Solution for Exercise: Week 4-5

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1 Liquidity Injections in the GK framework

Exercise: Consider the model of GK with the liquidity injections and use the code of the steady state for the original GK and extend this to include the liquidity injections framework we saw in class. Firstly, solve for the SS by pen and paper. Express M and R^M as function of variables you know $M^{ss} = \chi_m S^{ss}$ and $R^M = f(R^K)$. You will end up to the 2 equations for 2 unknowns (ϕ, R^K) as we did in the previous lecture. For the code you don't have to work with the `.mod` file and the `steady_state.m` file. Create a new Matlab file just for the SS. Copy paste the old GK SS file, delete the preamble and the ending that connects it with Dynare. Make the adjustments in the %% **Banker Solution (GK)** section. Set $\chi_m = 0.001$ and $\omega = 0.5$. Extend the `myfun_GK1.m` with the new components. Find ϕ, R^K, R^M in steady state and report those values.

Solution:

The incentive constraint of the bank in steady state is

$$QS = \phi^B N + \omega M, \tag{1}$$

where $M = \chi_m QS \rightarrow \frac{M}{S} = \chi_m$. We also know that in steady state Q=1.

By dividing (1) over loans we have $\frac{S}{S} = \phi^B \frac{(N)}{S} + \omega \frac{M}{S}$. Rearranging terms :

$$\frac{N^B}{S} = \frac{1}{\phi^B}(1 - \omega \chi_m). \tag{2}$$

From the bank's balance sheet constraint we have $D = S - N - M$. Dividing over S:

$$\frac{N^B}{S} = 1 - \frac{D}{S} - \chi_m. \tag{3}$$

The bank's net worth is $N = (\sigma^B + \xi^B)(R^K QS) - \sigma^B(RD + R^M M)$. Again dividing over S, setting $Q = 1$ and rearranging terms, yields:

$$\frac{N}{S} = [(\sigma^B + \xi^B)R^K - \sigma^B(R\frac{D}{S} + R^M\chi_m]. \tag{4}$$

Substituting (3) in (4) and using $R = 1/\beta$ we have

$$\frac{N^B}{S} = (\sigma^B + \xi^B)(R^K) - \sigma^B(\frac{1}{\beta}(1 - \frac{N}{S} - \chi_m) + R^M M)$$

Rearranging terms and substituting $R^M = \omega R^K + (1-\omega)R$

$$\frac{N^B}{S} = \frac{(\sigma^B + \xi^B)R^K - \sigma^B/\beta + \omega\sigma^B\chi_m(R - R^K)}{1 - \sigma^B/\beta} = \frac{1}{\phi^B}(1 - \omega\chi_m). \tag{5}$$

So we get the **first equation** for the steady state leverage,

$$\boxed{\phi^B = \frac{(1-\sigma^B/\beta)(1-\omega\chi_m)}{(\sigma^B + \xi)R^K - \sigma^B/\beta + \omega\sigma^B\chi_m(R - R^K)}} \tag{6}$$

Now I turn in finding the steady state value of the leverage using the definition of leverage. We know that

$$\phi^B = \frac{\nu_{d,j}}{\theta - spread} \tag{7}$$

We also know that $\nu_d = \Lambda\Omega R = \beta\Omega\frac{1}{\beta} = \Omega$ After substituting ν_d, the leverage (ϕ^B) becomes

$$\phi^B = \frac{\Omega}{\theta - \Lambda\Omega(R^K - R)}$$

.

Rearranging terms and substituting Ω given by

$$\Omega = (1 - \sigma^B) + \sigma^B\phi^B\theta \tag{8}$$

the leverage yields:

$$\boxed{\phi^B = \frac{(1-\sigma^B) + \sigma^B\phi^B\theta}{\theta - ((1-\sigma^B) + \sigma^B\phi^B\theta)(\beta R^K - 1)}} \tag{9}$$

being **the second equation** in the system.

Hence, we have 2 equations (6, 9) and 2 unknowns (ϕ^B, R^B). After solving this system it is straightforward to find $(\frac{N}{L}, \frac{D}{L})$.

Code:

We dont neet to work with the `.mod` file at all, just to rearrange the steady state file. Create a new SS file and copy -paste all the parameter values in the Matlab file and delete the preamble and the ending of the `steady_state.m` file so as to work without the `.mod` file at all.

The part we need is here

```
%% Banker Solution (GK)
R_1=[1.0122, 3.55];
%%Solution for the Loan Interest Rate(1) and phi(2) on SS%%
%%Uses function determined by the solution of SS divided by K.Solves a system%%
fun = @(c) myfun_GK_liq_inj(c,sigmab,ksi,betta,theta,omega,chim);
options=optimset('MaxFunEvals',10000,'MaxIter',10000,'Display','off');
c=fsolve(fun,R_1,options);

Rk=c(1);
phi=c(2);
Rm = omega*Rk +(1-omega)*R;
```

Where we have created a new function called `myfun_GK_liq_inj` almost identical with the `myfun_GK1`, just changed to include the new formulation with the liquidity. It includes the two equations for ϕ found above. It also needs to additional inputs, ω and the liquidity ratio in SS, a parameter, χ_m which we have set to 0.001.

Our function `myfun_GK_liq_inj` is now:

```
function F = myfun_GK_liq_inj(c,sigmab,ksi,betta,theta,omega,chim)
F(1)=c(2) - (1-sigmab/betta)*(1-omega*chim)/(c(1)*(sigmab+ksi)-sigmab/betta + omega*sigmab*chim*(1/betta-c(1))); %Equation 1
F(2)=c(2) - ((1-sigmab)+sigmab*c(2)*theta)/(theta-((1-sigmab)+sigmab*c(2)*theta)*(betta*c(1)-1));%Equation 2
end
```

Solving the system, the new value for `Rk` is 1.0122, `phi` is 3.5585 and `Rm` is 1.0112.

2 Countercyclical Buffer in the GK framework

Exercise: Consider the original model of GK and turn the parameter θ into a a time varying parameter (actually a variable in `Dynare` wording) that follows the countercyclical buffer equation as

$$\theta_t = \theta^* + \psi_k gap_{k,t}$$

where $\theta^* = 0.383$, $\psi_k = 0.15$ and the definition of the credit gap follows

$$gap_{k,t} = \frac{Q_t S_t}{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}} - \frac{Q^{ss} S^{ss}}{4Y^{ss}}$$

Finally, plot the impulse responses for $Y, C, I, N, \phi, spread$ after a capital quality shock of `std=1` for the cases of

- $\theta_t = \theta^*$ (original GK)

- $\theta_t = \theta^* + \psi_k gap_{k,t}$

Solution:

In the `.mod` file you now have to introduce a new parameter `theta_star` which is the SS value of `theta`. For the case of the original GK model `theta=theta_star` and nothing changes. For the case of countercyclical requirements in the mod file, add:

```
theta = theta_star + psi_k*((Q*S)/(Y+Y(-1)+Y(-2)+Y(-3)) - steady_state(Q)*
    steady_state(S)/(4*steady_state(Y)));
```

Also add in the parameter section `psi_k` = 0.5. Note that in the steady state, nothing changes here, `theta` is always equal to `theta_star` in SS.

Plotting IRFs:

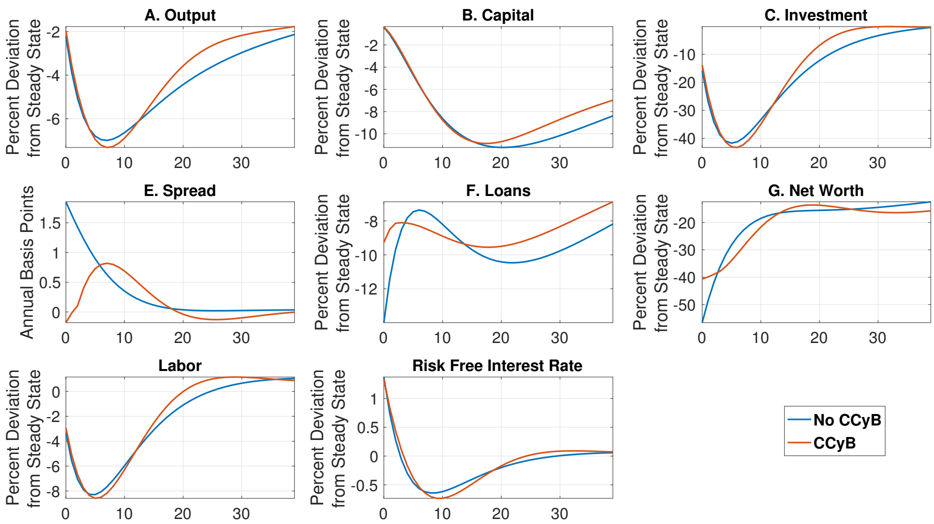

Figure 1: IRFs to capital quality shock with countercyclical capital requirements