ELEC353 Lecture Notes Set 4

The homework assignments are posted on the course web site.

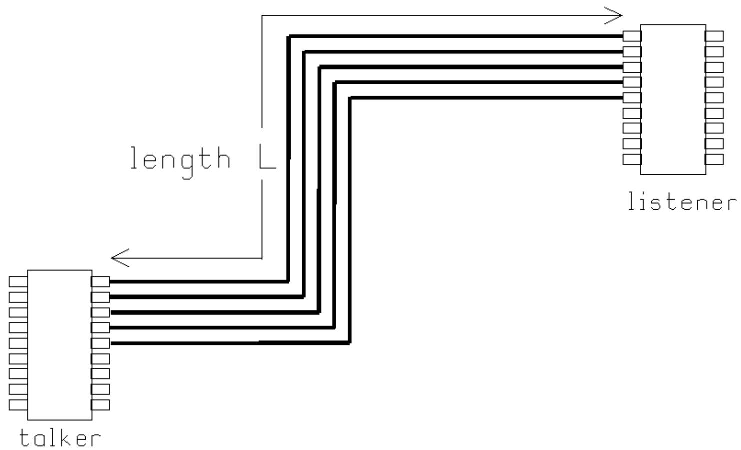
Homework #2: Do this assignment by January 25nd, 2019.
Homework #3: Do homework #3 by February 1, 2019.
Homework #4: Do homework #4 by February 8, 2019.
Mid-term test: Thursday February 14, 2019.

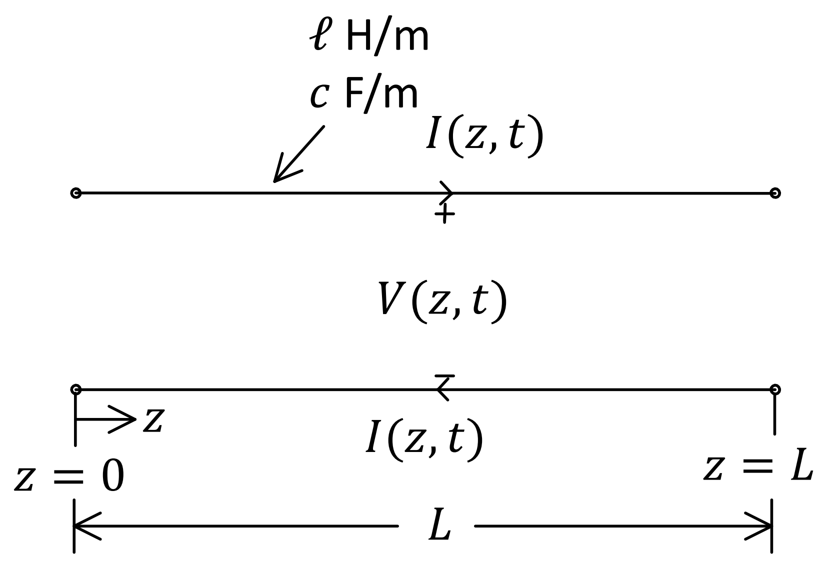
The course web site is:
www.ece.concorcia.ca/~trueman/web_page_353.htm

The course outline
The lecture notes
The homework assignments and solutions
Practice problems
Past midterm tests.
Software: BOUNCE, TRLINE

Review: Analysis of Transmission Lines

Inan, Inan and Said Section 2.2.2





$$\frac{\partial V}{\partial z} = -\ell \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -c \frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 V}{\partial t^2}$$

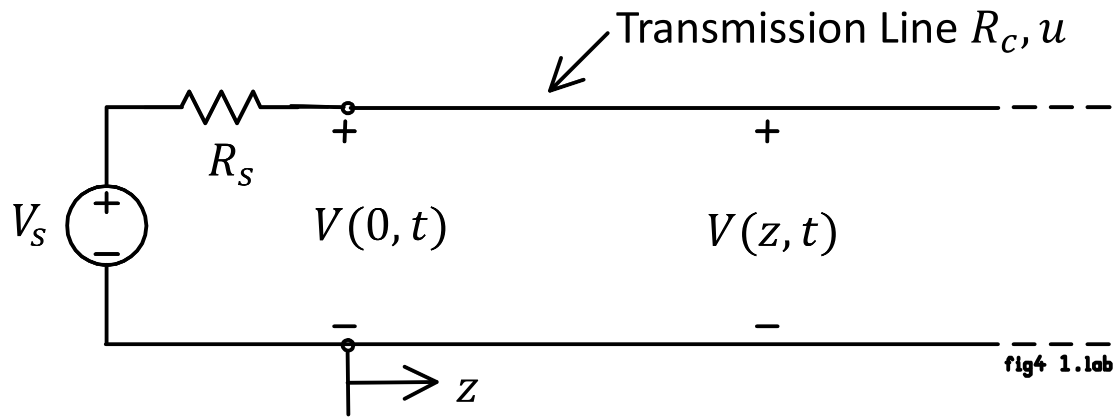
$$V(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-\left(t + \frac{z}{u}\right)$$

$$I(z,t) = \frac{1}{R_c} f^+\left(t - \frac{z}{u}\right) - \frac{1}{R_c} f^-\left(t + \frac{z}{u}\right)$$

$$u = \frac{1}{\sqrt{\ell c}} \text{ m/s}$$

$$R_c = \sqrt{\frac{\ell}{c}}$$

Transmission Line Driven by a Generator



$$V(z,t) = f^{+}\left(t - \frac{z}{u}\right) + f^{-}\left(t + \frac{z}{u}\right)$$

$$I(z,t) = \frac{1}{R_c} f^{+}\left(t - \frac{z}{u}\right) - \frac{1}{R_c} f^{-}\left(t + \frac{z}{u}\right)$$

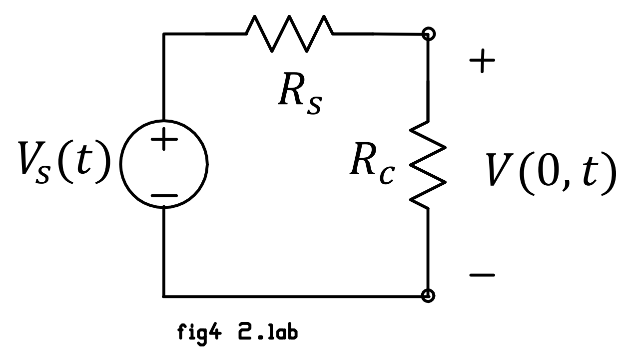
There is no reflected wave, so $f^{-} = 0$

$$V(z,t) = f^{+}\left(t - \frac{z}{u}\right)$$

$$I(z,t) = \frac{1}{R_c} f^{+}\left(t - \frac{z}{u}\right)$$

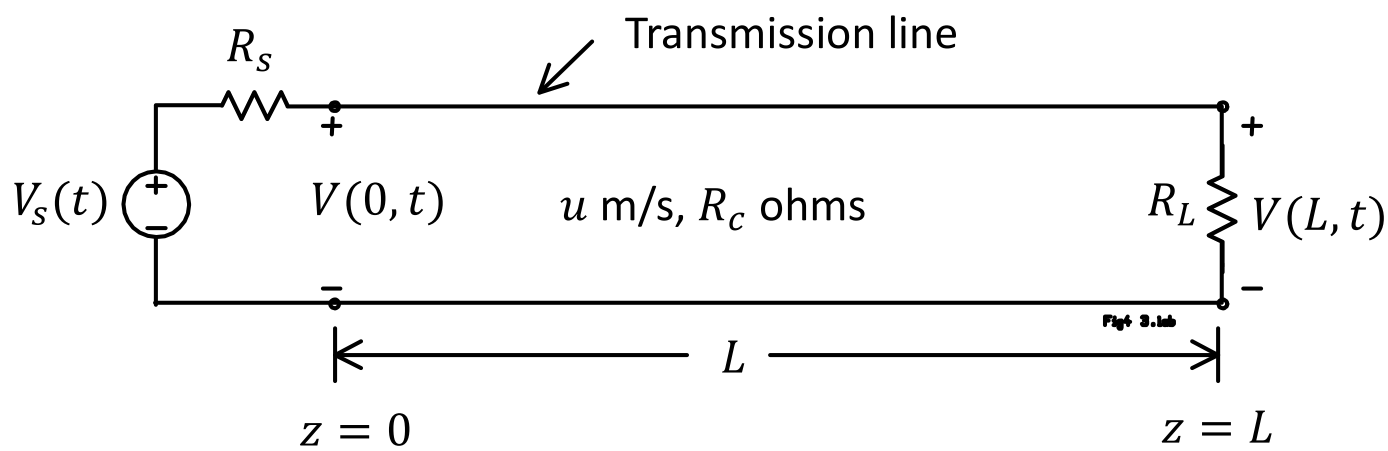
at the generator terminals at $z = 0$

$$\left.\begin{aligned} V(0,t) &= f^{+}(t) \\ I(0,t) &= \frac{1}{R_c} f^{+}(t) \end{aligned}\right\} \qquad R_{input} = \frac{V(0,t)}{I(0,t)} = R_c$$

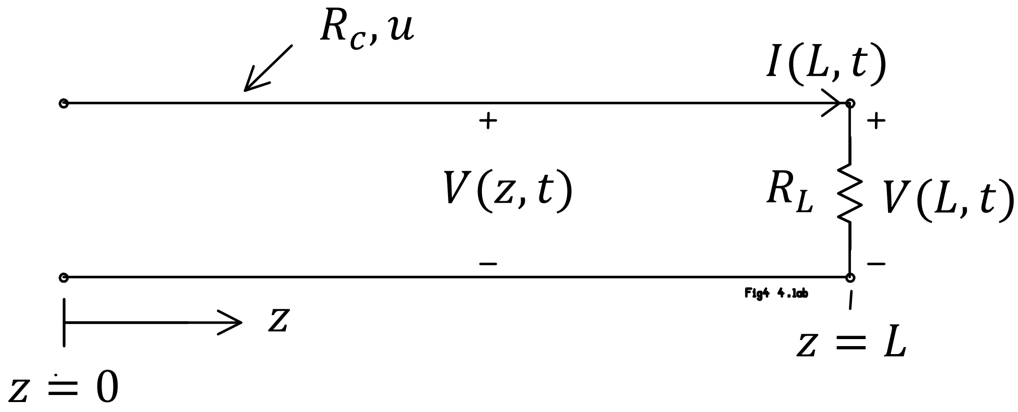


$$V(0,t) = \frac{R_c}{R_c + R_s} V_s(t)$$

Transmission Line Terminated with a Resistor



The Reflection Coefficient at the Load



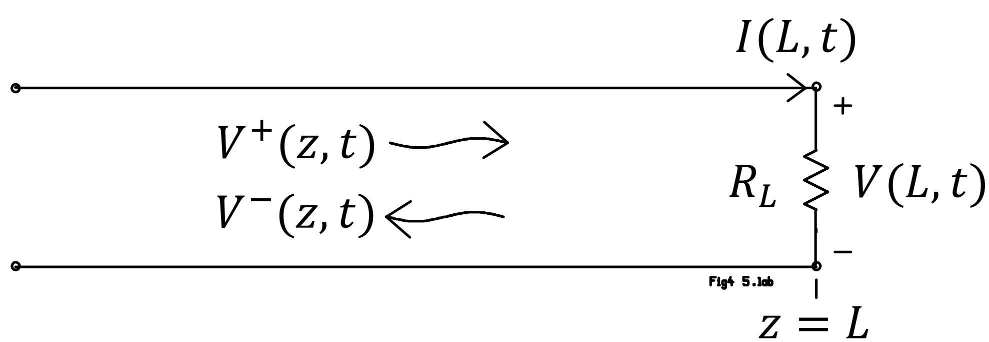
$$V^+(z,t) = f^+(t - \frac{z}{u})$$

$$V^-(z,t) = f^-(t + \frac{z}{u})$$

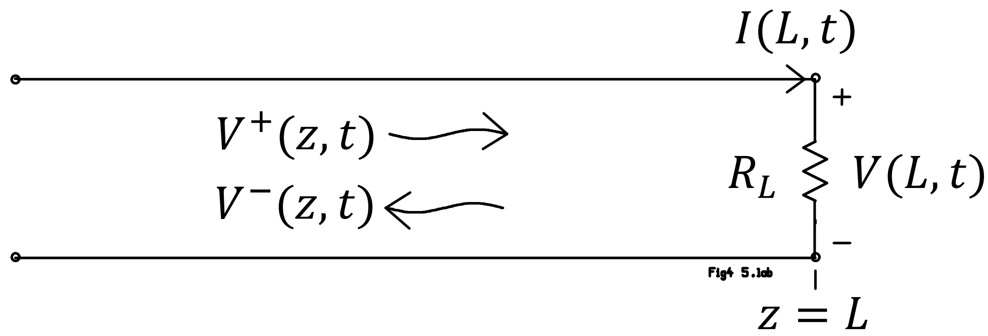
Simplify the notation

$$V(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-\left(t + \frac{z}{u}\right) \Rightarrow V(z,t) = V^+(z,t) + V^-(z,t)$$

$$I(z,t) = \frac{1}{R_c} f^+\left(t - \frac{z}{u}\right) - \frac{1}{R_c} f^-\left(t + \frac{z}{u}\right) \Rightarrow I(z,t) = \frac{1}{R_c} V^+(z,t) - \frac{1}{R_c} V^-(z,t)$$



Reflection from the Load



$$V(z,t) = V^+(z,t) + V^-(z,t)$$

$$I(z,t) = \frac{1}{R_c}V^+(z,t) - \frac{1}{R_c}V^-(z,t)$$

At the load, $z = L$, we must obey Ohm's Law:

$$V(L,t) = R_L I(L,t)$$

$$V^+(L,t) + V^-(L,t) = R_L\left(\frac{V^+(L,t)}{R_c} - \frac{V^-(L,t)}{R_c}\right)$$

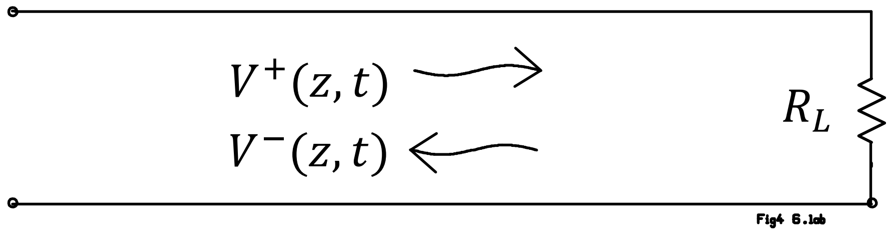
Solve for V^- :

$$V^- = \frac{R_L - R_c}{R_L + R_c}V^+$$

Define "reflection coefficient" Γ:

$$\Gamma = \frac{V^-}{V^+} \qquad \Gamma = \frac{R_L - R_c}{R_L + R_c} \qquad V^- = \Gamma V^+$$

Important Special Cases



$$\Gamma = \frac{R_L - R_c}{R_L + R_c}$$

$$V^- = \Gamma V^+$$

Matched Load: $R_L = R_c$ so $\Gamma = 0$.

- The "incident" wave is completely absorbed by the load.

$R_L > R_c$, $\Gamma = \dfrac{R_L - R_c}{R_L + R_c} > 0$

Extreme case:

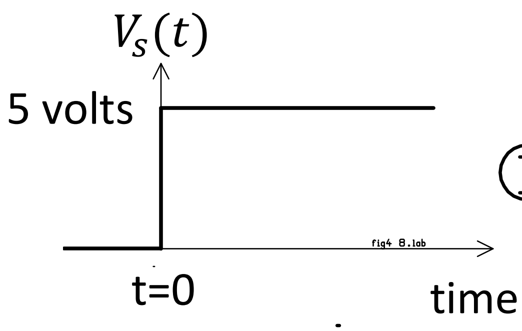
- If $R_L \to \infty$, then $\Gamma \to +1$, so the reflected voltage has the same amplitude as the incident voltage.
- $R_L \to \infty$ is an open-circuit load, for which $\Gamma \to +1$.

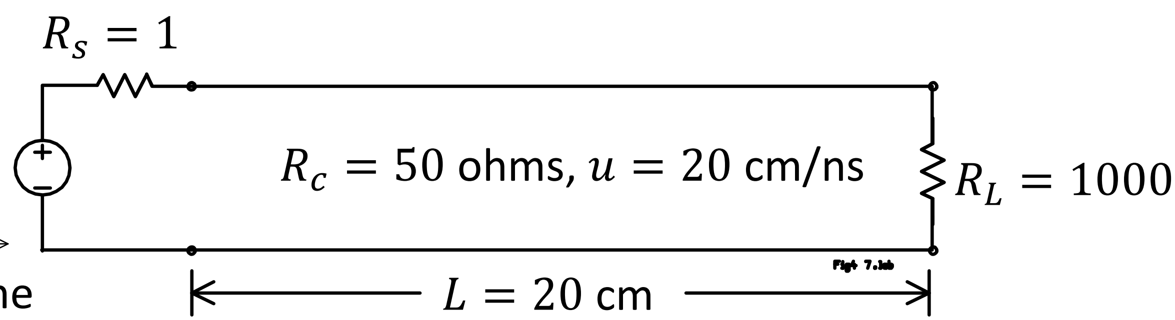
$R_L < R_c$, $\Gamma = \dfrac{R_L - R_c}{R_L + R_c} < 0$

Extreme case:

- If $R_L \to 0$, then $\Gamma \to -1$, so the reflected voltage is the negative of the incident voltage.
- $R_L \to 0$ is a short-circuit load, for which $\Gamma \to -1$.

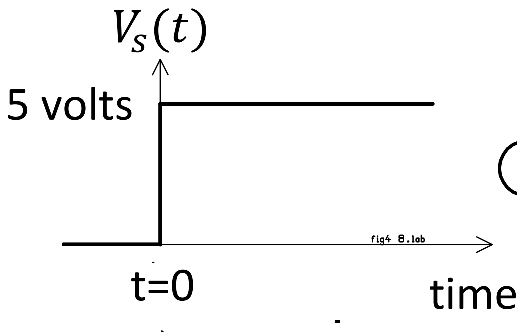
Solving Transmission-Line Circuits

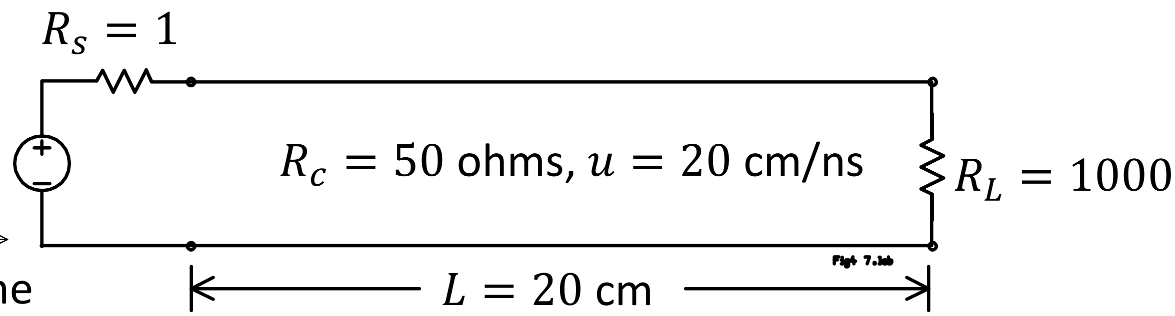




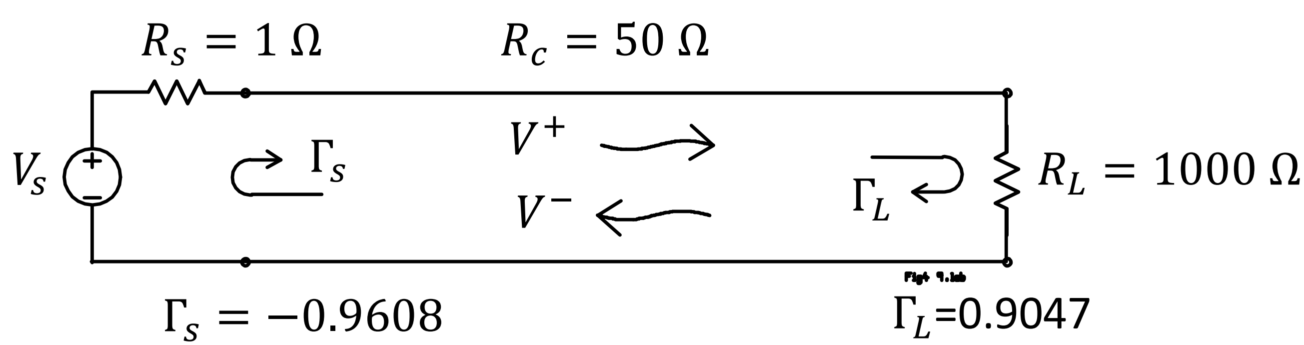
Find the voltage across the load resistor as a function of time.

Values of the coefficients:





- Transit time: $T_d = \dfrac{L}{u} = \dfrac{20}{20} = 1$ ns.
- Load reflection coefficient: $\Gamma_L = \dfrac{R_L - R_c}{R_L + R_c} = \dfrac{1000-50}{1000+50} = 0.9047$
- Source reflection coefficient: $\Gamma_s = \dfrac{R_s - R_c}{R_s + R_c} = \dfrac{1-50}{1+50} = -0.9608$



Initial Step Launched onto the Transmission Line

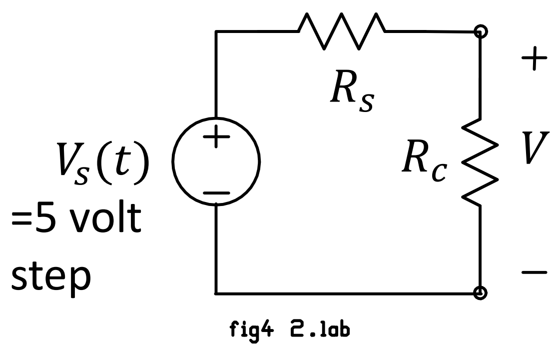
In general $V(z,t) = V^+(z,t) + V^-(z,t)$
At the generator terminals $V(0,t) = V^+(0,t) + V^-(0,t)$
Before $t = 2T$ when the reflection from the load arrives, $V^-(0,t) = 0$ so

$$V(0,t) = V^+(0,t)$$

So we can find V^+ using

$$V^+(0,t) = V(0,t)$$

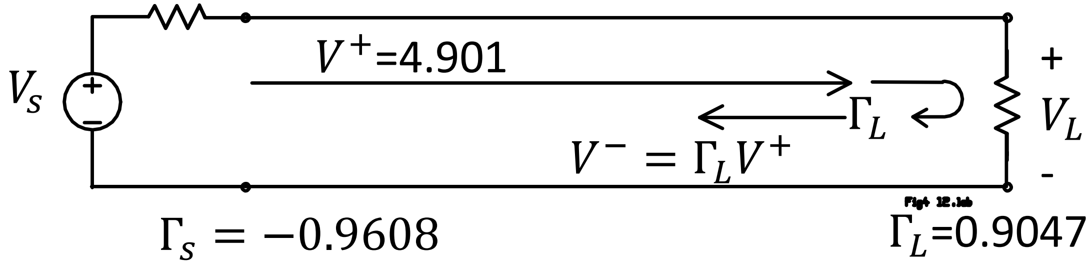
The input resistance of the transmission line is R_c.



$$V(0,t) = \frac{R_c}{R_s+R_c} V_s = \frac{50}{1+50} x5 = 4.901 \text{ volt step}$$

$$V^+ = V(0,t) = 4.901 \text{ volt step}$$

After one transit time:

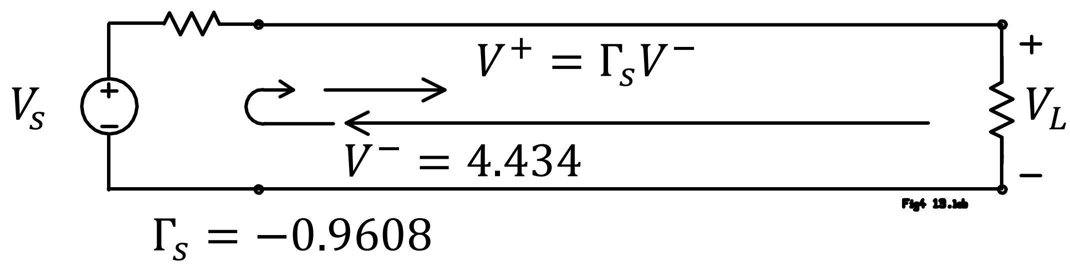


$V^- = \Gamma_L V^+ = 0.9047x4.901 = 4.434$ volts

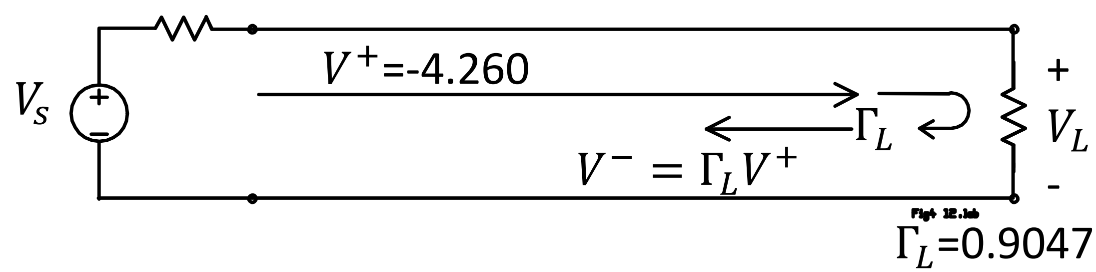
The voltage at the load after one transit time is the incident plus the reflected voltage:

$V(L,t) = V^+(L,t) + V^-(L,t) = 4.901 + 4.434 = 9.335$ volts

At $t = T_d$ the voltage is 4.901+4.434= 9.335 volts.



After two transit times: $V^+ = \Gamma_s V^- = -0.9608x4.434 = -4.260$ volts



After three transit times: $V^- = \Gamma_L V^+ = 0.9047x - 4.260 = -3.854$

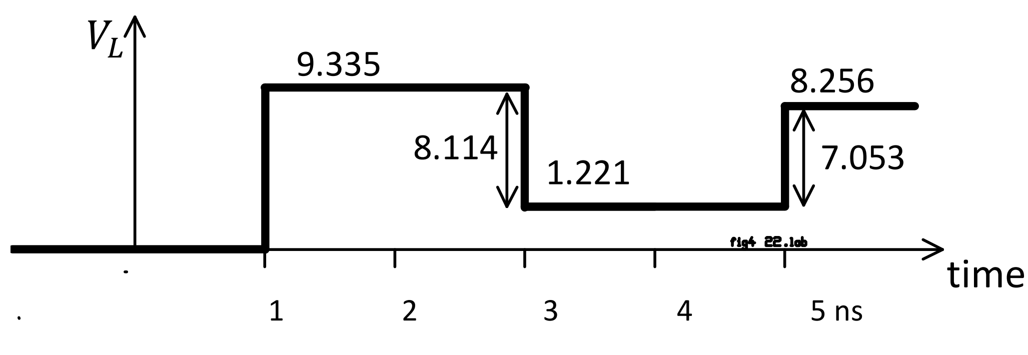
The voltage at the load after one transit time is the incident plus the reflected voltage:
$V(L,t) = V^+(L,t) + V^-(L,t)$

At $t = T_d$ the voltage is 4.901+4.434= 9.335 volts.

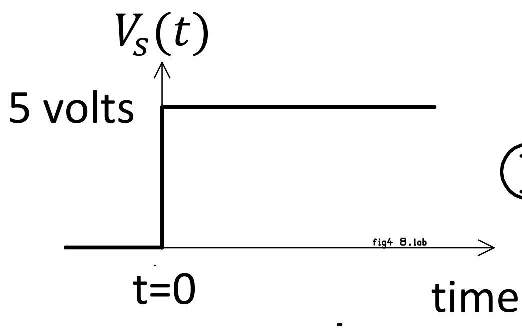
After three transit times, a new incident wave arrives and generates a new reflected wave.
The voltage is the previous value plus the new incident voltage plus the new reflected voltage.

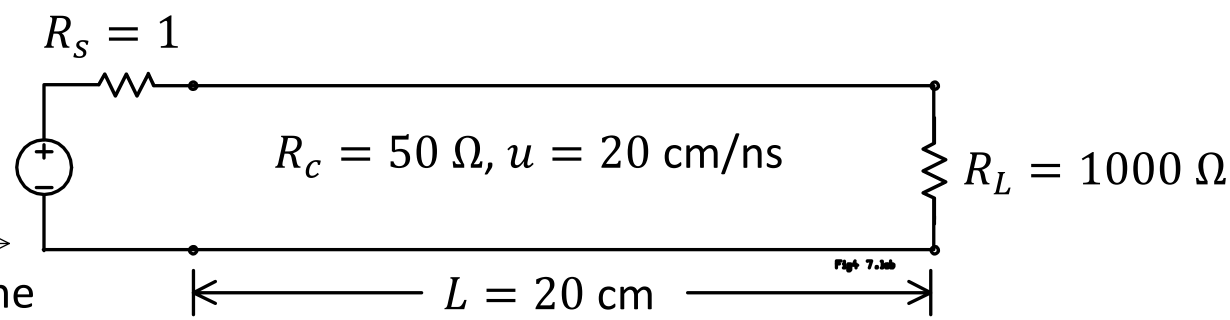
At $t = 3T_d$ the voltage is 9.335 -4.260-3.854=1.221 volts.

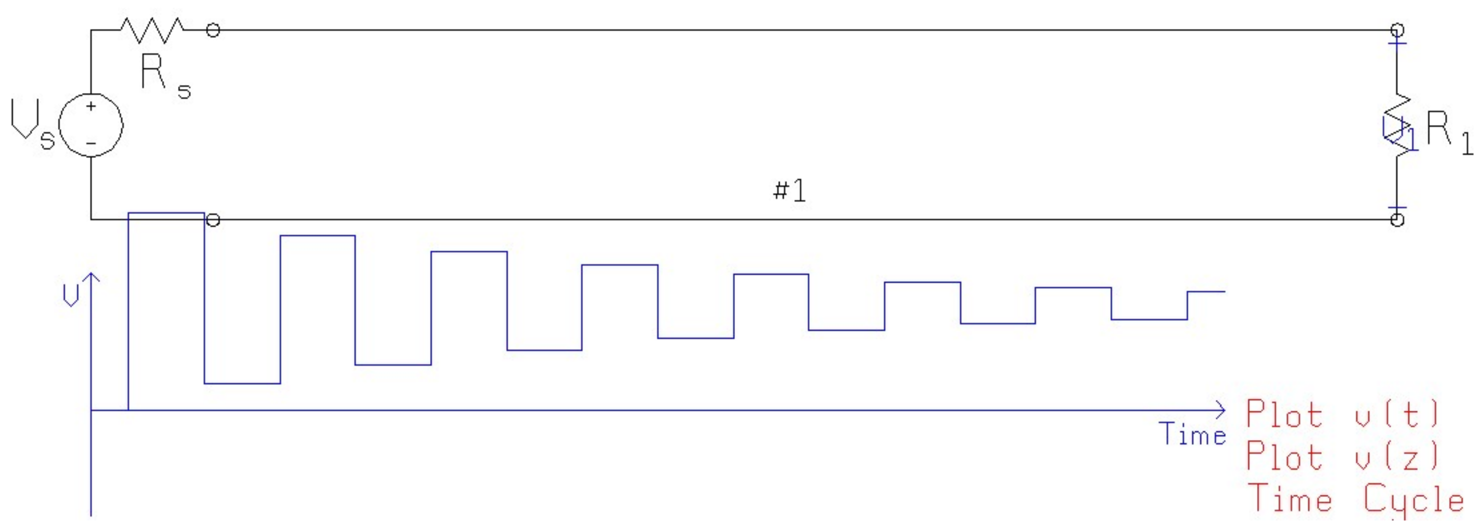
Homework: show that at $t = 5T_d$ the voltage is 1.221 + 7.035 = 8.256 volts



Removing the Reflections:

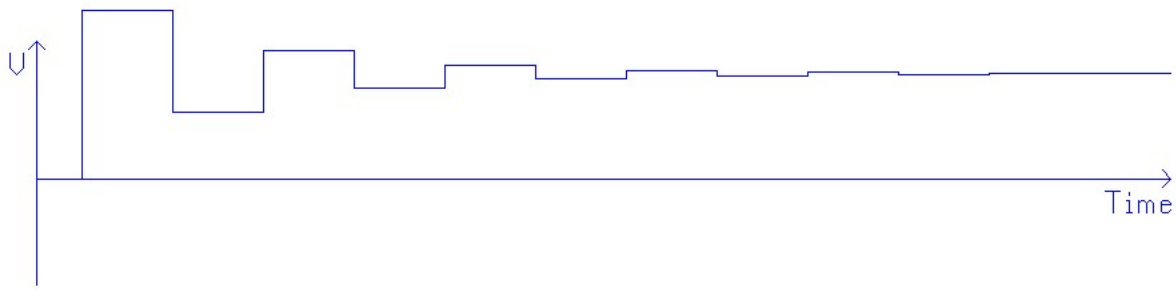




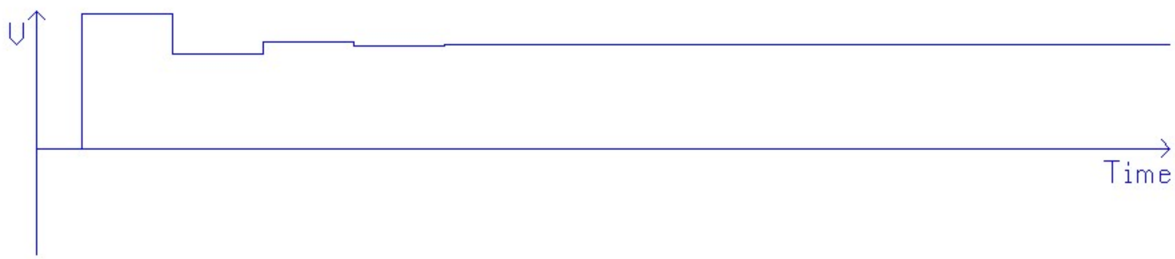


Better match at the source:

$R_s = 10\Omega$



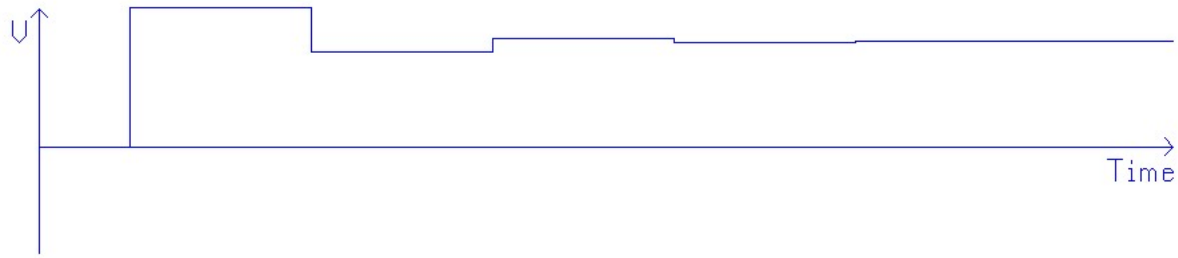
$R_s = 25\Omega$



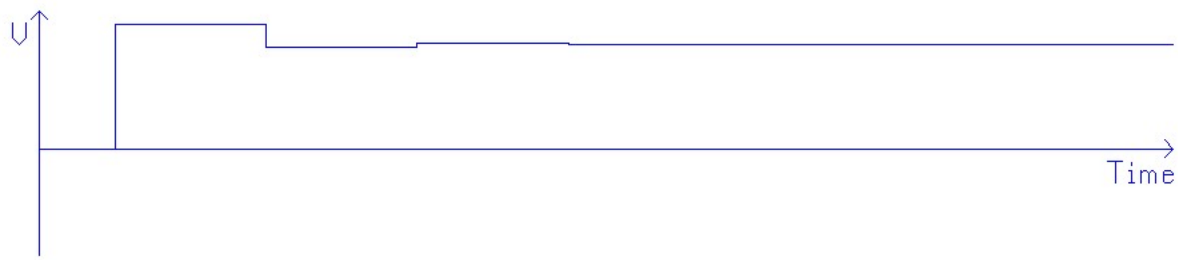
$R_s = 50\Omega$

Perfect match!

Better match at the load:

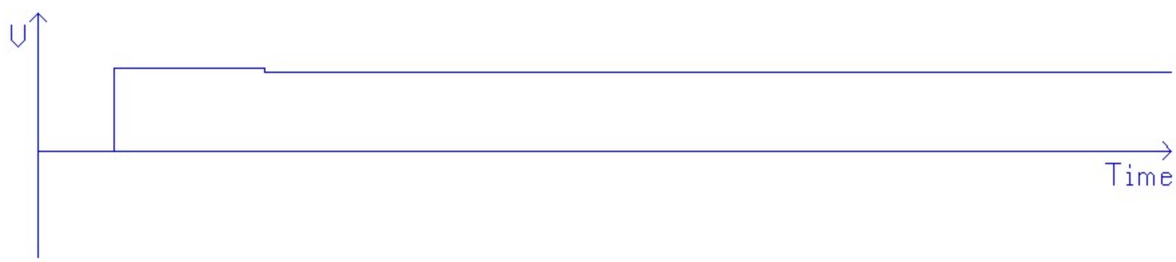


$R_s = 1$ ohm and $R_L = 100$ ohms



$R_L = 75$ ohms

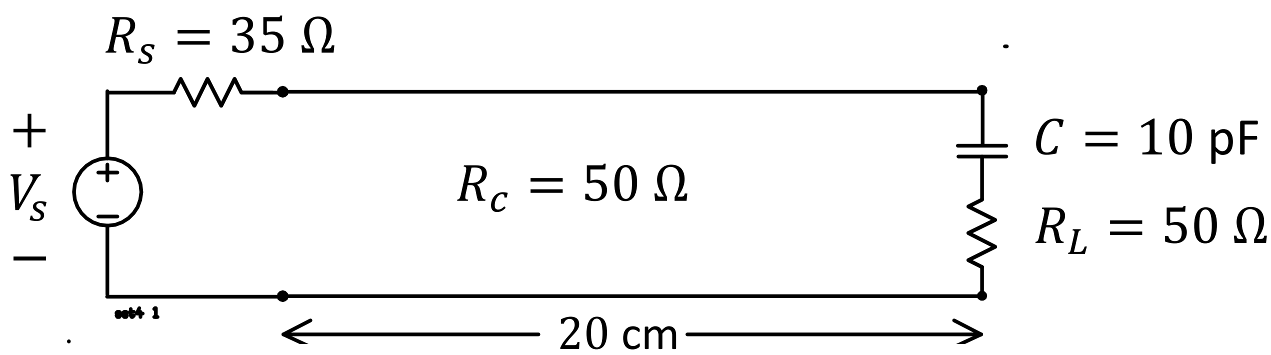
Better match at the generator and at the load:

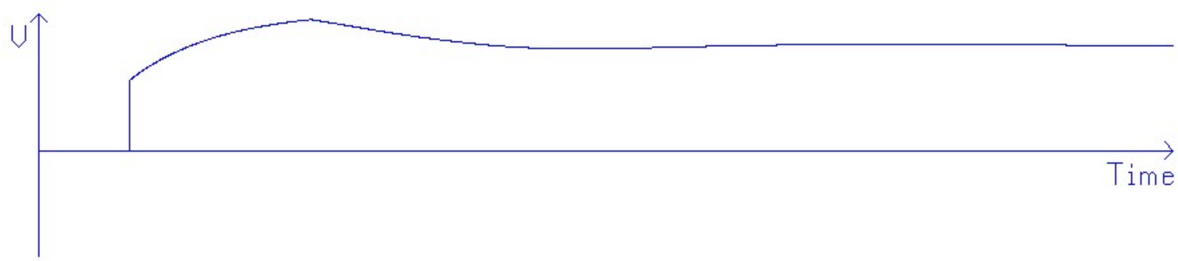


$R_s = 35$ ohms and $R_L = 100$ ohms

There is a small reflection at $3T_d$.

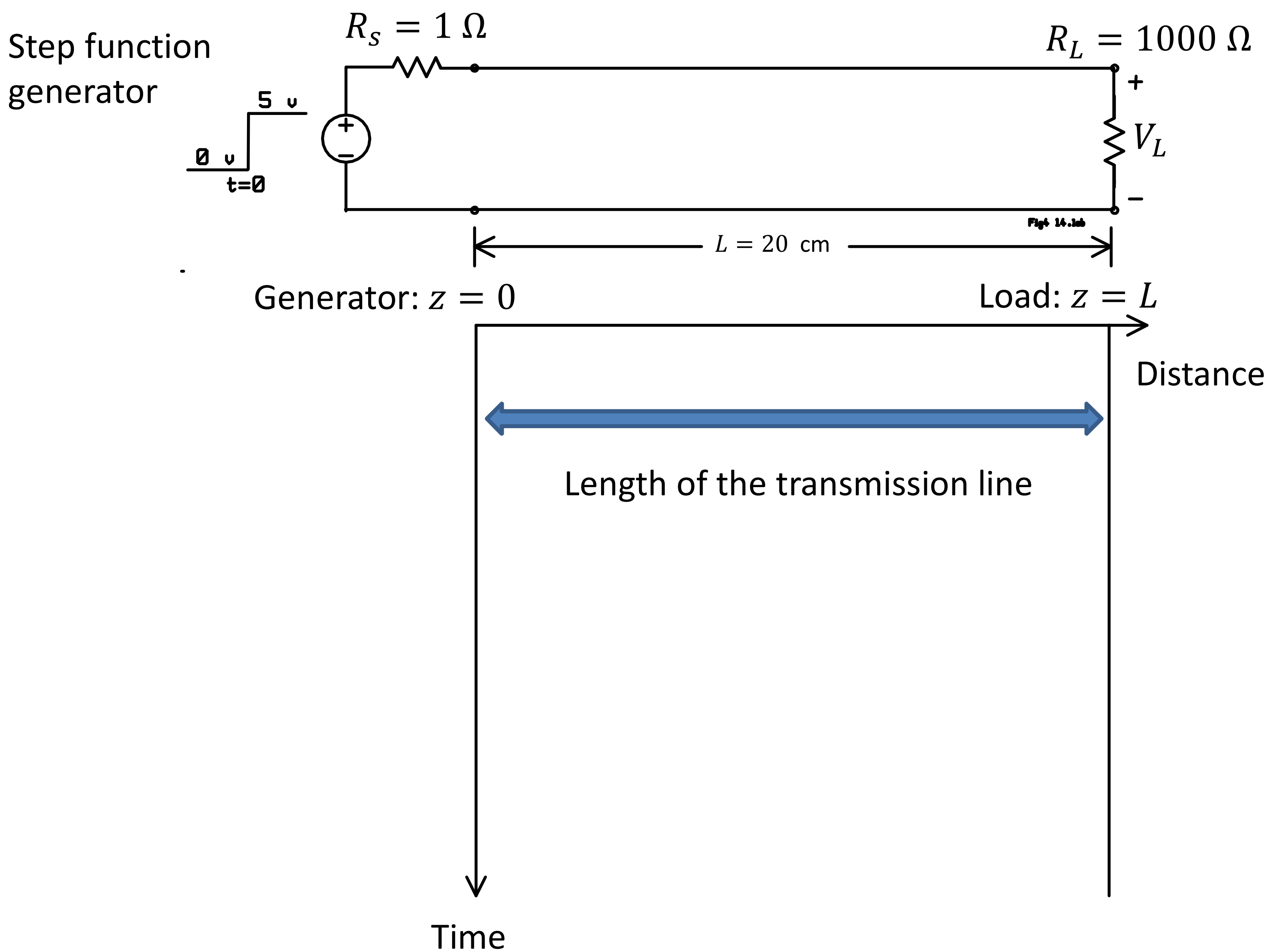
Removing DC Power Dissipation





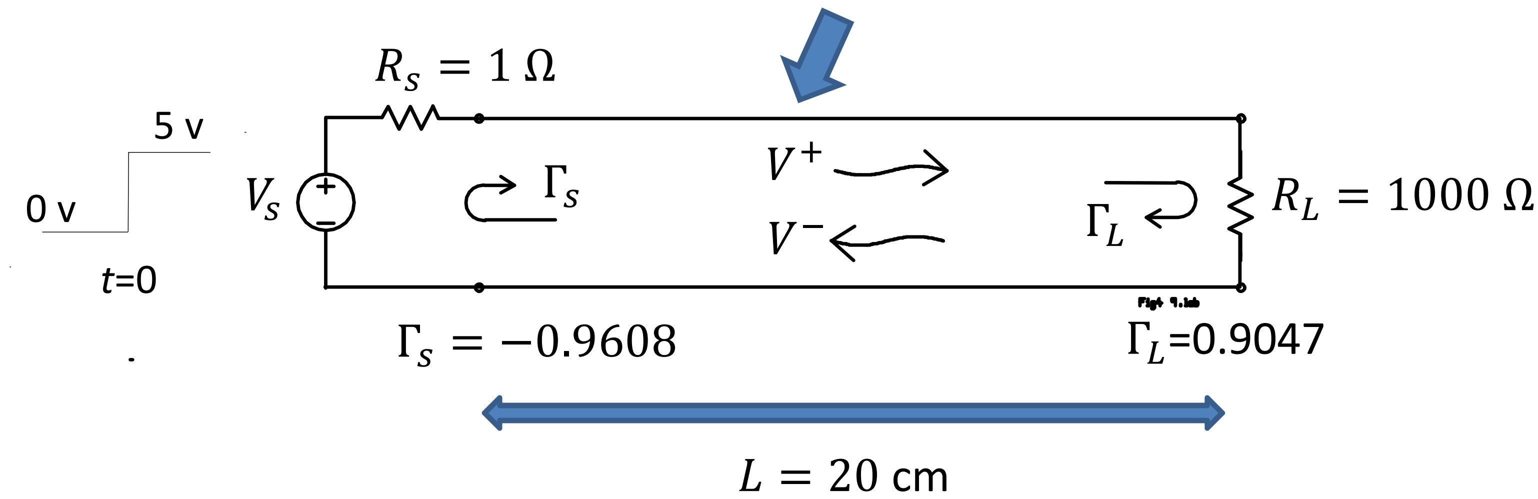
The Bounce Diagram

(Inan and Inan Section 2.3.1)



Problem setup:

$R_c = 50\ \Omega,\ u = 20$ cm/ns

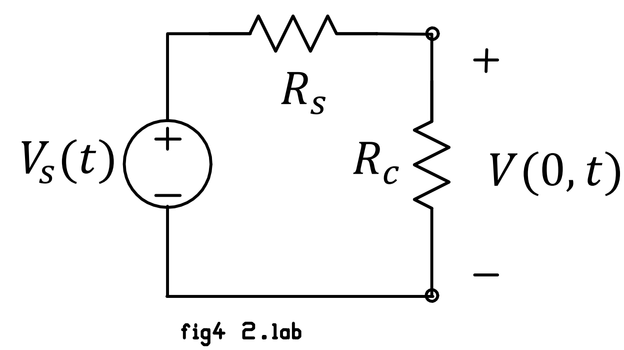


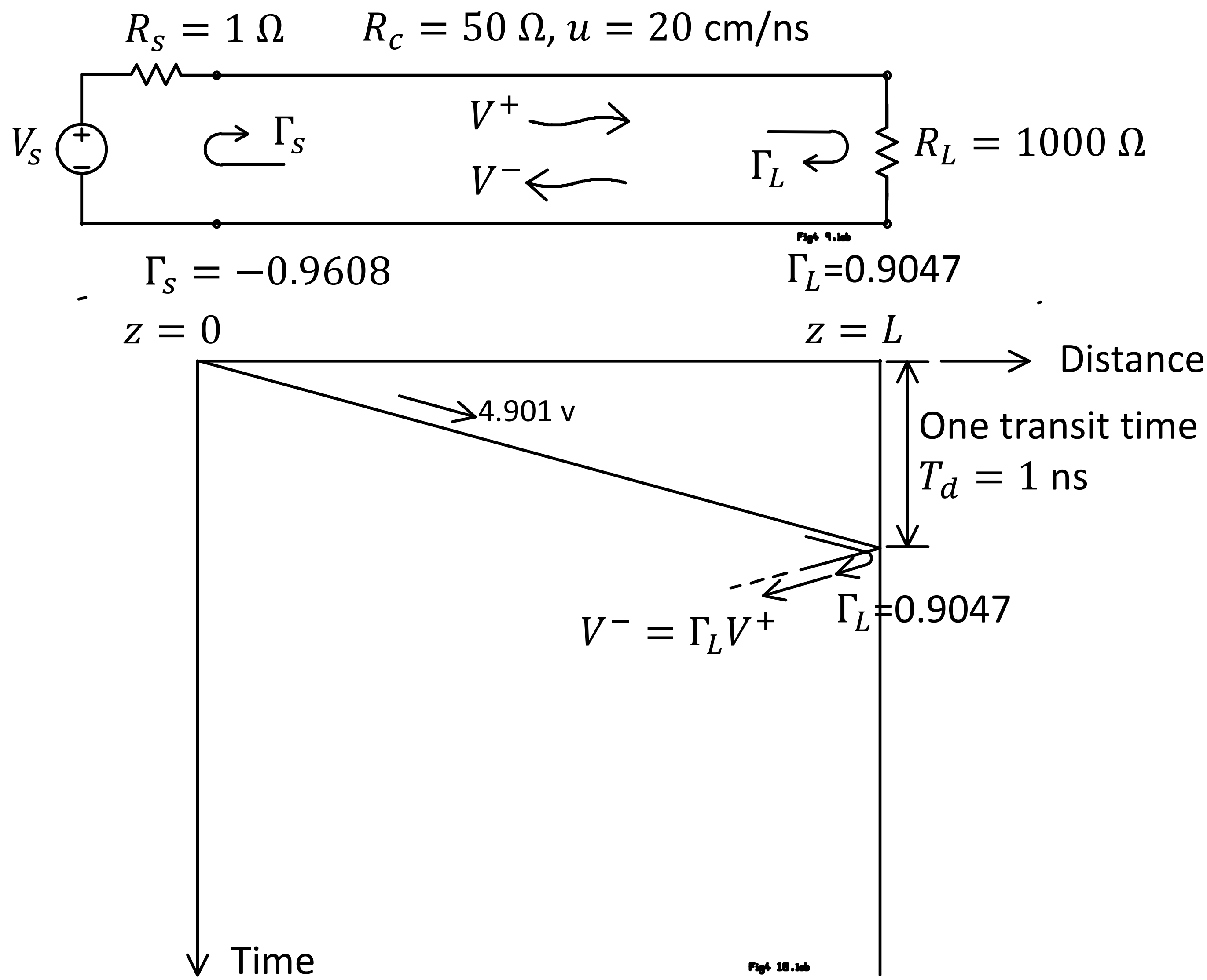
$$T_d = \frac{L}{u} = \frac{20}{20} = 1 \text{ ns}$$

$$V^+ = \frac{R_c}{R_s + R_c} V_s = \frac{50}{50+1} x5 = 4.901 \text{ volts}$$

Load reflection coefficient: $\Gamma_L = \dfrac{R_L - R_c}{R_L + R_c} = \dfrac{1000-50}{1000+50}$=0.9047

Source reflection coefficient: $\Gamma_s = \dfrac{R_s - R_c}{R_s + R_c} = \dfrac{1-50}{1+50}$=-0.9608



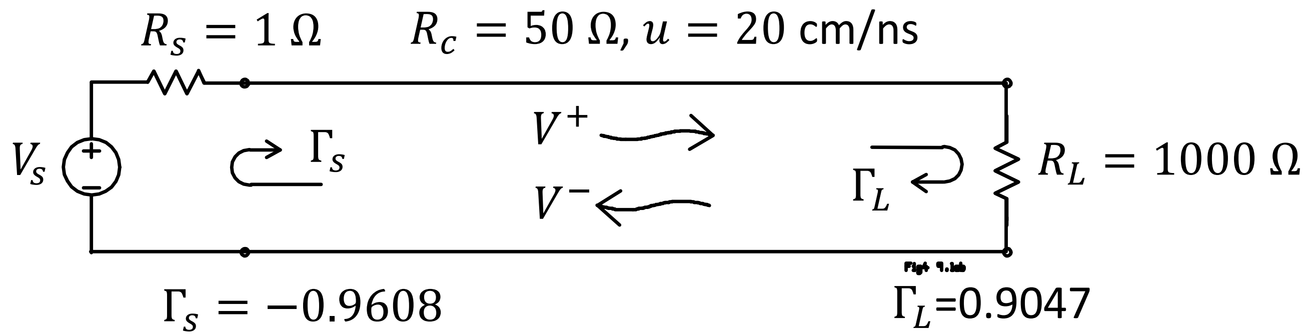


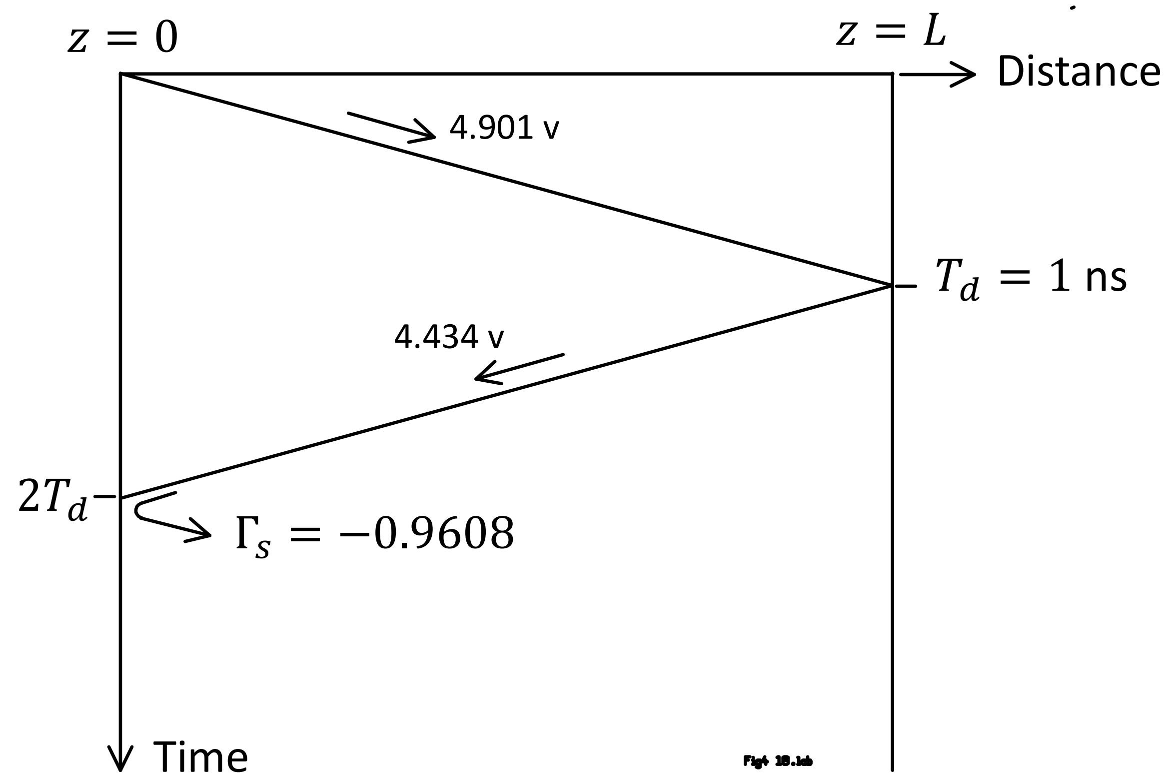
$$V^- = \Gamma_L V^+ = 0.9047x4.901 = 4.434 \text{ v}$$

at the load

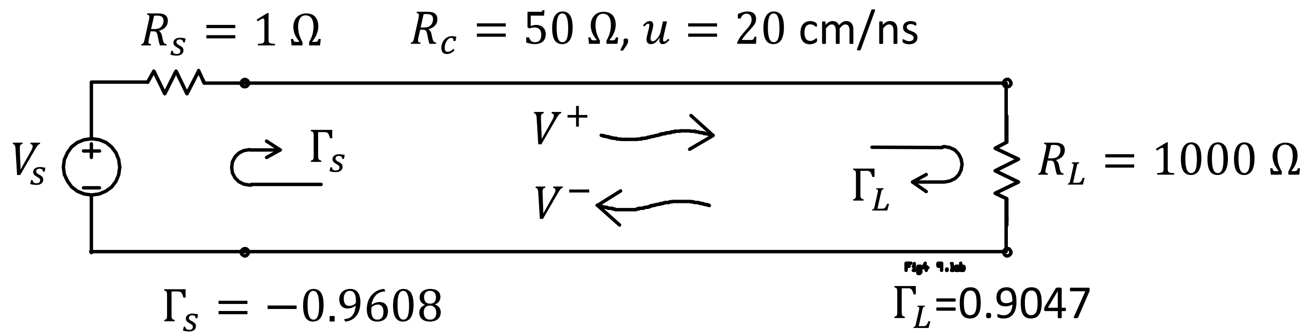
$$V(z=L) = V^+(z=L) + V^-(z=L)$$

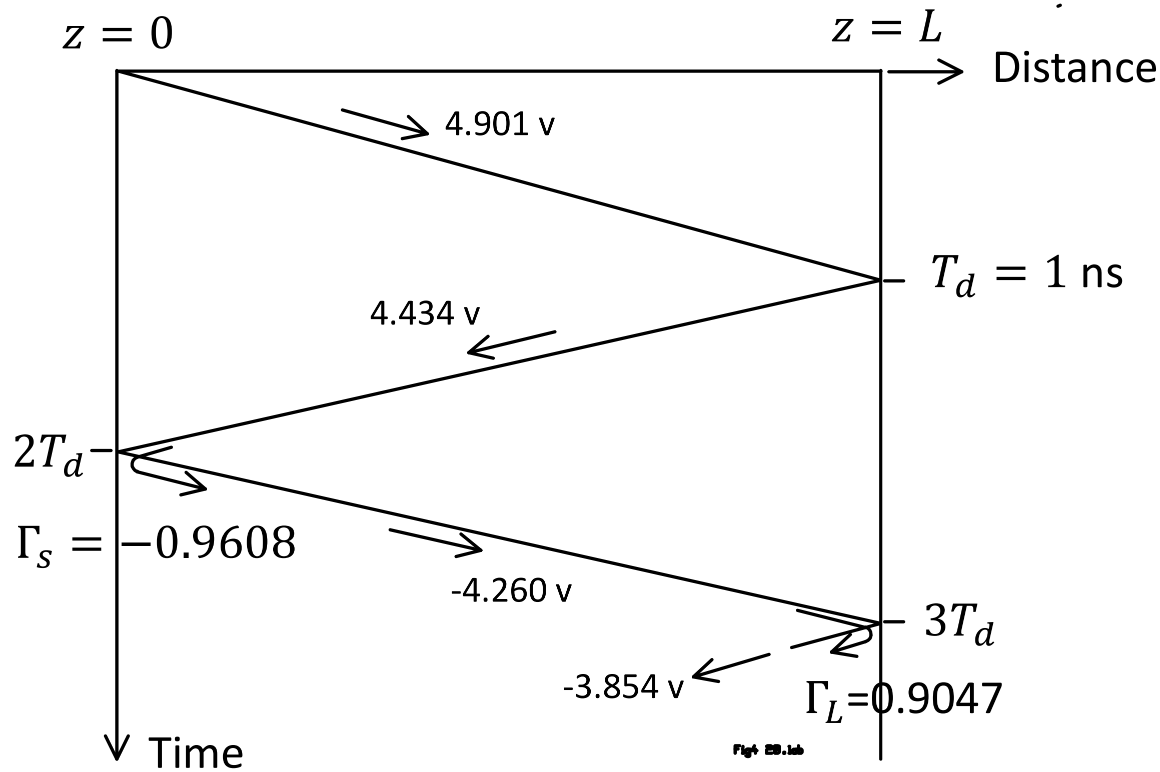
$$V(z=L) = 4.901 + 4.434 = 9.335 \text{ volts}$$



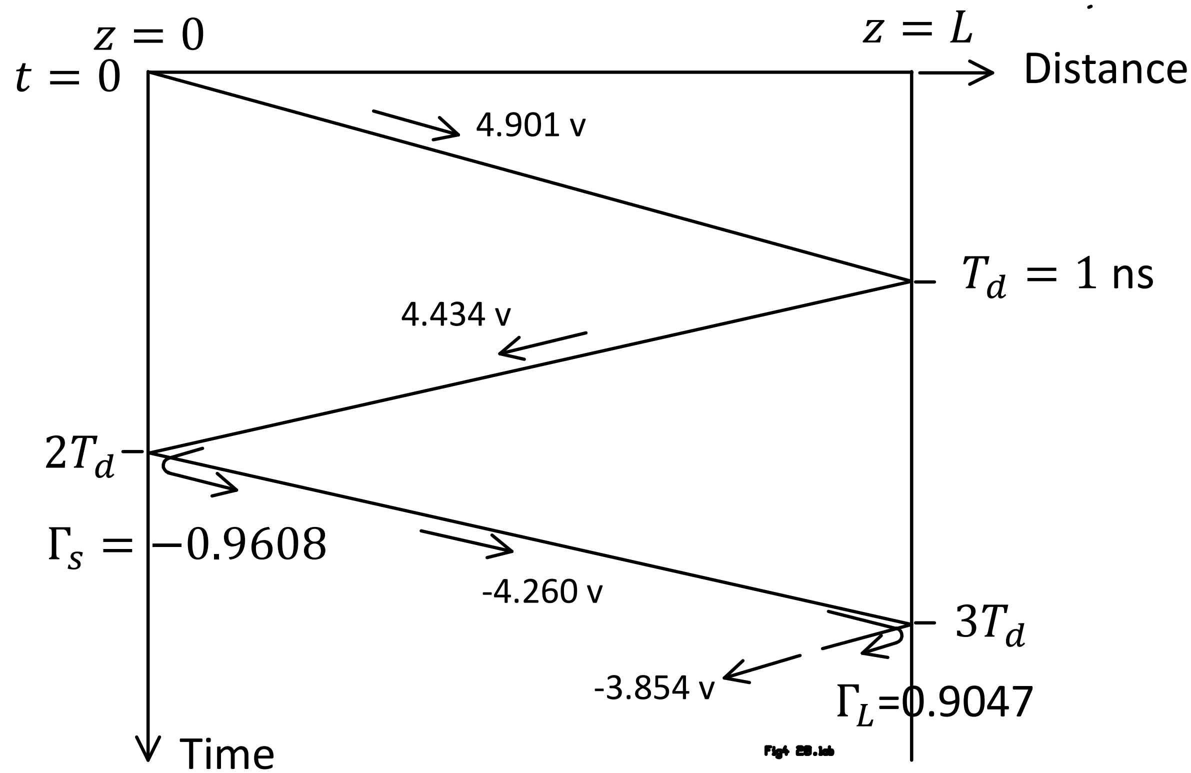


$$V^+ = \Gamma_S V^- = -0.9608x4.434 = -4.260 \text{ v}$$

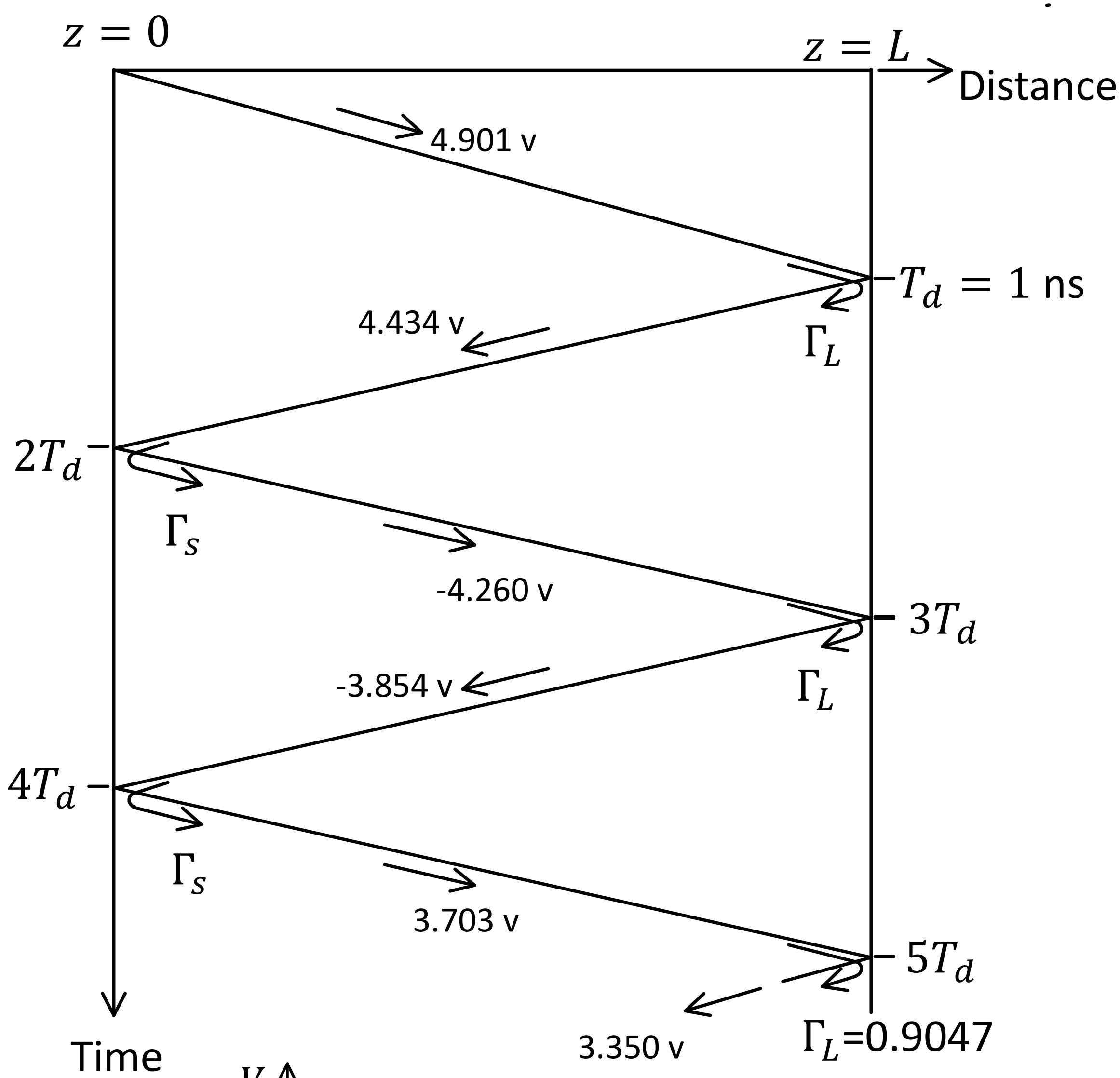


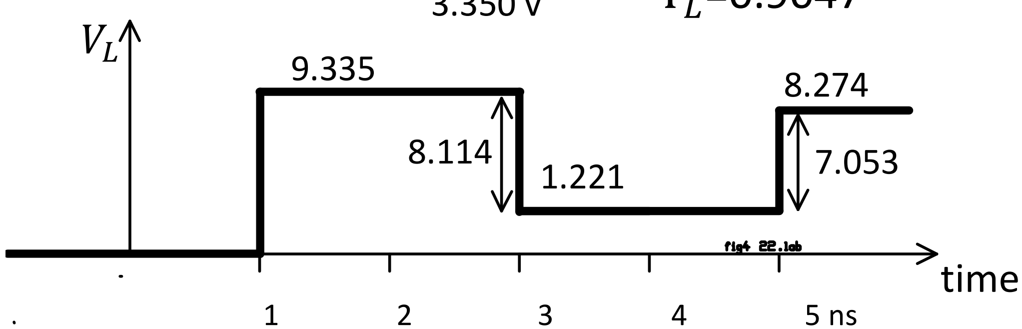


$$V^- = \Gamma_L V^+ = 0.9047x - 4.260 = -3.854 \text{ v}$$



Find the load voltage:
Read the $z = L$ axis downward from $t = 0$ at the top.
From *t=0* to T_d the voltage at the load is zero.
At $t = T_d$, voltage step V^+ =4.901 arrives, and generates voltage V^- =4.434 volts, so the load voltage steps up to 4.901+4.434=9.335 volts.
From $t = T_d$ to $t = 3T_d$ the voltage at the load is constant at 9.335 volts.
At $t = 3T_d$, voltage step V^+ =-4.260 volts arrives, and generates voltage step V^- =-3.854 volts, and the net CHANGE is $V^+ + V^-$=(-4.260)+(-3.854)=-8.114 volts.
So the load voltage changes to 9.335-8.114=1.221 volts.





At $T_d = 1$ ns

$V(z = L) = 4.901 + 4.434 = 9.335$

At $3T_d$:

The change is

$$-4.260 - 3.854 = -8.114 \text{ volts}$$

The load voltage is

9.335-8.114=1.221 volts

At $5T_d$:

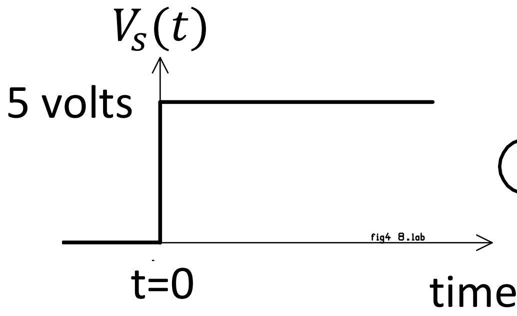
The change is

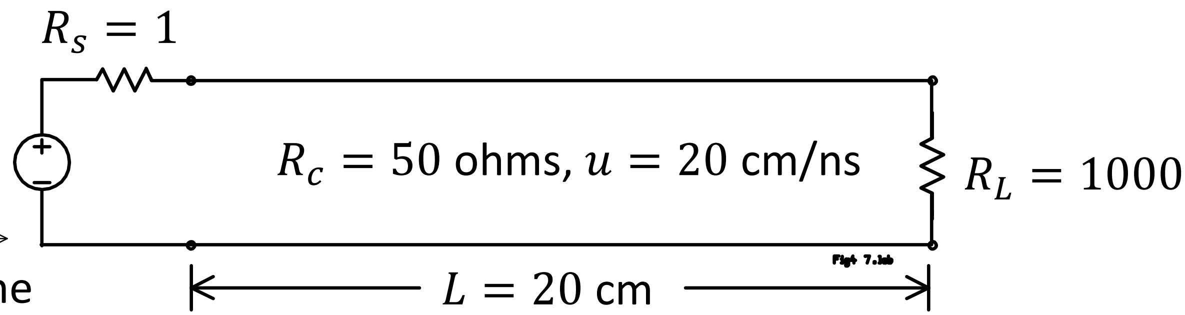
$$3.703 + 3.350 = 7.053 \text{ volts}$$

The load voltage is

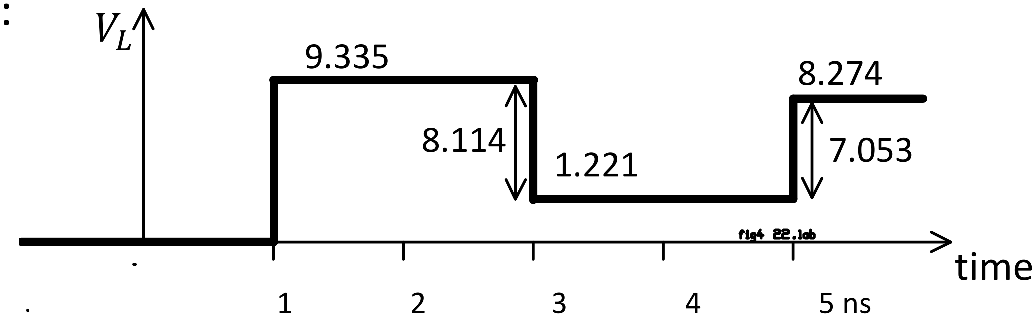
1.221+7.053=8.274 volts

Response to a Pulse Function

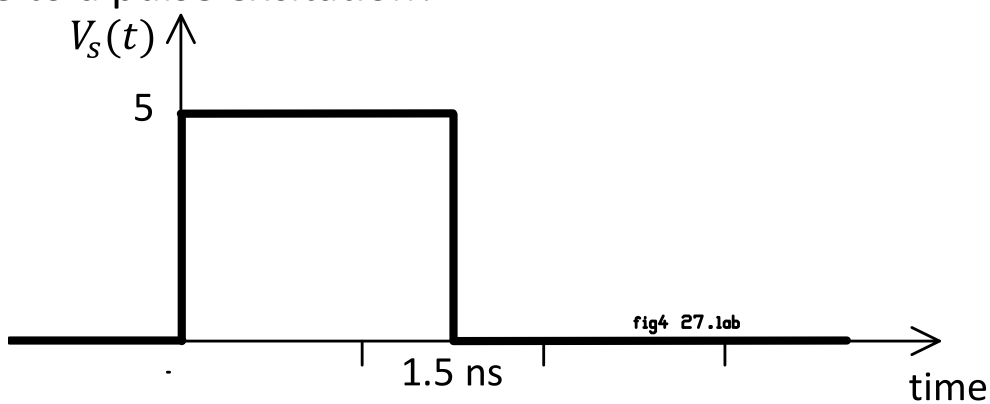




We calculated the step response:



How do we calculate response to a pulse excitation?



$$V_s(t) = 5u(t) - 5u(t-1.5)$$

Construct a unit pulse excitation from two step functions:

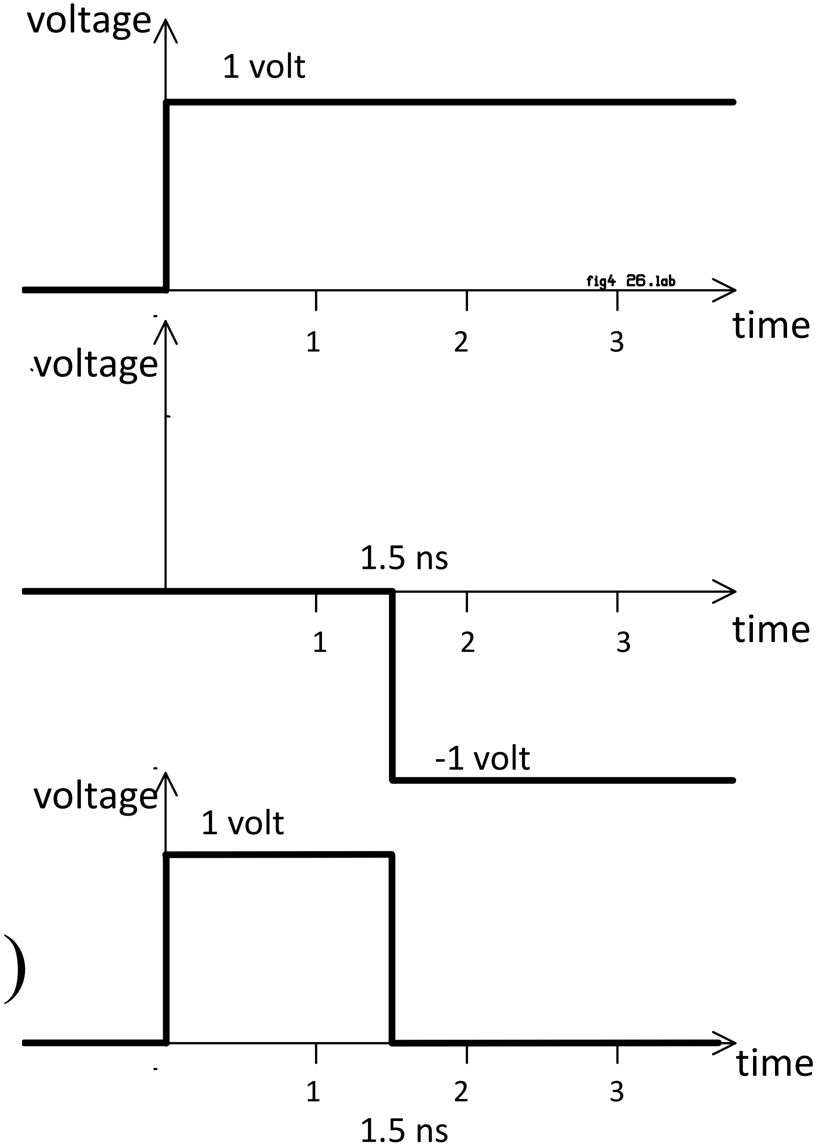
$$u(t)$$

$$+$$

$$-u(t-1.5)$$

$$=$$

$$u(t)-u(t-1.5)$$



Construct the Pulse Response from the Step Response

The response to a unit step function $u(t)$ is $V_{step}(t)$

The response to $u(t-1.5)$ is $V_{step}(t-1.5)$

The response to a unit pulse function $(u(t) - u(t-1.5))$ is $(V_{step}(t) - V_{step}(t-1.5))$

The response to $5u(t)-5u(t-1.5)$ is $5V_{step}(t)-5V_{step}(t-1.5)$

So the response to the pulse $5(u(t) - u(t-1.5))$ is

$$V_{pulse}(t) = 5[V_{step}(t) - V_{step}(t-1.5)]$$

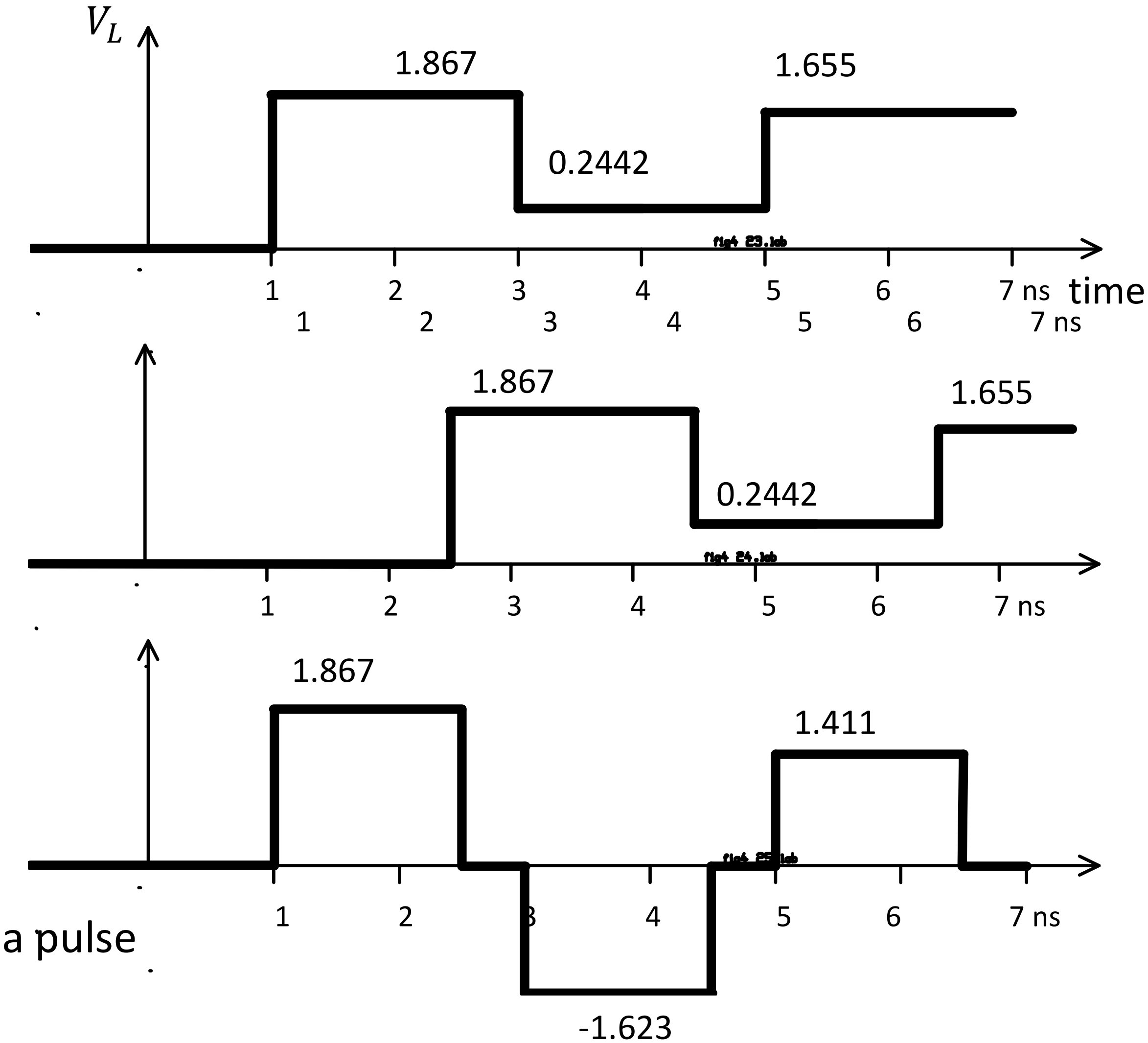
We can use this formula as a recipe for constructing the pulse response.

Construct the pulse response: $V_{pulse}(t) = 5[V_{step}(t) - V_{step}(t-1.5)]$

Unit step $V_{step}(t)$

$V_{step}(t-1.5)$

$V_{step}(t) - V_{step}(t-1.5)$

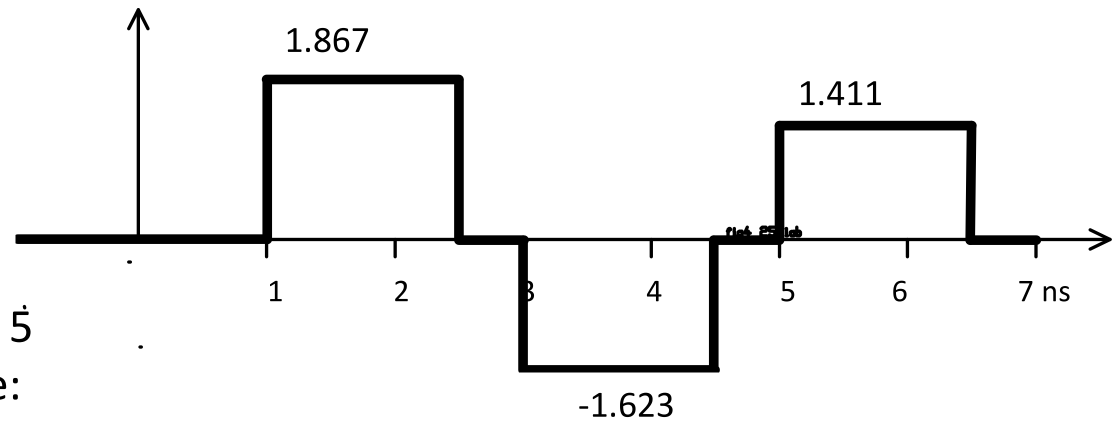


This is the response to a pulse of unit amplitude.

Multiply the unit pulse response by five: $V_{pulse}(t) = 5[V_{step}(t) - V_{step}(t-1.5)]$

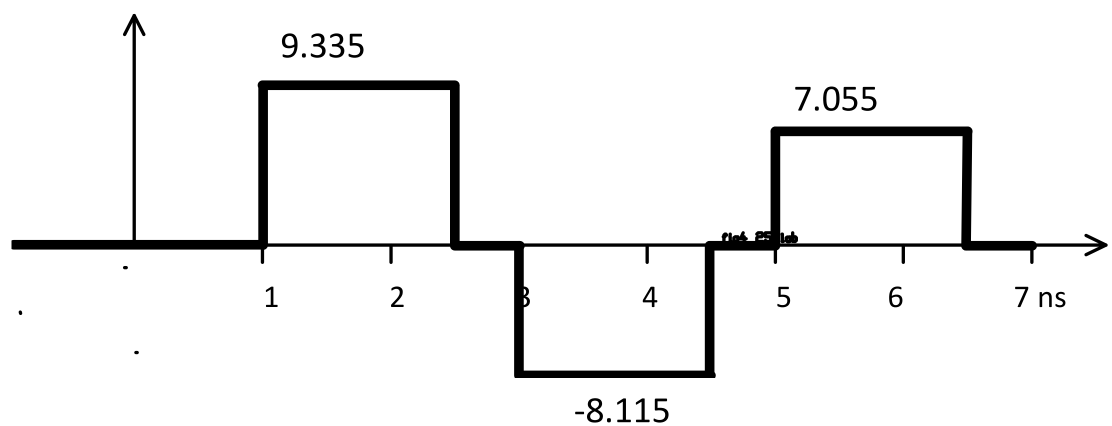
Unit pulse response:

$$V_{step}(t) - V_{step}(t-1.5)$$



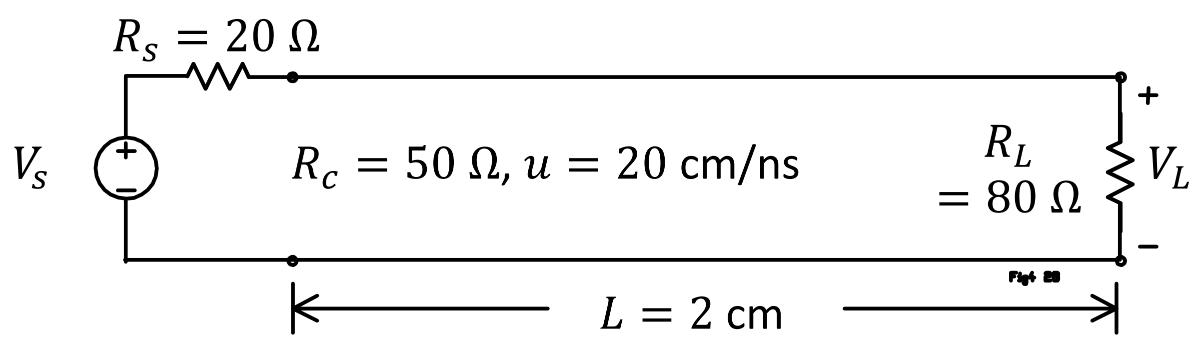
Multiply the unit pulse response by 5
to get the response to a 5 volt pulse:

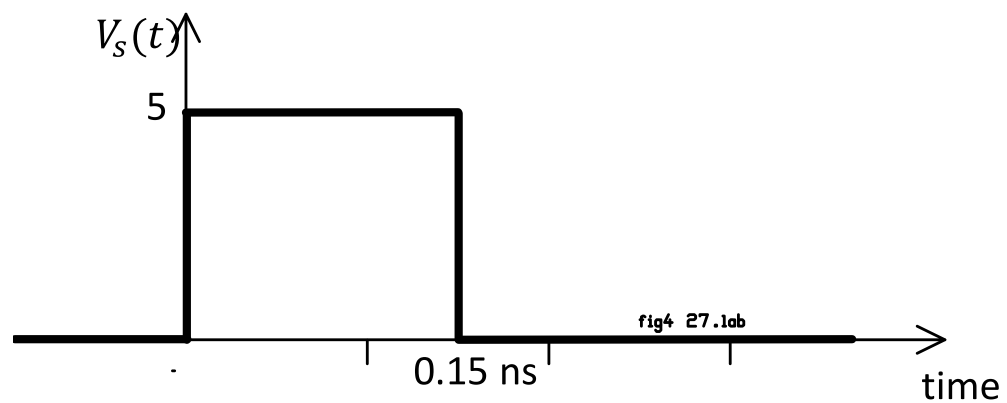
$$V_{pulse}(t) = 5[V_{step}(t) - V_{step}(t-1.5)]$$

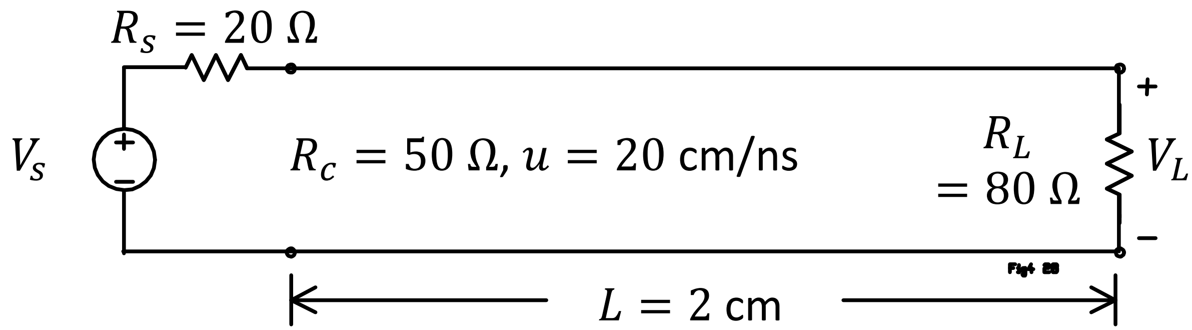


Tracking the Leading Edge and the Trailing Edge

Another way to find the pulse response is to draw a bounce diagram that keeps track of the leading edge of the pulse and of the trailing edge of the pulse.







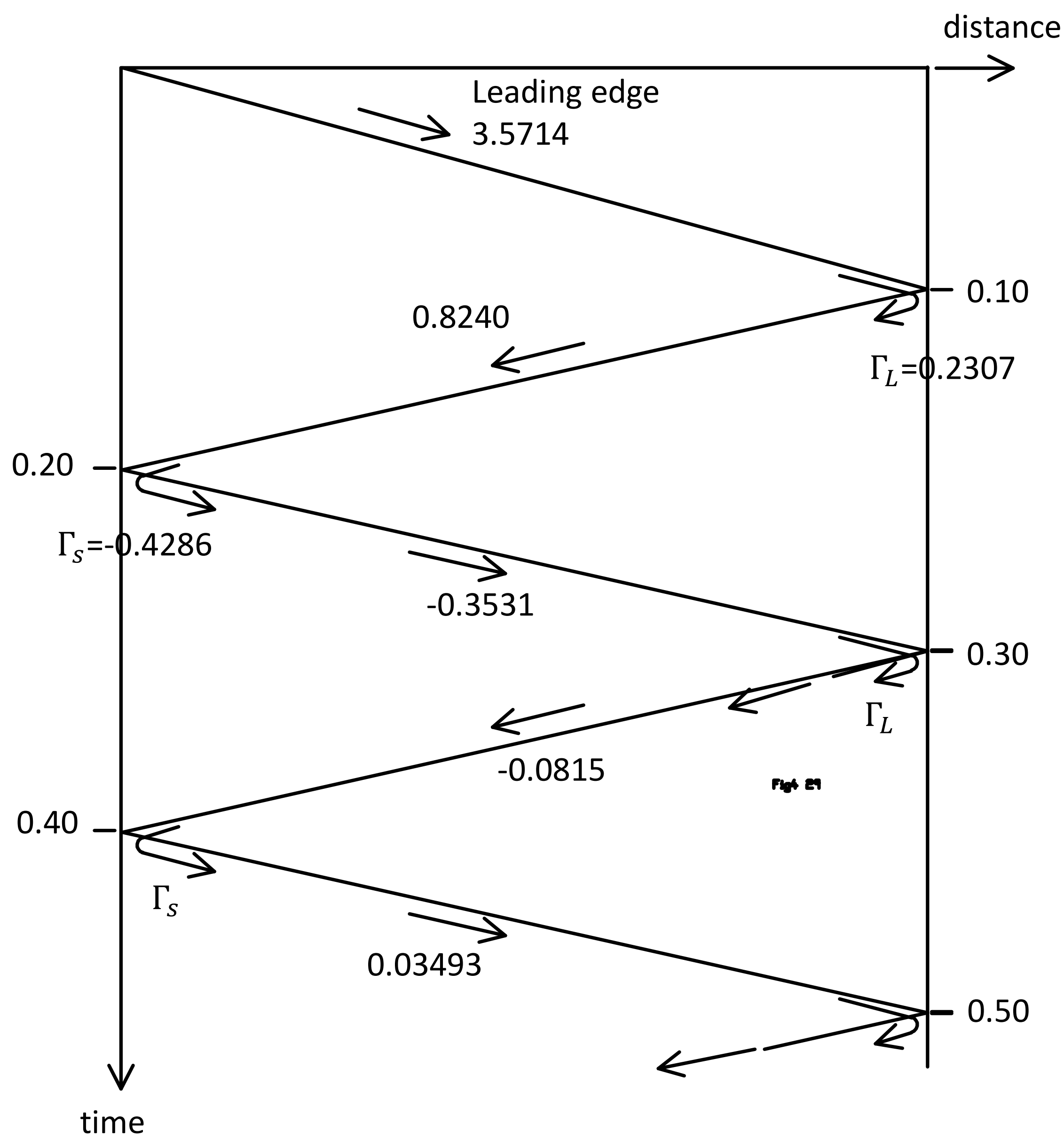
$$T_d = \frac{L}{u} = \frac{2}{20} = 0.1 \text{ ns.}$$

$$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{20-50}{20+50} = \text{-}0.4286$$

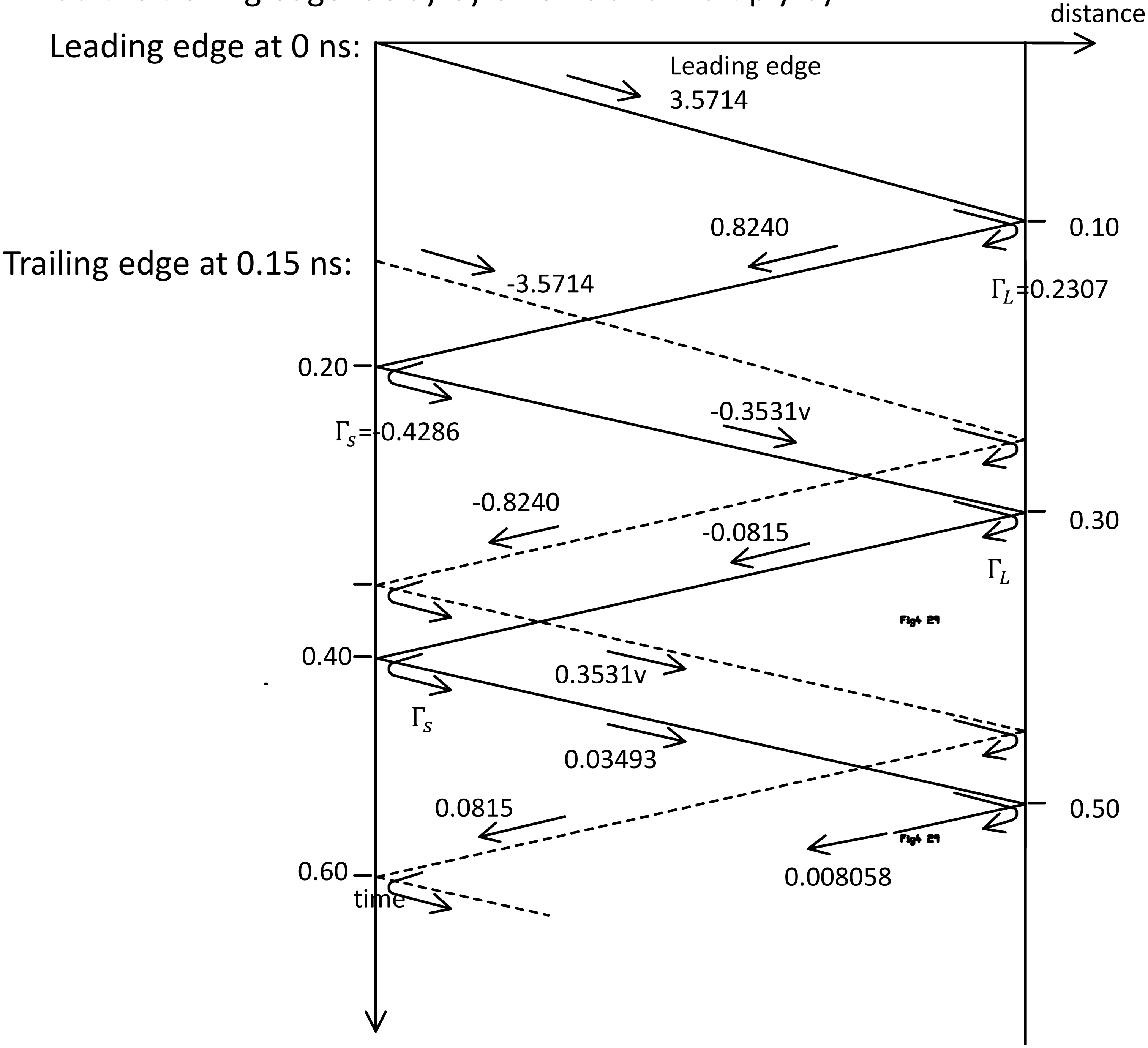
$$\Gamma_L = \frac{R_L - R_c}{R_L + R_c} = \frac{80-50}{80+50} = 0.2307$$

$$V(0,t) = \frac{R_c V_x}{R_c + R_s} = \frac{50x5}{50+20} = 3.57 \text{ volts.}$$

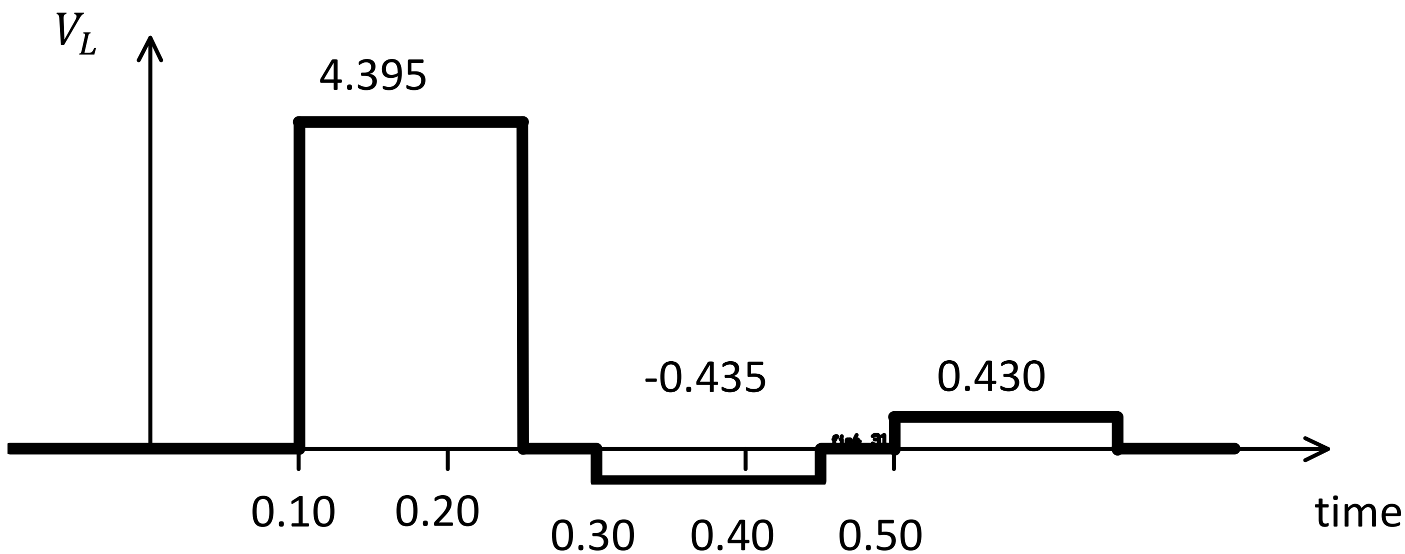
Track the leading edge of the pulse to find the step response:



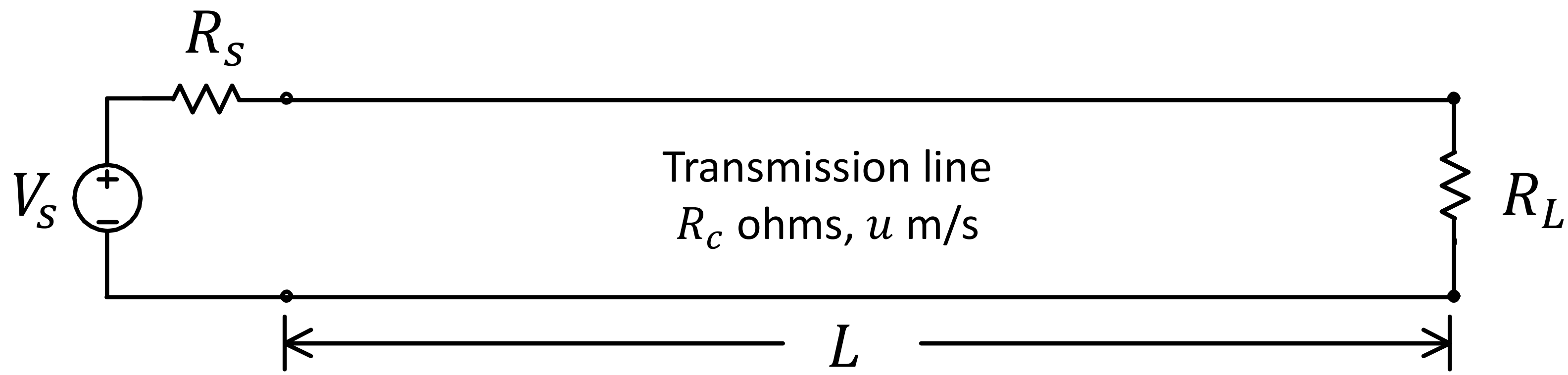
Add the trailing edge: delay by 0.15 ns and multiply by -1.

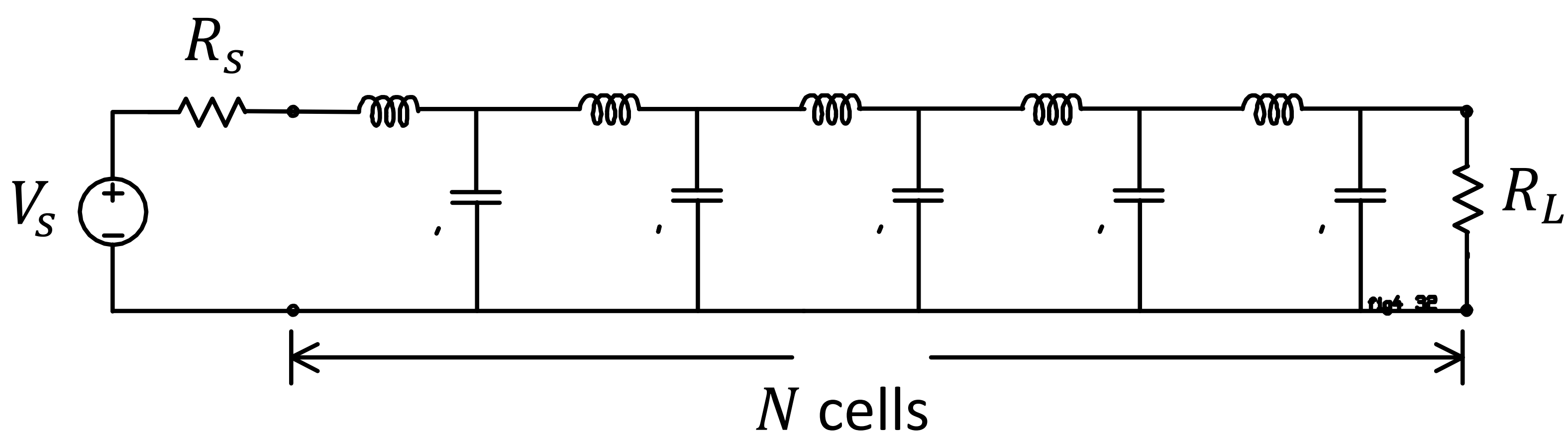


To find the voltage at the load, go down the time axis starting at the top at t=0:

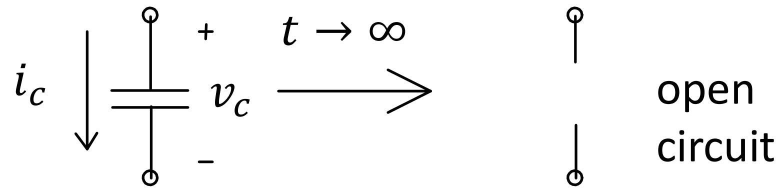


Final Values in Transmission-Line Circuits



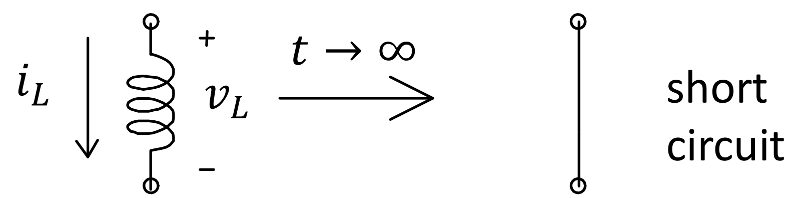


Final Values for Capacitors and Inductors



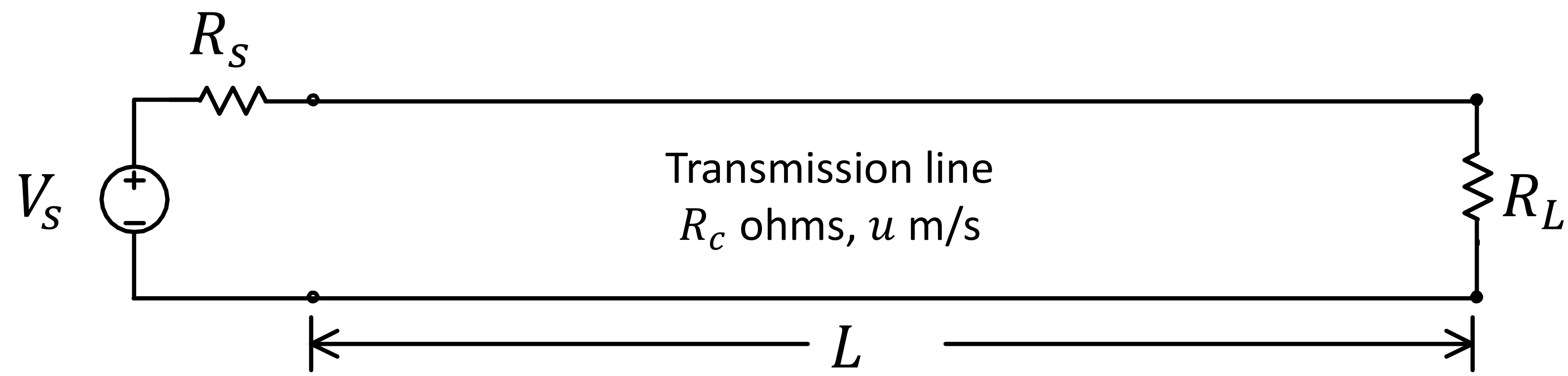
$$i_c = C\frac{dv_c}{dt}$$

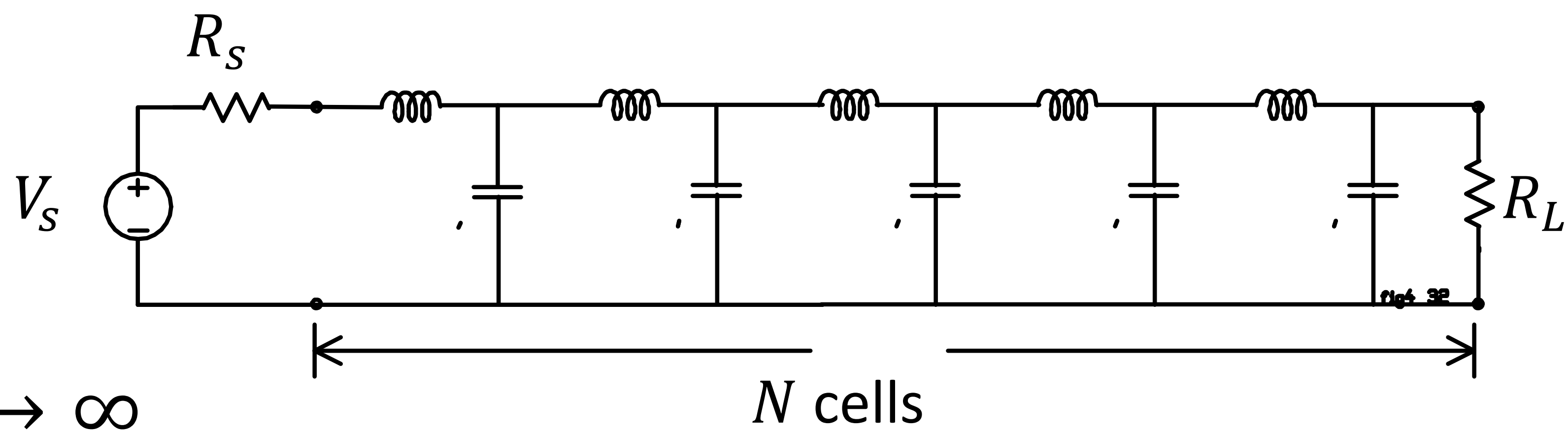
As $t \to \infty$, $\frac{dv_c}{dt} \to 0$ and $i_c = C\frac{dv_c}{dt} \to 0$ and the capacitor becomes an open circuit.



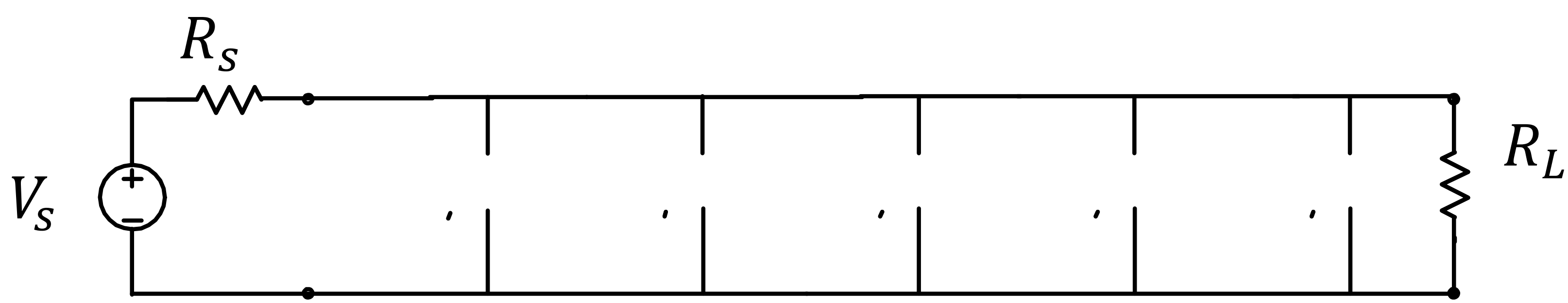
$$v_L = L\frac{di_L}{dt}$$

As $t \to \infty$, $\frac{di_L}{dt} \to 0$ and $v_L = L\frac{di_L}{dt} \to 0$ and the inductor becomes a short circuit.



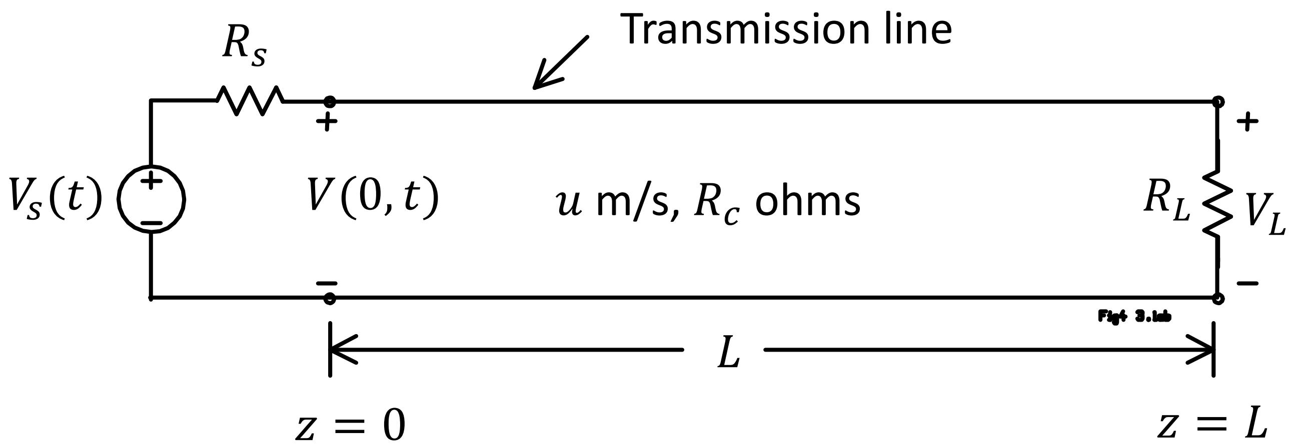


$t \to \infty$

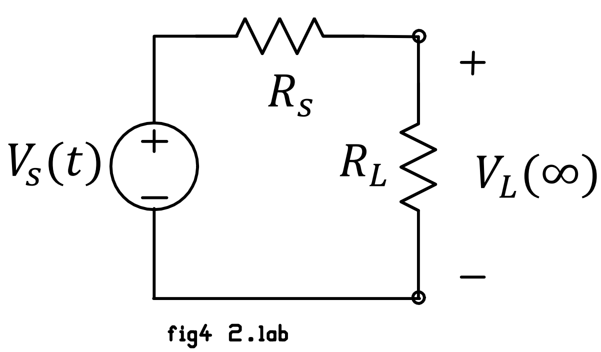


To calculate the final value of the voltages and currents, make C's open circuits and L's short circuits.

Final Values Circuit

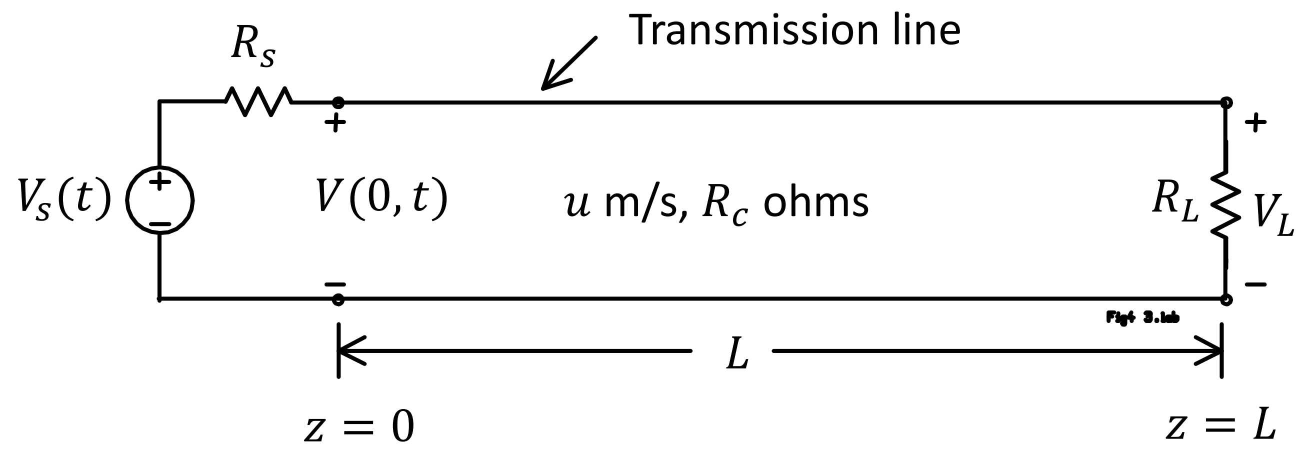


- Replace L by a short circuit
- Replace C by an open circuit
- Hence, the transmission line becomes an ideal short circuit.



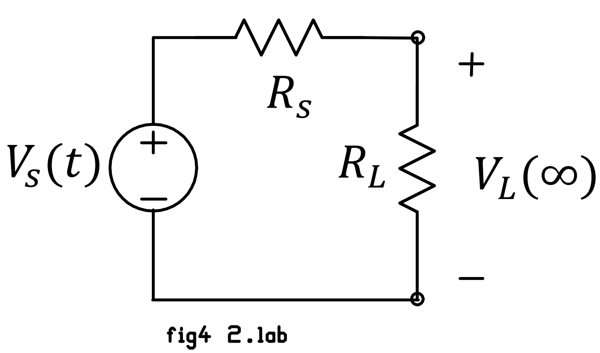
$$V_L(\infty) = \frac{R_L}{R_s + R_L} V_s(\infty)$$

Example



The generator $V_s(t)$ is a step function of height 5 volts starting at $t = 0$. The internal resistance is $R_s = 10$ ohms and the load resistor is $R_L = 100$ ohms. Find the final value of the voltage across the load V_L.

Final Values Circuit: the transmission line behaves like an ideal short circuit.



$$V_L(\infty) = \frac{R_L}{R_s+R_L} V_s(\infty) = \frac{100}{10} x5 = \frac{100}{110} x5 = 4.55 \text{ volts}$$