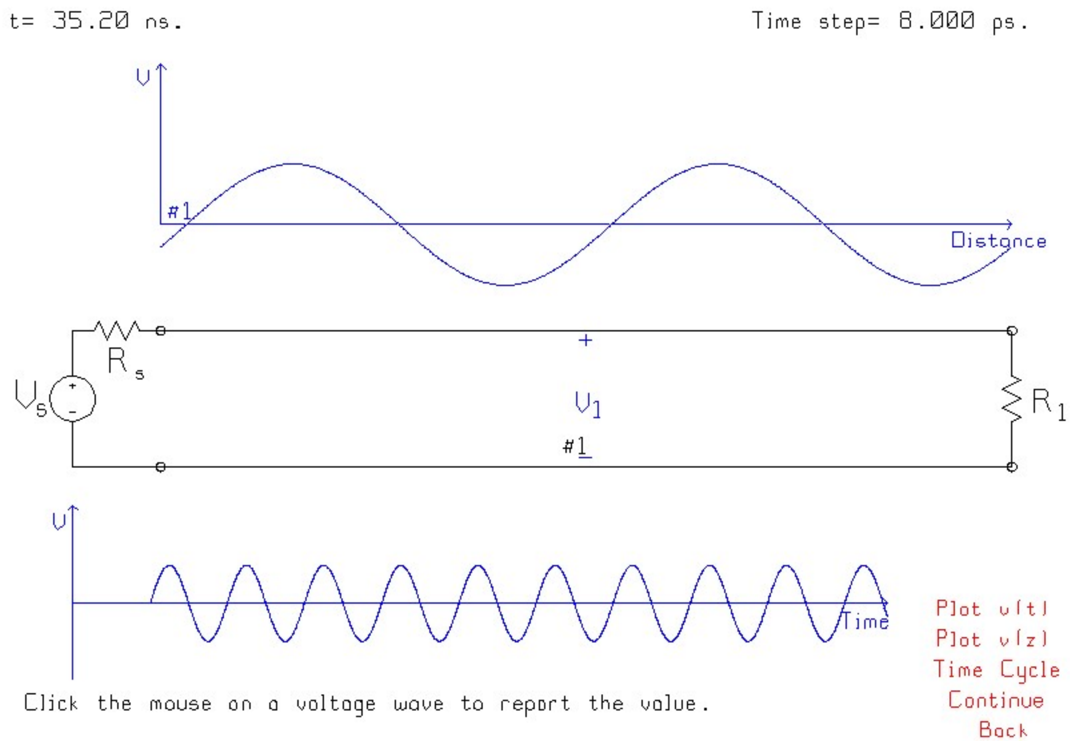
ELEC353 Lecture Notes Set 9

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #6: Do homework #6 by February 21, 2019.

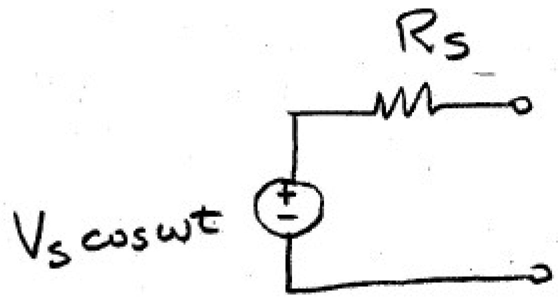
Tentative final exam date: Tuesday April 23, 2019, 9:00 to 12:00.

Transition to the Sinusoidal Steady State



- Travelling wave
- Reflection from an unmatched load
- Standing wave

A.C. Circuit Analysis



$$v(t) = V_s \cos \omega t$$

- The "amplitude" of the voltage is V_s .
- The "RMS value" of the voltage is $\dfrac{V_s}{\sqrt{2}}$.
- The frequency of operation is f Hertz, and the "radian frequency" is $\omega = 2\pi f$.

- In an LTI system, when the generator is sinusoidal, then at "steady state" all the voltages are sinusoidal *at the same frequency as the generator* and have the form

$$v(t) = A\cos(\omega t + \phi)$$

where

A is the *amplitude* of the voltage
ϕ is the *phase* of the voltage

Phasor Representation

$$v(t) = A\cos(\omega t + \phi) \quad \Longleftrightarrow \quad V = Ae^{j\phi}$$

The **magnitude** of the phasor is the **amplitude** of the cosine.
The **angle** of the phasor is the **phase angle** of the cosine.

We can "recover" the cosine wave $v(t)$ from the phasor V with the formula

$$v(t) = \mathrm{Re}(Ve^{j\omega t})$$

where the function $\mathrm{Re}(\ldots)$ means "take the real part of", so

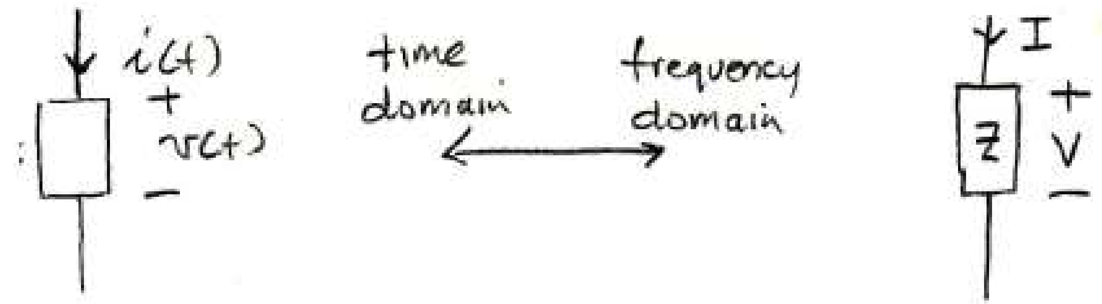
$$v(t) = \mathrm{Re}(Ae^{j\phi}e^{j\omega t}) = \mathrm{Re}(Ae^{j(\omega t+\phi)}) = \mathrm{Re}[A\cos(\omega t + \phi) + jA\sin(\omega t + \phi)]$$

and taking the real part

$$v(t) = A\cos(\omega t + \phi)$$

A.C. Circuit Analysis Notes
The course web site includes a set of notes entitled "AC Circuit Analysis Notes" which summarize AC circuit analysis and include a solved problem.

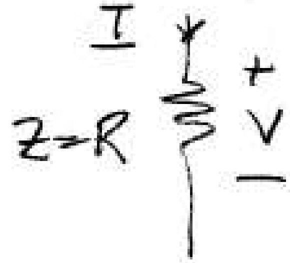
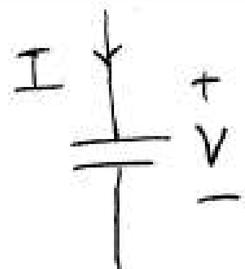
Impedance



- Let the voltage across a component be $v(t) = V_o \cos(\omega t + \theta)$ so that the phasor representing the voltage is $V = V_o e^{j\theta}$.
- Let the current flowing through the component be $i(t) = I_o \cos(\omega t + \phi)$ so that the phasor representing the current $I = I_o e^{j\phi}$.
- Then the "impedance" of the component is defined as the ratio of the voltage phasor to the current phasor:

$$Z = \frac{V}{I}$$

Impedance for R, L and C Elements

Component		Time Domain	Frequency Domain "Impedance"
Resistor		$v(t) = Ri(t)$	$Z = R$
Capacitance		$i(t) = C\frac{d}{dt}v(t)$	$Z = \frac{1}{j\omega C}$
Inductance		$v(t) = L\frac{d}{dt}i(t)$	$Z = j\omega L$

Homework:

Prove that if $i(t) = C\frac{d}{dt}v(t)$ then $Z = \frac{1}{j\omega C}$.

Prove that if $v(t) = L\frac{d}{dt}i(t)$ then $Z = j\omega L$.

Power in A.C. Circuits

- The "instantaneous power" is
$$p(t) = v(t)i(t) = V_0 \cos(\omega t + \phi) I_0 \cos(\omega t + \theta)$$
- The "average power" is the instantaneous power averaged over one A.C. cycle:
$$P_{av} = \frac{1}{T}\int_0^T p(t)dt$$
- You can evaluate the integral with trig identities to show that the average power is
$$P_{av} = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos(\phi - \theta)$$

Homework: evaluate the P_{av} integral to prove the formula.

Complex Power

The phasor for $v(t) = V_0 \cos(\omega t + \phi)$ is $V = V_0 e^{j\phi}$

The phasor for $i(t) = I_0 \cos(\omega t + \theta)$ is $I = I_0 e^{j\theta}$

The "complex power" is defined as

$$S = \frac{1}{2} V I^*$$

where I^* is the complex conjugate of the current phasor, $I^* = I_0 e^{-j\theta}$.
(To get the complex conjugate, replace j by $-j$.)

$$S = \frac{1}{2} V I^*$$

$$S = P_{av} + jQ$$

The real part P_{av} is the average power:

$$P_{av} = \mathrm{Re}(S) = \frac{1}{2}\mathrm{Re}(V I^*)$$

The imaginary part Q is called the "reactive power".

The average formula is the same as the one we wrote before:

$$P_{av} = \frac{1}{2}\mathrm{Re}(V I^*)$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left(V_0 e^{j\phi} I_0 e^{-j\theta}\right) = \frac{1}{2}\mathrm{Re}\left(V_0 I_0 e^{j(\phi-\theta)}\right)$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V_0 I_0 (\cos(\phi-\theta) + j\sin(\phi-\theta))\right]$$

$$P_{av} = \frac{1}{2} V_0 I_0 \cos(\phi-\theta)$$

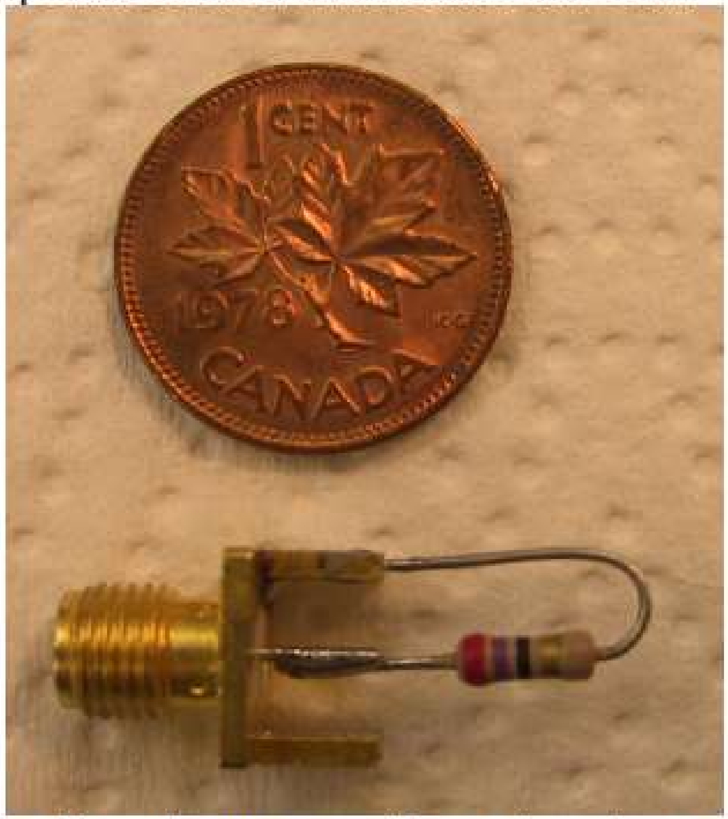
High Frequency Circuits

Objective:

- Learn to find the transfer function $H(j\omega)$ of an interconnection on a high-speed logic board.
- Learn to use the transfer function to identify shortcomings in the interconnection or "communication path".
- Learn to "fix" the circuit interconnection so that it provides acceptable performance.
- Learn some design principles for radio frequency circuits that apply equally well to logic circuits and to communication systems.

Resistors at Radio Frequencies

- Devices behave in unexpected ways at “radio” frequencies!
- What is the impedance of a 27 ohm resistor at 2 GHz?

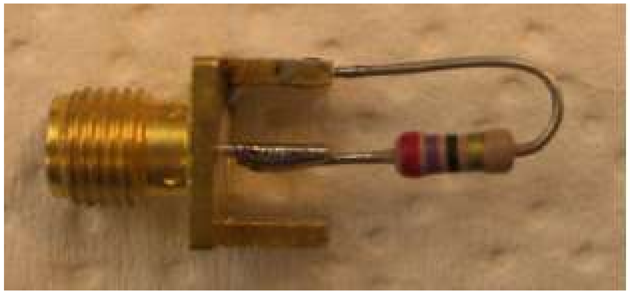


  - At D.C. this carbon-composition resistor has a “nominal” value of 27 ohms.
  - The resistor is shown here mounted on an “SMA” connector.
  - The SMA connector allows the resistor to be connected to a coaxial-cable “transmission line” of characteristic impedance $R_c = 50$ ohms.
  - What is the impedance of the resistor at 2 GHz?

HP8720 Network Analyzer

- A "network analyzer" can be used to measure the impedance of the resistor at 2 GHz.
- The 27-Ω resistor is used as the load on a 50-Ω coaxial "transmission line".
- The network analyzer measures the magnitude and phase of the **reflection coefficient** Γ of the 27-Ω resistor at the end of a 50-ohm cable.
- Then we can calculate the impedance using $\Gamma = \dfrac{Z_L - R_c}{Z_L + R_c}$, hence

$$Z_L = R_c \frac{1+\Gamma}{1-\Gamma}$$



The "27-Ω" resistor mounted on an SMA connector is measured to have an "impedance" of Z_L =100+j200 ohms at 2 GHz.

So R=100 ohms and $\omega L = 200$ ohms.

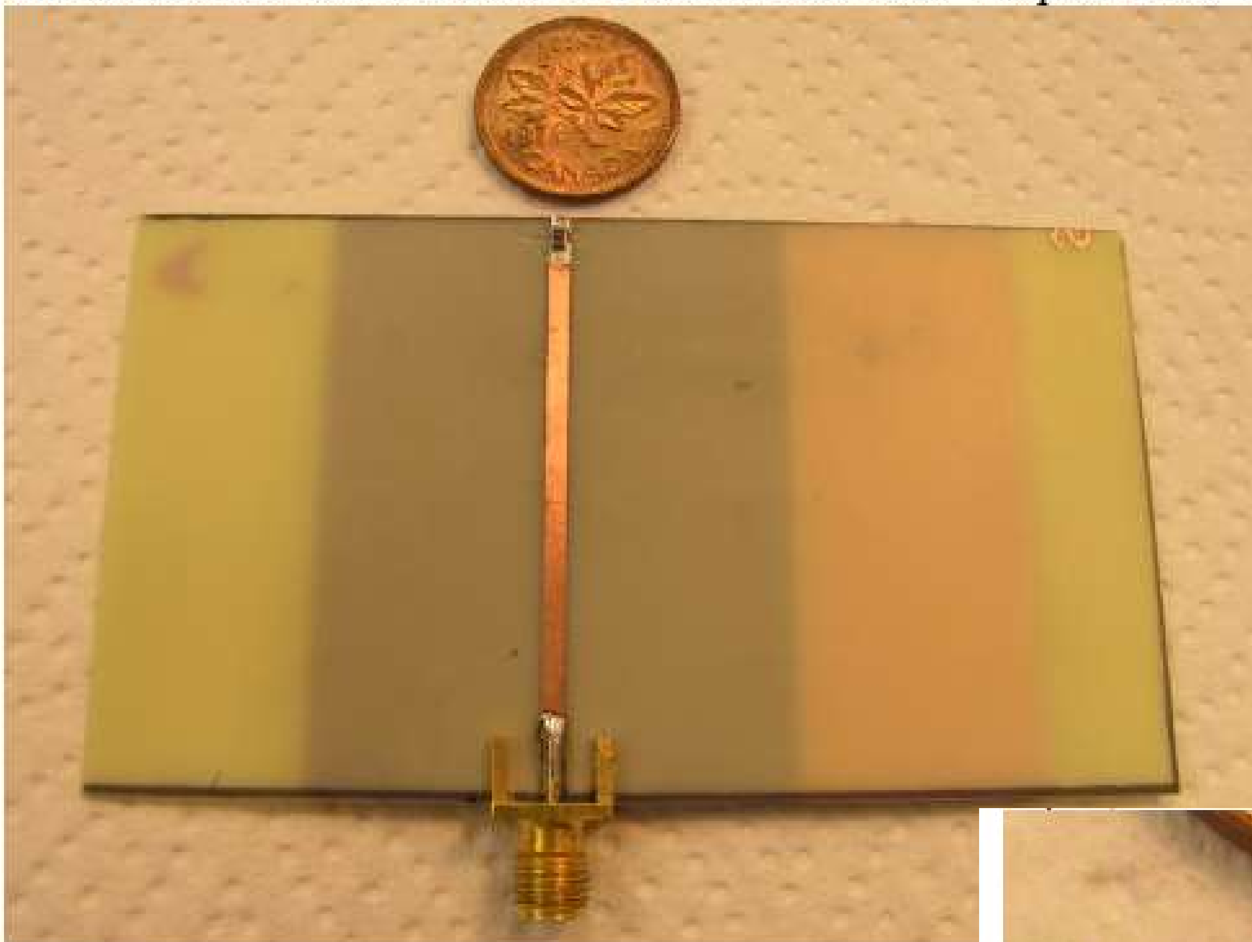
The resistance is much larger than 27 ohms-why?

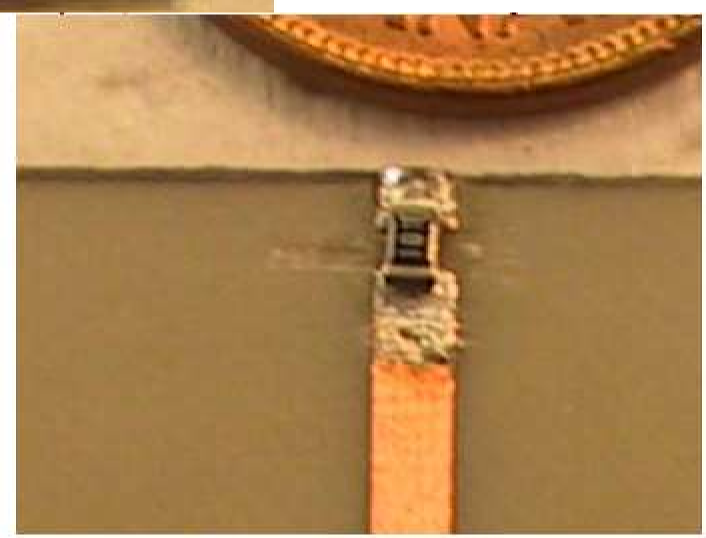
- Answer: the current flows in a thin layer at the surface called the "skin depth". So the cross-sectional area A of the resistor used for current flow is much smaller and the resistance $R = \dfrac{\rho L}{A}$ is higher.

There is an inductance of $j\omega L = j200$ ohms-why?

- Answer: at "radio" frequencies, the small one-turn loop formed with the lead wires of the resistor has inductance!

What is the resistance of a surface-mount 100 ohm "chip resistor" at 2 GHz?

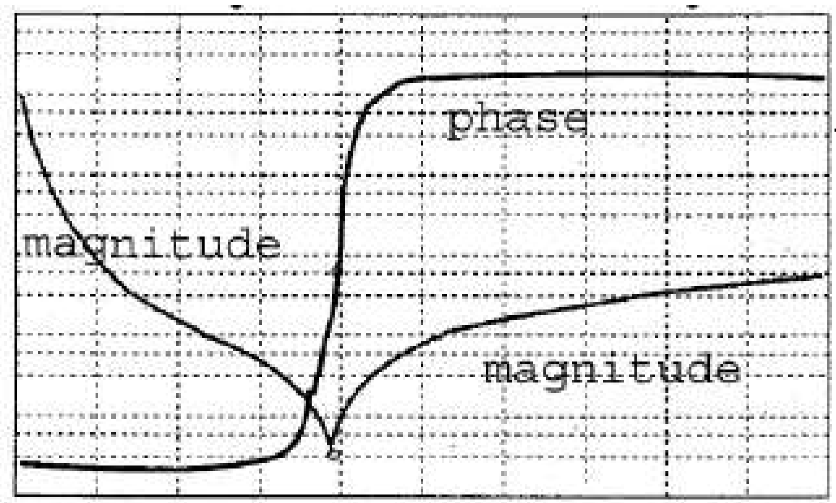




$$Z_L = 90 \text{ - j10 ohms}$$

- The resistance of 90 ohms is much closer to the "nominal" value of 100 ohms because this resistor is "designed" for use at 2 GHz.
- There is some capacitance, hence a reactance of -j10 ohms.
- This is associated with the construction of the "chip resistor".
- There will also be some "lead inductance" which is in series with the capacitance. But it is not the dominant effect at 2 GHz.

Impedance of a Capacitor as a function of Frequency



Angle $+90°$

Angle $-90°$

Horizontal: 1 MHz to 15 MHz
Vertical: log scale, 20 milliohms to 50 ohms
-100 deg to 100 deg

Capacitor: $$Z = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90°$$

Inductor: $$Z = j\omega L = \omega L \angle +90°$$