

A Course on DSGE Models with Financial Frictions

Part 2: Simple RBC & Dynare Introduction

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A simple Business Cycle Model

- There is a representative household
- Chooses optimally consumption and labour subject to its budget constraint
- Firms produce output according to a production technology and choose labour and capital inputs to minimize cost
- Labour, capital and output markets clear

Summary

- Households choose hours worked (H_t) and consumption (C_t) to maximize their utility
- Their utility is:

$$U = U(C_t, L_t)$$

where C_t is consumption and all variables are expressed in real terms relative to the price of retail output. We assume that

$$U_C > 0, U_L > 0 \quad U_{CC} \leq 0, U_{LL} \leq 0 \quad (1)$$

- In a stochastic environment, the **value function** of the representative household at time t is given by

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}, L_{t+s}) \right] ; \beta \in (0, 1) \quad (2)$$

The Household Optimization Problem

- Household chooses $\{C_t\}$, leisure, $\{L_t\}$, labour supply $\{H_t = 1 - L_t\}$, capital stock $\{K_t\}$ and investment $\{I_t\}$ to maximize V_t given by (2) given the budget constraint:

$$B_t = R_{t-1}B_{t-1} + r_t^K K_{t-1} + W_t H_t - C_t - I_t - T_t \quad (3)$$

- B_t is the value of the stock of one-period bonds (price \times number of bonds) at the end of period t .
- r_t^K is the rental rate for capital, W_t is the wage rate and R_{t-1} is the interest rate set in period $t - 1$ paid in period t on bonds held at the end of period $t - 1$. Note K_t is *end-of-period* capital stock.
- Capital stock accumulates according to

$$K_t = (1 - \delta)K_{t-1} + I_t$$

Solution to the Household Optimization Problem

First order conditions are

$$\text{Euler Consumption} \quad : \quad R_t \mathbb{E}_t [\Lambda_{t,t+1}] = 1$$

$$\text{Labour Supply} \quad : \quad \frac{U_{H,t}}{U_{C,t}} = -W_t$$

$$\text{Capital Supply} \quad : \quad \mathbb{E}_t [\Lambda_{t,t+1} R_{t+1}^K] = 1$$

where the gross return on capital is given by

$$R_t^K = [r_t^K + (1 - \delta)]$$

and $\Lambda_{t,t+1} \equiv \beta \frac{U_{C,t+1}}{U_{C,t}}$ is the *real stochastic discount factor* $[t, t + 1]$.

Solution to the Household Optimization Problem

- Then we have the arbitrage condition

$$1 = R_t \mathbb{E}_t [\Lambda_{t,t+1}] = \mathbb{E}_t [\Lambda_{t,t+1} R_{t+1}^K]$$

- In our financial frictions models $R_t \mathbb{E}_t [\Lambda_{t,t+1}] \neq \mathbb{E}_t [\Lambda_{t,t+1} R_{t+1}^K]$

Production Side and Closing the Model

- Output and the firm's behaviour is summarized by:

$$\begin{aligned}\text{Output} & : Y_t = A_t K_t^\alpha H_t^{1-\alpha} \\ \text{Labour Demand} & : \frac{\alpha Y_t}{H_t} = W_t \\ \text{Capital Demand} & : \frac{(1-\alpha)Y_t}{K_{t-1}} = r_t^K\end{aligned}$$

- The model is completed with an output equilibrium and a balanced budget constraint with lump-sum taxes (T_t).

$$\begin{aligned}Y_t &= C_t + G_t + I_t \\ G_t &= T_t\end{aligned}$$

where G_t is government spending and A_t follows an AR(1) process:

$$\ln A_t - \ln \bar{A}_t = \rho_A (\ln A_{t-1} - \ln \bar{A}_{t-1}) + \epsilon_{A,t}$$

The Steady State

- We assume a **zero-growth** steady state and a CRRA utility $U = \ln C_t - \omega \frac{H_t^{1+\phi}}{1+\phi}$ where $\omega > 0$ indicates how leisure is valued relative to consumption, and $\phi > 0$ is the inverse of the labour supply elasticity
- $\bar{A}_t = \bar{A}_{t-1} = A$, say and $\bar{G}_t = \bar{G}_{t-1} = G$. $K_t = K_{t-1} = K$ etc
- The zero-growth steady state in recursive form is given by:

$$\begin{aligned} R &= \frac{1}{\beta} \\ \frac{K}{Y} &= \frac{(1-\alpha)}{R-1+\delta} \\ \frac{I}{Y} &= \frac{\delta K}{Y} = \frac{(1-\alpha)\delta}{R-1+\delta} = \frac{(1-\alpha)\delta}{R-1+\delta} \\ \frac{C}{Y} &= 1 - \frac{I}{Y} - \frac{G}{Y} = 1 - \frac{I}{Y} - g_y \end{aligned}$$

Zero Growth Steady State in Recursive Form (cont)

$$H = \left(\frac{\alpha}{C/Y} \frac{1}{\omega} \right)^{\frac{1}{1+\phi}}$$

$$Y = (AH)^{\alpha} K^{1-\alpha} = (AH)^{\alpha} \left(\frac{K}{Y} \right)^{1-\alpha} (Y)^{1-\alpha}$$

$$\Rightarrow Y = (AH)(K/Y)^{\frac{1-\alpha}{\alpha}}$$

$$G = g_y Y = T$$

$$W = \alpha \frac{Y}{H}$$

$$I = \frac{I}{Y} Y; \quad C = \frac{C}{Y} Y; \quad K = \frac{K}{Y} Y$$

$$A = 1$$

Solve the Model with Dynare

- Dynare uses a technique called **perturbation**. For more : [Judd \(1998\)](#)
- Computes first, second and third order Taylor series approximation of the policy rules *around the steady state*

It also:

- Computes the steady state (numerically) of the model
- Computes the solution of deterministic models
- Estimates (either by maximum-likelihood(ML) or Bayesian approach) parameters of DSGE models and their distribution

Dynare Starters

- It is a big collection of Matlab functions that use Matlab in order to solve the model with perturbation
- We just have to set the external path of Matlab to the Dynare folder
- Download it at <https://www.dynare.org>
- Install it. Go to Matlab on menu File/Set Path to add the path to the Dynare subdirectory (to store all the subroutines), e.g. the path would be set to *c : \dynare\4.4.y\matlab*

Dynare Model [.mod] file

- The .mod file is the file where you write down your DSGE model
- It includes several blocks
 - Variable block
 - Parameter block
 - Parameter values block
 - Model block
 - Steady state block
 - Shocks block
 - Solution (or estimation) block

Super useful reading: [Adjemian *et al.* \(2011\)](#)

Variables and Parameters Block

- **var** block: Names of the endogenous variables
example:
`var K C G A;`
- **varexo** block: Names of the shocks
example:
`varexo epsA epsG;`
- **parameters** block: Names of the parameters ; Values of the parameters
example:
`parameters alpha beta delta ;`
`alpha=0.3;`
`beta=0.99;`
`delta=0.025;`

Model Block

- Starts with `model`; and ends with `end`;
- Type equations ending with ;
 - `x(-1)` for predetermined variables. The variable is decided in $t - 1$ (predetermined), e.g. the capital stock, write it as `x(-1)` instead of `x`
 - `x(+1)` for expectations

example:

```
K = (1-delta)*K(-1)+I;
```

Shocks Block

- Starts with `shocks;` and ends with `end;`
- In between declare shock standard deviations
example:
`shocks;`
`var epsA;`
`stderr 0.02;`
`end;`
- The variances (and covariances) of the shocks are defined within these commands
- Sets the std. error of this exogenous variable = 0.02

Some info

- **Note** that each instruction of the .mod file must be terminated by a semicolon
- Also Dynare uses 2 forward slashes (//) to comment out any line (whereas MATLAB uses %). (Note: for Dynare the two are equivalent!)
- There need to be as many equations as your endogenous variables declared (except for optimal policy)
- Names are case sensitive
- The stability “Blanchard-Kahn” conditions are met only if the number of jumpers equals the number of eigenvalues greater than one. (See Topic 2).

The Steady State Block

- It's the most difficult and time consuming part
- There are two options
 - Let Dynare calculate the steady state (sounds good, does not always work)
 - Calculate it ourselves and then add this as a Matlab function

The Steady State Block: Option 1

- Dynare solves for the steady state of the model, it just need (good!) initial values
- Starts with `initval`; and ends with `end`;
- In between, add initial values for all variables
- Initial values can be exact numbers or functions that depend on parameters or steady state variables
- Then, *steady* command computes the steady state
- If the model is quite complicated and the initial values not close to the truth there will be problems → **Option 2**

The Steady State Block: Option 2

- Find the analytical solution for the steady state
- Import it to a Matlab function doing the computation externally with a Matlab program `FILENAME_steadystate.m`
- Dynare understands that this function gives the steady state of the model
- Needs a specific preamble and ending that is provided in these files

Solution Block

- `stoch_simul` starts the solution routine for stochastic models and `simul` for deterministic simulations
example
`stoch simul(order=1,IRF=20, periods =10000) ;`
- There are many options for the stochastic simulation (see Dynare manual for more)
- *periods* - specifies the number of simulation periods
- *irf* sets the number of periods for which to compute impulse responses
- *order* = 1 sets the order of the Taylor approximation (default is two)

Solve your Model

- Just type in Matlab `dynare modfilename.mod`
- Dynare output is (among many others):
 - Policy rules
 - Moments
 - Impulse response functions
- Almost everything is in the folder `oo_`. You will find it in Matlab's workspace right after the solution takes place

Exercise - Introducing Investment Adjustment Costs

- Same problem as above BUT now we have cost $\Phi(\Xi_t)$ for any investment adjustment

$$\begin{aligned}K_t &= (1 - \delta)K_{t-1} + (1 - \Phi(\Xi_t))I_t \\ \Xi_t &\equiv \frac{I_t}{I_{t-1}}; \quad S', S'' \geq 0; \quad \Phi(1) = \Phi'(1) = 0\end{aligned}$$

- I_t units of output converts to $(1 - \Phi(\Xi_t))I_t$ of new capital sold at a real price Q_t
- We now have two constraints and two Lagrange multipliers: λ_t & μ_t .
- Express Q_t as μ_t/λ_t : the marginal value of capital measured in terms of consumption goods (this is Tobin's Q) and express the FOC to Investment in terms of Q_t .

Exercise - Introducing Investment Adjustment Costs

- For $\Phi(\Xi_t) = \phi_X(\Xi_t - 1)^2$ find the new equilibrium and solve the model in Dynare
- Do it first by setting $\phi_X = 2$
- Construct a `[0:4]` vector with `step=1` grid for ϕ_X and show the different impulse responses for every different value of ϕ_X after a positive TFP shock
- Important! At each iteration set `dynare modfile.mod noclearall`. Dynare does not clear up the workspace after each iteration

Exercise - Introducing Investment Adjustment Costs

What you need In Matlab

% Give to Phix the value of every iteration

```
phiX = Phix_value(i);
```

% Send the parameter value to Dynare Phix_value

```
save Phix_value phiX
```

% Run the mod file every time with noclearall option

```
dynare RBC1_inv_adj noclearall
```

In the parameter section of Dynare

```
load Phix_value;
```

```
set_param_value('phiX',phiX);
```


Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F.,
Mutschler, W., Perendia, G., Pfeifer, J., Ratto, M., and Villemot, S. (2011).
Dynare: Reference manual version 4. Dynare Working Papers 1, CEPREMAP.
Judd, K. L. (1998). *Numerical methods in economics*. MIT press.