Solution for Exercise: Week 1

Stylianos Tsiaras[1]

[1]European University Institute

April 3, 2021

The problem of the household is to maximize $U(C, L) = \ln C_t - \omega \frac{H_t^{1+\phi}}{1+\phi}$ subject to the constraints:

$$B_t = R_{t-1}B_{t-1} + r_t^K K_{t-1} + W_t H_t - C_t - I_t - T_t \tag{1}$$

and

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + (1 - \Phi(\Xi_t))I_t \\ \Xi_t &\equiv \frac{I_t}{I_{t-1}}\,;\;\; S',\, S'' \geqslant 0\,;\;\; \Phi(1) = \Phi'(1) = 0. \end{aligned}$$

where $\Phi(\Xi_t) = \phi_X(\Xi_t - 1)^2$. I_t units of output converts to $(1 - \Phi(\Xi_t))I_t$ of new capital sold at a real price Q_t.

The Lagrangian is:

$$\begin{aligned} \mathcal{L} &= \mathbb{E}_t\Big[\sum_{s=0}^{\infty} \beta^s \Big(U(C_{t+s}, L_{t+s}) \\ &+ \lambda_{t+s}\Big[R_{t+s-1}B_{t+s-1} + W_{t+s}(1 - L_{t+s}) + r_{t+s}^K K_{t+s-1} - C_{t+s} - I_{t+s} - T_{t+s} - B_{t+s}\big] \\ &+ \quad \mu_{t+s}[(1-\delta)K_{t+s-1} + (1 - S(X_{t+s}))I_{t+s} - K_{t+s}]\Big)\Big] \end{aligned}$$

Then the first-order conditions with respect to $\{C_{t+s}\}$, $\{B_{t+s-1}\}$, $\{K_{t+s-1}\}$, $\{I_{t+s}\}$ and $\{L_{t+s}\}$ are respectively

$$\{C_{t+s}\} \;:\; \mathbb{E}_t[U_{C,t+s} - \lambda_{t+s}] = 0\,;\;\; s \geqslant 0 \tag{2}$$

$$\{B_{t+s-1}\} \;:\; \mathbb{E}_t[\beta^s \lambda_{t+s} R_{t+s-1} - \beta^{s-1}\lambda_{t+s-1}] = 0\,; \tag{3}$$

$$\{K_{t+s-1}\} \;:\; \mathbb{E}_t[\beta^s \lambda_{t+s} r_{t+s}^K + \beta^s \lambda_{t+s}\mu_{t+s}(1-\delta) - \beta^{s-1}\mu_{t+s-1}] = 0\,; \tag{4}$$

$$\begin{aligned} \{I_{t+s}\} \;:\;& \mathbb{E}_t\Big[\mu_{t+s}\left(1 - S\left(I_{t+s}/I_{t+s-1}\right) - S'\left(I_{t+s}/I_{t+s-1}\right)\frac{I_{t+s}}{I_{t+s-1}}\right) - \lambda_{t+s} \\ -\;& \beta\lambda_{t,t+s+1} S'\left(I_{t+s}/I_{t+s-1}\right) \times \left(-\frac{I_{t+s+1}}{I_{t+s}^2} I_{t+s+1}\right)\Big] = 0\,; \end{aligned} \tag{5}$$

$$\{L_{t+s}\} \quad : \quad \mathbb{E}_t[U_{L,t+s} - \lambda_{t+s} W_{t+s}] = 0\,; \tag{6}$$

Putting $s = 0$ in (2), (5) and (6) and $s = 1$ in (3) and (4) and defining the stochastic discount factor as $\Lambda_{t,t+1} \equiv \beta \frac{U_{C,t+1}}{U_{c,t}}$ we now have:

$$R_t \mathbb{E}_t \left[\Lambda_{t,t+1}\right] = 1 \tag{7}$$

$$\frac{U_{H,t}}{U_{c,t}} = -W_t \tag{8}$$

$$Q_t(1 - S(X_t) - X_t S'(X_t)) + \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} S'(\Xi_{t+1}) \Xi_{t+1}^2\right] = 1 \tag{9}$$

$$\mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^K\right] = 1. \tag{10}$$

where R_t^K is the gross return on capital given by

$$R_t^K = \frac{\left[r_t^K + (1-\delta) Q_t\right]}{Q_{t-1}}$$

How we go from (5) to (9)? We need to define $Q_t = \frac{\mu_t}{\lambda_t}$. μ_t is the shadow value of having an extra unit of investment. Dividing this by λ_t (which is equal to the marginal utility of consumption) puts this in terms of consumption goods. In other words, Q_t is the marginal value of investment measured in terms of consumption goods.

Code to change in the `mod` file:

```
% Gross rate becomes
RK = (rK + (1-delta)*Q)/Q(-1);
%  Law of motion of capital
K = I*(1-phiX*(Xi-1)^2) + (1-delta)*K(-1);
%  Investment Adjustment. You have to add the respective parameters and variables
Xi=I/I(-1);
Q*(1-phiX*(Xi-1)^2-Xi*2*phiX*(Xi-1))+2*phiX*(Xi(+1)-1)*Xi(+1)^2*Q(+1)*LAMBDA(+1)=1;
%  In SS Xi = 1; Q=1; so there's nothing to change
```

The new `mod` file is `RBC1_inv_adj.mod`.

Final question: Construct a 0:4 with step=1 grid for ϕ_X and show the different impulse responses for every different value of ϕ_X after a positive TFP shock.

A simple Matlab code `Inv_adj_params.m` that does this the following:

```
clear all
clc
Phix_value     = (0:1:4)';           % Parameter value grid
rep            = size(Phix_value,1); % Setting index for the loop

for i = 1:rep
phiX    = Phix_value(i);             % Give to Phix the value of every iteration
```

```
save Phix_value phiX                    % Send the parameter value to Dynare Phix_value

dynare RBC1_inv_adj noclearall       % Run the mod file every time
% importantly set noclearall, otherwise
% Dynare erases all data in the workspace

Yres(i,:) = Y_epsA; % thake the IRF of every variable and add it to the new 40xrep matrix
Ires(i,:) = I_epsA;
Cres(i,:) = C_epsA;
Hres(i,:) = H_epsA;
end
```

At the same time change the Dynare code to receive the different ϕ_x at every iteration from Matlab. In the `mod` file add in the parameters block

```
load Phix_value;
set_param_value('phiX'    ,phiX);
%phiX = 2; % Investment adjustment costs. This was for the original model
```

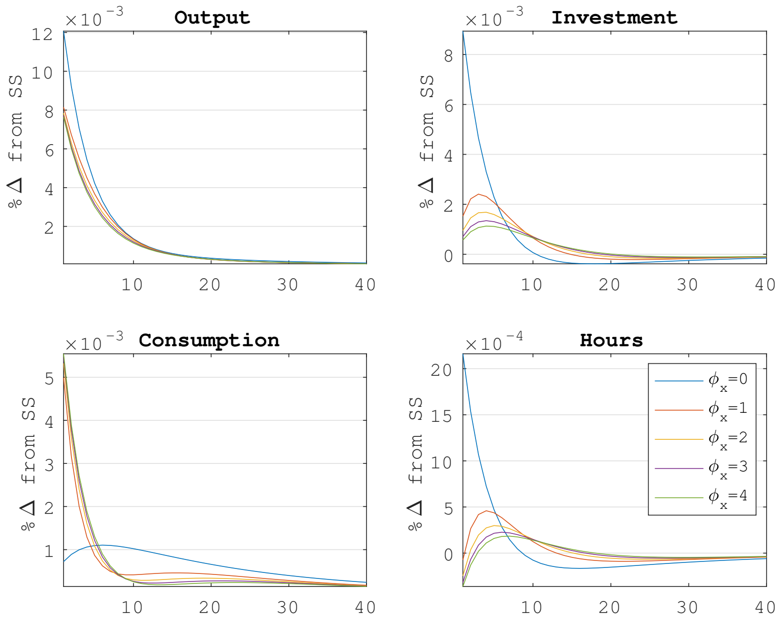

Figure 1: IRFs to different values for ϕ_x