a) Base case: 1 = 1^2

Inductive step: For some k >= 1, 1 + 3 +...+(2k-1)=k^2, then for k + 1: 1 + 3 + ...+ (2k-1) + (2k + 1) = k^2 + (2k + 1) = (k + 1)^2, so the statement holds for k + 1, by induction it holds or all n belonging to the naturals

b) Let N0 = number of leaves, n1 = number of nodes with 1 child, N2 = number wih 2 children. In any nonempty rooted tree the numbr of edges equals the number of nodes minus 1, each edge comes from exactly one parent-child relation, so the number of edges also equals the totla number of children across all nodes: edges = N1 + 2*N2 and edges = (N0 + N1 + N2) -1. Equating the two together we get N2 = N0 – 1

c) Proof by contradiction: Supose G is diconnected. Let C be a component with k vertices, 1<= k <= n-1. Every vertex in C has all its neighbors inside C, so deg(v)<= k -1 for all v belonging to C. But by hypothesis deg(v) >= n/2, hence k-1 > = n/2. Applying the same argument to any other compone C' it also must have size a least n/2 + 1. Two disjoint components each of size at least n/2 + 1 cannot fit into n vertices which is a contradiction. Therefore G is connected

d) Adjency:

Node	Neighbors
1	2, 10
2	1, 11, 3, 5
3	2, 10, 9, 4
4	3, 7, 8
5	2, 9
6	7, 10
7	4, 6, 8
8	4, 7, 9
9	3, 5, 8
10	1, 3, 6
11	2

Layer	Nodes	Relation
0	{5}	Start node
1	{2, 9}	Neighbors of 5
2	{1, 3, 11, 8}	Neighbors of 2 and 9 not yet discovered
3	{4, 10, 7}	Neighbors of 1, 3, 8 not yet discovered
4	{6}	Neighbors of 4, 7, 10 not yet discovered
5		No new nodes

e) Layer 0: 5 -> Color A
Layer 1: 2,9 -> color B
Layer 2: 1,3,11,8 -> color A
Layer 3: 4, 10, 7 -> color B
Layer 4: 6 -> color A
Since there is an edge between nodes 4 and 7 and they both have color B he graph is not bipartite.