A Course on DSGE Models with Financial Frictions
Part 3: Asymmetric Information

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From Complete Markets to Financial Frictions

- To introduce financial frictions we need two deviations from the complete markets representative agent setting
    - Incomplete markets
    - Heterogeneous Agents

- A representative agent environment is in a sense an infinite agents setting where they can trade as much as necessary to eliminate idiosyncratic risk

- An example of market incompleteness:

- The Kiyotaki-Moore style collateral constraint $b_t \leq \theta k_t$

Financial Frictions: The BGG Model

- The two main streams of introducing financial frictions in DSGE models are
  - Asymmetric information
  - Limited commitment
- In this lecture we will go through one of the most seminal papers in the literature the Bernanke *et al.* (1999)
- A canonical NK model with a friction between the banks and the firms (called entrepreneurs)
- Eliminates $\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1} R^K_{t+1}]$ of the RBC model and introduces a spread between the two discounted expected returns

Summary of the Model

- Main agents in the model: Households, entrepreneurs, retailers, capital producers and the government
- Households work, consume and save in deposits
- Entrepreneurs are risk neutral. They purchase physical capital from capital goods producers and produce goods combining capital and labour
- Finance capital purchases with their net worth and bank loans
- Retailers are present to introduce nominal rigidities without complicating the entrepreneurs problem
    - Production side: NK framework with monopolistic competition and sticky prices
    - We will not go through that today

Entrepreneurs: Overview

- The entrepreneur j borrows loans B_t from the financial intermediary at time t to purchase capital K_t at price Q_t
- Loans together with her net worth $N_{E,t}$ finance the expenditure on new capital $Q_t K_t$.
- Entrepreneur's balance sheet therefore is:

$$B_t = Q_t K_t - N_{E,t} \tag{1}$$

Entrepreneurs: The Friction

- At every t the entrepreneur receives an idiosyncratic shock ω_t
- Results in return on capital: $\omega_t R_t^K$
  - ω_t is iid across all entrepreneurs
  - Drawn from a density f(ω_t) with mean E(ω_t)=1
- ω_t^j is only observable by entrepreneur j
- Costly State Verification (CSV) Townsend (1979): Lenders to observe the state, need to pay a monitoring cost, a proportion μ of the gross return $\mu R_{t+1}^K Q_t K_t$

Entrepreneurs: No Aggregate Risk

- We start with the case of no aggregate risk
  - R^K_{t+1} is known in advance and only uncertainty is idiosyncratic to the firm
  - The optimal contract is a risky debt contract (see Appendix A of Bernanke *et al.* (1999) for more)

- Entrepreneurs choose B_t and $Q_t K_t$ given R^K_t

- The optimal contract can be characterized by a gross non-default loan rate Z_t and a threshold value for the idiosyncratic shock $\bar{\omega}_t$

$$\bar{\omega}_{t+1} = \frac{Z_t B_t}{R^K_{t+1} Q_t k_t} \tag{2}$$

Entrepreneurs: Optimal Contract

- Entrepreneurs who receive a $\omega_t \geq \bar{\omega}_t$
  - Repay the intermediary $Z_t B_t$
  - Keep the difference $\bar{\omega}_{t+1} R^K_{t+1} Q_t k_t - Z_t B_t$
- Entrepreneurs who receive a $\omega_t < \bar{\omega}_t$
  - Default and get nothing
  - Intermediary pays the monitoring costs and gets what it finds: $(1-\mu) R^K_{t+1} Q_t K_t$

Entrepreneurs: Optimal Contract

- Under the optimal contract the lender should receive an expected return equal to its opportunity cost of funds
- Here, our bank's balance sheet is $B_t = D_t$ where D_t (deposits by households) is remunerated at the riskless rate R_t
- The loan contract must satisfy the bank's incentive compatibility constraint at time t

$$\underbrace{(1-\mu)R^K_{t+1}Q_tK_t\int_0^{\bar{\omega}_{t+1}}\omega f(\omega)d\omega}_{\text{Default}} + \overbrace{(1-p(\bar{\omega}_{t+1}))Z_tB_t}^{\text{No default}}$$

$$= R_tB_t$$

where $p(\bar{\omega}_t$ is the default probability

Entrepreneurs: Optimal Contract

- Using the $Z_t B_t = \bar{\omega}_{t+1} R^K_{t+1} Q_t k_t$, the constraint becomes

$$R^K_{t+1} Q_t K_t \left((1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1}(1 - p(\bar{\omega}_{t+1})) \right) \geq R_{t+1} B_t \tag{3}$$

- To simplify notation a bit let $\Gamma(\bar{\omega}_{t+1})$ to be the fraction of net capital received by the lender (the bank)

- $\mu G(\bar{\omega}_{t+1})$ to be monitoring costs:

$$\begin{aligned} \Gamma(\bar{\omega}_{t+1}) &\equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1}(1 - p(\bar{\omega}_{t+1})) && (4) \\ G(\bar{\omega}_{t+1}) &\equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega && (5) \end{aligned}$$

Entrepreneurs: Introducing Aggregate Risk

- So far we were abstracting from aggregate risk
- Now price and return of capital are given but uncertain
- $\bar{\omega}_{t+1}$ will now generally depend on the ex-post realization of R^K_{t+1}
- Entrepreneur is willing to offer a return that is free of any aggregate risk
  - Conditional on the return R^K_{t+1} the borrower offers state contingent non-default payment guaranteeing an expected return equal to the riskless rate
  - IC now implies a set of contracts for each realization of R^K_{t+1}

Entrepreneurs: Net Worth and Optimal Choice of Capital

- The entrepreneur's payoff is

$$\mathbb{E}_t\left[R^K_{t+1}Q_tK_t\int_{\bar{\omega}_{t+1}}^{\infty}\omega f(\omega)d\omega-(1-p(\bar{\omega}_{t+1}))Z_tB_t\right]=$$

$$\mathbb{E}_t\left[\left(1-\Gamma(\bar{\psi}_{t+1})\right)R^K_{t+1}Q_tK_t\right] \tag{6}$$

- She maximizes (6) given the net worth $N_{E,t}$, subject to the incentive compatibility constraint (3) which, using (1), (4) and (5), can be rewritten as

$$\mathbb{E}_t\left[R^K_{t+1}Q_tK_t\left[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})\right]\geq R_{t+1}(Q_tK_t-N_{E,t})\right]$$

Entrepreneurs: Solution

- Let λ_t be the Lagrange multiplier associated with the IC constraint. Then the first order conditions are

$$\begin{aligned} K_t &: \quad \mathbb{E}_t\left[(1-\Gamma(\bar{\omega}_{t+1})R^K_{t+1} + \lambda_t\left[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))R^K_{t+1} - R_{t+1}\right]\right] = 0 \\ \bar{\omega}_{t+1} &: \quad \mathbb{E}_t\left[\Gamma'(\bar{\omega}_{t+1}) + \lambda_t(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))\right] = 0 \end{aligned}$$

  plus the binding IC condition if $\lambda_t > 0$ with $\lambda_t = 0$ if it does not bind.

- Combining these two conditions, we arrive at

$$\mathbb{E}_t[R^K_{t+1}] = \mathbb{E}_t[\rho(\bar{\omega}_{t+1})R_{t+1}]$$

  where the premium on external finance, $\rho(\bar{\omega}_{t+1})$ is given by

$$\rho(\bar{\omega}_{t+1}) = \frac{\Gamma'(\bar{\omega}_{t+1})}{[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))\Gamma'(\bar{\omega}_{t+1}) + (1-\Gamma(\bar{\omega}_{t+1}))(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))]}$$

External Finance Premium

- The external finance premium equation replaces the no arbitrage condition between R^K and R in NK and RBC models
- External finance is given by the safe rate, scaled up by a premium term
- Introduces the spread $R_t^K - R_t$
- Note that in the limiting case of $\bar{\omega}_{t+1}$ =0 and the probability of default tend to zero and $\mu = 0$, $\Gamma \rightarrow 0$ and the risk premium $\rho(\bar{\omega}_{t+1}) \rightarrow 1$ going back to the frictionless RBC

Aggregation

- To set a limit in net worth accumulation, entrepreneurs exit with a fixed probability $(1 - \sigma_E)$

- New entrants get a start-up transfer equal to ξ_E of the old entrepreneurs wealth

- Aggregate net worth is

$$N_{E,t} = (\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1} K_{t-1}$$

- Entrepreneur who exit consume

$$C_{E,t} = (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1} K_{t-1}$$

- The economy's resource constraint then is

$$Y_t = C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t) R_t^K Q_{t-1} K_{t-1}$$

General Equilibrium

- Households work, consume, save and pay taxes
  - Introduce habit in consumption

$$E_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} L_{t+i}^{1+\epsilon}] \tag{7}$$

  - Their budget constraint is:

$$C_t + D_t + T_t = W_t L_t + R_t D_{t-1}$$

- Banks are very simple, get deposits D_t from the households and provide loans B_t to the entrepreneurs at a rate Z_t

$$B_t = D_t$$

- Government has a balanced budget: $G_t = T_t$
- Production side in the BGG is NK
- Here we will just follow the RBC model structure on the real sector

General Equilibrium

- Capital accumulation with investment adjustment costs carried out by **Capital Producers**

$$
\begin{aligned}
K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X(X_t - 1)^2 \\
S'(X_t) &= 2\phi_X(X_t - 1) \\
Q_t(1 - S(X_t) - X_t S'(X_t)) &+ \mathbb{E}_t\left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2\right] = 1
\end{aligned}
$$

- $S(X_t)$ are investment adjustment costs equal to zero in a balance growth steady state

Summary of Equilibrium

$$
\begin{aligned}
Y_t &= C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1} \\
C_{E,t} &= (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
B_t &= Q_t K_t - N_{E,t} \\
\bar{\omega}_t &= \frac{Z_{t-1} B_{t-1}}{R_t^K Q_{t-1} K_{t-1}} \\
R_t^K &= \frac{r_t^K + (1 - \delta) Q_t}{Q_{t-1}} \\
r_t^K &= \frac{(1 - \alpha) Y_t}{K_{t-1}}
\end{aligned}
$$

Steady State Equilibrium

- The zero growth steady state is given by

$$\begin{aligned} R_k &= \rho(\bar{\psi})R \\ N_E &= (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))R_k QK \\ R_k QK\left[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})\right] &= R(QK - N_E) \text{ or} \\ R_k[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})] &= R(\phi - 1) \end{aligned}$$

- where $\phi \equiv \frac{QK}{N_E}$ and

$$\rho(\bar{\psi}) = \frac{\Gamma'(\bar{\psi})}{\left[(\Gamma(\bar{\psi}) - \mu G(\bar{\psi}))\Gamma'(\bar{\psi}) + (1 - \Gamma(\bar{\psi}))(\Gamma'(\bar{\psi}) - \mu G'(\bar{\psi}))\right]}$$

Steady State Equilibrium

- with resource constraint

$$\begin{aligned} Y &= C + C_E + G + I + \mu G(\bar{\psi}) R_k Q K \\ C_E &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi})) R_k Q K \end{aligned}$$

- and post-recursive equations

$$\begin{aligned} L &= QK - N_E \\ R_l &= \frac{\bar{\psi} R_k Q K}{L} \\ R_k &= \frac{Z + (1-\delta)Q}{Q} \\ Z &= \frac{(1-\alpha)Y}{K} \end{aligned}$$

The Density Function

- BGG choose a log-normal distribution
- Thus $\omega_t = e^{x_t}$ where $x_t \sim N(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$
- This guarantees that $\mathbb{E}[\omega_t] = 1$

Steady State Solution Strategy

- Steady state is not straightforward for this kind of models
- Firstly solve the financial sector problem
- Then go to the real sector and find all the variables as shares of capital

Steady State Solution Strategy

- Find two equations with only unknowns the entrepreneurial leverage (ϕ^E) and the return on capital (R^K).

$$\phi^E = \frac{QK}{N^E}$$

Rearranging,

$$\frac{N^E}{K} = \frac{1}{\phi^E}$$

- From entrepreneurs net worth N^E, divide with capital and substituting $Q = 1$ we have:

$$\frac{N_E}{K} = (\sigma^E + \xi^E)(1 - \Gamma(\bar{\omega}))R^k$$

Steady State Solution Strategy

- From entrepreneurs balance sheet constraint $L = QK - N^E$, dividing with $K^s s$ we have

$$\frac{N_E}{K} = 1 - \frac{L}{K} = \frac{1}{\phi^E}$$

- Hence and using the fact that $R^K = \rho(\omega)R$,

$$\boxed{\phi_E = \frac{1}{(\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}))\rho(\omega)R}}$$

which is **the first equation** for the system

Steady State Solution Strategy

- From the Zero Profit Condition , solving for ϕ_E we get

$$\phi_E = -\frac{R}{R^K(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) - R}$$

- Substituting again:

$$\boxed{\phi_E = -\frac{R}{\rho(\bar{\omega})R(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) - R}}$$

  yields **the second equation** for the system

- We have 2 equations and 2 unknowns $(\bar{\omega}, \phi_E)$

- Actually we treat ϕ_E as known equal to 2 in SS and we calibrate σ_E to match this leverage

Steady State Calibration

- Production parameters, same as in the simple RBC
- For NK extension see code in GitHub
- Additional financial parameters to calibrate are σ_ω, σ_E, and μ.
- These three parameters are calibrated to hit four targets:
  (1) μ such as a default probability $p(\bar{\psi}) = 0.03$,

  (2) σ_ω for $\rho(\bar{\psi}) = 1.0025$ corresponding to a credit spread of 100 basis points as in GK,

  (3) σ_E for an entrepreneur leverage $\frac{QK}{N_E} = 2$ as in BGG

Inside the Code: Steady State

```
R            = 1/betta;
Lambda       = betta;

% Targets
p_mom        =  0.03;      % steady state default
p            = p_mom;
rho_mom      = 1 + 0.02/4; % steady state premium
rho            = rho_mom;
phie_mom     = 2;          % steady state leverage
phie         = phie_mom;
```

Inside the Code: Steady State

```
% give initial values
x0=[0.98,0.3, 0.49, 0.008, 0.98, 0.30, 0.2, 0.49];
fun = @(c) myfun_BGG_calib(c,p_mom,ksie,rho,phie_mom,R);
options=optimset('MaxFunEvals',1000);
c     = fsolve(fun,x0,options);
sigmae  = c(1); %parameter
sigma_omega_ss = c(2);  %parameter
fnGam = c(3);
fnG   = c(4);
DGam  = c(5);
DG    = c(6);
mon   = c(7);  %parameter
omega  = exp(c(8));
```

Inside the Code: Steady State

```
function F = myfun_BGG_calib(c,p_mom,ksie,rho,phie_mom,R)
F=[phie_mom-1/((c(1)+ksie)*(1-c(3))*rho*R); %Eq 1
phie_mom + (R/((rho*R)*(c(3)-c(7)*c(4))-R)); %Eq 2
rho   - (c(5)/((c(3)-c(7)*c(4))*c(5)+(1-c(3))*(c(5)-c(7)*c(6)))); %rho
p_mom - (logncdf(exp(c(8)),-0.5*(c(2))^2,c(2))); %p
c(4)  - (1- normcdf((0.5*(c(2))^2 -log(exp(c(8))))/c(2),0,1)); %fnG
c(3)  - (c(4)+exp(c(8))*(1-p_mom)); %fnGam
c(5)  - (1-p_mom); %DGam
c(6)  - (1/(exp(c(8))*c(2)*sqrt(2*(4*atan(1))))*exp(-((log(exp(c(8)))
    +.5*(c(2))^2)^2)/(2*(c(2))^2)));]; %DG
end
```

Steady State: Real Sector

- To find Z we go to the only equation that has it:

$$Z = \bar{\omega} R^K Q \frac{K}{L}.$$

From the entrepreneurs leverage we know

$$\frac{K}{L} = \frac{1}{1 - \frac{1}{\phi^E}} = \frac{\phi^E}{\phi^E - 1}$$

$$r_K = R^k + \delta - 1$$

- For the real sector variables: define all variables as share of capital first

$$\frac{C_e^{ss}}{K^{ss}} = (1 - \sigma^e)(1 - \xi^e)(1 - \Gamma(\bar{\psi^{ss}}))R_{k,ss}Q^{ss}$$

Steady State: Real Sector

- The resource constraint is $Y^{ss} = C^{ss} + I^{ss} + G^{ss} + \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss} K^{ss}$

- Using the production function,

$$C^{ss} = L^{1-\alpha} K^{\alpha} - \delta K^{ss} - \eta L^{1-\alpha} K^{\alpha} - C_e - \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss} K^{ss}$$

  where $\eta = G^{ss}/Y^{ss}$

- Rearranging terms we get :

$$\frac{C^{ss}}{K^{ss}} = (1-\eta)(\frac{L^{ss}}{K^{ss}})^{1-\alpha} - \delta - \frac{C_e}{K^{ss}} - \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss}$$

- From the resource constraint we have

$$\frac{Y^{ss}}{K^{ss}} = \left(\frac{C^{ss}}{K^{ss}} + \delta + \frac{C_e}{K^{ss}} + \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss} \right) \frac{1}{1-\eta}$$

Steady State: Real Sector

- The labour leisure F.O.C in steady state is

$$(L^{ss})^{\epsilon}\chi = [W^{ss}((1-\gamma)c)^{-1}(1-\beta\gamma)]$$

- Substituting for the wage it yields:

$$(L^{ss})^{1+\epsilon} = \frac{(1-\beta\gamma)(1-\alpha)}{\chi(1-\gamma)}\frac{Y^{ss}}{C^{ss}}$$

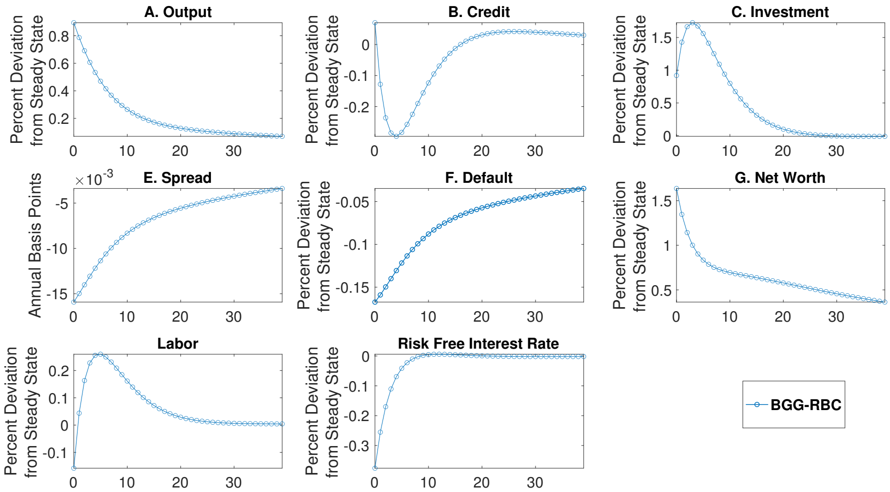
- By knowing $\frac{C^{ss}}{K^{ss}}$ and $\frac{Y^{ss}}{K^{ss}}$ is straightforward to calculate $\frac{Y^{ss}}{C^{ss}}$ and find L^{ss}.

- Finally, knowing L^{ss} and $\frac{Y^{ss}}{K^{ss}}$ from the production function we find the capital

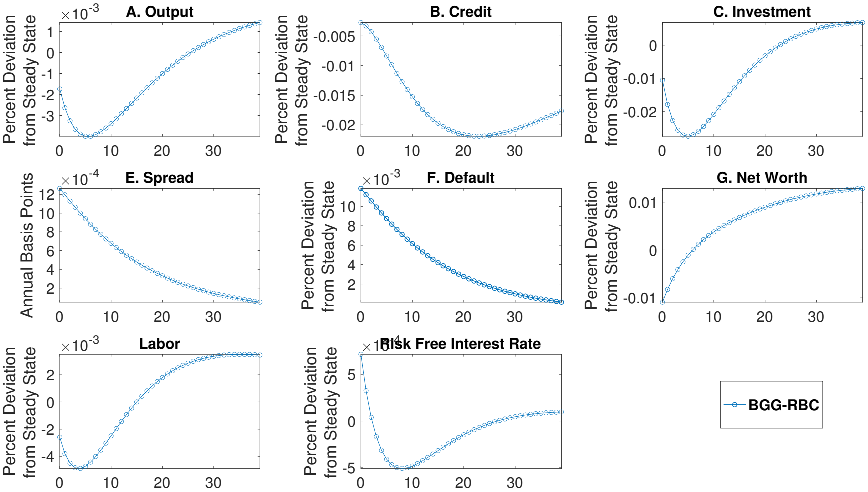
$$K^{ss} = \left(\frac{\frac{Y^{ss}}{K^{ss}}}{L^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}$$

- Hence, having capital, by reverse engineering we can find the values for $(I^{ss}, C^{ss}, C_e^{ss}, Y^{ss})$

Impulse Responses: Productivity Shock (positive)



Impulse Responses: Risk Shock (negative)



Christiano, Motto and Rostagno (2014) extension

- Identify the *Risk Shock*
- Built on the Bernanke *et al.* (1999)
- Under a NK framework
- Lots of additional frictions and shocks (capacity utilization rate, taxes, sticky wages, long-term bonds...)
- *Estimated* for the US economy

The Risk Shock

- They refer to as risk the time period t cross-sectional standard deviation of $\log(\omega)$
- Risk is high in periods when $\sigma_{\omega,t}$ is high, and there is substantial dispersion in the outcomes across entrepreneurs
- Model estimation assigns a large role to $\sigma_{\omega,t}$
- Disturbances in $\sigma_{\omega,t}$ trigger responses in the model that resemble actual business cycles
- They find that fluctuations in $\sigma_{\omega,t}$ account for 60 percent of the fluctuations in the growth rate of aggregate US output since the mid-1980s

The Risk Shock

- Essentially from being a parameter $\sigma_{\omega,t}$ now follows an AR(1) process `log(sigma_omega / sigma_omega_ss) = rhosigma * log(sigma_omega(-1) / sigma_omega_ss) + e_RS;`
- Where `e_RS` is the risk shock

Unanticipated and News Shocks

- A standard assumption in estimated equilibrium models is that a shock's statistical innovation becomes known to agents only when the innovation is realized
- Literature document that people receive information about the period t statistical innovation before the innovation is realized

Unanticipated and News Shocks

- CMR (2014) consider the following shock representation

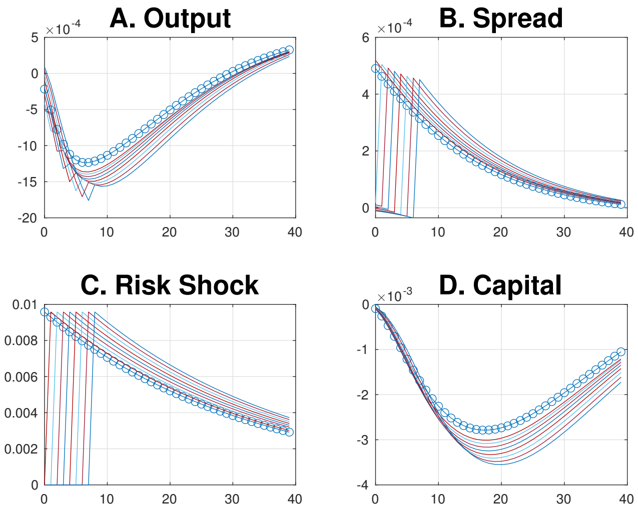
$$x_t = \rho_x x_{t-1} + \xi_0 + \xi_{1,t-1} + ... + \xi_{p,t-p}$$

- x_t is the log deviation of the shock from its non-stochastic steady state
- ξ_0 is the unanticipated component of the statistical innovation and $\xi_{1,t-1} + ... + \xi_{p,t-p}$ the anticipated, or news components
- They estimate the model for giving this structure to technology, monetary policy, government spending, equity, and risk shocks
- They find that the model that has the highest marginal likelihood is the one with news on the risk shock

Unanticipated and News Shocks

- This formulation is only in the `BBGsticky.mod` file
- For the flexible price economy there is only the unanticipated component
-

Risk Shock (Unanticipated + Anticipated Components)



Homework

- Continue with part 2 of previous week
- Work with `BGGflexi.mod` and `BBGsticky.mod`
- Provide the impulse responses of both models in the same graph for output, consumption, investment, spread, loans and entrepreneurs' new worth
- For the case of a positive productivity shock and a risk shock (only the unanticipated component)
- Make sure that they have the same std and AR(1) parameter
- File `BGGplots_lecture.m` is the file I used for the lecture plots, it will help!

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, **1**, 1341–1393.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, **21**(2), 265–293.