4]$$

The open knot vector are the standard for CAD(Computer Analysis Design).

1.2 Open Uniform B-splines[2]:

The B-Splines are the Blending-Mixture between the Open Uniform B-Splines and the Non-Uniform B-Splines. There are sometimes agree as some special Uniform B-Splines; nevertheless, there are classified as Non-uniform B-Splines.

Examples:

$$[0,0,1,2,3,3] \text{ for } \mathbf{P}(=1)+1=2 \text{ and } \mathbf{n}=3$$

$$[0,0,0,0,1,2,2,2,2] \text{ for } \mathbf{P}=3 \text{ and } \mathbf{n}=4$$

These Knot vectors can be normalized for the intervals $0-1; 0 \leq \mathbf{u} \leq 1$ in this way:

$$[0,0,0,33,0,67,1,1] \text{ for } \mathbf{P}=1 \text{ and } \mathbf{n}=3$$

$$[0,0,0,0,0,5,1,1,1,1] \text{ for } \mathbf{P}=3 \text{ and } \mathbf{n}=4$$

In general for any value of "$\mathbf{P}$" and "$\mathbf{n}$"($\mathbf{n}$ is the number of control points -1) could be generate a "open knot vector" with integer values through the following calculations:

$$\mathrm{u}_j = \begin{cases} 0 & \text{for} \quad 0 \leq j < \mathbf{P}+1 \\ j-\mathbf{P}+1+1 & \text{for} \quad \mathbf{P}+1 \leq j \leq \mathbf{n} \\ \mathbf{n}-\mathbf{P}+1+2 & \text{for} \quad j > \mathbf{n} \end{cases}$$

$$\text{Size of Knot Vectors is} = \mathbf{n}+\mathbf{P}+1+1. \tag{1}$$

For j that show from 0 until $\mathbf{n}+\mathbf{P}+1$. The first $\mathbf{P}+1$ Knots are assigned to zero 0; Otherwise, the last $\mathbf{P}+1$ knots have a value of $\mathbf{n}-p+3$.

1.3 Open Knot Vectors(Continuations):

"*The Open B-Splines have very similar characteristics to the* 'Bézier Splines'."[2] Even though, when ($\mathbf{P}+1=\mathbf{n}+1 \Rightarrow \mathbf{n}=\mathbf{P}$)there are polynomial order of control points(CP) +1 (CP=$\mathbf{P}+1$); the open B-Splines are reduced to the "Bézier Splines" and all of the values of the index vector "Knot Values" are 0 or 1. For instance, with a open B-Spline of four-4 control points($\mathbf{n}=3$) and cubic ($\mathbf{P}=3$) the index vector is:

$$\Xi = [0,0,0,0,1,1,1,1]$$

The polynomial curve of a Open B-Spline go through for the first and the last Control Point - CP. Figure 1.

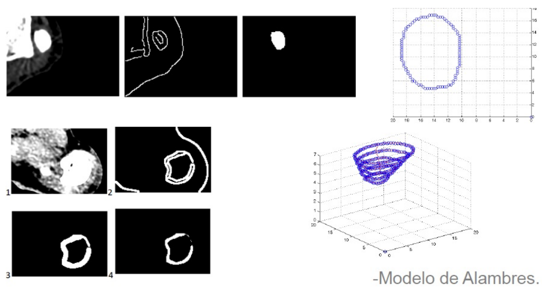

Figure 1: Results of the polynomial curve of the B-Spline Segmentation.

Even as in the Bézier curves; specify multiples control points in just one coordinate, push any B-Spline to that location. Therefore, it could be generate close curves with "open B-Splines" specifying the first and the last control points in the same position. Figure 2.

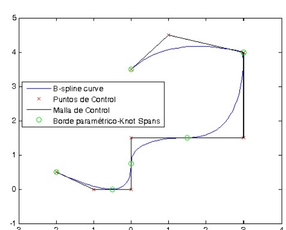

Figure 2: Polynomial curve of the Open B-Spline.

[1] As a consequence of the use of "Open Knot Vectors" in multiple dimensions; the border of one "B-Spline object" with "$\mathbf{d}$" parametric dimensions is that the "B-Spline object" will had "$\mathbf{d}$" -1 dimensions.
i.e. each border of one B-Spline surface is one B-Spline curve. Figure 3.

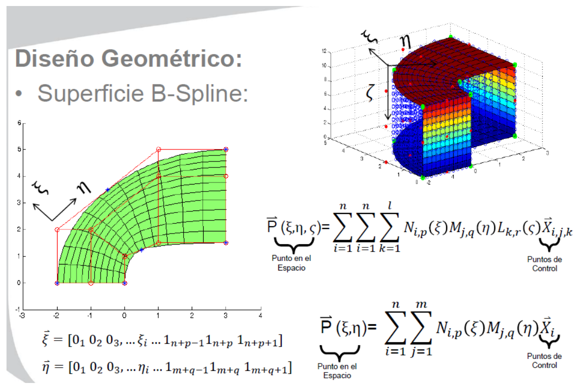

Figure 3: 2D-3D Polynomial B-Spline Geometry.

2 Basic Functions(A.K.A:"Blending Functions". Open Knot Vectors Continuations)[1]:

$$\mathbf{N}_{i,0}(\xi)=¿?\ \Leftrightarrow \mathrm{B}_{k,1}(\mathbf{u})$$

$i = \mathbf{n}$ control points $\Leftrightarrow$ control point i

$0 = \mathbf{P}$. Polynomial Order $\Leftrightarrow$ B-Spline or Basic Polynomial order degree zero 0

$k = \mathbf{n} + 1$

$1 =$ "$\mathbf{d}$" Parametric dimensions $\Leftrightarrow$ Parametric dimensions order one 1

2.1 Recursive Functions of Cox-de Boor(Basic Functions Continuations):

The Cox-de Boor recursion formula are functions to join the B-Splines curves. Usually, we can obtain a expression for the calculus to the positions along of a B-Spline curve, with the couple of functions of the "Blending-function"(Basic Functions) formulation thereby:

$$\mathbf{P}(\mathbf{u}) = \sum_{k=0,1}^{\mathbf{n}} \mathbf{P}_k \mathbf{B}_{k,\mathbf{P}}(\mathbf{u})$$
$$\mathbf{u}_{min} \leq \mathbf{u} \leq \mathbf{u}_{max}$$
$$1 \leq \mathbf{P} \leq \mathbf{N} \text{ (Control points)}$$

(2)

Where:

1. $\mathbf{P}_k$= There are $\mathbf{n}+1$ control points.
2. $\mathbf{B}_{k,\mathbf{d}}$= There are the polynomials of degree "$\mathbf{P}$". Where "$\mathbf{P}$" could be selected to any integer between 1 until the last control points ($\mathbf{N} = \mathbf{n} + 1$).

It could be achieve B-Spline local control, defining the join functions "Blending functions"(Basic Functions) about the subinterval of the total degree-rank of $\mathbf{u}$:

$$\mathbf{N}_{i,0}(\xi) \Leftrightarrow \mathbf{B}_{k,1}(\mathbf{u})=\begin{cases} 1 \quad \text{if} \quad \mathbf{u}_k \leq \mathbf{u} \leq \mathbf{u}_{k+1}\ (\mathbf{P}=0);(\mathbf{d}=1=\mathbf{P}+1) \\ 0 \quad \text{if} \quad \text{otherwise} \end{cases} \tag{3}$$

$$\mathbf{B}_{k,\mathbf{P}+1}(\mathbf{u})=\frac{\mathbf{u}+\mathbf{u}_k}{\mathbf{u}_{k+\mathbf{P}}-\mathbf{u}_k}\ \mathbf{B}_{k,\mathbf{P}}(\mathbf{u}) + \frac{\mathbf{u}_{k+\mathbf{P}+1}-\mathbf{u}}{\mathbf{u}_{k+\mathbf{P}+1}-\mathbf{u}_{k+1}}\ \mathbf{B}_{k+1,\mathbf{P}}(\mathbf{u})$$

$$\mathbf{P} = 1,2,3,4,...$$
$$\mathbf{d} = 2,3,4,5,...$$

(4)

Given that $\mathbf{u}_{k+\mathbf{P}} = \mathbf{u}_{\mathbf{P}}$; or $\mathbf{u}_{k+\mathbf{P}+1} = \mathbf{u}_{k+1} \Rightarrow$ any term evaluated as $\frac{0}{0} = 0$ will be equal to zero 0.
It is important is to show that even though the definitions before, we always need to take into a account that each Join Function or Basic Function constitute a part of unit i.e.:

$$\sum_{i=1}^{\mathbf{n}} \mathbf{N}_{i,\mathbf{P}}(\xi)= 1\ \forall\ \xi \text{ or } \sum_{k=0}^{\mathbf{n}} \mathbf{B}_{k,\mathbf{d}}(\mathbf{u})= 1\ \forall\ \mathbf{u}$$

(5)

2.2 Relation between continuity and multiplicity(Basic Functions Continuations):

In general the basic functions of order $\mathbf{P}$ have $\mathbf{P} - \mathbf{m}_i$ continuities derivatives on the "knot" index $\mathbf{u}_i$.

Where $\mathbf{m}_i$: is multiplicity of the value $\mathbf{u}_i$ in the "Knot vector".

We need to remember that the multiplicity in a polynomial is the number of times that the value of $\mathbf{u}_i$ is the base-root of the polynomial. For instance:

$$\mathbf{P}(x) = x^3 + 2x^2 - 7x + 4 = (x+4)(x-1)^2$$
Where: -4 = multiplicity 1; 1 = multiplicity 2

(6)

In B-Splines-IGA the multiplicity of a Index Knot value $\mathbf{m}_i$ is given by the number of times that this value is repeated in the "Knot Vector".

Example:

$\Xi = [0, 1, 2, 3, 4, 4, 5]$ in this case $0 = \textit{multiplicity}\ 1$; and $4, 4 = \textit{multiplicity}\ 2$

3 Medical Image Analysis Segmentation:

Medical Image Analysis Segmentation is make for each finger bone. In each finger of the one hand the Distal phalange, Proximal phalange, Metacarpal, and Trapezium, are segmented with the analysis of the pixels, Voxels of each DICOM(Digital Imaging and Communications in Medicine) Medical Image. See Figure 4.

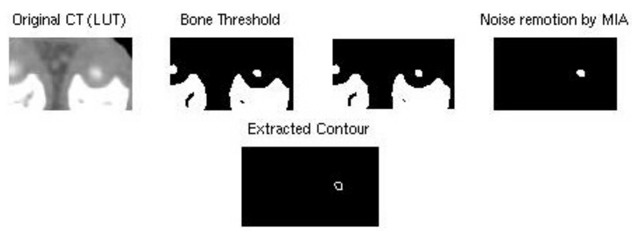

Figure 4: Pixel-Voxel Boundering Segmentation.

3.1 B-Spline curves models for bone segmentation:

A B-Spline curves was generated to understand the mesh and the control points of a DICOM bone Pixel-Voxel values of the images. First was generated a Circle and Oval curves to understand the cloud control points that was obtained for each DICOM image. See Figure 5.
Second, Oval deformed cloud control points was generated to understand this kind of segmentation of the DICOM bone image. See Figure 6.

4 B-Splines and NURBS Bones Modeling:

B-Splines and NURBS codes for bone segmentation process was generated to obtain solid bones of the fingers. B-Splines and NURBS formulate was make to obtain osteocytes cells of the bones. Index or Pointer Finger bones was segmented (see Figure 7) to obtain the solid of the bone to make stress - strain process.

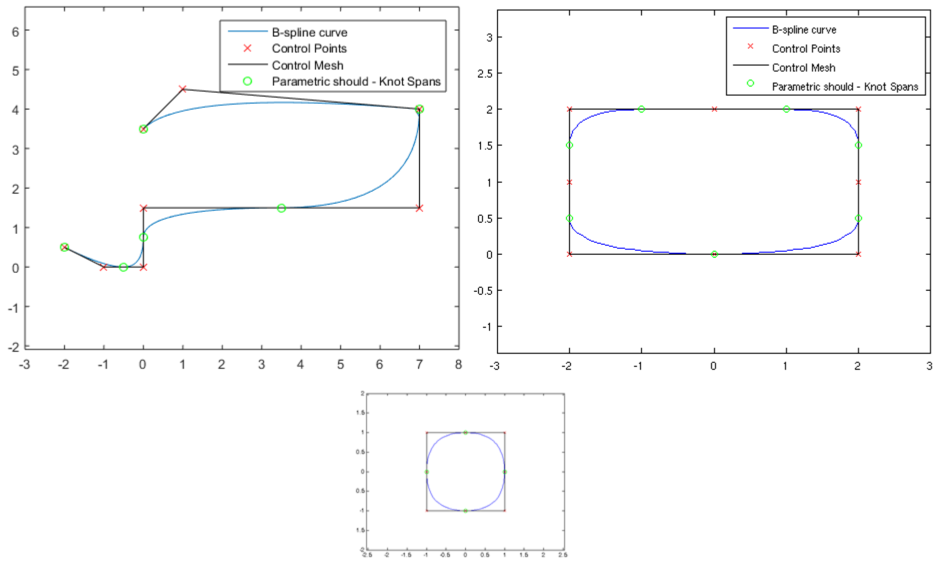

Figure 5: Left general B-Spline Curve. Right Oval B-Spline. Down Circle B-Spline.

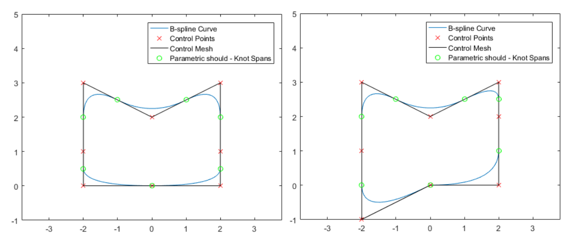

Figure 6: Oval Deformed B-Spline.

The codes was generated to segmented each bone of the finger: Distal phalange, Proximal phalange, Metacarpal, and Trapezium. However, the Medical Image Analysis was difficult to obtain the accurate segmentation of the bones. Therefore, Medical Image Analysis segmentation of others bones image its need to be done in order to obtain other accurate information.

Finally, Medical Image Analysis segmentation was make of the Index - Pointer Finger bones of the hand. A solid model of the bones was obtained to make stress - strain simulation to the forces in the bones. Future work is required for Stress - Strain simulation of the bones that was segmented.

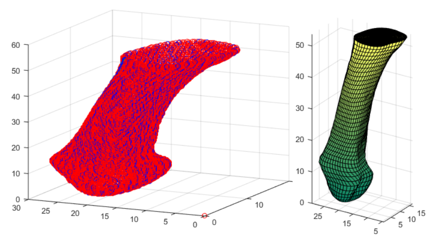

Figure 7: Left cloud control points of the Metacarpal bone. Right Solid results of the Metacarpal bone.

References

[1] J. A. Cottrell, T. J. R. Hughes, Y. Bazilevs, Isogeometric Analysis: Toward Integration of CAD and FEA, John Wiley & Sons, New York, 2009. 3, 4, 5

[2] D. Hearn, M. Baker, Computer Graphics, Prentice-Hall., Englewood Cliffs, NJ, 1994. 3, 4