

Solution for Exercise: Week 4-5

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1 Liquidity Injections in the GK framework

Exercise: Consider the model of GK with the liquidity injections and use the code of the **steady state** for the original GK and extend this to include the liquidity injections framework we saw in class. Firstly, solve for the SS by pen and paper. Express M and R^M as function of variables you know $M^{ss} = \chi_m S^{ss}$ and $R^M = f(R^K)$. You will end up to the 2 equations for 2 unknowns (ϕ, R^K) as we did in the previous lecture. For the code you don't have to work with the `.mod` file and the `steady_state.m` file. Create a new Matlab file just for the SS. Copy paste the old GK SS file, delete the preamble and the ending that connects it with Dynare. Make the adjustments in the `%% Banker Solution (GK)` section. Set $\chi_m = 0.001$ and $\omega = 0.5$. Extend the `myfun.GK1.m` with the new components. Find ϕ, R^K, R^M in steady state and report those values.

Solution:

The incentive constraint of the bank in steady state is

$$QS = \phi^B N + \omega M, \quad (1)$$

where $M = \chi_m QS \rightarrow \frac{M}{S} = \chi_m$. We also know that in steady state $Q=1$.

By dividing (1) over loans we have $\frac{S}{S} = \phi^B \frac{(N)}{S} + \omega \frac{M}{S}$. Rearranging terms :

$$\frac{N^B}{S} = \frac{1}{\phi^B} (1 - \omega \chi_m). \quad (2)$$

From the bank's balance sheet constraint we have $D = S - N - M$. Dividing over S :

$$\frac{N^B}{S} = 1 - \frac{D}{S} - \chi_m. \quad (3)$$

The bank's net worth is $N = (\sigma^B + \xi^B)(R^K QS) - \sigma^B(RD + R^M M)$. Again dividing over S , setting $Q = 1$ and rearranging terms, yields:

$$\frac{N}{S} = [(\sigma^B + \xi^B)R^K - \sigma^B(R\frac{D}{S} + R^M\chi_m)]. \quad (4)$$

Substituting (3) in (4) and using $R = 1/\beta$ we have

$$\frac{N^B}{S} = (\sigma^B + \xi^B)(R^K) - \sigma^B(\frac{1}{\beta}(1 - \frac{N}{S} - \chi_m) + R^M M)$$

Rearranging terms and substituting $R^M = \omega R^K + (1 - \omega)R$

$$\frac{N^B}{S} = \frac{(\sigma^B + \xi^B)R^K - \sigma^B/\beta + \omega\sigma^B\chi_m(R - R^K)}{1 - \sigma^B/\beta} = \frac{1}{\phi^B}(1 - \omega\chi_m). \quad (5)$$

So we get the **first equation** for the steady state leverage,

$$\boxed{\phi^B = \frac{(1 - \sigma^B/\beta)(1 - \omega\chi_m)}{(\sigma^B + \xi)R^K - \sigma^B/\beta + \omega\sigma^B\chi_m(R - R^K)}} \quad (6)$$

Now I turn in finding the steady state value of the leverage using the definition of leverage. We know that

$$\phi^B = \frac{\nu_{d,j}}{\theta - spread} \quad (7)$$

We also know that $\nu_d = \Lambda\Omega R = \beta\Omega\frac{1}{\beta} = \Omega$ After substituting ν_d , the leverage (ϕ^B) becomes

$$\phi^B = \frac{\Omega}{\theta - \Lambda\Omega(R^K - R)}$$

Rearranging terms and substituting Ω given by

$$\Omega = (1 - \sigma^B) + \sigma^B\phi^B\theta \quad (8)$$

the leverage yields:

$$\boxed{\phi^B = \frac{(1 - \sigma^B) + \sigma^B\phi^B\theta}{\theta - ((1 - \sigma^B) + \sigma^B\phi^B\theta)(\beta R^K - 1)}} \quad (9)$$

being **the second equation** in the system.

Hence, we have 2 equations (6, 9) and 2 unknowns (ϕ^B, R^B). After solving this system it is straightforward to find $(\frac{N}{L}, \frac{D}{L})$.

Code:

We dont neet to work with the .mod file at all, just to rearrange the steady state file. Create a new SS file and copy -paste all the parameter values in the Matlab file and delete the preamble and the ending of the `steady_state.m` file so as to work without the .mod

file at all.

The part we need is here

```

1 %% Banker Solution (GK)
2 R_1=[1.0122, 3.55];
3 %%Solution for the Loan Interest Rate(1) and phi(2) on SS%%
4 %%Uses function determined by the solution of SS divided by K.Solves a
   system%%
5 fun = @(c) myfun_GK_liq_inj(c,sigtab,ksi,betta,theta,omega,chim);
6 options=optimset('MaxFunEvals',10000,'MaxIter',10000,'Display','off');
7 c=fsolve(fun,R_1,options);
8
9 Rk=c(1);
10 phi=c(2);
11 Rm = omega*Rk +(1-omega)*R;

```

Where we have created a new function called `myfun_GK_liq_inj` almost identical with the `myfun_GK1`, just changed to include the new formulation with the liquidity. It includes the two equations for ϕ found above. It also needs to additional inputs, ω and the liquidity ratio in SS, a parameter, χ_m which we have set to 0.001.

Our function `myfun_GK_liq_inj` is now:

```

1 function F = myfun_GK_liq_inj(c,sigtab,ksi,betta,theta,omega,chim)
2 F(1)=c(2) - (1-sigtab/betta)*(1-omega*chim)/(c(1)*(sigtab+ksi)-sigtab/betta
   + omega*sigtab*chim*(1/betta-c(1))); %Equation 1
3 F(2)=c(2) - ((1-sigtab)+sigtab*c(2)*theta)/(theta-((1-sigtab)+sigtab*c(2)*
   theta)*(betta*c(1)-1));%Equation 2
4 end

```

Solving the system, the new value for `Rk` is 1.0122, `phi` is 3.5580 and `Rm` is 1.0112.

2 Countercyclical Buffer in the GK framework

Exercise: Consider the original model of GK and turn the parameter θ into a a time varying parameter (actually a variable in `Dynare` wording) that follows the countercyclical buffer equation as

$$\theta_t = \theta^* + \psi_k gap_{k,t}$$

where $\theta^* = 0.383$, $\psi_k = 0.15$ and the definition of the credit gap follows

$$gap_{k,t} = \frac{Q_t S_t}{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}} - \frac{Q^{ss} S^{ss}}{4Y^{ss}}$$

Finally, plot the impulse responses for $Y, C, I, N, \phi, spread$ after a capital quality shock of `std=1` for the cases of

- $\theta_t = \theta^*$ (original GK)

- $\theta_t = \theta^* + \psi_k gap_{k,t}$

Solution:

In the .mod file you now have to introduce a new parameter **theta_star** which is the SS value of **theta** and also turn **theta** into a variable. In this way it can follow the CCyB rule. For the case of the original GK model **theta=theta_star** and nothing changes. For the case of countercyclical requirements in the mod file, add:

```
1 theta = theta_star + psi_k*((Q*S)/(Y+Y(-1)+Y(-2)+Y(-3)) - steady_state(Q)*
    steady_state(S)/(4*steady_state(Y)));
```

Also add in the parameter section **psi_k = 0.15**. Note that in the steady state, nothing changes here, **theta** is always equal to **theta_star** in SS.

Plotting IRFs:

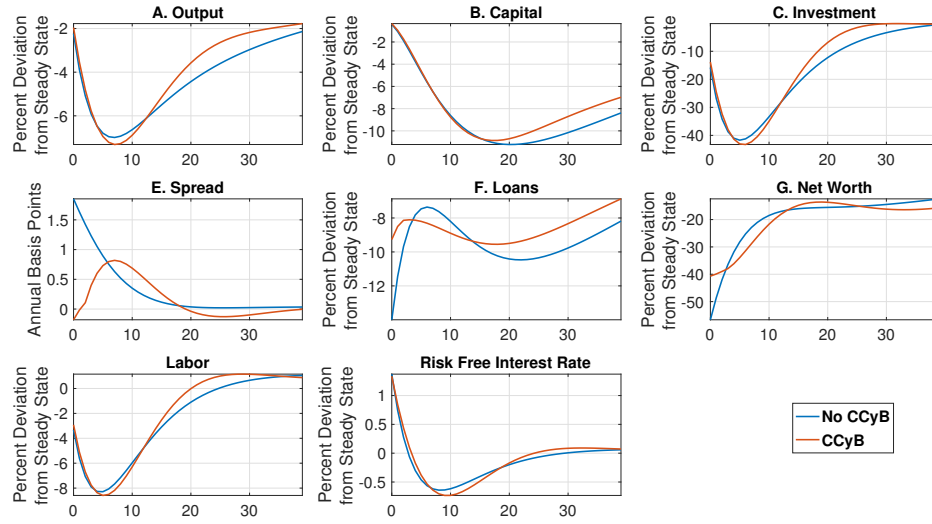


Figure 1: IRFs to capital quality shock with countercyclical capital requirements