1. Base case: 1 = 1^2

Inductive step: For some k >= 1, 1 + 3 +…+(2k-1)=k^2, then for k + 1: 1 + 3 + …+ (2k-1) + (2k + 1) = k^2 + (2k + 1) = ( k + 1)^2, so the statement holds for k + 1, by induction it holds or all n belonging to the naturals

1. Let N0 = number of leaves, n1 = number of nodes with 1 child, N2 = number wih 2 children. In any nonempty rooted tree the numbr of edges equals the number of nodes minus 1, each edge comes from exactly one parent-child relation, so the number of edges also equals the totla number of children across all nodes: edges = N1 + 2\*N2 and edges = (N0 + N1 + N2) -1. Equating the two together we get N2 = N0 – 1
2. Proof by contradiction: Supose G is diconnected. Let C be a component with k vertices, 1<= k <= n-1. Every vertex in C has all its neighbors inside C, so deg(v)<= k -1 for all v belonging to C. But by hypothesis deg(v) >= n/2, hence k-1 > = n/2. Applying the same argument to any other compone C’ it also must have size a least n/2 + 1. Two disjoint components each of size at least n/2 + 1 cannot fit into n vertices which is a contradiction. Therefore G is connected
3. Adjency:

|  |  |
| --- | --- |
| Node | Neighbors |
| 1 | 2, 10 |
| 2 | 1, 11, 3, 5 |
| 3 | 2, 10, 9, 4 |
| 4 | 3, 7, 8 |
| 5 | 2, 9 |
| 6 | 7, 10 |
| 7 | 4, 6, 8 |
| 8 | 4, 7, 9 |
| 9 | 3, 5, 8 |
| 10 | 1, 3, 6 |
| 11 | 2 |
|  |  |

| Layer | Nodes | Relation |
| --- | --- | --- |
| 0 | {5} | Start node |
| 1 | {2, 9} | Neighbors of 5 |
| 2 | {1, 3, 11, 8} | Neighbors of 2 and 9 not yet discovered |
| 3 | {4, 10, 7} | Neighbors of 1, 3, 8 not yet discovered |
| 4 | {6} | Neighbors of 4, 7, 10 not yet discovered |
| 5 |  | No new nodes |

1. Layer 0: 5 -> Color A

Layer 1: 2,9 -> color B

Layer 2: 1,3,11,8 -> color A

Layer 3: 4, 10, 7 -> color B

Layer 4: 6 -> color A

Since there is an edge between nodes 4 and 7 and they both have color B he graph is not bipartite.