A Course on DSGE Models with Financial Frictions
Part 3: Limited Commitment

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The BGG model

- In the previous lecture we saw in detail the Bernanke *et al.* (1999) (BGG model)
- In the core of the financial friction is an entrepreneur receiving an idiosyncratic shock
- A bank that lends as long the incentive compatibility constraint holds
- An external finance premium that creates a wedge between the capital return and the risk free rate
- The financial friction is between the firm and the bank

Limited Commitment

- Another strand on the FF literature builds on limited commitment
- Initiated by the Kiyotaki and Moore (1997) paper that introduces collateral constraints:
  - Borrowers cannot commit to repay debt and so must hold collateral as a guarantee: $b_t \leq \theta k_t$
- Similarly to the BGG: eliminates $\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1} R^K_{t+1}]$ of the RBC model and introduces a spread between the two discounted expected returns

Today

- We will see the seminal paper by Gertler and Kiyotaki (2010); Gertler and Karadi (2011)
- The financial friction is between the household and the bank
- Main friction: Limited commitment from the bank to not ”steal” its assets and return the funds to the households
- Extends the limited commitment problem of Kiyotaki and Moore (1997)
- Introduces an agency problem between depositors and banks when the value of bank capital declines, the borrowing constraint tightens
- Limits the amount of deposits the bank can raise and subsequently, the level of investment

Summary of the Model

- Main agents in the model: Households, banks, firms, capital goods producers and the government
- Households work, consume and save in deposits
- Banks are risk neutral. They provide loans to the firms(by buying claims of firms' capital)
- Finance their assets with their net worth and deposits
- Have to respect an incentive compatibility constraint

Banks: Overview

- The bank j receives deposits D_t from the households at time t to provide loans S_t at price Q_t to the firms where
- S_t is the number of claims to one unit of firms' capital, so the asset against which the loans are obtained is end-of-period capital K_t
- Deposits together with her net worth N_t finance the loan provision.
- Bank's balance sheet therefore is:

$$Q_t S_t = N_t + D_t$$

Banks: Overview

- Banks receive a gross return from the loans R_t^K and pay back the depositors at the risk-free interest rate R_t

- Bank's net worth at time t is:

$$N_t = R_t^K Q_{t-1} S_{t-1} - R_t D_{t-1}$$

- The GK model assumes that an agent can be worker or a banker

- Bankers exit and become workers with probability $1 - \sigma_B$ per period

- Workers become banks with the same probability keeping proportions fixed.

- New bankers get a start-up transfer ξ_B similarly to the BGG

The Banker's Objective

- The banker's objective at the end of period t is to maximize expected terminal wealth
$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma_B)\sigma_B^{i-1} \Lambda_{t,t+i} N_{t+i}$$
where $\Lambda_{t,t+i}$ is the $[t, t+i]$ stochastic discount factor corresponding to the consumer's optimization problem.

The Banker Constraint

- The banks manager may divert a fraction θ of her assets back to her family
- Cost: Lenders can force the intermediary into bankruptcy and recover the remaining fraction of assets
- In order for the creditors to continuer supply funds to the bank, the following incentive constraint must hold

$$\underbrace{V_t}_{\text{Value of the bank}} \geq \underbrace{\theta(Q_t S_t)}_{\text{Gain from diverting}}$$

The Banker's Problem

- The banker's decision problem is to maximize discounted future earnings

$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma_B)\sigma_B^{i-1}\Lambda_{t,t+i}N_{t+i}$$

subject to

- Balance sheet $Q_t S_t = N_t + D_t$
- Net worth $N_t = R_t^K Q_{t-1} S_{t-1} - R_t D_{t-1}$
- Incentive constraint $V_t \geq \theta(Q_t S_t)$ where we assume that it is always binding

Banker's Problem Solution

- To solve the problem we make a guess that a value function has the following linear form:

$$V_t(s_t, d_t) = \nu_{s,t} S_t - \nu_{d,t} D_t \tag{1}$$

  where $\nu_{s,t}$ is the marginal value from credit for the bank and $\nu_{d,t}$ the marginal cost of deposits

- Using the balance sheet constraint we can eliminate d_t from the value function, this yields

$$V_t(s_t, d_t) = \nu_{s,t} S_t - \nu_{d,t}(Q_t S_t - N_t)$$

  and

$$V_t = Q_t S_t \left(\frac{\nu_{s,t}}{Q_t} - \nu_{d,t}\right) + \nu_{d,t} N_t$$

Banker's Problem Solution

- Let $\mathcal{L}$ be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t[V_t - \theta(Q_t S_t)] = (1 + \lambda_t)V_t - \lambda_t \theta(Q_t S_t)$$

- The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta S_t} : (1 + \lambda_t)(\frac{\nu_{s,t}}{Q_t} - \nu_{d,t}) = \lambda_t \theta$$

$$KT : \lambda_t[(\frac{\nu_{s,j,t}}{Q_t} - \nu_{d,t})Q_{j,t}s_{j,t} + \nu_{d,j,t}N_{j,t} - \theta(Q_{j,t}S_{j,t})] = 0$$

Banker's Problem Solution

- Define the excess value of bank's financial claim holdings as

$$\mu_t = \frac{\nu_{s,t}}{Q_t} - \nu_{d,t}$$

- Given that banks are constrained in borrowing funds from the depositors from the KT and the FOC conditions we have:

$$Q_t S_t(\theta - \mu_t) = \nu_{d,t} N_t$$

- Rearranging terms, we have:

$$Q_t S_t = \phi_t N_t$$

where $\phi_t = \frac{\nu_{d,t}}{\theta - \mu_t}$

Banker's Problem Solution

- Substituting this into the guessed value function yields:

$$V_t = N_t \phi_t \mu_t + \nu_{d,t} N_t$$

- The Bellman equation now is:

$$V_{j,t-1}(s_{t-1}, d_t, m_{t-1}) = E_{t-1}\Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1-\sigma)N_t + \sigma(\phi_t \mu_t + \nu_{d,t})N_t\}$$

- By collecting terms with N_t the common factor and defining the variable Ω_t as

$$\Omega_{t+1} = (1-\sigma) + \sigma(\mu_{t+1}\phi_{t+1} + \nu_{d,t+1})$$

- The Bellman equation now becomes:

$$V_t(s_t, d_t) = E_t \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t} Q_{t-1} S_{t-1} - R_t D_t]$$

Banker's Problem Solution

- Using the method of undetermined coefficients and comparing (1) with (2) we have the final solutions for the coefficients:

$$\nu_{s,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} Q_t R_{k,t+1}$$
$$\nu_{d,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}$$
$$\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t+1} - R_{t+1}]$$

and

$$\phi_t = \frac{\mathbb{E}_t[\Omega_{t+1} \Lambda_{t,t+1} R_{t+1}]}{\Theta - \mathbb{E}_t[\Omega_{t+1} \Lambda_{t,t+1} (R_{k,t+1} - R_{t+1})]}$$

Aggregation

- At the aggregate level net worth is the sum of existing (old) bankers and new bankers:
$$N_t = N_{o,t} + N_{n,t}$$
where
$$N_{o,t} = \sigma_B\{R_{k,t}Q_tS_{t-1} - R_tD_{t-1}\}$$
- To allow new bankers to operate with some net worth, we assume that the family transfers to each one a fraction ξ_B of the value value of assets of the exiting bank implying:
$$N_{n,t} = \xi_B R_{k,t}Q_tS_{t-1}$$

Aggregation

- Given K_t, aggregate net worth accumulates according to

$$N_t = (\sigma_B + \xi_B) R_t^K Q_{t-1} S_{t-1} - \sigma_B R_t D_{t-1}$$

  where $D_t = Q_t S_t - N_t$

- Remember that the asset against which the loans S_t are obtained is end-of-period capital K_t. Therefore, $S_t = K_t$

General Equilibrium

- Households work, consume, save and pay taxes
  - Introduce habit in consumption

$$E_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} L_{t+i}^{1+\epsilon}]$$

  - Their budget constraint is:

$$C_t + D_t + T_t = W_t L_t + R_t D_{t-1}$$

- Government has a balanced budget: $G_t = T_t$

General Equilibrium

- Capital accumulation with investment adjustment costs carried out by **Capital Producers**

$$
\begin{aligned}
K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X (X_t - 1)^2 \\
S'(X_t) &= 2\phi_X (X_t - 1) \\
Q_t(1 - S(X_t) - X_t S'(X_t)) &+ \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2 \right] = 1
\end{aligned}
$$

- $S(X_t)$ are investment adjustment costs equal to zero in a balance growth steady state

Summary of Equilibrium

$$Y_t = C_t + G_t + I_t$$

$$\begin{aligned}
Q_t S_t &= \phi_t N_t \\
\phi_t &= \frac{\Omega_{t+1}\mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}]}{\theta - \Omega_{t+1}\mathbb{E}_t\Lambda_{t,t+1}[R_{k,t+1} - R_{t+1}]} \\
\Omega_t &= 1 - \sigma_B + \sigma_B\theta\phi_t \\
N_t &= (\sigma_B + \xi_B)R_t^K Q_{t-1}S_{t-1} - \sigma_B R_t D_{t-1} \\
D_t &= Q_t S_t - N_t
\end{aligned}$$

Summary of Equilibrium

The link with the real sector is provided by

$$
\begin{aligned}
S_t &= K_t \\
R_t^K &= \frac{r_t^K + (1-\delta)Q_t}{Q_{t-1}} \\
r_t^K &\equiv \frac{(1-\alpha)Y_t}{K_{t-1}}
\end{aligned}
$$

Steady State Solution Strategy

- Similarly to the BGG model we follow the same strategy here. But it's much simpler
- Firstly solve the financial sector problem
- Then go to the real sector and find all the variables as shares of capital

Steady State Solution Strategy

- Find two equations with only unknowns the bank leverage (ϕ) and the return on capital (R^K).

- From the capital producers problem we have that $Q^{ss} = 1$ and from the Euler equation we have that $R^{ss} = \frac{1}{\beta}$ and $\Lambda^{ss} = \beta$.

$$Q^{ss}K^{ss} = \phi^{ss}N^{ss}$$

  Rearranging,

$$\frac{N^{ss}}{K^{ss}} = \frac{1}{\phi}$$

- From the bank's balance sheet constraint we have $D^{ss} = K^{ss} - N^{ss}$. Dividing over K^{ss}:

$$\frac{N^{ss}}{K^{ss}} = 1 - \frac{D^{ss}}{K^{ss}} \tag{2}$$

Steady State Solution Strategy

- The bank's net worth is $N^{ss} = (\sigma + \xi)(R_k^{ss} Q^{ss} K^{ss}) - \sigma R^{ss} D^{ss}$.
- Dividing over K^{ss} and rearranging terms, yields:

$$\frac{N^{ss}}{K^{ss}} = (\sigma + \xi) R_k^{ss} - \sigma R \frac{D^{ss}}{K^{ss}} \tag{3}$$

Substituting (2) in (3) and using $R = 1/\beta$ we have

$$\frac{N^{ss}}{K^{ss}} = (\sigma + \xi) R_k^{ss} - \frac{\sigma}{\beta}(1 - \frac{N^{ss}}{K^{ss}})$$

Steady State Solution Strategy

- Rearranging terms

$$\frac{N^{ss}}{K^{ss}} = \frac{(\sigma + \xi)R_k^{ss} - \sigma/\beta}{1 - \sigma/\beta} = \frac{1}{\phi^{ss}}$$

- So we get the **first equation** for the steady state leverage,

$$\boxed{\phi^{ss} = \frac{1 - \sigma/\beta}{(\sigma + \xi)R_k^{ss} - \sigma/\beta}}$$

Steady State Solution Strategy

- Remember the definition of leverage

$$\phi_t = \frac{\nu_d}{\theta - \mu}$$

- Substituting in steady state

$$\phi_t = \frac{\Omega\Lambda R}{\theta - \Omega\Lambda[R_k - R]}$$

where Λ and R cancel out

- and Ω is given by

$$\Omega^{ss} = (1-\sigma) + \sigma\phi^{ss}\theta$$

Steady State Solution Strategy

- Thus the leverage yields:

$$\boxed{\phi^{ss} = \frac{(1-\sigma)+\sigma\phi^{ss}\theta}{\theta-((1-\sigma)+\sigma\phi^{ss}\theta)(\beta R_k^{ss}-1)}}$$

  being **the second equation** in the system.

- We have 2 equations and 2 unknowns (ϕ^{ss}, R_k^{ss})

- After solving this system it is straightforward to find $(\frac{N^{ss}}{K^{ss}}, \frac{D^{ss}}{K^{ss}})$

Steady State Calibration

- Production parameters, same as in the simple RBC
- In this code version I have just assigned parameter values that provide us with some moments we want to target
- No calibration takes place

Inside the Code

Files needed

- `GK.mod`
- `GK_steadystate.m`
- `myfun_GK1.m` is the function that solves the two equation system we just defined

Inside the Code: Steady State

```
%% Basics
R=1/betta;
%% Banker Solution (GK)

R_1=[1.0122, 3.55];
%%Solution for the Loan Interest Rate(1) and phi(2) on SS%%
fun = @(c) myfun_GK1(c,sigmab,ksi,betta,theta);
options=optimset('MaxFunEvals',10000,'MaxIter',10000,'Display','off');
c=fsolve(fun,R_1,options);

Rk=c(1);
phi=c(2);
```

Inside the Code: Steady State

```
function F = myfun_GK1(c,sigmab,ksi,betta,theta)

F(1)=c(2) - (1-sigmab/betta)/(c(1)*(sigmab+ksi)-sigmab/betta); %Equation 1
F(2)=c(2) - ((1-sigmab)+sigmab*c(2)*theta)/(theta-((1-sigmab)+sigmab*c(2)*theta)*(betta*c(1)-1)); %Equation 2
end
```

Steady State: Real Sector

- From the MPK
$$\frac{L}{K} = \left(\frac{r^K}{\alpha}\right)^{(1/(1-\alpha))}$$
- The resource constraint is $Y^{ss} = C^{ss} + I^{ss} + G^{ss}$, dividing with K
$$\frac{C}{K} = (1-\eta)\left(\frac{L}{K}\right)^{(1-\alpha)} - \delta$$
where $\eta = G^{ss}/Y^{ss}$

Steady State: Real Sector

- We also know that

$$\frac{Y}{K} = (\frac{C}{K} + \delta)/(1 - \eta)$$

- From the labour FOC we solve for L:

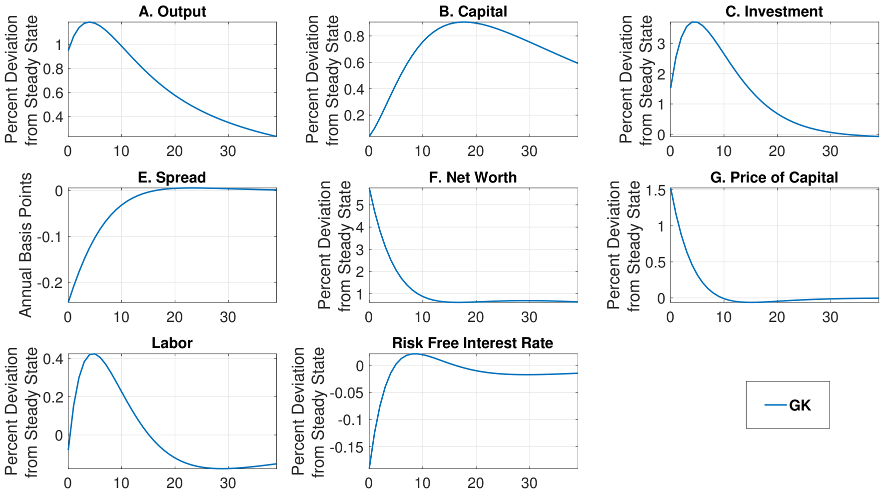
$$L = \left(\frac{(1 - \beta\gamma)(1 - \alpha)\frac{Y}{C}}{\chi(1 - \gamma)} \right)^{1/(1+\epsilon)}$$

- And finally from the production function:

$$K = \left(\frac{Y}{K} / L^{(1-\alpha)} \right)^{\frac{1}{(\alpha - 1)}}$$

- Since we know K we can then find all the variables

Impulse Responses: Productivity Shock (positive)



The Capital Quality Shock

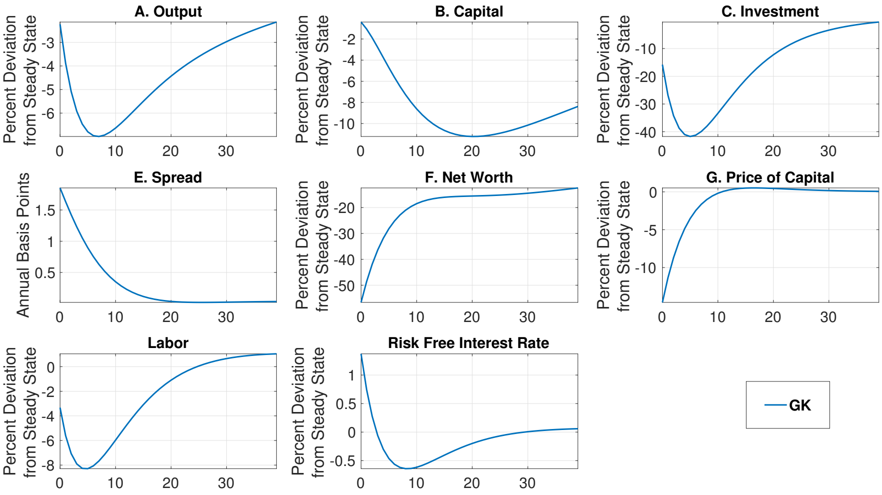
- Following Gertler and Karadi (2011), we add a capital quality shock, ψ_t, this reduces or increases the value of capital in period t going into period $t+1$
- In this way they tried to capture the devaluation of banks' assets at the 2008 financial crisis
- Capital now evolves as:

$$K_t = \psi_t\{(1-\delta)K_{t-1} + (1 - S(X_t))I_t\}$$

and

$$R_t^K = \psi_t \frac{r_t^K + (1-\delta)Q_t}{Q_{t-1}}$$

Impulse Responses: Capital Quality Shock (negative)



Note

- If you look at the Gertler and Kiyotaki (2010) they also assume interbank lending and borrowing
- This is essentially one more asset in the banks balance sheet
- For simplicity we abstract from that
- If you have time and interest see Gertler and Karadi (2011)
- It's exactly the same model in a NK setting with conventional and unconventional monetary policy

Homework

- In the model we saw, I have just set the parameter values to get some steady state moments
- What you have to do is, similarly to the BGG model, calibrate a key parameter to get exactly the value of a specific moment
- Instead of giving to the code a value for θ and getting a value for ϕ, treat θ as a free parameter
- Set `phi_mom = 4.5` and find the θ that produces that number
- Show that's the differences in IRFs after a capital quality shock with the non-calibrated version

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, **1**, 1341–1393.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of monetary Economics*, **58**(1), 17–34.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of monetary economics*, volume 3, pages 547–599. Elsevier.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of political economy*, **105**(2), 211–248.