# ELEC353 Lecture Notes Set 13

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #7: Do homework #7 by March 8, 2019.
Homework #8: Do homework #8 by March 15, 2019.
Homework #9: Do homework #9 by March 22, 2019.

Tutorial Workshop #8: Friday March 8, 2019.
Tutorial Workshop #9: Friday March 15, 2019.
Tutorial Workshop #10: Friday March 22, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

# Topics to be Covered

Transmission Lines (TLs)

- Wave Equation and Solution - done
- Solving a TL Circuit - done
- Standing Wave Patterns - done
- Impedance Matching – today's class
- Bandwidth of Digital Signal

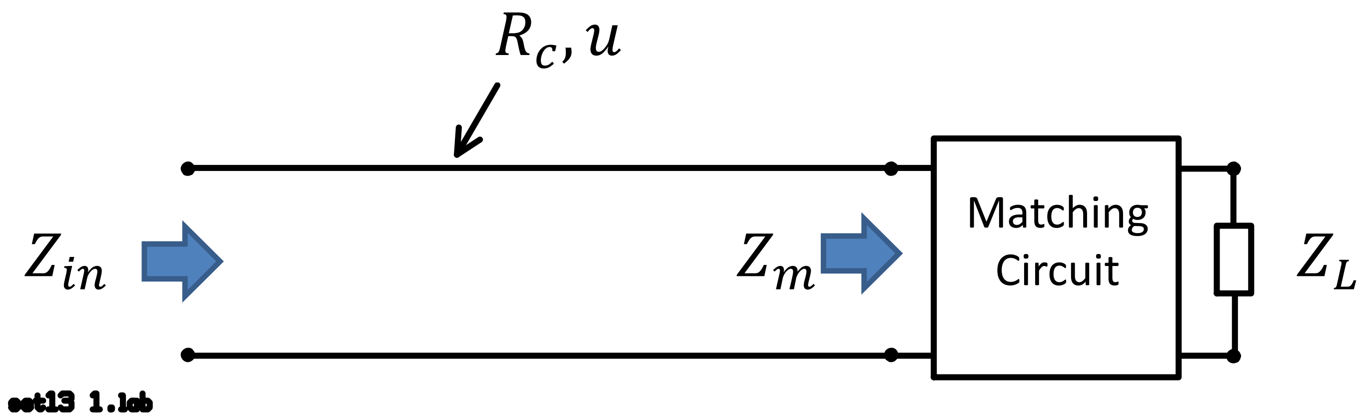
Plane Waves

- Maxwell's Equations and the Wave Equation
- Plane waves
- Material Boundaries
- Transmission Through a Wall

Antennas

# Impedance Matching

Inan and Inan Section 3.5



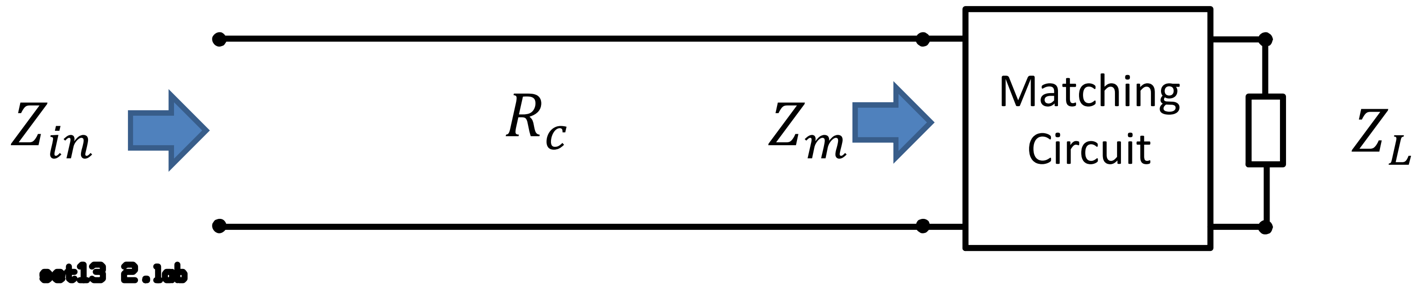

What can we use for the "matching circuit"?

- Quarter wave transformer
- Line plus quarter wave transformer
- Many other schemes are available!

The matching circuit is designed to provide a perfect match at the "center frequency" $f_0$ .

The **bandwidth** of the match is the range of frequencies over which the match is "sufficiently good".

# Return Loss



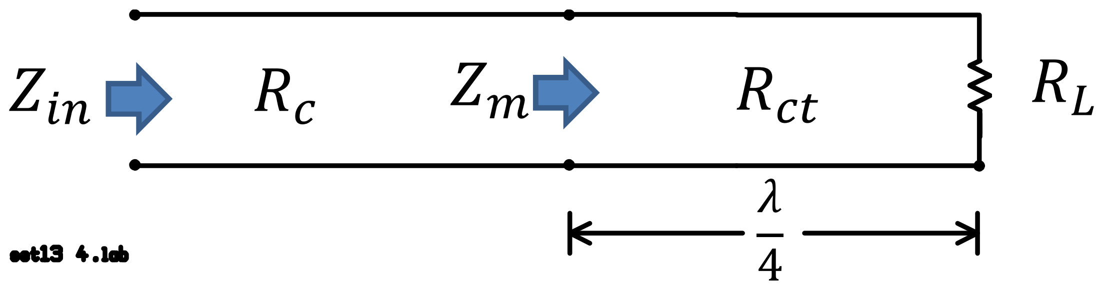

$$\Gamma_m(f) = \frac{Z_m(f) - R_c}{Z_m(f) + R_c}$$

- The "return loss" is defined as

  $R.L. = 20\log|\Gamma_m|$ decibels or "dB".
- For consumer electronics, the maximum reflection coefficient is often set at $|\Gamma_{max}| = 0.316$, so R.L. $\leq -10$ dB.
- For precision applications, $|\Gamma_{max}| = 0.1$ is often used, so R.L. ≤=-20 dB.
- The bandwidth is defined as the frequency range over which the reflection coefficient meets the standard that $|\Gamma_m| \leq |\Gamma_{max}|$.
- Connectors have a return loss of -30 to -40 dB.
- A "matched load" has a return loss of about -40 dB.

# Quarter-Wave Transformer

Inan and Inan Section 3.5.3




cet13 4.lab

$$Z_m = R_{ct}\frac{Z_L + jR_{ct}\tan\beta L}{R_{ct} + jZ_L\tan\beta L} = R_{ct}\frac{R_L + jR_{ct}\tan\beta L}{R_{ct} + jR_L\tan\beta L}$$

$$L = \frac{\lambda}{4} \qquad \beta L = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \qquad \tan\beta L = \tan\frac{\pi}{2} \to \infty$$

$$Z_m = R_{ct}\lim_{\tan\beta L \to \infty}\frac{R_L + jR_{ct}\tan\beta L}{R_{ct} + jR_L\tan\beta L}$$

$$Z_m = R_{ct}\lim_{\tan\beta L \to \infty}\frac{jR_{ct}\tan\beta L}{jR_L\tan\beta L} = \frac{R_{ct}^2}{R_L}$$

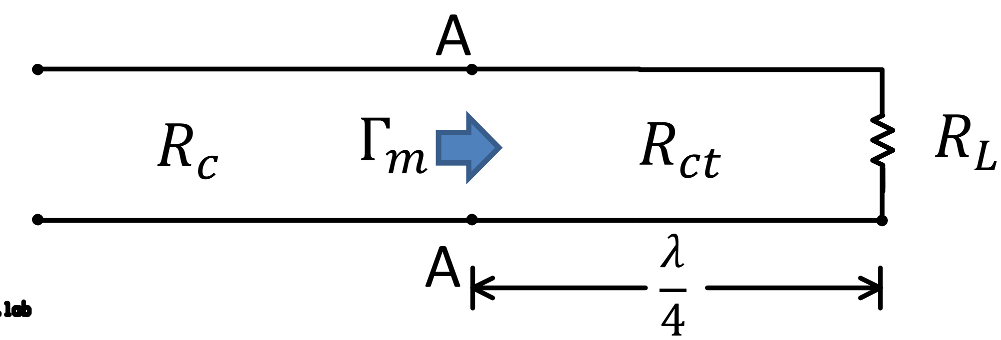
so $Z_m = \frac{R_{ct}^2}{R_L}$

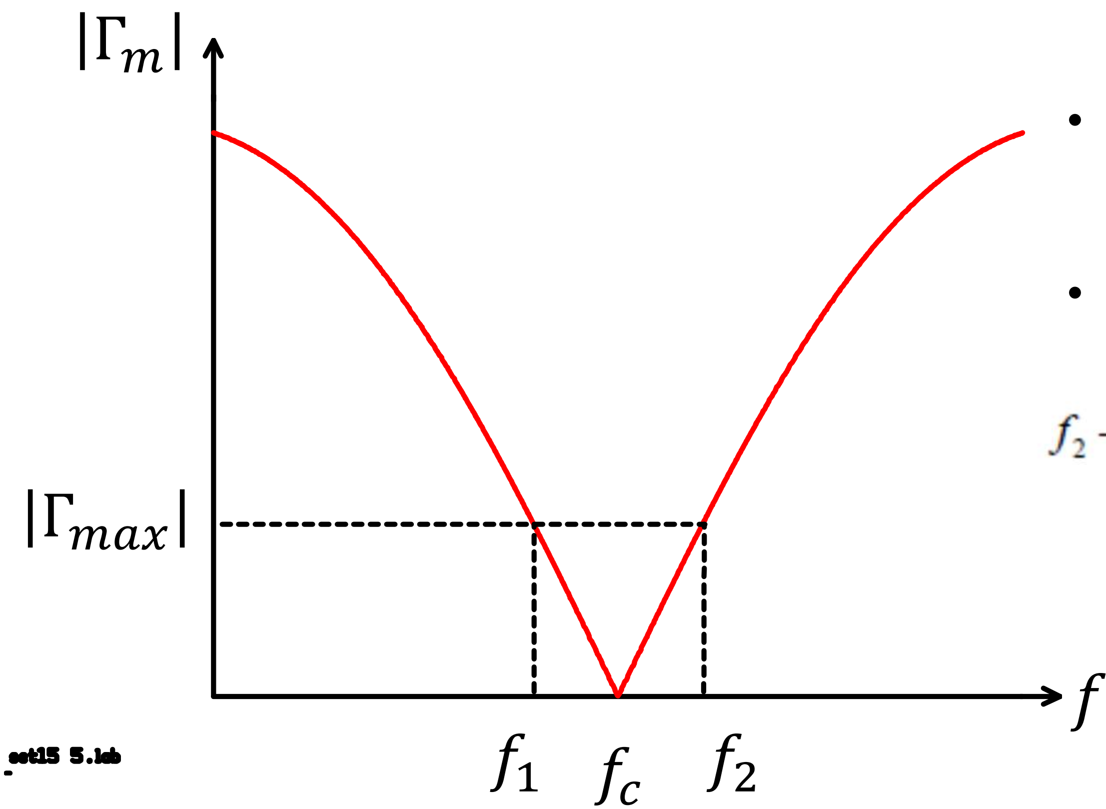
We want $Z_m = R_c$ so $\frac{R_{ct}^2}{R_L} = R_c$ so choose $R_{ct} = \sqrt{R_c R_L}$

# What is the bandwidth?

At terminals AA:

$$\Gamma_m(f) = \frac{Z_m(f) - R_c}{Z_m(f) + R_c}$$



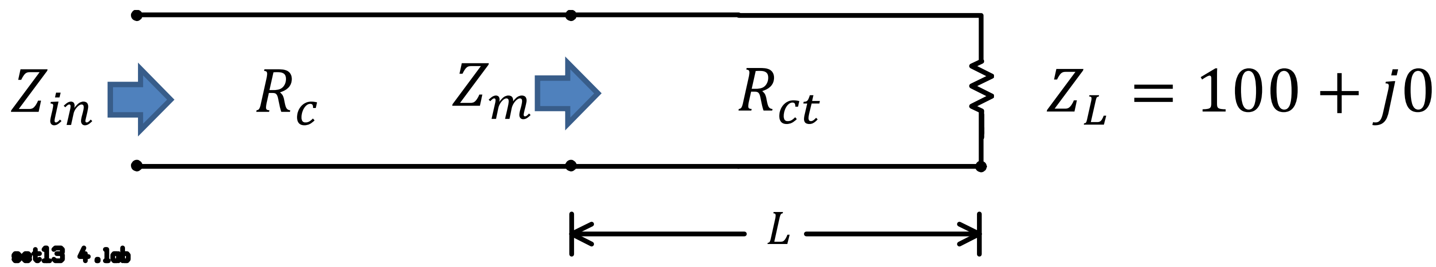




- The bandwidth is the frequency range for which $|\Gamma_m| \leq |\Gamma_{max}|$,
  $$BW = f_2 - f_1$$
- Formula for the bandwidth:

$$f_2 - f_1 \approx f_c\left[2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{R_c R_L}}{|R_L - R_c|}\right)\right]$$

David Pozar, "Microwave Engineering", Addison-Wesley, 1990.

# Example: quarter-wave transformer




At 850 MHz, an antenna with an input impedance of $Z_L = 100 + 0j$ ohms must be matched to a transmission line of characteristic impedance $R_c = 50$ ohms. The speed of travel on the transmission line is $u = 30$ cm/ns.

1. Design a quarter-wave transformer by choosing the length $L$ and the characteristic impedance $R_{ct}$.
2. Use Pozar's formula to find the bandwidth for a return loss of 20 dB or better.
3. Verify that your design works with TRLINE.
4. Use TRLINE to find the bandwidth of the match and compare with Pozar's value.

# Solution

$$f_c = 850 \text{ MHz} \qquad \lambda = \frac{u}{f_c} = \frac{300}{850} = 35.29 \text{ cm.}$$

$$L = \frac{\lambda}{4} = \frac{35.29}{4} = 8.82 \text{ cm.}$$

$$R_{ct} = \sqrt{R_c R_L} = \sqrt{50 \cdot 100} = 70.71 \text{ ohms.}$$

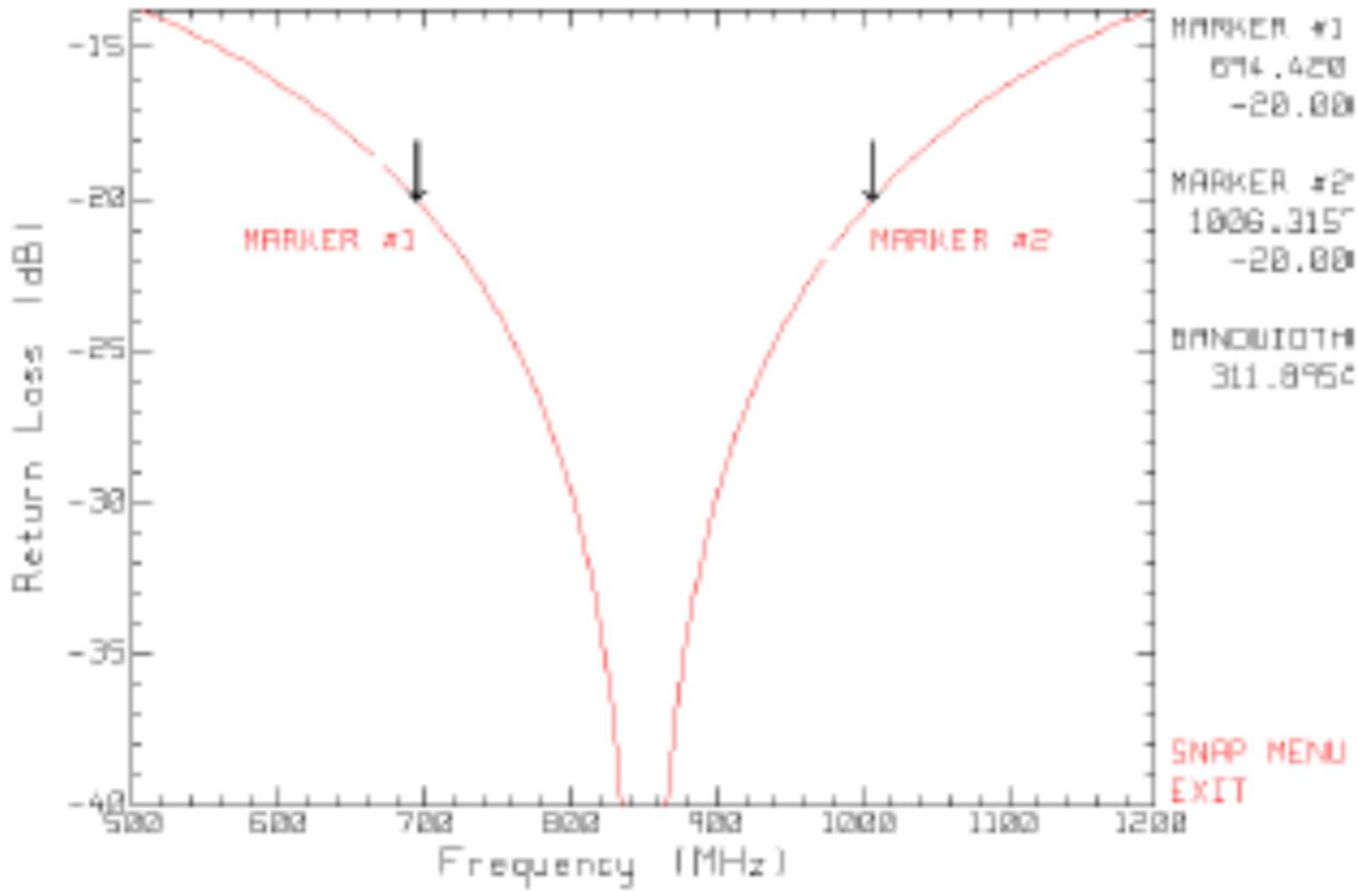
return loss of 20 dB $\qquad -20\log\Gamma_m = 20 \text{ dB} \qquad \Gamma_m = 0.1$

$$f_2 - f_1 \approx f_c\left[2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{R_c R_L}}{|R_L - R_c|}\right)\right]$$

$$f_2 - f_1 \approx 850\left[2 - \frac{4}{\pi}\cos^{-1}\left(\frac{0.1}{\sqrt{1-0.1^2}}\frac{2\cdot 70.71}{|100-50|}\right)\right]$$

$$f_2 - f_1 \approx 850\left[2 - \frac{4}{\pi}\cdot 1.28253\right] = 311.95 \text{ MHz}$$

# Use TRLINE to find the bandwidth

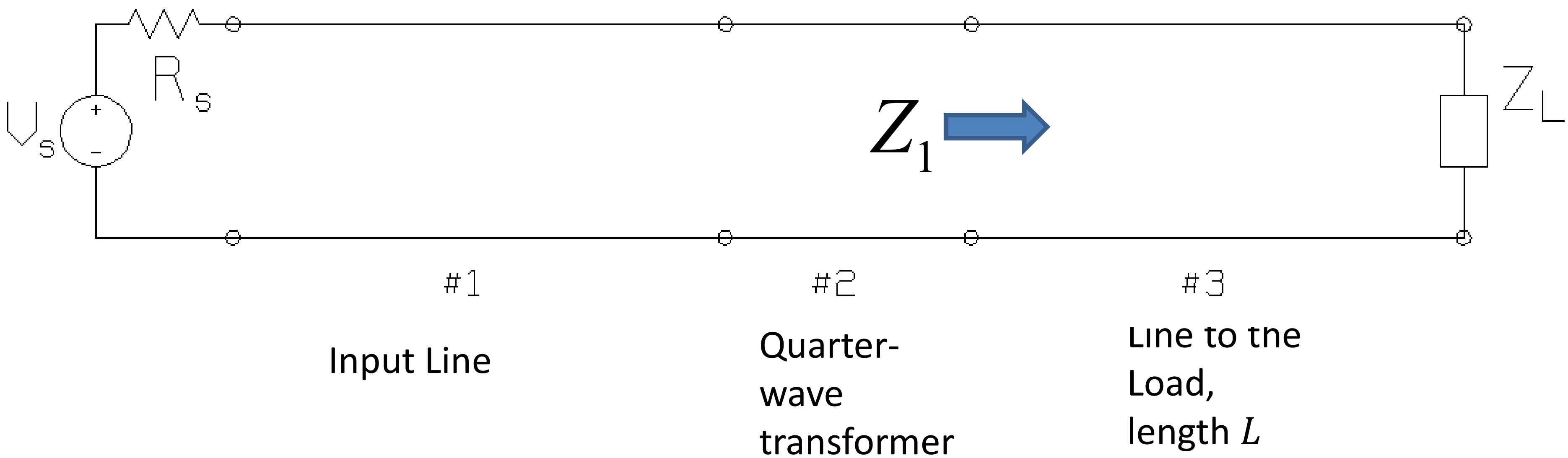



This agrees well with the Pozar formula, $f_2 - f_1 \approx 311.95$ MHz.

# Matching a Complex-Valued Load

## How do we match a complex-valued load?

Match a load of $Z_L$=100-j45 ohms to a transmission line with $R_c$=50 ohms at 850 MHz.  The speed of travel on the transmission lines is $u$=30 cm/ns.



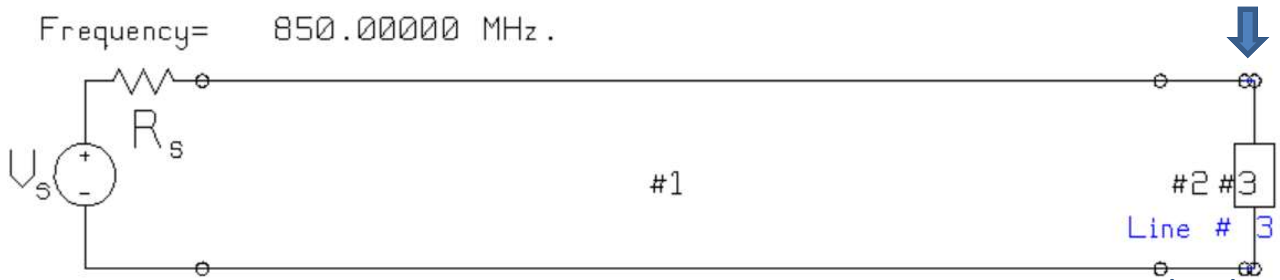

**Idea: put a transmission line in series with the load and adjust the length to get a real-valued input impedance.**

# Trial and Error Solution

The transformer length is $\frac{\lambda}{4} = \frac{u/f}{4} = \frac{300/850}{4} = 0.0882$ m

Start with line #3 short and gradually increase the length. Monitor $Z_1$, the input impedance of line #3.

Line #3 is very short.




$Z_L = 100 - j45$

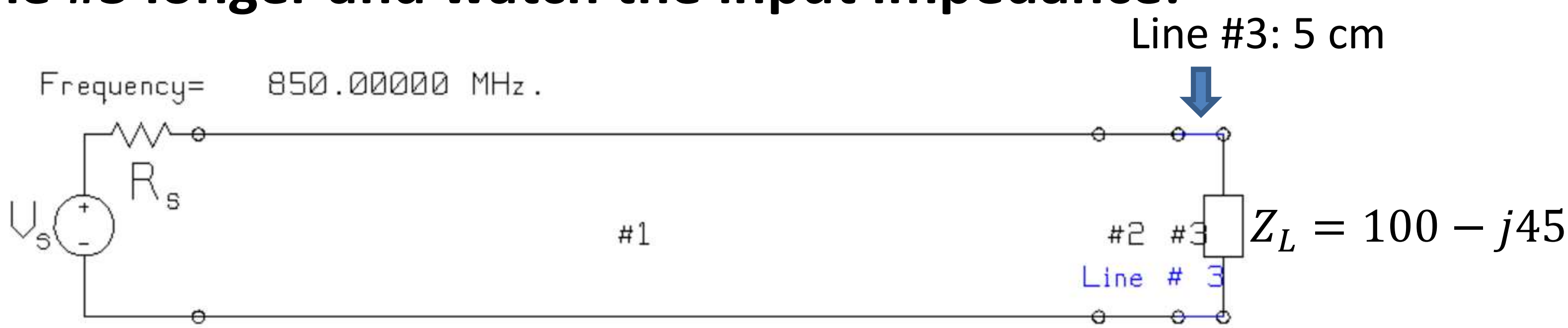
Quarter-wave transformer

```
Transmission Line #   3:
V+=500.0 angle -30.0;V-=214.8 angle -75.7 millivolts RMS.
At the input:
Voltage = 668.0 millivolts RMS, phase = -43.3  degrees.
Current = 7.644 milliamps RMS, phase = -6.3  degrees.
Power = 4.077 milliwatts.
Impedance =  69.774-j52.595  ohms
At the output:
Voltage = 700.2 millivolts RMS, phase = -47.7  degrees.
Current = 6.386 milliamps RMS, phase = -23.5  degrees.
Power = 4.077 milliwatts.
Impedance =  100.000-j45.000  ohms
```

The input impedance of line #3: $Z_1$

$Z_1 = 69.774 - j52.595$

# Make line #3 longer and watch the input impedance:



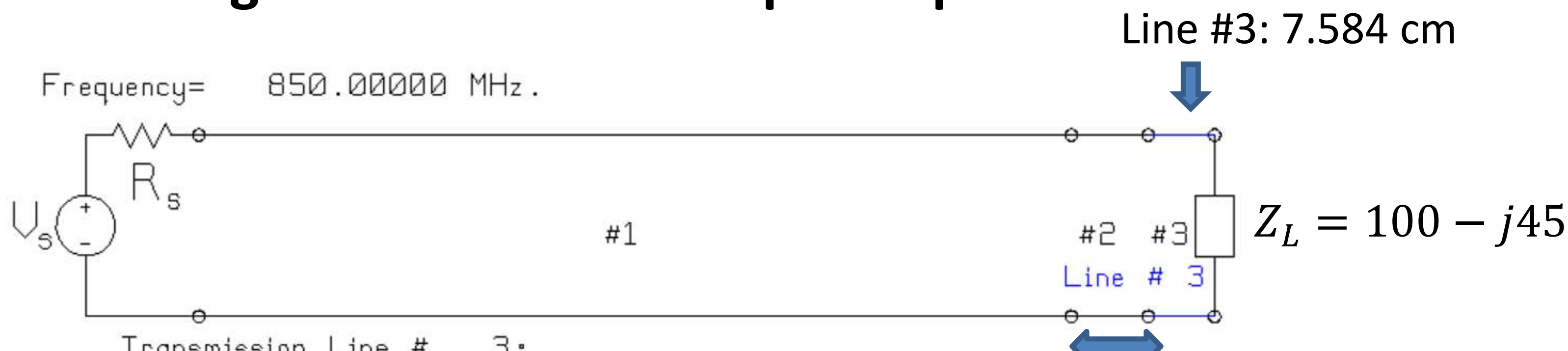

Quarter-wave transformer

```
Transmission Line #   3:
V+=500.0 angle -30.0;V-=214.8 angle -157.3 millivolts RMS.
At the input:
Voltage = 407.4 millivolts RMS, phase = -54.8  degrees.
Current = 13.06 milliamps RMS, phase = -14.8  degrees.
Power = 4.077 milliwatts.
Impedance =  23.915-j20.044   ohms
At the output:
Voltage = 700.2 millivolts RMS, phase = -88.5  degrees.
Current = 6.386 milliamps RMS, phase = -64.3  degrees.
Power = 4.077 milliwatts.
Impedance =  100.000-j45.000   ohms
```

$Z_1$

$$Z_1 = 23.915 - j20.044$$

# Make line #3 longer and watch the input impedance:




```
Transmission Line #   3:
V+=500.0 angle -30.0;V-=214.8 angle 150.0 millivolts RMS.
At the input:
Voltage = 285.2 millivolts RMS, phase = -30.0  degrees.
Current = 14.30 milliamps RMS, phase = -30.0  degrees.
Power = 4.077 milliwatts.
Impedance =  19.953+j0.001  ohms
At the output:
Voltage = 700.2 millivolts RMS, phase = -114.8  degrees.
Current = 6.386 milliamps RMS, phase = -90.6  degrees.
Power = 4.077 milliwatts.
Impedance =  100.000-j45.000  ohms
```

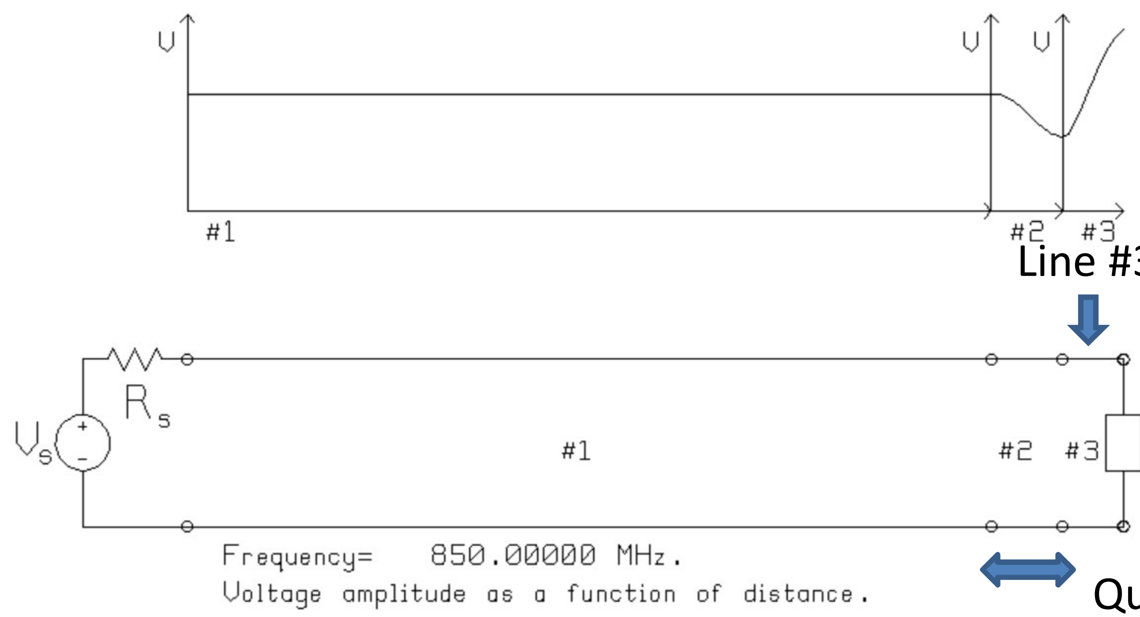
$Z_1$

$$Z_1 = 19.953 + j0.001$$

# Design the quarter-wave transformer:

Choose $R_{ct} = \sqrt{R_c R_L} = \sqrt{50x19.953} = 31.59$ ohms

Line # 1 VSWR= 1.0006
Line # 2 VSWR= 1.5833
Line # 3 VSWR= 2.5059



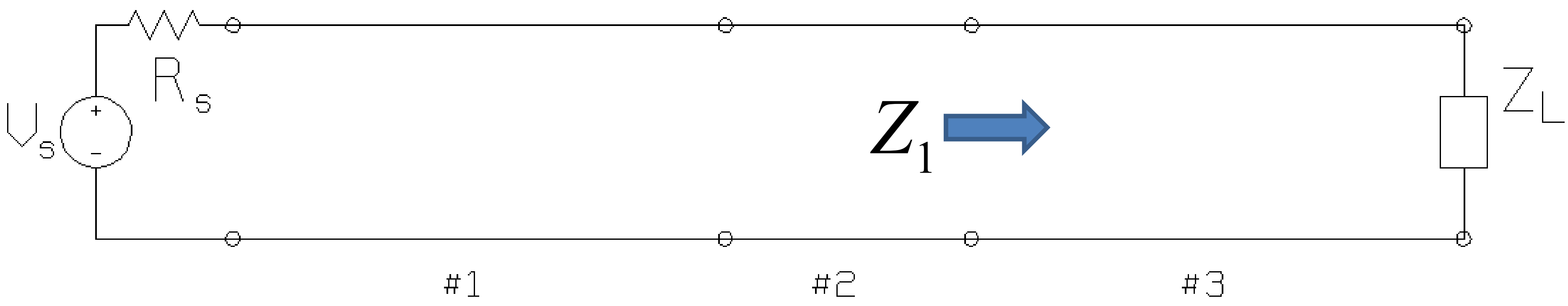

Line #3: 7.584 cm

$Z_L = 100 - j45$

Quarter-wave Transformer: 31.59 ohms

The voltage on the input transmission line (line #1) is constant with position. There is (almost) no reflected wave and the VSWR is 1.0006.

# Matching a Complex-Valued Load




Input Line

Quarter-wave transformer

Line to the Load, length $L$

IDEA:

•Put a transmission line in series with the load.

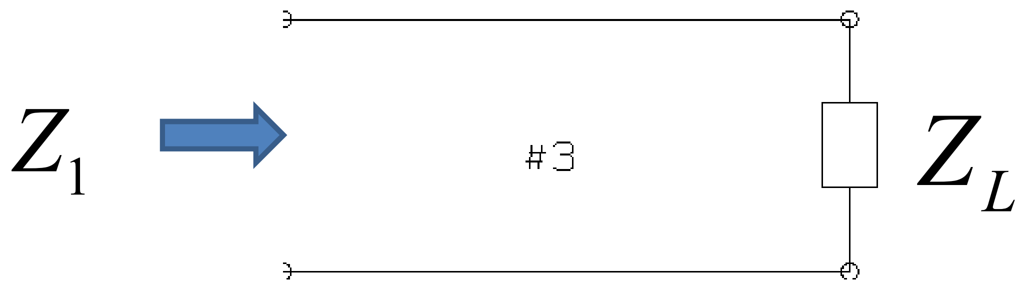
•Choose the length $L$ of line #3 so that the input impedance is resistive:

$$Z_1 = R_1 + j0$$

•Design a quarter-wave transformer to match $R_1$

$$Z_t = \sqrt{Z_0 R_1}$$

# Choose the line length so that $Z_1$ is real:




From a previous class (set 11):

$$Z(z) = \frac{V(z)}{I(z)} = R_c \frac{1 + \Gamma_L e^{-j2\beta L} e^{j2\beta z}}{1 - \Gamma_L e^{-j2\beta L} e^{j2\beta z}}$$

At the input at $z = 0$: Define $\Gamma_1 = \Gamma_L e^{-j2\beta L}$

$$Z_1 = R_c \frac{1 + \Gamma_L e^{-j2\beta L}}{1 - \Gamma_L e^{-j2\beta L}} = R_c \frac{1 + \Gamma_1}{1 - \Gamma_1}$$

where

$$\Gamma_1 = \Gamma_L e^{-j2\beta L}$$

If $\Gamma_1$ is real then $Z_1$ is real. Choose $L$ to make $\Gamma_1$ real:

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

then

$$\Gamma_1 = \Gamma_L e^{-j2\beta L} = |\Gamma_L| e^{j(\phi - 2\beta L)}$$

Make $\Gamma_1$ real:

$$\Gamma_1 = |\Gamma_L| e^{j(\phi - 2\beta L)}$$

$$\Gamma_1 = |\Gamma_L| \cos(\phi - 2\beta L) + j|\Gamma_L| \sin(\phi - 2\beta L)$$

- Make the imaginary part zero:

$$\sin(\phi - 2\beta L) = 0$$

$$\phi - 2\beta L = \pm n\pi$$

$$L = \frac{\phi \mp n\pi}{2\beta}$$

•This formula gives two line lengths that are different by a quarter of a wavelength.

# Find the input resistance $Z_1 = R_1$+j0, with $L = \dfrac{\phi \mp n\pi}{2\beta}$

From a previous slide:

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

$$Z_1 = R_c \frac{1 + \Gamma_L e^{-j2\beta L}}{1 - \Gamma_L e^{-j2\beta L}} = R_c \frac{1 + |\Gamma_L| e^{j\phi} e^{-j2\beta L}}{1 - |\Gamma_L| e^{j\phi} e^{-j2\beta L}}$$

$$Z_1 = R_c \frac{1 + |\Gamma_L| e^{j(\phi - 2\beta L)}}{1 - |\Gamma_L| e^{j(\phi - 2\beta L)}}$$

$$Z_1 = R_c \frac{1 + |\Gamma_L| \cos(\phi - 2\beta L) + j|\Gamma_L| \sin(\phi - 2\beta L)}{1 - |\Gamma_L| \cos(\phi - 2\beta L) - j|\Gamma_L| \sin(\phi - 2\beta L)}$$

$$Z_1 = R_c \frac{1+|\Gamma_L|\cos(\phi-2\beta L)+j|\Gamma_L|\sin(\phi-2\beta L)}{1-|\Gamma_L|\cos(\phi-2\beta L)-j|\Gamma_L|\sin(\phi-2\beta L)}$$

We chose $\phi - 2\beta L = \pm n\pi$ so $\sin(\phi-2\beta L) = \sin(\pm n\pi) = 0$

$$\cos(\phi-2\beta L) = \cos(\pm n\pi) = \pm 1$$

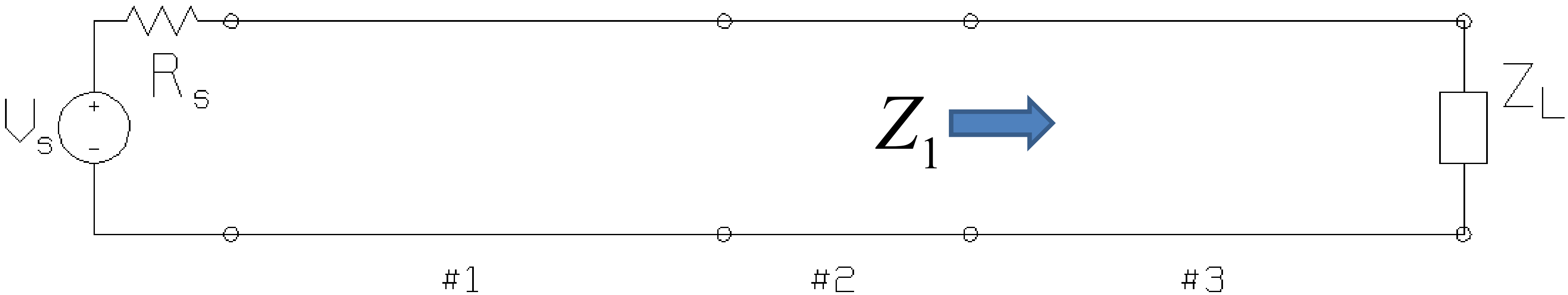
$$Z_1 = R_c \frac{1+|\Gamma_L|(\pm 1)}{1-|\Gamma_L|(\pm 1)}$$

$$Z_1 = R_c \frac{1 \pm |\Gamma_L|}{1 \mp |\Gamma_L|}$$

We choose $L$ to make $Z_1$ is real, $Z_1 = R_1 + j0$:

$$R_1 = R_c \frac{1 \pm |\Gamma_L|}{1 \mp |\Gamma_L|}$$

Design procedure: choose $L$ and $Z_t$




Input Line

Quarter-wave transformer
$L_t, Z_t$

Line to the Load, of length $L$

Reflection coefficient at the load: $\Gamma_L = |\Gamma_L| e^{j\phi}$

- Step 1: Choose the line length: $$L = \frac{\phi \mp n\pi}{2\beta}$$
- Find the input resistance $$R_1 = R_c \frac{1 + |\Gamma_L| \cos(\phi - 2\beta L)}{1 - |\Gamma_L| \cos(\phi - 2\beta L)}$$
- Find the transformer characteristic impedance: $Z_t = \sqrt{Z_0 R_1}$
- Find the transformer length, $L_t = {}^{\lambda}/_{4}$

## Example

Match a load of $Z_L$=100-j45 ohms to a transmission line with $R_c$=50 ohms at 850 MHz. The speed of travel on the transmission lines is $u$=30 cm/ns.

## Solution

Find the reflection coefficient:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - j45 - 50}{100 - j45 + 50} = \frac{50 - j45}{150 - j45} \frac{67.28\angle - 42.0°}{156.6\angle - 16.7°} = 0.4295\angle - 25.3°$$

$$\left|\Gamma_L\right| = 0.4295 \qquad \phi = \angle\Gamma_L = -25.3°$$

Find the phase constant:

$$\lambda = \frac{u}{f} = \frac{300}{850} = 0.3529 \text{ meters}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{360}{\lambda} = 1020 \qquad \text{degrees per meter}$$

Find the line length $L$:

$$L = \frac{\phi \mp n\pi}{2\beta} = \frac{-25.3 \mp 180n}{2(1020)}$$

$L = 7.584$ cm, and $L = 16.41$ cm.

For L=7.584 cm, find $R_1$ using the formula with cosine, so that the correct sign is obtained:

$$R_1 = R_c \frac{1 + |\Gamma_L| \cos(\phi - 2\beta L)}{1 - |\Gamma_L| \cos(\phi - 2\beta L)}$$

$|\Gamma_L| = 0.4295$

$\phi = \angle\Gamma_L = -25.3°$

$$\beta L = 1020x0.07584 = 77.36$$

$$\cos(\phi - 2\beta L) = \cos(-25.3° - 2x77.36) = \cos(-180.02) = -1$$

$$R_1 = 50\frac{1 + 0.4295x(-1)}{1 - 0.4295x(-1)} = 19.95$$

For L=16.41 cm:

$$\beta L = 1020x0.1641 = 167.38°$$

$$\cos(\phi - 2\beta L) = \cos(-25.3° - 2x167.38) = \cos(-360.06) = 1$$

$$R_1 = 50\frac{1+0.4295x(+1)}{1-0.4295x(+1)} = 125.3$$

Hence,

For L=7.584 cm, $R_1 = 19.95$ so $Z_t = \sqrt{Z_0 R_1} = \sqrt{50x19.95} = 31.58$

For L=16.41 cm, $R_1 = 125.3$ so $Z_t = \sqrt{Z_0 R_1} = \sqrt{50x125.3} = 79.15$

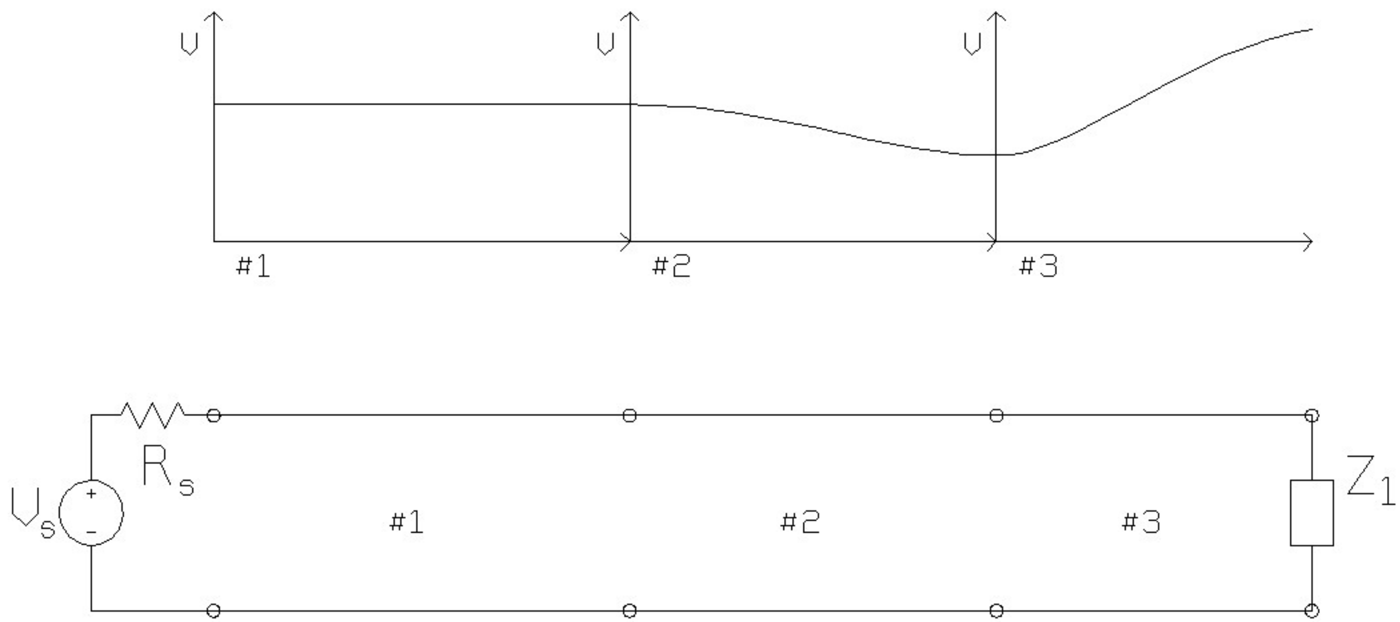
The wavelength is $\lambda = 0.3529$

The transformer length is $L_t = \frac{\lambda}{4} = \frac{0.3529}{4} = 0.0882$ m

# Use TRLINE to verify the design:

For L=7.584 cm, $R_1 = 19.95$ so $Z_t = \sqrt{Z_0 R_1} = \sqrt{50x19.95} = 31.58$

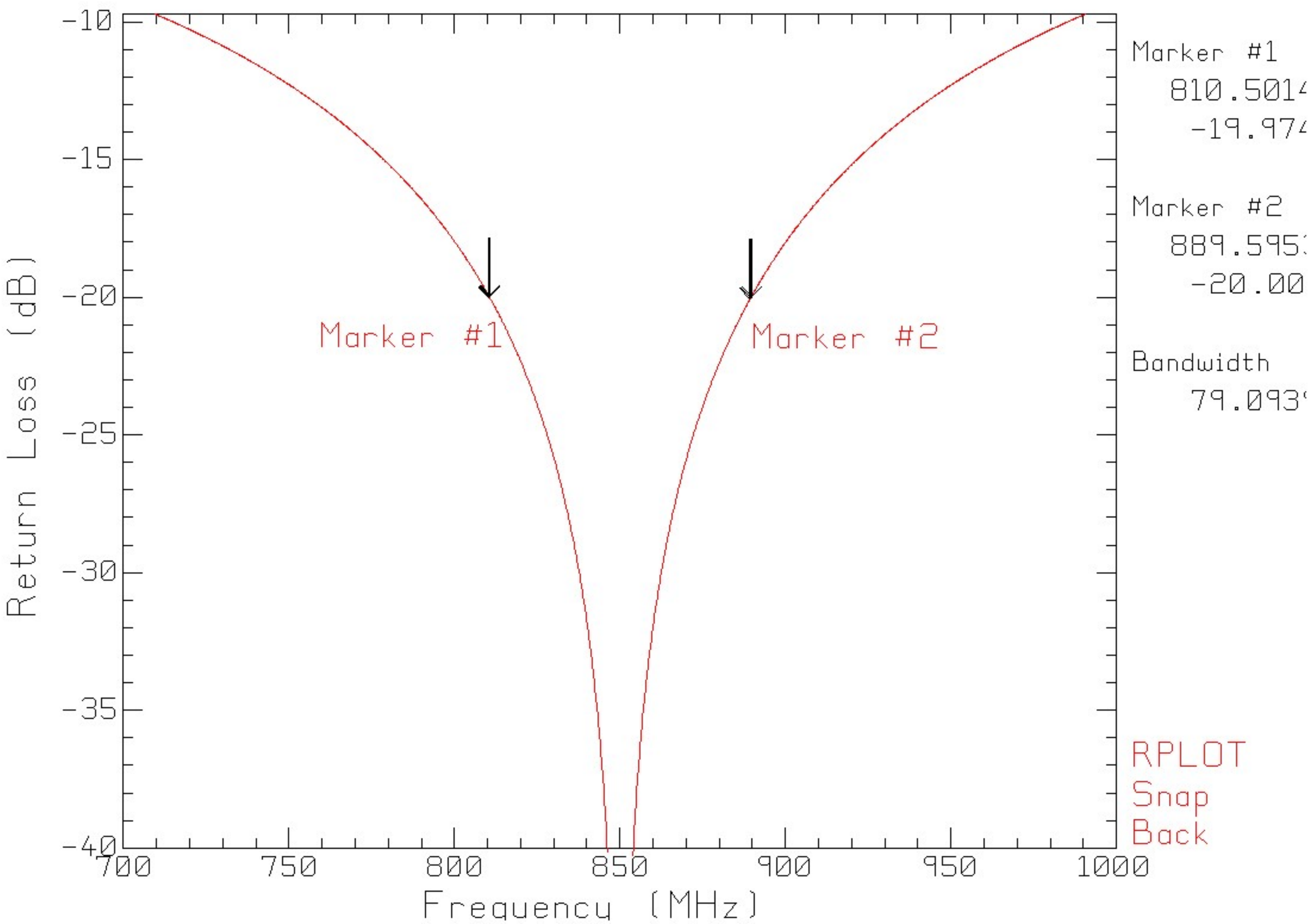
Line # 1 VSWR= 1.0007
Line # 2 VSWR= 1.5828
Line # 3 VSWR= 2.5059




Frequency= 850.000 MHz.
Voltage amplitude as a function of distance.

Back

# Use TRLINE to find the bandwidth:




For a return loss of 20 dB or better, the bandwidth is 79.0 MHz.