A Course on DSGE Models with Financial Frictions Part 3: Asymmetric Information and Limited Commitment

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From Complete Markets to Financial Frictions

- To introduce financial frictions we need two deviations from the complete markets representative agent setting
  - Incomplete markets
  - Heterogeneous Agents

- A representative agent environment is in a sense an infinite agents setting where they can trade as much as necessary to eliminate idiosyncratic risk

- An example of market incompleteness:

- The Kiyotaki-Moore style collateral constraint $b_t \leq \theta k_t$

Financial Frictions: The BGG Model

- The two main streams of introducing financial frictions in DSGE models are
  - Asymmetric information
  - Limited commitment

- In this lecture we will go through one of the most seminal papers in the literature the Bernanke *et al.* (1999)

- it is a canonical NK model with a friction between the banks and the firms (called entrepreneurs)

- Eliminates $\mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1}R^K_{t+1}]$ of the RBC model and introduces a spread between the two discounted expected returns

Summary of the Model

- Main agents in the model: Households, entrepreneurs, retailers and a government
- Households work, consume and save in deposits
- Entrepreneurs are risk neutral. They purchase physical capital and produce goods with capital and labour
- Finance capital purchases with their net worth and bank credit
- Retailers are present to introduce nominal rigidities without complicating the entrepreneurs problem
    - Production side: NK framework with monopolistically competition and sticky prices
    - We will not go through that today

Entrepreneurs: Overview

- The entrepreneur j borrows loans B_t from the financial intermediary at time t to purchase capital K_t
- Loans together with her net worth $N_{E,t}$ finance the expenditure on new capital $Q_t K_t$.
- Entrepreneur's balance sheet therefore is:

$$B_t = Q_t K_t - N_{E,t} \tag{1}$$

where Q_t is the price of capital

Entrepreneurs: The Friction

- At every t the entrepreneur receives an idiosyncratic shock ω_t
- Results in return on capital: $\omega_t R_t^K$
  - ω_t is iid across all entrepreneurs
  - Drawn from a density f(ω_t) with mean E(ω_t)=1
- ω_t^j is only observable by entrepreneur j
- Costly State Verification (CSV) Townsend (1979): Lenders to observe the state, need to pay a monitoring cost, a proportion mu of the gross return $\mu R_{t+1}^K Q_t K_t$

Entrepreneurs: No Aggregate Risk

- We start with the case of no aggregate risk
  - R^K_{t+1} is known in advance and only uncertainty is idiosyncratic to the firm
  - The optimal contract is a risky debt contract (see Appendix A of Bernanke *et al.* (1999) for more)

- Entrepreneurs choose B_t and $Q_t K_t$ given R^K_t

- The optimal contract can be characterized by a gross non-default loan rate Z_t and a threshold value for the idiosyncratic shock $\bar{\omega}_t$

$$\bar{\omega}_{t+1} = \frac{Z_t B_t}{R^K_{t+1} Q_t k_t} \tag{2}$$

Entrepreneurs: Optimal Contract

- Entrepreneurs who receive a $\omega_t \geq \bar{\omega}_t$
  - Repay the intermediary $Z_t B_t$
  - Keep the difference $\bar{\omega}_{t+1} R^K_{t+1} Q_t k_t - Z_t B_t$
- Entrepreneurs who receive a $\omega_t < \bar{\omega}_t$
  - Default and get nothing
  - Intermediary pays the monitoring costs and gets what it finds: $(1-\mu) R^K_{t+1} Q_t K_t$

Entrepreneurs: Optimal Contract

- Under the optimal contract the lender should receive an expected return equal to its opportunity cost of funds
- Here, our bank's balance sheet is $B_t = D_t$ where D_t (deposits by households) is remunerated at the riskless rate R_t
- The loan contract must satisfy the bank's incentive compatibility constraint at time t

$$\underbrace{(1-\mu)R^K_{t+1}Q_tK_t\int_0^{\bar{\omega}_{t+1}}\omega f(\omega)d\omega}_{\text{Default}} + \overbrace{(1-p(\bar{\omega}_{t+1}))Z_tB_t}^{\text{No default}}$$

$$= R_tB_t$$

Entrepreneurs: Optimal Contract

- Using the Z_t from $\bar{\omega}_{t+1} = \frac{Z_t B_t}{R^K_{t+1} Q_t k_t}$, the constraint becomes

$$R^K_{t+1} Q_t K_t \left((1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} (1 - p(\bar{\omega}_{t+1})) \right) \geq R_{t+1} B_t \tag{3}$$

- To simplify notation a bit let $\Gamma(\bar{\omega}_{t+1})$ to be the fraction of net capital received by the lender (the bank)

- $\mu G(\bar{\omega}_{t+1})$ to be monitoring costs:

$$\Gamma(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} (1 - p(\bar{\omega}_{t+1})) \tag{4}$$

$$G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \psi f(\omega) d\omega \tag{5}$$

Entrepreneurs: Introducing Aggregate Risk

- So far we were abstracting from aggregate risk
- Now price and return of capital are given but uncertain
- $\bar{\omega}_{t+1}$ will now generally depend on the ex-post realization of R^K_{t+1}
- Entrepreneur is willing to offer a return that is free of any aggregate risk
    - Conditional on the return R^K_{t+1} the borrower offers state contingent non-default payment guaranteeing an expected return equal to the riskless rate
    - IC now implies a set of contracts for each realization of R^K_{t+1}

Entrepreneurs: Net Worth and Optimal Choice of Capital

- The entrepreneur's payoff is

$$\mathbb{E}_t\left[R^K_{t+1}Q_tK_t\int_{\bar{\omega}_{t+1}}^{\infty}\omega f(\omega)d\omega-(1-p(\bar{\omega}_{t+1}))Z_tB_t\right]=$$

$$\mathbb{E}_t\left[\left(1-\Gamma(\bar{\psi}_{t+1})\right)R^K_{t+1}Q_tK_t\right] \tag{6}$$

- She maximizes (6) given the net worth $N_{E,t}$, subject to the IC constraint (3) which, using (1), (4) and (5), can be rewritten as

$$\mathbb{E}_t\left[R^K_{t+1}Q_tK_t\left[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})\right]\geq R_{t+1}(Q_tK_t-N_{E,t})\right]$$

Entrepreneurs: Solution

- Let λ_t be the Lagrange multiplier associated with the IC constraint. Then the first order conditions are

$$\begin{aligned} K_t &: \quad \mathbb{E}_t \left[(1-\Gamma(\bar{\omega}_{t+1}) R^K_{t+1} + \lambda_t \left[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})) R^K_{t+1} - R_{t+1} \right] \right] = 0 \\ \bar{\omega}_{t+1} &: \quad \mathbb{E}_t \left[\Gamma'(\bar{\omega}_{t+1}) + \lambda_t (\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})) \right] = 0 \end{aligned}$$

  plus the binding IC condition if $\lambda_t > 0$ with $\lambda_t = 0$ if it does not bind.

- Combining these two conditions, we arrive at

$$\mathbb{E}_t[R^K_{t+1}] = \mathbb{E}_t[\rho(\bar{\omega}_{t+1}) R_{t+1}]$$

  where the *premium on external finance*, $\rho(\bar{\omega}_{t+1})$ is given by

$$\rho(\bar{\omega}_{t+1}) = \frac{\Gamma'(\bar{\omega}_{t+1})}{[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))\Gamma'(\bar{\omega}_{t+1}) + (1-\Gamma(\bar{\omega}_{t+1}))(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))]}$$

External Finance Premium

- The external finance premium equation replaces the no arbitrage condition between R^K and R in NK and RBC models
- External finance is given by the safe rate, scaled up by a premium term
- Note that in the limiting case of $\bar{\omega}_{t+1}$ =0 and the probability of default tend to zero and $\mu = 0$, $\Gamma \to 0$ and the risk premium $\rho(\bar{\omega}_{t+1}) \to 1$ going back to the frictionless RBC

Aggregation

- Entrepreneurs exit with a fixed probability $(1-\sigma_E)$
- New entrants get a start-up transfer equal to ξ_E of the old entrepreneurs wealth
- Aggregate net worth is

$$N_{E,t} = (\sigma_E + \xi_E)(1-\Gamma(\bar{\omega}_t))R_t^K Q_{t-1}K_{t-1}$$

- Entrepreneur who exit (die) consume

$$C_{E,t} = (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\omega}_t))R_t^K Q_{t-1}K_{t-1}$$

- The economy's resource constraint then is

$$Y_t = C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t)R_t^K Q_{t-1}K_{t-1}$$

General Equilibrium

- Households work, consume, save and pay taxes
  - Their budget constraint therefore is:

$$C_t + D_t + T_t = W_t L_t + R_t D_{t-1}$$

- Banks are very simple, get deposits D_t from the households and provide loans B_t to the entrepreneurs at a rate Z_t

$$B_t = D_t$$

- Government has a balanced budget: $G_t = T_t$
- Production side in the BGG is NK
- Here we will just follow the RBC model structure

Summary of Equilibrium

$$
\begin{aligned}
Y_t &= C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1} \\
C_{E,t} &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
B_t &= Q_t K_t - N_{E,t} \\
\bar{\omega}_t &= \frac{Z_{t-1} B_{t-1}}{R_t^K Q_{t-1} K_{t-1}} \\
R_t^K &= \frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}} \\
r_t^K &= \frac{(1-\alpha) Y_t}{K_{t-1}}
\end{aligned}
$$

Steady State Equilibrium

- The zero growth steady state is given by

$$\begin{aligned} R_k &= \rho(\bar{\psi})R \\ N_E &= (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))R_k QK \\ R_k QK\left[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})\right] &= R(QK - N_E) \text{ or} \\ R_k[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})] &= R(\phi - 1) \end{aligned}$$

- where $\phi \equiv \frac{QK}{N_E}$ and

$$\rho(\bar{\psi}) = \frac{\Gamma'(\bar{\psi})}{\left[(\Gamma(\bar{\psi}) - \mu G(\bar{\psi}))\Gamma'(\bar{\psi}) + (1 - \Gamma(\bar{\psi}))(\Gamma'(\bar{\psi}) - \mu G'(\bar{\psi}))\right]}$$

Steady State Equilibrium

- with a resource constraint

$$
\begin{aligned}
Y &= C + C_E + G + I + \mu G(\bar{\psi}) R_k Q K \\
C_E &= (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\psi})) R_k Q K
\end{aligned}
$$

- and post-recursive equations

$$
\begin{aligned}
L &= QK - N_E \\
R_l &= \frac{\bar{\psi} R_k Q K}{L} \\
R_k &= \frac{Z + (1 - \delta) Q}{Q} \\
Z &= \frac{(1 - \alpha) P^W Y^W}{K}
\end{aligned}
$$

The Density Function

- BGG choose a log-normal distribution.
- Thus $\omega_t = e^{x_t}$ where $x_t \sim N(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$
- This guarantees that $\mathbb{E}[\psi_t] = 1$

Calibration

- Production parameters, same as in the simple RBC
- For NK extension see code in GitHub
- Additional financial parameters to calibrate are σ_ω, σ_E, and μ.
- These three parameters are calibrated to hit four targets:
  (1) a default probability $p(\bar{\psi}) = 0.05$,
  (2) $\rho(\bar{\psi}) = 1.0025$ corresponding to a credit spread of 100 basis points as in GK,
  (3) an entrepreneur leverage $\frac{QK}{N_E} = 2$ as in BGG

Calibration

- With these values the calibrated parameters are:

Parameter	Calibrated Value
σ_{ψ}	0.3135
σ_E	0.9764
μ	0.0284

Table: BGG Model with SW Preferences. Calibrated Parameters

- In the code we avoid a notational clash with μ already defined as a labour supply elasticity by denoting the monitoring cost parameter above as μ_{FF}.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, **1**, 1341–1393.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, **21**(2), 265–293.