

A Course on DSGE Models with Financial Frictions

Part 3: Asymmetric Information and Limited Commitment

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From Complete Markets to Financial Frictions

- To introduce financial frictions we need two deviations from the complete markets representative agent setting
 - Incomplete markets
 - Heterogeneous Agents
- A representative agent environment is in a sense an infinite agents setting where they can trade as much as necessary to eliminate idiosyncratic risk
- An example of market incompleteness:
- The Kiyotaki-Moore style collateral constraint $b_t \leq \theta k_t$

Financial Frictions: The BGG Model

- The two main streams of introducing financial frictions in DSGE models are
 - Asymmetric information
 - Limited commitment
- In this lecture we will go through one of the most seminal papers in the literature the [Bernanke *et al.* \(1999\)](#)
- it is a canonical NK model with a friction between the banks and the firms (called entrepreneurs)
- Eliminates $\mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}^K]$ of the RBC model and introduces a spread between the two discounted expected returns

Summary of the Model

- Main agents in the model: Households, entrepreneurs, retailers and a government
- Households work, consume and save in deposits
- Entrepreneurs are risk neutral. They purchase physical capital and produce goods with capital and labour
- Finance capital purchases with their net worth and bank credit
- Retailers are present to introduce nominal rigidities without complicating the entrepreneurs problem
 - Production side: NK framework with monopolistically competition and sticky prices
 - We will not go through that today

Entrepreneurs: Overview

- The entrepreneur j borrows loans B_t from the financial intermediary at time t to purchase capital K_t
- Loans together with her net worth $N_{E,t}$ finance the expenditure on new capital $Q_t K_t$.
- Entrepreneur's balance sheet therefore is:

$$B_t = Q_t K_t - N_{E,t} \tag{1}$$

where Q_t is the price of capital

Entrepreneurs: The Friction

- At every t the entrepreneur receives an idiosyncratic shock ω_t
- Results in return on capital: $\omega_t R_t^K$
 - ω_t is iid across all entrepreneurs
 - Drawn from a density $f(\omega_t)$ with mean $E(\omega_t)=1$
- ω_t^j is only observable by entrepreneur j
- Costly State Verification (CSV) [Townsend \(1979\)](#): Lenders to observe the state, need to pay a monitoring cost, a proportion mu of the gross return $\mu R_{t+1}^K Q_t K_t$

Entrepreneurs: No Aggregate Risk

- We start with the case of no aggregate risk
 - R_{t+1}^K is known in advance and only uncertainty is idiosyncratic to the firm
 - The optimal contract is a risky debt contract (see Appendix A of [Bernanke *et al.* \(1999\)](#) for more)
- Entrepreneurs choose B_t and $Q_t K_t$ given R_t^K
- The optimal contract can be characterized by a gross non-default loan rate Z_t and a threshold value for the idiosyncratic shock $\bar{\omega}_t$

$$\bar{\omega}_{t+1} = \frac{Z_t B_t}{R_{t+1}^K Q_t k_t} \quad (2)$$

Entrepreneurs: Optimal Contract

- Entrepreneurs who receive a $\omega_t \geq \bar{\omega}_t$
 - Repay the intermediary $Z_t B_t$
 - Keep the difference $\bar{\omega}_{t+1} R_{t+1}^K Q_t k_t - Z_t B_t$
- Entrepreneurs who receive a $\omega_t < \bar{\omega}_t$
 - Default and get nothing
 - Intermediary pays the monitoring costs and gets what it finds: $(1 - \mu) R_{t+1}^K Q_t K_t$

Entrepreneurs: Optimal Contract

- Under the optimal contract the lender should receive an expected return equal to its opportunity cost of funds
- Here, our bank's balance sheet is $B_t = D_t$ where D_t (deposits by households) is remunerated at the riskless rate R_t
- The loan contract must satisfy the bank's incentive compatibility constraint at time t

$$\underbrace{(1 - \mu)R_{t+1}^K Q_t K_t \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega}_{\text{Default}} + \overbrace{(1 - p(\bar{\omega}_{t+1})) Z_t B_t}^{\text{No default}} = R_t B_t$$

Entrepreneurs: Optimal Contract

- Using the Z_t from $\bar{\omega}_{t+1} = \frac{Z_t B_t}{R_{t+1}^K Q_t k_t}$, the constraint becomes

$$R_{t+1}^K Q_t K_t \left((1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} (1 - p(\bar{\omega}_{t+1})) \right) \geq R_{t+1} B_t \quad (3)$$

- To simplify notation a bit let $\Gamma(\bar{\omega}_{t+1})$ to be the fraction of net capital received by the lender (the bank)
- $\mu G(\bar{\omega}_{t+1})$ to be monitoring costs:

$$\Gamma(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} (1 - p(\bar{\omega}_{t+1})) \quad (4)$$

$$G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \psi f(\omega) d\omega \quad (5)$$

Entrepreneurs: Introducing Aggregate Risk

- So far we were abstracting from aggregate risk
- Now price and return of capital are given but uncertain
- $\bar{\omega}_{t+1}$ will now generally depend on the ex-post realization of R_{t+1}^K
- Entrepreneur is willing to offer a return that is free of any aggregate risk
 - Conditional on the return R_{t+1}^K the borrower offers state contingent non-default payment guaranteeing an expected return equal to the riskless rate
 - IC now implies a set of contracts for each realization of R_{t+1}^K

Entrepreneurs: Net Worth and Optimal Choice of Capital

- The entrepreneur's payoff is

$$\mathbb{E}_t \left[R_{t+1}^K Q_t K_t \int_{\bar{\omega}_{t+1}}^{\infty} \omega f(\omega) d\omega - (1 - p(\bar{\omega}_{t+1})) Z_t B_t \right] =$$
$$\mathbb{E}_t \left[(1 - \Gamma(\bar{\psi}_{t+1})) R_{t+1}^K Q_t K_t \right] \quad (6)$$

- She maximizes (6) given the net worth $N_{E,t}$, subject to the IC constraint (3) which, using (1), (4) and (5), can be rewritten as

$$\mathbb{E}_t \left[R_{t+1}^K Q_t K_t [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \right] \geq R_{t+1} (Q_t K_t - N_{E,t})$$

Entrepreneurs: Solution

- Let λ_t be the Lagrange multiplier associated with the IC constraint. Then the first order conditions are

$$\begin{aligned} K_t &: \mathbb{E}_t \left[(1 - \Gamma(\bar{\omega}_{t+1})R_{t+1}^K + \lambda_t \left[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))R_{t+1}^K - R_{t+1} \right] \right] = 0 \\ \bar{\omega}_{t+1} &: \mathbb{E}_t \left[\Gamma'(\bar{\omega}_{t+1}) + \lambda_t (\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})) \right] = 0 \end{aligned}$$

plus the binding IC condition if $\lambda_t > 0$ with $\lambda_t = 0$ if it does not bind.

- Combining these two conditions, we arrive at

$$\mathbb{E}_t[R_{t+1}^K] = \mathbb{E}_t[\rho(\bar{\omega}_{t+1})R_{t+1}]$$

where the *premium on external finance*, $\rho(\bar{\omega}_{t+1})$ is given by

$$\rho(\bar{\omega}_{t+1}) = \frac{\Gamma'(\bar{\omega}_{t+1})}{[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))\Gamma'(\bar{\omega}_{t+1}) + (1 - \Gamma(\bar{\omega}_{t+1}))(\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}))]}$$

External Finance Premium

- The external finance premium equation replaces the no arbitrage condition between R^K and R in NK and RBC models
- External finance is given by the safe rate, scaled up by a premium term
- Note that in the limiting case of $\bar{\omega}_{t+1} = 0$ and the probability of default tend to zero and $\mu = 0$, $\Gamma \rightarrow 0$ and the risk premium $\rho(\bar{\omega}_{t+1}) \rightarrow 1$ going back to the frictionless RBC

Aggregation

- Entrepreneurs exit with a fixed probability $(1 - \sigma_E)$
- New entrants get a start-up transfer equal to ξ_E of the old entrepreneurs wealth
- Aggregate net worth is

$$N_{E,t} = (\sigma_E + \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1}K_{t-1}$$

- Entrepreneur who exit (die) consume

$$C_{E,t} = (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\omega}_t))R_t^K Q_{t-1}K_{t-1}$$

- The economy's resource constraint then is

$$Y_t = C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t)R_t^K Q_{t-1}K_{t-1}$$

General Equilibrium

- Households work, consume, save and pay taxes
 - Their budget constraint therefore is:

$$C_t + D_t + T_t = W_t L_t + R_t D_{t-1}$$

- Banks are very simple, get deposits D_t from the households and provide loans B_t to the entrepreneurs at a rate Z_t

$$B_t = D_t$$

- Government has a balanced budget: $G_t = T_t$
- Production side in the BGG is NK
- Here we will just follow the RBC model structure

- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, **1**, 1341–1393.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, **21**(2), 265–293.