A Course on DSGE Models with Financial Frictions
Part 5: Simple Heterogeneity in DSGE models

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On the referee reports

- Write a short summary of the paper. Should contain a concise summary of the papers claimed results, contributions, and general line of reasoning
- It is important for this summary to be clear
- Find some possible problems in the paper (e.g. modelling, robustness etc)
- Provide solutions or alternatives for those problems that this is possible
- Identify any possible missing links with the literature
- Here is a helpful report for the AEA journals referees
  https://www.aeaweb.org/content/file?id=222

Solving a Financial Frictions model

- Find the equilibrium conditions and the steady state of the model
- Solve it in Dynare and replicate some of the paper's main results
- Write a 3-4 pages report with results and analysis
- Given that you have replicated the main results you might want to extend it in some other dimension
- You don't have to write all the papers equations in the final report, they are already in the paper
- Be specific though about any changes from the original paper specifications

In the previous lecture

- Gertler and Kiyotaki (2010); Gertler and Karadi (2011) liquidity injections to the banking system
- Gertler and Karadi (2013) QE framework
- Countercyclical bank capital requirements
- Introducing housing and households borrowing in the models: Models with Loan to value constraints, Iacoviello (2005)

Today

- We will see how to introduce heterogeneity among households
- In a tractable way
- Does not require heavy computational tools to keep track of the full households' distribution
- TANK: Two Agent New Keynesian (model)

Two agents model motivating facts

- Recent studies on the households' wealth distribution, originating from Campbell and Mankiw (1989) in the US, have shown the existence of a population share of 30-40% that does not smooth consumption across states and time

- A significant fraction of households have near-zero net worth
  → There is no an aggregate Euler equation representing all households Bilbiie and Straub (2013), Vissing-Jørgensen (2002)

Some of the tractable heterogeneity literature

- Mankiw (2000) calls for the systematic incorporation of non-Ricardian households in macroeconomic models and uses an TA-RBC model
- Galí *et al.* (2007) firstly introduced a quantitative TANK framework to study the effects of government spending on consumption
- Bilbiie (2008) uses a similar TANK model and provides analytical insights, Bilbiie and Straub (2013) estimate a similar model, Colciago (2011) adds sticky wages
- Kaplan *et al.* (2014) identify the wealthy hand to mouth consumers, introducing a three agents model
- Debortoli and Galí (2017) compare HANK to TANK
  → TANK is a good approximation for heterogeneity *between* constrained and unconstrained households, not *within* unconstrained households

Today

- We will see the seminal paper by Galí *et al.* (2007), which has been a basis for many papers that followed this route
- Bilbiie (2008) that uses a similar model to provide analytical insights
- Bilbiie (2019) that extends the two agent framework and compares it with HANK models

TANK

- Two types of households, optimizers and hand to mouth consumers
- Optimizing households work, consume and save
- Hand to mouth (HtM) consumers live paycheck to paycheck, do not borrow or save, consuming all of their disposable income
- Essentially, optimizers have an Euler equation, while the HtM do not
  $\rightarrow$ Cannot transfer wealth across states and time

Galí *et al.* (2007), GLS

- One of the first papers that introduces agents with and agents without access to financial markets
- Problem: The standard RBC model predicts a decline in consumption in response to a rise in government purchases
  - An increase in government spending lowers the present value of post-tax income, thus generating a negative wealth effect that induces a cut in consumption
- Firstly (not covered here): Provide empirical evidence using a SVAR and show that consumption rises in response to a positive government spending shock
- Their research question: What does a model need to produce a rise in consumption in response to a rise in government spending?
- They show that you need a two agents model and some form of non-perfect labour markets

GLS: Overview

The economy consists of:

- Two types of households
- A continuum of firms producing differentiated intermediate goods
- A perfectly competitive firm producing a final good
- A central bank in charge of monetary policy
- A fiscal authority

Except for the presence of rule-of-thumb consumers, their framework is the standard NK DSGE model with Calvo price stickiness

- With the addition of labour unions for introducing sticky wages

GLS: Households

- Optimizing or Ricardian: A fraction $1 - \lambda$ of households
  - Have access to capital markets
  - Work and pay taxes, and get the profits from the firms
- Rule of thumb or non-Ricardian: A fraction λ of households
  - Do not own any assets nor have any liabilities
  - Just consume their current labour income

Optimizers

- All households are assumed to have identical preferences, given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i}^s) - \frac{L_{t+i}^{1+\epsilon,s}}{1+\epsilon} \right],$$

  where $s = o, r$

- The budget constraint of the optimizing households (o) is:

$$C_t^o + B_{t+1}^o R_t^{-1} + T_t^r = W_t L_t^r + R_t^k K_t^o + B_t^o + \Pi$$

  and the capital accumulation equation is

$$K_{t+1}^o = (1-\delta) K_t^o + \phi(I_t^o / K_t^o) K_t^o$$

  where $\phi()$ is the capital adjustment cost function, which determines the change in the capital stock induced by investment spending, with $\phi(\delta) = \delta$

Optimizers FOCs

- The optimizers' FOC are

$$1 = R_t \, \mathbb{E}_t \, \Lambda_{t,t+1}$$

$$Q_t = \mathbb{E}_t \, \Lambda_{t,t+1}[R^k_{t+1} + Q_{t+1}((1-\delta) + \phi_{t+1} - \frac{I^o_{t+1}}{K^o_{t+1}} + \phi'_{t+1})]$$

$$Q_t = \frac{1}{\phi'(\frac{I^o_t}{K^o_t})}$$

and the labour FOC

$$W_t = C^o_t (N^o_t)^{\phi}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor

$$\Lambda_{t,t+1} = \beta(\frac{C_{t+1}}{C_t})^{-1}$$

Sticky Wages

- In their second labour market specification, wages are set in a centralized manner by an economy-wide union
- Hours are assumed to be determined by firms (instead of being chosen optimally by households), given the wage set by the union
- Households are willing to meet the demand from firms, since wages always remain above all households MRS
- Therefore $W_t = C_t^o(N_t^o)^\phi$ no longer holds

Rule of thumb

- They behave in a hand-to-mouth fashion, fully consuming their current labour income
- No consumption smoothing due to labour income fluctuations
- No intertemporal changes in consumption due to interest rates changes
- Rule of thumb households r maximize

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\log(C^r_{t+i}) - \frac{L^{1+\epsilon,r}_{t+i}}{1+\epsilon} \right],$$

- Subject to their budget constraint

$$C^r_t = W_t L^r_t - T^r_t$$

- Which also determines their consumption level

Rule of thumb FOCs

- The only FOC of these households is with respect to labour hours
- If labour markets are competitive and they decide optimally their labour hours

$$W_t = C_t^r (N_t^r)^{\phi}$$

- Alternatively, when the wage is set by a union, hours are determined by firms labour demand, and the above does not apply

Aggregation

- Aggregate consumption and hours are given by a weighted average of the corresponding variables for each consumer type

$$C_t = \lambda C_t^r + (1-\lambda) C_t^o$$

$$L_t = \lambda L_t^r + (1-\lambda) L_t^o$$

- Aggregate investment and capital are given by:

$$I_t = (1-\lambda) I_t^o$$

$$K_t = (1-\lambda) K_t^o$$

Firms

- There is a continuum of monopolistically competitive firms producing differentiated intermediate goods
- Those goods are used as inputs by a perfectly competitive firm producing a single final good
- Intermediate firms are assumed to set nominal prices in a staggered fashion following Calvo (1983)
- Each firm resets its price with probability $1 - \theta$ each period
- Thus, each period a measure $1 - \theta$ of producers reset their prices, while a fraction θ keep their prices unchanged.

Fiscal Policy

- The government budget constraint is

$$T_t + R_t^{-1} B_{t+1} = B_t + G_t$$

where

$$T_t = \lambda T_t^r + (1 - \lambda) T_t^o$$

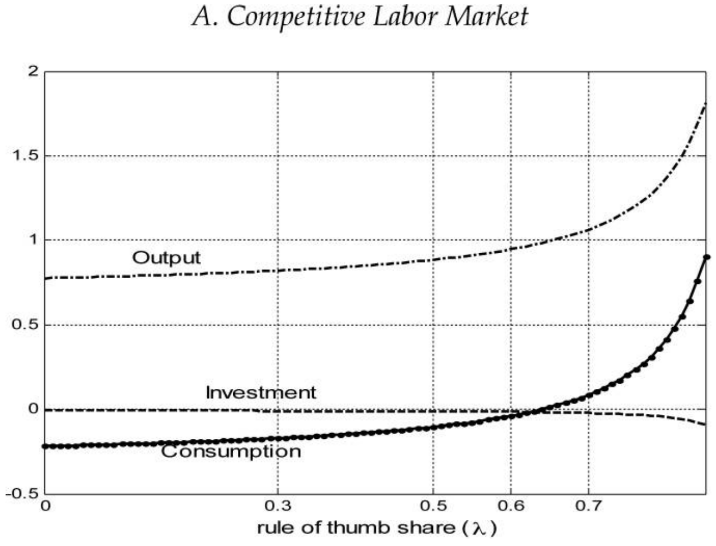
Government Spending and Consumption

- Back to the research question
- What does a model need to produce a rise in consumption in response to a rise in government spending?

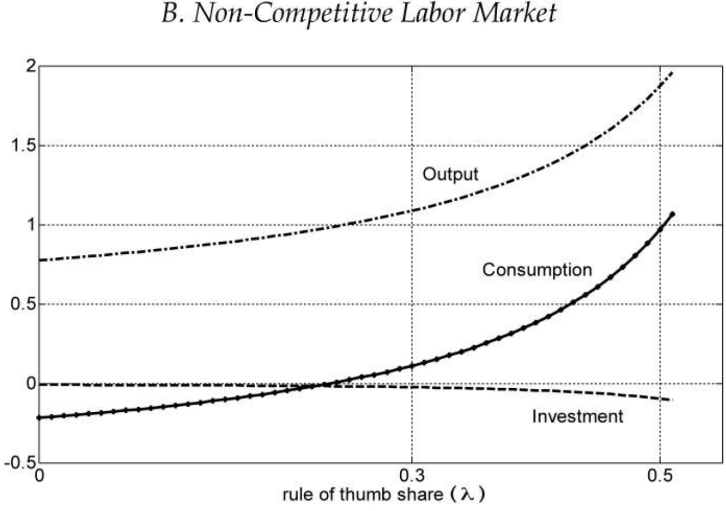
Perfect labour market case (no sticky wages):

- Consumption drops initially (as in the RBC models)
- As λ goes up (more HtM consumers) there is the offsetting role of rule-of-thumb behaviour on the conventional negative wealth and intertemporal substitution effects triggered by the fiscal expansion
- Rule of thumb people cannot smooth consumption or react to interest rate changes
- Therefore they spend and increase aggregate consumption

Impact multipliers: Government Spending Shock



Impact multipliers: Government Spending Shock



Government Spending and Consumption

- For non-competitive labour markets the positive effect on consumption comes even faster, threshold value of $\lambda = 0.25$
- Why?

Government Spending and Consumption

- Consider the following equilibrium condition

$$mpn_t = \mu_t + c_t + \phi l_t$$

  mpn_t is the log marginal product of labour, $c_t + \phi l_t$ the log marginal rate of substitution,
  μ_t is the wedge between the marginal rate of substitution and the marginal product of labour, ϕ measures the curvature of the labour disutility

- If $\mu_t = \mu$ (or 0 as in the RBC), when $g_t \uparrow$, according to the theory $l_t \uparrow, mpn_t \downarrow$, therefore $c_t \downarrow$

- A necessary condition for $c_t \uparrow$ is the existence of a simultaneous decline in the wedge μ_t, induces by the sticky wage setting

- Therefore: The income effect to the HtM and the decline in the labour wage lead to an increase in consumption after a gov. spending shock

Bilbiie (2008)

- Similar model with savers and hand to mouth consumers
- Incorporates limited asset markets participation (no access to capital markets) in a dynamic general equilibrium model
- Develops a simple analytical framework for monetary policy analysis

Main result:

- While moderate participation rates (lots of savers) strengthen the role of monetary policy
- Low enough participation causes an inversion of results dictated by conventional wisdom
  $\rightarrow$ A policy rate hike could lead in increase in output

Differences with GLV

- Both papers are very similar in their modelling structure
- Model the asset market explicitly and emphasize its interaction with the labour market
- Abstract from physical capital accumulation
- No sticky wages

Bilbiie (2008) main mechanism I

- Interest rate changes modify the intertemporal consumption and labour supply of asset holders
- This affects the real wage and hence the demand of agents who have no asset holdings but merely consume their wage income
- Changes in the real wage (marginal cost) lead to variations in profits and hence in the dividend income of asset holders
- These variations can overturn the initial impact of interest rates on aggregate demand
  - If the share of non-asset holders is high enough and/or
  - The elasticity of labour supply is low enough
- Variations in profit income offset the interest rate effects on the demand of asset holders

Bilbiie (2008) main mechanism II

- If participation is below a certain threshold, as the share of non-asset holders increases, the link between interest rates and aggregate demand becomes stronger, and monetary policy is more effective

  $\rightarrow$ This is what they call as standard aggregate demand logic, (SADL)

- When participation is restricted beyond a given threshold, standard theoretical prescriptions or predictions are reversed

  $\rightarrow$ Current aggregate output is positively related to real interest rates leading to the inverted aggregate demand logic, (IADL)

Bilbiie's Model

- Standard cashless dynamic general equilibrium model, augmented for limited assets market participation (LAMP)
- Some households are excluded from asset markets, while others trade in complete markets for state-contingent securities
- A perfectly competitive final-good producer
- A continuum of monopolistically competitive intermediate-goods producers setting prices on a staggered basis following Calvo (1983)
- Sticky prices $\rightarrow$ role for monetary policy
- A monetary authority setting its policy instrument, the nominal interest rate

Households - Optimizers

- A share of $1-\lambda$, represented by households who are forward looking and smooth consumption

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\log(C^s_{t+i}) - \omega \frac{L^{1+\phi,s}_{t+i}}{1+\phi} \right],$$

  where $s = o, r$

- subject to

$$C^o_t + B^o_{t+1} + V_t \Omega_{t+1} = W_t L_t + R_t B^o_t + \Omega_t (V_t + D_t)$$

- V_t is the average market value at time t of shares in intermediate good firms
- D_t are real dividend payoffs of these shares
- Ω_t are share holdings.

Optimizers FOCs

- The optimizers' FOC are

$$1 = R_t \, \mathbb{E}_t \, \Lambda_{t,t+1}$$

$$V_t = \mathbb{E}_t \, \Lambda_{t,t+1} [V_{t+1} + D_{t+1}]$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor

$$\Lambda_{t,t+1} = \beta (\frac{C_{t+1}}{C_t})^{-1}$$

and the labour FOC

$$\omega (N_t^o)^{\phi} = \frac{1}{C_t^o} W_t$$

Households - Rule of thumb

- A share of λ, represented by households who have no assets

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i}^s) - \omega \frac{L_{t+i}^{1+\phi,s}}{1+\phi} \right],$$

  where $s = o, r$

- Subject to their budget constraint

$$C_t^r = W_t L_t^r$$

Rule of thumb FOCs

- The only FOC of these households is with respect to labour hours
- If labour markets are competitive and they decide optimally their labour hours

$$W_t = C_t^r (L_t^r)^{\phi}$$

- Due to this utility function, hours are constant for these agents:

$$L_t^r = \omega^{-\frac{1}{1+\phi}}$$

- Consumption tracks the real wage to exhaust the budget constraint

Firms and Market Clearing

- Firms structure follows exactly Galí *et al.* (2007) but without capital, $Y_t = AL_t - F$, where F is a fixed cost
- Here, no government spending neither capital, therefore $Y_t = C_t$
- Facilitates analytical results
- Equity market clearing implies that share holdings of each asset holder are

$$\Omega = \frac{1}{1-\lambda}$$

$$C_t = \lambda C_t^r + (1-\lambda) C_t^o$$

$$L_t = \lambda L_t^r + (1-\lambda) L_t^o$$

Steady State and Linearization

- Assumes a fixed cost in production such as profits in steady state (D) is zero
- Given that both types have the same preferences, this implies that hours and consumption are the same in steady state
- Remember

$$C_t^o + B_{t+1}^o + V_t\Omega_{t+1} = W_t L_t + R_t B_t^o + \Omega_t(V_t + D_t)$$

- Under $D = 0$, and $B_t = 0$ optimizers budget constraint becomes

$$C^o = WL^o$$

- Same thing for the constrained agents

$$C^r = WL^r$$

Steady State and Linearization

- The intratemporal optimality conditions evaluated at steady-state imply that for $j \in (o, r)$

$$\omega(L_t^j)^{\phi} = \frac{1}{C_t^j} W_t$$

- Solving for hours and consumption, we find that these will be equal across groups

$$L^o = L^r = \omega^{-\frac{1}{1+\phi}}$$

$$C^o = C^r = \omega^{-\frac{1}{1+\phi}} W$$

- Also $C^o = C^r = C = Y$

- Let a small-case letter denote the log-deviation of a variable from its steady-state value, $y_t = \log(\frac{Y_t}{Y})$

Aggregate Dynamics

- To derive the Inverted Aggregate Demand Logic result we need to have an analytical expression for the aggregate Euler equation or "IS curve"
- Express consumption of savers in in terms of aggregate consumption/output
- Note that since hours of non-asset holders are constant $l_t^r = 0$, their consumption tracks real wage, $c_t^r = w_t$
- Total labour supply is $l_t = (1 - \lambda) l_t^o$

Inverted Aggregate Demand Logic

- Using these, asset holders labour supply equation, the production function and the goods market clearing condition into the definition of total consumption, and the savers' Euler equation we find the aggregate Euler equation or "IS curve"

$$y_t = \mathbb{E}_t\, y_{t+1} - \delta^{-1}[r_t - \mathbb{E}_t\, \pi_{t+1}] + (1+\mu)(1-\delta^{-1})[a_t - a_{t+1}]$$

  where

$$\delta = 1 - \phi \frac{\lambda}{1-\lambda} \frac{1}{1+\mu}$$

- Due to LAMP, δ is modifying the elasticity of aggregate demand to real interest rates

- Note that in RANK $\delta = 1$ because $\lambda = 0$

Inverted Aggregate Demand Logic

- Solving $\delta = 0$, there exists a threshold value of the share of non-asset holders beyond which the parameter δ changes sign

$$\lambda^* = \frac{1}{1 + \phi/(1+\mu)}$$

- For high enough participation rates, $\lambda < \lambda^*$, δ is positive and we are in the standard aggregate demand logic, positive change in the policy rate, leads to reduction in output

- As $\lambda \to \lambda^*$ the sensitivity of aggregate demand to interest rates increases in absolute value, making policy more effective in containing demand

- When $\lambda > \lambda^*$ we move to the IADL region where increases in real interest rates become expansionary

Inverted Aggregate Demand Logic

- The IADL case occurs when enough agents consume their wage income w_t (λ_t high) and/or wage is sensitive enough to real income y_t (ϕ high)

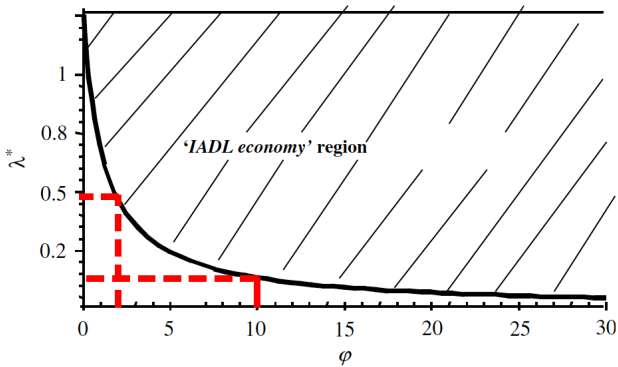


Fig. 1. Threshold share of non-asset holders as a function of inverse labor supply elasticity.

Intuition, $\lambda < \lambda^*$

How can an increase in interest rates become expansionary when asset market participation is restricted enough?

- In the standard, full-participation economy, an increase in interest rates leads to a fall in aggregate demand today
- Asset holders are willing to work more at a given real wage (labour supply shifts rightward)
- Labour demand from firms shifts left because of sticky prices; not all the fall in demand can be accommodated via cutting prices
- The new equilibrium is one with lower output, consumption, hours and real wage

Intuition, $\lambda < \lambda^*$

Introducing rule of thumb agents

- The fall in real wage means a further fall in demand, since non-asset holders consume their wage income
- This generates a further shift in labour demand, so the new equilibrium is one with even lower (compared to the full-participation one) output, consumption, hours and real wage

Intuition, $\lambda > \lambda^*$

- Why this effect is not monotonic over the λ domain?
- The further demand effect that occurs because of non-asset holders has an effect on profits: both marginal cost (wage) and sales (output and hours) fall
- Profits are $d_t = -w_t$, a reduction in real wages lead to an increase in profits occurs which generates a positive income effect on asset holders
- *Remember* asset holders hold $\frac{1}{1-\lambda}$ shares, so as long as λ increases they get more dividends
- This expansionary effect contradicts the initial contractionary effects
- This is an equilibrium whereby consumption, output, hours and the real wage increasehence 'expansionary monetary contractions'

Bilbiie (2019)

- Builds on the 2008 paper and enhances its heterogeneity, still focusing on the analytical insights
- Identifies a key parameter χ: the constrained agents income elasticity to aggregate incomedepends on fiscal redistribution
- When it is larger (smaller) than one, the effects of policies and shocks are amplified (dampened)
- A TANK model version as in the 2008 paper captures some of the key mechanisms of HANK models
- Also identifies the cyclical inequality component in the TANK models

The Model

- The model is essentially the same with the 2008
- Difference with the 2008 model: Hand to mouth consumers receive transfers, a function of the savers' payoffs from the firms
- Hand to mouth budget constraint is

$$C_t^r = W_t L_t^r + Transfer_t^r$$

where $Transfer_t^r$ are fiscal transfers

Fiscal Transfers

- The government conducts fiscal policy, which consists of a simple endogenous redistribution scheme
- Taxing profits at rate τ_D and rebating the proceedings lump-sum to the hand to mouth consumers

$$Transfer_t^r = \frac{\tau_D}{\lambda} D_t$$

- Remember that in the 2008 paper, Optimizers receive all profits, here this is a redistribution mechanism

Dynamics

- The hand-to-mouth consume all their income, $c_t^r = y_t^r$
- While their consumption co-moves one-to-one with their income, it co-moves more or less than one-to-one with aggregate income y_t
- This is the model's key contribution, extension from the 2008 version
- Start from the hand to mouth loglinearized budget constraint:

$$c_t^r = w_t + l_t^r + \frac{\tau_D}{\lambda} d_t$$

- Substituting $w_t = (\phi + \sigma^{-1}) c_t$ and $d_t = -w_t$
- We obtain

$$c_t^r = \chi y_t$$

where $\chi = 1 + \phi(1 - \frac{\tau_D}{\lambda})$

The parameter χ

- Denotes the elasticity of r households consumption (and income) to aggregate income
- In the 2008 paper, $\chi = 1 + \phi > 1$, a constant. Here depending on the profits share
- It depends on fiscal redistribution and labour market characteristics (ϕ), and determines the amplification properties of monetary and fiscal policy and shocks
- Using the savers log-linearized income, we have: $y_t^o = w_t + n_t^o + \frac{1-\tau_D}{1-\lambda} d_t$
- Replacing $d_t = -w_t$:

$$c_t^o = \frac{1 - \lambda\chi}{1 - \lambda} y_t$$

- Whenever ones income elasticity to aggregate income is larger than 1, the others is lower than 1

The cyclical inequality channel

- When $\chi > 1$, corresponds countercyclical income inequality, i.e. the poor get poorer in recession
- When $\chi < 1$, corresponds procyclical income inequality, i.e. falls in recessions

The cyclical inequality channel

- In RANK the same agents that receives a higher wage due to a shock is the same that loses from a reduction in profits because the wage is marginal cost
- Since the same agent incurs both the labour gain and the profit loss, the distribution of income between the two is neutral
- In TANK when $\tau_D = 0$, when wages go up, the hand to mouth consumer has no negative income effect from a profit drop and increase their demand, leading also to the IADL we saw above
- Introducing $\tau_D > 0$ dampens this channel, profits are shared by the two households types
- HtM start internalizing through the transfer some of the negative income effect of profits and do not increase demand by as much

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