A Course on DSGE Models with Financial Frictions Part 3: Asymmetric Information and Limited Commitment

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From Complete Markets to Financial Frictions

- To introduce financial frictions we need two deviations from the complete markets representative agent setting
  - Incomplete markets
  - Heterogeneous Agents

- A representative agent environment is in a sense an infinite agents setting where they can trade as much as necessary to eliminate idiosyncratic risk

- An example of market incompleteness:

- The Kiyotaki-Moore style collateral constraint $b_t \leq \theta k_t$

Financial Frictions: The BGG Model

- The two main streams of introducing financial frictions in DSGE models are
  - Asymmetric information
  - Limited commitment

- In this lecture we will go through one of the most seminal papers in the literature the Bernanke *et al.* (1999)

- it is a canonical NK model with a friction between the banks and the firms (called entrepreneurs)

- Eliminates $\mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1}R^K_{t+1}]$ of the RBC model and introduces a spread between the two discounted expected returns

Summary of the Model

- Main agents in the model: Households, entrepreneurs, retailers and a government
- Households work, consume and save in deposits
- Entrepreneurs are risk neutral. They purchase physical capital and produce goods with capital and labour
- Finance capital purchases with their net worth and bank credit
- Retailers are present to introduce nominal rigidities without complicating the entrepreneurs problem
  - Production side: NK framework with monopolistically competition and sticky prices
  - We will not go through that today

Entrepreneurs: Overview

- The entrepreneur j borrows loans B_t from the financial intermediary at time t to purchase capital K_t
- Loans together with her net worth $N_{E,t}$ finance the expenditure on new capital Q_tK_t.
- Entrepreneur's balance sheet therefore is:

$$B_t = Q_tK_t - N_{E,t} \tag{1}$$

where Q_t is the price of capital

Entrepreneurs: The Friction

- At every t the entrepreneur receives an idiosyncratic shock ω_t
- Results in return on capital: $\omega_t R_t^K$
  - ω_t is iid across all entrepreneurs
  - Drawn from a density f(ω_t) with mean E(ω_t)=1
- ω_t^j is only observable by entrepreneur j
- Costly State Verification (CSV) Townsend (1979): Lenders to observe the state, need to pay a monitoring cost, a proportion μ of the gross return $\mu R_{t+1}^K Q_t K_t$

Entrepreneurs: No Aggregate Risk

- We start with the case of no aggregate risk
  - R_{t+1}^K is known in advance and only uncertainty is idiosyncratic to the firm
  - The optimal contract is a risky debt contract (see Appendix A of Bernanke *et al.* (1999) for more)

- Entrepreneurs choose B_t and $Q_t K_t$ given R_t^K

- The optimal contract can be characterized by a gross non-default loan rate Z_t and a threshold value for the idiosyncratic shock $\bar{\omega}_t$

$$\bar{\omega}_{t+1} = \frac{Z_t B_t}{R_{t+1}^K Q_t k_t} \tag{2}$$

Entrepreneurs: Optimal Contract

- Entrepreneurs who receive a $\omega_t \geq \bar{\omega}_t$
  - Repay the intermediary $Z_t B_t$
  - Keep the difference $\bar{\omega}_{t+1} R^K_{t+1} Q_t k_t - Z_t B_t$
- Entrepreneurs who receive a $\omega_t < \bar{\omega}_t$
  - Default and get nothing
  - Intermediary pays the monitoring costs and gets what it finds: $(1-\mu) R^K_{t+1} Q_t K_t$

Entrepreneurs: Optimal Contract

- Under the optimal contract the lender should receive an expected return equal to its opportunity cost of funds
- Here, our bank's balance sheet is $B_t = D_t$ where D_t (deposits by households) is remunerated at the riskless rate R_t
- The loan contract must satisfy the bank's incentive compatibility constraint at time t

$$\underbrace{(1-\mu)R^K_{t+1}Q_tK_t\int_0^{\bar{\omega}_{t+1}}\omega f(\omega)d\omega}_{\text{Default}} + \overbrace{(1-p(\bar{\omega}_{t+1}))Z_tB_t}^{\text{No default}}$$

$$= R_tB_t$$

Entrepreneurs: Optimal Contract

- Using the Z_t from $\bar{\omega}_{t+1} = \frac{Z_t B_t}{R^K_{t+1} Q_t k_t}$, the constraint becomes

$$R^K_{t+1} Q_t K_t \left((1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1}(1 - p(\bar{\omega}_{t+1})) \right) \geq R_{t+1} B_t \tag{3}$$

- To simplify notation a bit let $\Gamma(\bar{\omega}_{t+1})$ to be the fraction of net capital received by the lender (the bank)
- $\mu G(\bar{\omega}_{t+1})$ to be monitoring costs:

$$\Gamma(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1}(1 - p(\bar{\omega}_{t+1})) \tag{4}$$

$$G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \psi f(\omega) d\omega \tag{5}$$

Entrepreneurs: Introducing Aggregate Risk

- So far we were abstracting from aggregate risk
- Now price and return of capital are given but uncertain
- $\bar{\omega}_{t+1}$ will now generally depend on the ex-post realization of R_{t+1}^K
- Entrepreneur is willing to offer a return that is free of any aggregate risk
  - Conditional on the return R_{t+1}^K the borrower offers state contingent non-default payment guaranteeing an expected return equal to the riskless rate
  - IC now implies a set of contracts for each realization of R_{t+1}^K

Entrepreneurs: Net Worth and Optimal Choice of Capital

- The entrepreneur's payoff is

$$\mathbb{E}_t\left[R^K_{t+1}Q_tK_t\int_{\bar{\omega}_{t+1}}^{\infty}\omega f(\omega)d\omega-(1-p(\bar{\omega}_{t+1}))Z_tB_t\right]=$$

$$\mathbb{E}_t\left[\left(1-\Gamma(\bar{\psi}_{t+1})\right)R^K_{t+1}Q_tK_t\right] \tag{6}$$

- She maximizes (6) given the net worth $N_{E,t}$, subject to the IC constraint (3) which, using (1), (4) and (5), can be rewritten as

$$\mathbb{E}_t\left[R^K_{t+1}Q_tK_t\left[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})\right]\geq R_{t+1}(Q_tK_t-N_{E,t})\right]$$

Entrepreneurs: Solution

- Let λ_t be the Lagrange multiplier associated with the IC constraint. Then the first order conditions are

$$\begin{aligned} K_t &: \mathbb{E}_t\left[(1-\Gamma(\bar{\omega}_{t+1})R^K_{t+1} + \lambda_t\left[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))R^K_{t+1} - R_{t+1}\right]\right] = 0 \\ \bar{\omega}_{t+1} &: \mathbb{E}_t\left[\Gamma'(\bar{\omega}_{t+1}) + \lambda_t(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))\right] = 0 \end{aligned}$$

plus the binding IC condition if $\lambda_t > 0$ with $\lambda_t = 0$ if it does not bind.

- Combining these two conditions, we arrive at

$$\mathbb{E}_t[R^K_{t+1}] = \mathbb{E}_t[\rho(\bar{\omega}_{t+1})R_{t+1}]$$

where the *premium on external finance*, $\rho(\bar{\omega}_{t+1})$ is given by

$$\rho(\bar{\omega}_{t+1}) = \frac{\Gamma'(\bar{\omega}_{t+1})}{[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))\Gamma'(\bar{\omega}_{t+1}) + (1-\Gamma(\bar{\omega}_{t+1}))(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))]}$$

External Finance Premium

- The external finance premium equation replaces the no arbitrage condition between R^K and R in NK and RBC models
- External finance is given by the safe rate, scaled up by a premium term
- Note that in the limiting case of $\bar{\omega}_{t+1}$ =0 and the probability of default tend to zero and $\mu = 0$, $\Gamma \to 0$ and the risk premium $\rho(\bar{\omega}_{t+1}) \to 1$ going back to the frictionless RBC

Aggregation

- Entrepreneurs exit with a fixed probability $(1 - \sigma_E)$
- New entrants get a start-up transfer equal to ξ_E of the old entrepreneurs wealth
- Aggregate net worth is

$$N_{E,t} = (\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1} K_{t-1}$$

- Entrepreneur who exit (die) consume

$$C_{E,t} = (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1} K_{t-1}$$

- The economy's resource constraint then is

$$Y_t = C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t) R_t^K Q_{t-1} K_{t-1}$$

General Equilibrium

- Households work, consume, save and pay taxes
  - Introduce habit in consumption

$$E_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} L_{t+i}^{1+\epsilon}] \tag{7}$$

  - Their budget constraint is:

$$C_t + D_t + T_t = W_t L_t + R_t D_{t-1}$$

- Banks are very simple, get deposits D_t from the households and provide loans B_t to the entrepreneurs at a rate Z_t

$$B_t = D_t$$

- Government has a balanced budget: $G_t = T_t$
- Production side in the BGG is NK
- Here we will just follow the RBC model structure

General Equilibrium

- Capital accumulation with investment adjustment costs carried out by **Capital Producers**

$$
\begin{aligned}
K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X (X_t - 1)^2 \\
S'(X_t) &= 2\phi_X (X_t - 1) \\
Q_t(1 - S(X_t) - X_t S'(X_t)) &+ \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2\right] = 1
\end{aligned}
$$

- $S(X_t)$ are investment adjustment costs equal to zero in a balance growth steady state

Summary of Equilibrium

$$
\begin{aligned}
Y_t &= C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1} \\
C_{E,t} &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
B_t &= Q_t K_t - N_{E,t} \\
\bar{\omega}_t &= \frac{Z_{t-1} B_{t-1}}{R_t^K Q_{t-1} K_{t-1}} \\
R_t^K &= \frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}} \\
r_t^K &= \frac{(1-\alpha) Y_t}{K_{t-1}}
\end{aligned}
$$

Steady State Equilibrium

- The zero growth steady state is given by

$$\begin{aligned} R_k &= \rho(\bar{\psi})R \\ N_E &= (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))R_k QK \\ R_k QK\left[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})\right] &= R(QK - N_E) \text{ or} \\ R_k[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})] &= R(\phi - 1) \end{aligned}$$

- where $\phi \equiv \frac{QK}{N_E}$ and

$$\rho(\bar{\psi}) = \frac{\Gamma'(\bar{\psi})}{\left[(\Gamma(\bar{\psi}) - \mu G(\bar{\psi}))\Gamma'(\bar{\psi}) + (1 - \Gamma(\bar{\psi}))(\Gamma'(\bar{\psi}) - \mu G'(\bar{\psi}))\right]}$$

Steady State Equilibrium

- with a resource constraint

$$
\begin{aligned}
Y &= C + C_E + G + I + \mu G(\bar{\psi}) R_k Q K \\
C_E &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi})) R_k Q K
\end{aligned}
$$

- and post-recursive equations

$$
\begin{aligned}
L &= QK - N_E \\
R_l &= \frac{\bar{\psi} R_k Q K}{L} \\
R_k &= \frac{Z + (1-\delta) Q}{Q} \\
Z &= \frac{(1-\alpha) Y}{K}
\end{aligned}
$$

The Density Function

- BGG choose a log-normal distribution.
- Thus $\omega_t = e^{x_t}$ where $x_t \sim N(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$
- This guarantees that $\mathbb{E}[\omega_t] = 1$

Steady State Solution Strategy

- Find two equations with only unknowns the entrepreneurial leverage (ϕ^E) and the return on capital (R^K).

$$\phi^E = \frac{QK}{N^E}$$

  Rearranging,

$$\frac{N^E}{K} = \frac{1}{\phi^E}$$

- From entrepreneurs net worth N^E, divide with capital and substituting $Q = 1$ we have:

$$\frac{N_E}{K} = (\sigma^E + \xi^E)(1 - \Gamma(\bar{\omega}))R^k$$

Steady State Solution Strategy

- From entrepreneurs balance sheet constraint $L = QK - N^E$, dividing with $K^s s$ we have

$$\frac{N_E}{K} = 1 - \frac{L}{K} = \frac{1}{\phi^E}$$

- Hence and using the fact that $R^K = \rho(\omega)R$,

$$\boxed{\phi_E = \frac{1}{(\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}))\rho(\omega)R}}$$

which is **the first equation** for the system

Steady State Solution Strategy

- From the Zero Profit Condition , solving for ϕ_E we get

$$\phi_E = -\frac{R}{R^K(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) - R}$$

- Substituting again:

$$\boxed{\phi_E = -\frac{R}{\rho(\bar{\omega})R(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) - R}}$$

  yields **the second equation** for the system

- We have 2 equations and 2 unknowns $(\bar{\omega}, \phi_E)$

- Actually we treat ϕ_E as known equal to 2 in SS and we calibrate σ_E to match this leverage

Steady State Calibration

- Production parameters, same as in the simple RBC
- For NK extension see code in GitHub
- Additional financial parameters to calibrate are σ_ω, σ_E, and μ.
- These three parameters are calibrated to hit four targets:
  (1) μ such as a default probability $p(\bar{\psi}) = 0.03$,

  (2) σ_ω for $\rho(\bar{\psi}) = 1.0025$ corresponding to a credit spread of 100 basis points as in GK,

  (3) σ_E for an entrepreneur leverage $\frac{QK}{N_E} = 2$ as in BGG

Inside the Code: Steady State

```
R              = 1/betta;
Lambda         = betta;

% Targets
p_mom          =  0.03;      % steady state default
p              = p_mom;
rho_mom        = 1 + 0.02/4; % steady state premium
rho              = rho_mom;
phie_mom       = 2;          % steady state leverage
phie           = phie_mom;
```

Inside the Code: Steady State

```
% give initial values
x0=[0.98,0.3, 0.49, 0.008, 0.98, 0.30, 0.2, 0.49];
fun = @(c) myfun_BGG_calib(c,p_mom,ksie,rho,phie_mom,R);
options=optimset('MaxFunEvals',1000);
c      = fsolve(fun,x0,options);
sigmae  = c(1); %parameter
sigma_omega_ss = c(2);  %parameter
fnGam = c(3);
fnG   = c(4);
DGam  = c(5);
DG    = c(6);
mon   = c(7);  %parameter
omega  = exp(c(8));
```

Inside the Code: Steady State

```
function F = myfun_BGG_calib(c,p_mom,ksie,rho,phie_mom,R)
F=[phie_mom-1/((c(1)+ksie)*(1-c(3))*rho*R); %Eq 1
phie_mom + (R/((rho*R)*(c(3)-c(7)*c(4))-R)); %Eq 2
rho   - (c(5)/((c(3)-c(7)*c(4))*c(5)+(1-c(3))*(c(5)-c(7)*c(6)))); %rho
p_mom - (logncdf(exp(c(8)),-0.5*(c(2))^2,c(2))); %p
c(4)  - (1- normcdf((0.5*(c(2))^2 -log(exp(c(8))))/c(2),0,1)); %fnG
c(3)  - (c(4)+exp(c(8))*(1-p_mom)); %fnGam
c(5)  - (1-p_mom); %DGam
c(6)  - (1/(exp(c(8))*c(2)*sqrt(2*(4*atan(1))))*exp(-((log(exp(c(8)))
    +.5*(c(2))^2)^2)/(2*(c(2))^2)));]; %DG
end
```

Steady State: Real Sector

- To find Z we go to the only equation that has it:

$$Z = \bar{\omega} R^K Q \frac{K}{L}.$$

From the entrepreneurs leverage we know

$$\frac{K}{L} = \frac{1}{1 - \frac{1}{\phi^E}} = \frac{\phi^E}{\phi^E - 1}$$

$$r_K = R^k + \delta - 1$$

- For the real sector variables: define all variables as share of capital first

$$\frac{C_e^{ss}}{K^{ss}} = (1 - \sigma^e)(1 - \xi^e)(1 - \Gamma(\bar{\psi^{ss}}))R_{k,ss}Q^{ss}$$

Steady State: Real Sector

- The resource constraint is $Y^{ss} = C^{ss} + I^{ss} + G^{ss} + \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss} K^{ss}$

- Using the production function,

$$C^{ss} = L^{1-\alpha} K^{\alpha} - \delta K^{ss} - \eta L^{1-\alpha} K^{\alpha} - C_e - \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss} K^{ss}$$

  where $\eta = G^{ss}/Y^{ss}$

- Rearranging terms we get :

$$\frac{C^{ss}}{K^{ss}} = (1-\eta)(\frac{L^{ss}}{K^{ss}})^{1-\alpha} - \delta - \frac{C_e}{K^{ss}} - \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss}$$

- From the resource constraint we have

$$\frac{Y^{ss}}{K^{ss}} = \left(\frac{C^{ss}}{K^{ss}} + \delta + \frac{C_e}{K^{ss}} + \mu G(\bar{\psi_t}) R_k^{ss} Q^{ss} \right) \frac{1}{1-\eta}$$

Steady State: Real Sector

- The labour leisure F.O.C in steady state is

$$(L^{ss})^{\epsilon}\chi = [W^{ss}((1-\gamma)c)^{-1}(1-\beta\gamma)]$$

- Substituting for the wage it yields:

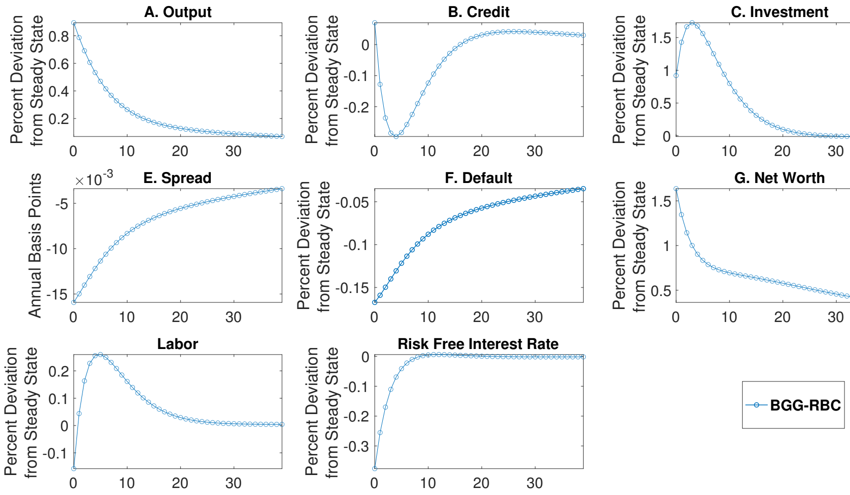
$$(L^{ss})^{1+\epsilon} = \frac{(1-\beta\gamma)(1-\alpha)}{\chi(1-\gamma)}\frac{Y^{ss}}{C^{ss}}$$

- By knowing $\frac{C^{ss}}{K^{ss}}$ and $\frac{Y^{ss}}{K^{ss}}$ is straightforward to calculate $\frac{Y^{ss}}{C^{ss}}$ and find L^{ss}.
- Finally, knowing L^{ss} and $\frac{Y^{ss}}{K^{ss}}$ from the production function we find the capital

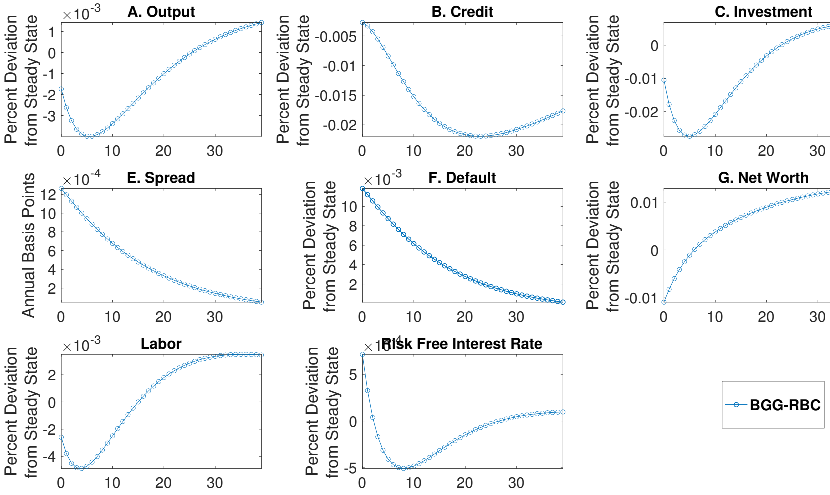
$$K^{ss} = \left(\frac{\frac{Y^{ss}}{K^{ss}}}{L^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}$$

- Hence, having capital, by reverse engineering we can find the values for $(I^{ss}, C^{ss}, C_e^{ss}, Y^{ss})$

Impulse Responses: Productivity Shock (positive)



Impulse Responses: Risk Shock (negative)



Christiano, Motto and Rostagno (2014) extension

- Identify the *Risk Shock*
- Built on the Bernanke *et al.* (1999)
- Under a NK framework
- Lots of additional frictions and shocks (capacity utilization rate, taxes, long-term bonds...)
- *Estimated* for the US economy

The Risk Shock

- They refer to as risk the time period t cross-sectional standard deviation of $\log(\omega)$
- Risk is high in periods when $\sigma_{\omega,t}$ is high, and there is substantial dispersion in the outcomes across entrepreneurs
- Model estimation assigns a large role to $\sigma_{\omega,t}$
- Disturbances in $\sigma_{\omega,t}$ trigger responses in the model that resemble actual business cycles
- They find that fluctuations in $\sigma_{\omega,t}$ account for 60 percent of the fluctuations in the growth rate of aggregate US output since the mid-1980s

The Risk Shock

- Essentially from being a parameter $\sigma_{\omega,t}$ now follows an AR(1) process
  `log(sigma_omega / sigma_omega_ss) = rhosigma * log(sigma_omega(-1) / sigma_omega_ss) + e_RS;`
- Where `e_RS` is the risk shock

Unanticipated and News Shocks

- A standard assumption in estimated equilibrium models is that a shock's statistical innovation becomes known to agents only when the innovation is realized

- Literature document that people receive information about the period t statistical innovation before the innovation is realized

Unanticipated and News Shocks

- CMR (2014) consider the following shock representation

$$x_t = \rho_x x_{t-1} + \xi_0 + \xi_{1,t-1} ... + \xi_{p,t-p}$$

- x_t is the log deviation of the shock from its non-stochastic steady state
- ξ_0 is the unanticipated component of the statistical innovation and $\xi_{1,t-1} ... + \xi_{p,t-p}$ the anticipated, or news components
- They estimate the model for giving this structure to technology, monetary policy, government spending, equity, and risk shocks
- They find that the model that has the highest marginal likelihood is the one with news on the risk shock

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, **1**, 1341–1393.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, **21**(2), 265–293.