Predictable Interest Rate Movements and Their Implications for Emerging Markets*

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Abstract

US monetary policy announcements and news about fundamentals gives rise to *expected interest rate movements* in the international financial center. We study their consequences for emerging markets in terms of domestic activity, borrowing costs, and welfare. In the data, tight monetary policy in the US is associated with higher spreads and depressed activity in emerging markets. However, a standard sovereign default model augmented with (news) shock to the risk-free short-term rate cannot replicate these patterns: production is disconnected from financial conditions and, in equilibrium, yields rise but spreads often fall, as sovereigns reduces new issuance when bond prices are low. We solve this disconnect by introducing domestic financial frictions in production, which allows the model to generate recessions and debt crises caused by higher (expected) interest rates in the financial center. These working capital loans have a side-effect of eliminating the "consumption boom before default" puzzle. We argue that the interplay between sovereign and domestic financial frictions is key for replicating patterns in the data.

Keywords: emerging markets, sovereign default, news shocks, global financial cycle
JEL classification: F34, F41, E52

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1 Introduction

We revisit the consequences of changes in default-risk-free rates in the international financial center for emerging markets' sovereign cost of borrowing, level of activity, and welfare. Complementary to prior work, we focus on the impact of *expected interest rate movements* induced by news in the financial center and the role domestic financial frictions play in shaping the response of production to sovereign distress. Whether movements in US rates account for a sizable share of emerging market volatility has been the topic of an extensive and influential literature, which we briefly reference below, but relatively less is known about the impact of monetary policy announcements or other "early warning" information available to market participants on emerging markets vulnerable to crises.

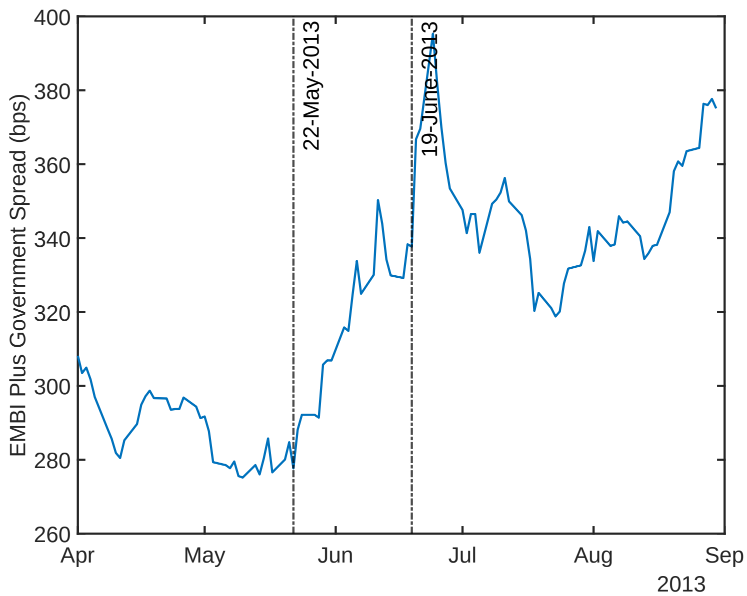


Figure 1: Spreads Around US FRB Announcements (2013 "Taper Tantrum" Episode)

Figure 1 illustrates one of the events motivating our work. We plot the EMBI+ spread for the entire set of emerging markets during the "taper tantrum" episode in 2013. In May 22$^{\text{nd}}$, Federal Reserve Chairman Bernanke suggested that tapering of the ongoing Quantitative Easing program is imminent, causing the start of a "taper tantrum" in US bond markets. On June 19$^{\text{th}}$, the FRB announced officially the tapering of the program, quickly leading to a new increase in the spreads of emerging markets.

The empirical literature has broadly conclude that tight monetary policy in the US is associated with a heightened risk of recession and higher spreads in emerging markets, e.g., as exemplified recently by Kalemli-Özcan (2019). We make two related contributions. First, we investigate whether a standard sovereign default model in the tradition of Eaton and Gersovitz (1981) and

Arellano (2008), once augmented with news shocks to the risk-free rate, can replicate this evidence. We find that, following an increase in the risk-free rate, such a model exhibits lower default risk and spreads in equilibrium, and no change in output, in contrast to the data. We explain the mechanism behind this disconnect. Second, we argue that the missing model ingredients are domestic financial frictions, along the lines of Neumeyer and Perri (2005), more precisely the interplay between sovereign borrowing, household consumption, and the interest rate on working capital loans necessary for production. A final by-product of our work is that, by incorporating financially-constrained domestic production, we eliminate the "consumption booms before default" puzzle discussed by, among others, Hatchondo, Martinez, and Sosa-Padilla (2016), whereby sovereign default models with positive recovery rates give an incentive for "full dilution" and a spike in consumption in the period just prior to default.

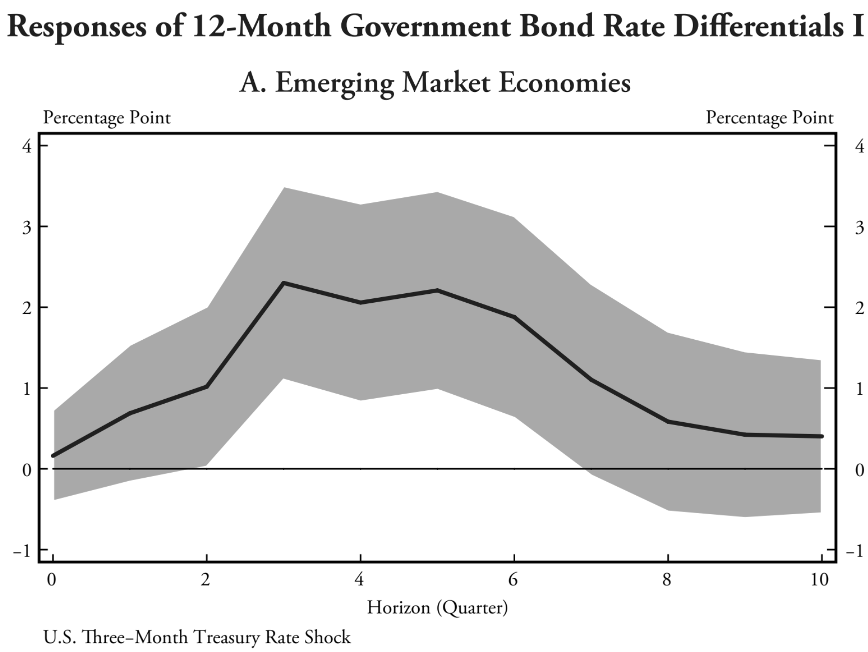


Figure 2: Emerging Markets' Spreads, Response to US Shock. Source: Kalemli-Özcan (2019)

We formulate two version of our model: a baseline model with financially-constrained production and interest rate news, as well as an endowment version of this baseline, in which income is stochastic and uncorrelated to the international risk-free rate. We calibrate our models and illustrate their dynamics though selected business cycle statistics and by plotting fully nonlinear impulse response functions, using the method of Koop, Pesaran, and Potter (1996). Our model assumes a constant expected haircut rate and a delay to resumption of international borrowing following default. The inclusion of a positive recovery rate leads to our endowment model to exhibit the "consumption boom before default" puzzle, unless borrowing is constrained by, e.g., an underwriting standard as in Chatterjee and Eyigungor (2015). We show that the full model, with domestic production subject to working capital constraints, endogenously prevents these "booms" even absent any constraint on the bond price or the default probability, as common in prior work. This is because a high level of borrowing prior to default would severely distort domestic production, enough for the sovereign to choose default immediately rather than postponing one period and enjoying the net present value of eventual recovery as bond sale proceeds today.

Related Literature. Earlier contribution such as Neumeyer and Perri (2005) and Uribe and Yue (2006) studied both theoretically and empirically the role of *interest rate shocks* for the behavior of small open economies, but abstracted from endogenous default risk and preannounced or predetermined interest rate movements, i.e., news. Fernández-Villaverde et al. (2011) document the importance of interest rate volatility and Johri, Khan, and Sosa-Padilla (2020) revisit this feature in a model with endogenous sovereign default. Wolf and Zessner-Spitzenberg (2021) study the consequences of interest rate hikes in a monetary union, for a two-goods economy with downward rigid wages, where tight monetary policy is recessionary and does lead to high spreads. In contrast with these works, we focus on *news* about the near-term path of interest rates in the financial center and the role domestic financial frictions play in translating these expected rate movements into recessionary pressures, as well as the sovereign's ability to use borrowing to mitigate their impact.

Our work here also relates to the broader question of international comovement, the "*global financial cycle*," as discussed by Longstaff et al. (2011), Bai, Perri, and Kehoe (2019), and Miranda-Agrippino and Rey (2021) among others. This line of research stresses the role of a small number of global factors in explaining the cross-section and comovement of spreads. Other spillover channels from which we abstract include lenders' risk-aversion and wealth effects, as in Lizarazo (2013) and Arellano, Bai, and Lizarazo (2017).

News shocks were studied extensively in applications to total factor productivity, both theoretical and empirical, as in Beaudry and Portier (2004, 2006) and more recently Görtz, Tsoukalas, and Zanetti (2016). We abstract from news about productivity and instead allow only early warning about future interest rate movements in the financial center, leaving the question of their interaction for future work.

2 Risk-free Rate Shocks in a Tractable Default Model

In this section,[1] we characterize the consequences of risk-free rate shocks in a highly tractable model, where optimal issuance and default behavior are summarized by a simple shock threshold and a first-order condition. This setting allows us to highlight the key role of shock persistence for the dynamics of sovereign spreads and the inherent limitation of models without domestic frictions.

1. The development of this section has benefited greatly from Yan Bai's careful suggestions, for which we are particularly thankful.

Model. A sovereign receives a *constant* endowment y and trades a one-period bond with risk-neutral, competitive lenders, with an option to default of stochastic value. There is no recovery and market exclusion is permanent. In standard recursive notation, the sovereign's value function is

$$V(b) = \max_{b'} \left\{ u\left[\bar{y} - b + q(b')b'\right] + \beta \mathbb{E}_\nu \max\left[V(b'), V^d - \nu\right] \right\} \tag{1}$$

where V^d is a constant and ν is an iid shock with cdf Φ and pdf ϕ.

Default and Bond Prices. The default policy is characterized by a threshold value for ν, $\nu^*(b) \equiv V^d - V(b)$, so that the sovereign default whenever the start-of-period debt level is b and the realization of the iid shock ν is below $\nu^*(b)$. The resulting bond price schedule is then

$$q(b') = \frac{1 - \Phi\left[\nu^*(b')\right]}{1 + r^{\mathrm{rf}}}. \tag{2}$$

Issuance Behavior. Given the threshold form of the default policy, the value function can be expressed as

$$V(b) = \max_{b'} \left\{ u\left[\bar{y} - b + q(b')b'\right] + \beta\left[V(b') + \Phi\left(\nu^*(b')\right)\left(V^d - V(b')\right) + \int_{-\infty}^{\nu^*(b')} \nu d\Phi\right] \right\} \tag{3}$$

and the associated first-order condition for b' is

$$\left[q(b') + b'q'(b')\right] u'(c) = \beta\left[1 - \Phi(\nu^*(b'))\right] u'(c') \tag{4}$$

or, using the equilibrium q schedule,

$$\left[1 - h\left(\nu^*(b')\right) b'\right] u'(c) = \beta\left(1 + r^{\mathrm{rf}}\right) u'(c') \tag{5}$$

where $h(\nu) = \phi(\nu)/\left[1 - \Phi(\nu)\right]$ is the hazard function.

The Linear Utility Case. Equation (5) suggests that the response of b' to shocks to r^{rf} could be shaped by consumption-smoothing incentives, due to the presence of current and future marginal utility of consumption, we therefore further restrict the environment by consider the case of linear utility, under which $u'(c) = 1$ and $V'(b) = -1$. Now, the first-order condition becomes

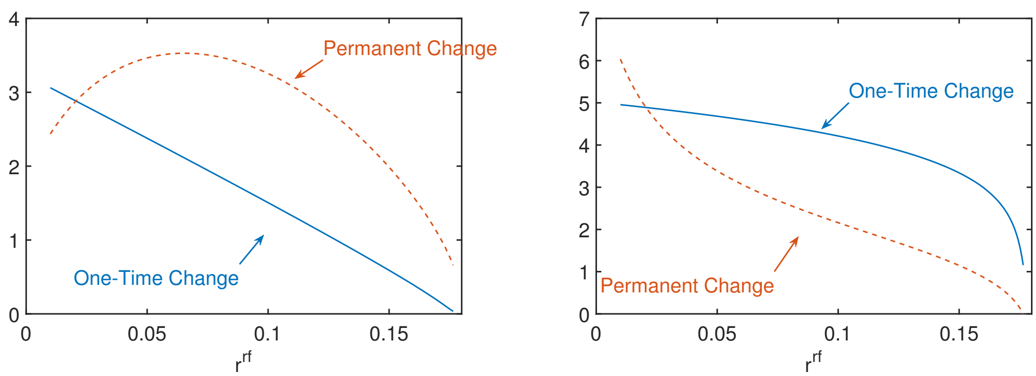
$$1 - h\left[\nu^*(b')\right] b' = \beta\left(1 + r^{\mathrm{rf}}\right). \tag{6}$$

Note that, since $V(b)$ is affine in b with slope -1, the default threshold $\nu^*(b)$ is affine in b with slope 1. The first-order condition is independent of b so that the debt levels "jumps" and remains at a "pseudo-steady-state" level b_{SS} until the country eventually default, with constant probability $\Phi\left(\nu^*(b_{\mathrm{SS}})\right)$.

Proposition 1 (One-Time, Unexpected r^{rf} Shock.)**.** *If the distribution of ν has a weakly increasing hazard function h, a one-time, unexpected increase in r^{rf} reduced borrowing, default risk, and the spread.*

Proof. See Appendix A. □

Intuitively, when r^{rf} increases unexpectedly today, but returns to its initial value permanently starting next period, the right hand side of equation (6) increases which implies that $h(\nu^*(b'))b'$ must decrease. With linear utility, $V(b) = \overline{v} - b$ and therefore $\nu^*(b) = V^d - \overline{v} + b$, where $\overline{v}$ is a constant reflecting models parameters including the initial risk-free rate r^{rf}. Since next period's r^{rf} is expected to return to the pre-shock level, the default threshold function $\nu^*(b)$ is unaltered by today's one-time change in r^{rf}. Given the weakly increasing hazard function[2] h, the new solution of the first-order condition must yield a lower b', thus a lower $\nu^*(b')$ and therefore lower default risk $\Phi(\nu^*(b'))$.



(a) Default Probability ($\Phi(\nu^*(b'))$)

(b) Borrowing Choice (b')

Figure 3: One-Time and Permanent Changes in r^{rf}

Permanent Changes in r^{rf}. With permanent changes in r^{rf}, the impact on b' and spreads is ambiguous, because the new risk-free rate alters the value function's intercept $\overline{v}$ and therefore shifts the intercept of the default threshold function $\nu^*(b)$ as well. Consider a numerical example plotted in Figure 3, for $\beta = 0.85$, $\overline{y} = 1$, ν is Normal with mean 0 and standard deviation 2.5, with a reference r^{rf} of 2%. As we vary r^{rf} on the horizontal axes, either permanently or one-time, the choice of borrowing is always lower, in the right (b) panel, but the overall impac for the one-period ahead default probability (and thus the spread) is markedly different across the two cases: with a one-time change, consistent with Proposition 1, default risk falls monotonically in the r^{rf} shock, whereas for permanent changes default risk first increases then eventually decreases, due to shifts in the $\overline{v}$ intercept of the value function.

Taken together, these findings suggest that a standard endowment default model could exhibit higher spreads in response to increases in risk-free rates if either the increase is persistent enough

2. Note that we have restricted attention to shock distributions with a weakly increasing hazard function. This is true for, among others, the Uniform distribution and the Normal distribution. See Baricz (2008).

or if consumption-smoothing forces are strong enough, as in, e.g., Bocola, Bornstein, and Dovis (2019), who study the role of subsistence consumption for borrowing behavior during debt crises.

Given the limitation of the standard endowment approach, we pursue a complementary mechanism based on distorted domestic production, with the aim of rationalizing not only the response of spreads to rate changes in the financial center but also domestic income and financial conditions. We turn to such a model next.

3 Model

This section introduces our model with defaultable, long-term debt, as in Chatterjee and Eyigungor (2012), augmented with stochastic movements in the risk-free rate, which is the opportunity cost for international lenders, as in Johri, Khan, and Sosa-Padilla (2020). Domestic financial frictions follow Neumeyer and Perri (2005). We argue that these elements are essential for replicating the impact of expected and unexpected movements of risk-free rate in the international financial center on emerging markets' production and spreads. We relegate the endowment version of our model to Appendix C and the strategy for computing the model, using discrete choice methods, to Appendix D.

3.1 The Dynamics of the Risk-free Rate

The opportunity cost for international lenders is the short-term default-risk-free rate in the financial center, denoted by r_t^{rf}. We assume it is governed by the following process with news shocks,

$$\begin{aligned} r_{t+1}^{\mathrm{rf}} &= (1-\rho_r)\nu_t + \rho_r r_t^{\mathrm{rf}} + \sigma_{r,\varepsilon}\varepsilon_{t+1} \\ \nu_{t+1} &\sim F(\nu_{t+1}|\nu_t) \end{aligned} \tag{7}$$

where ν_t is a persistent news shock, *observed at time t*, which shifts forecasts of near-term risk-free rates, and ε_{t+1} is a standard Normal innovation. The ν news shock about the trend of the AR(1) process for r^{rf} follows Markov chain with transition F. We think of it as highly persistent, capturing, e.g., the Fed's eventual preferred interest rate level in 1-3 years. This process is a regime-switching AR(1), where the regime shifts the intercept, the long-run level of the variable, with the twist that the long-run level for next period is known this period.

Figure 4 provide an illustrative example for the evolution of r^{rf} and ν. Following a change in the level of ν, r^{rf} exhibits predictable transition dynamics. Initially, rates are roughly 2% but suddenly agents come to expect them to eventually reach 3%, after several periods, following a jump in ν, perhaps a policy announcement. This news about the evolution of risk-free short-term rates has sharp implications for long-term rates in the financial center and for lenders' opportunity cost of funds when buying emerging market sovereign bonds.

The short-term rate r^{rf} induces a price for a random-maturity, *long-term* default-risk-free bond

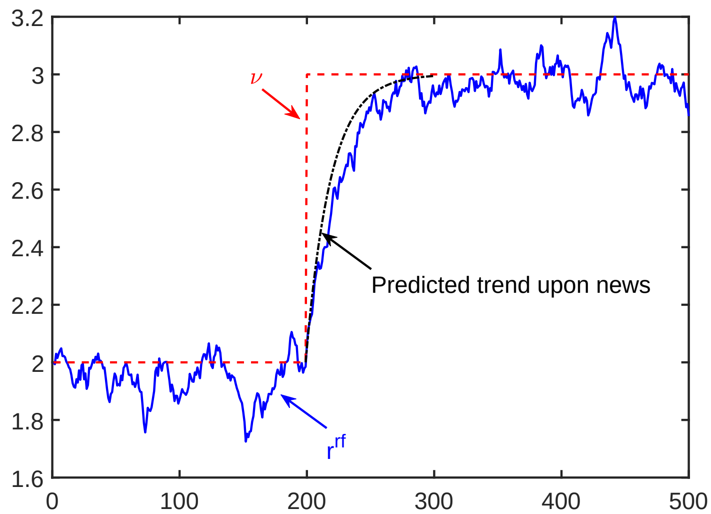


Figure 4: Example Paths for News Shock

in the financial center through the no-arbitrage condition,

$$q_t^{\mathrm{rf}} = \frac{1}{1 + r_t^{\mathrm{rf}}} \left[\kappa + (1 - \delta) \, \mathbb{E}_t q_{t+1}^{\mathrm{rf}} \right], \tag{8}$$

where we have assumed that each unit of debt calls for a service payment κ each period and a fraction δ "matures," so that a $1 - \delta$ share will remain outstanding the following period. The implied yield-to-maturity is given by $\kappa / q_t^{\mathrm{rf}} - \delta$. In this simple setting the expectations hypothesis holds and the yield implied by q_t^{rf} is an expected weighted average of current and future short-term rates. This "κ & δ" maturity structure will also be the one for the defaultable debt issued by the sovereign, so that the default-risk-free long-term yield constructed here is the relevant one for the computation of sovereign spreads.

For tractability, we write down a process for r^{rf} directly rather than, e.g., introduce an exponentially affine pricing kernel, as in Hatchondo, Martinez, and Sosa-Padilla (2016), since our emphasis is on outcomes in borrower countries. In so doing we avoid having to take a stand on any risk premium associated with, e.g., productivity shocks in emerging markets, as common in the literature. This could be mimicked via correlating the shocks to r^{rf} with those to domestic productivity, for which the data might call.

3.2 The Domestic Private Sector

The domestic private sector consists of a representative household, a representative firm, and a representative financial intermediary.

Household. The representative household supplies labor competitively and receives the profits of domestic firms and financial intermediaries, as well as a lump-sum tax or transfer from the fiscal authority. Its problem is static and given by

$$\max_{\ell_t} u\left(c_t, \ell_t\right) \text{ s.t. } c_t = w_t \ell_t + \Pi_t + \Pi_t^f + T_t \tag{9}$$

where c_t is domestic consumption, w_t is the wage rate, ℓ_t is labor supply, Π_t are the profits of domestic final goods producers, Π_t^f are the profits of financial intermediaries, and T_t is the fiscal authority's lump-sum transfer or tax.

Even though the household's labor supply decision is static, it discounts the future with factor β, which is relevant for welfare assessments and for the construction of its intertemporal marginal rate of substitution.

Domestic Producer. Competitive domestic firms hire labor to produce final goods, using a decreasing return technology. A fraction θ of the wage bill must be financed with an *intra-period* working capital loan from the domestic financial intermediary.[3] The profit maximization problem of the producer is thus

$$\Pi_t = \max_{\ell_t} \left\{ A_t \ell_t^{\alpha} - \left[\left(1-\theta\right) w_t \ell_t + \theta\left(1+i_t\right) w_t \ell_t\right]\right\} \tag{10}$$

where i_t is the domestic interest rate carried by the intra-period working capital loan, ℓ_t is the resulting labor demand, and A_t is a productivity level common to all firms in the economy.

The resulting demand for labor is given by

$$\ell_t = \left(\frac{\alpha}{1+\theta i_t} \cdot \frac{A_t}{w_t}\right)^{1/(1-\alpha)} \tag{11}$$

which implies that tighter domestic financial conditions, as captured by a higher interest rate for working capital loans, depress production.

We assume that the productivity level A_t is stochastic and endogenously lowered by the sovereign default decision, as described later. Net of any such reductions induced by sovereign default, the productivity term evolves exogenously, according to

$$\log A_{t+1} = \rho_A \log A_t + \sigma_{A,\epsilon} \epsilon_{t+1} \tag{12}$$

and $\epsilon_{t+1} \sim \mathcal{N}(0,1)$ is standard Normal.

3. A similar constraint is used by Mendoza and Yue (2012) to expose the import of intermediate goods to default risk, to endogenize the resource costs of default. Modeling such intra-period loans follows Fuerst (1992).

Domestic Financial Intermediary. The domestic financial intermediary extends working capital loans to producers and transfers the profits resulting from this activity to the household

$$\Pi_t^f = -a_t + (1 + i_t)\, a_t = i_t a_t, \tag{13}$$

where a_t is the working capital loan size. In equilibrium firms take loans of size $a_t = \theta w_t \ell_t$.

The rate for this intra-period loan is given by the *domestic interest rate*

$$i_t = \frac{u_c\,(c_t, \ell_t)}{\beta \mathbb{E}_t u_c\,(c_{t+1}, \ell_{t+1})} - 1, \tag{14}$$

which is the domestic cost of funding in this economy, measured by the expected marginal rate of intertemporal substitution of the household. This is consistent with common timing assumptions made in modeling intra-period loans, e.g., the loan is extended in the "morning" of period t while production and repayments are done in the "evening" of period t.

We use this structure to combine the benefit from having the marginal rate of substitution of the representative household give the domestic cost of borrowing, with the tractability of the intra-period loan, so that no additional state variables are required.

3.3 Sovereign Borrowing

The benevolent fiscal authority can borrow abroad, lacking commitment over future repayment and borrowing decisions. It starts each period t with an initial debt level b_t, which, if it chooses not to default, it must service with a payment of κb_t, at which point it can issue or retire units of its long-term bond, inducing next period's debt level b_{t+1}. The resulting proceeds from operating in international markets are transferred lump sum to the domestic household[4]

$$T_t = -\kappa b_t + q_t \left[b_{t+1} - (1 - \delta)\, b_t \right]. \tag{15}$$

If the sovereign chooses to default, we encode this decision in a policy variable $d_t = 1$, no debt service payments are made and no new issuance or buyback of bonds are possible, resulting in $T_t = 0$. If instead the sovereign services its debt, we write $d_t = 0$.

We assume that following a decision to default, the sovereign spends a random length of time being excluded from international financial markets. With a constant probability λ the sovereign resumes international market access and applies a haircut to its creditors, so that upon reentry the outstanding debt level is ϕb_t with $\phi < 1$. Furthermore, a state of default carries a direct productivity reduction for domestic producers, that is $A_t^d \leq A_t$.

We consider a setting with centralized borrowing, as standard in the quantitative sovereign default literature, by assuming that all external lending or borrowing is done by the government, some of which on behalf of the private sector. Possible alternatives, which we do not pursue here,

4. As common in the literature, we could accommodate the presence of a stochastic or constant public spending level G_t in this budget constraint. We abstract from such an extension.

are combinations of (de)centralized borrowing and (de)centralized default, with the caveat that, with flexible enough capital controls policy instruments, the government might still be able to induce the outcomes studied here.

International Lenders. Competitive international lenders price the sovereign's bonds, given an opportunity cost represented by the short-term rate in the financial center r_t^{rf},

$$q_t = \frac{1}{1+r_t^{\mathrm{rf}}}\mathbb{E}_t\left\{(1-d_{t+1})\left[\kappa+(1-\delta)q_{t+1}\right]+d_{t+1}q_{t+1}^d\right\}. \tag{16}$$

The price q_t^d is the secondary market value of defaulted debt units, which satisfies

$$q_t^d = \frac{1}{1+r_t^{\mathrm{rf}}}\mathbb{E}_t\left\{(1-\lambda)\,q_{t+1}^d+\lambda\phi\left[d_{t+1}q_{t+1}^d+(1-d_{t+1})\,(\kappa+(1-\delta)q_{t+1})\right]\right\}, \tag{17}$$

where the right-hand side reflects the assumption that the sovereign has the option to immediately default again after returning to market and applying the haircut. ϕ is the ex-post recovery rate, assumed constant here. It is not also the ex-ante recovery rate because of the random nature of the length of time required to "return to markets."

The yield-to-maturity spread of the sovereign bond is given by $\kappa/q_t - \kappa/q_t^{\mathrm{rf}}$ where q_t^{rf} is the price of the default-risk-free bond trade in the financial center, with the same maturity structure with the sovereign's bond, defined in Section 3.1.

3.4 A Recursive Formulation

We consolidate our model and provide a recursive formulation, using exogenous state variables $s = \left\langle r^{\mathrm{rf}}, \nu, A\right\rangle$ and the endogenous state variable b. Implicitly, whether the sovereign has or lacks market access is also recorded as a state variable but we choose to express this dependency by writing down separate value and policy functions for a country in default. As standard in the study of equilibrium default, we restrict attention to Markov Perfect Equilibria.

Domestic Outcomes. In state $\langle s,b\rangle$, if the sovereign chooses b' as next period's debt level, domestic outcomes are summarized by policy functions $\mathcal{C}(s,b,b')$ for consumption, $\mathcal{L}(s,b,b')$ for employment, $\mathcal{I}(s,b,b')$ for the domestic interest rate, the wage rate $\mathcal{W}(s,b,b')$, and the consolidated profits of firms and financial intermediaries $\mathcal{P}(s,b,b')$. These satisfy the labor supply condition

$$-\frac{u_\ell\left(\mathcal{C},\mathcal{L}\right)}{u_c\left(\mathcal{C},\mathcal{L}\right)} = \mathcal{W} \tag{18}$$

the labor demand condition

$$\mathcal{L} = \left(\frac{\alpha}{1+\theta\mathcal{I}}\cdot\frac{A}{\mathcal{W}}\right)^{1/(1-\alpha)} \tag{19}$$

the household's budget constraint

$$\mathcal{C} = \mathcal{W}\mathcal{L} + P - \kappa b + q\left(s, b'\right)\left[b' - (1-\delta)b\right] \tag{20}$$

a profit condition for $\mathcal{P}$ consistent with the problems in Section 3.2, and the domestic interest rate condition

$$u_c\left(\mathcal{C}, \mathcal{L}\right) = \beta\left(1 + \mathcal{I}\right) H(s, b'), \tag{21}$$

for any state and choice $\langle s, b, b' \rangle$, where q is the bond price schedule and H is expected future marginal utility of consumption, both to be defined shortly.

Domestic Outcomes in Default. Analogously, domestic outcomes in default are characterized by functions of $\langle s, b \rangle$ alone, as there is no borrowing choice, e.g., $\mathcal{C}^d(s, b), \mathcal{L}^d(s, b), \ldots$, and an expected marginal utility of consumption while in default, $H^d(s, b)$.

Forward-looking Functions. We can facilitate the construction of the bond price schedules by introducing an auxiliary function

$$Q(s, b) = \mathcal{D}(s, b)q^d(s, b) + \left[1 - \mathcal{D}(s, b)\right]\left\{\kappa + (1-\delta)q(s, \mathcal{B}(s, b))\right\} \tag{22}$$

so that the bond price schedule with market access is simply

$$q(s, b') = \frac{1}{1 + r^{\text{rf}}}\mathbb{E}_{s'|s}Q(s', b') \tag{23}$$

and the secondary market value of defaulted bonds is

$$q^d(s, b) = \frac{1}{1 + r^{\text{rf}}}\mathbb{E}_{s'|s}\left\{(1-\lambda)q^d(s', b) + \lambda\phi Q(s', \phi b)\right\} \tag{24}$$

and $\mathcal{B}(s, b)$ and $\mathcal{D}(s, b)$ are the sovereign's equilibrium issuance and default policies function, to be described shortly.[5]

Similarly, the expected marginal utility functions, with market access and in default, can be expressed parsimoniously by introducing

$$J(s, b) = \left[1 - \mathcal{D}(s, b)\right]u_c\left(\mathcal{C}(s, b, \mathcal{B}(s, b)), \mathcal{L}(s, b, \mathcal{B}(s, b))\right) + \mathcal{D}(s, b)u_c\left(\mathcal{C}^d(s, b), \mathcal{L}^d(s, b)\right), \tag{25}$$

as

$$H(s, b') = \mathbb{E}_{s'|s}J(s', b') \tag{26}$$

5. This construction is slightly more convoluted than standard bond pricing schedules in the literature due to the presence of recovery and the option of the sovereign to immediate default again, upon returning to market and applying a haircut. If the initial debt level is very high, the sovereign will default and apply haircuts several times before resuming normal market access. Such scenarios are not part of the model's ergodic set.

and

$$H^d(s,b) = \mathbb{E}_{s'|s}\left\{\lambda J(s',\phi b) + (1-\lambda)u_c\left(\mathcal{C}^d(s',b),\mathcal{L}^d(s',b)\right)\right\}. \tag{27}$$

The expected marginal utilities here account for the risk of default and the associated low levels of consumption, as in Arellano, Bai, and Mihalache (2020). The default risk, through the H function, weighs heavily on the domestic interest rate i in key states of the world.

Sovereign. The sovereign chooses whether to default,

$$V(s,b) = \max_{d\in\{0,1\}}\left\{d\,V^d(s,b) + (1-d)V^r(s,b)\right\} \tag{28}$$

with its choice encoded in the policy function $\mathcal{D}(s,b)$, and how much debt to carry into next period, conditional on not defaulting,

$$V^r(s,b) = \max_{b'}\left\{u\left(\mathcal{C}(s,b,b'),\mathcal{L}(s,b,b')\right) + \beta\mathbb{E}_{s'|s}V(s',b')\right\} \tag{29}$$

with solution encoded in the policy function $\mathcal{B}(s,b)$. The value in default satisfies

$$V^d(s,b) = u\left(\mathcal{C}^d(s,b),\mathcal{L}^d(s,b)\right) + \beta\mathbb{E}_{s'|s}\left\{\lambda V(s',\phi b) + (1-\lambda)V^d(s',b)\right\} \tag{30}$$

so that the sovereign has no choice variables but understands that domestic outcomes must be consistent with labor and working capital market clearing.

Definition of Equilibrium. A *Markov Perfect Equilibrium* consists of

- Value functions for the sovereign, V, V^r, V^d
- Policy functions for the sovereign, $\mathcal{D}, \mathcal{B}$
- Domestic outcome functions with market access, $\mathcal{C}, \mathcal{L}, \ldots$
- Domestic outcome functions in default, $\mathcal{C}^d$, $\mathcal{L}^d, \ldots$
- Bond price schedules, q, q^d
- Expected marginal utility of consumption functions, H, H^d

such that

- The sovereign's policy functions solve the maximization problems of the value function, given domestic outcomes
- Bond prices are consistent with the sovereign's policy functions, lenders break even
- The expected marginal utility functions are consistent with the sovereign's policies and domestic outcomes

- Domestic outcomes are consistent with market clearing for labor and working capital, in default and with market access, respectively.

4 Quantitative Analysis

We calibrate our model and compare its quantitative properties to the data and to a version without domestic production subject to financial frictions, an endowment economy version, describe in detail in Appendix C.

4.1 Calibration and Quantitative Properties

We assume household preferences are CRRA and GHH, following Greenwood, Hercowitz, and Huffman (1988),

$$u(c,\ell) = \frac{\left(c - \psi \frac{\ell^{1+\mu}}{1+\mu}\right)^{1-\sigma} - 1}{1-\sigma}. \tag{31}$$

The functional form for the cost of default in terms of productivity follows Chatterjee and Eyigungor (2012),

$$h(A) = A - \max\left\{0,\, \lambda_0 A + \lambda_1 A^2\right\} \leq A \tag{32}$$

and we focus on the concave case, $\lambda_0 \leq 0 \leq \lambda_1$.

We use the Shadow Rate of Wu and Xia (2016) as a measure of the risk-free rate in the financial center, to circumvent issues arising from the extended period at the zero lower bound (ZLB/ELB). We estimate an AR(1) process for r^{rf}, with a Markov-switching intercept (ν) with n_ν states.

The ν news shock follows a Markov chain with support $[0.005, 0.01, 0.02]$, quarterly, and transition matrix

$$F(\nu'|\nu) = \begin{bmatrix} 0.950 & 0.035 & 0.015 \\ 0.020 & 0.960 & 0.020 \\ 0.015 & 0.035 & 0.950 \end{bmatrix}$$

Table 1 summarizes tentative parameter values for our quarterly calibration. [TODO]

We compute this model with defaultable long-term debt using *discrete choice methods*, as discussed in Appendix D. Recent applications of these methods for default models and their rationale can be found in, among others, Mihalache (2020), Dvorkin et al. (2020), Dvorkin et al. (2021), or Wolf and Zessner-Spitzenberg (2021), including comparisons with prior methods based on other iid perturbations, such as Chatterjee and Eyigungor (2012).

4.2 Impulse Response Functions

We produce stochastic impulse response functions for our model following the method of Koop, Pesaran, and Potter (1996). This approach is particularly well suited for models of sovereign default due to the highly nonlinear nature of policy functions around the dichotomous default

	Value	Comment
σ	2.0	CRRA
β	0.9725	Discounting
δ	0.05	5 year debt Macaulay duration
κ	$\delta + \mathbb{E}\nu$	Normalization
ψ	1.5	Normalization, mean ℓ
μ	0.5	Inverse of Frisch elasticity
λ_0	-0.24	Penalty, linear
λ_1	$+0.27$	Penalty, quadratic
λ	0.13	2 year exclusion
ϕ	0.7	Recovery rate
ρ_r	0.75	Autocorrelation of international rate
ρ_y	0.9	Autocorrelation of endowment
σ_ε	0.0025	Variance of interest rate shock
σ_y	0.01	Variance of endowment shock
ρ_D	$1e^{-4}$	Default shock
ρ_B	$3e^{-5}$	Borrowing shock

Table 1: Parameter Values

[TODO]

Table 2: Moments

decision.[6] We construct these impulse response panels by simulating a wide and long panel of uncorrelated countries, such that after a long enough time the cross-sectional distribution converges to the ergodic distribution of the model. We then shock at time $t = 0$ all economies by the same amount (except if the shock's grid/support is binding) and plot average dynamics over time. We include in all calculation the simulated economies that are in a state of default. Note that in such a state, the spread variable reflects the secondary market price of defaulted bonds, so that the variable has a valid value at all times.

Productivity Shocks. Figure 5 plots impulse responses to shock to A, the productivity in the small open economy. Low productivity is associated with a drop in output and consumption, and a Current Account reversal, as the Trade Balance spikes, as seen in the top row of panels. A large share of economies in the ergodic distribution immediately default (bottom center panel) and the sovereign spread rises sharply (top right panel). Because of the depressed consumption, domestic interest rates are high, to discourage "borrowing" (bottom right panel) and, as a consequence, working capital constraints are tight, activity is depressed and labor inputs falls sharply. These patterns are in line with previous results from sovereign default models with GHH preferences

6. For other applications of this approach to impulse response functions to sovereign default models see, among others, Arellano, Bai, and Mihalache (2018) and Dvorkin et al. (2020).

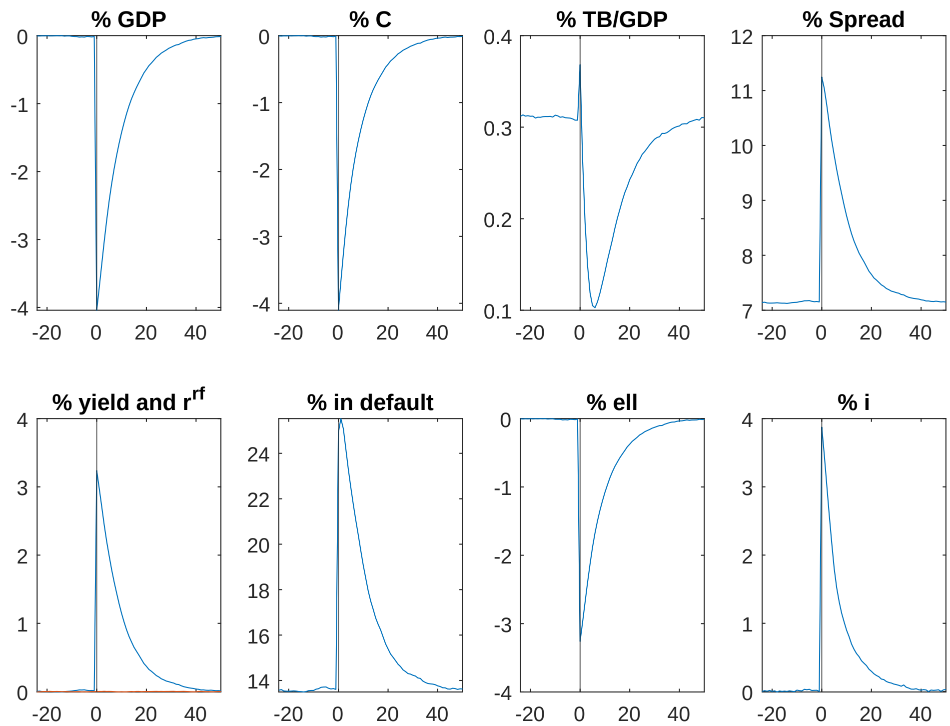


Figure 5: Impulse Response Functions, Domestic Frictions Model, *Endowment Shock*

and production, e.g., Cuadra, Sanchez, and Sapriza (2010), here further strengthened by the working capital constraint.

In our Trade Balance to GDP panel, the Trade Balance spikes on impact and falls. This is in part a composition effect, since the economies that are in a state of default have a Trade Balance of zero, "balanced trade." Alternatives such as eliminating from the cross-section all economies in default, or just the economies that default at the time of the shock, while easily implementable, give rise to other composition biases, such as selecting only "survivors" with high productivity prior to the shock. Overall, we conjecture that the impulse response functions which include all economies are the most informative, and we complement our results here with a full set of plots on policy functions in the following section, to better mitigate any composition issues.

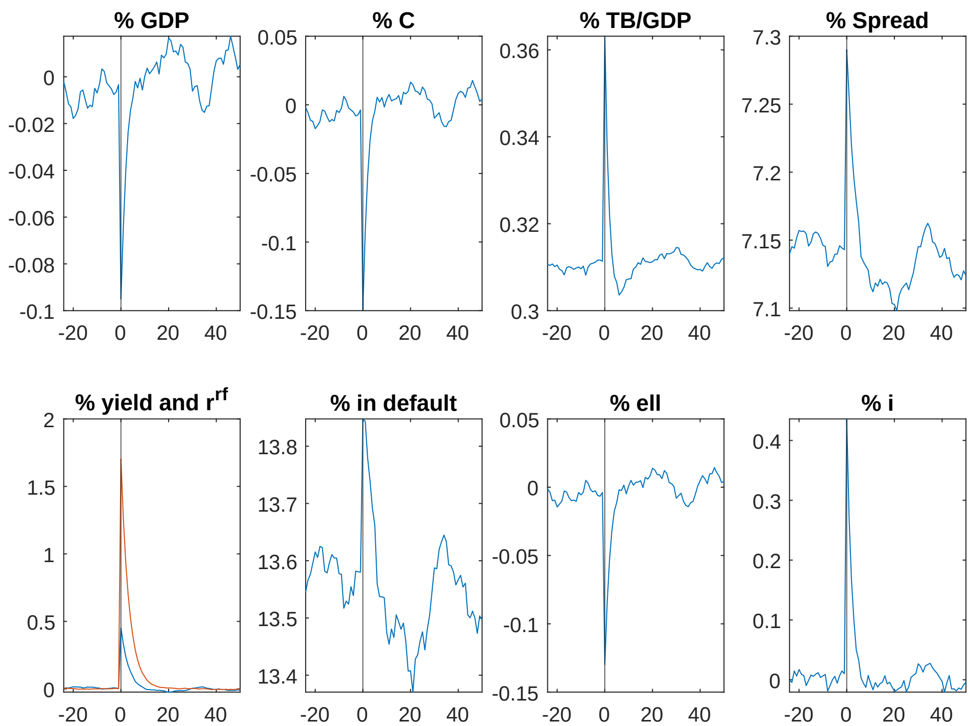


Figure 6: Impulse Response Functions, Domestic Frictions Model, *Risk-free Rate Shock*

Risk-free Rate Shocks. In Figure 6 we turn to the impact of shocks to the risk-free rate, ε_t in the notation of Section 3.1. These are unexpected and relatively transitory movements in the risk-free rate. An increase in r^{rf} is associated with a very modest increase in the share of economies in default, high domestic interest rates (bottom right) which in turn distort production and labor markets, ℓ falls and output follows, even though productivity is unchanged. The domestic recession leads to low consumption (top left panels) and, through the standard sovereign default mechanism, a Current Account reversal and a spike in spreads (top right panels).

The left-most bottom panel compared the response of the sovereign yield (in blue) to the path of r^{rf}. Yields increase shy of 0.5%, less that the increase in the short-term risk-free rate. Note that for spreads (top right panel) the relevant risk-free rate is the long-term rate from Section 3.1, which "averages" over current and future r^{rf} values. With transitory shocks to r^{rf} the long-run rate moves little, in contrast to the trend-like shocks which we consider next.

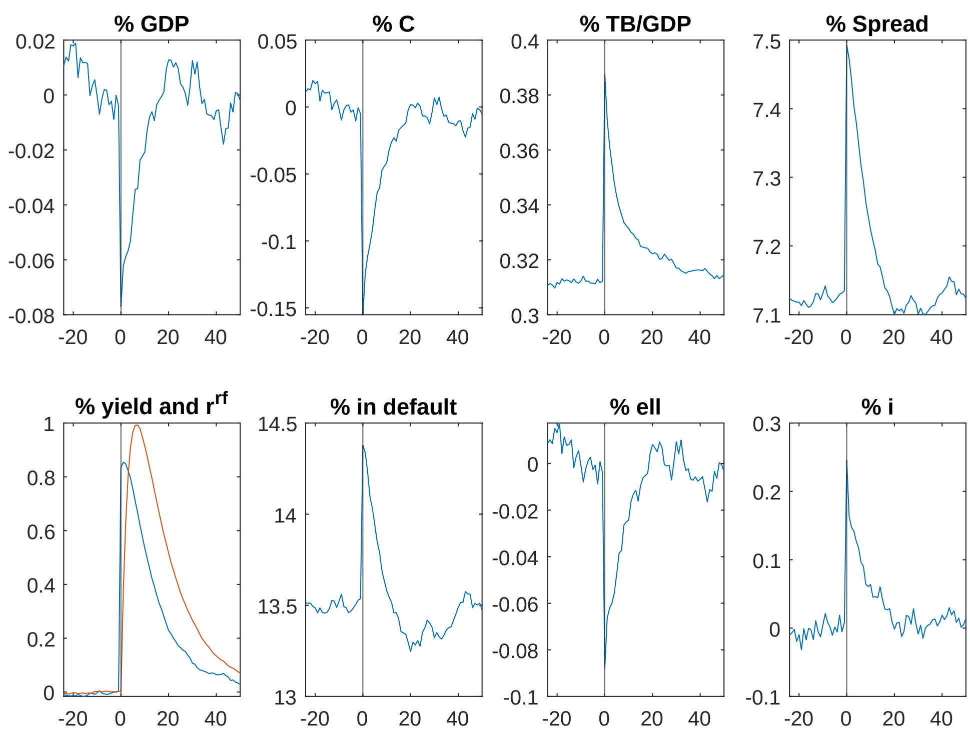


Figure 7: Impulse Response Functions, Domestic Frictions Model, *News Shock*

Risk-free Rate *News* Shocks. Figure 7 plots impulse responses to an increase in ν, a higher average level of short-term rates for the foreseeable future. In the left-most bottom panel of the Figure we plot the time path of r^{rf} following the ν shock in red. It first increases then, as ν mean-reverts, it returns to its long-run ergodic mean level. These dynamics are *fully anticipated* in expectation. Following this shock, lenders and the sovereign know that financial conditions in the financial center will be tighter for many quarters.

The consequences of this shock for the sovereign include a lengthy spell of higher spreads (right-most top panel), high domestic rates which depress production (right bottom panels), a fall in output and consumption (left top panels) and a persistent Current Account increase, as the sovereign must cut debt levels in the face of persistently higher costs of borrowing. A small share of economies immediately default, about 1%.

In our results, the impact of shocks is strongest immediately, unlike the hump-shared response in the data, from Figure 2. We conjecture that this is a consequence of the absence of adjustment costs and capital accumulation, slow-moving state variables more broadly, the inclusion of which would be quite challenging in our model which already features 4 state variables. Alternatively, consumption habits or subsistence consumption, as in Bocola, Bornstein, and Dovis (2019) has the potential to delay debt adjustment in response to shocks.

4.3 Characterization of Domestic Outcomes

In order to further explore the mechanism of domestic financial frictions and its impact on the sovereign's bond issuance choices, we now study policy functions for arbitrary b' levels. In Figure 8 we fix all shocks to their means, fix b slightly lower than the pseudo-steady-state $b^* = \mathcal{B}(\bar{s}, b^*)$, and vary b' on the horizontal axis. We include a vertical dashed line at $b' = b$ and mark with a red circle the equilibrium outcomes based on the sovereign's choice of b'. By borrowing more, the

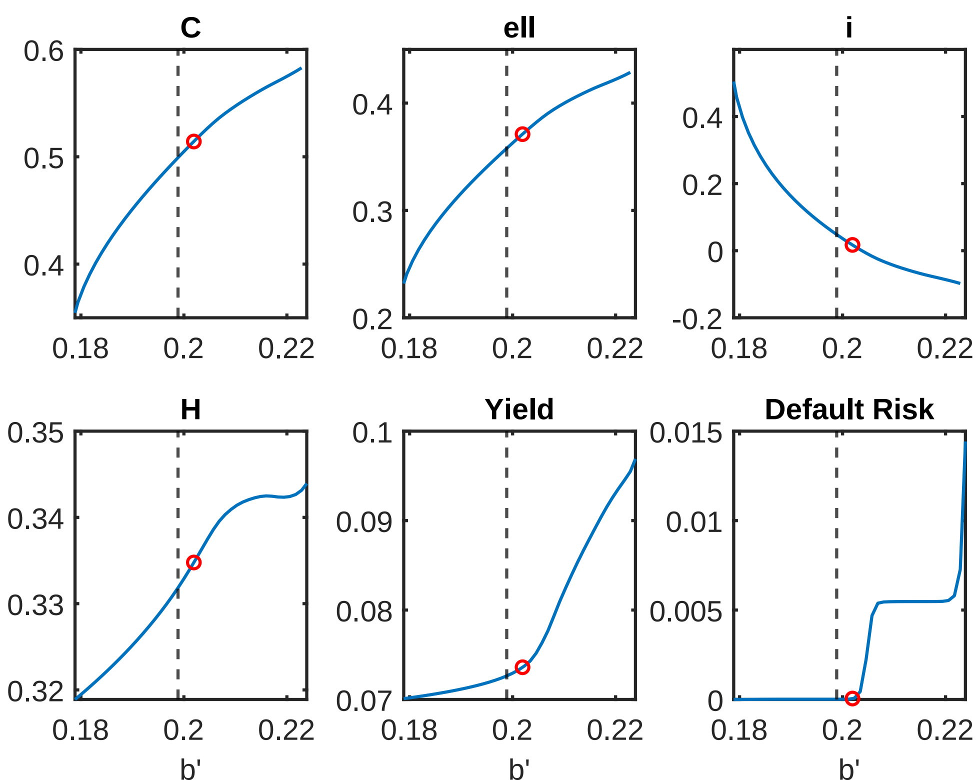


Figure 8: Domestic Outcomes, the Role of b'

sovereign and deliver a larger lump sum transfer to the household, increasing consumption this period, but the higher debt is associated with higher default risk (bottom right panel) and thus higher yields (bottom middle panel), as well as a higher expected marginal utility next period, with $H(s, b')$ increasing in b', in the bottom left panel. This expectation of high marginal utility in the future, i.e., low expected consumption, translate into a lower domestic interest rate this period, and i falls with higher b' (top right panel). The lower domestic rates stimulate employment and production, inducing *expansionary capital inflows* in our model. This further increases current consumption and depresses the domestic rate, until a fixed point is reached.

Figure 9 plots equilibrium outcomes, as a function of the initial debt level b, for shocks fixed at their mean levels, using the equilibrium b' policies of the sovereign. We place a vertical dashed line at the pseudo-steady-state associated with mean shock levels and focus on debt slightly lower and around this level. As debt increases, default becomes more likely, so that yields rise (bottom

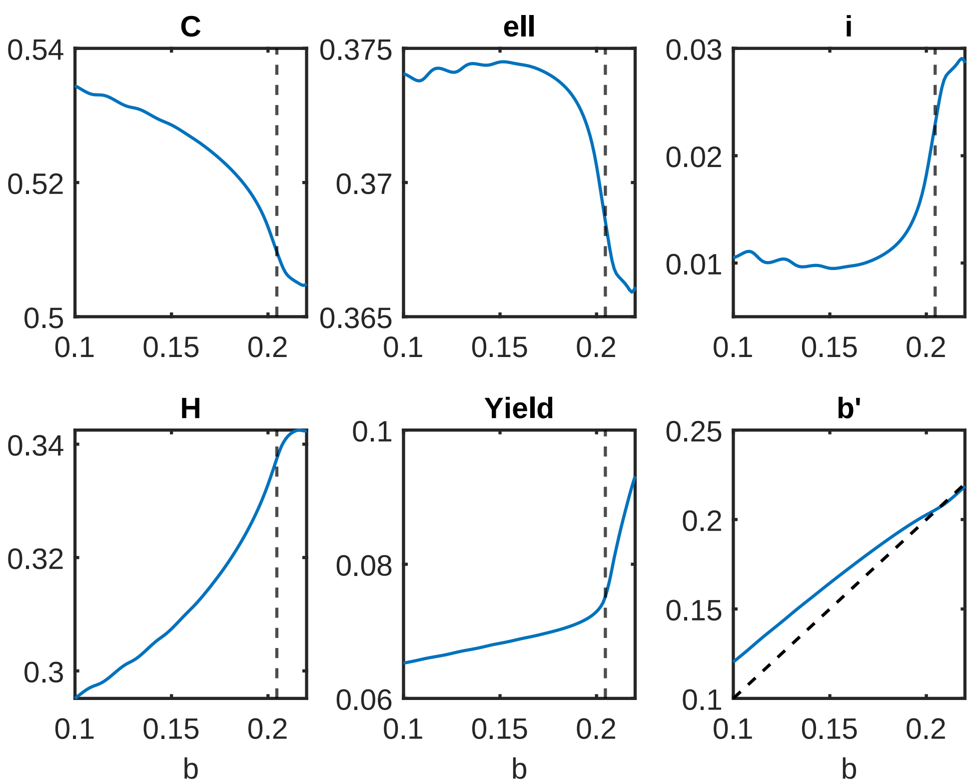


Figure 9: Equilibrium Outcomes, the Role of b

middle panel) and household come to expect higher marginal utility next period (bottom left panel).[7] As a result, domestic rates are high, production is depressed, and consumption falls (top panels).

7. Note that the figure plots $H(s, \mathcal{B}(s, b))$ and yields based on $q(s, \mathcal{B}(s, b))$, respectively.

4.4 The Role of Shock Persistence

[TODO]

5 Conclusion

[TODO]

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A Proof of Proposition 1

[TODO]

B Data Sources

We use the Shadow Rate of Wu and Xia (2016) as a measure for the default-risk-free rate in the US. We rely on National Accounts data from the IMF's IFS database. The EMBI+ spread series is from JP Morgan, via Bloomberg. [TODO]

C Endowment Model

The endowment economy has state variables $s = \langle r^{\text{rf}}, \nu, y \rangle$ and b. s are exogenous state variables, including the endowment y, the risk-free rate r^{rf}, and news ν. b is the debt level of the sovereign. The sovereign's policies are the default decision $\mathcal{D}(s, b)$ and the borrowing choice $\mathcal{B}(s, b)$. Bond pricing schedules are determined in the equilibrium with international lenders, $q(s, b')$ for the price at initial issuance and $q^d(s, b)$ for defaulted bonds, priced in secondary market.

The sovereign's value function are, at the start of the period,

$$V(s, b) = \max_{d \in \{0,1\}} \left\{ (1-d) \cdot V^r(s, b) + d \cdot V^d(s, b) \right\} \tag{33}$$

for repayment

$$V^r(s, b) = \max_{b'} \left\{ u \left[y - \kappa b + q(s, b')(b' - (1-\delta)b) \right] + \beta \mathbb{E}_{s'|s} V(s', b') \right\} \tag{34}$$

and in default, respectively,

$$V^d(s, b) = u \left[h\left(y \right) \right] + \beta \mathbb{E}_{s'|s} \left\{ (1-\lambda) \, V^d(s', b) + \lambda V(s', \phi b) \right\}. \tag{35}$$

As for our baseline model, we can facilitate the construction of the bond price schedules by introducing an auxiliary function

$$Q(s, b) = d(s, b) q^d(s, b) + [1 - d(s, b)] \left\{ \kappa + (1-\delta) q(s, \mathcal{B}(s, b)) \right\} \tag{36}$$

which allow us to write

$$q(s, b') = \frac{1}{1 + r^{\text{rf}}(s)} \mathbb{E}_{s'|s} Q(s', b') \tag{37}$$

and

$$q^d(s, b) = \frac{1}{1 + r^{\text{rf}}(s)} \mathbb{E}_{s'|s} \left\{ (1-\lambda) \, q^d(s', b) + \lambda \phi Q(s', \phi b) \right\}. \tag{38}$$

This model abstracts from domestic production and labor supply. The choice of GHH preferences in our baseline imply that, absent the financial friction, domestic production endogenously resembles an endowment economy, as labor supply monotonically amplified productivity shocks and is free from wealth effects. This aids the comparison with reference models in the literature.

D Computation

We compute our model with defaultable, long-term debt using discrete choice methods, following Mihalache (2020) and Dvorkin et al. (2021). We introduce Extreme Value Type I shocks for the choices over b' and d, so that ex-ante there is a choice probability function over b' levels, as well as an interior default probability for any ex-post state. These "taste shocks" facilitate convergence in models with defaultable long-term debt and smooth policies and values. The appendix of Arellano, Bai, and Mihalache (2020) provides additional discussion for these methods.

We describe here the taste shocks version of our endowment economy, from Appendix C, to keep notation simple. The full model follows an analogous strategy, with minor complications raised by the forward-looking functions H and H^d. With taste shocks, the default decision becomes

$$V(s,b) = \rho_D \log \left\{ \exp\left[\frac{V^d(s,b)}{\rho_D}\right] + \exp\left[\frac{V^r(s,b)}{\rho_D}\right] \right\} - \rho_D \cdot \mathrm{em} \tag{39}$$

where em is the Euler-Mascheroni constant, and the default probability after observing the state s but before observing the taste shocks is

$$d(s,b) = \frac{1}{1 + \exp\left[\dfrac{V^r(s,b) - V^d(s,b)}{\rho_D}\right]}. \tag{40}$$

The parameter ρ_D controls the amount of "noise" in the taste shock.

Analogously, for the new debt level, define the value for an arbitrary choice of b' as

$$W(s,b,b') = u\left[y - \kappa b + q\left(s,b'\right)\left(b' - (1-\delta)b\right)\right] + \beta \mathbb{E}_{s'|s} V(s',b') \tag{41}$$

and set $W(s,b,b') = -\infty$ for all b' value that yield negative consumption. Then,

$$V^r(s,b) = \rho_B \log \left\{ \sum_{b'} \exp\left[\frac{W(s,b,b')}{\rho_B}\right] \right\} - \rho_B \cdot \mathrm{em} \tag{42}$$

and the probability of choosing any particular $b' = x$ after observing s is given by

$$\Pr(b' = x|s,b) = \frac{\exp\left[\dfrac{W(s,b,x)}{\rho_B}\right]}{\sum_j \exp\left[\dfrac{W(s,b,j)}{\rho_B}\right]}. \tag{43}$$

The value of default is unaltered, as

$$V^d(s,b) = u\left[h(y)\right] + \beta \mathbb{E}_{s'|s}\left\{(1-\lambda)V^d(s',b) + \lambda V(s',\phi b)\right\}.$$

The choice probabilities for default and borrowing are reflected in the lenders' pricing of the debt. Define as before the "helper" function

$$Q(s,b) = d(s,b)q^d(s,b) + [1-d(s,b)]\left[\kappa + (1-\delta)\sum_{b'} \Pr(b'|s,b)q(s,b')\right] \tag{44}$$

and use it to write

$$q(s,b') = \frac{1}{1+r^{\mathrm{rf}}}\mathbb{E}_{s'|s}Q(s',b') \tag{45}$$

and

$$q^d(s,b) = \frac{1}{1+r^{\mathrm{rf}}}\mathbb{E}_{s'|s}\left\{(1-\lambda)q^d(s',b) + \lambda\phi Q(s',\phi b)\right\}. \tag{46}$$

Note that now, in the definition of Q, the continuation value is given by an expectation over future bond prices, across taste shock realization, and that d now takes value in $(0,1)$ rather than in $\{0,1\}$.

As ρ_B and ρ_D go to zero, we recover the original model, without taste shocks, albeit at the cost of poor convergence properties. $\rho_B > 0$ in particular is key for near-uniform convergence, while ρ_D can only play a quantitative role.