A Course on DSGE Models with Financial Frictions Part 3: Asymmetric Information

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From Complete Markets to Financial Frictions

- To introduce financial frictions we need two deviations from the complete markets representative agent setting
 - Incomplete markets
 - Heterogeneous Agents
- A representative agent environment is in a sense an infinite agents setting where they can trade as much as necessary to eliminate idiosyncratic risk
- An example of market incompleteness:
- The Kiyotaki-Moore style collateral constraint $b_t \leq \theta k_t$

Financial Frictions: The BGG Model

- The two main streams of introducing financial frictions in DSGE models are
 - Asymmetric information
 - Limited commitment
- In this lecture we will go through one of the most seminal papers in the literature the Bernanke et al. (1999)
- A canonical NK model with a friction between the banks and the firms (called entrepreneurs)
- Eliminates $\mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}^K]$ of the RBC model and introduces a spread between the two discounted expected returns

Summary of the Model

- Main agents in the model: Households, entrepreneurs, retailers, capital producers and the government
- Households work, consume and save in deposits
- Entrepreneurs are risk neutral. They purchase physical capital from capital goods producers and produce goods combining capital and labour
- Finance capital purchases with their net worth and bank loans
- Retailers are present to introduce nominal rigidities without complicating the entrepreneurs problem
 - Production side: NK framework with monopolistic competition and sticky prices
 - We will not go through that today

Entrepreneurs: Overview

- The entrepreneur j borrows loans B_t from the financial intermediary at time t to purchase capital K_t at price Q_t
- Loans together with her net worth $N_{E,t}$ finance the expenditure on new capital Q_tK_t .
- Entrepreneur's balance sheet therefore is:

$$B_t = Q_t K_t - N_{E,t} \tag{1}$$

Entrepreneurs: The Friction

- At every t the entrepreneur receives an idiosyncratic shock ω_t
- Results in return on capital: $\omega_t R_t^K$
 - ω_t is iid across all entrepreneurs
 - Drawn from a density $f(\omega_t)$ with mean $E(\omega_t)=1$
- ω_t^j is only observable by entrepreneur j
- Costly State Verification (CSV) Townsend (1979): Lenders to observe the state, need to pay a monitoring cost, a proportion μ of the gross return $\mu R_{t+1}^K Q_t K_t$

Entrepreneurs: No Aggregate Risk

- We start with the case of no aggregate risk
 - R_{t+1}^{K} is known in advance and only uncertainty is idiosyncratic to the firm
 - The optimal contract is a risky debt contract (see Appendix A of Bernanke *et al.* (1999) for more)
- Entrepreneurs choose B_t and Q_tK_t given R_t^K
- The optimal contract can be characterized by a gross non-default loan rate Z_t and a threshold value for the idiosyncratic shock $\bar{\omega_t}$

$$\bar{\omega}_{t+1} = \frac{Z_t B_t}{R_{t+1}^K Q_t k_t} \tag{2}$$

Entrepreneurs: Optimal Contract

- Entrepreneurs who receive a $\omega_t \geq \bar{\omega_t}$
 - Repay the intermediary Z_tB_t
 - Keep the difference $\bar{\omega}_{t+1} R_{t+1}^K Q_t k_t Z_t B_t$
- Entrepreneurs who receive a $\omega_t < \bar{\omega_t}$
 - Default and get nothing
 - Intermediary pays the monitoring costs and gets what it finds: $(1 \mu)R_{t+1}^K Q_t K_t$

Entrepreneurs: Optimal Contract

- Under the optimal contract the lender should receive an expected return equal to its opportunity cost of funds
- Here, our bank's balance sheet is $B_t = D_t$ where D_t (deposits by households) is remunerated at the riskless rate R_t
- The loan contract must satisfy the bank's incentive compatibility constraint at time t

$$(1 - \mu)R_{t+1}^K Q_t K_t \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + (1 - p(\bar{\omega}_{t+1})) Z_t B_t$$
Default
$$= R_t B_t$$

where $p(\bar{\omega}_t)$ is the default probability

Entrepreneurs: Optimal Contract

• Using the $Z_tB_t = \bar{\omega}_{t+1}R_{t+1}^KQ_tk_t$, the constraint becomes

$$R_{t+1}^K Q_t K_t \left((1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} (1 - p(\bar{\omega}_{t+1})) \right) \ge R_{t+1} B_t$$
 (3)

- To simplify notation a bit let $\Gamma(\bar{\omega}_{t+1})$ to be the fraction of net capital received by the lender (the bank)
- $\mu G(\bar{\omega}_{t+1})$ to be monitoring costs:

$$\Gamma(\bar{\omega}_{t+1}) \equiv \int_{0}^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} (1 - p(\bar{\omega}_{t+1})) \tag{4}$$

$$G(\bar{\omega}_{t+1}) \equiv \int_{0}^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega \tag{5}$$

Entrepreneurs: Introducing Aggregate Risk

- So far we were abstracting from aggregate risk
- Now price and return of capital are given but uncertain
- $\bar{\omega}_{t+1}$ will now generally depend on the ex-post realization of R_{t+1}^K
- Entrepreneur is willing to offer a return that is free of any aggregate risk
 - Conditional on the return R_{t+1}^K the borrower offers state contingent non-default payment guaranteeing an expected return equal to the riskless rate
 - IC now implies a set of contracts for each realization of R_{t+1}^K

Entrepreneurs: Net Worth and Optimal Choice of Capital

• The entrepreneur's payoff is

$$\mathbb{E}_{t} \left[R_{t+1}^{K} Q_{t} K_{t} \int_{\bar{\omega}_{t+1}}^{\infty} \omega f(\omega) d\omega - (1 - p(\bar{\omega}_{t+1})) Z_{t} B_{t} \right] =$$

$$\mathbb{E}_{t} \left[\left(1 - \Gamma(\bar{\psi}_{t+1}) \right) R_{t+1}^{K} Q_{t} K_{t} \right]$$
(6)

• She maximizes (6) given the net worth $N_{E,t}$, subject to the incentive compatibility constraint (3) which, using (1), (4) and (5), can be rewritten as

$$\mathbb{E}_{t} \left[R_{t+1}^{K} Q_{t} K_{t} \left[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \right] \ge R_{t+1} (Q_{t} K_{t} - N_{E,t}) \right]$$

Entrepreneurs: Solution

• Let λ_t be the Lagrange multiplier associated with the IC constraint. Then the first order conditions are

$$K_{t} : \mathbb{E}_{t} \left[(1 - \Gamma(\bar{\omega}_{t+1}) R_{t+1}^{K} + \lambda_{t} \left[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})) R_{t+1}^{K} - R_{t+1} \right] \right] = 0$$

$$\bar{\omega}_{t+1} : \mathbb{E}_{t} \left[\Gamma'(\bar{\omega}_{t+1}) + \lambda_{t} (\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})) \right] = 0$$

plus the binding IC condition if $\lambda_t > 0$ with $\lambda_t = 0$ if it does not bind.

• Combining these two conditions, we arrive at

$$\mathbb{E}_t[R_{t+1}^K] = \mathbb{E}_t[\rho(\bar{\omega}_{t+1})R_{t+1}]$$

where the premium on external finance, $\rho(\bar{\omega}_{t+1})$ is given by

$$\rho(\bar{\omega}_{t+1}) = \frac{\Gamma'(\bar{\omega}_{t+1})}{[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))\Gamma'(\bar{\omega}_{t+1}) + (1 - \Gamma(\bar{\omega}_{t+1}))(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))]}$$

External Finance Premium

- The external finance premium equation replaces the no arbitrage condition between R^K and R in NK and RBC models
- External finance is given by the safe rate, scaled up by a premium term
- Introduces the spread $R_t^K R_t$
- Note that in the limiting case of $\bar{\omega}_{t+1} = 0$ and the probability of default tend to zero and $\mu = 0$, $\Gamma \to 0$ and the risk premium $\rho(\bar{\omega}_{t+1}) \to 1$ going back to the frictionless RBC

Aggregation

- To set a limit in net worth accumulation, entrepreneurs exit with a fixed probability $(1 \sigma_E)$
- New entrants get a start-up transfer equal to ξ_E of the old entrepreneurs wealth
- Aggregate net worth is

$$N_{E,t} = (\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1} K_{t-1}$$

• Entrepreneur who exit consume

$$C_{E,t} = (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1} K_{t-1}$$

• The economy's resource constraint then is

$$Y_t = C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t) R_t^K Q_{t-1} K_{t-1}$$

General Equilibrium

- Households work, consume, save and pay taxes
 - Introduce habit in consumption

$$E_{t} \sum_{i=0}^{\infty} \beta^{i} \left[\ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} L_{t+i}^{1+\epsilon} \right]$$
 (7)

• Their budget constraint is:

$$C_t + D_t + T_t = W_t L_t + R_t D_{t-1}$$

• Banks are very simple, get deposits D_t from the households and provide loans B_t to the entrepreneurs at a rate Z_t

$$B_t = D_t$$

- Government has a balanced budget: $G_t = T_t$
- Production side in the BGG is NK
- Here we will just follow the RBC model structure on the real sector

General Equilibrium

• Capital accumulation with investment adjustment costs carried out by Capital Producers

$$K_t = (1 - \delta)K_{t-1} + (1 - S(X_t))I_t$$

$$X_t \equiv \frac{I_t}{I_{t-1}}$$

$$S(X_t) = \phi_X(X_t - 1)^2$$

$$S'(X_t) = 2\phi_X(X_t - 1)$$

$$Q_t(1 - S(X_t) - X_tS'(X_t)) + \mathbb{E}_t \left[\Lambda_{t,t+1}Q_{t+1}S'(X_{t+1})X_{t+1}^2\right] = 1$$

• $S(X_t)$ are investment adjustment costs equal to zero in a balance growth steady state

Summary of Equilibrium

$$Y_{t} = C_{t} + C_{E,t} + G_{t} + I_{t} + \mu G(\bar{\psi}_{t})) R_{t}^{K} Q_{t-1} K_{t-1}$$

$$C_{E,t} = (1 - \sigma_{E}) (1 - \xi_{E}) (1 - \Gamma(\bar{\psi}_{t})) R_{t}^{K} Q_{t-1} K_{t-1}$$

$$B_{t} = Q_{t}K_{t} - N_{E,t}$$

$$\bar{\omega}_{t} = \frac{Z_{t-1}B_{t-1}}{R_{t}^{K}Q_{t-1}K_{t-1}}$$

$$R_{t}^{K} = \frac{r_{t}^{K} + (1 - \delta)Q_{t}}{Q_{t-1}}$$

$$r_{t}^{K} = \frac{(1 - \alpha)Y_{t}}{K_{t-1}}$$

Steady State Equilibrium

• The zero growth steady state is given by

$$R_k = \rho(\bar{\psi})R$$

$$N_E = (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))R_kQK$$

$$R_kQK\left[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})\right] = R(QK - N_E) \text{ or }$$

$$R_k[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})] = R(\phi - 1)$$

• where $\phi \equiv \frac{QK}{N_E}$ and

$$\rho(\bar{\psi}) = \frac{\Gamma'(\bar{\psi})}{\left[(\Gamma(\bar{\psi}) - \mu G(\bar{\psi}))\Gamma'(\bar{\psi}) + (1 - \Gamma(\bar{\psi}))(\Gamma'(\bar{\psi}) - \mu G'(\bar{\psi})) \right]}$$

Steady State Equilibrium

• with resource constraint

$$Y = C + C_E + G + I + \mu G(\bar{\psi}) R_k QK$$

$$C_E = (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\psi})) R_k QK$$

• and post-recursive equations

$$L = QK - N_E$$

$$R_l = \frac{\bar{\psi}R_kQK}{L}$$

$$R_k = \frac{Z + (1 - \delta)Q}{Q}$$

$$Z = \frac{(1 - \alpha)Y}{K}$$

The Density Function

- BGG choose a log-normal distribution
- Thus $\omega_t = e^{x_t}$ where $x_t \sim N(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$
- This guarantees that $\mathbb{E}[\omega_t] = 1$

- Steady state is not straightforward for this kind of models
- Firstly solve the financial sector problem
- Then go to the real sector and find all the variables as shares of capital

• Find two equations with only unknowns the entrepreneurial leverage (ϕ^E) and the return on capital (R^K) .

$$\phi^E = \frac{QK}{N^E}$$

Rearranging,

$$\frac{N^E}{K} = \frac{1}{\phi^E}$$

• From entrepreneurs net worth N^E , divide with capital and substituting Q=1 we have:

$$\frac{N_E}{K} = (\sigma^E + \xi^E)(1 - \Gamma(\bar{\omega}))R^k$$

• From entrepreneurs balance sheet constraint $L = QK - N^E$, dividing with $K^s s$ we have

$$\frac{N_E}{K} = 1 - \frac{L}{K} = \frac{1}{\phi^E}$$

• Hence and using the fact that $R^K = \rho(\omega)R$,

$$\phi_E = \frac{1}{(\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}))\rho(\omega)R}$$

which is **the first equation** for the system

• From the Zero Profit Condition , solving for ϕ_E we get

$$\phi_E = -\frac{R}{R^K(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) - R}$$

• Substituting again:

$$\phi_E = -\frac{R}{\rho(\bar{\omega})R(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) - R}$$

yields the second equation for the system

- We have 2 equations and 2 unknowns $(\bar{\omega}, \phi_E)$
- Actually we treat ϕ_E as known equal to 2 in SS and we calibrate σ_E to match this leverage

Steady State Calibration

- Production parameters, same as in the simple RBC
- For NK extension see code in GitHub
- Additional financial parameters to calibrate are σ_{ω} , σ_{E} , and μ .
- These three parameters are calibrated to hit four targets:
 - (1) μ such as a default probability $p(\bar{\psi}) = 0.03$,
 - (2) σ_{ω} for $\rho(\bar{\psi}) = 1.0025$ corresponding to a credit spread of 100 basis points as in GK,
 - (3) σ_E for an entrepreneur leverage $\frac{QK}{N_E} = 2$ as in BGG

Inside the Code: Steady State

```
R
            = 1/betta:
Lambda
            = betta;
% Targets
            = 0.03; % steady state default
p_{mom}
            = p_mom;
            = 1 + 0.02/4; % steady state premium
rho_mom
rho
              = rho_mom;
phie_mom
            = 2; % steady state leverage
            = phie_mom;
phie
```

Inside the Code: Steady State

```
% give initial values
   x0=[0.98,0.3, 0.49, 0.008, 0.98, 0.30, 0.2, 0.49];
   fun = @(c) mvfun_BGG_calib(c.p_mom.ksie.rho.phie_mom.R);
   options=optimset('MaxFunEvals'.1000):
   c = fsolve(fun,x0,options);
   sigmae = c(1): %parameter
   sigma_omega_ss = c(2); %parameter
   fnGam = c(3):
   fnG = c(4):
   DGam = c(5):
   DG = c(6):
12
   mon = c(7); %parameter
13
   omega = exp(c(8));
```

Inside the Code: Steady State

```
function F = myfun_BGG_calib(c,p_mom,ksie,rho,phie_mom,R)
   F=[phie\_mom-1/((c(1)+ksie)*(1-c(3))*rho*R): %Ea 1
   phie_mom + (R/((rho*R)*(c(3)-c(7)*c(4))-R)); %Eq 2
   rho -(c(5)/((c(3)-c(7)*c(4))*c(5)+(1-c(3))*(c(5)-c(7)*c(6)))); %rho
   p_{mom} - (logncdf(exp(c(8)), -0.5*(c(2))^2, c(2))); %p
   c(4) - (1- \text{normcdf}((0.5*(c(2))^2 - \log(\exp(c(8))))/c(2),0,1)); %fnG
   c(3) - (c(4) + exp(c(8))*(1-p_mom)); %fnGam
   c(5) - (1-p_mom); %DGam
   c(6) - (1/(exp(c(8))*c(2)*sqrt(2*(4*atan(1))))*exp(-((log(exp(c(8))))
       +.5*(c(2))^2)^2)/(2*(c(2))^2))):1: %DG
10
   end
```

Steady State: Real Sector

• To find Z we go to the only equation that has it:

$$Z = \bar{\omega} R^K Q \frac{K}{L}.$$

From the entrepreneurs leverage we know

$$\frac{K}{L} = \frac{1}{1 - \frac{1}{\phi^E}} = \frac{\phi^E}{\phi^E - 1}$$

$$r_K = R^k + \delta - 1$$

• For the real sector variables: define all variables as share of capital first

$$\frac{C_e^{ss}}{K^{ss}} = (1 - \sigma^e)(1 - \xi^e)(1 - \Gamma(\bar{\psi}^{ss}))R_{k,ss}Q^{ss}$$

Steady State: Real Sector

- The resource constraint is $Y^{ss} = C^{ss} + I^{ss} + G^{ss} + \mu G(\bar{\psi}_t) R_k^{ss} Q^{ss} K^{ss}$
- Using the production function,

$$C^{ss}=L^{1-\alpha}K^\alpha-\delta K^{ss}-\eta L^{1-\alpha}K^\alpha-C_e-\mu G(\bar{\psi}_t)R_k^{ss}Q^{ss}K^{ss}$$
 where $\eta=G^{ss}/Y^{ss}$

• Rearranging terms we get :

$$\frac{C^{ss}}{K^{ss}} = (1 - \eta)(\frac{L^{ss}}{K^{ss}})^{1 - \alpha} - \delta - \frac{C_e}{K^{ss}} - \mu G(\bar{\psi}_t) R_k^{ss} Q^{ss}$$

• From the resource constraint we have

$$\frac{Y^{ss}}{K^{ss}} = \left(\frac{C^{ss}}{K^{ss}} + \delta + \frac{C_e}{K^{ss}} + \mu G(\bar{\psi}_t) R_k^{ss} Q^{ss}\right) \frac{1}{1 - \eta}$$

Steady State: Real Sector

• The labour leisure F.O.C in steady state is

$$(L^{ss})^{\epsilon} \chi = [W^{ss}((1-\gamma)c)^{-1}(1-\beta\gamma)]$$

• Substituting for the wage it yields:

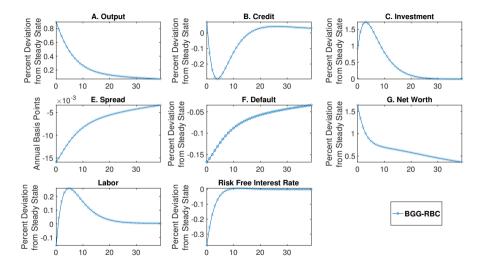
$$(L^{ss})^{1+\epsilon} = \frac{(1-\beta\gamma)(1-\alpha)}{\chi(1-\gamma)} \frac{Y^{ss}}{C^{ss}}$$

- By knowing $\frac{C^{ss}}{K^{ss}}$ and $\frac{Y^{ss}}{K^{ss}}$ is straightforward to calculate $\frac{Y^{ss}}{C^{ss}}$ and find L^{ss} .
- Finally, knowing L^{ss} and $\frac{Y^{ss}}{K^{ss}}$ from the production function we find the capital

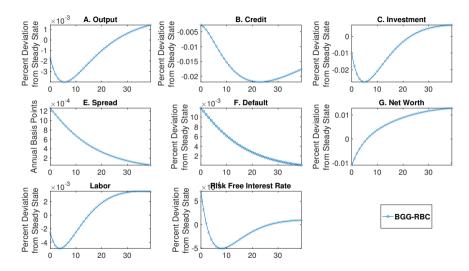
$$K^{ss} = \left(\frac{\frac{Y^{ss}}{K^{ss}}}{L^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}$$

• Hence, having capital, by reverse engineering we can find the values for $(I^{ss}, C^{ss}, C^{ss}_e, Y^{ss})$

Impulse Responses: Productivity Shock (positive)



Impulse Responses: Risk Shock (negative)



Christiano, Motto and Rostagno (2014) extension

- Identify the *Risk Shock*
- Built on the Bernanke et al. (1999)
- Under a NK framework
- Lots of additional frictions and shocks (capacity utilization rate, taxes, sticky wages, long-term bonds...)
- Estimated for the US economy

The Risk Shock

- They refer to as risk the time period t cross-sectional standard deviation of $\log(\omega)$
- Risk is high in periods when $\sigma_{\omega,t}$ is high, and there is substantial dispersion in the outcomes across entrepreneurs
- Model estimation assigns a large role to $\sigma_{\omega,t}$
- Disturbances in $\sigma_{\omega,t}$ trigger responses in the model that resemble actual business cycles
- They find that fluctuations in $\sigma_{\omega,t}$ account for 60 percent of the fluctuations in the growth rate of aggregate US output since the mid-1980s

The Risk Shock

- Essentially from being a parameter $\sigma_{\omega,t}$ now follows an AR(1) process log(sigma_omega / sigma_omega_ss) = rhosigma * log(sigma_omega(-1) / sigma_omega_ss) + e_RS;
- Where e RS is the risk shock

Unanticipated and News Shocks

- A standard assumption in estimated equilibrium models is that a shock's statistical innovation becomes known to agents only when the innovation is realized
- Literature document that people receive information about the period t statistical innovation before the innovation is realized

Unanticipated and News Shocks

• CMR (2014) consider the following shock representation

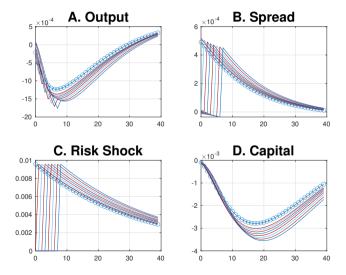
$$x_t = \rho_x x_{t-1} + \xi_0 + \xi_{1,t-1} + \dots + \xi_{p,t-p}$$

- x_t is the log deviation of the shock from its non-stochastic steady state
- ξ_0 is the unanticipated component of the statistical innovation and $\xi_{1,t-1} + ... + \xi_{p,t-p}$ the anticipated, or news components
- They estimate the model for giving this structure to technology, monetary policy, government spending, equity, and risk shocks
- They find that the model that has the highest marginal likelihood is the one with news on the risk shock

Unanticipated and News Shocks

- This formulation is only in the BBGsticky.mod file
- For the flexible price economy there is only the unanticipated component
- Have a look at the code to get familiar with this
- To turn of the anticipated component of the risk shocks <code>@#define stopsignal= 0</code> at the top of the code

Risk Shock (Unanticipated + Anticipated Components)



Homework

- Work with BGGflexi.mod and BBGsticky.mod
- Provide the impulse responses of both models in the same graph for output, consumption, investment, spread, loans and entrepreneurs' new worth
- For the case of a positive productivity shock and a risk shock (only the unanticipated component)
- Make sure that they have the same std (1) and AR(1) parameters: $\rho_A=0.75, \rho_\sigma=0.97$
- File BGGplots_lecture.m is the file I used for the lecture plots, it will help!

- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, **1**, 1341–1393.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, **21**(2), 265–293.