ELEC353 Lecture Notes Set 18

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #10: Do homework #10 by March 29, 2019.
Homework #11: Do homework #11 by April 5, 2019.
Homework #12: Do homework #12 by April 12, 2019.

Tutorial Workshop #11: Friday March 29, 2019.
Tutorial Workshop #12: Friday April 5, 2019.
Tutorial Workshop #13: Friday April 12, 2019

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

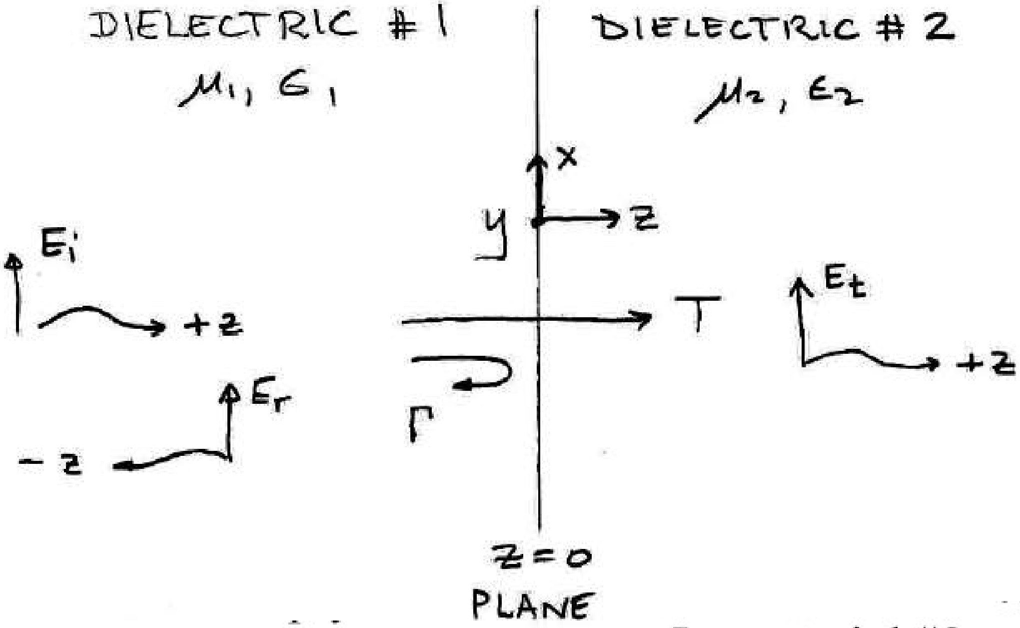
Topics to be Covered

Plane Waves

- Maxwell's Equations and the Wave Equation - done
- Plane waves - done
- Material Boundaries - done
- Transmission Through a Wall - today

Antennas

Review: Reflection from a Material Boundary



In material #1,

$$E_{1x} = E_i e^{-j\beta_1 z} + E_r e^{j\beta_1 z}$$

$$H_{1y} = \frac{1}{\eta_1} E_i e^{-j\beta_1 z} - \frac{1}{\eta_1} E_r e^{j\beta_1 z}$$

In material #2,

$$E_{2x} = E_t e^{-j\beta_2 z}$$

$$H_{2y} = \frac{1}{\eta_2} E_t e^{-j\beta_2 z}$$

What are the fields very near the interface between the two dielectrics?
At z=0:

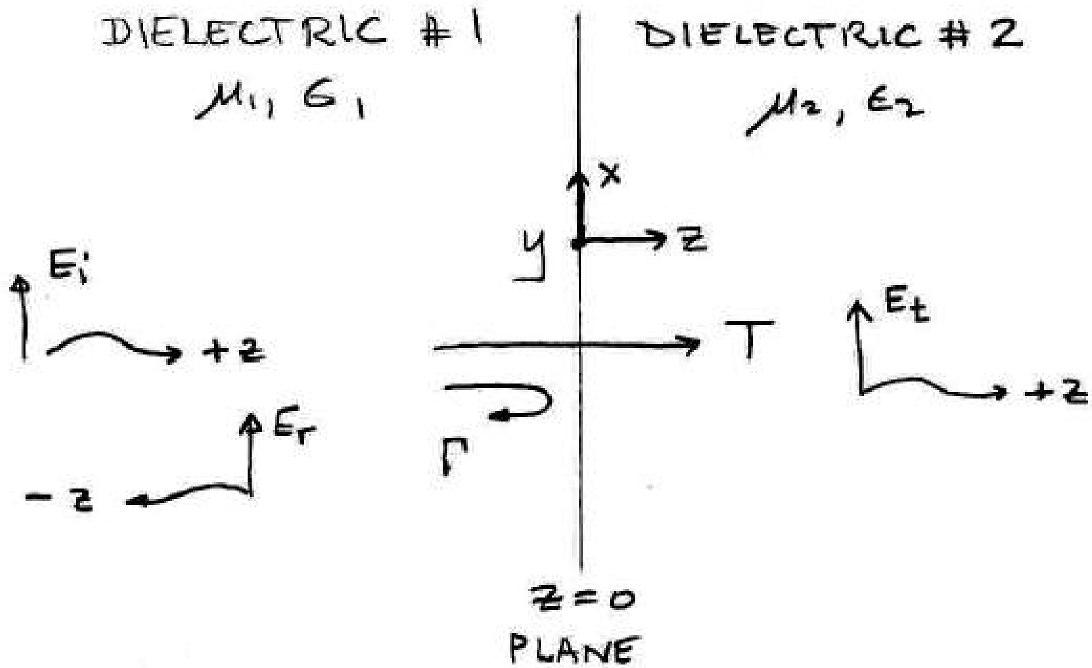
$$E_{1x} = E_i + E_r$$

$$H_{1y} = \frac{1}{\eta_1} E_i - \frac{1}{\eta_1} E_r$$

$$E_{2x} = E_t$$

$$H_{2y} = \frac{1}{\eta_2} E_t$$

Review: Reflection and Transmission Coefficients



Boundary Conditions:

$$E_i + E_r = E_t$$

and

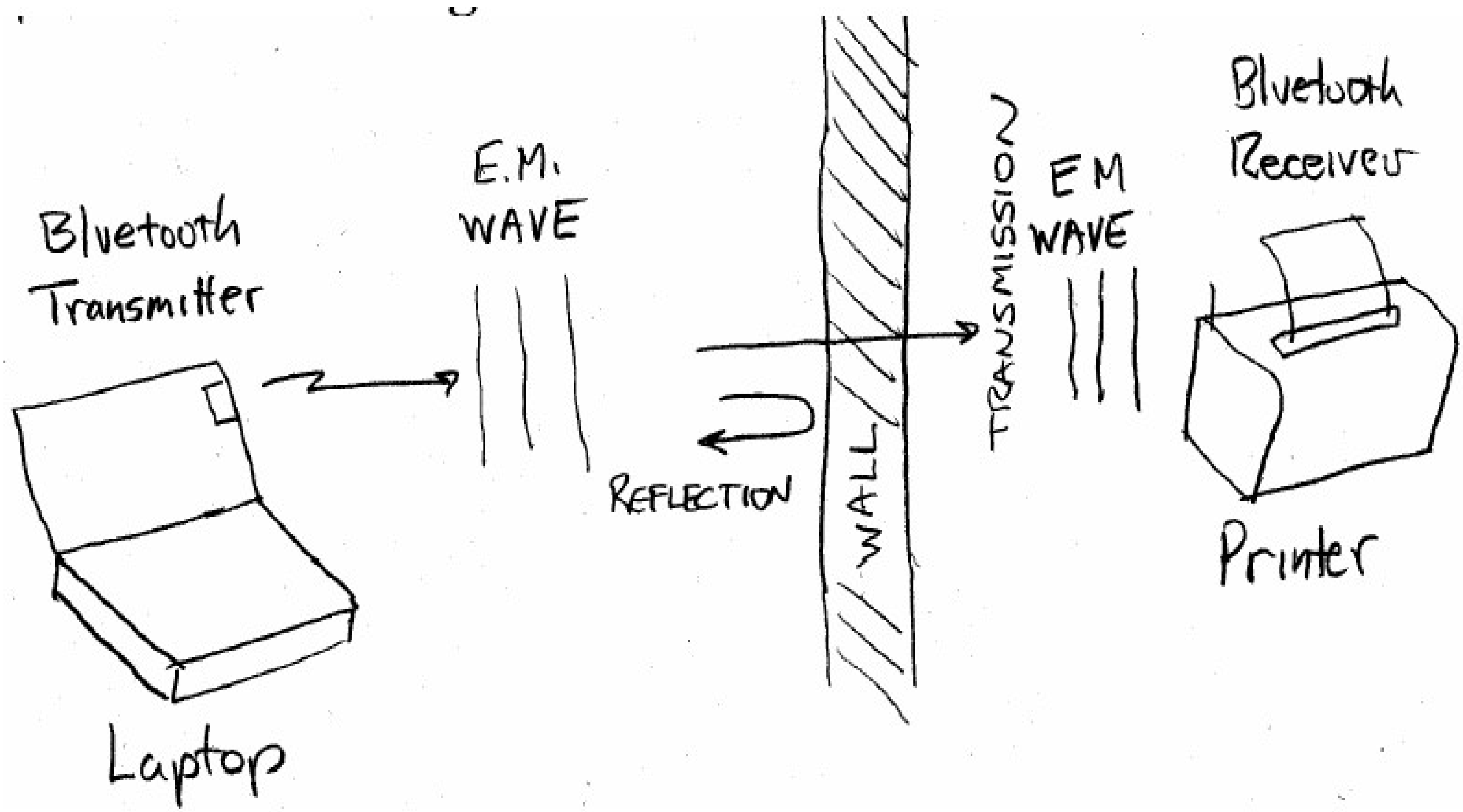
$$\frac{1}{\eta_1}E_i - \frac{1}{\eta_1}E_r = \frac{1}{\eta_2}E_t$$

Solve the equations:

$$E_r = \Gamma E_i \qquad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1};$$

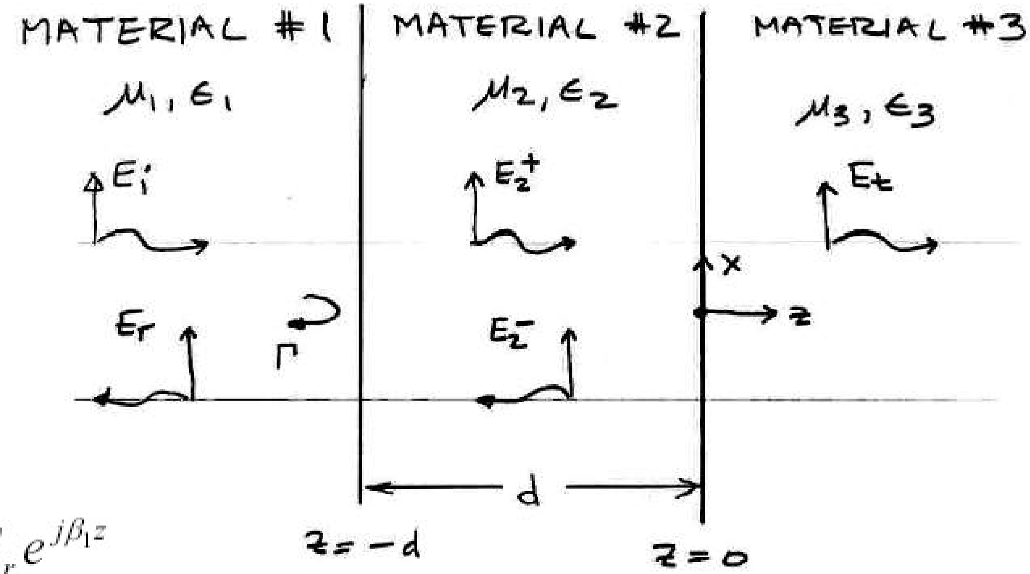
$$E_t = T E_i \qquad T = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Transmission Through a Wall



Transmission through a Wall

(Inan and Inan Section 8.2.3 page 705)



$$E_{1x} = E_i\, e^{-j\beta_1 z} + E_r e^{j\beta_1 z}$$

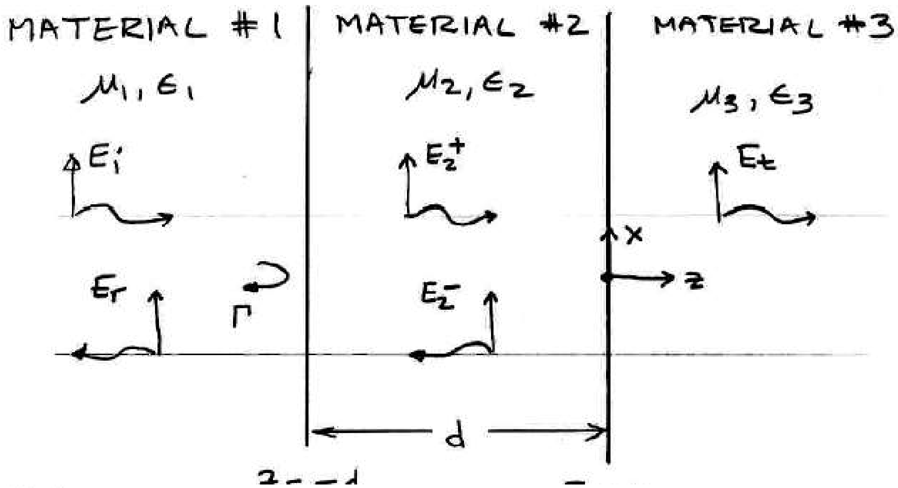
$$H_{1y} = \frac{1}{\eta_1} E_i\, e^{-j\beta_1 z} - \frac{1}{\eta_1} E_r e^{j\beta_1 z}$$

$$E_{2x} = E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}$$

$$H_{2y} = \frac{1}{\eta_2} E_2^+ e^{-j\beta_2 z} - \frac{1}{\eta_2} E_2^- e^{j\beta_2 z}$$

$$E_{3x} = E_t\, e^{-j\beta_3 z}$$

$$H_{3y} = \frac{1}{\eta_3} E_t\, e^{-j\beta_3 z}$$



The boundary conditions are:

- the electric field must be continuous across a material interface
- the magnetic field must be continuous across a material interface

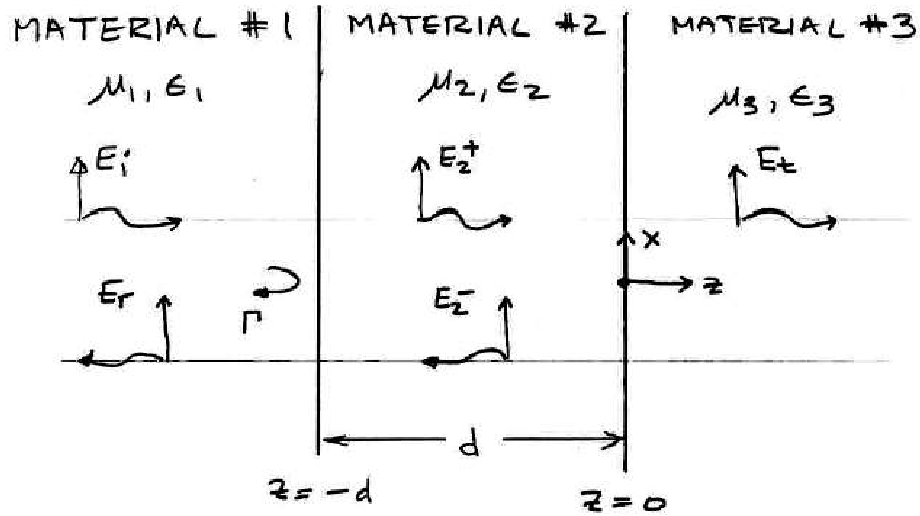
$$z = -d$$

$$E_{1x}(z=-d) = E_{2x}(z=-d)$$

$$E_i\, e^{-j\beta_1(-d)} + E_r\, e^{j\beta_1(-d)} = E_2^+ e^{-j\beta_2(-d)} + E_2^- e^{j\beta_2(-d)}$$ Equation 1

$$H_{1y}(z=-d) = H_{2y}(z=-d)$$

$$\frac{1}{\eta_1} E_i\, e^{-j\beta_1(-d)} - \frac{1}{\eta_1} E_r\, e^{j\beta_1(-d)} = \frac{1}{\eta_2} E_2^+ e^{-j\beta_2(-d)} - \frac{1}{\eta_2} E_2^- e^{j\beta_2(-d)}$$ Equation 2



$$z = 0$$

$$E_{2x}(z=0) = E_{3x}(z=0)$$

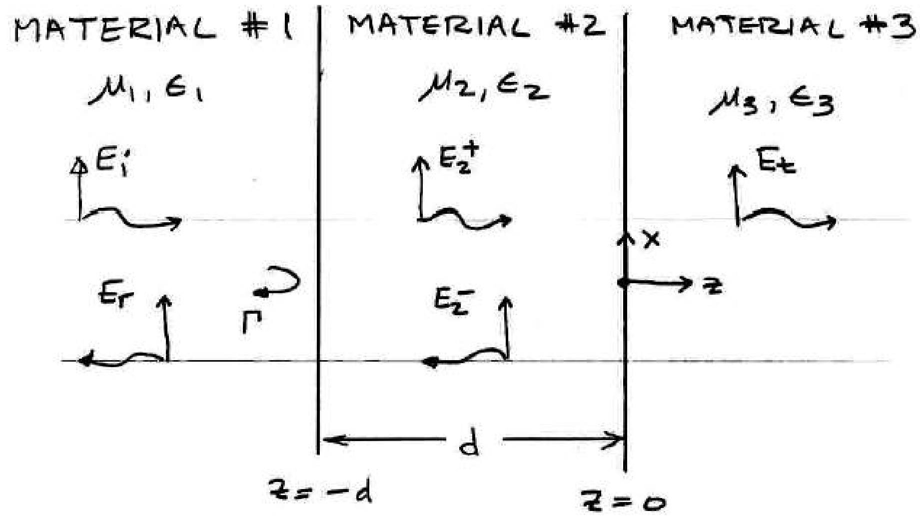
$$E_2^+ e^{-j\beta_2(0)} + E_2^- e^{j\beta_2(0)} = E_t\, e^{-j\beta_3(0)}$$

$$E_2^+ + E_2^- = E_t$$ Equation 3

$$H_{2y}(z=0) = H_{3y}(z=0)$$

$$\frac{1}{\eta_2} E_2^+ e^{-j\beta_2(0)} - \frac{1}{\eta_2} E_2^- e^{j\beta_2(0)} = \frac{1}{\eta_3} E_t\, e^{-j\beta_3(0)}$$

$$\frac{1}{\eta_2} E_2^+ - \frac{1}{\eta_2} E_2^- = \frac{1}{\eta_3} E_t$$ Equation 4



We can solve the set of four equations in four unknowns to find the reflection coefficient $\Gamma = \dfrac{E_r}{E_i}$ and the transmission coefficient $T = \dfrac{E_t}{E_i}$:

$$\Gamma = \frac{(\eta_2 - \eta_1)(\eta_3 + \eta_2) + (\eta_2 + \eta_1)(\eta_3 - \eta_2)e^{-j2\beta_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}$$

$$T = \frac{4\eta_2\eta_3 e^{-j\beta_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}$$

Remark: **lossy materials**

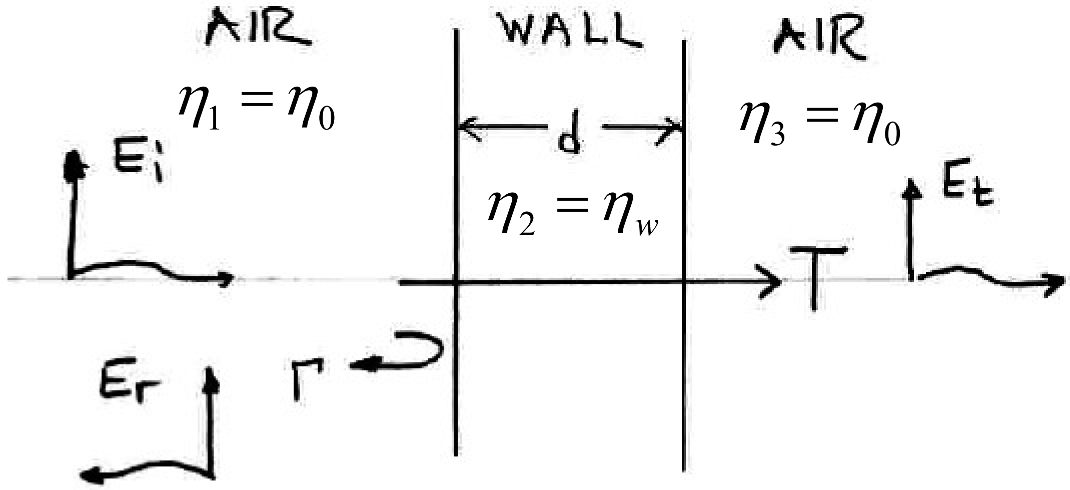
- Although we assumed the materials were lossless with zero conductivity, the formulas are in fact correct for lossy materials as well.
- If the materials are lossy, use the "complex permittivity" $\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$ when calculating the intrinsic impedances η_1, η_2, η_3.

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} = \sqrt{\frac{j\omega\mu_2}{j\omega\left(\varepsilon_2 - j\frac{\sigma_2}{\omega}\right)}} = \sqrt{\frac{\mu_2}{\varepsilon_{c2}}}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$
$$\gamma = \sqrt{j\omega\mu j\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)}$$
$$\gamma = \sqrt{j\omega\mu j\omega\varepsilon_c}$$
$$\gamma = j\omega\sqrt{\mu\varepsilon_c}$$

$$\Gamma = \frac{(\eta_2 - \eta_1)(\eta_3 + \eta_2) + (\eta_2 + \eta_1)(\eta_3 - \eta_2)e^{-2\gamma_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2\gamma_2 d}}$$
$$T = \frac{4\eta_2\eta_3 e^{-\gamma_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2\gamma_2 d}}$$

Air-Wall-Air Interface



$$\Gamma = \frac{(\eta_2 - \eta_1)(\eta_3 + \eta_2) + (\eta_2 + \eta_1)(\eta_3 - \eta_2)e^{-j2\beta_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}$$

$$T = \frac{4\eta_2\eta_3 e^{-j\beta_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}$$

$$\eta_1 = \eta_0$$

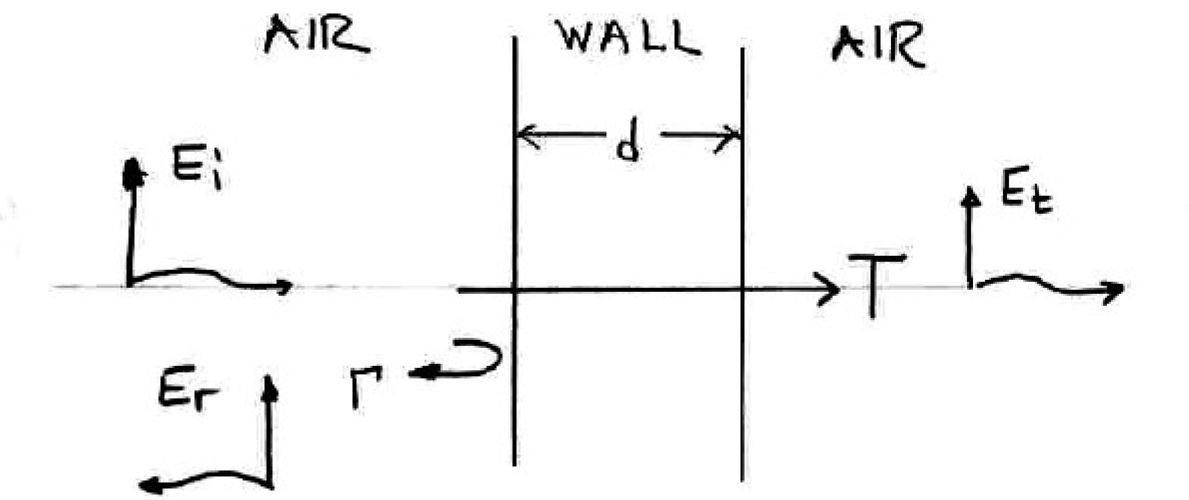
$$\eta_2 = \eta_w \qquad \beta_2 = \beta_w$$

$$\eta_3 = \eta_0$$

$$\Gamma = \frac{\left(\eta_0^2 - \eta_w^2\right)\left(e^{-2j\beta_w d} - 1\right)}{(\eta_0 + \eta_w)^2 - (\eta_0 - \eta_w)^2 e^{-2j\beta_w d}}$$

$$T = \frac{4\eta_0\eta_w e^{-j\beta_w d}}{(\eta_0 + \eta_w)^2 - (\eta_0 - \eta_w)^2 e^{-2j\beta_w d}}$$

Example: Indoor Propagation



A Bluetooth transmitter at 2450 MHz radiates field strength of $E_i = 3$ V/m evaluated at the surface of the wall in the figure above. The wave is normally incident on the surface of a wall of thickness $d = 14$ cm. The wall is made of brick with $\varepsilon_r = 5.1$ and $\sigma = 0$. Find the field strength transmitted through the wall.

Solution

- In the air, $\eta_0 = 377$ ohms
- In the wall, $\eta_w = \dfrac{\eta_0}{\sqrt{\varepsilon_r}} = \dfrac{377}{\sqrt{5.1}} = 167$ ohms
- In the wall, $\beta_w = \dfrac{\omega}{u_w} = \dfrac{2\pi f}{u_w}$ where the speed of travel in the wall material is

$$u_w = \frac{c}{\sqrt{\varepsilon_r}} = \frac{300}{\sqrt{5.1}} = 132.8 \text{ meters per microsecond}$$

- Evaluate $\beta_w = \dfrac{2\pi f}{u_w} = \dfrac{2\pi \cdot 2450x10^6}{132.8x10^6} = 115.9$ radians/meter = 6,642 deg/meter

Thickness $d = 14$ cm, so $\beta_w d = 6{,}642x0.14 = 929.8$ degrees.
Use angles between -180 and 180 degrees:

$$\beta_w d = 929.8° \rightarrow -150.2°$$

$$e^{-j\beta_w d} = 1\angle 150.2°$$

$$e^{-2j\beta_w d} = 1\angle\left(-2x - 150.2\right) = 1\angle 300.4° \rightarrow 1\angle -59.6°$$

$$\eta_0 = 377 \qquad \eta_w = 167 \qquad e^{-j\beta_w d} = 1\angle 150.2^\circ \qquad e^{-2j\beta_w d} = 1\angle -59.6^\circ$$

Evaluate the transmission coefficient: (messy!)

$$T = \frac{4\eta_0\eta_w e^{-j\beta_w d}}{(\eta_0 + \eta_w)^2 - (\eta_0 - \eta_w)^2 e^{-2j\beta_w d}}$$

$$T = \frac{4\cdot 377\cdot 167\cdot 1\angle 150.2^\circ}{(377+167)^2 - (377-167)^2 \cdot 1\angle -59.6^\circ}$$

$$T = \frac{251{,}836\angle 150.2^\circ}{295{,}936 - 44{,}100\angle -59.6^\circ}$$

$$T = \frac{251{,}836\angle 150.2^\circ}{276{,}251\angle 7.9^\circ} = 0.9116\angle -217.7^\circ = 0.9116\angle 142.3^\circ$$

Evaluate the transmitted wave complex amplitude:

$$E_t = TE_i = (0.9116\angle 142.3^\circ)\cdot 3 = 2.73\angle 142.3^\circ$$

The amplitude of the transmitted wave is

$$|E_t| = |T||E_i| = 0.9116\cdot 3 = 2.73 \text{ V/m.}$$

Transmission Loss

$$TL = -20\log|T| = -20\log(0.9116) = 0.8 \quad \text{dB}$$

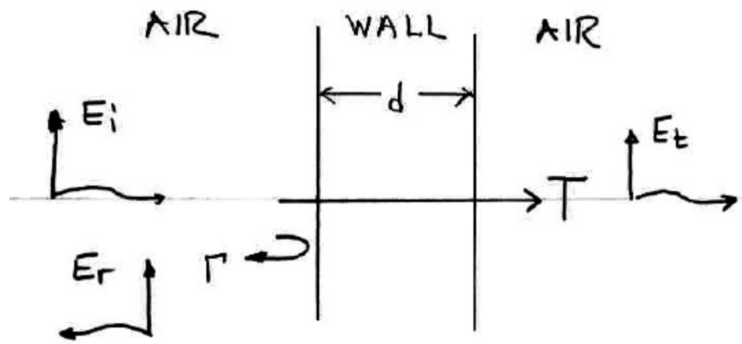
The incident field in decibels is $E_i^{dB} = 20\log E_i = 20\log(3) = 9.54 \text{ dB}$

The transmitted field in dB:

$$E_t^{dB} = 20\log|E_t| = 20\log(|T||E_i|) = 20\log(E_i) + 20\log|T|$$

$$E_t^{dB} = E_i^{dB} - TL = 9.54 - 0.8 = 8.74 \text{ dB}$$

Perfectly-Transparent Walls: The "Radome Effect"



- If all of the wave is transmitted, then none of the wave is reflected and so we must have a reflection coefficient of zero:

$$\Gamma = \frac{(\eta_0^2 - \eta_w^2)(e^{-2j\beta_w d} - 1)}{(\eta_0 + \eta_w)^2 - (\eta_0 - \eta_w)^2 e^{-2j\beta_w d}} = 0$$

- We can make the reflection coefficient zero by making

$$e^{-2j\beta_w d} - 1 = 0$$
$$e^{-2j\beta_w d} + e^{-j\pi} = 0$$

- The two terms cancel when the difference in phase between them is 180 degrees or π radians:

$$-2\beta_w d - (-\pi) = \pi \pm 2n\pi$$
$$-2\beta_w d + \pi = \pi \pm 2n\pi$$
$$-2\beta_w d = \pm 2n\pi$$

$$-2\beta_w d = \pm 2n\pi$$

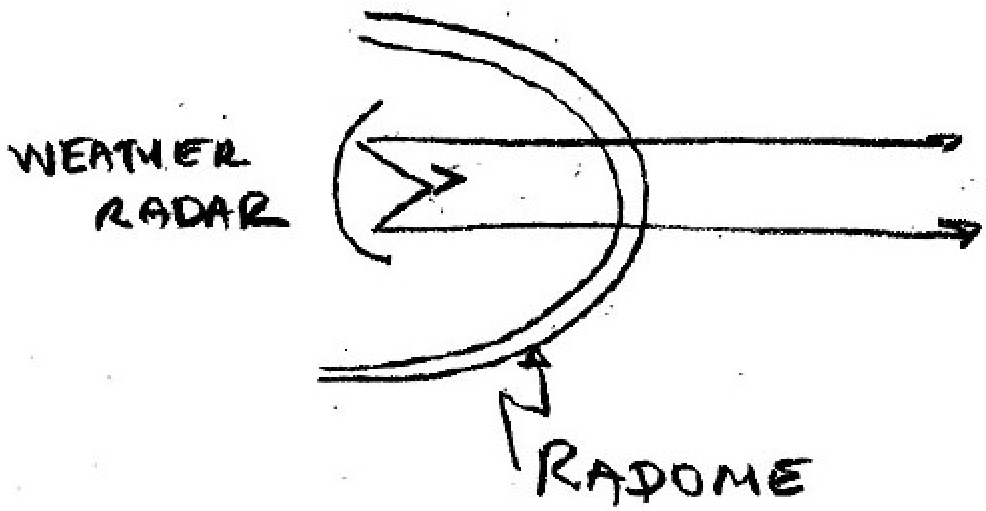
Since d must be positive,

$$2\beta_w d = 2n\pi$$

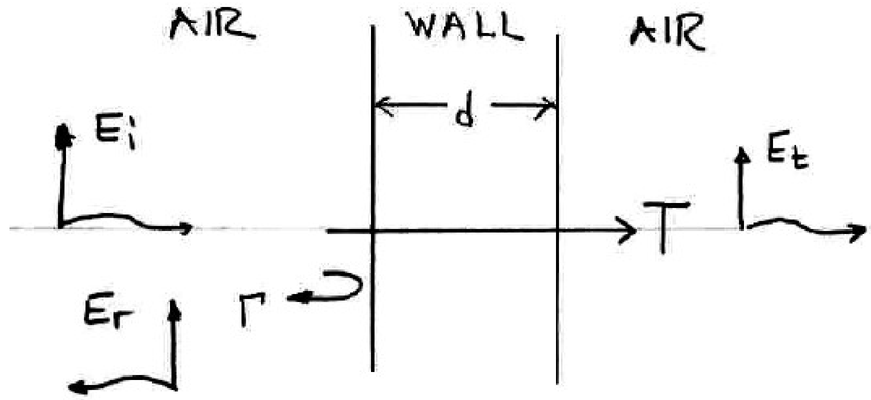
$$\beta_w d = n\pi$$

$$d = \frac{n\pi}{\beta_w} = \frac{n\pi}{2\pi/\lambda_w} = n\frac{\lambda_w}{2}$$

So when the thickness of the wall is integer multiples of the half-wavelength, the wall is perfectly transparent!



- A “radome” is a plastic cover that goes over an antenna to protect it from the weather.
- For example, the nose of an aircraft contains a weather radar antenna that looks ahead of the aircraft for thunderstorms. The radar uses a small dish antenna.
- The antenna is protected by a plastic radome that forms the nose of the aircraft.
- The radome material and the thickness of the radome are designed for a transmission coefficient close to unity.
- The radome is made a half-wavelength thick.
- The wall behaves like a radome: when it is integer multiples of the half-wavelength in thickness, it is perfectly transparent.



Example

For a brick wall with $\varepsilon_r = 5.1$ and σ =0, find the wall thicknesses d which make the wall perfectly transparent at 2450 MHz.

Solution

- Calculate the wavelength λ_w in brick at 2450 MHz.
- The speed of travel in brick is

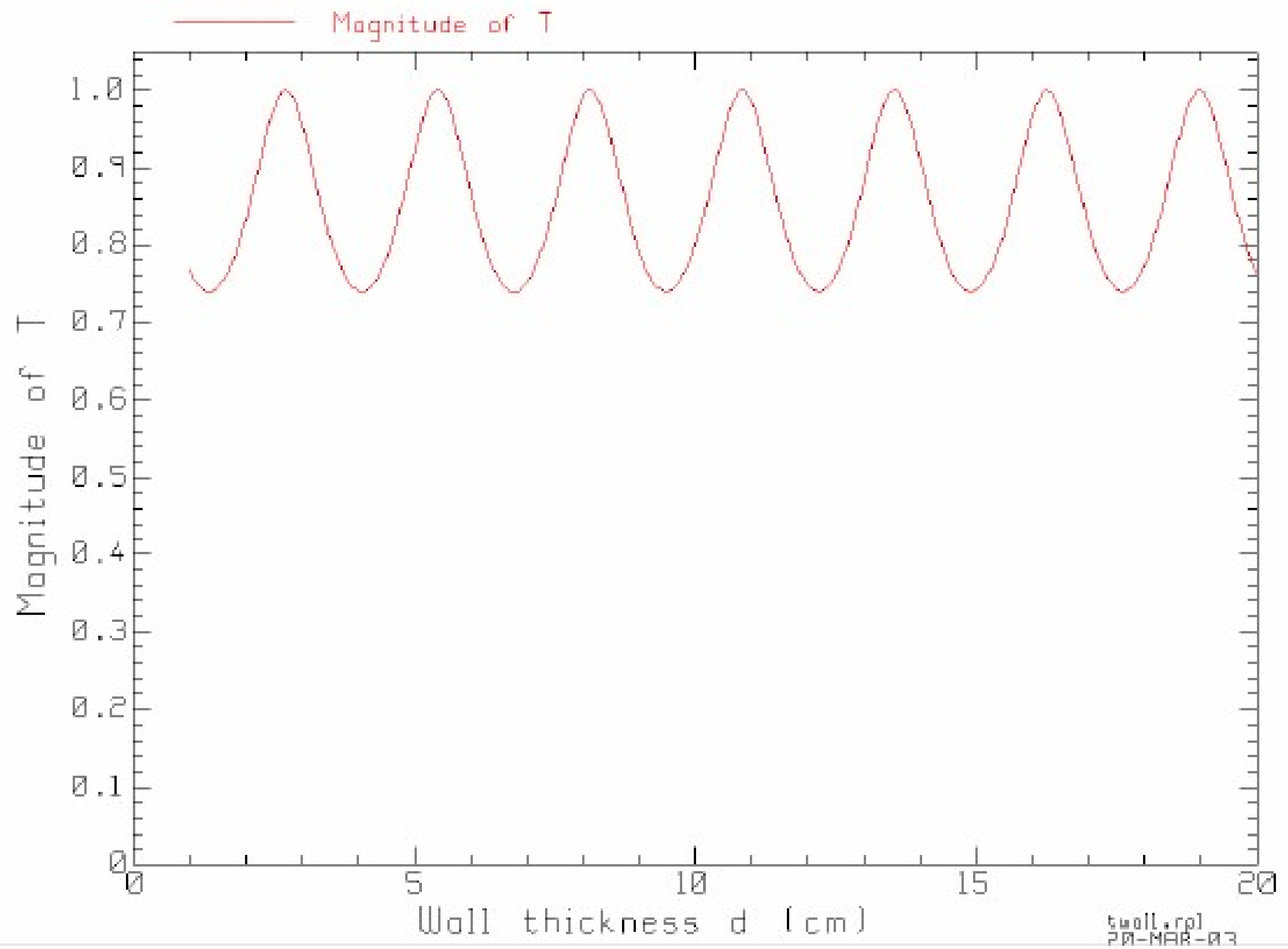
$$u_w = \frac{1}{\sqrt{\mu_0 \varepsilon_r \varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}} \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{300}{\sqrt{5.1}} = 132.8 \text{ meters per microsecond}$$

hence the wavelength in brick is

$$\lambda_w = \frac{u_w}{f} = \frac{132.8}{2450} = 5.422 \text{ cm}$$

- The wall thicknesses which make the wall transparent are integer multiples of the half-wavelength:

$$d = \frac{5.422}{2} = 2.711 \text{ cm, } 5.422 \text{ cm, } 8.133 \text{ cm, } 10.844 \text{ cm, } 13.555 \text{ cm, } \ldots$$



- It is instructive to graph $|T|$ as a function of the wall thickness:

$$|T(d)| = \left| \frac{4\eta_0 \eta_w e^{-j\beta_w d}}{(\eta_0 + \eta_w)^2 - (\eta_0 - \eta_w)^2 e^{-2j\beta_w d}} \right|$$

- It is easy to write a short computer program that evaluates this formula for various values of thickness d .

Remark: Lossy Materials

- Brick is *not* a lossless material with zero conductivity.
- The conductivity of brick is approximately σ = 10 mS/m and is dependent on the moisture content of the brick.
- At 2450 MHz, the loss tangent is

$$\tan\delta = \frac{\sigma}{\omega\varepsilon} = \frac{10x10^{-3}}{2\pi \cdot 2450x10^{6} \cdot 5.1 \cdot 8.854x10^{-12}} = 0.01439$$

- So brick is an insulator but not a really good one!
- The propagation constant is $\gamma = 0.8341 + j116.0$ so the attenuation constant is $\alpha = 0.8341$ Np/m
- The penetration depth is $\delta = \frac{1}{\alpha} = 1.19$ m
- So the conductivity of the wall is not the dominant factor in assessing the transmission coefficient.
- We can evaluate the complex value of η_w for the lossy brick material:

$$\eta_w = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon_r\varepsilon_0}}$$

and the phase constant using

$$\gamma_w = \sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_r\varepsilon_0)}$$

- Then the transmission coefficient is given by

$$T = \frac{4\eta_0\eta_w e^{-\gamma_w d}}{(\eta_0+\eta_w)^2 - (\eta_0-\eta_w)^2 e^{-2\gamma_w d}}$$

- Hence we can graph the magnitude of the transmission coefficient T as a function of wall thickness to obtain the following: