1 BGG Model Equations

Production

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\begin{aligned}
K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X (X_t - 1)^2 \\
S'(X_t) &= 2\phi_X (X_t - 1) \\
Q_t(1 - S(X_t) - X_t S'(X_t)) + \mathbb{E}_t\left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2\right] &= 1 \\
Y_t &= A_t K_t^{\alpha} L_t^{1-\alpha} \\
W_t &= (1-\alpha)\left(\frac{K_t}{Lb_t}\right)^{\alpha} \\
r_{K,t} &= \alpha \left(\frac{Lb_t}{K_t}\right)^{1-\alpha} \\
R_{k,t+1} &= \frac{\left[r_{K,t+1} + (1-\delta)Q_{t+1}\right]}{Q_t}
\end{aligned}
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Households

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\begin{aligned}
u_{c,t} &= (C(1-\gamma))^{-1} - \beta\gamma (C(1-\gamma))^{-1} \\
\Lambda_{t,t+1} &\equiv \beta \frac{u_{c,t+1}}{u_{c,t}} \\
\Lambda_{t,t+1} R_{t+1} &= 1 \\
u_{c,t} W_t &= \chi L b_t^{\epsilon} \\
u_{c,t}(1-\alpha) Y / Lb &= \chi (Lb^{\epsilon})
\end{aligned}
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Entrepreneurs & Debt Contract

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\begin{aligned}
Q_t K_t &= L_t + N_t^E \\
N_t^E &= R_{k,t} Q_t K_{i,t} - R_{l,t} L_t \\
R_{l,t} L_t &= \bar{\psi}_t R_{k,t} Q_t K_t \\
N_t^E &= (\sigma_{E,t} + \xi^e)(1 - \Gamma(\bar{\psi}_{t+1})) R_{k,t+1} Q_t K_{t+1} \\
N_t^E \phi_t^E &= Q_t K_t \\
R_{k,t} Q_t K_t [\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})] &\geqslant R_{t+1}(Q_t K_t - N_t^E) \\
R_{k,t+1} &= \mathbb{E}_t\, \rho(\bar{\psi}_{t+1}) R_{t+1}
\end{aligned}
$$

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\begin{aligned}
p(\bar{\psi}_t) &= \int_0^{\bar{\psi}_t} f(\psi, -0.5(\sigma_\psi)^2, \sigma_\psi^2) d\psi \\
\Gamma(\bar{\psi}_t) &= G(\bar{\psi}_t) + \bar{\psi}_t(1-p) \\
G(\bar{\psi}_t) &= \int_0^{\bar{\psi}_t} \psi f(\psi, -0.5(\sigma_\psi)^2, \sigma_\psi^2) d\psi \\
\Gamma'(\bar{\psi}_t) &= (1 - p(\bar{\psi}_t)) \\
G'(\bar{\psi}_t) &= \frac{1}{\sigma_\psi \sqrt{\pi}} \exp\Big[- \frac{(\log(\bar{\psi}) + 0.5\sigma_\psi^2)^2}{2\sigma_\psi^2} \Big] \\
\rho(\bar{\psi}_{t+1}) &= \frac{\Gamma'(\bar{\psi}_{t+1})}{[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1})(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))]} \\
L_t &= D_t
\end{aligned}
$$

Resource Constraint

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\begin{aligned}
Y_t &= C_t + I_t + G_t + \mu G(\psi_t) R_{k,t} Q_t K_t \\
G_t &= \eta Y_t
\end{aligned}
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