Bargaining over Taxes and Entitlements in the Era of Unequal Growth

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Abstract

Entitlement programs have become an increasing component of total government spending in the US over the last six decades. To some observers, this growth of the welfare state is excessive and unwarranted. To others, it is a welcome counter-acting force to the rapid increase in income inequality. Using a political-economy model where parties bargain over taxes and entitlements, we argue that such dynamics can be explained by two factors. The first one is that institutional features of policy determination, in particular budget rules, make the status quo levels of taxes and entitlements difficult to change. The second one is that the country experienced a process of "unequal growth," where top earners became richer while the income levels of the bottom 50 percent remained stagnant. Richer agents would like the government to provide more public goods as the economy grows. Low-income earners are willing to support such policies only in exchange for an expansion of entitlement programs. Sustained bargaining power by a party that represents the latter, amid budget rules, results in a rising share of entitlements. We explain how parties can take advantage of budget rules to tilt the evolution of policy in their favor in a simple two-period model. We then calibrate an infinite horizon version of the model to the US, and show that it delivers dynamics consistent with the data. Through counter-factual experiments, we find that while entitlements programs are sub-optimally large, welfare outcomes are better than those under alternative budget rules and in scenarios without rules, making it explicit that the type of budget rule matters for both welfare and equity.

Keywords: Unequal growth, entitlements, inequality, redistribution, dynamic legislative bargaining, endogenous status quo, political economy, mandatory spending, budget rules.

JEL Classification: C7, D6, E6

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1 Introduction

Prior to the Great Depression, nearly all federal expenditures in the United States were discretionary. That is, spending did not occur in a given year unless Congress provided funding through an annual appropriations' bill. Following the Social Security Act of 1935, an increasing percentage of the federal budget became devoted to financing mandatory spending programs. This trend accelerated in the mid 1960s, and continued until the present day, as illustrated by Figure 1. The solid line represents the share of mandatory spending in total outlays (excluding debt interest payments), whereas the dotted line corresponds to discretionary spending.[1]

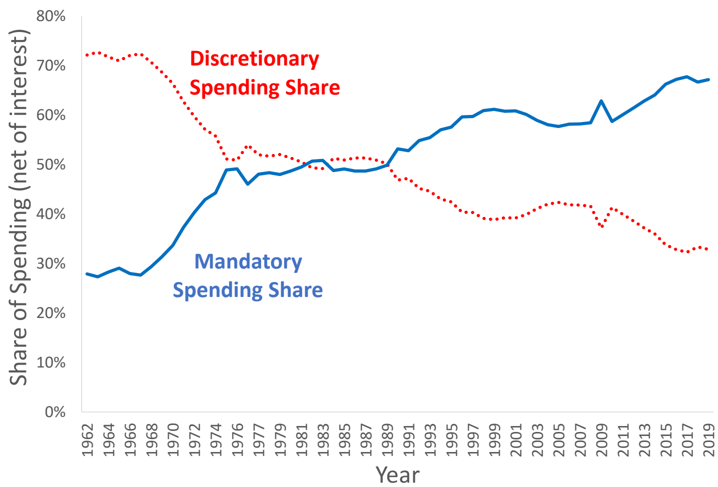


Figure 1: Mandatory and Discretionary Spending, 1962-2019.

By 2019, mandatory spending represented over 60% of all government spending, as shown in the left panel of Figure 2. A key characteristic is that these programs need to be established under authorization laws and their generosity can only be modified with approval of a majority (or super-majority) of members of Congress. The resulting bargaining process is similar to the one engaged by legislators when attempting to modify the tax code.

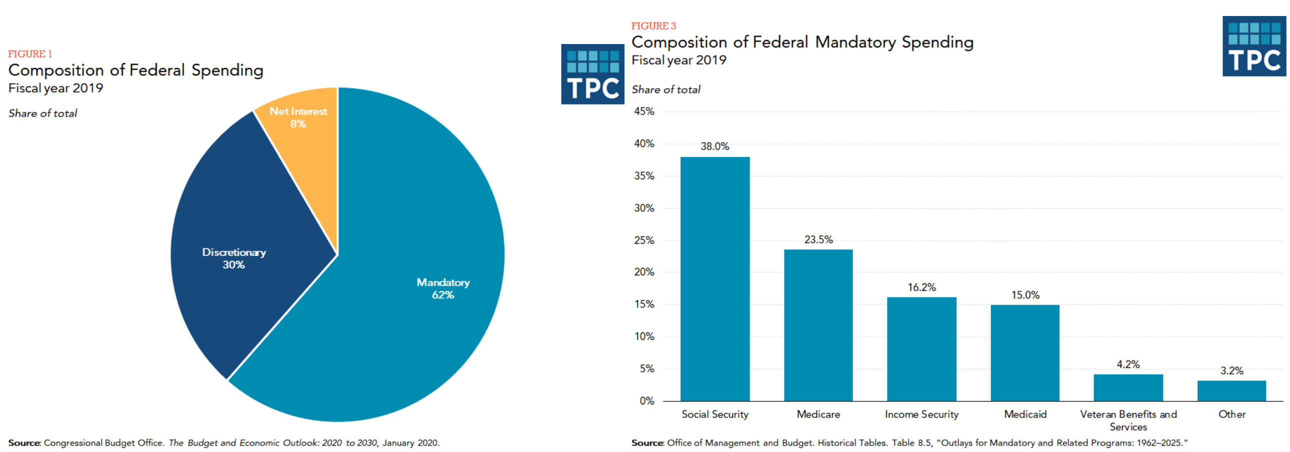


Figure 2: Composition of Federal Spending (left) and of Mandatory Spending (right), 2019

[1]Data sources are described in Section 7.1

While there is a wide range of goods financed by the government which are mandatory, the largest ones are *entitlement programs*, such as Social Security, Medicaid, Income Security, and Medicare (see right panel of Figure 2), which provide private transfers to qualified individuals. These social welfare programs have specific criteria set by Congress, such as eligibility and benefit generosity, which generate significant redistribution towards eligible recipients from wealthier individuals. Explaining what could have caused the sustained expansion of what is known as the "welfare state," and showing that the type of budget rule in place matters for welfare and equity, are the two main objectives of this paper.

Our explanation relates the expansion of entitlement programs to the process of *unequal growth* experienced by the US over the same period. Figure 3, replicated from Piketty, Saez and Zucman (2016), shows the evolution of pre-tax income for the average worker in the US (green line) between 1962 and 2016, together with that of the bottom 50% of earners (dark-brown line). While average income grew by 61% in real terms, the income of the bottom half of the population stayed relatively flat. This process of unequal growth, where the productivity gains only benefited the richest individuals in society, resulted in a significant increase in income inequality. The increase in entitlement programs documented in Figure 1, together with the asymmetric evolution of income across individuals observed in Figure 3, suggest that the two phenomena (more inequality and larger entitlement programs) may be related.

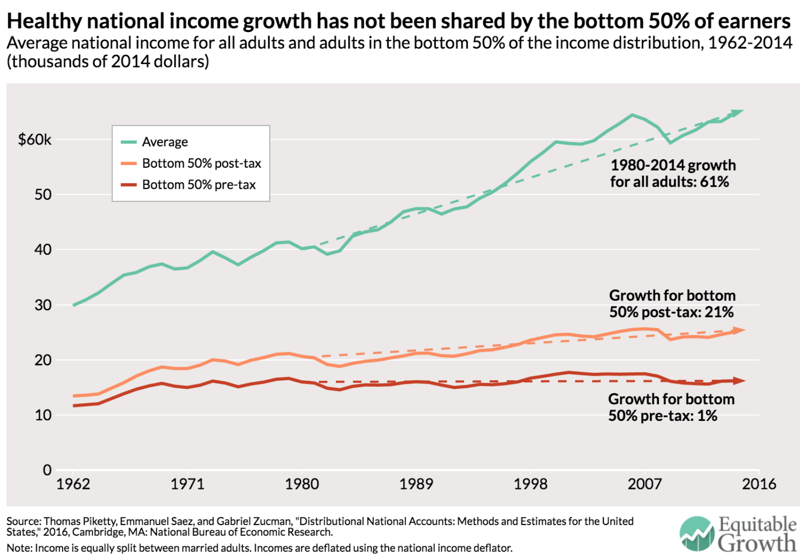


Figure 3: Pre-tax income growth in the US, 1962-2016

In this paper, we show that the process of unequal growth can lead to an expansion of entitlements in an economy with budget rules, composed by a *tax code* and *entitlement programs*. We do this in a dynamic political-economy model featuring disagreement over the distribution of resources and an endogenous status quo. We study an endowment economy where two parties representing individuals from different income groups (rich and poor) must decide how to finance (pure) public goods and whether to redistribute resources through entitlement programs. While both agents have the same preferences, they disagree on the degree of redistribution. This is the case because rich agents are initially endowed with more resources than poor ones. As a result, the rich prefer a small government whose main role is to provide public goods, whereas the poor would like the government to provide large entitlement programs – even if this involves high taxes. We do not restrict the set of instruments that policymakers have access to, so in principle the political equilibrium can be efficient. Our main friction, thus, is not distortionary taxation, but instead disagreement regarding *where* on the Pareto frontier society should be. Under the 'veil of ignorance' all agents agree on policies that decentralize the first best, which is both efficient and equitable. However, once they learn their type—at birth— each group prefers less equitable allocations that benefit them disproportionately. Political parties channel this through policy choices once they gain decision-making power. Their ability to do so depends on the budgetary arrangements regulating how fiscal policy is determined.

We characterize a political equilibrium with bargaining over taxes and entitlements. We follow the protocol used in the bargaining literature under endogenous status quo to represent policy determination (see, for example, Bowen, Chen, and Eraslan (2014)). Every period, a party is selected at random to make a budget proposal involving public good spending, entitlement provision, and taxes. The proposer makes a take-it-or-leave-it offer to the opposition. The proposal is only implemented if the other party accepts. If the proposal is rejected, last period's taxes and entitlements (e.g., the status-quo) are implemented instead. What makes this environment different from one in which there are no budget rules is that taxes and entitlement programs are hard to change.

We show that budget rules make fiscal policy more persistent, reducing the inefficiently high volatility of unconstrained environments. This is beneficial for welfare. In addition, these rules provide insurance against expropriation. By choosing the level of entitlements appropriately when in power, poor agents ensure a minimum degree of consumption through redistributive policies. By choosing taxes appropriately, rich agents shield themselves against excessively high taxes. In the long-run, however, budget rules result in allocations involving too much private consumption and under-provision of public goods. Because of this, introducing budget rules may not generate a Pareto improvement relative to an unconstrained environment.

In order to understand how the process of unequal growth affects the share of entitlements over time, we conduct a quantitative experiment in which the level of income of rich individuals increases (permanently and unexpectedly), whereas the income of the poor stays the same. We calibrate the model parameters to the US economy, and impose a change in relative income between 1960 and 2010 consistent with Figure 3. Using time series for the share of seats in Congress by Democrat and Republican legislators, we back out a sequence of power alternation consistent with the data. Our simulation delivers an increase in the share of entitlements to total spending that is consistent with Figure 1. The increase in this share arises for two reasons. One of them is that Democrats have had proposal power more often than Republicans during that interval of time. A poor proposer always tries to increase entitlement programs (whereas a rich agent would try to reduce it). The second reason is that growth increases the size of the pie, and hence the fiscal capacity of the government. In a bargaining equilibrium, more inequality results in an expansion of the set of policies that can arise in this model. Under unequal growth, the additional resources are endowed only to rich agents. Hence, even a planner would choose to expand entitlements in order to redistribute the extra income and attain a more equal allocation. A planner would, however, change the entitlements instantaneously. In the bargaining equilibrium, this change is inefficiently slow, and, when combined with a long sequence of the poor in power, results in a program that is inefficiently large.

Our estimates indicate that these inefficiencies are quantitatively relevant. The outcome is, nonetheless, significantly better (in welfare terms) than what would be attained under alternative budget rules or in a world without any rules. To show this, we evaluate how alternative budget rules fare for the equilibrium. We show that for the current calibration of the model, budget rules on taxes and entitlements are significantly better for society than rules that would only affect one of these policies. Interestingly, low-income earners are significantly worse off when entitlements are mandatory but taxes are easy to change (e.g. when they are not status-quo variables). This happens because the rich have a significant advantage by owning a larger share of income in society, which in turn endows them with relatively more bargaining power. If tax cuts are easy to implement when in power, high-income earners can effectively keep the size of the government small and entitlement programs at bay. Hence, budget rules in both, taxes and entitlements, are needed to explain a sustained increase in the share of entitlements. For completeness, we also consider a rule making public goods mandatory spending and show that, while such goods are more efficiently provided, the equilibrium is associated with welfare losses relative to our benchmark case. This is primarily due to the increased volatility of private consumption, which matters when agents are risk averse.

Our paper makes three important contributions to the existing literature. First, we analyze the bargaining process over entitlement programs and taxes in an infinite horizon model with concave utilities. This allows us to evaluate how budget rules affect allocations in an environment where agents are heterogeneous in income levels and have preferences for smooth consumption profiles. A key finding is that introducing budget rules is not always beneficial to society. Our second contribution, is to point out that the type of fiscal policy targeted by a budget rule matters. This feature of budgetary design has been overlooked by most of the literature on dynamic bargaining. Our third contribution is to provide a rationale for the sustained increase in the share of entitlements in total spending over the last six decades. We show that a process of unequal growth, paired with long incumbency by the Democratic party can result in such dynamics in a bargaining model with budget rules on taxes and entitlements. We see that as a novel contribution to existing literature. Finally, we also make a methodological contribution. Because we relax the linearity assumption on preferences used in most of the endogenous status-quo literature, characterization of the symmetric Markov-perfect equilibrium in the infinite-horizon dynamic game requires a numerical approach. We propose a numerical method that can robustly compute the equilibrium for a wide range of parameters. Computation is complex, because we have a multi-dimensional state space (e.g. two endogenous status-quo variables). Our method is inspired by Duggan and Kalandrakis (2012), and uses advances in the quantitative macroeconomics literature, such as those in Dvorkin et al. (2021).

A brief literature review can be found in the next section. Section 3 defines the economic environment, while Section 4 characterizes theoretically efficient allocations for arbitrary Pareto weights and defines our concept of equity. In Section 5 we define the bargaining protocol and the political equilibrium. Section 6 characterizes a two-period model example to illustrate how these rules shape the equilibrium. The infinite-horizon dynamic model is solved quantitatively in Section 7 for a benchmark economy. Finally, Section 8 concludes and points venues for future research.

2 Literature Review

Our paper contributes to the literature studying redistributive policies and public good provision in the presence of political fictions. Similarly to Lizzeri and Persico (2001), we analyze how alternative arrangements can improve on allocations obtained under discretionary spending. While they focus on 'winner-take-all' systems versus proportional systems, we consider budget rules in legislative models of bargaining instead.

We model budget rules following the literature on legislative bargaining with endogenous status quo, along the lines of Baron and Ferejohn (1989). In our model, the tax code and the level of entitlement programs represent a status quo which remains in place unless some political group proposes an alternative allocation that is acceptable to the opposition. This mechanism creates a dynamic strategic link between the groups by impacting the trade-off faced by current policy-makers, and imposes limits on the degree of redistribution, in the spirit of Diermeier, Egorov, and Sonin (2017). Because fiscal policy directly impacts private allocations—and the size of the endowment is fixed— our work is, more generally, related to papers analyzing divide-the-dollar games with multilateral bargaining. One branch of this literature focuses on the continuous space (e.g., Kalandrakis (2004), Kalandrakis (2010), Baron and Herron (2003), Anesi and Seidmann (2013), Nunnari (2018)), as in our theoretical analysis of the finite horizon model. The other branch restricts attention to choices in a discrete state space (e.g., Anesi (2010), Diermeier and Fong (2011), Diermeier, Prato, and Vlaicu (2016), and Duggan and Kalandrakis (2012)), as we do in the numerical implementation. A thorough discussion of the recent developments in the legislative bargaining literature with endogenous status quo can be found in Eraslan, Evdokimov, and Zápal (2020). One important departure from these papers is that we consider government policies that affect both, public and private goods. Moreover, we emphasize the distortions on public good provision arising from the bargaining process over private transfers. [2]

Our paper is also related to the literature determining the optimal provision of public goods in legislative bargaining models.[3] The closest paper to ours is Bowen, Chen, and Eraslan (2014), who analyze the welfare implications of mandatory spending rules on public goods. A key departure from their work is that we consider budget rules affecting private consumption allocations (through the determination of taxes and entitlements), where public good spending is discretionary. This, paired with initial income inequality, allows us to study how policies evolve in response to a process of unequal growth. Our paper thus complements their findings by pointing out that the type of good targeted by the budget rule has important implications for its associated welfare gains in an environment where agents are risk averse. This finding is relevant because the largest mandatory spending programs in the United States are entitlements, which are mostly provided in the form of private transfers. There is an additional, and more subtle, difference between our paper and Bowen, Chen, and Eraslan (2014)'s work. A key underlying assumption in their model is the linearity in the utility of private goods. Because of linearity, fluctuations in private consumption / transfers resulting from bargaining do not generate welfare losses (level changes affect utility, but volatility does not). In our environment, because of concavity in private goods' utility, agents prefer smooth consumption profiles. Therefore, volatility of private consumption can generate significant welfare losses. [4]

Bouton, Lizzeri, and Persico (2020) study the effect of introducing entitlement programs in an environment with public good provision, but using an alternating dictator approach. This delivers similar results to a bargaining environment in which all spending is discretionary and taxes are determined residually from the budge constraint of the government. While they focus on the effects of entitlement programs on debt (which we abstract from by considering a balanced budget), we center on how the legislative bargaining process affects the evolution of the share of entitlements under unequal growth.

The discussion of rules versus discretion (see Amador, Werning, and Angeletos (2006)), is also salient to our results. For example, Halac and Yared (2014) study the optimal level of discretion in

[2] Battaglini and Coate (2007) and Battaglini and Coate (2008) do study legislative bargaining between private and public goods, but under the assumption of an exogenous status quo.

[3] For a cooperative bargaining approach to optimal public good provision, check Davila, Eeckhout, and Martinelli (2009).

[4] Moreover, when we replicate their model with concave utility on private consumption, we find that mandatory spending rules on public goods do not restore Pareto efficiency (they do involve Pareto improvements, though). The solution under mandatory spending on public goods with concave utilities is detailed in a previous version of this paper, see Azzimonti et al. (2020).

fiscal policy when the economy faces persistent shocks. They show that when shocks are not i.i.d., an ex-ante optimal fiscal rule can create incentives for governments to accumulate maximal debt, becoming immiserated. Azzimonti, Battaglini, and Coate (2016) and Martin (2020) consider the welfare implications of balanced budget rules instead. We depart from these papers by considering mandatory rather than discretionary spending, but restricting the government ability to issue debt. Allowing for sovereign debt would be an interesting extension to our work. An excellent summary of the recent literature on budget rules in economies featuring debt in presidential systems can be found in Yared (2019). For recent work on the effect of budget rules on debt mitigation in a bargaining game see Piguillem and Riboni (2020).

Our paper is also related to the literature studying the effects of power alternation on government policy, which includes Persson and Svensson (1989), Alesina and Tabellini (1990), Persson and Tabellini (2000), Acemoglu, Golosov, and Tsyvinski (2011) or Azzimonti (2011). These papers emphasize that political turnover introduces inefficiencies in a political equilibrium with no budget rules. We contribute to this literature by considering how budget rules can affect welfare in a model with legislative bargaining.[5] Considering concave utility functions over private consumption is key to our findings because individuals have preferences for smooth sequences of private and public consumption. This is an important departure from other papers studying the welfare consequences of budget rules.[6]

Our contribution is both qualitative and computational. Adding concavity embeds more macroeconomic realism to the setting but also raises technical challenges for the computation of optimal policies.[7] This is particularly the case when studying entitlement programs because it calls for solving for a large number of value and policy functions over a multidimensional state space. We add to the computational bargaining literature by complementing the work of Duggan and Kalandrakis (2012), and to the macroeconomic literature by extending the techniques of Gordon (2019) and Chatterjee and Eyigungor (2020) to a political-economy environment with legislative bargaining.

3 Environment

Consider a discrete-time infinite horizon economy populated by two types of agents, R (rich) and P (poor), of equal measure but different income levels, $y_R > y_P$. A draw of nature at the beginning of time determines the agent's type (with equal probability), and their type is fully persistent thereafter. Agents value private goods c and public goods g according to an additively separable instantaneous utility function

$$U(c,g) = u(c) + \theta u(g),$$

where u is strictly increasing and concave in both arguments, and satisfies $\lim_{x\to 0} u'(x) = \infty$. The constant $\theta > 0$ represents the relative importance of private to public goods in utility.

The government finances g with lump-sum taxes on the rich, denoted by τ_t, and on the poor, denoted by κ_t, and can redistribute income through an entitlement program, implemented as a

[5] Agenda setting power is exogenously determined in our model. See Agranov, Cotton, and Tergiman (2020) of an environment where it is endogenous instead.

[6] Bowen, Chen, Eraslan, and Zápal (2017) discuss how concavity affects the desirability of mandatory spending rules, but also abstract from entitlements.

[7] There are few papers in the legislative bargaining models with endogenous status quo which introduce macroeconomic features. Piguillem and Riboni (2011) considers taxation as an endogenous status quo in the neoclassical growth model, whereas Grechyna (2017) considers endogenous resources.

cash transfer $\tilde{e}_t \geq 0$ to poor agents at each point in time. Denoting by e_t the net transfer that poor agents receive from the government, $e_t = \tilde{e}_t - \kappa_t$, the government budget constraint (GBC) satisfies

$$g_t + e_t \leq \tau_t. \tag{1}$$

The rich agents' consumption satisfies

$$c_{R,t} = y_R - \tau_t. \tag{2}$$

Whereas the poor's consumption is given by

$$c_{P,t} = y_P + e_t. \tag{3}$$

Note that whereas entitlements $\tilde{e}_t$ are positive, net transfers e_t need not be. We assume that there are constraints on the fiscal system ensuring a minimum level of private and public consumption, $c_{i,t} \geq \bar{x}$ for $i \in \{R, P\}$ and $g_t \geq \bar{x}_g$ with $\bar{x}$, $\bar{x}_g \geq 0$. We can interpret $\bar{x}$ as minimum consumption and $\bar{x}_g$ as the minimum amount of resources needed to run government operations and maintain law and order in society. Hence, spending on public goods takes this value unless a policymaker chooses to spend more. These constraints impose restrictions on the net transfers received by poor agents and the level of taxes paid by the rich,

$$e_t \geq \bar{x} - y_P, \qquad \tau_t \leq y_R - \bar{x} \quad \text{and} \quad \tau_t - e_t \geq \bar{x}_g. \tag{4}$$

For example, with $\bar{x} = \bar{x}_g = 0$, these bounds just restrict consumption of each agent to be non-negative. In such case, net transfers to the poor can be negative because entitlements are not enough to cover their taxes $e_t = \tilde{e}_t - \kappa_t < 0$. With $\bar{x} \geq y_P$, we capture an environment where the poor never pay taxes and net transfers are $e \geq 0$, so public goods are financed solely by the rich. In that case, we refer to e simply as entitlements. The values of $\bar{x}$ and $\bar{x}_g$, together with the degree of income inequality $y_R - y_P$, therefore, jointly determine the *fiscal capacity* of the government to finance public goods and the degree of redistribution that can be achieved.

Equations (1)-(3) imply that the total income in the economy $Y = y_R + y_P$ must be enough to cover private and public consumption levels, as stated in the resource constraint below.

$$c_{R,t} + c_{P,t} + g_t \leq Y. \tag{5}$$

With these, we can define allocations and evaluate lifetime utility as follows.

Definition 1. *An allocation* $\mathbf{a}$ *is a sequence of private and public goods throughout the lifetime of an individual,* $\mathbf{a} = \{c_{R,t}, c_{P,t}, g_t\}_{t=0}^{\infty}$*. These allocations induce lifetime welfare* $\mathcal{V}_i(\mathbf{a})$*, for each individual type* $i \in \{R, P\}$*,*

$$\mathcal{V}_i(\mathbf{a}) = \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, g_t),$$

given the discount factor $\beta \in (0, 1)$*.*

4 Efficient, Equitable, and Optimal Allocations

Before describing the political environment, it is useful to characterize the set of *Pareto Efficient* allocations.

Proposition 1 *The Pareto efficient allocations are time invariant and implicitly defined by*

$$Y - g^* = c_P^* + c_R^*$$
$$\theta\, u'(g^*) = \lambda u'(c_P^*) = (1-\lambda)u'(c_R^*),$$

with $\lambda \in (0,1)$ denoting the Pareto-weight of poor agents.

When utility is logarithmic,

$$g^* = \frac{\theta Y}{1+\theta}, \quad c_P^* = \frac{\lambda Y}{1+\theta}, \quad \textit{and} \quad c_R^* = \frac{(1-\lambda)Y}{1+\theta}. \tag{6}$$

Proof. See Appendix 9.1. □

The efficient level of the public good satisfies the "Samuelson rule," which requires that the social marginal benefit of providing the public good (e.g. the sum of private marginal benefits) is equated to the social marginal cost. The solid blue line in Figure 4 illustrates the Pareto Frontier, corresponding to combinations of lifetime welfare given different values of $\lambda \in (0,1)$. Note that the planner is not subject to the constraints on minimum private consumption $\bar{x}$. [8] We made this assumption precisely to emphasize that the concept of Pareto optimality may render solutions in which some agents are allocated infinitesimal levels of private consumption. In other words, we interpret $\bar{x}$ as a constitutional constraint limiting the set of allocations that society considers desirable, and it's value is unrelated to efficiency considerations.[9]

It is standard in studies of optimal taxation to focus on Pareto efficient allocations in order to measure distortions from alternative fiscal policy plans. The underlying assumption is that if the policymaker had access to a complete set of instruments, she would be able to redistribute resources to achieve desirable societal outcomes. In other words, we typically work under the assumption that the Second Welfare Theorem holds and that all points in the Pareto frontier are, in principle, desirable.

The point of departure of this paper is different. We assume that policymakers have access to a complete set of fiscal policy instruments but that: (i) not all points in the frontier are desirable (as described above), and (ii) individuals disagree on the direction of policy once they know their type (i.e. at birth, in the first period). Note that in our economy, Pareto efficient allocations can be decentralized with time-invariant taxes and entitlements,

$$\tau^* = y_R - c_R^* \qquad \text{and} \qquad e^* = c_P^* - y_P.$$

These would be associated with some degree of redistribution, which, at the end of the day, depends on the value of the Pareto-weights. For example, with $\lambda = 0.5$, consumption would be equated $c_R^* = c_P^*$, resulting in significant redistribution from rich to poor agents. Clearly, R and P agents would disagree on value of λ that should be adopted by the government. Rich agents would like a policymaker with $\lambda \to 0$, whereas poor agents would prefer one with $\lambda \to 1$. Such extreme allocations would be efficient (e.g. at the Pareto Frontier), but they would not be equitable, as they would be associated with very different levels of welfare for both agents.

[8] The constraint on g is non-binding in this environment, so it can be ignored.

[9] Adding this constraint shrinks the frontier in the direction of the optimal (efficient and equitable) allocation.

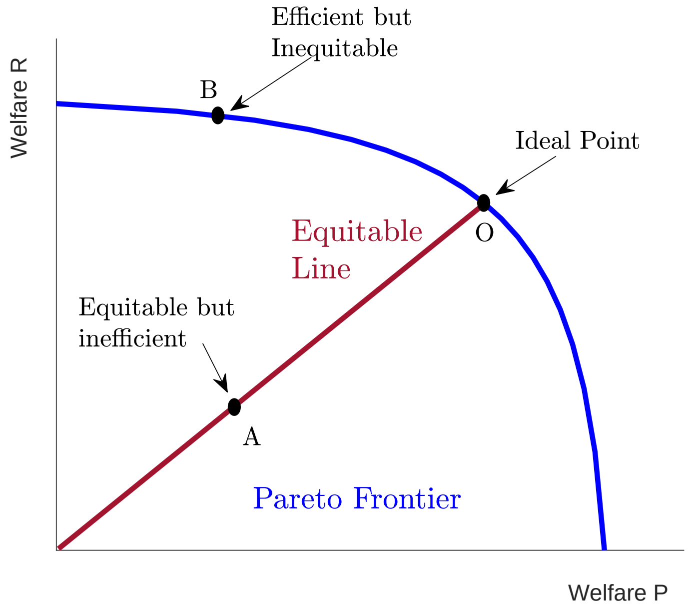


Figure 4: Efficient vs Equitable Allocations

We define an allocation $\mathbf{a}^e$ to be *equitable* when it is feasible and associated with the same level of lifetime welfare for all agents in society, $\mathcal{V}_R(\mathbf{a}^e) = \mathcal{V}_P(\mathbf{a}^e) \equiv \mathcal{V}(\mathbf{a}^e)$. These are illustrated in Figure 4 as points along the Equitable Line (solid red).

From the figure, we can see that there are allocations which are efficient but not equitable (such as B), while others are equitable but inefficient (such as A). What is, then, optimal from society's point of view? We call an allocation $\mathbf{a}^O$ *optimal* when it is both, efficient and equitable. In the plot, this corresponds to point O, where the Equitable Line intersects the Pareto Frontier. Optimal allocations can be found by simply setting $\lambda = 0.5$ in the expressions of Proposition 1.

When $u(x) = \ln(x)$ and $\theta = 1$, the optimal allocation prescribes that half of 'the pie' Y is devoted to private goods and the rest is split evenly between the two agents,

$$g_t^o = \frac{Y}{2}, \quad \text{and} \quad c_{i,t}^o = \frac{Y}{4}. \tag{7}$$

Note that if agents were asked about their most preferred allocation under a veil of ignorance, i.e. before their type was revealed at birth, they would both agree on O being the ideal point. This is the case because we assumed that agents could be rich or poor with equal probability. Ex-post, on the other hand, they prefer to be closer to the extremes of the Pareto Frontier. This inconsistency in preferences, paired with an inability to commit to institutions ensuring an efficient and equitable allocation, is what gives raise to political parties in our model. Hence, our main friction is not distortionary taxation, but instead disagreement between individuals in society. The political equilibrium is discussed next.

5 Political Equilibrium

Because of income inequality, individuals of different income groups disagree on fiscal policy. As a result, political parties naturally arise in this environment. We assume that there are two parties: R and P representing the interests of agents in each group. These parties bargain over policy each period and are subject to budget rules. More specifically, we assume that taxes and entitlements are governed by criteria set by enacted law. The latter corresponds to mandatory spending on transfers to the poor, directly affecting their level of private consumption. Unless a majority of legislators choose to change such laws, τ and e must take last period's values. We refer to them as the *tax code* and *entitlement programs*. Public goods, on the other hand, are considered to be discretionary spending. As such, there is no pre-determined value for g, which takes the minimum value $\bar{x}_g$ unless there is an explicit agreement by the two parties to spend more.

The protocol to change taxes and entitlements is similar to that in the legislative bargaining literature (see Baron and Ferejohn (1989)). A representative of one of the parties is selected at random to make a policy proposal, which consists of a triple $\{\tau, e, g\}$. The opposition party can accept or reject this proposal. If the proposal is accepted, it is implemented this period. If it is rejected, then taxes and entitlements must take last period's values and $g = \bar{x}_g$. This implies that there are two relevant state variables, $\mathbf{s} = \{\bar{\tau}, \bar{e}\}$, as they determine status-quo values in case a proposal is rejected. The current proposer takes them as given when choosing the triple $\{\tau, e, g\}$.

To fix ideas, suppose that the proposer is of type P. Party R's choices are summarized by $d_R(\mathbf{s}) \in \{0, 1\}$, where $d_R(\mathbf{s}) = 1$ denotes that the proposal has been accepted. When a proposal is accepted, it becomes the new state, $\mathbf{s}' = \{\tau, e\}$. If the proposal is rejected, the status quo allocation from $\mathbf{s}$ is implemented, and next period's state remains the same, $\mathbf{s}' = \mathbf{s}$. There is a key dynamic component in this environment which was absent in the determination of Pareto efficient and optimal policies, as the outcome of the bargaining problem becomes the *endogenous status quo* for next period.

5.1 Markov Perfect Equilibrium

Throughout our analysis, we assume that proposers alternate in power following a Markov process, where q denotes the probability of being the proposer tomorrow given that the party has proposal power today. We focus on a Markov Perfect Equilibrium. Under this equilibrium concept, policy functions only depend on payoff-relevant states of the economy, given by $\mathbf{s} = \{\bar{\tau}, \bar{e}\}$. Due to the existence of income inequality, equilibrium policy rules depend on the identity of the proposer. We denote by $\mathcal{G}_i(\mathbf{s})$ the MPE policy for public goods chosen by proposer i, whereas $\Psi_i(\mathbf{s})$ denotes the rule determining taxes imposed on the rich and $\mathcal{E}_i(\mathbf{s})$ the equilibrium policy determining net transfers to the poor. Consumption allocations of agent j determined by proposer i are denoted with $\mathcal{C}_{j,i}(\mathbf{s})$. The associated continuation utilities are $V_i(\mathbf{s}')$ if the proposer remains the proposer next period and $W_i(\mathbf{s}')$ if out of power.

Suppose that P is the current proposer, her maximization problem can be written as

$$\max_{\{\tau,e,g\}} u(c_P) + \theta u(g) + \beta \Big\{ q V_P(\pi_P) + (1-q) W_P(\pi_P) \Big\} \tag{8}$$

where we used the fact that on the equilibrium path the proposal is accepted, $\mathbf{s}' = \{\tau, e\} \equiv \pi_P$. The constraints are eqs. (1)-(4), and the *acceptance constraint*

$$\begin{aligned} u(c_R) + \theta u(g) + \beta \Big\{ (1-q) V_R(\pi_P) + q W_R(\pi_P) \Big\} \quad &\geq \\ u(y_R - \bar{\tau}) + \theta u(\bar{x}_g) + \beta \Big\{ (1-q) V_R(\boldsymbol{s}) + q W_R(\boldsymbol{s}) \Big\} & \end{aligned} \tag{9}$$

The bottom part of the equation denotes the dynamic payoff to the opposition party when the proposal is rejected, that is, the payoff from keeping the status quo. Recall that when no agreement is reached, $g = \bar{x}_g$ and $c_R = y_R - \bar{\tau}$. The acceptance constraint ensures that the proposal is accepted if and only if the payoff from the proposal is individually rational for the respondent, i.e., it provides a payoff that is at least as high as the payoff under the status quo s. The expression makes it clear that the budget rule defines a lower bound for the welfare level attained by the opposition R. It also makes it clear that the lower bound $\bar{x}_g \geq 0$ controls how binding equation (9) is at every point in time. When $\bar{x}_g$ is very small, rejecting a proposal can be very costly for the opposition, so larger deviations from the status quo will become attainable for the proposer.

The acceptance constraint of party P when R is in power is similarly defined,

$$\begin{aligned} u(c_P) + \theta u(g) + \beta\Big\{(1-q)V_P(\pi_R) + qW_P(\pi_R)\Big\} \quad &\geq \\ u(y_P + \bar{e}) + \theta u(\bar{x}_g) + \beta\Big\{(1-q)V_P(s) + qW_P(s)\Big\} \end{aligned}$$

since $c_P = y_P + \bar{e}$ when the proposal is rejected.

Finally, we have that in the MPE, the value function of a type-P proposer satisfies

$$V_P(\mathbf{s}) = u\Big(\mathcal{C}_{P,P}(\mathbf{s})\Big) + \theta u\Big(\mathcal{G}_P(\mathbf{s})\Big) + \beta\Big\{qV_P\Big(\Pi_P(\mathbf{s})\Big) + (1-q)W_P\Big(\Pi_P(\mathbf{s})\Big)\Big\} \tag{10}$$

with next period's status quo given by today's equilibrium choices by proposer P, namely $\Pi_P(\mathbf{s}) = \{\Psi_P(\mathbf{s}), \mathcal{E}_P(\mathbf{s})\}$. The value function of type P when out of power satisfies

$$W_P(\mathbf{s}) = u\Big(\mathcal{C}_{P,R}(\mathbf{s})\Big) + \theta u\Big(\mathcal{G}_R(\mathbf{s})\Big) + \beta\Big\{(1-q)V_P\Big(\Pi_R(\mathbf{s})\Big) + qW_P\Big(\Pi_R(\mathbf{s})\Big)\Big\} \tag{11}$$

as policies are chosen by party R in such case, with $\Pi_R(\mathbf{s}) = \{\Psi_R(\mathbf{s}), \mathcal{E}_R(\mathbf{s})\}$. We can now formally define the Markov perfect equilibrium of this game.

Definition 2. *A MPE with legislative bargaining is a set of value functions $\{V_i(\mathbf{s}), W_i(\mathbf{s})\}$, policy functions $\Pi_i(\mathbf{s}) = \{\Psi_i(\mathbf{s}), \mathcal{E}_i(\mathbf{s})\}$, allocations $\Big\{\mathcal{C}_{j,i}(\mathbf{s}), \mathcal{G}_i(\mathbf{s})\Big\}$, and acceptance rules $d_j(\mathbf{s})$ for proposer i and opposition $j \neq i$ where $i, j \in \{R, P\}$, such that*

- *Proposer i chooses public good g and fiscal policy $\pi_i = \{\tau, e\}$ to maximize problem (8) subject to the budget constraints eqs. (1)-(3), the bounds eq. (4), and the acceptance constraint, eq. (9). Given the value functions $V_i(\mathbf{s})$ and $W_i(\mathbf{s})$, the acceptance decision $d_j(\mathbf{s})$, and the rules chosen by the opposition party j, $\Pi_j(\mathbf{s})$, these define the policy functions $\Pi_i(\mathbf{s})$. The problem of proposer j is analogously defined.*

- *Given the policy functions $\Pi_j(\mathbf{s})$ and $\Pi_i(\mathbf{s})$, the value functions $V_i(\mathbf{s})$ and $W_i(\mathbf{s})$ satisfy equations (10) and (11), respectively. The value functions $V_j(\mathbf{s})$ and $W_j(\mathbf{s})$ are analogously defined.*

- *Given $V_i(\mathbf{s})$ and $W_i(\mathbf{s})$, for any proposal π_i and status quo $\mathbf{s}$, the acceptance strategy $d_j(\mathbf{s}) = 1$ if and only if eq. (9) holds. The acceptance rule $d_i(\mathbf{s})$ is analogously defined.*

The first condition states that policy rules are the ones that solve the problem of the proposer, given continuation utilities and an acceptance rule for the opposition party. The second condition defines value functions as a fixed point using policy functions under an accepted proposal. The last condition determines that the opposition party accepts the proposal whenever its welfare is at least as large as under the status quo.

5.2 No Budget Rules

Before characterizing the equilibrium with budget rules, it is useful to briefly discuss the proposer's optimal choices in an environment with no budget rues. There is no pre-determined value for expenditures in public goods or entitlements in the tax system, other than those that guarantee minimum consumption levels. Hence, if a policy proposal is rejected, spending on public goods takes the minimum value $\bar{x}_g$, entitlements equal the lower bound $e = \bar{x} - y_P$. We also assume that taxes are residually determined from the government's budget constraint. The optimization problem is the same as before, but without imposing the acceptance constraints eq. (9).

The solution to this problem is analogous to the ones in the citizen candidate models of Osborne and Slivinski (1996) or Besley and Coate (1997) (also known as "alternating dictator" models). Because total income is constant over time and the government is subject to a balanced budget, there is no dynamic state variable. Therefore, the problem of a proposer choosing policies not subject to budget rules is static (with continuation utility being irrelevant). Proposer i sets

$$\theta u'(g^D) = u'(c^D_{i,i}), \tag{12}$$

equating the marginal benefit of public good provision to her private marginal cost. Proposition 2 further characterizes the solution for the tractable case of $\theta = 1$ and we relegate the more cumbersome general case for Section 2 of the Online Appendix.

Proposition 2 *Under logarithmic utility, $\theta = 1$, $\bar{x} = \bar{x}_g \equiv \bar{x}$, and absent budget rules, fiscal policy satisfies*

$$\textit{P in power:} \quad \tau^D_P = y_R - \bar{x} \quad \textit{and} \quad e^D_P = \frac{\Delta - \bar{x}}{2}.$$

$$\textit{R in power:} \quad \tau^D_R = \frac{\Delta + \bar{x}}{2} \quad \textit{and} \quad e^D_R = \bar{x} - y_P,$$

where $\Delta = y_R - y_P$ denotes income inequality. The resulting allocations when i is the incumbent are given by

$$g^D = \frac{Y - \bar{x}}{2}, \quad c^D_{i,i} = g^D, \quad \textit{and} \quad c^D_{j,i} = \bar{x} \quad \textit{for } j \neq i.$$

Proof. See Appendix 9.2 □

The proposer expropriates the other group as much as possible, providing them with the minimum feasible level of consumption when in power. When P is the proposer, taxes on the rich are maximal, $\tau^D_P = y_R - \bar{x}$ whereas their consumption hits the lower bound $c^D_{R,P} = \bar{x}$. The remaining of the budget is divided between the public good and the consumption of the poor. When private and public goods have the same weight, $\theta = 1$, the budget is split evenly, $c^D_{P,P} = g^D$. When R is the proposer, net transfers to the poor are minimal, so they consume at $c^D_{P,R} = \bar{x}$. Note that while the two parties disagree on the burden of taxation, they both choose the same provision of public goods, g^D and policies that result in the same total level of private consumption $c^D = c^D_{R,i} + c^D_{P,i}$. Hence, public good provision and total consumption do not depend on the identity of the party in power.

Assume both types of agents have the same Pareto-weight $\lambda = \frac{1}{2}$ (in this case, $c^D > c^*$ for sure). There are two sources of Pareto inefficiency when the proposer is not subject to budget rules and there is power alternation, $q < 1$. The first one is a static inefficiency which arises because, to the extent that $\bar{x} > 0$, the incumbent under-provides public goods relative to the planner,

$$g^D < g^* \quad \text{and} \quad c^D > c^*.$$

The solution when $\overline{x} > 0$ never satisfies the Samuelson rule because the incumbent equates the marginal cost of public goods to her private marginal benefit, whereas the planner would equate it to the social marginal benefit. In other words, incumbent i ignores the welfare gains to group j of providing g and this results in over-provision of private goods and under-provision of public goods, regardless of the identity of the incumbent.

The second source of inefficiency is dynamic and arises because private consumption fluctuates between $\frac{Y-\bar{x}}{2}$ and $\bar{x}$ with the identity of the incumbent, whereas the efficient ones are constant. Curvature in the utility function of agents implies that they would prefer a smooth consumption sequence to a volatile one. In a political equilibrium without rules, the volatility induced by power alternation reduces lifetime utility, and it is a dynamic source of inefficiency in this model. Allowing for power alternation, however, does have a benefit to society: By changing the decision-maker every period, the political system enables more equitable allocations. Without it, one type would always consume the lower bound $\bar{x}$.

6 Two-Period Model

The easiest way to understand the effect of budget rules on allocations is through the analysis of a two-period version of the model, where analytical results can be obtained. The infinite-horizon dynamic model is studied in Section 7.1. To find the solution to the Markov Perfect equilibrium, we solve the problem backwards, starting from the second period.[10]

6.1 Second-Period Characterization.

The second-period proposer takes the status quo $\mathbf{s} = \{\bar{\tau}, \bar{e}\}$ as given. Because the economy ends this period, there is no continuation utility. The analysis allows us to understand how the status quo affects the choice set and the relative bargaining power of the two groups, while ignoring the dynamic consequences of this choice.

Suppose that party P has proposal power this period. The incumbent proposes $\{g_2, \tau_2, e_2\}$, given status quo $\boldsymbol{s}$, in order to maximize her static payoff,

$$\max_{\{\tau_2, e_2, g_2\}} u(c_{P,2}) + \theta u(g_2) \quad \text{s.t.}$$

$$u(c_{R,2}) + \theta u(g_2) \geq u(y_R - \bar{\tau}) + \theta u(\bar{x}_g), \tag{13}$$

and eqs. (1)-(4),

The acceptance constraint, eq. (13), ensures that the proposal is accepted if and only if the payoff from the proposal weakly exceeds the payoff under the status quo $\boldsymbol{s}$. In the analysis that follows, we assume that the utility is logarithmic, $\theta = 1$, and $\bar{x} = \bar{x}_g \equiv \bar{x}$, as it greatly facilitates exposition.

When the acceptance constraint is not binding, the solution is akin to that under no budget rules, characterized in Proposition 2. This solution corresponds to the unconstrained best from P's point of view. When eq. (13) binds, the solution is more involved and depends on the status-quo value of taxes on the rich, $\bar{\tau}$. The full result is characterized in the following proposition.

Proposition 3 *Let utility be logarithmic $u(.) = \ln(.)$, $\theta = 1$, and $x_g = \bar{x}$. In the last period, the unique equilibrium proposal for proposer P satisfies:*

[10]Given that this is a two-person, two-period, complete information extensive form game, we focus on its unique subgame perfect equilibrium (SPE). Second-period strategies do not depend on histories except through the status quo.

$$\mathcal{E}_{P,2}(\boldsymbol{s}) = \begin{cases} \frac{\Delta}{2} - \frac{2\bar{x}[y_R - \bar{\tau}]}{Y}, & \textit{if} \quad \bar{\tau} < \frac{\Delta}{2} \\ \bar{\tau} - \bar{x}, & \textit{if} \quad \bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ e_P^D, & \textit{if} \quad \bar{\tau} \geq \tau_R^D. \end{cases} \qquad \Psi_{P,2}(\boldsymbol{s}) \quad = \begin{cases} y_R - \frac{2\bar{x}[y_R - \bar{\tau}]}{Y}, & \textit{if} \quad \bar{\tau} < \frac{\Delta}{2} \\ \tau_P^D, & \textit{if} \quad \bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ \tau_P^D, & \textit{if} \quad \bar{\tau} \geq \tau_R^D. \end{cases}$$

and

$$\mathcal{G}_{P,2}(\boldsymbol{s}) = \begin{cases} g^*, & \textit{if} \quad \bar{\tau} < \frac{\Delta}{2} \\ y_R - \bar{\tau}, & \textit{if} \quad \bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ g^D, & \textit{if} \quad \bar{\tau} \geq \tau_R^D. \end{cases}$$

The associated private consumption allocations are

$$\mathcal{C}_{P,P,2}(\boldsymbol{s}) = y_P + \mathcal{E}_{P,2}(\boldsymbol{s}) \quad \textit{and} \quad \mathcal{C}_{R,P,2}(\boldsymbol{s}) = y_R - \Psi_{P,2}(\boldsymbol{s}).$$

Proof. See Appendix 9.3. □

The only relevant state variable for proposer P is the status quo level of taxes on the rich, $\bar{\tau}$. This is the case because the rich use their veto power to block policy changes that deliver welfare levels below those obtained under the status quo. The effect of alternative $\bar{\tau}$ values is illustrated in Figure 5 for a numerical example with $Y_R = 1.3$, $Y_P = 0.1$, and $\bar{x} = 0.1$, where we plot equilibrium policies (top panel) and allocations (bottom panel) as functions of $\bar{\tau}$. When $\bar{\tau} \geq \tau_R^D$, status quo taxes on the rich are so large that the proposer is able to implement the unconstrained solution, τ_P^D and e_P^D. Because the acceptance constraint is not binding, the proposer just equates her marginal utility of private consumption to that of public consumption, implying $\mathcal{G}_{P,2}(\boldsymbol{s}) = g^D$ and $\mathcal{C}_{P,P,2}(\boldsymbol{s}) = c_{P,P}^D$.

When intermediate taxes are established in the tax code, $\bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right)$, proposing the unconstrained policies is no longer acceptable for the opposition. Group R is better off rejecting the proposal and keeping taxes and entitlements at their status quo values. Anticipating this, proposer P offers an alternative mix that guarantees the opposition's minimum welfare under the status quo, so that constraint (13) becomes binding, but that make P slightly better off. In order to induce the opposition to accept the proposal, proposer P needs to either impose lower taxes (e.g. increase $c_{R,2}$ above the minimum $\bar{x}$) or provide more public goods at the expense of her own consumption (through lower entitlements). Given that P enjoys consuming public goods (but derives no utility from the opposition's private consumption), it is best to offer τ_P^D and instead reduce entitlements below e_P^D. This results in higher provision of public goods and a slightly lower c_P than under discretionary spending (e.g. no budget rules environment). The rich are willing to accept this proposal even though their consumption is set at the lower bound. When taxes are below $\frac{\Delta}{2}$, the opposition has so much bargaining power, that proposer P is forced to reduce taxes and entitlements even further. The only proposal that would give P high consumption involves $g_2 = g^*$, the Samuelson level of the public good provision. Interestingly, consumption inequality is minimal when taxes on the rich are low, despite the built in mechanism to have more equality through entitlement programs. This is the case because low taxes give more bargaining power to rich agents when P is in power, limiting the poor's ability to expropriate the rich.

For completeness, it is useful to characterize the policy rules that would be chosen by proposer type R if in power in the second period under the assumptions of Proposition 3.

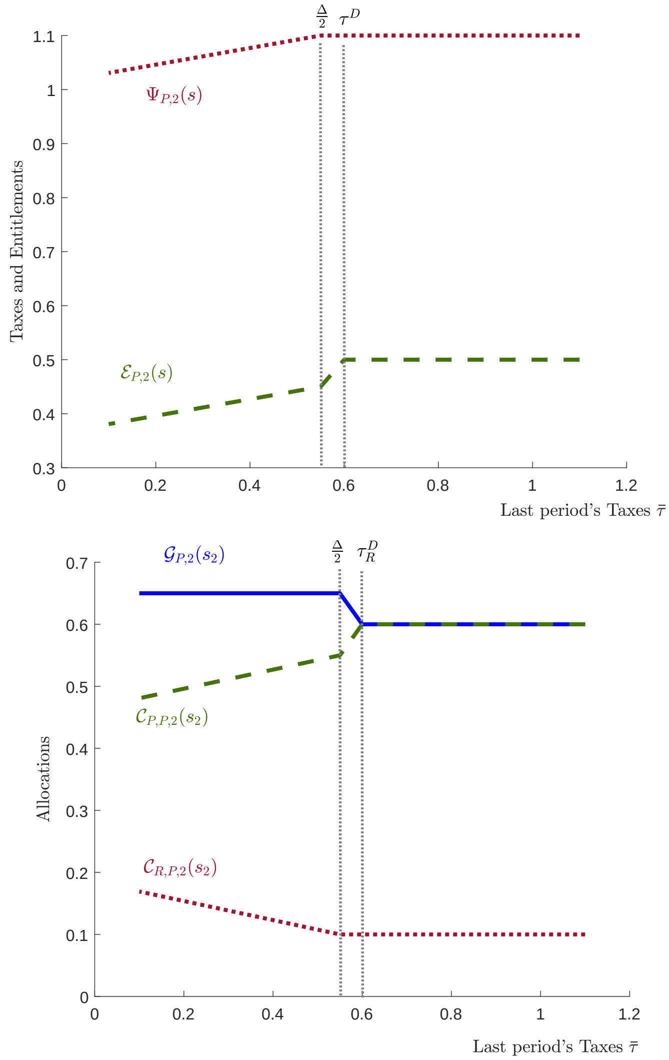


Figure 5: Second period policies and allocations under proposer P. Parameters: $y_R = 1.3$, $y_P = 0.1$, and $\overline{x} = 0.1$

Proposition 4 *Let utility be logarithmic $u(.) = \ln(.)$, $\theta = 1$, and $\bar{x} = \bar{x}_g \equiv \bar{x}$. In the last period, the unique equilibrium proposal for proposer R satisfies:*

$$\mathcal{E}_{R,2}(\mathbf{s}) = \begin{cases} e_R^D, & \text{if} \quad \bar{e} < e_P^D \\ e_R^D, & \text{if} \quad \bar{e} \in \left[e_P^D, \frac{\Delta}{2}\right) \\ \frac{2\bar{x}[y_P - \bar{e}]}{Y} - y_P, & \text{if} \quad \bar{e} \geq \frac{\Delta}{2}. \end{cases} \qquad \Psi_{R,2}(\mathbf{s}) = \begin{cases} \tau_R^D, & \text{if} \quad \bar{e} < e_P^D \\ \bar{x} - \bar{e}, & \text{if} \quad \bar{e} \in \left[e_P^D, \frac{\Delta}{2}\right) \\ \frac{\Delta}{2} + \frac{2\bar{x}[y_P - \bar{e}]}{Y} & \text{if} \quad \bar{e} \geq \frac{\Delta}{2}, \end{cases}$$

$$\mathcal{G}_{R,2}(\mathbf{s}) = \begin{cases} g^D, & \text{if} \quad \bar{e} < e_P^D \\ y_P + \bar{e}, & \text{if} \quad \bar{e} \in \left[e_P^D, \frac{\Delta}{2}\right) \\ g^*, & \text{if} \quad \bar{e} \geq \frac{\Delta}{2}. \end{cases}$$

with associated consumption $\mathcal{C}_{P,R,2}(\mathbf{s}) = y_P + \mathcal{E}_{R,2}(\mathbf{s})$ and $\mathcal{C}_{R,R,2}(\mathbf{s}) = y_R - \Psi_{R,2}(\mathbf{s})$.

Proof. See Appendix 9.4. □

For an R proposer, the relevant thresholds are determined by the status quo level of net transfers to the poor $\bar{e}$. Under the assumption that $\bar{x} = y_P$, these are just thresholds on entitlement levels. The intuition behind the solution above is analogous to the one described before. When existing law establishes a low level of entitlements, a rich proposer chooses her best unconstrained choice (e.g. the one under no budget rules). As $\bar{e}$ exceeds the first threshold (but not the second), the acceptance constraint of the poor becomes binding, so R finds it optimal to propose a policy combo that results in higher public good provision but lower consumption to herself, so as to keep entitlements as low as possible. Once $\bar{e} \geq \frac{\Delta}{2}$, the best proposal involves the Samuelson level of public good provision g^* and the smallest possible entitlement level and taxes that would induce P to accept the proposal. The highest consumption equality is achieved when $\bar{e}$ is large under a proposer of type R in the last period of the finite horizon game.

Summarizing, the second period choices depend on status quo values of entitlements and taxes. Low existing taxes force poor proposers to provide private consumption allocations which are more equitable and public goods which are closer to the first best. This is because under low $\bar{\tau}$, the opposition has high bargaining power and can veto allocations that would expropriate their type's income significantly in order to generate redistribution. On the other hand, when R is the proposer, higher status quo values of entitlements achieve more equitable allocations instead. By having a relatively high minimum consumption to the poor (as established by existing laws), the rich can only make themselves better off by providing public goods above the unconstrained levels. The reason being that high tax cuts can be easily be vetoed by poor agents when $\bar{e}$ is large. In other words, *entitlement laws protect the poor against policies preferred by the rich whereas a tax code protects the rich against excessive taxation preferred by the poor.* Discretionary spending, in the form of public goods, in our model, serves as a bargaining chip for a proposer to change the status quo in her favor. This is accepted by the opposition because both types of agents value public goods.

One important remark is in place before we move to the first period. That only one of the elements in $\boldsymbol{s}$ is relevant for each policymaker is not a general result. It only holds in the last period of a finite horizon game when there is no continuation utility. In the infinite horizon model, on the other hand, both status quo values are going to be payoff relevant. This is discussed at length in Section 7.2, where we illustrate how policy rules depend on both states.

6.2 First-Period Characterization.

We now characterize first-period allocations and ask whether an unconstrained P-proposer (e.g. whose acceptance constraint is slack) would find it beneficial to choose g^D and corresponding consumption allocations or deviate from them– knowing her choices in the first-period will impact her continuation value in the second-period by changing the status quo—. Building on our previous results, we work under the assumption that $\theta = 1$ and utility is logarithmic.

A proposer facing $q = 1$ would find it optimal to choose the unconstrained policies in both periods. There is no gain to introduce a tax code or an entitlement program that would move allocations away from g^D and $c^D_{P,P}$, as these achieve the highest level of welfare for the proposer at every point in time. This is no longer the case under uncertainty, as the appropriate choice of taxes and entitlements could 'tie the hands' of her successor. By choosing alternative taxes and entitlements today, a proposer can manipulate future decisions through the endogenous status quo channel. More importantly, altering the optimal policy mix today can provide insurance against expropriation in the event that the opposition gains proposal power.

To show this, consider a situation where P proposes a public good provision allocation g_1 and policies $\pi_{P,1} = \{\tau_1, e_1\}$. The acceptance constraint is slack today (by assumption), but the proposer understands that these policies become the status quo next period, $\boldsymbol{s} = \pi_{P,1}$. Her maximization problem is

$$\max_{\{\tau_1,e_1,g_1\}} \quad \ln(c_{P,1}) + \ln(g_1) + \beta\left\{qV_P(\pi_{P,1}) + (1-q)W_P(\pi_{P,1})\right\} \tag{14}$$
$$\text{s.t. eqs. (1) and (3)} - (4),$$

where we have used the fact that $s = \pi_{P,1}$. The value functions $V_P(\pi_{P,1})$ and $W_P(\pi_{P,1})$ can be obtained by evaluating the solution characterized in Propositions 3 and 4 into the utility in the second period. The continuation utility of proposer P if she stays in power tomorrow is given by

$$V_P(\tau_1) = \begin{cases} \ln(g^*) + \ln\left(\frac{Y^2 - 4\bar{x}[y_R - \tau_1]}{2Y}\right), & \text{if} \quad \tau_1 < \frac{\Delta}{2}. \\ \ln(y_R - \tau_1) + \ln(y_P + \tau_1 - \bar{x}), & \text{if} \quad \tau_1 \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ \ln\left(g^D\right) + \ln\left(c_{P,P}^D\right), & \text{if} \quad \tau_1 \geq \tau_R^D \end{cases}$$

This is because only τ_1, the level of entitlements received by the opposition, may constrain future decisions when P remains in power. If R becomes next period's proposer, then it is the current entitlement level e_1 what will constrain R's decisions instead. The continuation utility for P in such case would be

$$W_P(e_1) = \begin{cases} \ln(\bar{x}) + \ln(g^D), & \text{if} \quad e_1 < e_P^D \\ \ln(\bar{x}) + \ln(y_P + e_1), & \text{if} \quad e_1 \in \left[e_P^D, \frac{\Delta}{2}\right) \\ \ln\left(\frac{2\bar{x}[y_P + e_1]}{Y}\right) + \ln(g^*), & \text{if} \quad e_1 \geq \frac{\Delta}{2}. \end{cases}$$

This function is computed by replacing R's optimal choices on P's utility next period.

Proposer P chooses allocations in the first period to maximize eq. (14), which can be re-written as

$$\max_{\{\tau_1,e_1\}} \ \ln(y_P + e_1) + \ln(\tau_1 - e_1) + \beta\left\{qV_P(\tau_1) + (1-q)W_P(e_1)\right\},$$

subject to the lower bound constraints. We have used eqs. (1)-(3) to write down all the allocations in the first period in terms of policy.

Inspection of the problem above reveals that it is optimal for proposer P to set current taxes as high as possible, $\tau_1 = y_R - \bar{x}$. This maximizes resources in the current period and improves P's bargaining power tomorrow. The first order condition with respect to e_1 is

$$\underbrace{\frac{1}{c_{P,1}}}_{MU_c} - \underbrace{\frac{1}{g_{P,1}}}_{MU_g} = \underbrace{\beta(1-q)\frac{\partial W_P(e_1)}{\partial e_1}}_{\text{wedge}_c > 0}. \tag{15}$$

In the absence of uncertainty (e.g. $q = 1$), the proposer would set the left hand side of the equation to zero, which achieves the unconstrained solution. At that point, the private marginal costs and benefits of entitlements are equated. When $q < 1$, the proposer finds it optimal to distort the solution because, by choosing $e_1 > e_P^D$ it is possible to affect the status quo inherited by the opposition if group R becomes the proposer next period, therefore increasing her own welfare, $W_P(e_1)$, in that state of the world. The solution to the first period allocations under budget rules is characterized in Proposition 5.

Proposition 5 *Suppose that the acceptance constraint is not binding in $t = 1$. The unique proposal strategy for proposer P under budget rules is:*

$$\mathcal{E}_{P,1} = \frac{2e_P^D + \beta(1-q)\tau_P^D}{2+\beta(1-q)}, \qquad \Psi_{P,1} = \tau_P^D, \quad \textit{and} \qquad \mathcal{G}_{P,1} = \frac{2}{2+\beta(1-q)} g^D,$$

with associated allocations

$$\mathcal{C}_{P,P,1} = \frac{2\Big(1+\beta(1-q)\Big)}{2+\beta(1-q)} c_{P,P}^D \qquad \textit{and} \qquad \mathcal{C}_{R,P,1} = \bar{x}.$$

Proof. See Appendix 9.5 □

The solution above determines s, and hence the constraint faced by tomorrow's policymaker. Her preferred level of entitlements depends on the probability q. Under uncertainty, the proposer sets $\mathcal{E}_{P,1} > e_P^D$, understanding that this results in too little public good provision today and too much private consumption. This is a current cost because it distorts the allocation relative to the unconstrained case. The gain arises in the future: by establishing an overly generous entitlement program today, it forces the opposition to offer a better policy mix tomorrow. By over-spending on entitlement programs, the current proposer ensures a better bargaining position next period.

It is easy to show that if R was the first-period proposer, she would choose taxes and entitlements to favor her own private consumption while sacrificing the provision of public goods. In particular, she would set entitlements to their lowest possible level $\mathcal{E}_{R,1} = e_R^D$, in order to ensure maximum bargaining power in case she remains the proposer in the second period. She would adjust taxes trading off current distortions against insurance against future expropriation in response to alternative values of q. The resulting allocations are summarized in Proposition 6.

Proposition 6 *Suppose that the acceptance constraint is not binding in $t = 1$. The unique proposal strategy for proposer R under an entitlement rule is:*

$$\mathcal{E}_{R,1} = \overline{x} - y_P, \qquad \Psi_{R,1} = \frac{y_R + (1+\beta\,(1-q))\,e_R^D}{2+\beta\,(1-q)}, \quad \textit{and} \qquad \mathcal{G}_{R,1} = \frac{2}{2+\beta(1-q)} g^D,$$

with associated allocations

$$\mathcal{C}_{R,R,1} = \frac{2\Big(1+\beta(1-q)\Big)}{2+\beta(1-q)} c_{R,R}^D \qquad \textit{and} \qquad \mathcal{C}_{P,R,1} = \bar{x}.$$

Proof. See Appendix 9.6 □

The symmetry (e.g. $\mathcal{C}_{R,R,1} = \mathcal{C}_{P,P,1}$ and $\mathcal{G}_{R,1} = \mathcal{G}_{P,1}$) arises because we have assumed the two types have equal utility functions and because the bounds on taxes and entitlements imply that both agents face a common minimum consumption level $\bar{x}$. If we had tightened the upper bound on taxes, for example, allowing rich agents a higher minimum level of consumption, her choices would be significantly different from P's in the first period. While this is an interesting case to study, we leave it for future research. The symmetric case eases the exposition of our results.

The left panel of Figure 6 depicts private consumption allocations in the bargaining equilibrium for proposer i (solid line) and the allocation under no budget rules (dashed line) as functions

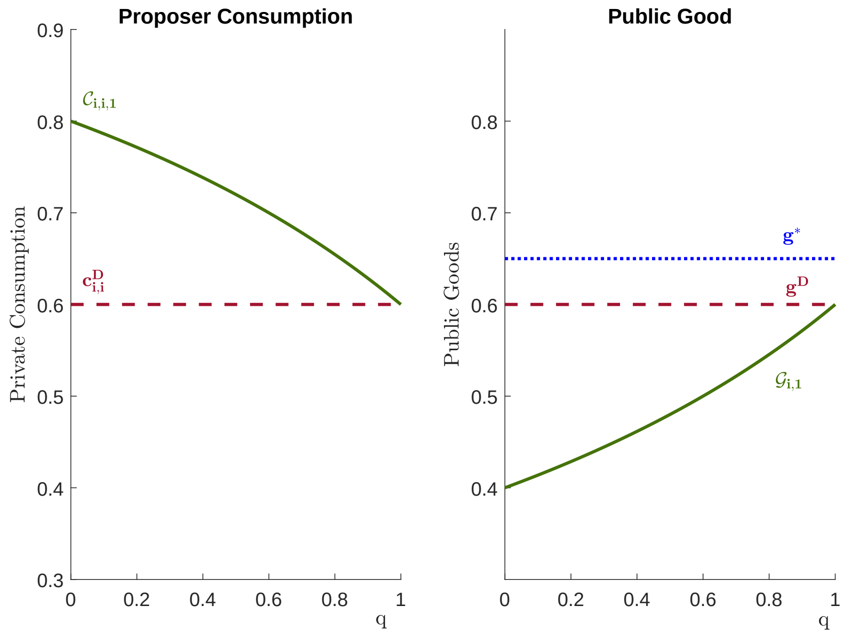


Figure 6: Allocations under Budget Rules as functions of q.

of q, whereas the right panel depicts public good provision in the two cases, together with the Samuelson level q^*. The figure—which was constructed using the same parameters as Figure 5—illustrates that the proposer has incentives to increase consumption relative to $c^D_{i,i}$ when $q < 1$. As long as she faces uncertainty, the proposer will use taxes or entitlements to ensure consumption above $\bar{x}$ when out of power. This comes at the cost of under-providing public goods (see right panel of the picture) relative to her preferred value of g under certainty. The net benefit derived from distorting the allocations from $c^D_{i,i}$ diminishes in q, and as a result $\mathcal{C}_{i,i,1}$ decreases with the probability of retaining proposal power, q.

6.3 The importance of budget rules

When $q < 1$, and proposers is initially unconstrained (e.g. the acceptance constraint is slack in the first period), both proposer types choose entitlement and tax levels such that $\mathcal{G}_{i,1} < g^*$, indicating that the bargaining equilibrium delivers under-provision of public goods relative to the Samuelson level (dotted blue line in Figure 6). Moreover, aggregate consumption—computed as the sum between the proposer's and the opposition's consumption—is inefficiently large. The degree of inefficiency exacerbates with political uncertainty. In addition to these static sources of inefficiency, individual consumption changes over time, as it varies with the identity of the proposer. Volatility, hence, is a second form of inefficiency of the political equilibrium. Recall that the Planner's solution is time invariant.

What are the implications in terms of equity? Interestingly, when the acceptance constraint is not binding, budget rules exacerbate inequities in the first period.[11] Proposer i sets consumption of the opposition to $\bar{x}$, whereas her own consumption is given by $\mathcal{C}_{i,i,1} > c^D_{i,i}$. As a result, the consumption gap is larger than the one where all spending is discretionary and taxes are residually

[11]We will relax the assumption that the acceptance constraint is slack when we analyze the infinite horizon problem. In the two-period problem, relaxing this constraint is not trivial – and bring little intuition, since there are potentially 8 cases to be analyzed given the bi-dimensionality of the endogenous status quo.

determined for $q < 1$ and increases with higher political turnover. In the second period, the budget rule reduces the consumption gap in expectation. This is because the proposer can actually use her first-mover advantage amid a favorable starting point to lock the society in an equilibrium that favors her. She chooses policy in order to minimize the degree of expropriation of the other party by setting a status quo which ensures consumption tomorrow to exceed $\bar{x}$.

Is the introduction of budget rules, then, beneficial or detrimental to society? It depends. Consider a stark example where P is the proposer in the first period and R is the proposer in the second period (e.g. under $q = 0$). Moreover, assume that the acceptance constraint is not binding in period 1 (that is, the first period proposer is unconstrained).

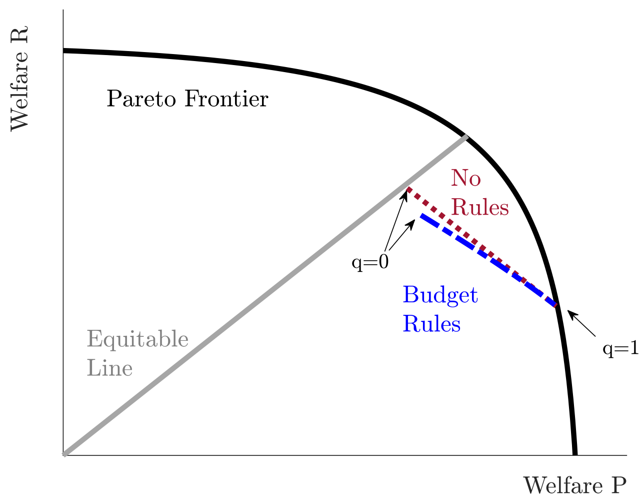


Figure 7: Adopting a Budget Rule (P in power) for $q \in [0, 1]$

Lemma 1. *Let $q = 0$ and assume P is in power in period 1. Moreover, assume that the acceptance constraint is not binding in $t = 1$. As $x \to 0$, group R is worse off if budget rules are introduced.*

Proof. See Appendix 9.7 □

The Lemma highlights that *introducing budget rules does not necessarily lead to Pareto improvements*. In Figure 7 we show, numerically, that this is also the case for $q < 1$ and $\bar{x} > 0$. In particular, we have used $\bar{x} = 0.1$ (and other parameters described in Figure 5). Assuming that P is the proposer in the first period, we plot the lifetime welfare pairs under budget rules (dashed-blue line) vs those under no budget rules (dotted-garnet line). While having a budget rule is clearly better for the proposer, the opposition is worse off: the dashed-blue line is at or below the dotted-garnet line. Moreover, the resulting allocations are not only less efficient than the ones under no budget rules (e.g. further away from the Pareto Frontier) but also less equitable (e.g. further away from the Equity line). This is clearest at the point where $q = 0$, where parties alternate in power deterministically. As $q \to 1$, there is less incentive for the first period proposer to take advantage of budget rules to ensure a good bargaining position for the second period, so taxes and entitlements converge to g^D and $c^D_{i,i}$. As a result, the blue and garnet lines approach each other (and get closest to the Parento frontier). In this example, we see that the introduction of budget rules (a tax code and an entitlement program), from an initial situation without such rules: (i) favors the party that introduces them, (ii) results in less equitable welfare pairs, and (iii) may involve more inefficiencies (e.g. it is further away form the Pareto Frontier).

The status-quo effect: First period allocations were characterized in Proposition 5 under the assumption that proposer P was completely unconstrained (e.g. that the acceptance constraint was slack in $t = 1$). We showed that in such case the proposer would under-provide public goods. This does not necessarily hold when the acceptance constraint binds.

Consider Problem 14, as before, but now subject to the acceptance constraint eq. (9). The latter establishes that the opposition will not accept proposals that make their constituents worse off than under the status-quo policies.

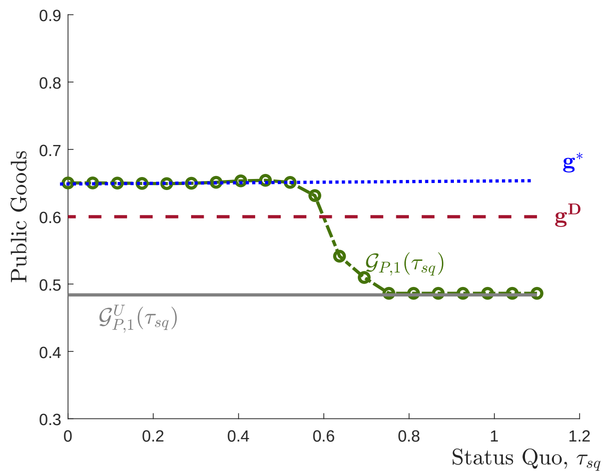


Figure 8: Public good provision as a function of the initial status quo τ_{sq}.

Figure 8 shows the equilibrium level of public good provision in period 1, $\mathcal{G}_{P,1}$ (green line with circles), under an initial status quo value of entitlements that is advantageous for the opposition: $e_0 = 0$. The horizontal axis corresponds to feasible initial status-quo values for the tax code, $\tau_0 = \tau_{sq}$. Note that the proposer no longer has full discretion to choose allocations, as we depart from a scenario without a entitlements. The plot is constructed using the same parameters as Figure 6, but fixing $p = 0.5$. When τ_{sq} is small, public goods are provided at the optimal level g^* (dashed blue line). Party P can no longer get way by taxing R at the maximum possible rate, so in order to be able to increase the level of entitlements, it must offer that the expansion in the size of the government is associated to a rise in g. At the other extreme, when τ_{sq} is higher than a threshold, the proposer can implement the unconstrained solution (solid gray line, $\mathcal{G}^U_{P,1}$) characterized in Proposition 5. The bargaining effect, thus, counteracts the tendency of the budget rule to worsen the under-provision of the public good.

Entitlements program (no tax code): Would these results be any different if P were to only introduce an entitlement program? Absent a tax code, P would not be subject to the acceptance constraint, eq. (13), in the second period. As a result, she would choose g^D, τ_P^D, and e_P^D in $t = 2$. This, in turn, would imply that V_P would be independent of τ_1. Because first period welfare would still be strictly increasing in τ_1, the optimal choice in the first period for a P-proposer would be at the upper bound $\tau_1 = y_R - \bar{x}$. Thus, the existence of a tax code does not change P's incentives to tax R at the maximum feasible rate. The choice of e_1 is also unchanged because in the two period model $W_P(e_1)$ is independent of τ_1. This result is specific to the two period model, as only one of the two state variables is relevant in the second period. In an infinite horizon, however, whether we have both an entitlement program and a tax code, or just one of them would have significant implications for long-run outcomes (see Section 7.2).

Mandatory spending on g: We could also ask how a budget rule on g, rather than on taxes and entitlements, affects the degree of efficiency and equity of the political equilibrium. This is the case that has been analyzed in detail by Bowen et al. (2014). A key difference between the two budget rules is that while those on $\tau\&e$ affect private consumption, a budget rule on g affects the provision of public goods. This has important implications for the strategic intertemporal decisions undertaken by poor and rich proposers when in power. We develop this comparison further in Appendix 9.8, where we present analytical and quantitative results in an environment closest to Bowen et al. (2014). In particular, we consider utility to be quasi-linear and $\theta \neq 1$.[12] Our main finding is that, under these assumptions, a budget rule on g delivers allocations that put society closer to the optimal solution than budget rules on $\tau\&e$ do.

That result is, unfortunately, not general. In Section 7.5 we compare the welfare gains (losses) of alternative budget rules in an infinite horizon economy with concave utilities calibrated to the US economy. We find that budget rules on taxes and entitlements bring society closer to optimal than mandatory spending rules on g do. There are two reasons for this apparently contradictory finding. The first one is that consumption profiles are much more volatile under a g-rule. With quasi-linear utility, this factor is irrelevant, as agents do not care about smooth private consumption profiles. These are much more important when utility is concave. The second reason is that the analysis in Appendix 9.8 is performed for a two-period model where the incumbent is unconstrained in the first period. Budget rules on taxes and entitlements are associated with significantly more redistribution in the long run, an effect that is abstracted from in a two-period model.

6.4 Unequal Growth

As discussed in the introduction, the US experienced a process of "unequal growth" since the 1960s, where most of the gains where enjoyed by the top 50% of income earners and there was little to no growth in the income of individuals in the bottom half of the distribution. In this section, we want to highlight how this affects the level of taxes and entitlements in the bargaining model, and compare them to the optimal allocations.

To capture a process of unequal growth resembling the US experience, we assume that the income of the rich grows unexpectedly between periods 1 and 2: $y_{R,2} > y_{R,1} = y_R$, whereas the income of the poor remains constant: $y_{P,t} = y_P$. As a result, aggregate GDP is higher and inequality rises. The rest of the environment is identical to that in the previous sections. The intuition is cleanest in the case where $\theta = 1$ and utility is logarithmic. Under these assumptions, it is optimal to set public and private consumption according to equation (7). The change in optimal allocations when y_R increases is

$$\frac{\partial g^o}{\partial y_R} = \frac{1}{2} \quad \text{and} \quad \frac{\partial c_i^o}{\partial y_R} = \frac{1}{4}.$$

Half of the the total GDP growth is allocated to public good provision and the rest divided evenly among the two types of agents. Decentralizing this through a tax and entitlement system implies that the size of the government should grow accordingly,

$$\frac{\partial \tau^*}{\partial y_R} = \frac{3}{4} \quad \text{and} \quad \frac{\partial e^*}{\partial y_R} = \frac{1}{4}.$$

[12] We also briefly discuss the case where utility in both goods is concave in Appendix 9.8.2.

Because only the rich become richer in the competitive equilibrium, a planner—who wants to equate the consumption of both agents— will tax the rich at a higher rate so that more public good provision and higher entitlements can be provided.

In the political equilibrium, poor and rich individuals disagree on how to distribute the gains from growth. If a rich proposer were in power in the second period, she would choose a much smaller increase in taxes than optimal, and no expansion of the entitlement program. Inspection of Proposition 4 reveals that when R is unconstrained (e.g. $\bar{e} < e_R^D$)

$$\frac{\partial \Psi_{R,2}(\boldsymbol{s})}{\partial y_R} = \frac{1}{2} \quad \text{and} \quad \frac{\partial \mathcal{E}_{R,2}(\boldsymbol{s})}{\partial y_R} = 0.$$

Hence, a rich agent consumes half of the increase and devotes the rest to additional public good provision. A poor proposer has incentives to tax at a higher rate and to over-provide entitlements. Inspection of Proposition 3 reveals that when P is unconstrained (e.g. $\bar{\tau} > \tau_P^D$), she will choose

$$\frac{\partial \Psi_{P,2}(\boldsymbol{s})}{\partial y_R} = 1 \quad \text{and} \quad \frac{\partial \mathcal{E}_{P,2}(\boldsymbol{s})}{\partial y_R} = \frac{1}{2}.$$

When in power, an unconstrained P proposer would fully expropriate the additional resources and use them to increase her consumption and the provision of public goods.

In this environment, thus, unequal growth paired with a long sequence of periods in which P is in power may explain the growth in the size of the government and the level of entitlements. While this analysis is limited because of the two-period environment, we show that the result holds in an infinite horizon version of our model calibrated to the US experience. This is developed in Section 7.4.

7 Infinite Horizon Model

The two period model allowed us to build intuition on how budget rules work in a simple environment. We learned that a proposer can strategically use budget rules to position herself advantageously if the opposition were to gain proposal power in the future. In the case of a poor proposer, this is translated into high entitlements. A rich proposer would try to keep taxes and entitlements low, instead. The exact nature of the budget rule, whether the status quo consists of taxes τ, entitlements e, both, or public goods provision alone g, has sharp consequences for the welfare and equity properties of the resulting allocations. Under unequal growth, where only top-income earners see their incomes rise, poor proposers have further incentives to increase the size of the government through expansions of entitlement programs. These results were derived under the extreme assumption that the proposer in period 1 is completely unconstrained (e.g. the acceptance constraint is slack). In this section, we show that an infinite horizon version of the model generates a time path similar to the one in the US data. In particular, as income inequality grows (see Figure 3), the share of entitlements in total public spending also rises (as in Figure 1).

7.1 Parameterization

There is no analytical solution to the proposer's problem in the infinite horizon model. The main reason, and in contrast to most of papers in the bargaining literature, is that we are considering risk-averse agents and multiple endogenous status-quo variables. Our analysis from now on is, thus, numerical. Because of this, we need to choose a reasonable parameterization for the model, which is described next. Our computational strategy is discussed in Appendix 9.9.

Utility is logarithmic, $u(x) = \ln(x)$. The value of $\theta = 0.5$ is chosen to match the share of entitlements to total government spending $\frac{e}{e+g} = 0.28$ in 1962. This share is computed using information contained in the historical tables published by the Office of Management and Budget of the White House.[13] The value of g is obtained from Table 8.7, under the item "Total outlays for discretionary programs." The data for e corresponds to the item "Total mandatory programs" in Table 8.5. These are also the series used to construct Figure 1. The discount factor is set to $\beta = 0.97$, consistent with a 3% real interest rate (annual model), and implying a standard value for the degree of impatience in the literature. The probability of retaining proposal power is $q = 0.8$, chosen to generate an expected 5-year incumbency by a proposer. This is also the value used in related literature, such as Bowen et al. (2014).

Income in our baseline economy is calibrated to match US data during the early 1960s, a low inequality period. Because of this, we will refer to it as the Δ_{low} environment. The value of y_P is chosen to match the pre-tax income (in 100-thousand dollars) of the bottom 50% of earners in the US in 1960, which can be read off Figure 3 (dark red line). The income of the rich is computed such that average income is 0.3, which corresponds to the value (in 100-thousand dollars) attained in 1960 (see green line in Figure 3). This delivers $y_P = 0.1$ and $y_{R,low} = 0.5$, with corresponding inequality of $\Delta_{low} = y_{R,low} - y_L = 0.4$. Minimum consumption is set to $\bar{x} = \bar{x}_g = 0.1$, as in the two period model, in order to ensure a lower bound for entitlement programs equal to zero.[14]

Parameter	Value	Comment
q	0.8	5 year expected incumbency
β	0.97	3% real interest rate
θ	0.5	Share of e in tot spending $(e+g)$ in 1960
$\overline{x}$	0.1	Normalize lowest $e_l = 0$
Income		
y_P	0.1	Bottom 50% income share
$y_{R,low}$	0.5	Top 50% income share (1960s), Δ_{low} environment
$y_{R,high}$	1.1	Top 50% income share (2010s), Δ_{high} environment

Table 1: Parameter Values

The main experiment consists on increasing, once and for all, the level of inequality to match the observed value in the US during the early 2010s. We will refer to this as the high inequality, or Δ_{high}, environment. To do so, we consider a process of *unequal growth*, where y_R rises to $y_{R,high} = 1.1$ and y_P stays the same. The income level of the rich is chosen to obtain an average income of 0.6, which corresponds to the value (in 100-thousand dollars) attained in the early 2010 (see green line in Figure 3). As a result, $\Delta_{high} = 1.1 - 0.1 = 1$ in this scenario.

We assume that the increase in inequality is sudden and unexpected. We do this for two reasons. First, because it greatly simplifies the computation of the bargaining equilibrium: the equilibrium functions for the two regimes (Δ_{low} and Δ_{high}) can be solved for separately.[15] Second, because

[13]See `https://www.whitehouse.gov/omb/historical-tables/`

[14]This choice is not without loss of generality. The larger the value of $\bar{x}$, the less important budget rules will be for welfare. At the other extreme, as $\bar{x} \to 0$, almost any proposal is accepted by the opposition because, due to the logarithmic utility function assumption, not reaching an agreement becomes arbitrarily costly. Given that the optimal solution is constant, it can trivially be implemented with $\bar{x} = g^*$. With this value, policymakers would always be locked at the optimum and the problem would become uninteresting.

[15]If agents were to foresee the increase in inequality, they would adjust policies in advance. The two period model

it allows us to emphasize that budget rules induce an inefficiently slow adjustment in policies. A benevolent planner would jump to the new allocations (and policies which decentralize them) instantaneously.

7.2 Quantitative Analysis

There are two relevant states in the Markov-Perfect endogenous bargaining equilibrium, namely last period's taxes $\bar{\tau}$ and entitlements $\bar{e}$. In the two-period model, only one of them was relevant for the proposer: proposer P cared about taxes to the rich whereas proposer R cared about entitlements, because these determined the outside option for the opposition. In the infinite horizon model, on the other hand, both states are relevant when choosing a proposal. To fix ideas, suppose that P is the proposer. The level of taxes established in the code matters because it directly affects how likely the opposition is to accept a change in taxes (as in the two-period example). The value of $\bar{e}$ is also important because, if the proposal is rejected, $\bar{e}$ determines next period's status quo. This, in turn, affects P's bargaining power if R were to become the proposer tomorrow. Through continuation utilities, then, $\bar{e}$ affects today's decisions by proposer P (which did not happen in the finite horizon example).

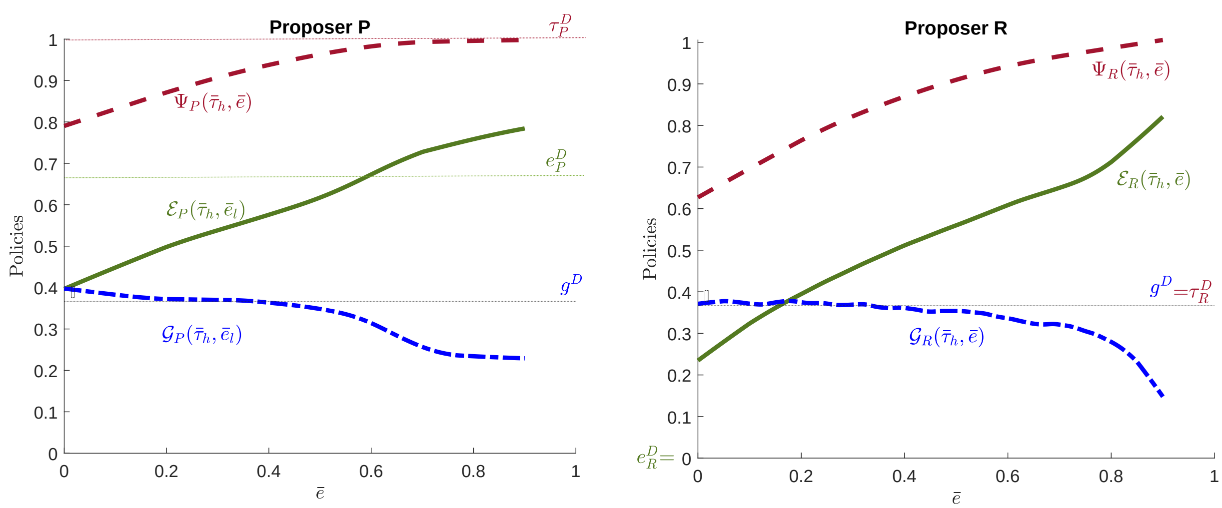


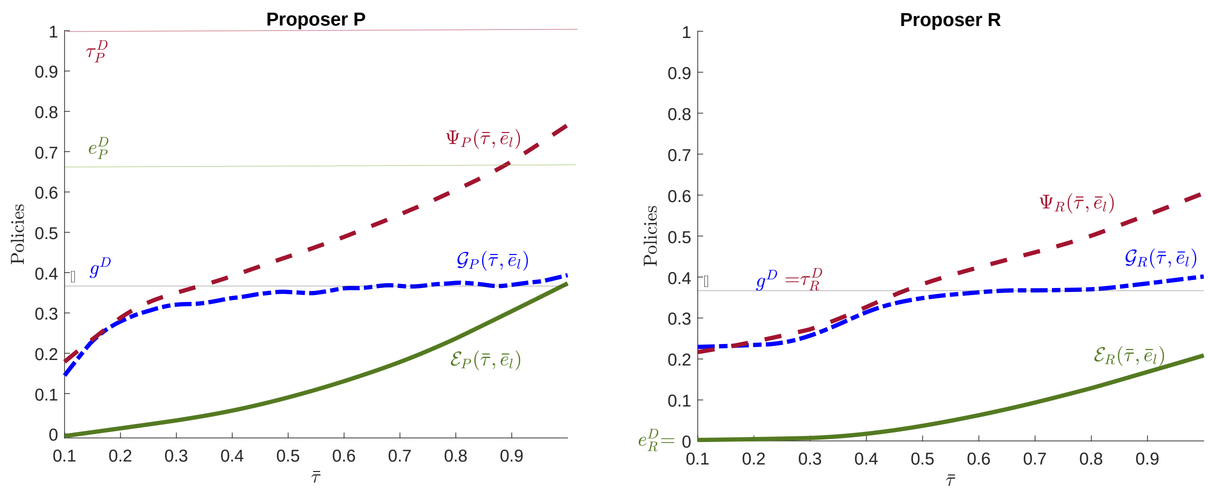
Figure 9: Policy functions for P (left) and R (right) as functions of $\bar{e}$ (fixing $\bar{\tau}_h = 1$). Computed for Δ_{high} environment.

To make the analysis cleaner, we first analyze policy rules as functions of status-quo taxes fixing $\bar{\tau}$, and then let status-quo entitlements vary while fixing taxes. The left panel of Figure 9 depicts P's equilibrium policy rules as functions of $\bar{e}$, fixing taxes at their maximum possible value: $\bar{\tau} = \bar{\tau}_h = 1$. Taxes (dot-dashed garnet line) and entitlements (dashed green line) are increasing in the status-quo level of entitlements $\bar{e}$, whereas public good provision (solid blue line) declines with it. When $\bar{e} > 0.5$, entitlements are above their value under no rules (e.g, when all spending is discretionary and taxes are residually determined), $\mathcal{E}_P > e_P^D$, whereas public good provision is significantly below it, $\mathcal{G}_P < g^D$. This is in contrast with the two period model, where allocations where independent of $\bar{e}$. That they depend on this state is purely a result of the effect of continuation values on current choices. Incumbent P sets a large entitlement program today in order to

indicates that poor proposers would want to strategically increase entitlements *before* the income of the rich rises. While interesting, this case is significantly more complicated to compute, as there is a change in regime entering expectations.

make it difficult for the opposition to reduce it in the future. As a result, there is over-provision of private consumption (through entitlements) and under-provision of public goods, even relative to the case with no rules. When $\bar{e}$ is low, P's bargaining power is very limited (remember that entitlements are set at zero at the outset of the period), making unconstrained entitlement choices unfeasible. In this situation, R's bargaining power is higher. A poor proposer is forced to offer more public goods and a smaller entitlement program than she would like to, because anything else would be rejected by the opposition.

The right panel shows R's policy variables for the same set of states. Given that status-quo taxes are high (at their maximum values, actually), when entitlements are low, R finds it beneficial to propose a significant tax-cut. P only accepts this in exchange of an expansion of the entitlement program. The expansion is, however, smaller than what would be attained if P were the proposer (to see this, compare the value of entitlements at the origin in the left and right panels). The tax cut is also smaller than R would choose in an unconstrained environment with no rules. This can be seen because τ_R^D is always below $\Psi(\bar{\tau}_h, \bar{e})$. In the infinite horizon model, then, a rich proposer is willing to distort her choices away from those under no rules in order to ensure herself a better barging position in the future. By keeping taxes very low, it can tie the hands of a successor from the P group who wants a larger government size. As the status-quo value of entitlements grows, R finds it more difficult to slash such programs, and finances them at the expense of public goods (that eventually fall below g^D).



(a) Proposer P

(b) Proposer R

Figure 10: Policy functions for P (left) and R (right) as functions of $\bar{\tau}$ (fixing $\bar{e}_l = 0$). Computed for Δ_{high} environment

In Figure 10, we keep status-quo level of entitlements fixed at their minimum feasible level $\bar{e} = \bar{e}_l = 0$ and vary potential status-quo variables for $\bar{\tau}$. Public good provision (solid blue line) increases for low values of $\bar{\tau}$, but remains flat once it reaches g^D. It is also worth pointing out that the unconstrained value of τ_P^D is unfeasible in this scenario. As a result, even when $\bar{\tau}$ is close to the upper bound, a poor proposer is unable to reach an agreement that delivers her preferred value of entitlements, $\mathcal{E}_P < e_P^D$. [16] When $\bar{\tau}$ is low, P's bargaining power is very limited (remember

[16]This result is mainly due to the low value of θ, combined with a high q. We have experimented with scenarios where $\theta = 1$ and $q = 0.5$ in which entitlements can be significantly above their unconstrained value $\mathcal{E}_P > e_P^D$ whereas

that entitlements are set at zero at the outset of the period). A poor proposer is forced to offer a minimal entitlement program, because any increase in taxes not used to provide public goods would be rejected by the opposition.

The right hand side of Figure 10 shows the policy rules of a rich proposer that starts the period with $\bar{e} = 0$. Clearly, R has no incentives to start an entitlement program and chooses $\mathcal{E}_R = 0$ for low values of $\bar{\tau}$. Because of the effect of potentially losing elections in the future, she limits the size of the government by choosing lower taxes and public good provision than in the scenario without budget rules. This is an attempt to tie the hands of a potential poor successor who would like to expand the entitlement program. Overall, an R proposer tends to keep the size of the government smaller than a poor one. This can be seen by the fact that all policy rules are flatter under an R proposer.

7.3 Evolution of Policy

The analysis above illustrates that budget rules that make taxes and entitlement programs difficult to change affect a proposers' incentives dynamically. By forcing changes to have a bipartisan support, these rules restrict policymakers' choices, potentially smoothing their evolution over time. Because they have consequences in the future, the strategic incentives may, however, significantly distort the optimal mix between private and public goods. In this subsection, we conduct a series of simulations to further study how policies and allocations evolve over time.

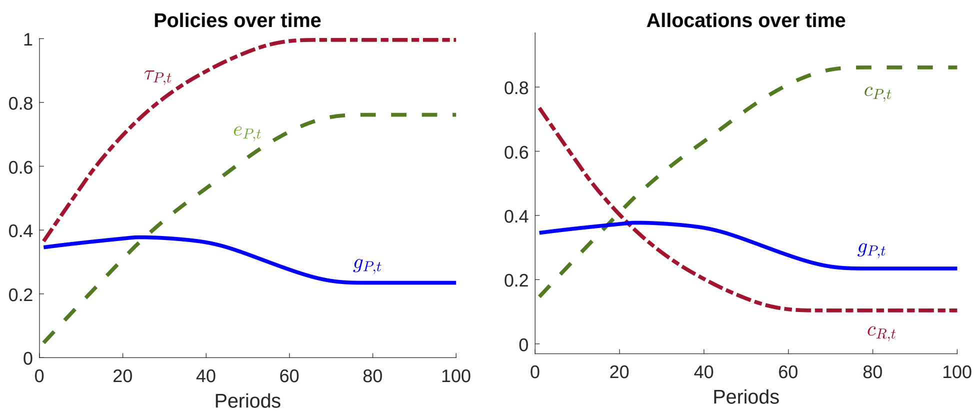


Figure 11: Taxes and entitlements under proposer P. Computed for Δ_{high} environment

When P is in power a long time: We first consider a scenario where proposer P starts period 0 with status-quo taxes $\bar{\tau} = \tau_R^D = 0.366$ and $\bar{e} = e_R^D = 0$, inherited from period -1. This would be the state of the world chosen by R in -1 if unconstrained. We assume that proposer P makes choices under uncertainty, but the realizations of the shock are such that there is no turnover, so P happens to make proposals in all subsequent periods. Policies are shown over time on the left panel of Figure 11, whereas the right panel depicts allocations. Proposer P would like to expand the size of the entitlement program significantly, but it is only able to increase e marginally. The rich are willing to accept this because their consumption of public goods would reduce to $\bar{x}$ if

public good provision can be significantly below it, $\mathcal{G}_P < g^D$.

they were to reject the proposal. To the extent that P remains in power, she will slowly expand the entitlement program—and hence her own consumption of private goods (both dashed green lines),— through rises in taxes. Eventually, P will be able to secure herself a good enough bargaining position (through high $\bar{e}$) to start reducing g and keep high taxes in order to finance increases in e. If P is in power long enough, she would reach a steady-state level of private consumption which is significantly higher than what she would choose under no rules. In that new steady state, public good provision would be inefficient and there would be a high degree of ex-post income inequality, as seen by the distance between the two consumption levels (which reflect after-tax income).

When P and R alternate in power: The economy would only reach a steady state if one of the parties were in power forever. Because the proposer changes stochastically, the direction of policy does as well. Over time, the economy reaches an *ergodic set* determining a range in which policy fluctuates forever-after. To compute the ergodic set, we simulated the economy for $1,000,000$ periods, and eliminated the first $1,000$ periods. It is worth noticing that the economy converges to the same set regardless of initial conditions.

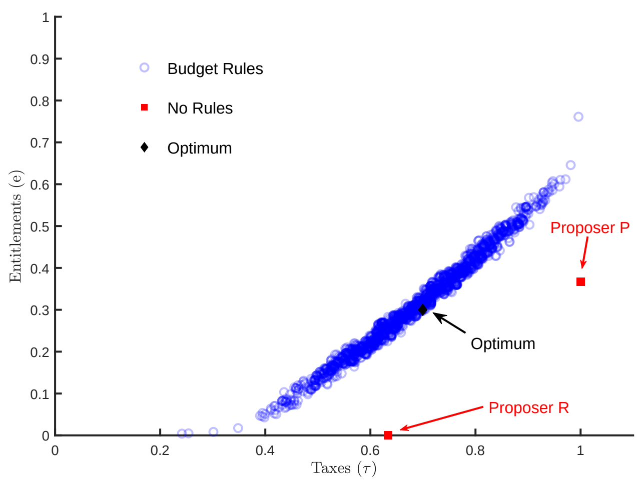


Figure 12: Scatter plot of taxes and entitlements in the simulation. Computed for Δ_{high} environment.

Figure 12 depicts a scatter plot of taxes and entitlement pairs for each period in the simulation (marked with blue circles), together with the pairs that would be obtained under no budget rules for each type of proposer (marked with red squares) and the optimal solution (black diamond). Interestingly, the bargaining solution does not overlap with the solution under discretionary spending (e.g. no budget rules): having to agree on policies reduces the ability of each proposer to reach their most preferred unconstrained value. This brings the solution closer to the optimal one (which is actually reached from time to time in our simulation). On average, taxes, spending, and entitlements (as percentages of total output) are close to the optimal amounts (see last column of Table 2).

Policies span a significant portion of the state-space in the bargaining solution, whereas it would be optimal to keep them constant. This implies that their evolution (and hence, that of private and public consumption) is less persistent and more volatile than optimal, as evident from Table 2. As a result, welfare is slightly below optimal in the decentralized solution with budget

	Δ_{low}		Δ_{high}	
	Optimal	Budget Rules	Optimal	Budget Rules
Long-run Shares (%)				
g/Y	33.3	32.5	33.3	31.1
τ/Y	50.0	49.6	58.3	57.2
e/Y	16.7	17.1	25.0	26.1
$e/(e+g)$	33.3	33.6	42.9	43.9
Coefficient of Variance (%)				
c_P	0	16.8	0	28.4
c_R	0	16.8	0	28.4
g	0	3.4	0	5.2
Autocorrelation (%)				
c_P	100	99.3	100	99.1
c_R	100	99.3	100	99.1
g	100	23.3	100	43.9
Welfare gains				
Per person		−0.24%		−2.9%

Table 2: Moments of the Ergodic Distribution, Budget Rules vs Optimal.

rules. More specifically, it is 2.9% lower than welfare attained under optimal allocations.[17] The welfare losses arise for two reasons: (i) slight under-provision of public goods and over-provision of private goods and (ii) volatility in resulting allocations.

The first two columns of Table 2 report the same long-run statistics but for the baseline economy with Δ_{low}. When inequality is lower, policies are even closer, on average, to the optimal ones. Because volatility is also lower, the welfare attained is just 0.24% below the optimal level. Why does this happen? A potential reason is that there is less room to maneuver in the bargaining solution under Δ_{low}. Given a fixed value of $\bar{x}$, lower inequality implies that agents have a lower ability to exploit budget rules in their advantage. This increases the gridlock region, making changes in policy towards the unconstrained solution less likely.

Finally, Table 2 uncovers a dramatic difference in the long run share of entitlements to total spending $\frac{e}{e+g}$ between the Δ_{low} and the Δ_{high} environments. This suggests that our model could explain the sharp increase in the share of entitlements depicted in Figure 1. We analyze the transition from the low-inequality to the high-inequality environment next.

[17] Welfare gains (and losses) are computed in consumption equivalent terms. Note that $V = (1-\beta)(\log c + \theta \log g) + \beta V$, so average welfare V is equal to $\log c + \theta \log g$. This can be written as $\log(cg^\theta)$ with cg^θ as a "consumption composite." By computing $ce = \exp V$, we obtain welfare levels as consumption equivalent units. In the table, we report the percentage loss between the optimal value ce_o and the one obtained under budget rules ce_r. Welfare gains are computed as $\frac{ce_o - ce_r}{ce_r}$.

7.4 Unequal Growth and Entitlements

The previous section considered two alternative environments, one with low inequality, Δ_{low}, resembling the US in the early 1960s and one with high inequality, Δ_{high}, capturing the US in the 2010s. In this section, we perform the following experiment: suppose that the economy has been in a Δ_{low} environment for a long period of time (so that we are initially somewhere in the ergodic set). Assume that in period 1 there is a permanent and unexpected increase in inequality where only y_R rises (e.g. we move to the Δ_{high} environment). Finally, imagine that proposers alternate in power consistently with the US experience. What would our model predict for the share of entitlements $\frac{e}{e+g}$ over time?

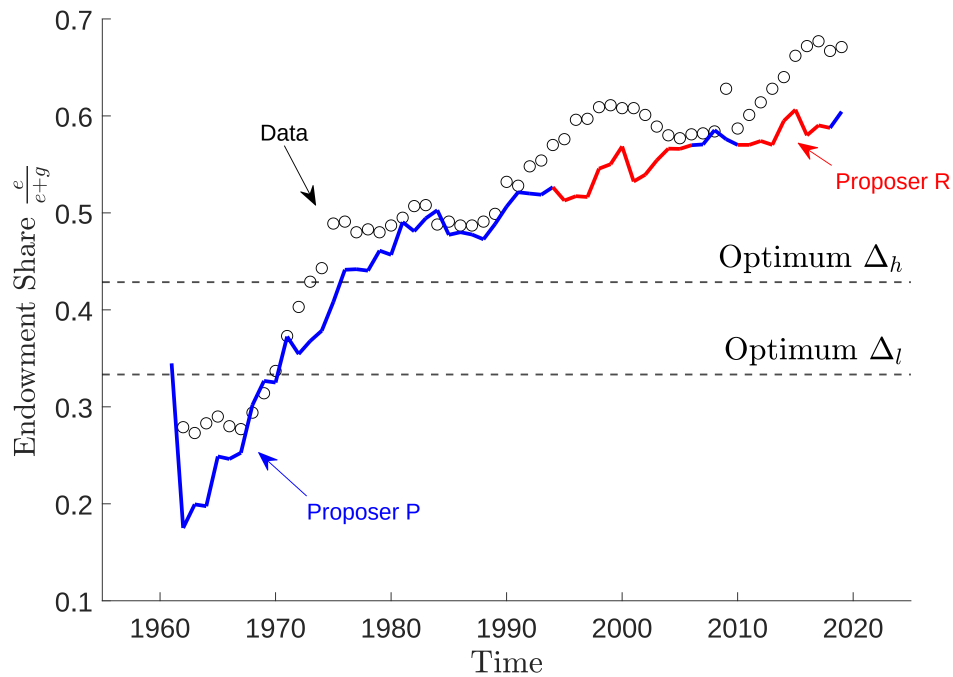
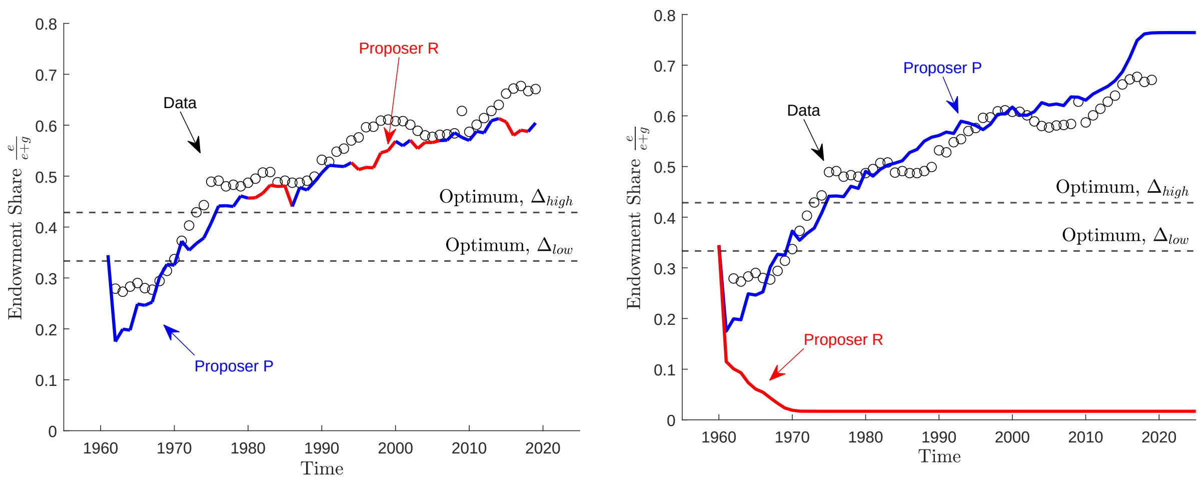


Figure 13: Share of entitlement in total spending under unequal growth

A key calibration detail consists on determining the time-path for alternation of proposers of the two parties during the transition. To do so, we first computed which party had a majority in the House of Representatives between 1960 and 2010. We then assigned proposer power to the party with a majority in the data, where the Democratic party corresponds to P and the Republican party corresponds to R. This mapping was assumed because Democrats are typically in favor of expanding welfare programs at the expense of discretionary spending (such as defense) and higher taxes, whereas Republicans generally propose tax-cuts and attempt to control the size of the government. Through this procedure, we obtained a sequence of P and R proposers consistent with the data which was inputted in our model simulation. The resulting path for the share of entitlements is shown in Figure 13 (solid line). The corresponding value from the data (analogous to that in Figure 1) is represented by circles in the same figure. A blue color in the share of entitlements indicates that P has proposer power, whereas a red color indicates that R has proposer power instead. While the fit is not perfect, the model is able to generate an increasing path in the share of entitlements to total spending, which is in line with the data.

For robustness, we re-computed the experiment using the time-path of majorities in the Senate by each party. The result is shown in the left panel of Figure 14, and except for minor differences, is consistent with the previous findings. For contrast, we also computed the evolution of policy assuming, conter-factually, that R was the proposer over the whole period. The resulting time-path is shown in the right panel of Figure 14. If allowed to, proposer R would drive the share of entitlements to zero, despite the increase in inequality.



(a) Proposer Based on Majority in Senate

(b) Unexpected Permanent Incumbency

Figure 14: Experiment for alternative proposer sequences

Our model then fits the data remarkably well. We are able to explain the sharp increase in entitlements as a percentage of total spending in the data using a model in which parties bargain over taxes and entitlements. From the figures, we can see that it is the combination of budget rules and persistence of party P with proposal power what leads to such increase.

It is worth noticing that a benevolent planner would also expand the entitlement program when inequality rises (as shown in the two period model). In our calibrated economy, the planner would jump from the dashed horizontal line in the figure corresponding to Δ_{low} to the line under Δ_{high} instantaneously. In the bargaining solution, the transition takes many periods instead. Moreover, we see that the program becomes inefficiently large over time, as the entitlement share is significantly above the optimal level after the early 1980s. In the counter-factual situation in which R was in power, the program would shrink inefficiently instead.

7.5 The importance of budget rules, revisited

The results in Table 2 indicate that budget rules on taxes and entitlements can deliver long-run allocations which are relatively close to optimal (for both, the low-inequality and high-inequality environments). Power alternation has advantages and disadvantages. On the one hand, turnover brings upon volatility (which is sub-optimal). On the other hand, it limits the ability of each group to move the economy towards very unequal equilibria. In this section, we ask whether society would be better off without budget rules or adopting alternative types of rules. For example, we could consider a scenario where only taxes are written in a code, but entitlements are discretionary (a τ-only rule). Alternatively, we could consider a situation where entitlements are mandatory and taxes are determined residually (an, e-only rule). Finally, we could impose a rule in which only public good spending is mandatory (a g-only rule). Moments associated with these possibilities are shown in Table 3.

	Optimal	Budget Rules				
		No Rules	τ & e	τ	e	g
Long-run Shares (%)						
g/Y	33.3	30.6	31.1	26.2	34.9	36.2
τ/Y	58.3	56.9	57.2	38.3	45.3	59.8
e/Y	25.0	26.4	26.1	12.1	10.4	23.6
$e/(e+g)$	42.9	31.6	43.9	22.3	22.2	29.6
Coefficient of Variance (%)						
c_P	0	75.9	28.4	68.9	27.3	70.1
c_R	0	75.9	28.4	34.2	14.3	70.1
g	0	0	5.2	19.4	13.8	4.8
Autocorrelation (%)						
c_P	100	60	99	90	98	60
c_R	100	60	99	93	75	60
g	100	100	44	89	63	99
Welfare gains						
Average		−32%	−3%	−20%	−3%	−23%
Rich		−32%	−3%	20%	26%	−23%
Poor		−32%	−3%	−59%	−32%	−23%

Table 3: Alternative budget rules. Computed for Δ_{high} environment

Welfare gains (losses if negative) are computed relative to the optimal value. A political environment without rules (where all spending is discretionary and taxes are residually determined) is associated with a welfare loss of 32% per-capita, symmetrically distributed across rich and poor agents. As in the two-period model, this arises in part because of the distortions in allocations. Note that g is slightly under-provided and c is slightly over-provided. Most of the losses, however, come from the high volatility of consumption. The optimal solution dictates that c_R and c_P must be constant, but they fluctuate between c_R^D and c_P^D in the scenario with no rules, instead. In this environment, private consumption is valued more than public consumption, so deviations from a smooth profile of c are particularly painful for agents with concave utility. Budget rules on τ&e, our benchmark case, reduce volatility and have more efficient allocations on average (see the third column in the table), reaching welfare levels which are only 3% shy of the optimal ones. For this calibration, then, rules on taxes and entitlements generate a Pareto improvement relative to a case where no rules are imposed.

A τ-only rule (fourth column) would result in welfare gains relative to the no-rules scenario, but not relative to the benchmark case with budget rules on taxes and entitlements. Interestingly, welfare gains are no longer symmetric. Absent the ability to set an advantageous status-quo for entitlements, poor agents have very little tools to move the equilibrium in their favor. Rich agents, who start with high wealth, are able to keep most of it by keeping the size of the government small. This results in long-run welfare gains of 20% for the rich and losses of almost 60% for the poor. The equilibrium with only a tax code results in a much more unequal distribution of resources. This is in contrast to the two-period analysis, where a τ-only rule was inconsequential for a proposer of type P. The big difference arises because of the ergodic set that can now be reached, uderscoring the importance of considering longer horizons in models with endogenous status-quo variables.

An economy that only had an entitlement rule (fifth column) would be better for poor agents than one with only a tax code. Their welfare is now just 32% lower than optimal. Rich agents would again be better off than in the optimal case because this rule reduces the amount of redistribution that would be feasible in the political equilibrium. As poor agents cannot grow taxes, their level of entitlements does not rise above a certain level. However it takes longer to be reduced by rich agents when in power, and this explains why the poor fair better than in a τ-only rule. For them, however, having a e-only rule delivers the same long-run welfare as an environment with no rules. This result is again different from the one in the two-period model, and stresses the importance of incorporating continuation utilities to the welfare analysis.

Finally, we consider a scenario where taxes and entitlements are freely chosen, but public goods are subject to a g-rule. Because both agents derive utility from g, we would expect this rule to be beneficial for society. This is, after all, the main finding from Appendix 9.8. Interestingly, we find that it does not achieve a solution closer to the optimal value than the benchmark case. The reason being that public goods are over-provided, while private goods are under-provided in this case. Because agents derive more utility from the consumption of c than from the consumption of g (recall that $\theta = 0.5$ in the calibrated example), this distortion is costly. In addition, while the g rule makes public good provision smoother (the autocorrelation coefficient is 99%), it makes private consumption more volatile (with an autocorrelation coefficient of 60%, closer to the no-rule scenario). This is a second source of welfare loss in this model.

Overall, we find that budget rules over taxes and entitlements allow us to reach a situation which is closer to the optimal one than the other possible rules. This result hinges on the specific parameterization. In an environment where θ is higher and q is lower, a mandatory spending rule on g could generate higher welfare gains (details available upon request).

8 Conclusion

We showed that the nature of budget rules, the fiscal instruments constituting the status quo, has first-order consequences for welfare, equity, and thus for the design of optimal budgetary institutions. One surprising finding is that budget rules determined by taxes and entitlements can yield worse outcomes than a regime under which all fiscal policy is discretionary. Whether budget rules can improve outcomes depends critically on the type of goods subjected to mandatory spending, public or private consumption, and the primitives of the environment, e.g., incumbency or tastes for public provision.

We leveraged these insights to argue that the increase in the share of entitlements in the US over the last six decades can be rationalized through a political economy model with bargaining over taxes and entitlements, combined with a process of unequal growth. Low-income earners who are excluded from the benefits associated with an expansion of resources can guarantee redistribution through their legislators. As the size of the economy rises, high-income earners would like more provision of public goods. Poor agents are willing to accept this in exchange for an expansion of entitlement programs. If they retain proposer power over a long enough period of time, this results in an increasing share of entitlements to total spending. We show that a calibrated model of the US economic and political environments delivers dynamics consistent with the data. Using the model, we evaluate how agents fare under counter-factual budget rules. If only public goods were mandatory, welfare of the two types of agents would be lower. This is the case because of the high volatility of allocations that would result in such environment. As expected, eliminating entitlement programs would only benefit high-income earners. Surprisingly, keeping entitlement programs but making taxes easier to change (e.g. no longer a status-quo variable) would have similar effects. This is the case because the rich are initially endowed with more resources than the poor and would oppose financing redistribution through tax increases over long periods of time. With flexibility in their ability to implement tax-cuts, high-income earners can keep the size of the government small. This shows that what generates the dynamics we observe in the data requires budget rules on both, taxes and entitlements.

Our model can be extended in several interesting dimensions. We assumed away the distortionary costs of taxation associated to implementing equilibrium allocations. If taxes were distortionary, the marginal gains from expanding entitlement programs would be significantly smaller, plausibly impacting the welfare gains of alternative budget rules. We have also assumed that the government is subject to a balanced budget. The only way in which low-income earners can increase the share of entitlements—or high-income earners introduce a tax-cut—is by reducing the provision of public goods. If the government were able to issue debt, we would expect parties to potentially agree to borrow in order to achieve their desired allocations. However, as Bouton, Lizzeri, and Persico (2020) pointed out, entitlement programs may be a good substitute for debt in these types of models. It would be interesting, thus, to augment our model to consider the possibility of issuing public debt in order to study whether policymakers would use all these instruments in the political equilibrium.

We have considered a situation where the income level of each agent is fully persistent over time. This makes the preferred fiscal policy by each group starkly different. If agents were subject to idiosyncratic risk, however, entitlement programs would also provide some insurance making it a desirable mandatory program for all agents in society. Other possible extensions involve studying how volatility in the endowment or changes in the degree of income inequality could affect our findings.[18]

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[18] See Dziuda and Loeper (2016) and Dziuda and Loeper (2018) for environments where legislators are subject to preference shocks.

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9 Appendix

9.1 Proof of Proposition 1.

The set of efficient allocations $\mathbf{a}^*$ can be traced-out by choosing sequences of private and public consumption in order to maximize a weighted sum of the lifetime utility of agents,

$$\max_{\{\mathbf{a}\}} \left\{ \lambda \mathcal{V}_P(\mathbf{a}) + (1-\lambda)\mathcal{V}_R(\mathbf{a}) \right\}, \tag{16}$$

subject to the resource constraint, eq. (5). The parameter $\lambda \in (0,1)$ denotes the Pareto-weight of poor agents. For $\lambda \in (0,1)$, the Lagrangian is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \lambda U(c_{P,t}, g_t) + (1-\lambda) U(c_{R,t}, g_t) + \psi_t \left[Y - c_{P,t} - c_{R,t} - g_t \right] \right\}$$

Recalling $U(c_{i,t}, g_t) = u(c_{i,t}) + \theta u(g_t)$, the first order and Kuhn-Tucker conditions for this problem are $c_{P,t}, c_{R,t}, g_t, \psi_t \geq 0$ and

$$\begin{aligned}
[c_{P,t}] &\quad \lambda u'(c_{P,t}) - \psi_t \leq 0 \\
&\quad c_{P,t}[\lambda u'(c_{P,t}) - \psi_t] = 0 && (17) \\
[c_{R,t}] &\quad (1-\lambda) u'(c_{R,t}) - \psi_t \leq 0 \\
&\quad c_{R,t}[(1-\lambda) u'(c_{R,t}) - \psi_t] = 0 && (18) \\
[g_t] &\quad \theta u'(g_t) - \psi_t \leq 0 \\
&\quad g_t[\theta u'(g_t) - \psi_t] = 0 && (19) \\
[RC] &\quad [Y - c_{P,t} - c_{R,t} - g_t]\psi_t = 0 && (20)
\end{aligned}$$

Since $u(.)$ is increasing in its argument, $\psi_t > 0$ and eq. (20) holds with equality. Because $u(.)$ satisfies Inada conditions and $\lambda \in (0,1)$, $g_t, c_{P,t}, c_{R,t} > 0$. This implies that

$$(1-\lambda) u'(c^*_{R,t}) = \lambda u'(c^*_{P,t}) = \theta u'(g^*_t).$$

Under logarithmic utility,

$$\frac{\theta}{g^*_t} = \frac{\lambda}{c^*_{P,t}} = \frac{1-\lambda}{c^*_{R,t}}.$$

which, after some manipulation, delivers the expressions in the main text.

9.2 Proof to Proposition 2

Because taxes and transfers are non-distortionary, the maximization problem is equivalent to one where the government chooses allocations directly. In other words, we can re-write the problem as

$$V^D = \max_{\{c_{i,i}, c_{j,i}, g\}} \left\{ u(c_{i,i}) + u(g) + \beta \left[q V^D + (1-q) W^D \right] \right\}, \tag{21}$$

subject to the resource constraint (5) and the bounds on consumption $c_{i,i}, c_{i,j} \geq \bar{x}$. Moreover, the problem of an incumbent is independent of her type when written in terms of allocations, so $V^D_i = V^D$. Because there is no dynamic state variable, the problem is equivalent to

$$\max_{\{c_{j,i}, g\}} \left\{ u(Y - c_{j,i} - g) + u(g) \right\}$$

subject to $c_{j,i} \geq \bar{x}$. Clearly, it is optimal for an incumbent to set $c_{j,i} = \bar{x}$. The optimal g equates her private marginal benefit of g to her marginal cost. Taking first order conditions yields the result in the proposition. Policies can be decentralized using eq. (4).

9.3 Proof of Proposition 3

Let $u(x) = \ln(x)$. Since endowments and taxes are non-distortionary, we can solve the problem in terms of allocations. This requires a transformation for the state space from status-quo policy pairs $\{\overline{\tau}, \overline{e}\}$ into status-quo consumption levels $\{\overline{c_P}, \overline{c_R}\}$. The consumer budget constraints provide a simple linear mapping to do this. Given $\mathbf{s} = \{\overline{c_P}, \overline{c_R}\}$, party's P Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = \ln(c_{P,2}) + \ln(g_2) + \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi \left(\ln(c_{R,2}) + \ln(g_2) - \ln(\overline{c_R}) - \ln(\overline{x})\right)$$

The first-order and Kuhn-Tucker conditions are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
[c_{P,2}] \quad & \frac{1}{c_{P,2}} - \lambda \leq 0. \\
& (c_{P,2} - \bar{x}) \left[\frac{1}{c_{P,2}} - \lambda\right] = 0. && (22) \\
[c_{R,2}] \quad & \frac{\psi}{c_{R,2}} - \lambda \leq 0. \\
& (c_{R,2} - \bar{x}) \left[\frac{\psi}{c_{R,2}} - \lambda\right] = 0. && (23) \\
[g_2] \quad & \frac{1+\psi}{g_2} - \lambda \leq 0. \\
& (g_2 - \bar{x}) \left[\frac{1+\psi}{g_2} - \lambda\right] = 0. && (24) \\
[RC] \quad & \left[Y - c_{P,2} - c_{R,2} - g_2\right] \geq 0. \\
& \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] = 0. && (25) \\
[IRC] \quad & \left[\ln(c_{R,2}) + \ln(g_2) - \ln(\overline{c_R}) - \ln(\overline{x})\right] \geq 0. \\
& \psi \left[\ln(c_{R,2}) + \ln(g_2) - \ln(\overline{c_R}) - \ln(\overline{x})\right] = 0. && (26)
\end{aligned}$$

The solution to this system depends on the status quo vector $\mathbf{s} = \{\overline{c_R}, \overline{c_P}\}$. Denote the solution by functions $\mathcal{C}_{i,P,2}(\mathbf{s})$ for private consumption, $\mathcal{G}_{P,2}(\mathbf{s})$ for public goods, and $\Psi_{P,2}(\mathbf{s})$ and $\mathcal{E}_{P,2}(\mathbf{s})$ for policy variables.

First note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. In addition, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{C}_{i,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. We have three cases to consider:

- Case 1: $\psi = 0$. By eq. (23) we have that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$. By eq. (22), eq. (24) and eq. (25), we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = \mathcal{G}_{P,2}(\mathbf{s}) = \frac{Y-\overline{x}}{2}$. Using eq. (3) to solve for transfers delivers $\mathcal{E}_{P,2}(\mathbf{s}) = \frac{\Delta-\overline{x}}{2} = e^D_P$. The expression for $\Psi_{P,2}(\mathbf{s})$ can be obtained from eq. (2). By eq. (26), this case holds if and only if $\overline{c_R} = y_R - \overline{\tau} < \frac{Y-\overline{x}}{2}$. In terms of policies, this happens when $\overline{\tau} > \frac{\Delta+\overline{x}}{2} = \tau^D_R$.
- Case 2: $\psi > 0$, $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{P,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. By eq. (26) we have that $\mathcal{G}_{P,2}(\mathbf{s}) = \overline{c_R} = y_R - \overline{\tau}$. By eq. (25), we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = Y - \overline{x} - \overline{c_R}$. From eq. (3), we have that $\mathcal{E}_{P,2}(\mathbf{s}) = \overline{\tau} - \overline{x}$. From eq. (2), $\Psi_{P,2}(\mathbf{s}) = \tau^D_P$. By eq. (22), eq. (24), eq. (23) and the fact that $\mathcal{C}_{R,P,2}(\mathbf{s}) \geq \overline{x}$, this case holds if and only if $\frac{\Delta}{2} < \overline{\tau} \leq \tau^D_R$.

- Case 3: $\psi > 0$ and $\mathcal{C}_{i,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. By eq. (22), eq. (23), eq. (24) and eq. (25), we have that $\mathcal{G}_{P,2}(\mathbf{s}) = \frac{Y}{2} = g^*$. By eq. (26), we have that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \frac{2\overline{x}\overline{c_R}}{Y}$. From eq. (2), we have that $\Psi_{P,2}(\mathbf{s}) = y_R - \frac{2\overline{x}\overline{c_R}}{Y} = y_R - \frac{2\overline{x}(y_R - \overline{\tau})}{Y}$. The expression for $\mathcal{E}_{P,2}(\mathbf{s})$ follows by using eq. (3). By eq. (22), eq. (24) and eq. (23), this case holds if and only if $\overline{\tau} < \frac{\Delta}{2}$.

9.4 Proof of Proposition 4

Party's R Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = \ln(c_{R,2}) + \ln(g_2) + \lambda\left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi\left(\ln(c_{P,2}) + \ln(g_2) - \ln(\overline{c_P}) - \ln(\overline{x})\right)$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
[c_{P,2}] \quad & \frac{\psi}{c_{P,2}} - \lambda \leq 0. \\
& (c_{P,2} - \bar{x})\left[\frac{\psi}{c_{P,2}} - \lambda\right] = 0. && (27)\\
[c_{R,2}] \quad & \frac{1}{c_{R,2}} - \lambda \leq 0. \\
& (c_{R,2} - \bar{x})\left[\frac{1}{c_{R,2}} - \lambda\right] = 0. && (28)\\
[g_2] \quad & \frac{1+\psi}{g_2} - \lambda \leq 0. \\
& (g_2 - \bar{x})\left[\frac{1+\psi}{g_2} - \lambda\right] = 0. && (29)\\
[RC] \quad & \left[Y - c_{P,2} - c_{R,2} - g_2\right] \geq 0. \\
& \lambda\left[Y - c_{P,2} - c_{R,2} - g_2\right] = 0. && (30)\\
[IRC] \quad & \left[\ln(c_{P,2}) + \ln(g_2) - \ln(\overline{c_P}) - \ln(\overline{x})\right] \geq 0. \\
& \psi\left[\ln(c_{P,2}) + \ln(g_2) - \ln(\overline{c_P}) - \ln(\overline{x})\right] = 0. && (31)
\end{aligned}$$

First note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{C}_{i,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. We have three cases to consider:

- Case 1: $\psi = 0$. By eq. (27) we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$. By eq. (28), eq. (29) and eq. (30), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = \mathcal{G}_{R,2}(\mathbf{s}) = \frac{Y - \overline{x}}{2}$. The expressions for $\mathcal{E}_{R,2}(\mathbf{s})$ and $\Psi_{R,2}(\mathbf{s})$ can be obtained by replacing allocations into the consumers' budget constraints. By eq. (31), this case holds if and only if $\overline{c_P} = y_P + \overline{e} < \frac{Y - \overline{x}}{2}$. This happens only when $\overline{e} < \frac{\Delta - \overline{x}}{2} = e_P^D$.

- Case 2: $\psi > 0$, $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{R,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (26) we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \overline{c_P} = y_P + \overline{e}$. By eq. (25), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \overline{x} - \overline{c_P}$. The expressions for $\mathcal{E}_{R,2}(\mathbf{s})$ and $\Psi_{R,2}(\mathbf{s})$ can be obtained by replacing allocations into the consumers' budget constraints. By eq. (22), eq. (24), eq. (23) and the fact that $\mathcal{C}_{P,R,2}(\mathbf{s}) \geq \overline{x}$, this case holds if and only if $\frac{\Delta - \overline{x}}{2} = e_P^D < \overline{e} \leq \frac{\Delta}{2}$.

- Case 3: $\psi > 0$ and $\mathcal{C}_{i,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (22), eq. (23), eq. (24) and eq. (25), we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \frac{Y}{2} = g^*$. By eq. (26), we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \frac{2\overline{x}\overline{c_P}}{Y}$. The expressions for $\mathcal{E}_{R,2}(\mathbf{s})$ and $\Psi_{R,2}(\mathbf{s})$ can be obtained by replacing allocations into the consumers' budget constraints. By eq. (22), eq. (24) and eq. (23), this case holds if and only if $\overline{e} \geq \frac{\Delta}{2}$.

9.5 Proof of Proposition 5

Since taxes and entitlements are all non-distortionary, we can write the whole problem in terms of allocations. Continuation values in terms of allocations satisfy:

$$V_P(c_{R,1}) = \begin{cases} \ln\left(\frac{Y-\bar{x}}{2}\right) + \ln\left(\frac{Y-\bar{x}}{2}\right), & \text{if} \quad c_{R,1} < \frac{Y-\bar{x}}{2}. \\ \ln(Y - \bar{x} - c_{R,1}) + \ln(c_{R,1}), & \text{if} \quad c_{R,1} \in \left[\frac{Y-\bar{x}}{2}, \frac{Y}{2}\right) \\ \ln\left(\frac{Y}{2} - \frac{2\bar{x}c_{R,1}}{Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad c_{R,1} \geq \frac{Y}{2} \end{cases}$$

and

$$W_P(c_{P,1}) = \begin{cases} \ln(\bar{x}) + \ln\left(\frac{Y-\bar{x}}{2}\right), & \text{if} \quad c_{P,1} < \frac{Y-\bar{x}}{2}. \\ \ln(\bar{x}) + \ln(c_{P,1}), & \text{if} \quad c_{P,1} \in \left[\frac{Y-\bar{x}}{2}, \frac{Y}{2}\right) \\ \ln\left(\frac{2\bar{x}c_{P,1}}{Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad c_{P,1} \geq \frac{Y}{2} \end{cases}$$

Note that the only relevant state for party P when in power is $c_{R,1}$ and for when out of power is $c_{P,1}$. Therefore, party's P Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{P,1}) + \ln(g_1) + \beta\left[qV_P(c_{R,1}) + (1-q)W_P(c_{P,1})\right] + \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party P are $c_{P,1}, c_{R,1}, g_1 \geq \bar{x}$, $\lambda, \psi \geq 0$ and

$$[c_{P,1}] \quad \frac{1}{c_{P,1}} + \beta(1-q)\frac{dW^P}{dc_{P,1}} - \lambda \leq 0.$$

$$(c_{P,1} - \bar{x})\left[\frac{1}{c_{P,1}} + \beta(1-q)\frac{dW^P}{dc_{P,1}} - \lambda\right] = 0. \tag{32}$$

$$[c_{R,1}] \quad \beta q \frac{dV^P}{dc_{R,1}} - \lambda \leq 0.$$

$$(c_{R,1} - \bar{x})\left[\beta q \frac{dV^P}{dc_{R,1}} - \lambda\right] = 0. \tag{33}$$

$$[g_1] \quad \frac{1}{g_1} - \lambda \leq 0.$$

$$(g_1 - \bar{x})\left[\frac{1}{g_1} - \lambda\right] = 0. \tag{34}$$

$$[RC] \quad [Y - c_{P,1} - c_{R,1} - g_1] \geq 0.$$

$$\lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right] = 0. \tag{35}$$

Again, first note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\bar{x}$ is relatively small relative to Y, $\mathcal{C}_{P,P,1}, \mathcal{G}_{P,1} > \bar{x}$. Also, by eq. (33), we have that $\mathcal{C}_{R,P,1} = \bar{x}$. This implies that $\frac{dV^P(c_{R,1})}{dc_{R,1}} = 0$. By eq. (35), we have that $c_{P,1} = Y - \bar{x} - g_1$ and $\frac{dW^P(c_{P,1})}{dc_{P,1}} = \frac{1}{c_{P,1}}$. By eq. (32) and eq. (34), we have that $\mathcal{G}_{P,1} = \frac{Y-\bar{x}}{2+\beta(1-q)} = \frac{2}{2+\beta(1-q)}g^D$. Back to eq. (35), we have that $\mathcal{C}_{P,P,1} = (1 + \beta(1-q))\frac{Y-\bar{x}}{2+\beta(1-q)}$. Since $\mathcal{C}_{P,P,1} = y_P + \mathcal{E}_{P,1}$ we have that $\mathcal{E}_{P,1} = \frac{2e_P^D + \beta(1-q)\tau_D^P}{2+\beta(1-q)}$. Also, since $\mathcal{C}_{R,P,1} = \bar{x} = y_R + \Psi_{P,1}$, we have that $\Psi_{P,1} = y_R - \bar{x} = \tau_P^D$.

9.6 Proof of Proposition 6

We follow as for the problem of party P. Party's R Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{R,1}) + \ln(g_1) + \beta\left[qV_R(c_{P,1}) + (1-q)W_R(c_{R,1})\right] + \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,1}, c_{R,1}, g_1 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
[c_{R,1}] \quad & \frac{1}{c_{R,1}} + \beta(1-q)\frac{dW^R}{dc_{R,1}} - \lambda \leq 0. \\
& (c_{R,1} - \bar{x})\left[\frac{1}{c_{R,1}} + \beta(1-q)\frac{dW^R}{dc_{R,1}} - \lambda\right] = 0. && (36) \\
[c_{P,1}] \quad & \beta q\frac{dV^R}{dc_{P,1}} - \lambda \leq 0. \\
& (c_{P,1} - \bar{x})\left[\beta q\frac{dV^R}{dc_{P,1}} - \lambda\right] = 0. && (37) \\
[g_1] \quad & \frac{1}{g_1} - \lambda \leq 0. \\
& (g_1 - \bar{x})\left[\frac{1}{g_1} - \lambda\right] = 0. && (38) \\
[RC] \quad & \left[Y - c_{P,1} - c_{R,1} - g_1\right] \geq 0. \\
& \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right] = 0. && (39)
\end{aligned}$$

Again, first note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{C}_{R,R,1}, \mathcal{G}_{R,1} > \overline{x}$. Also, by eq. (37), we have that $\mathcal{C}_{P,R,1} = \overline{x}$. This implies that $\frac{dV^R(c_{P,1})}{dc_{P,1}} = 0$. By eq. (35), we have that $c_{R,1} = Y - \overline{x} - g_1$ and $\frac{dW^R(c_{R,1})}{dc_{R,1}} = \frac{1}{c_{R,1}}$. By eq. (36) and eq. (38), we have that $\mathcal{G}_{R,1} = \frac{Y-\overline{x}}{2+\beta(1-q)} = \frac{2}{2+\beta(1-q)}g^D$, as in party's P problem. Back to eq. (35), we have that $\mathcal{C}_{R,R,1} = (1+\beta(1-q))\frac{Y-\overline{x}}{2+\beta(1-q)}$. Since $\mathcal{C}_{R,R,1} = y_R - \Psi_1$ we have that $\Psi_1 = \frac{y_R+(1+\beta(1-q))e^D_R}{2+\beta(1-q)}$. Also, since $\mathcal{C}_{P,R,1} = \overline{x} = y_P + \mathcal{E}_{R,1}$, we have that $\mathcal{E}_{R,1} = \overline{x} - y_P = e^D_R$.

9.7 Proof to Lemma 1

Let P be the proposer in the first period, since $q = 0$, R is the proposer in $t = 2$. When all spending is discretionary and taxes are residually determined, the lifetime welfare of R and P are given, respectively, by

$$U^D_P = (2+\beta)\ln(c^D_{P,P}) + \beta\ln(\bar{x})$$

$$U^D_R = \ln(\bar{x}) + (1+2\beta)\ln(c^D_{R,R})$$

where $c^D_{i,i} = \frac{Y-\bar{x}}{2}$, as shown in previous results. To find lifetime welfare under budget rules, first use Proposition 5, evaluated at $q = 0$, to find first period allocations.

$$\mathcal{C}_{P,P,1} = \frac{2(1+\beta)}{2+\beta}c^D_{P,P} \quad \mathcal{G}_{P,1} = \frac{2}{2+\beta}c^D_{P,P} \quad \text{and} \quad \mathcal{C}_{R,P,1} = \bar{x}.$$

Under the assumption that $\bar{x} \to 0$, the relevant state variable for proposer R at the outset of the second period is given by $c_{P,1} = \mathcal{C}_{P,P,1} \geq \frac{Y}{2}$. Hence, second period allocations satisfy

$$\mathcal{C}_{P,R,2} = \frac{2\bar{x}c_{P,1}}{Y} \quad \mathcal{G}_{R,2} = \frac{Y}{2} \quad \text{and} \quad \mathcal{C}_{R,R,2} = \frac{Y}{2} - \frac{2\bar{x}c_{P,1}}{Y}.$$

The lifetime welfare of R, under budget rules, is thus

$$U_R^{BR} = \ln(\bar{x}) + \ln\left(\frac{2}{2+\beta}c_{P,P}^D\right) + \beta\left\{\ln\left(\frac{Y}{2}\right) + \ln\left(\frac{Y}{2} - \frac{2\bar{x}c_{P,1}}{Y}\right)\right\}$$

Noting that $\frac{Y}{2} - \frac{2\bar{x}c_{P,1}}{Y} = \frac{Y}{2}\left[1 - \frac{4\bar{x}c_{P,1}}{Y^2}\right]$, and simplifying, the welfare gain obtained by R from imposing budget rules is given by

$$U_R^{BR} - U_R^D = \ln\left(\frac{2}{2+\beta}\right) + 2\beta\left[\ln\left(\frac{Y}{2}\right) - \ln(c_{R,R}^D)\right] + \beta\ln\left(1 - \frac{4\bar{x}c_{P,1}}{Y^2}\right).$$

The values inside the first and last terms are smaller than 1, so their natural logarithm is negative. Taking limits when $\bar{x} \to 0$, the second and third terms go to 0 since $c_{R,R}^D = \frac{Y-\bar{x}}{2}$. As a result, R's lifetime welfare is lower under budget rules than in the case without rules. Numerically, it is easy to see that this holds for $\bar{x} > 0$.

9.8 Mandatory Spending on Private vs Public Goods

Bowen, Chen, and Eraslan (2014) show that mandatory spending on public goods can prevent under-provision of public goods that happen when all spending is discretionary. Moreover, they derive conditions under which such budget rule can restore efficiency in a bargaining equilibrium. We show that the type of budget rule matters for whether mandatory spending is beneficial or detrimental for society. In this section, we compare two types of rules: (1) mandatory spending on public goods (*g-rule*) versus (2) a tax code and a mandatory spending rule in entitlements (*τ&e-rule*). The first rule is the one studied in detail in Bowen, Chen, and Eraslan (2014). The second one is our benchmark case. The key difference between them is that (1) targets public consumption whereas (2) targets private consumption. The objective of this section is to compare how these rules affect the political equilibrium.

It is easy to see that the tax code establishing a status-quo value for τ is equivalent to a rule establishing a status-quo value on the level of consumption of the rich. This is the case because $c_R = y_R - \tau$, and y_R is exogenous (the equivalence would break if agents were to choose labor effort, for example). Analogously, mandatory spending on entitlements e is equivalent to considering the consumption of the poor as a status-quo value, since $c_P = y_P + e$. Because of this, we can re-define our benchmark case as one in which we have mandatory spending in *private goods* (and refer to it as a c-rule) and the one in Bowen, Chen, and Eraslan (2014) as one in which there is mandatory spending on public goods (and refer to theirs as a g-rule). In terms of the bargaining game, under the g-rule the state space is given by $\mathbf{s} = \bar{g}$, whereas there is no predetermined value of e (e.g. it is equal to zero unless policymakers choose $e > 0$) and taxes are determined residually from the government budget constraint $\tau = e + g$. Under the c-rule, the state can be written as $\mathbf{s} = \{\bar{c}_R, \bar{c}_P\}$ (which is a simple re-normalization relative to our benchmark $\{\bar{e}, \bar{\tau}\}$ in the main text).

9.8.1 Quasi-linear utility

We present our results for a simple case where the utility function of the parties is quasi-linear

$$u(c_{i,t}, g_t) = c_{i,t} + \theta\ln(g_t), \quad \text{for } t = \{1, 2\}.$$

We are deviating from the log-log benchmark specification in order to make the comparison with Bowen, Chen, and Eraslan (2014) more transparent. The mathematical derivation of second period allocations and first period welfare is contained in the Online Appendix. Here, we just want to emphasize how first-period allocations are affected by the type of budget rule. For reference, note that with quasi-linear utility the Samuelson-level of public good provision is constant and equal to

$$g^* = 2\theta.$$

In addition, the level of g when there are no budget rules is $g^D = \theta$ in this case. That is, when all spending is discretionary, the proposer sets the private marginal cost of providing g to its private marginal benefit, θ. The latter is smaller than the social marginal benefit of 2θ, implying that the public goods are under provided when there are no rules.

To study how budget rules affect allocations, we depart from a situation where the proposer is unconstrained (e.g. the acceptance constraint is slack) in the first period (consistently with in the main text). This allows us to have closed-form solutions that can provide the necessary intuition for our comparison between spending rules that affect public good provision and spending rules that affect private goods.

Under a c-rule, it is optimal for an R-type proposer to set in $t = 1$

$$1 = \frac{\theta}{g_1} - \beta(1-q)W_R'(c_{R,1}).$$

Since $W_R'(c_{R,1}) \geq 0$, mandatory spending on private goods create a negative wedge on public good provision. After some manipulations, we find that

$$\text{c-rule:} \quad g_1 = \frac{\theta}{1+\beta(1-q)},$$

which is increasing in q. Under a g-rule (e.g. mandatory spending on public goods), proposer R sets

$$1 = \frac{\theta}{g_1} + \beta q V_R'(g_1) + \beta(1-q)W_R'(g_1)$$

Since $W_R'(g_1) \geq 0$ and $V_R'(g_1) \leq 0$, the total wedge on public good provision depends on the relative size of $\beta q V_R'(g_1)$ and $\beta(1-q)W_R'(g_1)$. As shown in the Online Appendix,

$$\text{g-rule:} \quad g_1 = \frac{(1+\beta)\theta}{1+\beta q},$$

with g_1 *decreasing* in q.

Mandatory spending rules create a positive wedge on the provision of the good that is being made mandatory. Under a *c-rule*, it is more beneficial to provide relatively more private consumption because this ensures a strong bargaining power in the second period. Because of this, g is under-provided: $g_1 < g^*$. As q increases, and the current proposer is more likely to remain in power, the benefits of such distortions decrease. Therefore, g_1 gets closer to the efficient level g^* when q rises under a *c-rule*. The left panel of Figure (15) illustrates this (see dashed blue line). In the right panel we can see that private consumption decreases with q instead. Note, however, that even when q is large, $g_1 \leq g^D < g^*$.

In the case of the *g-rule*, the proposer has incentives to provide relatively more public goods. The higher the q, the lower is the benefit of distorting public good provision upwards. The dotted

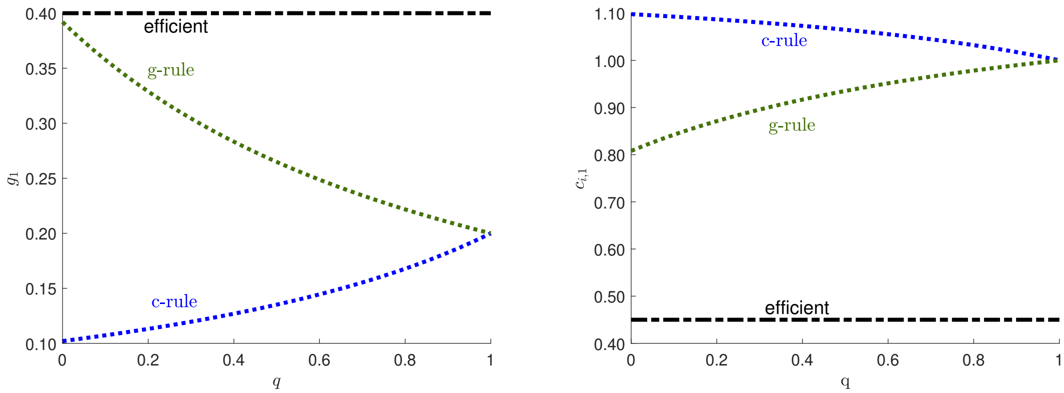

Figure 15: First-period allocations for all $q's$. $Y = 1.3$, $\bar{x} = 0.1$, $\beta = 0.96$ and $\theta = 0.2$.

green line in the left panel of Figure (15) illustrates this. Private consumption, on the other hand, rises with q. Consider the extreme case where $\beta = 1$ (perfectly patient society) and $q = 0$ (deterministic turnover). In such case, the $g - rule$ would restore optimality in period 1, with $g_1 = 2\theta$. This is in line with the results in Bowen, Chen, and Eraslan (2014), where mandatory spending rules tend to be beneficial for society: as long as the rule is set on the pure public good, it will push the proposer towards providing more of it. We find, however, that a $c - rule$ pushes to proposer away from the efficient value when turnover is high.

The impact of these budget rules on welfare depends on the identity of the party and on the level of political turnover. In this specific numerical example, the *g-rule* is always be better than the *c-rule* for society as a whole. This is clear in Figure (16), which shows the *g-rule* (bar green) always above the no-budget-rules line (dotted magenta) and the *c-rule* (dashed blue) for all values of q.[19]

9.8.2 The role of concavity

Consider the case where utility is logarithmic in private and public goods.

$$u(c_{i,t}, g_t) = \ln(c_{i,t}) + \theta \ln(g_t), \quad \text{for } t = \{1, 2\}.$$

This is the benchmark case analyzed in the paper, but under a generic value of θ. The allocations arising under budget rules on taxes and entitlements are characterized in the Online Appendix, Section 2.

Concavity is important for two reasons. First, because it affects the level of distortions in the provision of private in public goods deferentially for alternative rules. Second, because agents prefer scenarios exhibiting low volatility of private good consumption. These two forces are discussed next.

Concavity and distortions, a first-moment effect: In the first period, under a c-rule, the proposer sets

[19]Our numerical inspection suggests the same result is true for the case of log-log utility, i.e., the *g-rule* is always better than the *c-rule* when we start with a slack acceptance constraint. This is intuitive, since with a slack acceptance constraint the proposer has the advantage of distorting allocations to her favor in the second period. If the proposer starts from a binding constraint, however, it is easy to see numerically that the *c-rule* can be better than the *g-rule*.

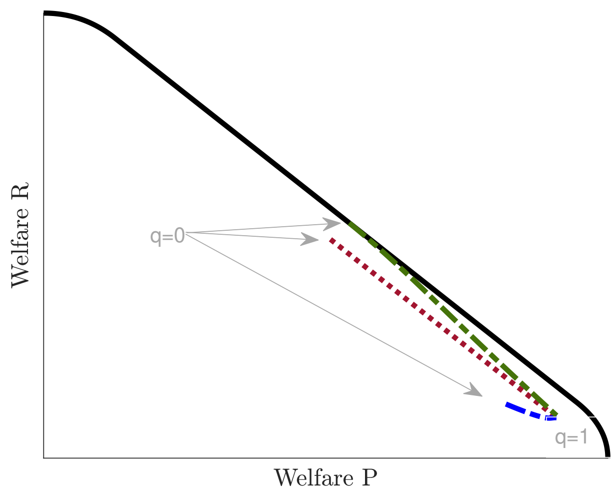

Figure 16: Pareto frontier (black), *c-rule* (dashed blue), *g-rule* (bar green), no rules (dotted red) for all $q's$. $Y = 1.3$, $\bar{x} = 0.1$, $\beta = 0.96$ and $\theta = 0.2$.

$$\frac{1}{c_{P,1}} + \beta(1-q)W'_P(c_{P,1}) = \frac{\theta}{g_1}.$$

Under a g-rule, it sets

$$\frac{1}{c_{P,1}} = \frac{\theta}{g_1} + \beta\left\{qV'_P(g_1) + (1-q)W'_P(g_1)\right\}.$$

As in the quasi-linear case, mandatory spending on different goods creates distortions on the first-period's proposer optimal conditions. These induce the policymaker to provide relatively more of the good subject to the budget rule. With concave utility in both goods, the marginal utility of private consumption is not always 1 (as it was in the linear case); it now depends on the value of c. The following table summarizes public good allocations for the *c-rule* and the *g-rule*, in the quasi-linear and concave utility cases. Optimal and no-rules solutions are also provided for reference.

The level of distortions in the provision of public goods, then, is different qualitatively and quantitatively. For example, under a g-rule public good provision is closer to optimal (e.g., an improvement relative to the no-rules scenario) only when utility is quasi-linear.

Concavity and volatility, a second-moment effect: In addition to whether the rule induces more or less provision of public goods, the volatility of private consumption allocations over time matters when utility is concave, but it does not when utility is linear in c. Because of concavity in private goods' utility, agents prefer smooth consumption profiles.

The easiest way to understand how volatility affects welfare is by looking at the solution of a two period model under no budget rules, assuming that one party is the proposer in the first period and the other party is the proposer in the second period. For illustration, suppose additionally that $\beta = 1$. In that case, proposer i sets consumption to the opposition at its lowest value,

g_1 under	**Quasi-linear**	**Concave**
Optimal:	2θ	$\frac{\theta Y}{1+\theta}$
No rules:	$g_l^D = \theta$	$g_c^D = \frac{\theta(Y-\bar{x})}{1+\theta}$
g-rule:	$\frac{(1+\beta)\theta}{1+\beta q} > g_l^D$	g_c^D
c-rule:	$\frac{\theta}{1+\beta(1-q)} < g_l^D$	$\frac{\theta(Y-\bar{x})}{1+\theta+\beta(1-q)} < g_c^D$

Table 4: Provision of g_1 under alternative budget arrangements and utility functions.

$c_j = \bar{x}$ at each point in time and its own level of consumption is $c_{i,i}^D > \bar{x}$ for every t. Public good provision is g^D regardless of the identity of the proposer.

Compare the proposer's welfare under two alternative sequences of private consumption $\mathbf{a_1}$ and $\mathbf{a}_2$, defined by $\mathbf{a}_1 = \{c_{i,i}^D, \bar{x}\}$ and $\mathbf{a}_2 = \{\tilde{x}, \tilde{x}\}$ with $\tilde{x} = \frac{c_{i,i}^D + \bar{x}}{2}$. The first value denotes consumption in the first period and the second value consumption in the second period. These two allocations give the same welfare under linear utility, as lifetime utility becomes $c_{i,i}^D + \bar{x}$ in both cases. Under concave utility, instead, the proposer would be better off under $\mathbf{a_2}$ than it would be under $\mathbf{a_1}$, because the former involves a smoother consumption profile. A similar argument holds for the opposition, with $\mathbf{a}_1 = \{\bar{x}, c_{j,j}^D\}$. Budget rules affect how volatile sequences of c and g are in the bargaining equilibrium. More volatile sequences in c generate welfare losses only if $u(c)$ is concave. Therefore, using c as a bargaining chip to move the status-quo value of g (e.g. under a g-rule) is "cheaper" when utility is quasi-linear than it is when utility is concave in both goods.

9.9 Computation with Taste Shocks

A well-known issue in dynamic legislative bargaining games with endogenous status quo is that standard algorithms are not always successful in computing Markov-perfect equilibria. This paper is no exception, as standard value-function iteration procedures do not converge for arbitrary parameterizations of the model. It is worth noticing that the convergence problems are not caused by the bounds of taxes and entitlement or the specific form of the utility function. We have experimented with quadratic utility and no minimum consumption and the issue remains (details available upon request). The fact that the computation of models in this class is notoriously challenging has been documented by Duggan and Kalandrakis (2012), Martin (2009), Chatterjee and Eyigungor (2012), and others. A common strategy, also adopted here, is to slightly perturb the choices of the proposing agent through the introduction of small, independent and identically distributed shocks. These shocks may apply to fundamentals, as in Chatterjee and Eyigungor (2012), or to the agent's payoff directly, as in Eyigungor and Chatterjee (2019) or Dvorkin et al. (2021). We follow Gordon (2019)[20] and use the functional forms and assumptions employed with

[20]Related, recent applications to fiscal policy and sovereign default include Dvorkin et al. (2021), Mihalache (2020), and Arellano et al. (2020).

discrete choice methods. The resulting randomization over options with payoff of comparable value greatly eases the computation of the model, induces smooth value functions and policy functions, and induces near-monotone convergence via standard value function iteration[21].

We perturb the proposer's problem by augmenting it with choice-specific taste shocks, distributed Gumbel (Extreme Value Type I), as commonly employed with discrete choice methods, e.g. Rust (1987). To simplify notation, let the status quo be given by $\bar{s} = \langle \bar{\tau}, \bar{e} \rangle$ and any potential choice denoted by $s = \langle \tau, e, g \rangle$. As before, $c_P = y_P + e$ and $c_R = yR - \tau$, so we suppress any explicit dependency of consumption levels on the proposal s.

Define the acceptance set for agent $j \in \{P, R\}$ as

$$\mathcal{A}_j(\bar{s}) = \left\{ s \in \mathbb{S} \,\middle|\, (1-\beta)u(c_j, g) + \beta \left[(1-q)V_j(s) + qW_j(s) \right] \geq K(\bar{s}) \right\}. \tag{40}$$

We write the value to the proposer $i \in \{P, R\}$ from proposing s, net of taste shocks, as

$$\mathcal{J}_i(\bar{s}, s) = \begin{cases} (1-\beta)u(c_i, g) + \beta \left\{ qV_i(s) + (1-q)W_i(s) \right\}, & \text{if } s \in \mathcal{A}_j(\bar{s}) \\ -\infty, & \text{otherwise} \end{cases} \tag{41}$$

and record the greatest value over s by

$$\overline{\mathcal{J}}_i(\bar{s}) = \max_{s \in \mathbb{S}} \mathcal{J}_i(\bar{s}, s) \tag{42}$$

The value to the proposer—given realized iid taste shocks $\{\varepsilon_s\}_s$, one for each possible proposal—is

$$\mathcal{V}_i(\bar{s}, \{\varepsilon_s\}_s) = \max_{s \in \mathbb{S}} \left\{ \mathcal{J}_i(\bar{s}, s) + \rho\, \varepsilon_s \right\}. \tag{43}$$

Following the standard proofs for discrete choice methods, e.g. McFadden (1973), it can be shown that ex-ante, before taste shocks are realized, the probability of choosing a particular option $\hat{s}$ is given by

$$\Pr(s = \hat{s} | \bar{s}) = \frac{\exp\left[\mathcal{J}_i(\bar{s}, \hat{s}) / \rho \right]}{\sum_z \exp\left[\mathcal{J}_i(\bar{s}, z) / \rho \right]} = \frac{\exp\left[\left(\mathcal{J}_i(\bar{s}, \hat{s}) - \overline{\mathcal{J}}_i(\bar{s}) \right) / \rho \right]}{\sum_z \exp\left[\left(\mathcal{J}_i(\bar{s}, z) - \overline{\mathcal{J}}_i(\bar{s}) \right) / \rho \right]} \tag{44}$$

and the expected value to the proposer, before observing the taste shocks, is

$$V_i(\bar{s}) = \mathbb{E}_{\{\varepsilon_s\}_s} \left\{ \mathcal{V}_i(\bar{s}, \{\varepsilon_s\}_s) \right\} = \overline{\mathcal{J}}_i(\bar{s}) + \rho \ln \left\{ \sum_z \exp\left[\left(\mathcal{J}_i(\bar{s}, z) - \overline{\mathcal{J}}_i(\bar{s}) \right) / \rho \right] \right\} \tag{45}$$

while the value of the agent receiving the proposal is

$$W_j(\bar{s}) = \sum_z \left\{ \Pr(s = z | \bar{s}) \left[u(c_j, g) + \beta \left((1-q)V_j(z) + qW_j(z) \right) \right] \right\}. \tag{46}$$

We remark briefly on key properties of the choice probabilities and of the solution. First, the probability of choosing s is strictly increasing in the value net of taste shocks for s, $\mathcal{J}_i(\bar{s}, s)$, so that better options are picked with higher probability. Second, given our use of the acceptance set, all s that would not be accepted are proposed to probability zero. Third, the mean level of the Gumbel tastes shocks is non-zero, yet this does not alter choice probabilities: what matters for the likelihood of choosing s over e.g. z is the difference between ε_s and ε_z, which is Logistic(0,

[21]The method requires the introduction of a parameter governing the importance of these taste shocks for the agent's behavior. We set this parameter to the smallest value consistent with convergence within 1,000 iterations, at $\rho = 5e^{-4}$.

1), as well as values net of taste shocks. We can and do remove the mean contribution of taste shocks to welfare by subtracting the Euler-Mascheroni constant times ρ. The parameter ρ scales the importance of the taste shocks for the proposal decision. If we take the value $\rho \to 0$, tastes shocks no longer play a role and the underlying best option is picked with probability 1. In turn, for arbitrarily high ρ values, all members of the acceptance set would be proposed with equal probability. Small values of ρ, such as our $5e^{-4}$ are sufficient to induce convergence when iterating over V_i and W_j.

Online Appendix to “Bargaining over Taxes and Entitlements in the Era of Unequal Growth”

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29th April 2022

1 Quasilinear Case

1.1 Budget rules on private goods

Party’s R Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = c_{R,2} + \theta \ln(g_2) + \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi \left(c_{P,2} + \theta \ln(g_2) - \overline{c_P} - \ln(\overline{x})\right)$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
[c_{R,2}] \quad & 1 - \lambda \leq 0. \\
& (c_{R,2} - \bar{x}) \left[1 - \lambda\right] = 0. && (1)\\
[c_{P,2}] \quad & \psi - \lambda \leq 0. \\
& (c_{P,2} - \bar{x}) \left[\psi - \lambda\right] = 0. && (2)\\
[g_2] \quad & \frac{\theta(1+\psi)}{g_2} - \lambda \leq 0. \\
& (g_2 - \bar{x}) \left[\frac{\theta(1+\psi)}{g_2} - \lambda\right] = 0. && (3)\\
[RC] \quad & \left[Y - c_{P,2} - c_{R,2} - g_2\right] \geq 0. \\
& \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] = 0. && (4)\\
[IRC] \quad & \left[c_{P,2} + \ln(g_2) - \overline{c_P} - \ln(\overline{x})\right] \geq 0. \\
& \psi \left[c_{P,2} + \ln(g_2) - \overline{c_P} - \ln(\overline{x})\right] = 0. && (5)
\end{aligned}$$

The solution to this problem depends on the status quo vector $\mathbf{s} = \{\overline{c_R}, \overline{c_P}\}$. Denote the solution by functions $\mathcal{C}_{i,R,2}(\mathbf{s})$ for private consumption of party i and $\mathcal{G}_{P,2}(\mathbf{s})$ for public goods.

First note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. We have four cases to consider:

- Case 1: $\psi = 0$. By eq. (2) we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$. By eq. (1) and eq. (3) we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \theta = g^D$. By eq. (4), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \bar{x} - \theta$. By eq. 5, we see this case holds if and only if $\overline{c_P} < \bar{x} - \theta \ln\left(\frac{\theta}{\bar{x}}\right)$.

- Case 2: $\psi > 0$, $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{R,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (5) we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \bar{x}e^{\frac{\overline{c_P}-\bar{x}}{\theta}}$. By eq. (4), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \overline{x} - e^{\frac{\overline{c_P}-\bar{x}}{\theta}}$. By eqs. (1)-(3) and the fact that $\mathcal{C}_{P,R,2}(\mathbf{s}) \geq \overline{x}$, this case holds if and only if $\bar{x} - \theta \ln\left(\frac{\theta}{\bar{x}}\right) \leq \overline{c_P} < \bar{x} + \theta \ln\left(\frac{2\theta}{\bar{x}}\right)$.
- Case 3: $\psi > 0$ and $\mathcal{C}_{i,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eqs. (1)-(4), we have that $\mathcal{G}_{R,2}(\mathbf{s}) = 2\theta = g^*$. By eq. (5), we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{c_P} + \theta \ln\left(\frac{\bar{x}}{2\theta}\right)$. By eq. (4) we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - 2\theta - \overline{c_P} - \theta \ln\left(\frac{\bar{x}}{2\theta}\right)$. This case holds if and only if $\bar{x} + \theta \ln\left(\frac{2\theta}{\bar{x}}\right) \leq \overline{c_P} \leq Y - 2\theta - \bar{x} - \theta \ln\left(\frac{\bar{x}}{2\theta}\right)$.
- Case 4: $\psi > 0$, and $\mathcal{C}_{R,R,2}(\mathbf{s}) = \bar{x}$, $\mathcal{C}_{P,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \bar{x}$. By eq. (4) and eq. (5), we have that $\mathcal{G}_{R,2}(\mathbf{s})$ solves

$$Y - \bar{x} - g_2 + \theta \ln g_2 - \overline{c_P} - \theta \ln(\bar{x}) = 0 \tag{6}$$

Note that by the implicit function theorem, $\mathcal{G}'_{R,2}(\mathbf{s}) = \frac{g_2}{\theta - g_2}$. Therefore, we have

$$\mathcal{G}_{R,2}(\boldsymbol{s}) = \begin{cases} \theta = g^D, & \text{if } \overline{c_P} < \xi \\ \bar{x}e^{\frac{\overline{c_P}-\bar{x}}{\theta}}, & \text{if } \overline{c_P} \in [\xi, \omega) \\ 2\theta = g^*, & \text{if } \overline{c_P} \in [\omega, Y - 2\theta - \omega) \\ G_2(\overline{c_P}), & \text{if } \overline{c_P} \geq Y - 2\theta - \omega \end{cases}$$

$$\mathcal{C}_{P,R,2}(\boldsymbol{s}) = \begin{cases} \bar{x}, & \text{if } \overline{c_P} < \xi \\ \bar{x}, & \text{if } \overline{c_P} \in [\xi, \omega) \\ \overline{c_P} + \theta \ln\left(\frac{\bar{x}}{2\theta}\right), & \text{if } \overline{c_P} \in [\omega, Y - 2\theta - \omega) \\ Y - \bar{x} - G_2(\overline{c_P}), & \text{if } \overline{c_P} \geq Y - 2\theta - \omega \end{cases}$$

where $\xi = \bar{x} - \theta \ln\left(\frac{\theta}{\bar{x}}\right)$, $\omega = \bar{x} + \theta \ln\left(\frac{2\theta}{\bar{x}}\right)$, $G_2(\overline{c_P})$ is given by eq. (6) and $\mathcal{C}_{R,R,2}(\boldsymbol{s})$ is given by eq. (4).

Now we follow back to period $t = 1$. It is convenient to first write continuation values. Let´s assume party R is in power at $t = 1$.

$$V_R(c_{P,1}) = \begin{cases} Y - \bar{x} - \theta + \theta \ln(\theta), & \text{if } c_{P,1} < \xi \\ Y - \bar{x} - \bar{x}e^{\frac{c_{P,1}-\bar{x}}{\theta}} + \theta \ln(\theta) + \frac{c_{P,1}-\bar{x}}{\theta}, & \text{if } c_{P,1} \in [\xi, \omega) \\ Y - c_{P,1} - \theta \ln(\bar{x}) - 2\theta + 2\theta \ln(2\theta), & \text{if } c_{P,1} \in [\omega, Y - 2\theta - \omega) \\ \bar{x} + \theta \ln\left(G_2(c_{P,1})\right), & \text{if } c_{P,1} \geq Y - 2\theta - \omega \end{cases}$$

and

$$W_R(c_{R,1}) = \begin{cases} \bar{x} + \theta \ln(\theta), & \text{if } c_{R,1} < \xi \\ c_{R,1} + \theta \ln(\theta) + \frac{c_{R,1}-\bar{x}}{\theta}, & \text{if } c_{R,1} \in [\xi, \omega) \\ c_{R,1} + \theta \ln(\bar{x}), & \text{if } c_{R,1} \in [\omega, Y - 2\theta - \omega) \\ Y - \bar{x} - G_2(c_{R,1}) + \theta \ln\left(G_2(c_{R,1})\right), & \text{if } c_{R,1} \geq Y - 2\theta - \omega \end{cases}$$

Note that the only relevant state for party R when in power is $c_{P,1}$ and for when out of power is $c_{R,1}$. Therefore, party's R Lagrangian for this problem at $t = 1$ when the acceptance constraint is not binding is given by:

$$\mathcal{L} = c_{R,1} + \theta \ln(g_1) + \beta \left[q V_R(c_{P,1}) + (1-q) W_R(c_{R,1})\right] + \lambda \left[Y - c_{R,1} - c_{P,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party R are $c_{R,1}, c_{P,1}, g_1 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
&[c_{P,1}] \quad -\lambda + \beta q V_R'(c_{P,1}) \leq 0. \\
&\qquad (c_{P,1} - \bar{x}) \left[-\lambda + \beta q V_R'(c_{P,1})\right] = 0. && (7) \\
&[c_{R,1}] \quad 1 - \lambda + \beta(1-q) W_R'(c_{R,1}) \leq 0. \\
&\qquad (c_{R,1} - \bar{x}) \left[1 - \lambda + \beta(1-q) W_R'(c_{R,1})\right] = 0. && (8) \\
&[g_1] \quad \frac{\theta}{g_1} - \lambda \leq 0. \\
&\qquad (g_1 - \bar{x}) \left[\frac{\theta}{g_1} - \lambda\right] = 0. && (9) \\
&[RC] \quad [Y - c_{R,1} - c_{P,1} - g_1] \geq 0. \\
&\qquad \lambda \left[Y - c_{R,1} - c_{P,1} - g_1\right] = 0. && (10)
\end{aligned}$$

Since $u(.)$ is increasing in its arguments, $\lambda > 0$. Also, by eq. (8), we have that $\mathcal{C}_{P,1} = \overline{x}$. By eq. (7) and eq. (9), we have that

$$1 + \beta(1-q) W_R'(c_{R,1}) - \frac{\theta}{g_1} = 0 \tag{11}$$

Since $W_R'(c_{R,1}) = 0$ if $c_{R,1} < \xi$ and $W_R'(c_{R,1}) = 1$ otherwise, we have two cases to consider.

- Case 1: $c_{R,1} < \xi$. By eq. (11), we have that $g_1 = \theta = g^D$. By eq. (10), we have that $c_{R,1} = Y - \bar{x} - \theta$. For this case to hold, we would require $c_{R,1} = Y - \bar{x} - \theta < \xi = \bar{x} - \theta \ln\left(\frac{\theta}{\bar{x}}\right)$, which is a contradiction.

- Case 2: $c_{R,1} \geq \xi$. By eq. (11), we have that $g_1 = \frac{\theta}{1+\beta(1-q)} < \theta = g^D$. $c_{R,1} = Y - \bar{x} - \theta$can be found by eq. (10).

Define g_1^c as the solution for the above problem, in which mandatory spending is on private goods.

1.2 Budget rules on public goods

Party's R Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = c_{R,2} + \theta \ln(g_2) + \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi \left(c_{P,2} + \theta \ln(g_2) - \overline{x} - \theta \ln(\overline{g})\right)$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
[c_{R,2}] \quad & 1 - \lambda \leq 0. \\
& (c_{R,2} - \bar{x})\,[1 - \lambda] = 0. \qquad (12) \\
[c_{P,2}] \quad & \psi - \lambda \leq 0. \\
& (c_{P,2} - \bar{x})\,[\psi - \lambda] = 0. \qquad (13) \\
[g_2] \quad & \frac{\theta(1+\psi)}{g_2} - \lambda \leq 0. \\
& (g_2 - \bar{x})\left[\frac{\theta(1+\psi)}{g_2} - \lambda\right] = 0. \qquad (14) \\
[RC] \quad & [Y - c_{P,2} - c_{R,2} - g_2] \geq 0. \\
& \lambda\,[Y - c_{P,2} - c_{R,2} - g_2] = 0. \qquad (15) \\
[IRC] \quad & [c_{P,2} + \theta\ln(g_2) - \overline{x} - \theta\ln(\overline{g})] \geq 0. \\
& \psi\,[c_{P,2} + \theta\ln(g_2) - \overline{x} - \theta\ln(\overline{g})] = 0. \qquad (16)
\end{aligned}$$

The solution to this problem depends on the status quo $\mathbf{s} = \{\overline{g}\}$. Denote the solution by functions $\mathcal{C}_{i,R,2}(\mathbf{s})$ for private consumption of party i and $\mathcal{G}_{P,2}(\mathbf{s})$ for public goods.

Since $u(.)$ is increasing in its arguments, $\lambda > 0$. We have four cases to consider:

- Case 1: $\psi = 0$. By eq. (13) we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$. By eq. (12) and eq. (14) we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \theta = g^D$. By eq. (15), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \overline{x} - \theta$. By eq. 16, we see this case holds if and only if $\overline{g} < \theta = g^D$.

- Case 2: $\psi > 0$, $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{R,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (16) we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \bar{g}$. By eq. (15), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \overline{x} - \bar{g}$. By eq. (12), eq. (14), eq. (2) and the fact that $\mathcal{C}_{P,R,2}(\mathbf{s}) \geq \overline{x}$, this case holds if and only if $g^D \leq \overline{g} \leq 2\theta = g^*$.

- Case 3: $\psi > 0$ and $\mathcal{C}_{i,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eqs. (12)-(15), we have that $\mathcal{G}_{R,2}(\mathbf{s}) = 2\theta = g^*$. By eq. (16), we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \bar{x} + \theta\ln\left(\frac{\bar{g}}{2\theta}\right)$. By eq. (15) we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \bar{x} - 2\theta - \theta\ln\left(\frac{\bar{g}}{2\theta}\right)$. This case holds if and only if $g^* < \overline{g} \leq \theta e^{\left(\frac{Y - 2(\bar{x}+\theta)}{\theta}\right)}$.

- Case 4: $\psi > 0$, and $\mathcal{C}_{R,R,2}(\mathbf{s}) = \bar{x}$, $\mathcal{C}_{P,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (15) and eq. (16), we have that $\mathcal{G}_{R,2}(\mathbf{s})$ solves

$$Y - \bar{x} - g_2 + \theta\ln g_2 - \bar{x} - \theta\ln\overline{g} = 0 \qquad (17)$$

Note that by the implicit function theorem, $\mathcal{G}'_{R,2}(\mathbf{s}) = \frac{\theta}{\bar{g}} \frac{g_2}{\theta - g_2}$. Therefore, we have

$$\mathcal{G}_{R,2}(\boldsymbol{s}) = \begin{cases} \theta = g^D, & \text{if} \quad \overline{g} < g^D \\ \overline{g}, & \text{if} \quad \overline{g} \in \left[g^D, g^*\right) \\ 2\theta = g^*, & \text{if} \quad \overline{g} \in \left[g^*, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}\right) \\ G_2(\overline{g}), & \text{if} \quad \overline{g} \geq, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)} \end{cases}$$

$$\mathcal{C}_{P,R,2}(\boldsymbol{s}) = \begin{cases} \bar{x}, & \text{if} \quad \overline{g} < g^D \\ \bar{x}, & \text{if} \quad \overline{g} \in \left[g^D, g^*\right) \\ \bar{x} + \theta \ln\left(\frac{\overline{g}}{2\theta}\right), & \text{if} \quad \overline{g} \in \left[g^*, \theta \ln\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)\right) \\ Y - \bar{x} - G_2(\overline{g}), & \text{if} \quad \overline{g} \geq \theta \ln\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right) \end{cases}$$

where $G_2(\boldsymbol{s})$ is given by eq. (17) and $\mathcal{C}_{R,R,2}(\boldsymbol{s})$ is given by eq. (15).

Now we follow back to period $t = 1$. It is convenient to first write continuation values:

$$V_R(\boldsymbol{s}) = \begin{cases} Y - \bar{x} - \theta + \theta \ln(\theta), & \text{if} \quad \overline{c_P} < g_i^D \\ Y - \bar{x} - \bar{g} + \theta \ln(\bar{g}), & \text{if} \quad \overline{c_P} \in \left[g_i^D, g^*\right) \\ Y - \bar{x} - \theta \ln(\bar{g}) - 2\theta\left(1 - \ln(2\theta)\right), & \text{if} \quad \overline{c_P} \in \left[g^*, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}\right) \\ \bar{x} + \theta \ln(G_2(\boldsymbol{s})), & \text{if} \quad \overline{c_P} \geq, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)} \end{cases}$$

$$W_R(\boldsymbol{s}) = \begin{cases} \bar{x} + \theta \ln(\theta), & \text{if} \quad \overline{c_P} < g_i^D \\ \bar{x} + \theta \ln(\bar{g}), & \text{if} \quad \overline{c_P} \in \left[g_i^D, g^*\right) \\ \bar{x} + \theta \ln(\bar{g}), & \text{if} \quad \overline{c_P} \in \left[g^*, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}\right) \\ Y - \bar{x} - G_2(\boldsymbol{s}) + \theta \ln(G_2(\boldsymbol{s})), & \text{if} \quad \overline{c_P} \geq, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)} \end{cases} \tag{18}$$

Party's R Lagrangian for this problem at $t = 1$ when the individual rationality constraint is not binding is given by:

$$\mathcal{L} = c_{R,1} + \theta \ln(g_1) + \beta\left[qV_R(g_1) + (1-q)W_R(g_1)\right] + \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,1}, c_{R,1}, g_1 \geq \bar{x}$, $\lambda, \psi \geq 0$ and

$$[c_{R,1}] \quad 1 - \lambda \leq 0.$$
$$(c_{R,1} - \bar{x})\left[1 - \lambda\right] = 0. \tag{19}$$
$$[c_{P,1}] \quad -\lambda \leq 0.$$
$$(c_{P,1} - \bar{x})\left[-\lambda\right] = 0. \tag{20}$$
$$[g_1] \quad \frac{\theta}{g_1} - \lambda + \beta q V'_R(g_1) + \beta(1-q)W'_R(g_1) \leq 0.$$
$$(g_1 - \bar{x})\left[\frac{\theta}{g_1} - \lambda + \beta q V'_R(g_1) + \beta(1-q)W'_R(g_1)\right] = 0. \tag{21}$$
$$[RC] \quad \left[Y - c_{P,1} - c_{R,1} - g_1\right] \geq 0.$$
$$\lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right] = 0. \tag{22}$$

Since $u(.)$ is increasing in its arguments, $\lambda > 0$. Also, by eq. (20), we have that $\mathcal{C}_{P,R,1} = \overline{x}$. By eq. (19) and eq. (21), we have that

$$\frac{\theta}{g_1} + \beta q V_R'(g_1) + \beta(1-q) W_R'(g_1) - 1 = 0 \tag{23}$$

$W_R'(c_{R,1}) = 0$ if $g_1 < \theta$, $W_R'(g_1) = \frac{\theta}{g_1}$ otherwise, $V_R'(g_1) = 0$ if $g_1 < \theta$, $V_R'(g_1) = 0$ if $g_1 < \theta$, $V_R'(g_1) = \frac{\theta}{g_1} - 1$ if $g_1 \in [\theta, 2\theta)$, $V_R'(g_1) = -\frac{\theta}{g_1}$ if $g_1 \in \left[2\theta, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}\right)$ and $V_R'(g_1) = \frac{\theta^2}{g_1(\theta G_2(g_1)}$ if $g_1 \geq \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}$ we have four cases to consider.

- Case 1: $g_1 < \theta$. By eq. (23), we have that $g_1 = \theta = g_i^D$, a contradiction.
- Case 2: $g_1\, [\theta, 2\theta)$. By eq. (23), we have that $g_1 = \frac{(1+\beta)\theta}{1+\beta q}$. This case holds if and only if $p \geq \frac{\beta-1}{2\beta}$ which is always true, since $p \in [0,1]$ and $\beta \in [0,1]$.
- Case 3: $g_1 \in \left[2\theta, \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}\right)$. By eq. (23), we have that $g_1 = (1+\beta(1-2q))\theta$. This case holds if and only if $p < \frac{\beta-1}{2\beta}$, which never holds.
- Case 4: $g_1 \geq \theta e^{\left(\frac{Y-2(\bar{x}+\theta)}{\theta}\right)}$. Eq. (23) and eq. (17) form a system of two non-linear equations that solve jointly g_1 and g_2.

Define g_1^g as the solution for the above problem, in which mandatory spending is on the public good.

2 Budget rules on private goods – concave case

The Lagrangian of the transformed problem of proposer P in the second period in the generic θ case is given by

$$\mathcal{L} = \ln(c_{P,2}) + \theta \ln(g_2) + \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi \left(\ln(c_{R,2}) + \theta \ln(g_2) - \ln(\overline{c_R}) - \theta \ln(\overline{x})\right)$$

The first-order and Kuhn-Tucker conditions are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$
\begin{aligned}
[c_{P,2}] \quad & \frac{1}{c_{P,2}} - \lambda \leq 0. \\
& (c_{P,2} - \bar{x}) \left[\frac{1}{c_{P,2}} - \lambda \right] = 0. && (24) \\
[c_{R,2}] \quad & \frac{\psi}{c_{R,2}} - \lambda \leq 0. \\
& (c_{R,2} - \bar{x}) \left[\frac{\psi}{c_{R,2}} - \lambda \right] = 0. && (25) \\
[g_2] \quad & \frac{(1+\psi)\theta}{g_2} - \lambda \leq 0. \\
& (g_2 - \bar{x}) \left[\frac{(1+\psi)\theta}{g_2} - \lambda \right] = 0. && (26) \\
[RC] \quad & [Y - c_{P,2} - c_{R,2} - g_2] \geq 0. \\
& \lambda \left[Y - c_{P,2} - c_{R,2} - g_2 \right] = 0. && (27) \\
[IRC] \quad & [\ln(c_{R,2}) + \theta \ln(g_2) - \ln(\overline{c_R}) - \theta \ln(\overline{x})] \geq 0. \\
& \psi \left[\ln(c_{R,2}) + \theta \ln(g_2) - \ln(\overline{c_R}) - \theta \ln(\overline{x}) \right] = 0. && (28)
\end{aligned}
$$

The solution to this problem depends on the status quo vector $\mathbf{s} = \{\overline{c_R}, \overline{c_P}\}$. Denote the solution by functions $\mathcal{C}_{i,R,2}(\mathbf{s})$ for private consumption of party i, $\mathcal{G}_{P,2}(\mathbf{s})$ for public goods, and $\Psi_{P,2}(\mathbf{s})$ and $\mathcal{E}_{P,2}(\mathbf{s})$ for policy variables.

First note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. We have four cases to consider:

- Case 1: $\psi = 0$. By eq. (25) we have that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$. By eqs. (24)-(27), we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = \frac{Y-\bar{x}}{1+\theta}$ and $\mathcal{G}_{P,2}(\mathbf{s}) = \frac{\theta(Y-\bar{x})}{1+\theta}$. By eq. (28), this case holds if and only if $\overline{c_R} < \bar{x}\frac{\xi(Y-\bar{x})^\theta}{Y^\theta}$, where $\xi = \frac{(\theta Y))^\theta}{\bar{x}^\theta(1+\theta)^\theta}$. Using P's budget constraint to solve for entitlement transfers delivers $\mathcal{E}_{P,2}(\mathbf{s}) = \frac{\Delta_P - \bar{x}}{1+\theta} = e_P^D$, where $\Delta_P = y_R - \theta y_P$. The expression for $\Psi_{P,2}(\mathbf{s})$ can be obtained from the budget constraint of rich agents. In terms of policies, this case holds if and only if $\overline{\tau} > y_R - \bar{x}\frac{\xi(Y-\bar{x})^\theta}{Y^\theta}$.

- Case 2: $\psi > 0$, $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{P,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. By eq. (28) we have that $\mathcal{G}_{P,2}(\mathbf{s}) = \bar{x}\left(\frac{\overline{c_R}}{\bar{x}}\right)^{\frac{1}{\theta}}$. By eq. (27), we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = Y - \overline{x} - \bar{x}\left(\frac{\overline{c_R}}{\bar{x}}\right)^{\frac{1}{\theta}}$. By eqs. (24)-(26) and the fact that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$, this case holds if and only if $y_R - \bar{x}\frac{\xi(Y-\bar{x})^\theta}{Y^\theta} \leq \overline{c_R} < \bar{x}\xi$. From P's budget constraint, we have that $\mathcal{E}_{P,2}(\mathbf{s}) = y_R - \bar{x}\left(1 + \left(\frac{\overline{c_R}}{\bar{x}}\right)\right)^{\frac{1}{\theta}}$. From the rich's budget constraint, $\Psi_{P,2}(\mathbf{s}) = \tau_P^D$. In terms of policies, this case holds if and only if $y_R - \bar{x}\xi \leq \bar{\tau} < y_R - \bar{x}\frac{\xi(Y-\bar{x})^\theta}{Y^\theta}$.

- Case 3: $\psi > 0$ and $\mathcal{C}_{i,P,2}(\mathbf{s}), \mathcal{C}_{j,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. By eqs. (24)-(27), we have that $\mathcal{G}_{P,2}(\mathbf{s}) = \frac{\theta Y}{1+\theta} = g^*$. By eq. (28), we have that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{c_R}\frac{1}{\xi}$ and from eq. (27) we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = \frac{Y}{1+\theta} - \overline{c_R}\frac{1}{\xi}$. This case holds if and only if $\mathcal{C}_{P,P,2}(\mathbf{s}) > \bar{x}$, which implies $\overline{c_R} < \omega\xi$, where $\omega = \frac{Y}{1+\theta} - \bar{x}$. From the rich's budget constraint, we have that $\Psi_{P,2}(\mathbf{s}) = y_R - \overline{c_R}\xi$. The expression for $\mathcal{E}_{P,2}(\mathbf{s})$ follows by using P's budget constraint. In terms of policies, this case holds if and only if $\bar{\tau} > y_R - \omega\xi$.

- Case 4: $\psi > 0$ and $\mathcal{C}_{i,P,2}(\mathbf{s}) = \bar{x}$. By eq. (27) and eq. (28), we have that g_2 is the solution for

$$\ln(Y - \bar{x} - g_2) + \theta \ln(g_2) - \ln(c_{j,1}) - \theta \ln(\bar{x}) = 0 \tag{29}$$

  By P's budget constraint we have that $\mathcal{E}_{P,2}(\mathbf{s}) = \bar{x} - y_P = e_R^D$. $\Psi_{P,2}(\mathbf{s}) = \tau_P^D$ can be obtained from the budget constraint of rich agents. This case holds if and only if $\bar{\tau} \leq y_R - \omega\xi$.

Therefore, we have

$$\mathcal{E}_{P,2}(\boldsymbol{s}) = \begin{cases} e_R^D, & \text{if} \quad \bar{\tau} < y_R - \omega\xi \\ e_P^D + \frac{\bar{x}}{1+\theta} - (y_R - \bar{\tau})\xi, & \text{if} \quad \bar{\tau} \in [y_R - \omega\xi, y_R - \bar{x}\xi) \\ y_R - \bar{x}\left(1 + \left(\frac{y_R - \bar{\tau}}{\bar{x}}\right)^{\frac{1}{\theta}}\right), & \text{if} \quad \bar{\tau} \in \left[y_R - \bar{x}\xi, y_R - \bar{x}\xi\frac{(Y-\bar{x})^\theta}{Y}\right) \\ e_P^D, & \text{if} \quad \bar{\tau} \geq y_R - \bar{x}\xi\frac{(Y-\bar{x})^\theta}{Y} \end{cases}$$

$$\Psi_{P,2}(\boldsymbol{s}) = \begin{cases} \bar{x} + G_2(\bar{\tau}) - y_P, & \text{if} \quad \bar{\tau} < y_R - \omega\xi \\ y_R - (y_R - \bar{\tau})\frac{\bar{x}}{\xi}, & \text{if} \quad \bar{\tau} \in [y_R - \omega\xi, y_R - \bar{x}\xi) \\ \tau_P^D, & \text{if} \quad \bar{\tau} \in \left[y_R - \bar{x}\xi, y_R - \bar{x}\xi\frac{(Y-\bar{x})^\theta}{Y}\right) \\ \tau_P^D, & \text{if} \quad \bar{\tau} \geq y_R - \bar{x}\xi\frac{(Y-\bar{x})^\theta}{Y} \end{cases}$$

and

$$\mathcal{G}_{P,2}(\boldsymbol{s}) = \begin{cases} G_2(\bar{\tau}), & \text{if} \quad \bar{\tau} < y_R - \omega\xi \\ g^*, & \text{if} \quad \bar{\tau} \in [y_R - \omega\xi, y_R - \bar{x}\xi) \\ \bar{x}\left(\frac{y_R - \bar{\tau}}{\bar{x}}\right)^{\frac{1}{\theta}}, & \text{if} \quad \bar{\tau} \in \left[y_R - \bar{x}\xi, y_R - \bar{x}\xi\frac{(Y-\bar{x})^\theta}{Y}\right) \\ g^D, & \text{if} \quad \bar{\tau} \geq y_R - \bar{x}\xi\frac{(Y-\bar{x})^\theta}{Y} \end{cases}$$

with $G_2(\boldsymbol{s})$ is the unique solution to eq. (29).
The associated private consumption allocations are

$$\mathcal{C}_{P,P,2}(\boldsymbol{s}) = y_P + \mathcal{E}_{P,2}(\boldsymbol{s}) \quad \text{and} \quad \mathcal{C}_{R,P,2}(\boldsymbol{s}) = y_R - \Psi_{P,2}(\boldsymbol{s}).$$

In the first-period, assuming the case in which the individual rationality constraint is not binding, party's P Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{P,1}) + \ln(g_1) + \beta\left[qV_P(c_{R,1}) + (1-q)W_P(c_{P,1})\right] + \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party P are $c_{P,1}, c_{R,1}, g_1 \geq \bar{x}$, $\lambda, \psi \geq 0$ and

$$
\begin{aligned}
[c_{P,1}] \quad & \frac{1}{c_{P,1}} + \beta(1-q)\frac{dW^P(c_{P,1})}{dc_{P,1}} - \lambda \leq 0. \\
& (c_{P,1} - \bar{x}) \left[\frac{1}{c_{P,1}} + \beta(1-q)\frac{dW^P(c_{P,1})}{dc_{P,1}} - \lambda \right] = 0. \qquad (30) \\
[c_{R,1}] \quad & \beta q \frac{dV^P(c_{R,1})}{dc_{R,1}} - \lambda \leq 0. \\
& (c_{R,1} - \bar{x}) \left[\beta q \frac{dV^P(c_{R,1})}{dc_{R,1}} - \lambda \right] = 0. \qquad (31) \\
[g_1] \quad & \frac{\theta}{g_1} - \lambda \leq 0. \\
& (g_1 - \bar{x}) \left[\frac{\theta}{g_1} - \lambda \right] = 0. \qquad (32) \\
[RC] \quad & [Y - c_{P,1} - c_{R,1} - g_1] \geq 0. \\
& \lambda \left[Y - c_{P,1} - c_{R,1} - g_1 \right] = 0. \qquad (33)
\end{aligned}
$$

where

$$
W_P(c_{P,1}) = \begin{cases} \ln(\bar{x}) + \theta \ln\left(\frac{\theta(Y-\bar{x})}{1+\theta}\right), & \text{if} \quad c_{P,1} < \frac{\bar{x}\xi(Y-\bar{x})^\theta}{Y^\theta}. \\ \ln(c_{P,1}) + \theta \ln(\bar{x}), & \text{if} \quad c_{P,1} \in \left[\frac{\bar{x}\xi(Y-\bar{x})^\theta}{Y^\theta}, \omega\xi\right) \\ \ln(Y - \bar{x} - G_2(c_{P,1})) + \theta \ln(G_2(c_{P,1})), & \text{if} \quad c_{P,1} \geq \omega\xi \end{cases}
$$

Note that the only relevant state for party P when in power is $c_{R,1}$ and for when out of power is $c_{P,1}$. By eq. (31), we have that $c_{R,1} = \bar{x}$.

Since $u(.)$ is increasing in its arguments, $\lambda > 0$. By eq. (33), we have that $c_{P,1} = Y - \bar{x} - g_1$. Also, since $W'_P(c_{P,1}) = 0$ if $c_{P,1} < \bar{x}\frac{\xi(Y-\bar{x})^\theta}{Y^\theta}$ and $W'_P(c_{P,1}) = \frac{1}{c_{P,1}}$ otherwise. Therefore, we have two cases to consider:

- Case 1: By eq. (30) and eq. (9) we have that $g = \frac{\theta(Y-\bar{x})}{1+\theta} = g^D$ and $c_{P,1} = \frac{Y-\bar{x}}{1+\theta} = c^D_{i,i} \geq \bar{x}\frac{\xi(Y-\bar{x})^\theta}{Y^\theta}$, a contradiction. Therefore, this case never holds.

- Case 2: By eq. (30) and eq. (32) we have that $g = \frac{\theta(Y-\bar{x})}{1+\theta+\beta(1-q)} = \frac{1+\theta}{1+\theta+\beta(1-q)} g^D \leq g^D$ and $c_{P,1} = \frac{(1+\beta(1-q))(Y-\bar{x})}{1+\theta+\beta(1-p)} = \frac{(1+\beta(1-q))(1+\theta)}{1+\theta+\beta(1-q)} c^D_{i,i} \geq c^D_{i,i}$. By P's budget constraint, we have that $\mathcal{E}_{P,1} = \frac{\Delta_P - \bar{x} + \beta(1-q)(y_P - \bar{x})}{1+\theta+\beta(1-q)} = \frac{\beta}{1+\theta+\beta(1-q)} \left(\beta\tau^D_P + (1+\theta)\mathcal{E}^D_P\right)$. Using the rich's budget constraint, we have that $\tau_{P,1} = y_R - \bar{x} = \tau^D_P$.

  It is worth noting that, since the problem for the first-period considers a case in which the individual rationality constraint is not binding, the respondent R will always receive the constitutional minimum $\bar{x}$, which is the value R would receive if P is in power with all spending discretionary. This implies a tax policy that also follows the case in which all fiscal instruments are discretionary. The political wedge created by the mandatory spending on private goods is manifested in the distortion of spending between public goods and entitlements, as we can see above. Even when the individual rationality constraint is not binding, P will always provide a public good level such that $g_{P,1} \leq g^D$ and $c_{P,1} \geq c^D_{i,i}$.