# ELEC353 Lecture Notes Set 19

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #11: Do homework #11 by April 5, 2019.
Homework #12: Do homework #12 by April 12, 2019.

Tutorial Workshop #12: Friday April 5, 2019.
Tutorial Workshop #13: Friday April 12, 2019

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

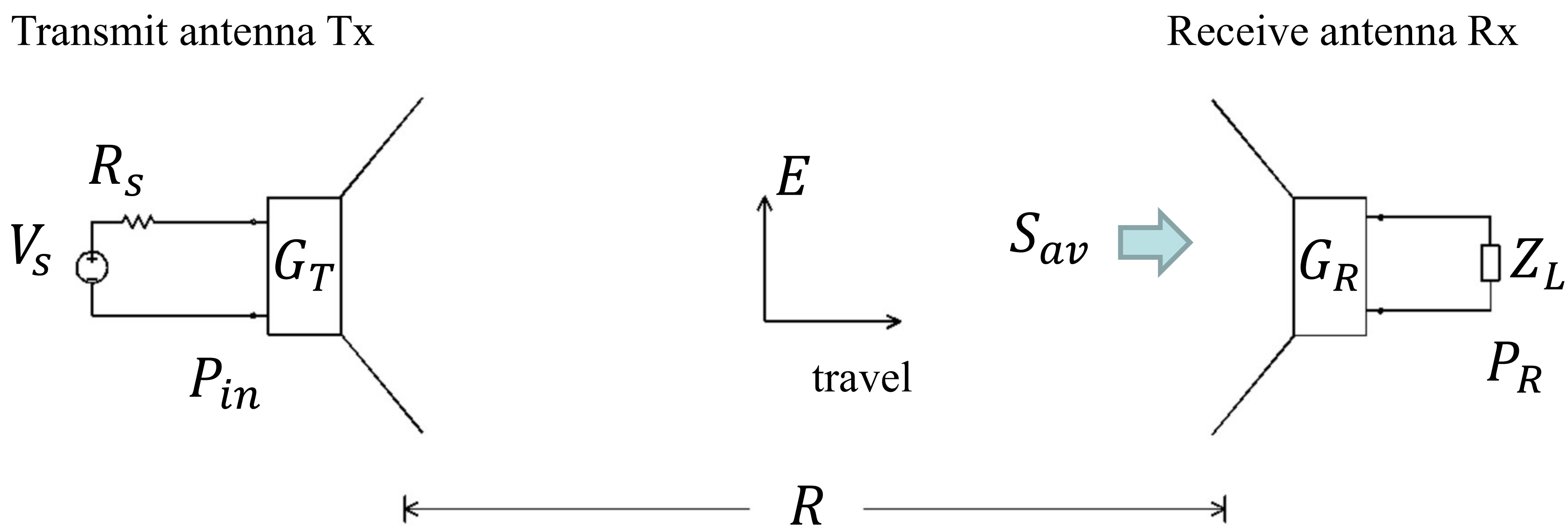
# Antenna Topics

Antennas

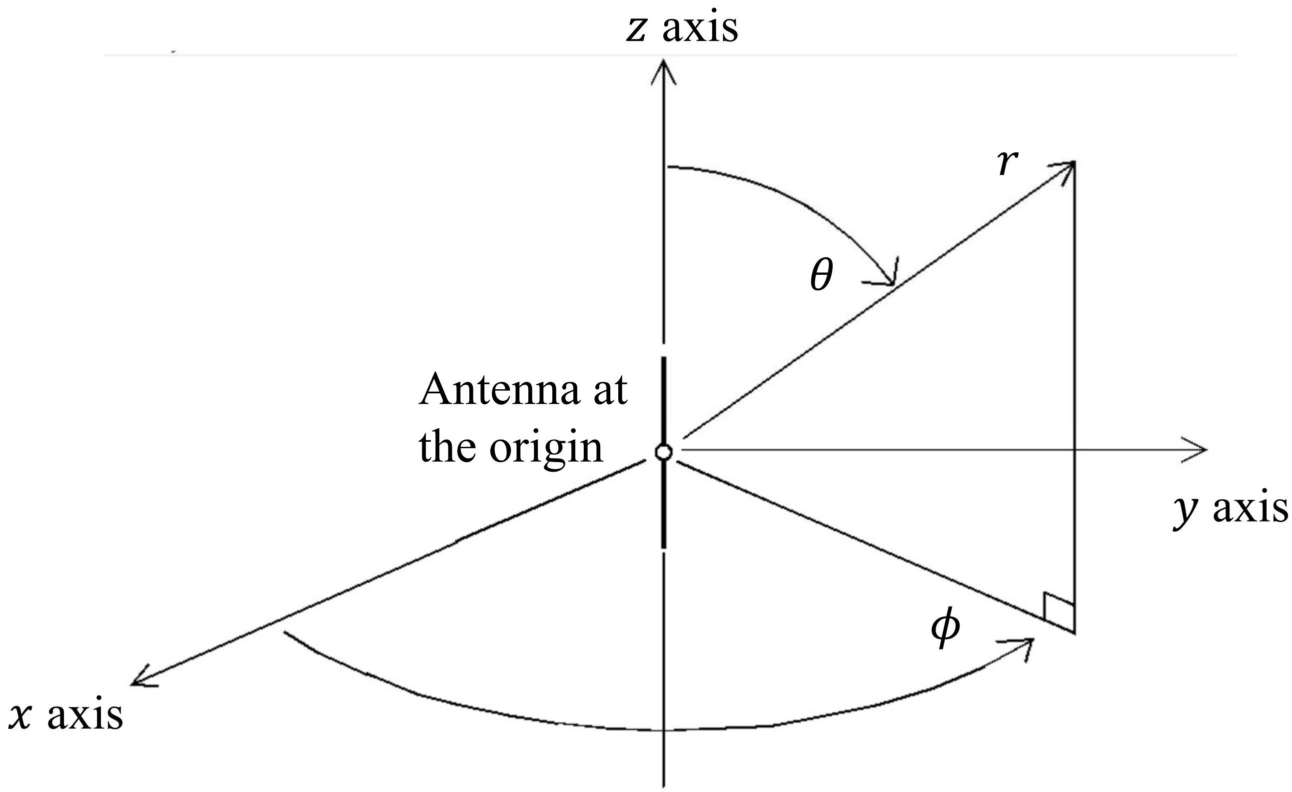
- The Far Fields of an Antenna and Power Flow Density
- The Dipole Antenna
- Radiation Patterns
- Array of Two Dipoles
- Radiated Power, Directivity and Gain
- Receiving Antennas, Effective Area
- The Friis Transmisision Equation

# Antennas and Wireless Links

Good reference: C.R. Paul, K.W. Whites, and S.A. Nasar, "Introduction to Electromagnetic Fields", 3rd edition, McGraw-Hill, 1998.

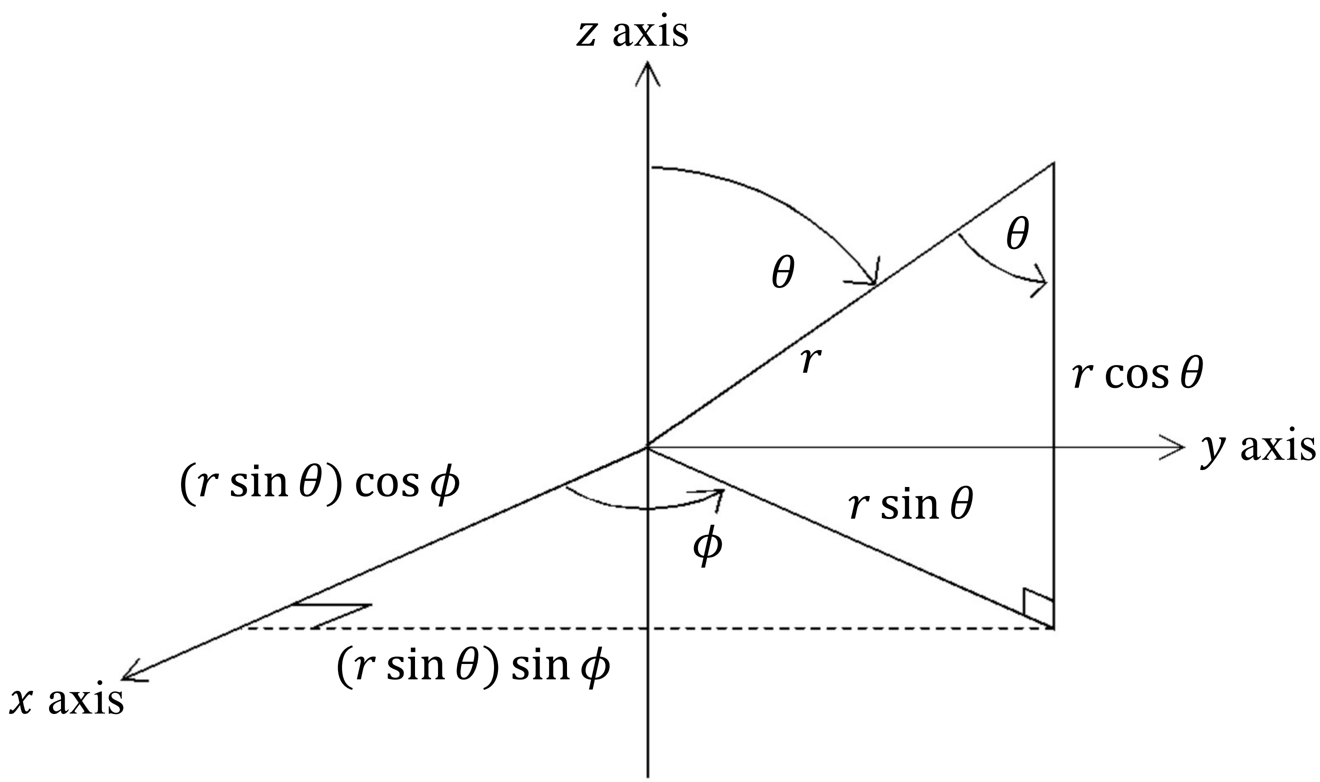


# Spherical Coordinates




$r$= the distance from the origin
$\theta$= the angle that the radial line from the origin makes to the z axis.
$\phi$= the angle that the projection of the radial line onto the xy plane makes to the x axis.

# Relation to Rectangular Coordinates




$$z = r\cos\theta$$
$$x = (r\sin\theta)\cos\phi$$
$$y = (r\sin\theta)\sin\phi$$

# Unit Vectors in Spherical Coordinates

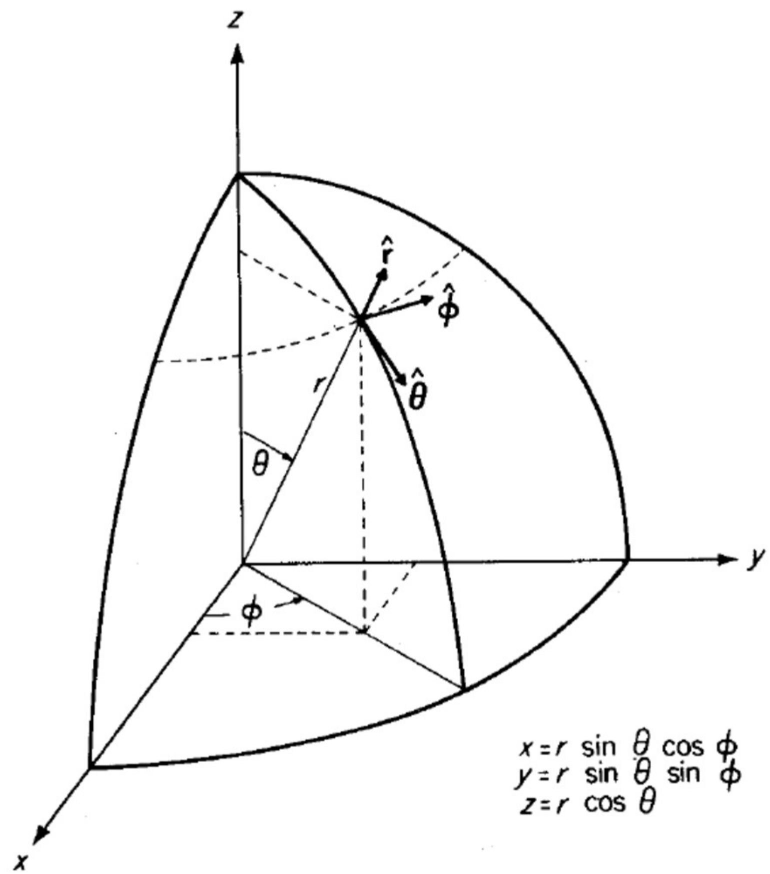
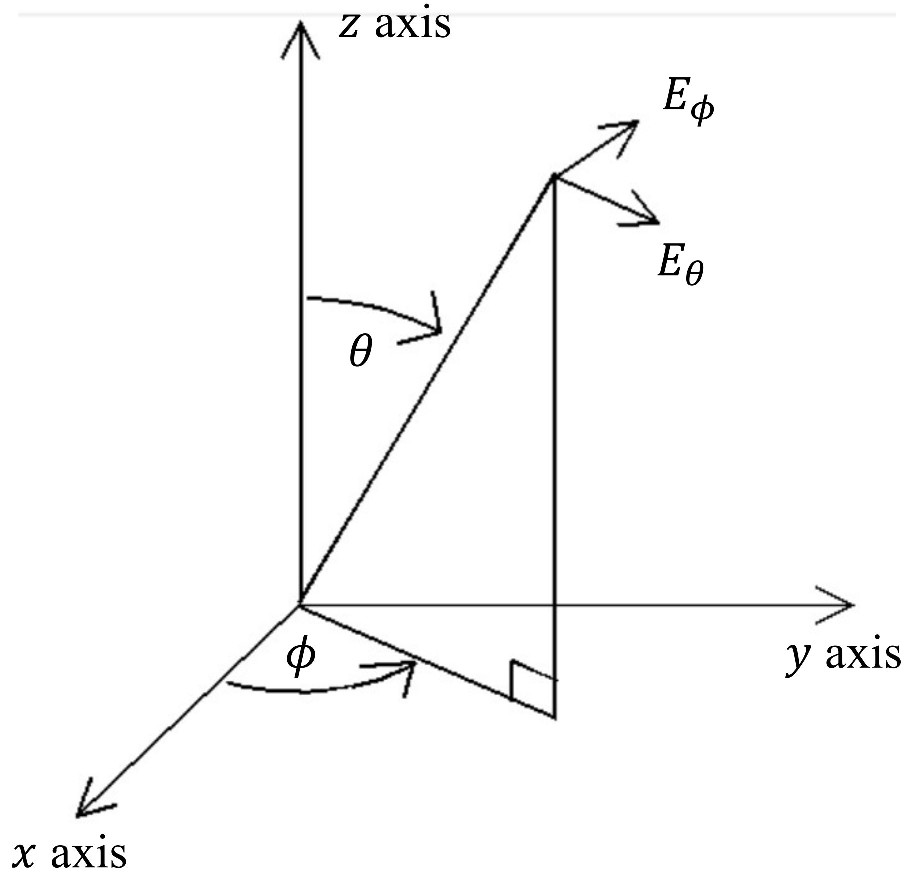



**Figure 1-9.** A spherical co-ordinate system.

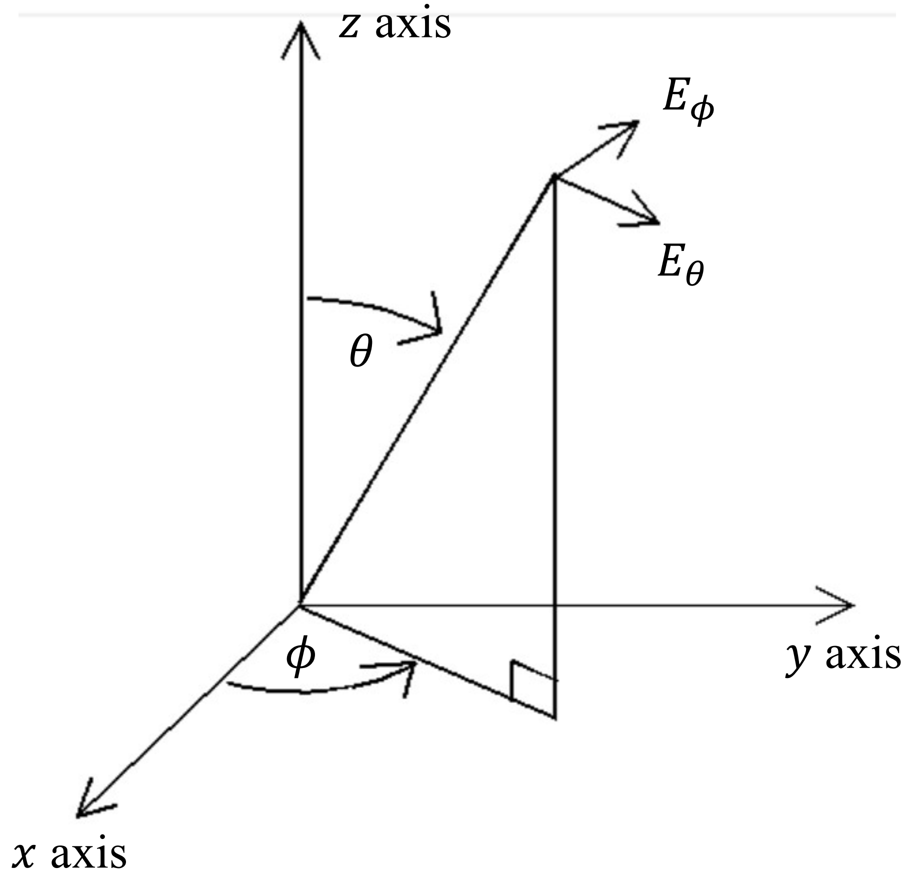
E.C. Jordan and K.C. Balmain, “Electromagnetic Waves and Radiating Systems”, Prentice-Hall, 1968.

# Electric Field Vector in Spherical Coordinates




$$\overline{E}(r,\theta,\phi)=\hat{a}_r E_r(r,\theta,\phi)+\hat{a}_\theta E_\theta(r,\theta,\phi)+\hat{a}_\phi E_\phi(r,\theta,\phi)$$

# The Far Fields of an Antenna




$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

# Find the Magnetic Field

In the far field the electric field is:

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

What is the magnetic field in the far field?

Faraday's Law: $\nabla x \overline{E} = -j\omega\mu\overline{H}$

so

$$\overline{H} = \frac{-1}{j\omega\mu}\nabla \times \overline{E}$$

- Do the curl in spherical coordinates.
- Neglect the terms in $1/r^2$ and $1/r^3$ because they are "near field".

Electric field:

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Curl in spherical coordinates:

$$\nabla \times \overline{E} = \hat{a}_r \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(E_\phi \sin\theta\right) - \frac{\partial}{\partial\phi}E_\theta\right)$$

$$+\hat{a}_\theta \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}E_r - \frac{\partial}{\partial r}\left(rE_\phi\right)\right)$$

$$+\hat{a}_\phi \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rE_\theta\right) - \frac{\partial}{\partial\theta}E_r\right)$$

The components of the far field are:

$$E_r = 0 \qquad E_\theta = e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} \qquad E_\phi = e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Work on the $\hat{a}_r$ component:

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(E_\phi\sin\theta\right)-\frac{\partial}{\partial\phi}E_\theta\right)=\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(e_\phi\frac{e^{-j\beta r}}{r}\sin\theta\right)-\frac{\partial}{\partial\phi}\left(e_\theta\frac{e^{-j\beta r}}{r}\right)\right)$$

$$=\frac{1}{r\sin\theta}\frac{e^{-j\beta r}}{r}\left(\frac{\partial}{\partial\theta}\left(e_\phi\sin\theta\right)-\frac{\partial}{\partial\phi}\left(e_\theta\right)\right)$$

$$=\frac{1}{\sin\theta}\frac{e^{-j\beta r}}{r^2}\left(\frac{\partial}{\partial\theta}\left(e_\phi\sin\theta\right)-\frac{\partial}{\partial\phi}\left(e_\theta\right)\right)$$

The $\hat{a}_r$ component varies as $\frac{1}{r^2}$ so we can neglect it.

The components of the far field are:

$$E_r = 0 \qquad E_\theta = e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} \qquad E_\phi = e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Work on the $\hat{a}_\theta$ component:

$$\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}E_r - \frac{\partial}{\partial r}\left(rE_\phi\right)\right) = \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}(0) - \frac{\partial}{\partial r}\left(re_\phi\frac{e^{-j\beta r}}{r}\right)\right)$$

$$= \frac{1}{r}\left(-\frac{\partial}{\partial r}\left(e_\phi e^{-j\beta r}\right)\right) = \frac{1}{r}\left(-\left(-j\beta e_\phi e^{-j\beta r}\right)\right) = \frac{1}{r}j\beta e_\phi e^{-j\beta r}$$

$$= j\beta e_\phi\frac{e^{-j\beta r}}{r}$$

The components of the far field are:

$$E_r = 0 \qquad E_\theta = e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} \qquad E_\phi = e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Work on the $\hat{a}_\phi$ component:

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial \theta}E_r\right) = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(re_\theta\frac{e^{-j\beta r}}{r}\right) - \frac{\partial}{\partial \theta}(0)\right)$$

$$= \frac{1}{r}\left(\frac{\partial}{\partial r}\left(e_\theta e^{-j\beta r}\right)\right) = \frac{1}{r}\left(-j\beta e_\theta e^{-j\beta r}\right)$$

$$= -j\beta e_\theta\frac{e^{-j\beta r}}{r}$$

So the curl is evaluated as:

$$\nabla \times \overline{E} = \hat{a}_\theta \left( j\beta e_\phi \frac{e^{-j\beta r}}{r} \right) + \hat{a}_\phi \left( -j\beta e_\theta \frac{e^{-j\beta r}}{r} \right)$$

And the magnetic field is:

$$\overline{H} = \frac{-1}{j\omega\mu} \nabla \times \overline{E}$$

$$\overline{H} = \frac{-1}{j\omega\mu} \left[ \hat{a}_\theta \left( j\beta e_\phi \frac{e^{-j\beta r}}{r} \right) + \hat{a}_\phi \left( -j\beta e_\theta \frac{e^{-j\beta r}}{r} \right) \right]$$

$$\overline{H} = \frac{-j\beta}{j\omega\mu} \left[ \hat{a}_\theta e_\phi \frac{e^{-j\beta r}}{r} - \hat{a}_\phi e_\theta \frac{e^{-j\beta r}}{r} \right]$$

$$\frac{j\beta}{j\omega\mu} = \frac{\beta}{\omega\mu} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu} = \frac{\sqrt{\mu\varepsilon}}{\mu} = \sqrt{\frac{\mu\varepsilon}{\mu^2}} = \sqrt{\frac{\varepsilon}{\mu}} = \frac{1}{\eta}$$

So the magnetic field is:

$$\overline{H} = -\hat{a}_\theta \frac{1}{\eta} e_\phi \frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta \frac{e^{-j\beta r}}{r}$$

Hence in the far field of any antenna:

$$\overline{E} = \hat{a}_\theta e_\theta(\theta,\phi) \frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi) \frac{e^{-j\beta r}}{r}$$

$$\overline{H} = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi) \frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi) \frac{e^{-j\beta r}}{r}$$

# Power Flow Density

$$\overline{E}(r,\theta,\phi)=\hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}+\hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi)=-\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}+\hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

conjugate

$$\overline{S}_{av}=\frac{1}{2}\mathrm{Re}\left[\overline{E}\times\overline{H}^*\right]$$

where phasors $E$ and $H$ are written relative to amplitude.

$$\overline{S}_{av}=\frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta e_\theta \frac{e^{-j\beta r}}{r}+\hat{a}_\phi e_\phi \frac{e^{-j\beta r}}{r}\right)\times\left(-\hat{a}_\theta \frac{e_\phi}{\eta}\frac{e^{-j\beta r}}{r}+\hat{a}_\phi \frac{e_\theta}{\eta}\frac{e^{-j\beta r}}{r}\right)^*\right]$$

$$\overline{S}_{av}=\frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta e_\theta \frac{e^{-j\beta r}}{r}+\hat{a}_\phi e_\phi \frac{e^{-j\beta r}}{r}\right)\times\left(-\hat{a}_\theta \frac{e_\phi^*}{\eta}\frac{e^{j\beta r}}{r}+\hat{a}_\phi \frac{e_\theta^*}{\eta}\frac{e^{j\beta r}}{r}\right)\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta e_\theta \frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi \frac{e^{-j\beta r}}{r}\right) \times \left(-\hat{a}_\theta \frac{e_\phi^*}{\eta}\frac{e^{j\beta r}}{r} + \hat{a}_\phi \frac{e_\theta^*}{\eta}\frac{e^{j\beta r}}{r}\right)\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\begin{vmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ 0 & e_\theta \frac{e^{-j\beta r}}{r} & e_\phi \frac{e^{-j\beta r}}{r} \\ 0 & -\frac{e_\phi^*}{\eta}\frac{e^{j\beta r}}{r} & \frac{e_\theta^*}{\eta}\frac{e^{j\beta r}}{r} \end{vmatrix}\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_r\left(\frac{1}{\eta}e_\theta e_\theta^* \frac{e^{-j\beta r}}{r}\frac{e^{j\beta r}}{r} + \frac{1}{\eta}e_\phi e_\phi^* \frac{e^{-j\beta r}}{r}\frac{e^{j\beta r}}{r}\right)\right]$$

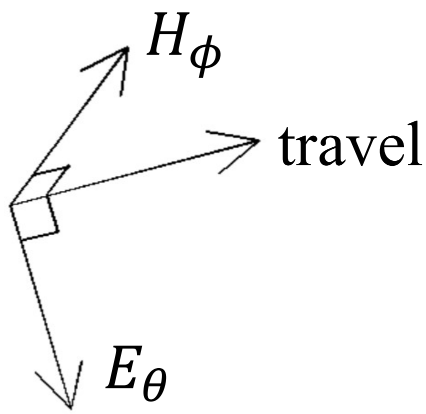
$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_r\left(\frac{\left|e_\theta\right|^2}{\eta}\frac{1}{r^2} + \frac{\left|e_\phi\right|^2}{\eta}\frac{1}{r^2}\right)\right]$$

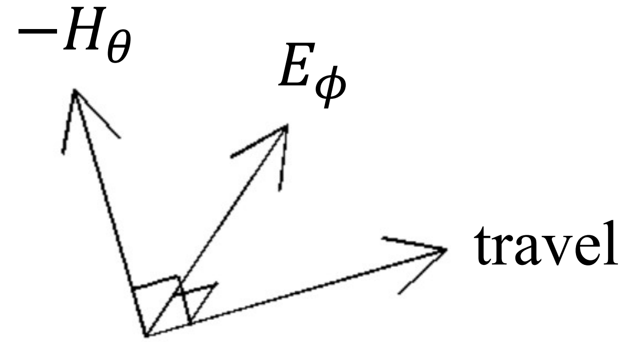
$$\overline{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{\left|e_\theta\right|^2}{2\eta} + \frac{\left|e_\phi\right|^2}{2\eta}\right]$$ Watts per square meter

# Quasi-Plane Waves

$$\overline{E}(r,\theta,\phi)=\hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}+\hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi)=-\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}+\hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$




$E_\theta$ and $H_\phi$

$$\overline{E}(r,\theta,\phi)=\hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi)=\hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{S}_{av}=\hat{a}_r \frac{1}{r^2}\left[\frac{\left|e_\theta\right|^2}{2\eta}\right]$$

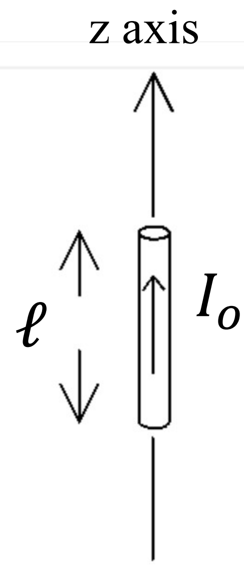
$E_\phi$ and $-H_\theta$

$$\overline{E}(r,\theta,\phi)=\hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi)=-\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{S}_{av}=\hat{a}_r \frac{1}{r^2}\left[\frac{\left|e_\phi\right|^2}{2\eta}\right]$$

# The Hertzian Dipole

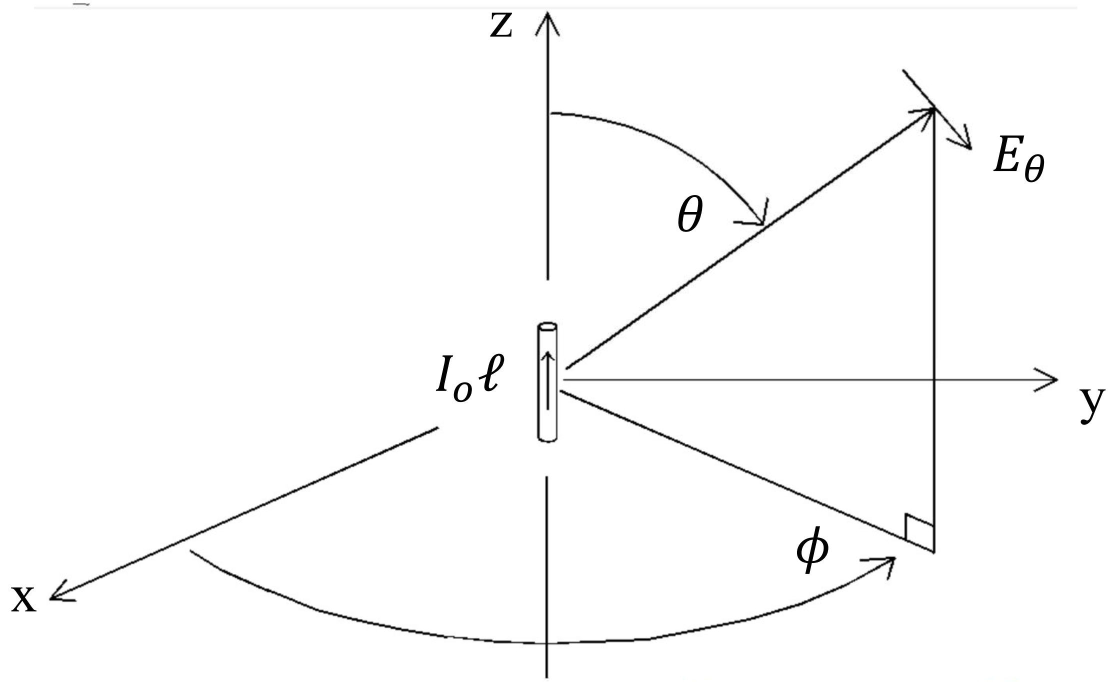



A "current element" or "Hertzian Dipole" is a short length of wire $\ell \ll \lambda$ carrying an AC current of frequency $\omega$ rad/sec and amplitude $I_o$

$$\ell \ll \lambda$$

$$i(t) = I_0 \cos(\omega t).$$

# The Fields of the Hertzian Dipole




The exact fields of a Hertzian dipole are:

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2}\right]$$

$$E_r = \frac{2\beta^2 I_0 \ell}{4\pi} \eta_0 \cos\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

# Near Fields and Far Fields

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2}\right]$$

$$E_r = \frac{2\beta^2 I_0 \ell}{4\pi} \eta_0 \cos\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

These formulas contain three sets of terms:

- $\frac{1}{\beta r} = \frac{1}{2\pi}\frac{1}{r/\lambda}$
- $\frac{1}{(\beta r)^2} = \frac{1}{(2\pi)^2}\frac{1}{(r/\lambda)^2}$
- $\frac{1}{(\beta r)^3} = \frac{1}{(2\pi)^3}\frac{1}{(r/\lambda)^3}$

# The Near Field $\frac{r}{\lambda} << 1$

- Consider "close" distances, where $r << \lambda$
- then $\beta r = 2\pi \frac{r}{\lambda}$ is "small"
- since $\frac{1}{(\text{small})^2} >> \frac{1}{\text{small}}$, we have $\frac{1}{(\beta r)^2} >> \frac{1}{\beta r}$
- and $\frac{1}{(\text{small})^3} >> \frac{1}{\text{small}}$, we have $\frac{1}{(\beta r)^3} >> \frac{1}{\beta r}$
- The dominant terms are those in $\frac{1}{(\beta r)^2}$ and $\frac{1}{(\beta r)^3}$
- Then the fields are given by

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2}\right]$$

$$E_r = \frac{2\beta^2 I_0 \ell}{4\pi}\eta_0 \cos\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi}\eta_0 \sin\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$e^{-j\beta r} \approx e^{j0} = 1$$

# The Far Field $\frac{r}{\lambda} >> 1$

- Consider "far" distances, where $r >> \lambda$
- then $\beta r = 2\pi \frac{r}{\lambda}$ is "large"
- since $\frac{1}{(\text{large})^2} << \frac{1}{\text{large}}$, we have $\frac{1}{(\beta r)^2} << \frac{1}{\beta r}$
- and $\frac{1}{(\text{large})^3} << \frac{1}{\text{large}}$, we have $\frac{1}{(\beta r)^3} << \frac{1}{\beta r}$
- The dominant terms are those in $\frac{1}{\beta r}$
- Then the "far fields" of the Hertzian dipole are given by

$$E_r = 0$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r} \frac{j}{\beta r}$$

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r} \frac{j}{\beta r}$$

# Angle Dependence

$$E_\theta = \frac{j\eta_0 \beta I_0 \ell}{4\pi} \sin\theta \frac{e^{-j\beta r}}{r}$$

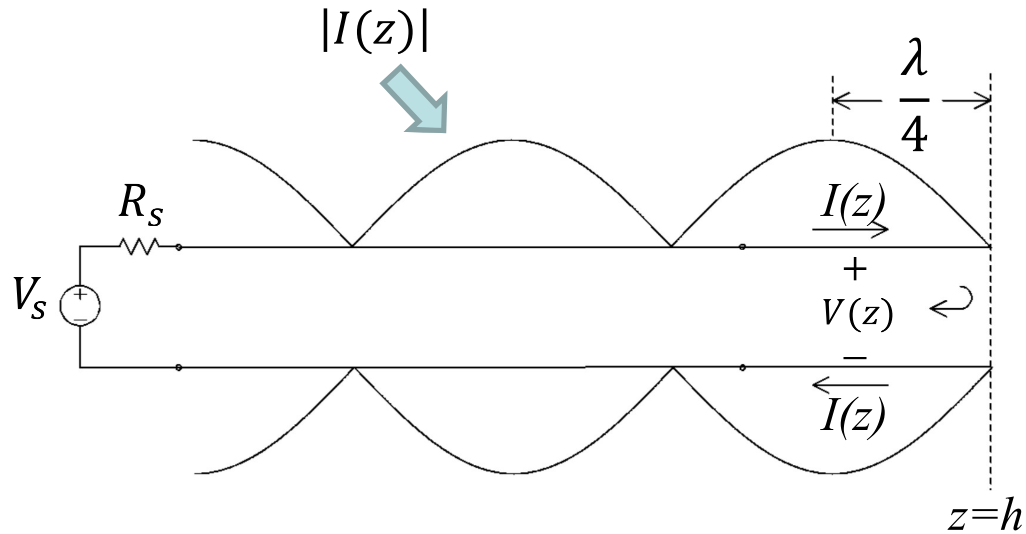
$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$e_\theta(\theta,\phi) = \frac{j\eta_0 \beta I_0 \ell}{4\pi} \sin\theta$$

and

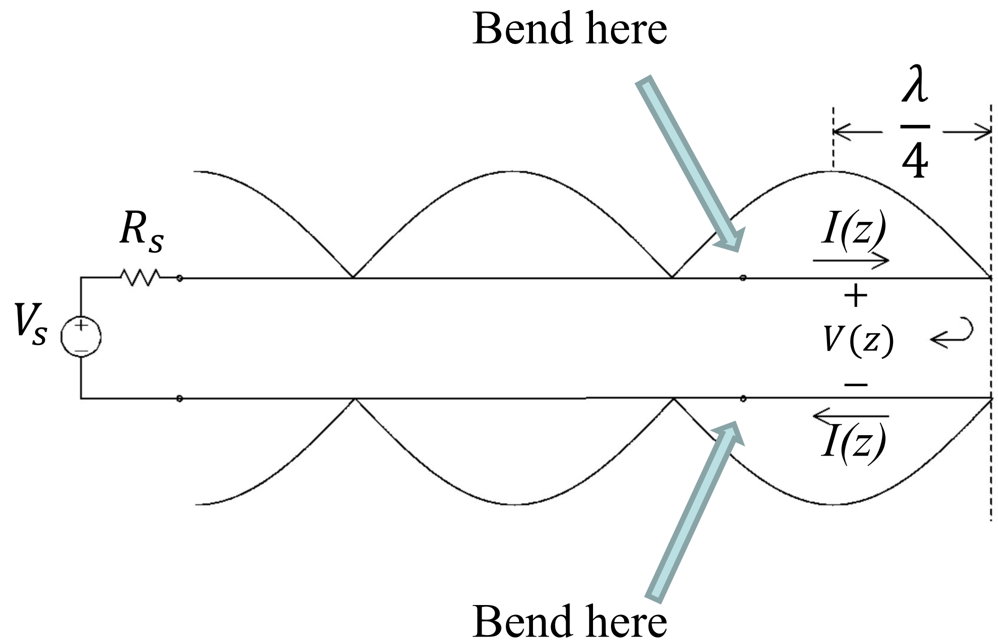
$$e_\phi = 0$$

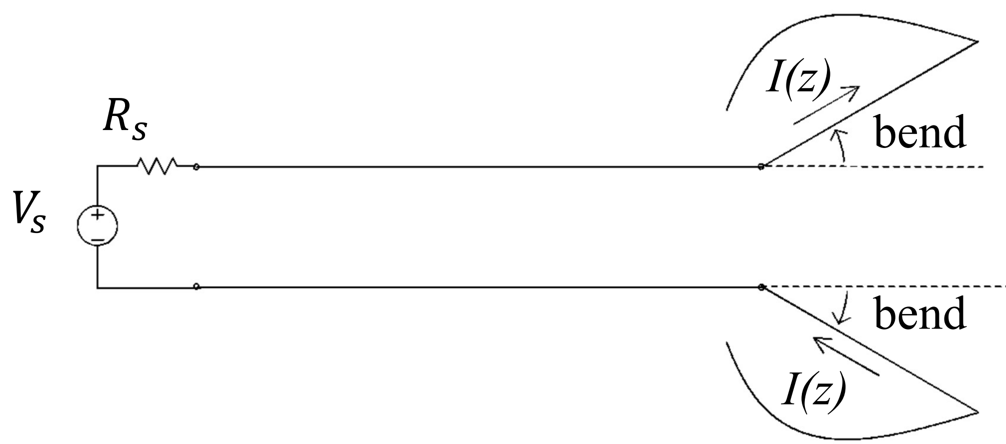
# The Dipole Antenna




- The amplitude of the current on the open-circuited transmission line is
$$|I(z)| = I_o \sin\big(\beta(z-h)\big)$$
- The end of the line is at $z = h$
- At $z = h$, we expect the current to be zero:
$$|I(h)| = I_o \sin\big(\beta(h-h)\big) = I_o \sin(0) = 0$$

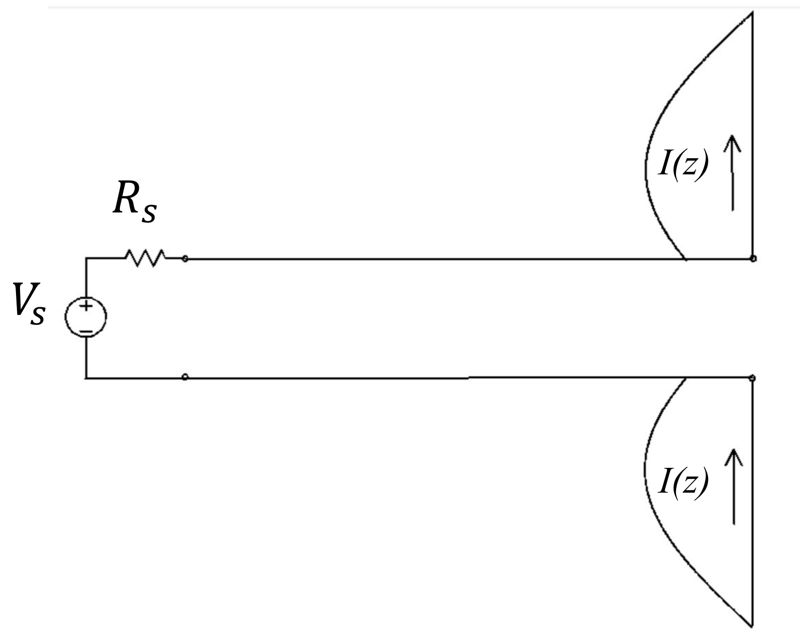
# Bend the wires to make dipole arms

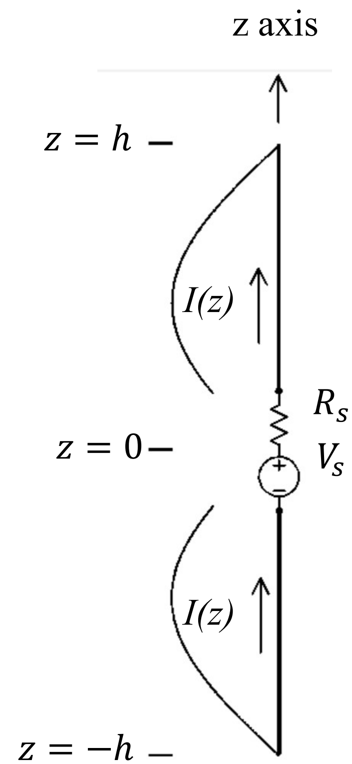


# Bend the arms to be vertical

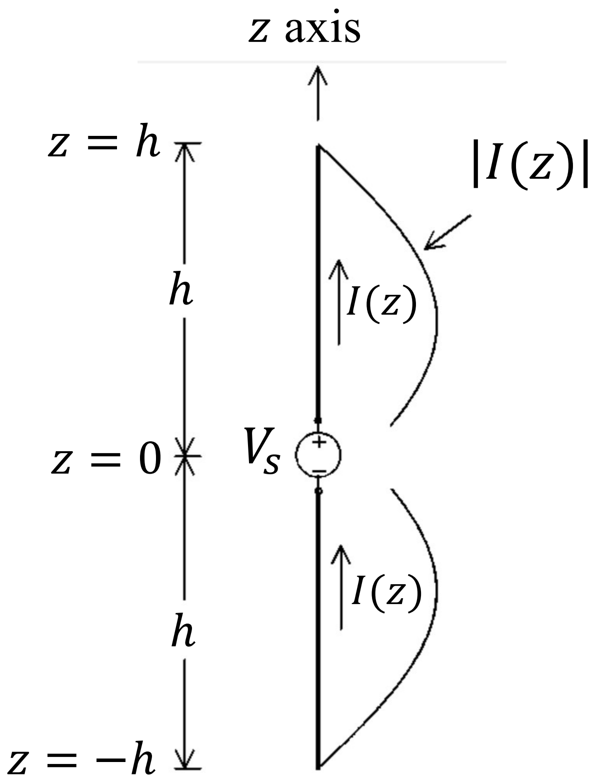

The result is a transmission line feeding a dipole antenna:




$$I(z) = I_o \sin\left(\beta(|z| - h)\right)$$

# The Far Fields of a Dipole Antenna



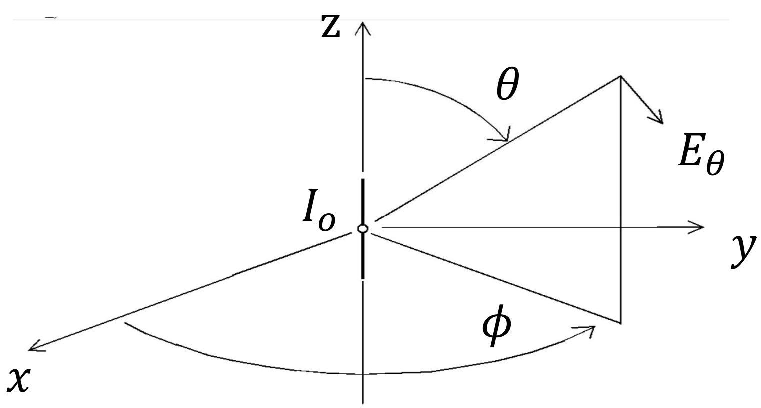

$$I(z) = I_o \sin\big(\beta(|z| - h)\big)$$

It May Be Shown that:

$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

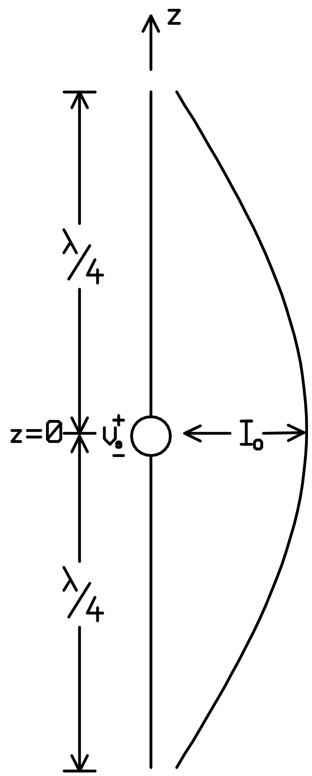
$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi) \frac{e^{-j\beta r}}{r}$$

$$e_\theta(\theta,\phi) = \frac{jI_0\eta_0}{2\pi} F(\theta)$$




$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$

# Half-Wave Dipole Antenna




In general:

$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$

Half-wave dipole:

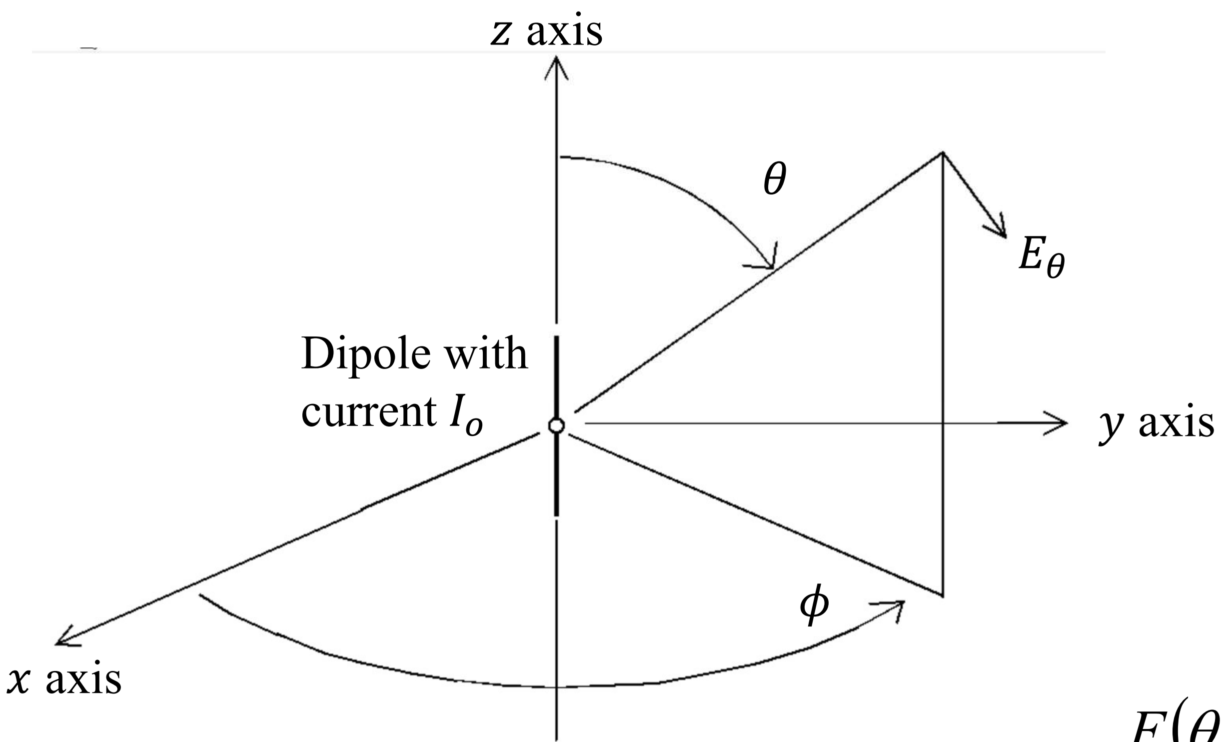
$$2h = \frac{\lambda}{2} \qquad h = \frac{\lambda}{4}$$

$$\beta = \frac{2\pi}{\lambda} \qquad \beta h = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right) - \cos\left(\frac{\pi}{2}\right)}{\sin\theta}$$

# Half-Wave Dipole Antenna Far Field




$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right) - \cos\left(\frac{\pi}{2}\right)}{\sin\theta}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

# Example

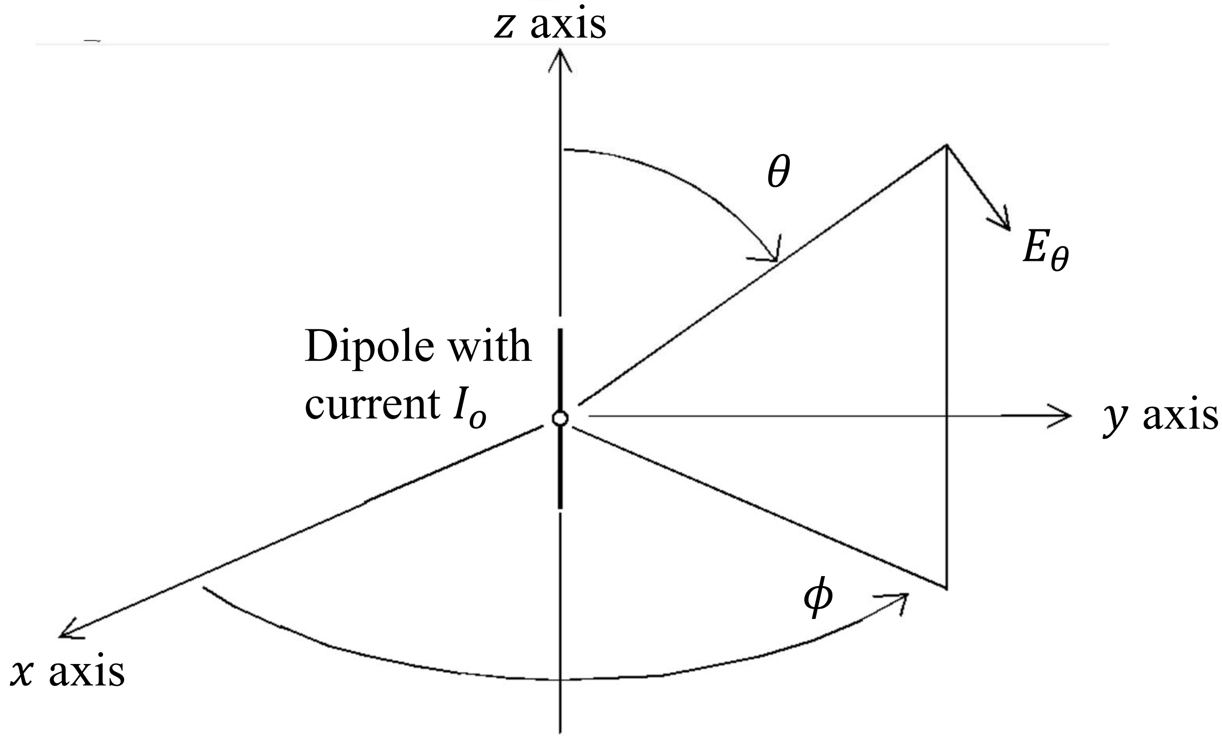
A vertical, half-wave dipole antenna is used to radiate a signal at 2450 MHz. At a perpendicular distance of 1 m from the center of the dipole, the field strength is 10 V/m.

1) What is the amplitude $I_0$ of the current on the dipole?

2) What is the field strength 30 m from the antenna, in the horizontal plane?

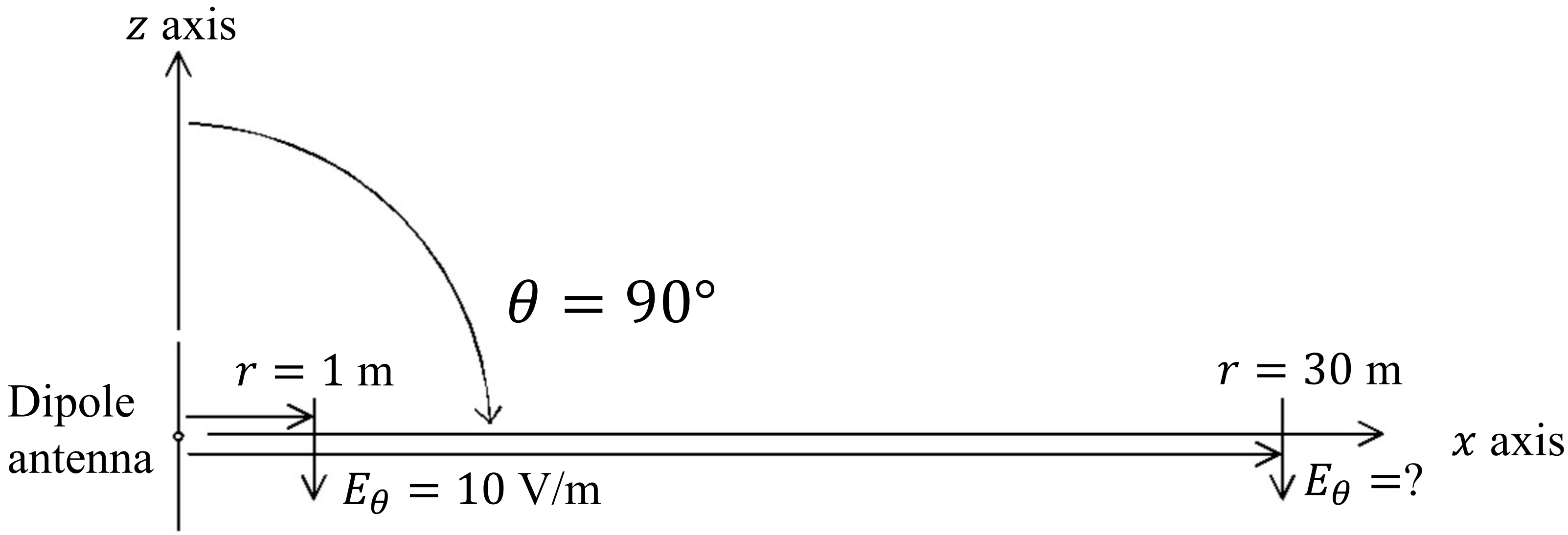
3) What is the field strength 200 m from the antenna, at an elevation of 30 degrees above the horizontal plane?

# Orientation- “vertical dipole” and “horizontal plane”




$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

- The dipole is at the origin oriented along the z axis, which is the “vertical” axis.
- The horizontal plane is the xy plane, $\theta = 90°$.




The electric field of the dipole is given by:

$$E_\theta(r,\theta) = \frac{jI_O\eta_0}{2\pi}\frac{e^{-j\beta r}}{r}F(\theta)$$

The amplitude of the electric field is given by the magnitude of the phasor:

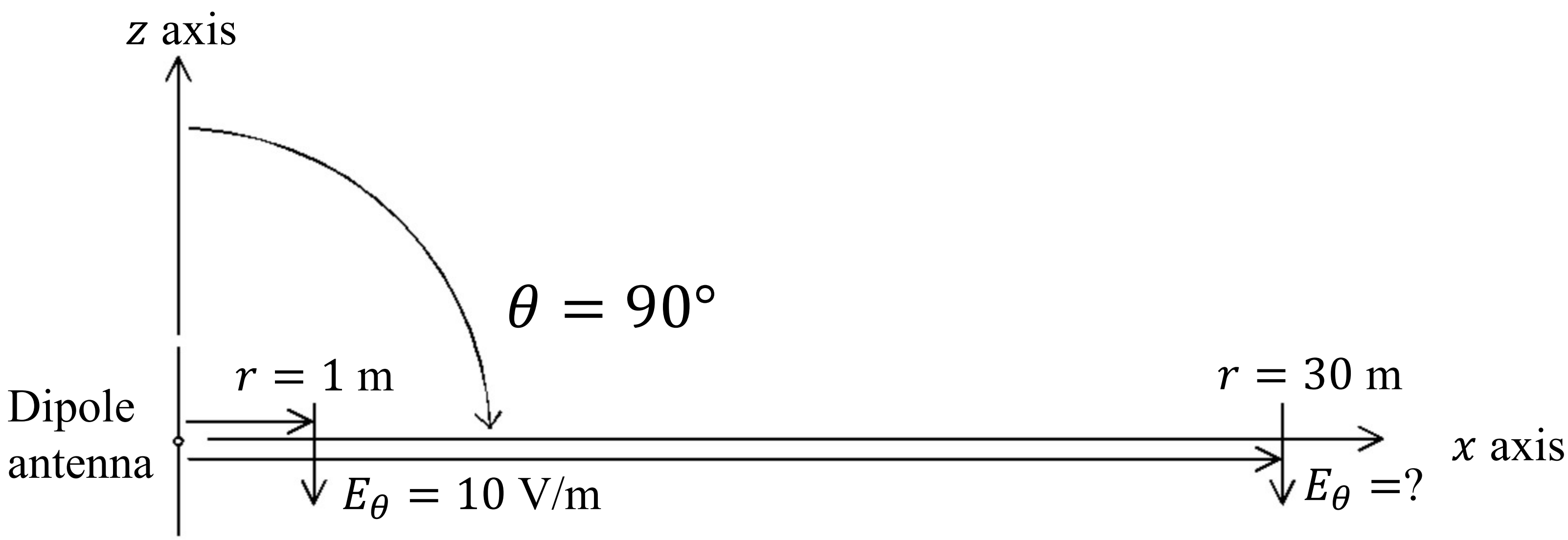
$$|E_\theta(r,\theta)| = \frac{I_O\eta_0}{2\pi r}|F(\theta)|$$

We can specialize the field to the $\theta = 90°$ plane:

$$F\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right)\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{\cos(0)}{1} = 1$$

Hence at $r = 1$ m the field strength is 10 V/m:

$$\left|E_\theta\left(1,\frac{\pi}{2}\right)\right| = \frac{I_O\eta_0}{2\pi x 1}x1 = 10$$




$$\frac{I_O \eta_0}{2\pi} = 10$$

And we can simplify $|E_\theta(r,\theta)| = \frac{I_O \eta_0}{2\pi r}|F(\theta)|$ as

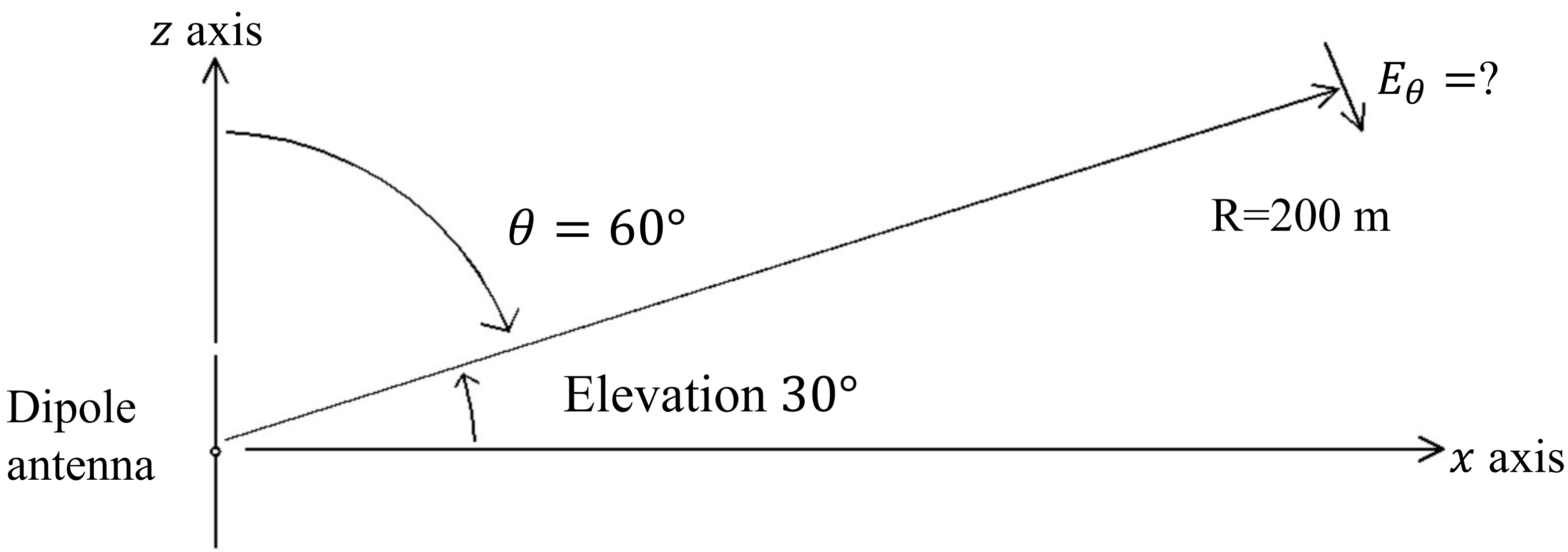
$$|E_\theta(r,\theta)| = \frac{10}{r}|F(\theta)|$$

What is the field strength the field in the $\theta = 90°$ plane at $r = 30$ m?

$$\left|E_\theta\left(30, \frac{\pi}{2}\right)\right| = \frac{10}{30} x1 = 0.333$$

The field strength at 30 m distance is 0.333 V/m or 33.3 mV/m.

What is the field strength 200 m from the antenna, at an elevation of 30 degrees above the horizontal plane?




In general the field strength is given by

$$|E_\theta(r,\theta)| = \frac{10}{r}|F(\theta)|$$

For $\theta = \frac{\pi}{3}$, we must evaluate

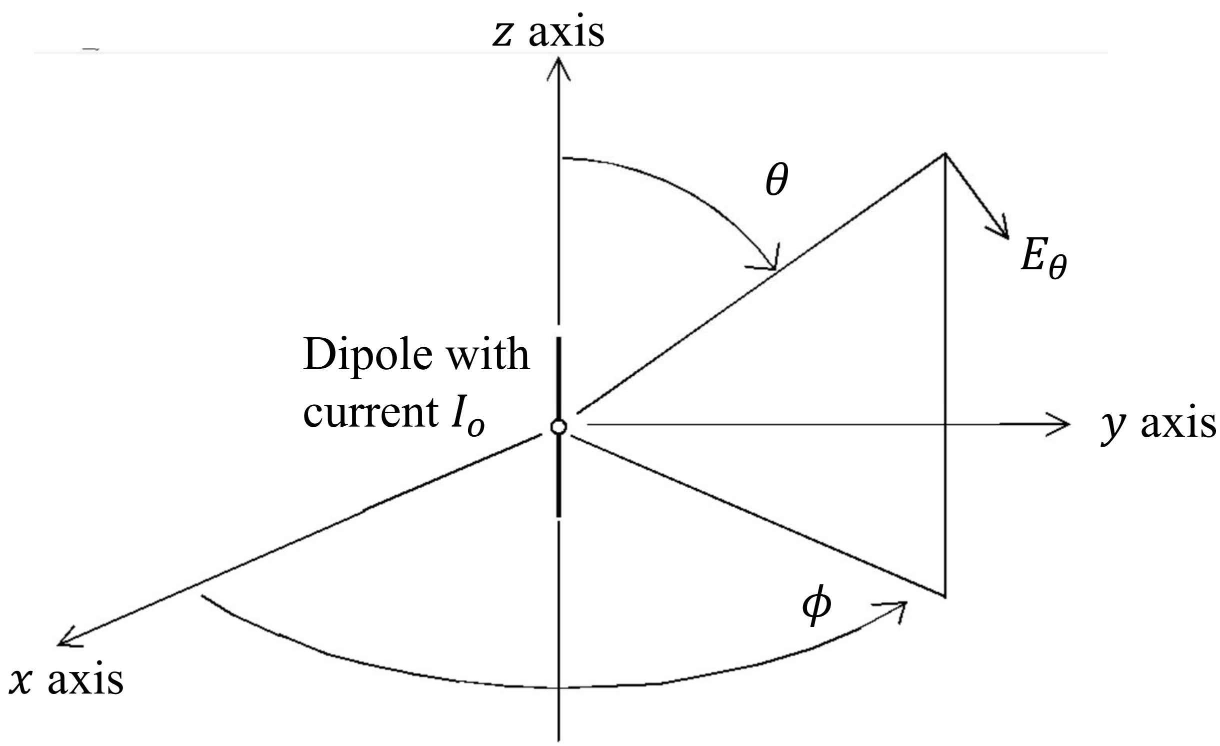
$$F\left(\frac{\pi}{3}\right) = \frac{\cos\left(\frac{\pi}{2}\cos\left(\frac{\pi}{3}\right)\right)}{\sin\left(\frac{\pi}{3}\right)} = 0.8165$$

Hence at $r = 200$ m and $\theta = \frac{\pi}{3}$, we must evaluate

$$\left|E_\theta\left(200, \frac{\pi}{3}\right)\right| = \frac{10}{200} x 0.8165 = 0.040825$$

The field strength at $\theta = 60°$ and 200 m distance is 40.825 mV/m.

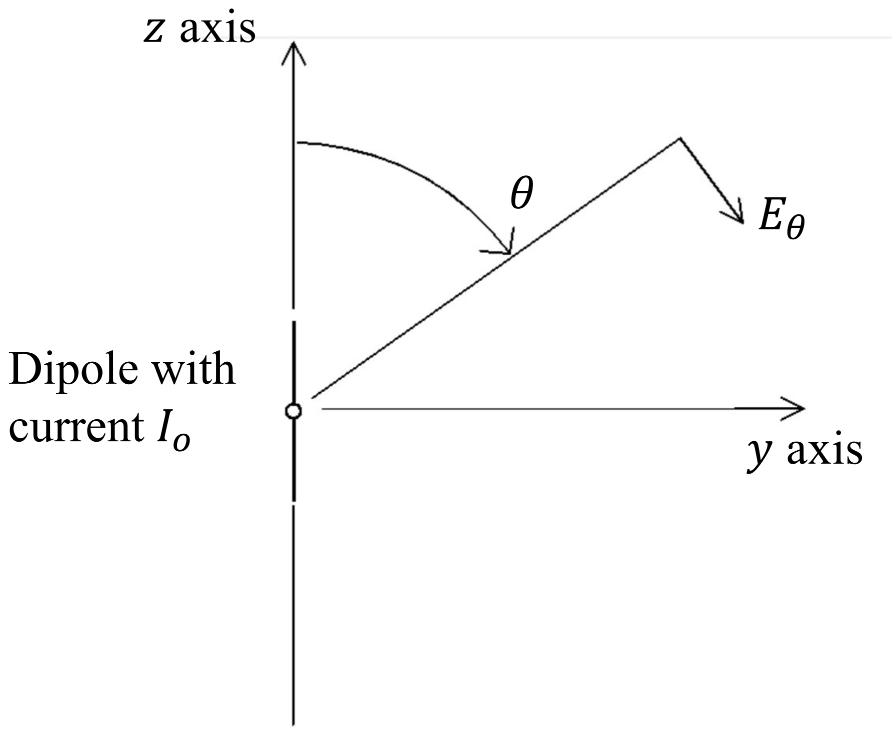
**Radiation Patterns:** How does the electric field strength vary with $\theta$ and $\phi$?



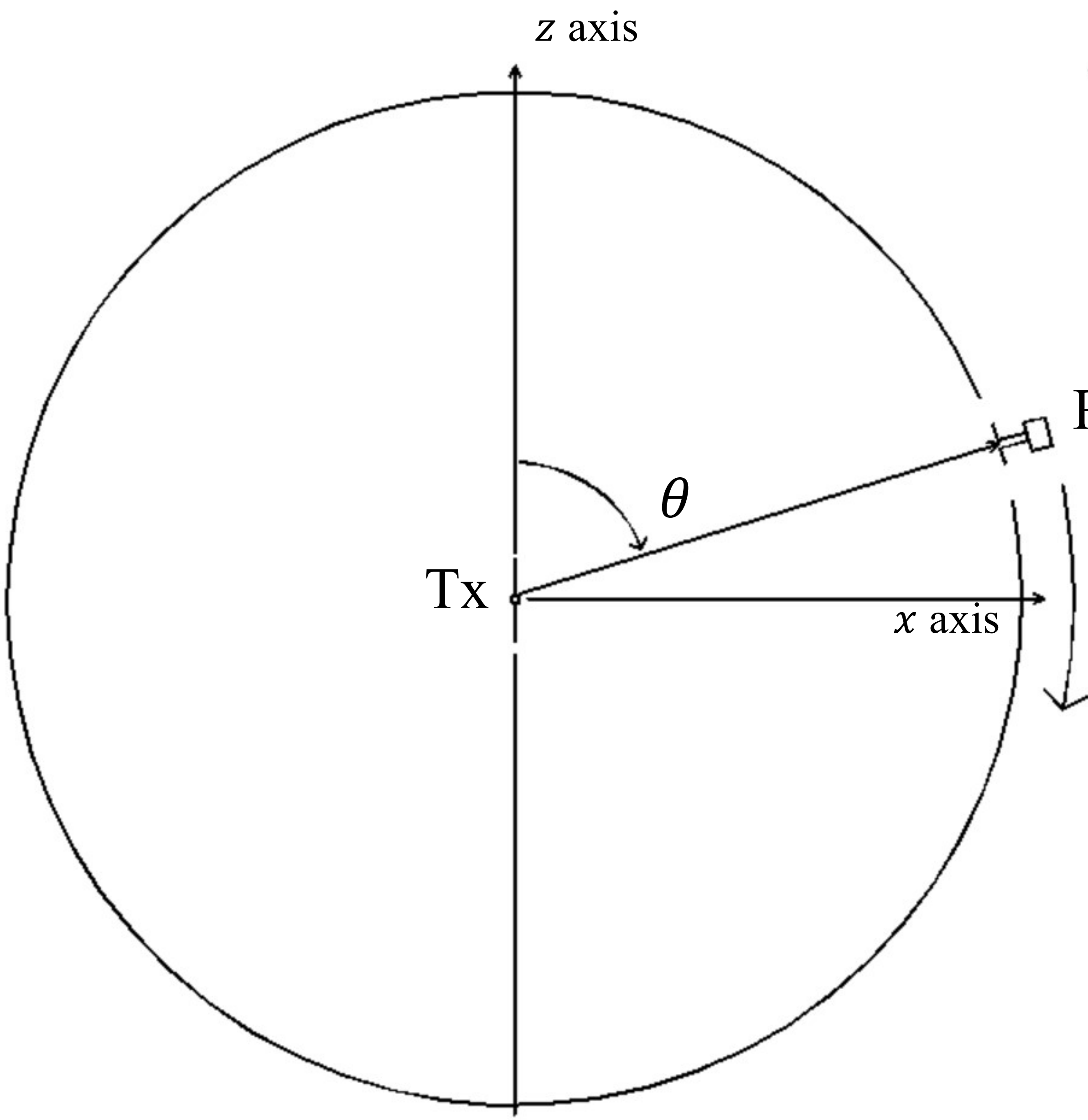

Azimuth pattern: With $\theta = 90°$, plot the field strength $|E_\theta|$as a function of $\phi$.

Elevation pattern: With $\phi = 0°$ or $\phi = 90°$, plot the field strength $|E_\theta|$as a function of $\theta$.

Elevation pattern: With $\phi = 90°$, plot the field strength $|E_\theta|$as a function of $\theta$.



Measure the field strength on a circle surrounding the antenna:




Move around the Tx in a circle of radius R and measure the field strength as a function of angle $\theta$

Measure the field strength as a function of angle:

Half-wave dipole:

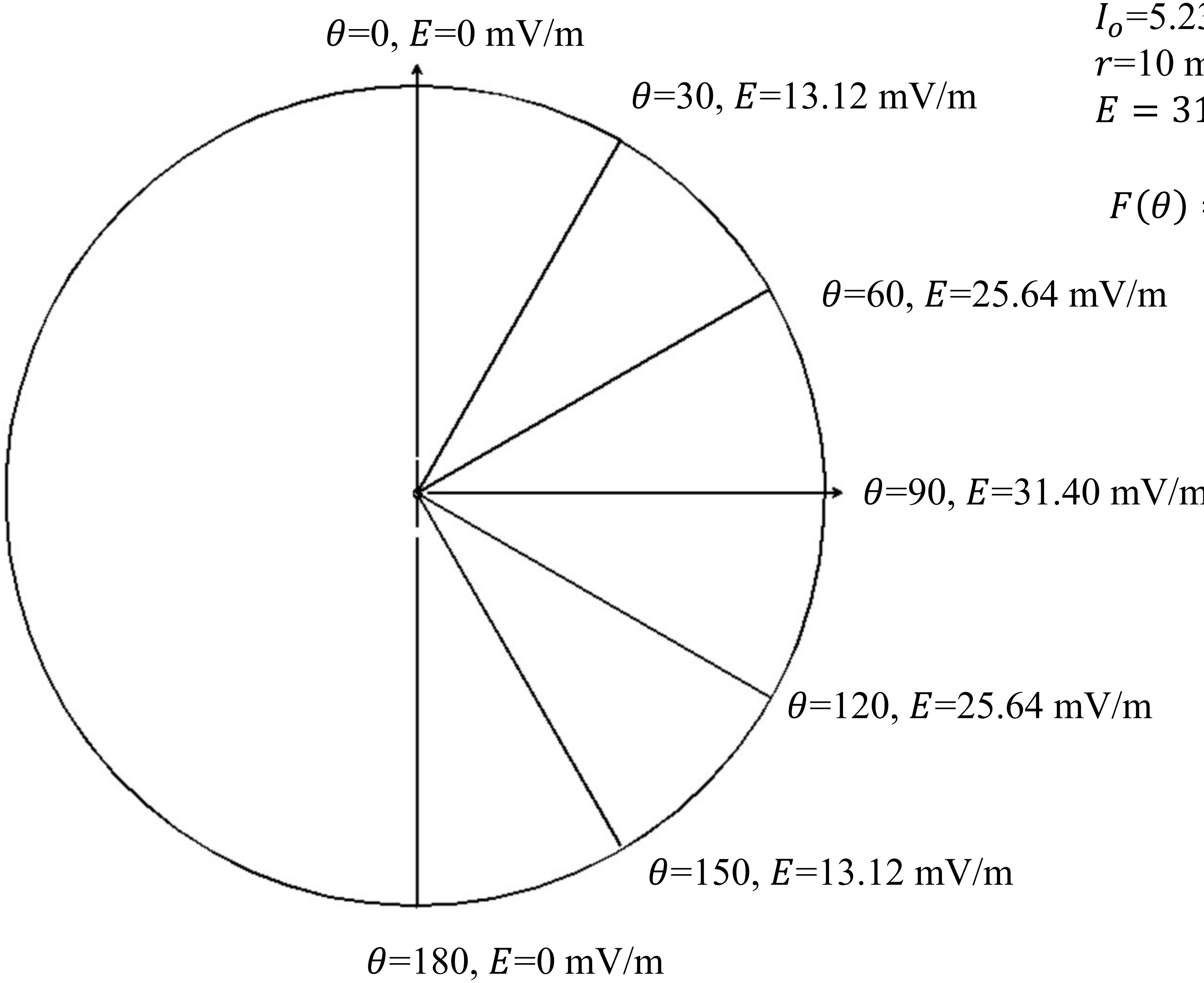
$P_r = 1$ mW

$I_o$=5.234 mA

$r$=10 m

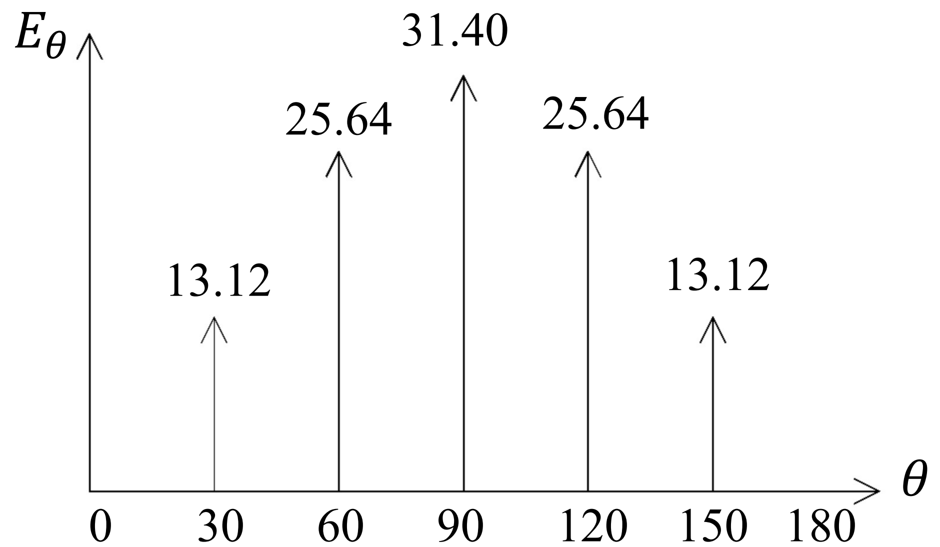
$E = 31.4F(\theta)$ mV/m

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

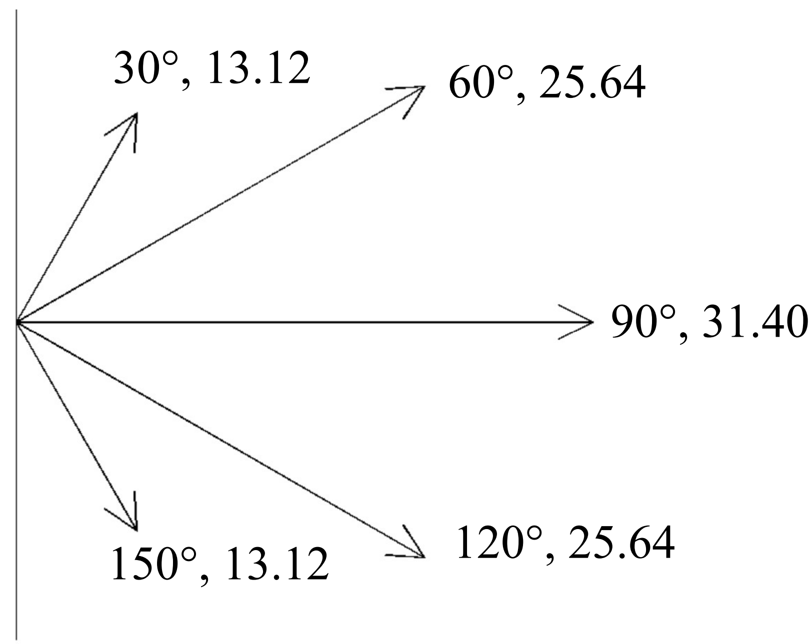


# Graph the Field strength as a function of angle:

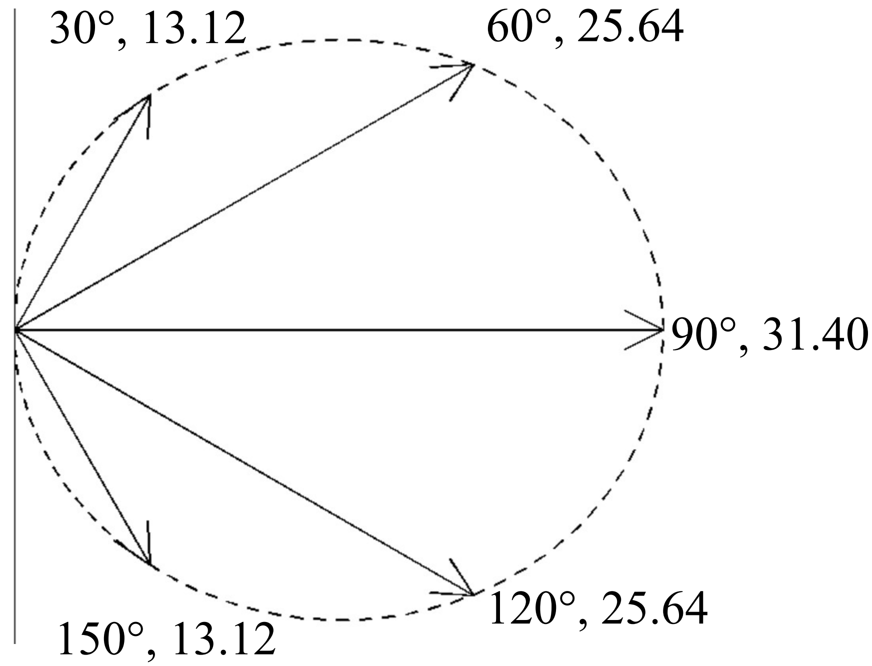
| $\theta$ | $E(\theta)$ |
|---|---|
| 0 | 0 |
| 30 | 13.12 mV/m |
| 60 | 25.64 |
| 90 | 31.40 |
| 120 | 25.64 |
| 150 | 13.12 |
| 180 | 0 |



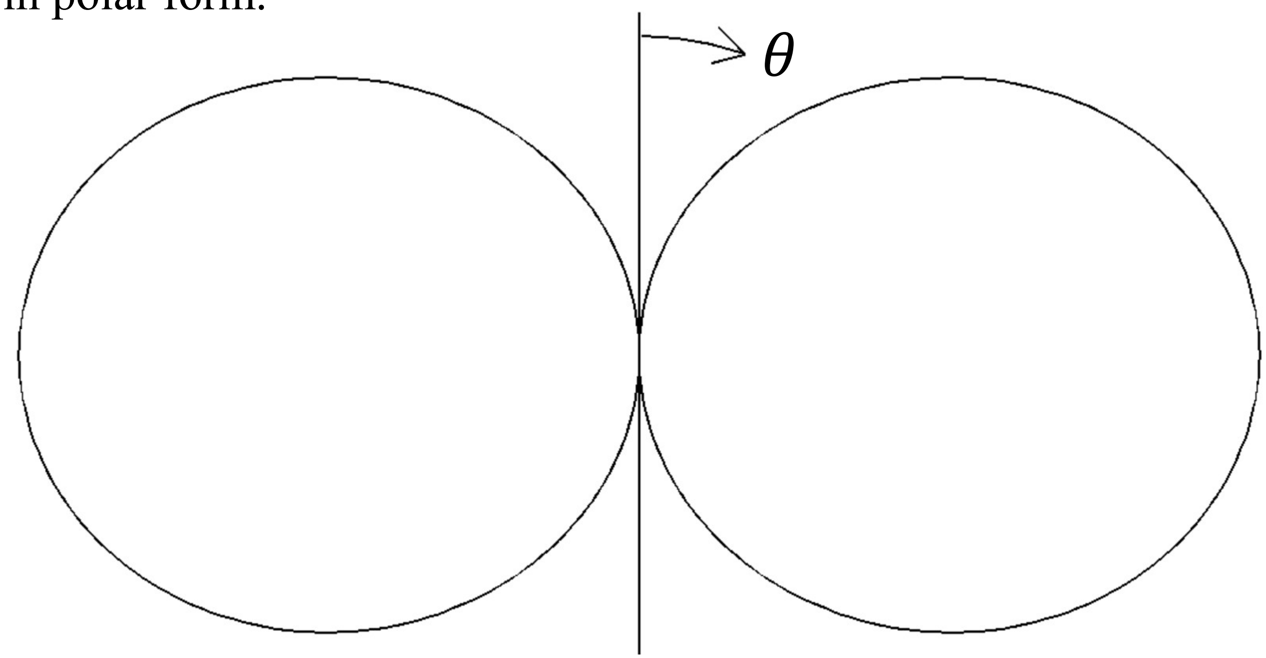

A *radation pattern* is a graph of the field strength in polar form:

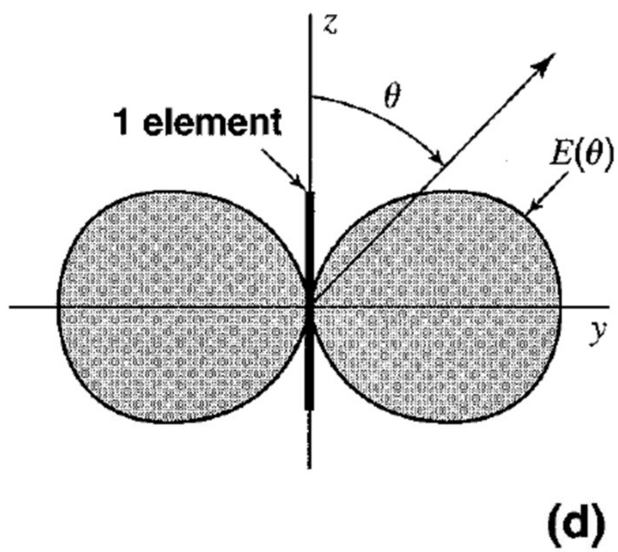


Measure the field strength at many theta angles to form a smooth curve.




A *radiation pattern* is a graph of field strength as a function of angle in polar form.

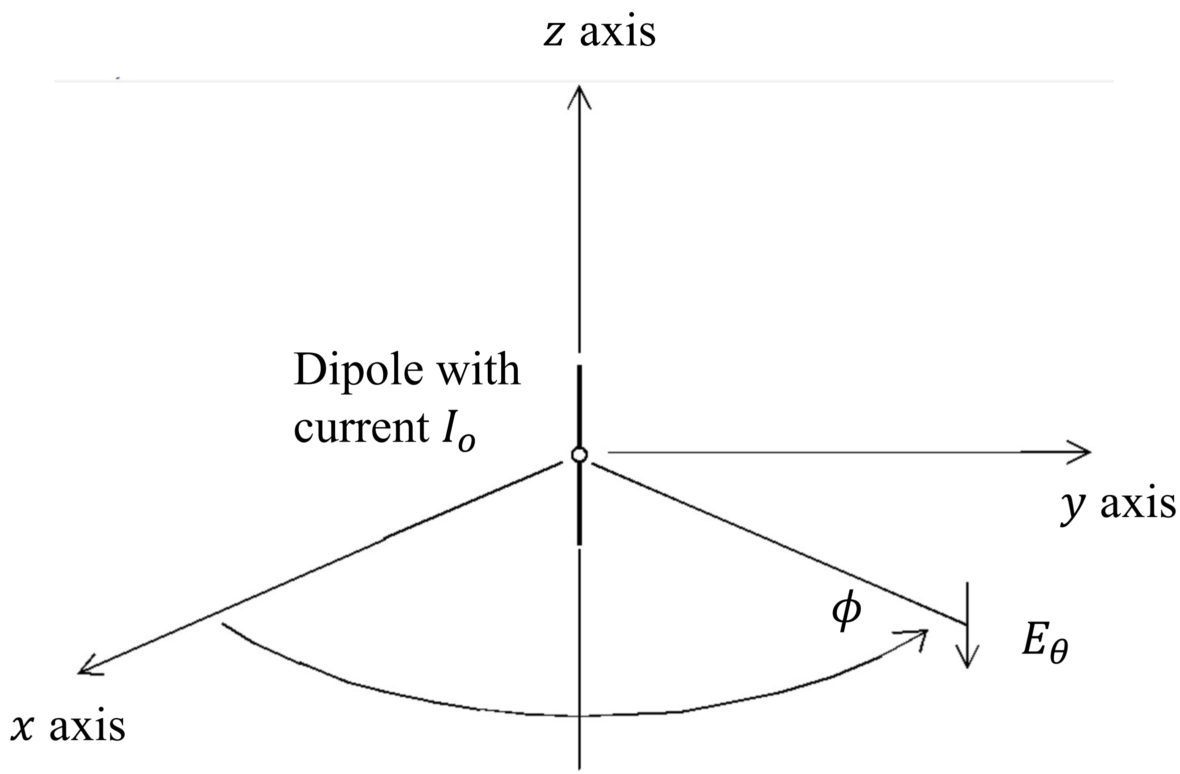


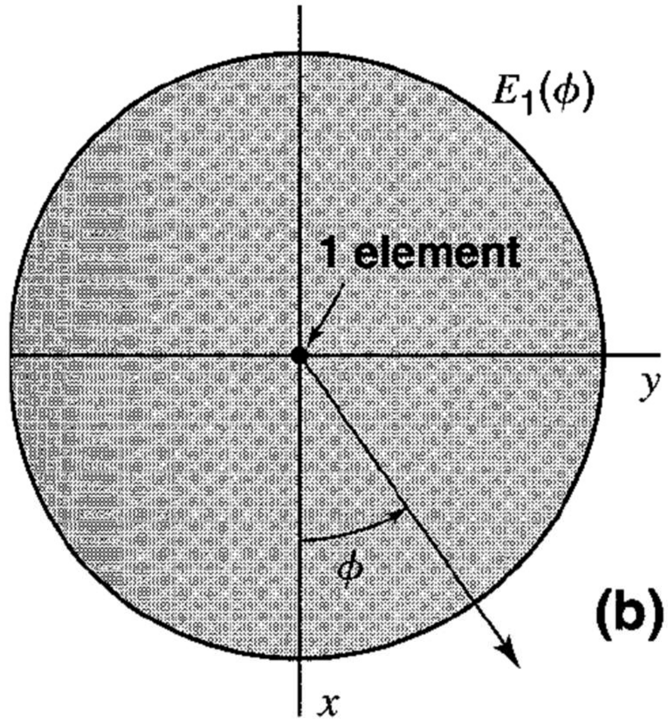



Kraus and Marhefka Fig. 16-2

- The x,z or y,z radiation pattern or "elevation plane" pattern is a figure-eight with zero radiation along the z axis.

Azimuth pattern: With $\theta = 90°$, plot the field strength $|E_\theta|$ as a function of $\phi$.



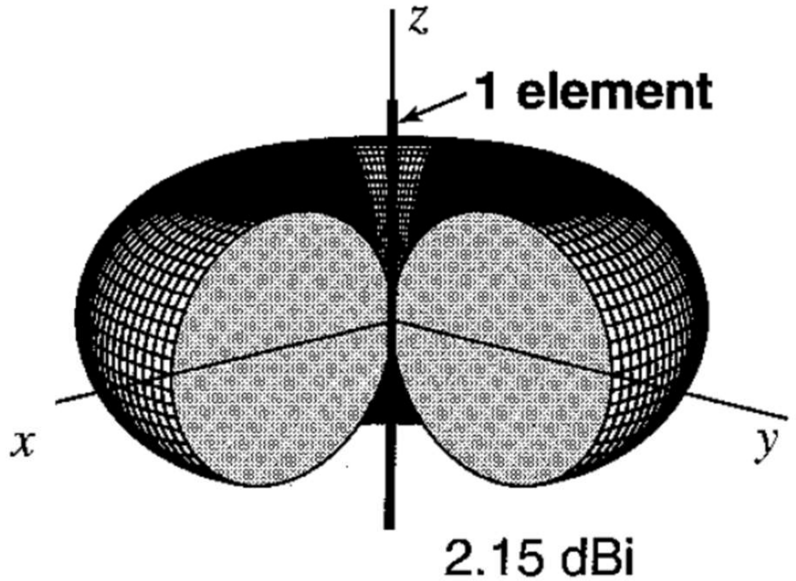



Kraus and Marhefka Fig. 16-2

- The radiation pattern in the x,y or $\theta = 90°$ plane or "azimuth plane" is a circle.
- A circular "radiation pattern" is said to be "omnidirectional".

John D. Kraus and Ronald J. Marhefka, "Antennas for All Applications", 3rd edition, McGraw-Hill, 2002.

# 3D Visualization of the Radiation Patterns of a Dipole Antenna




Kraus and Marhefka Fig. 16-2

# ELEC353 Lecture Notes Set 19

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #11: Do homework #11 by April 5, 2019.
Homework #12: Do homework #12 by April 12, 2019.

Tutorial Workshop #12: Friday April 5, 2019.
Tutorial Workshop #13: Friday April 12, 2019

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

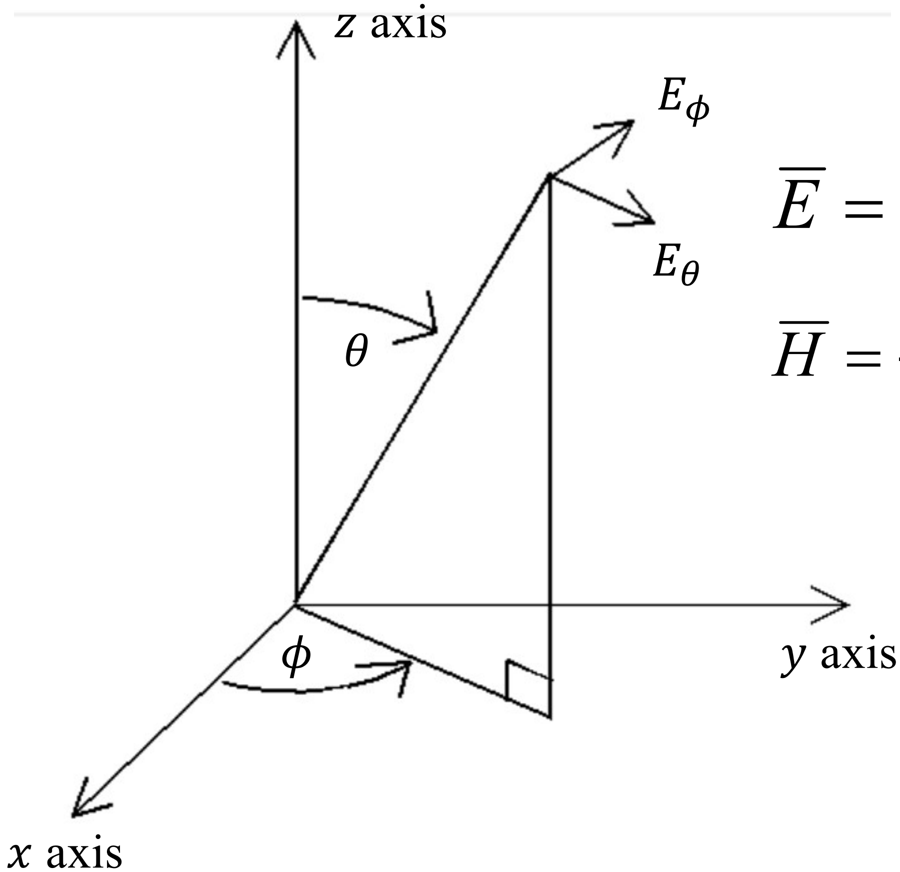
# Antenna Topics

Antennas

- The Far Fields of an Antenna and Power Flow Density -done
- The Dipole Antenna -done
- Radiation Patterns -done
- Array of Two Dipoles –today

- Radiated Power, Directivity and Gain
- Receiving Antennas, Effective Area
- The Friis Transmisision Equation

# Review: Far Field of an Antenna



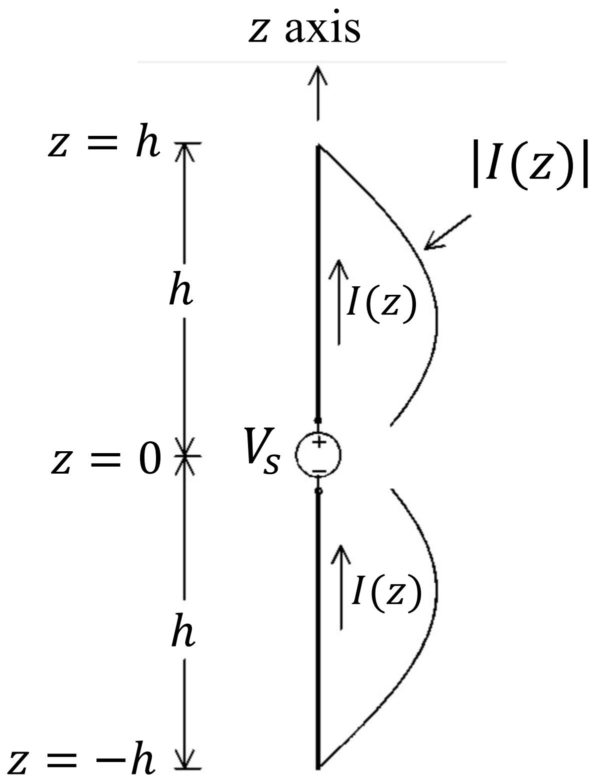

$$\overline{E} = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H} = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

The power flow density:

$$\overline{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right] \quad \text{Watts/meter}^2$$

# Review: The Far Fields of a Dipole Antenna




$$I(z) = I_o \sin\big(\beta(|z| - h)\big)$$

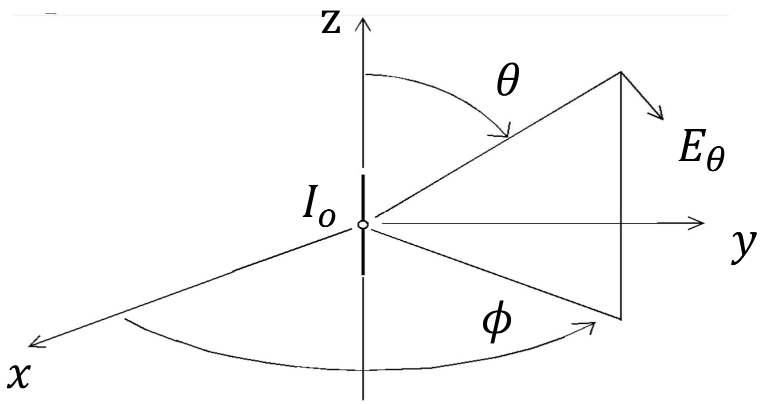
It May Be Shown that:

$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

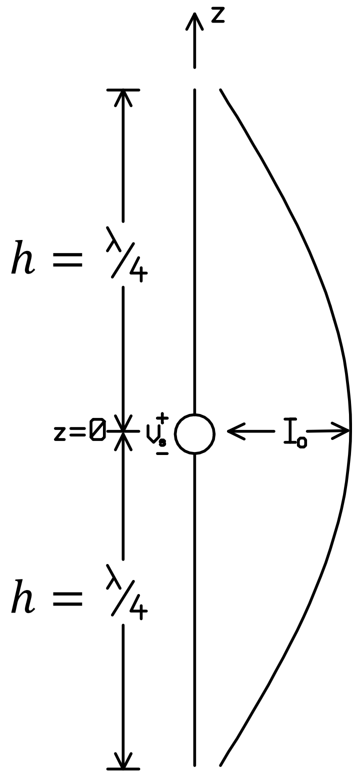
$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi) \frac{e^{-j\beta r}}{r}$$

$$e_\theta(\theta,\phi) = \frac{jI_0\eta_0}{2\pi} F(\theta)$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$



# Review: Half-Wave Dipole Antenna



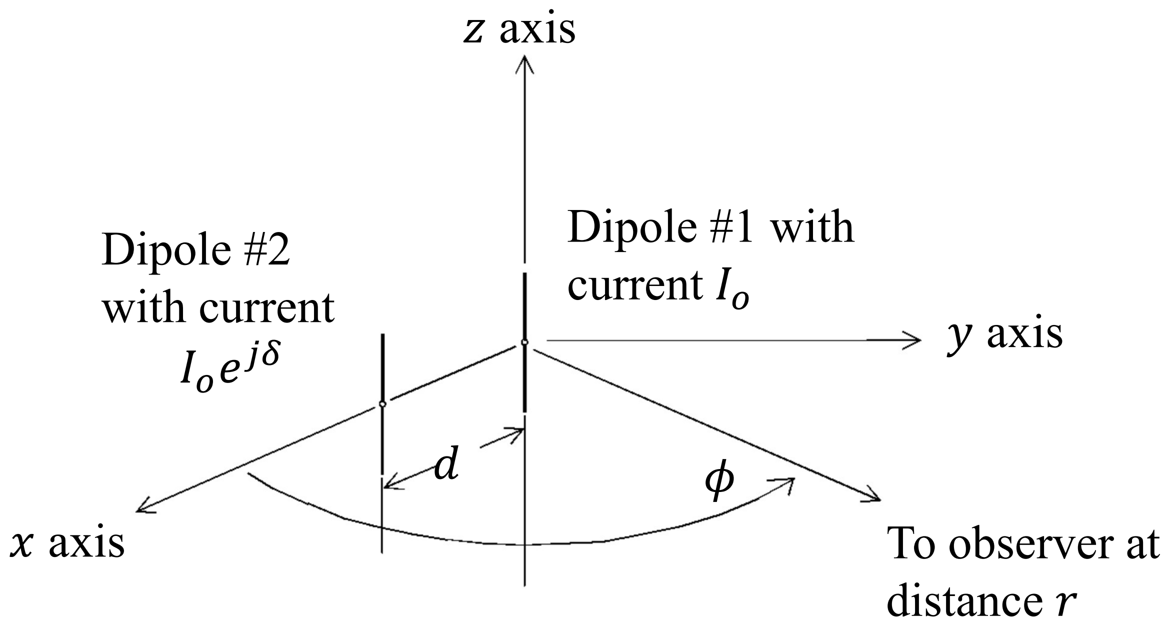

Choose length $2h = \frac{\lambda}{2}$:

$$I(z) = I_o \sin\left(\beta\left(|z| - \frac{\lambda}{4}\right)\right)$$

$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

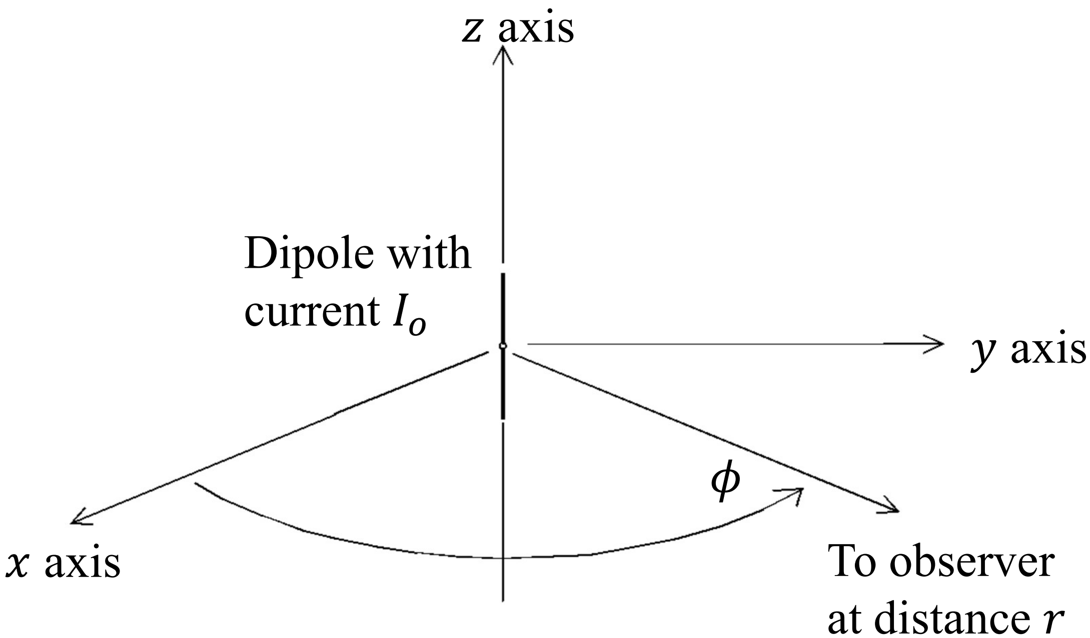
$$F(\theta) = \frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

# Array of Two Dipoles




- An antenna array is made up of two vertical, half-wave dipole antennas.
- Antenna #1 is at the origin and carries current $I_o$.
- Antenna #2 is at $x = d$ and carries current $I_o e^{j\delta}$, with a phase shift of $\delta$ relative to antenna #1.
- Find the far field for distance $r \gg d$.

# For Each Individual Dipole




For each individual dipole:

$$E_\theta = \frac{j\eta}{2\pi} I_o F(\theta) \frac{e^{-j\beta r}}{r}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

In the azimuth or $\theta = 90$ degrees plane,

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = \frac{\cos(0)}{1} = 1$$

Hence we can write $E_\theta$ for one dipole more simply as

$$E_\theta = C I_o \frac{e^{-j\beta r}}{r}$$

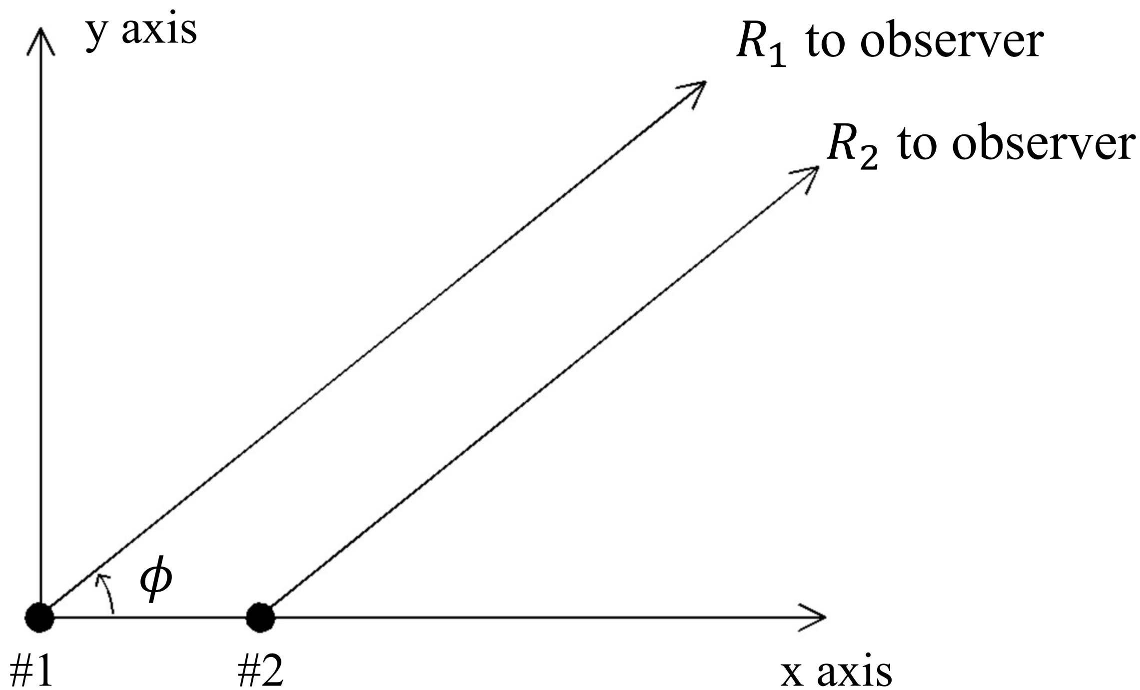
where

$$C = \frac{j\eta}{2\pi}$$

And

$r$ =the distance from the dipole to the observer.

# Array of Two Dipoles: Superposition




For one dipole "acting alone":

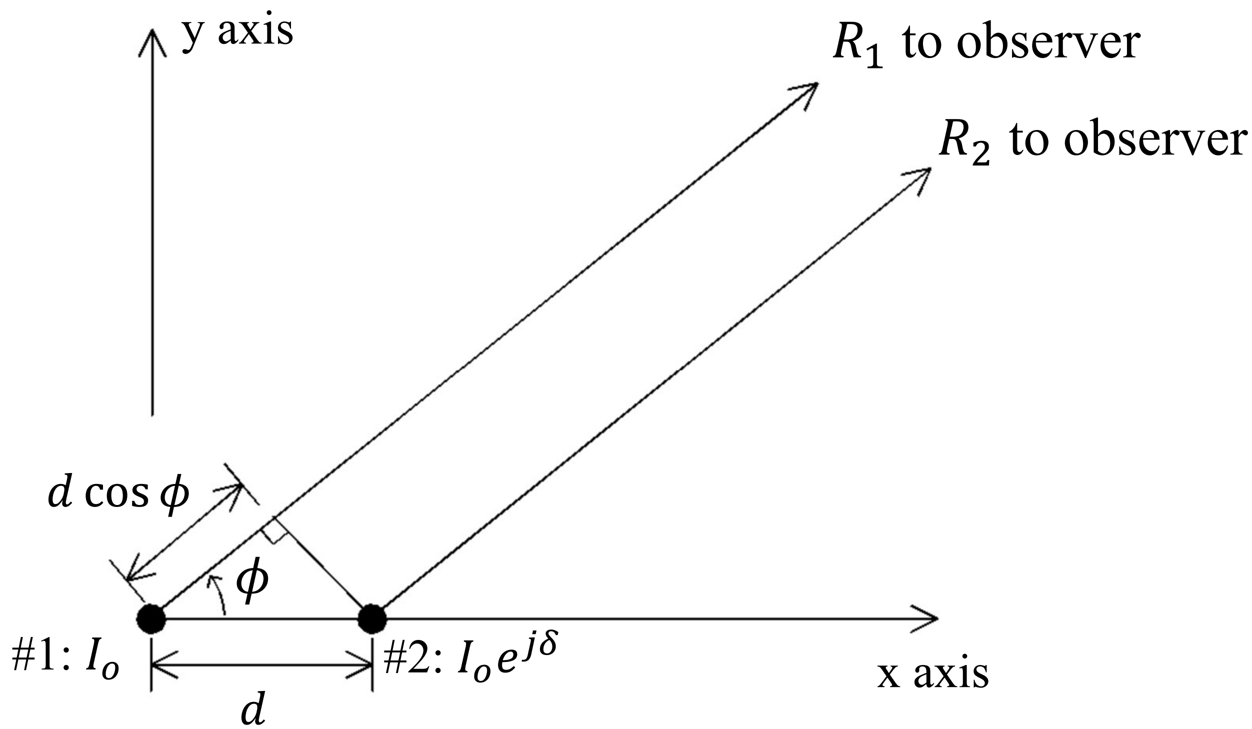
$$E_\theta = C I_o \frac{e^{-j\beta r}}{r}$$

where $C = \frac{j\eta}{2\pi}$ and $r$ =the distance from the dipole to the observer.

For two dipoles "acting together":

$$E_\theta = C I_o \frac{e^{-j\beta R_1}}{R_1} + C I_o e^{j\delta} \frac{e^{-j\beta R_2}}{R_2}$$

# Array of Two Dipoles: Far Field Approximation




$$E_\theta = CI_o \frac{e^{-j\beta R_1}}{R_1} + CI_o e^{j\delta} \frac{e^{-j\beta R_2}}{R_2}$$

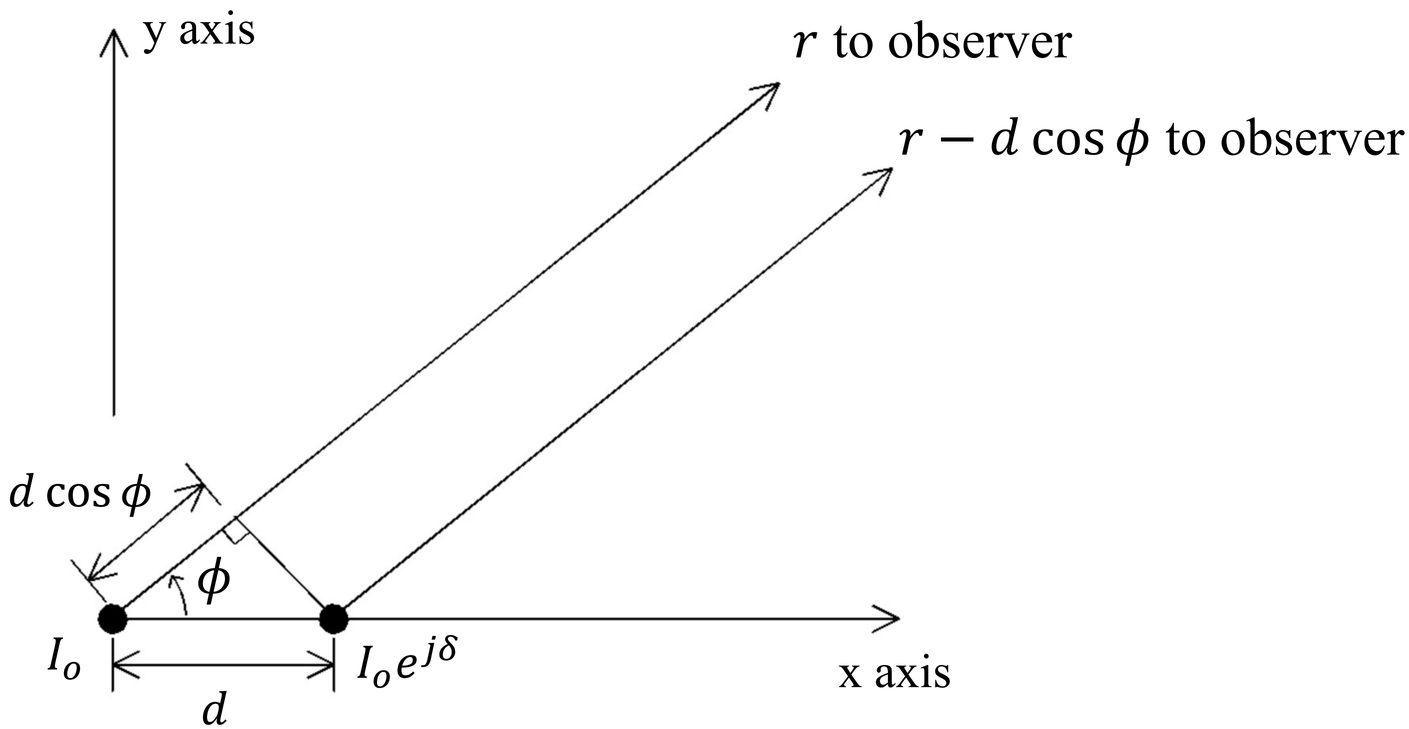
$R_1 = r$

If $R_1 \gg d$ then $R_1$ and $R_1$ are almost parallel so

$R_2 \approx r - d\cos\phi$

$$E_\theta \approx CI_o \frac{e^{-j\beta r}}{r} + CI_o e^{j\delta} \frac{e^{-j\beta(r-d\,\mathrm{co}\quad)}}{r-d\cos\phi}$$

# Array of Two Dipoles: Far Field Approximation




$$E_\theta \approx C I_o \frac{e^{-j\beta r}}{r} + C I_o e^{j\delta} \frac{e^{-j\beta(r-d\,\mathrm{co}\quad)}}{r-d\cos\phi}$$

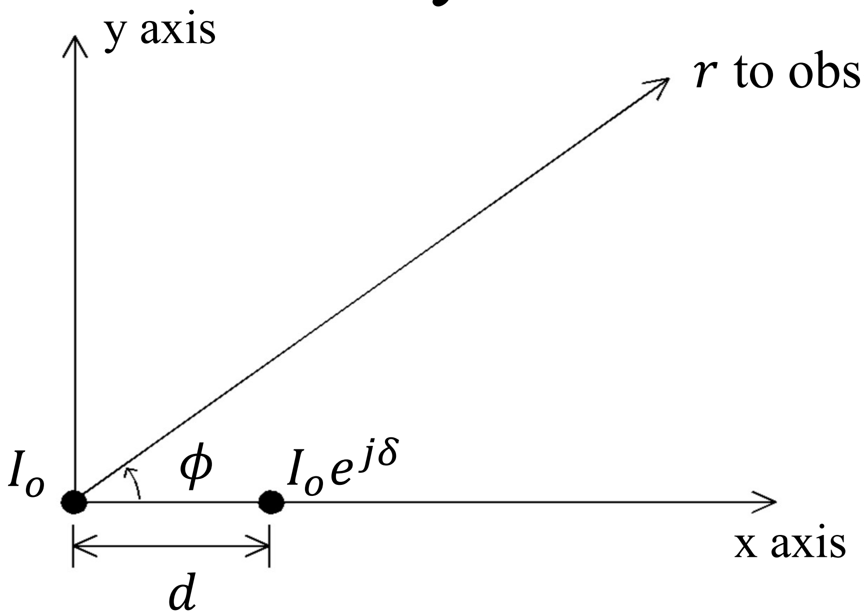
For amplitude if $r \gg d$ then $\frac{1}{r-d\,\mathrm{co}} \approx \frac{1}{r}$

However we cannot neglect $d\cos\phi$ for phase:

$$E_\theta \approx C I_o \frac{e^{-j\beta r}}{r} + C I_o e^{j\delta} \frac{e^{-j\beta(r-d\cos\phi)}}{r}$$

$$E_\theta(\phi) = C I_o \left(1 + e^{j\delta} e^{j\beta d\cos\phi}\right) \frac{e^{-j\beta r}}{r}$$

# End Fire Array: radiation in the +x direction.




$$E_\theta(\phi) = C I_o \left(1 + e^{j\delta} e^{j\beta d \,\mathrm{co}} \quad \right) \frac{e^{-j\beta r}}{r}$$

Choose $\delta = -\dfrac{\pi}{2}$ Ninety degrees out of phase.

$$d = \frac{\lambda}{4}$$ Space the antennas a quarter-wavelength apart.

$$\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$E_\theta(\phi) = C I_o \frac{e^{-j\beta r}}{r}\left(1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\phi}\right)$$

# Field Strength as a Function of Angle

$$E_\theta(\phi) = CI_o\left(1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\phi}\right)\frac{e^{-j\beta r}}{r}$$

$$|E_\theta(\phi)| = CI_o\left|1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\phi}\right|\left|\frac{e^{-j\beta r}}{r}\right|$$

$$|E_\theta(\phi)| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\phi}\right|$$

1.On the $+x$ axis, $\phi = 0$ degrees, $\cos\phi = \cos 0 = 1$ and

$$|E_\theta(0)| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos 0}\right| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}}\right|$$

$$= \frac{CI_o}{r}|1 + 1| = 2\frac{CI_o}{r}$$

The field strength at $\phi = 0$ is a maximum.

# Field Strength as a Function of Angle, continued

$$|E_\theta(\phi)| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{2}\cos\phi}\right|$$

2.On the $+y$ axis, $\phi = 90$ degrees, $\cos\phi = \cos 90 = 0$ and

$$|E_\theta(90)| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{2}\cos 90}\right| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}}e^{j0}\right|$$

$$= \frac{CI_o}{r}|1 - j| = \sqrt{2}\frac{CI_o}{r}$$

The field strength is reduced from $2\frac{CI_o}{r}$ at $\phi = 0$ to $\sqrt{2}\frac{CI_o}{r}$ at $\phi = 90$ .

# Field Strength as a Function of Angle, continued

$$|E_\theta(\phi)| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\phi}\right|$$

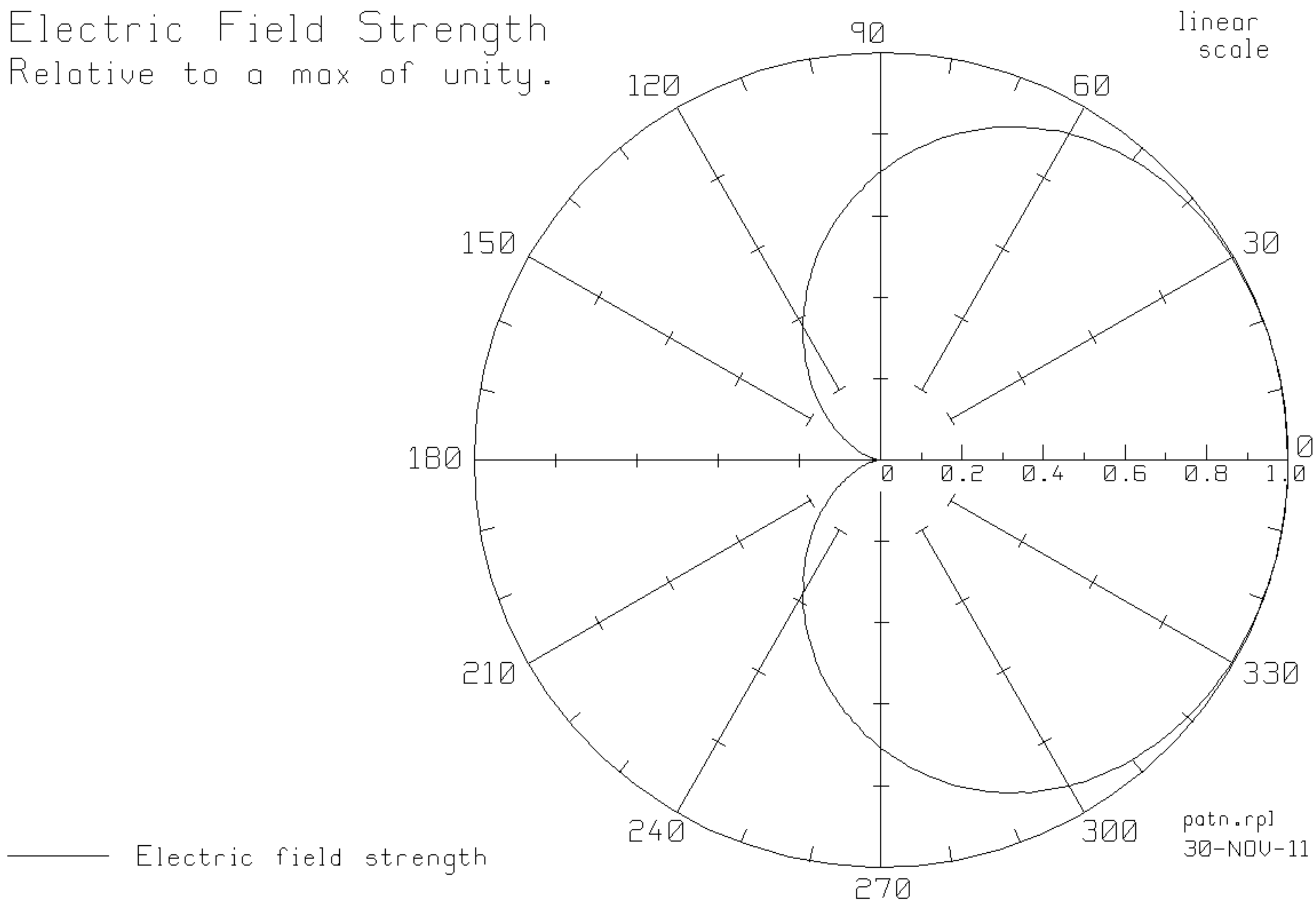
3.On the $-x$ axis, $\phi = 180$ degrees, $\cos\phi = \cos 180 = -1$ and

$$|E_\theta(0)| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos 180}\right| = \frac{CI_o}{r}\left|1 + e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{2}}\right|$$

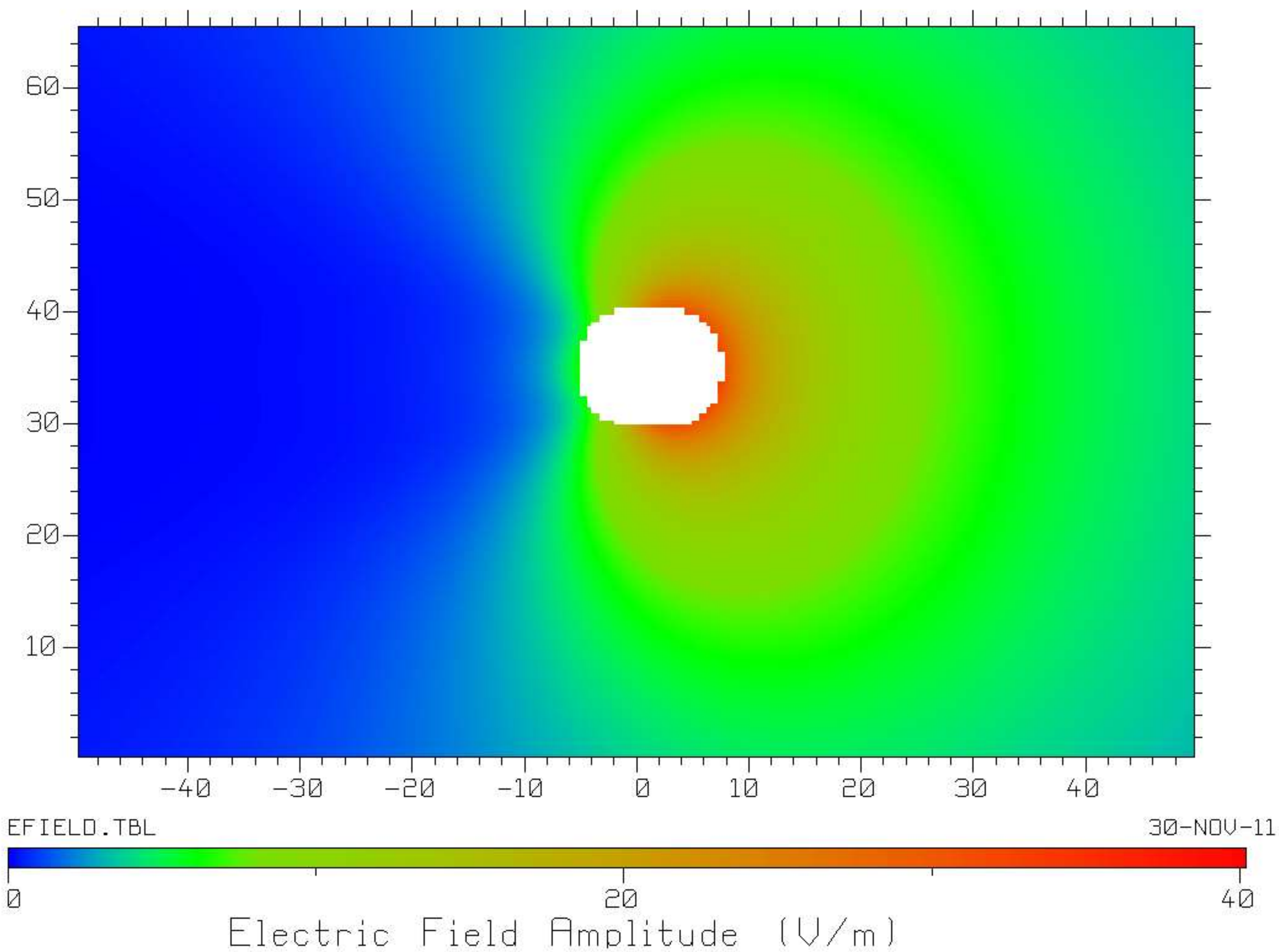
$$= \frac{CI_o}{r}\left|1 + e^{-j\pi}\right| = \frac{CI_o}{r}|1 - 1| = 0$$

The field strength at $\phi = 180$ degrees is zero.

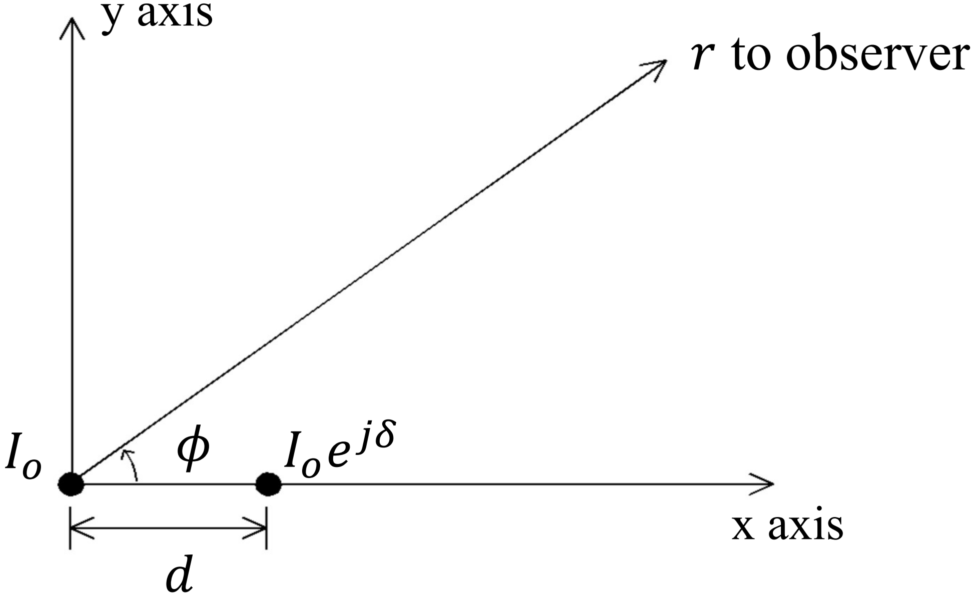
# Cardioid Radiation Pattern



# Amplitude vs. Position



# Broadside Array: radiation in the +y and –y direction.




$$E_\theta(\phi) = CI_o\left(1 + e^{j\delta} e^{j\beta d \text{ co}} \quad\right)\frac{e^{-j\beta r}}{r}$$

Choose: $\delta = 0$ Operate the antennas "in phase" $e^{j\delta} = e^{j0} = 1$

$$d = \frac{\lambda}{2}$$

Space the antennas a half-wavelength apart

$$\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

$$E_\theta(\phi) = CI_o\left(1 + e^{j0} e^{j\pi \cos\phi}\right)\frac{e^{-j\beta r}}{r} = CI_o\left(1 + e^{j\pi \text{ co}} \quad\right)\frac{e^{-j\beta r}}{r}$$

# Field Strength as a Function of Angle

$$E_\theta(\phi) = CI_o\left(1 + e^{j\pi\cos\phi}\right)\frac{e^{-j\beta r}}{r}$$

$$|E_\theta(\phi)| = CI_o\left|1 + e^{j\pi\cos\phi}\right|\left|\frac{e^{-j\beta r}}{r}\right|$$

$$|E_\theta(\phi)| = \frac{CI_o}{r}\left|1 + e^{j\pi\cos\phi}\right|$$

1.On the $+x$ axis, $\phi = 0$ degrees, $\cos\phi = \cos 0 = 1$ and

$$|E_\theta(0)| = \frac{CI_o}{r}\left|1 + e^{j\pi\cos 0}\right| = \frac{CI_o}{r}\left|1 + e^{j\pi}\right| = \frac{CI_o}{r}|1 - 1| = 0$$

The field strength is zero in the $+x$ direction.

# Field Strength as a Function of Angle, continued

$$|E_\theta(\phi)| = \frac{CI_o}{r}\left|1 + e^{j\pi\cos\phi}\right|$$

2. On the $+y$ axis, $\phi = 90$ degrees, $\cos\phi = \cos 90 = 0$ and

$$|E_\theta(90)| = \frac{CI_o}{r}\left|1 + e^{j\pi\cos 90}\right| = \frac{CI_o}{r}\left|1 + e^{j0}\right|$$

$$= \frac{CI_o}{r}|1 + 1| = 2\frac{CI_o}{r}$$

The field strength is a maximum along the *y* direction.

# Field Strength as a Function of Angle, continued

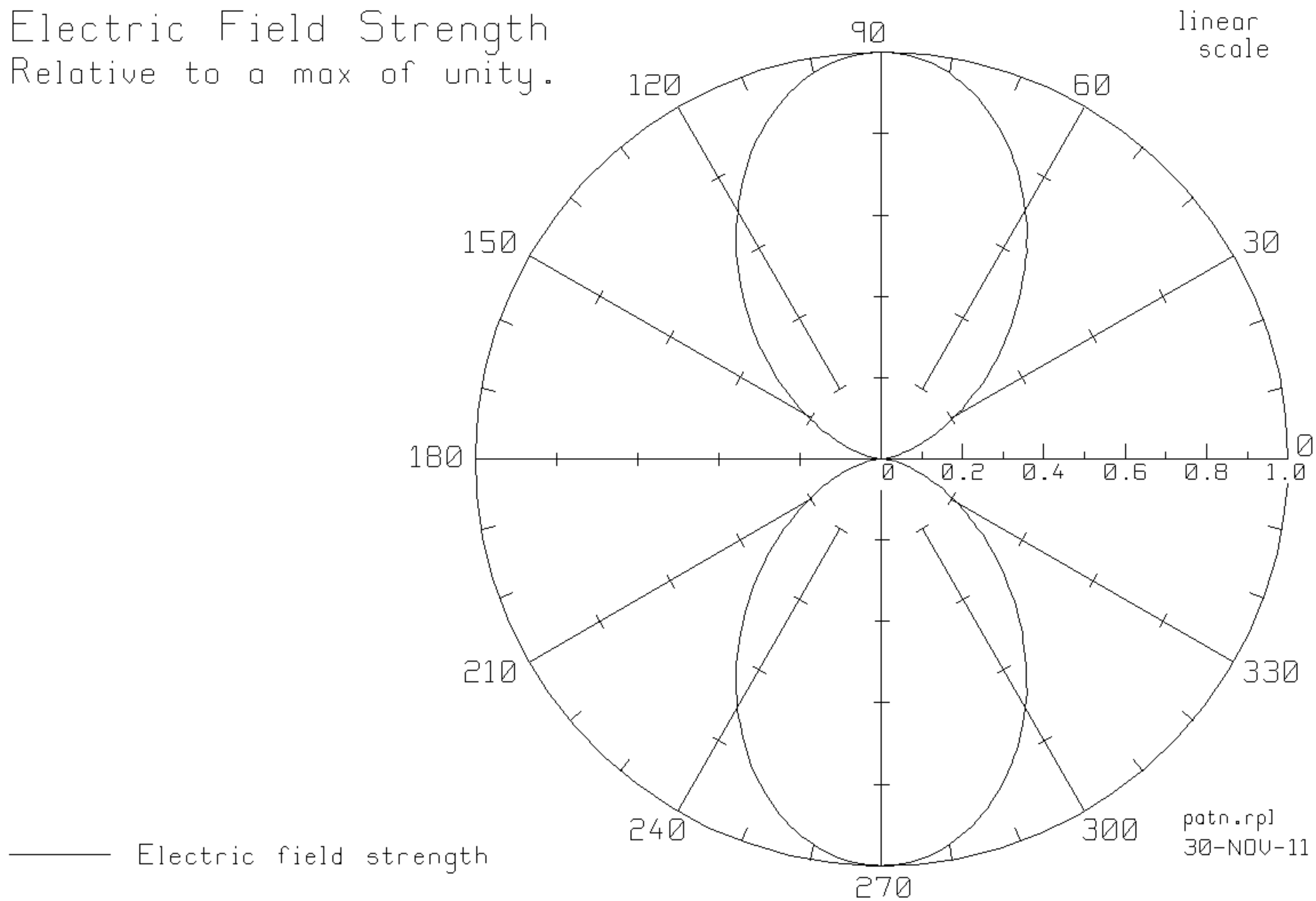
$$|E_\theta(\phi)| = \frac{CI_o}{r}\left|1 + e^{j\pi\cos\phi}\right|$$

3.On the $-x$ axis, $\phi = 180$ degrees, $\cos\phi = \cos 180 = -1$ and

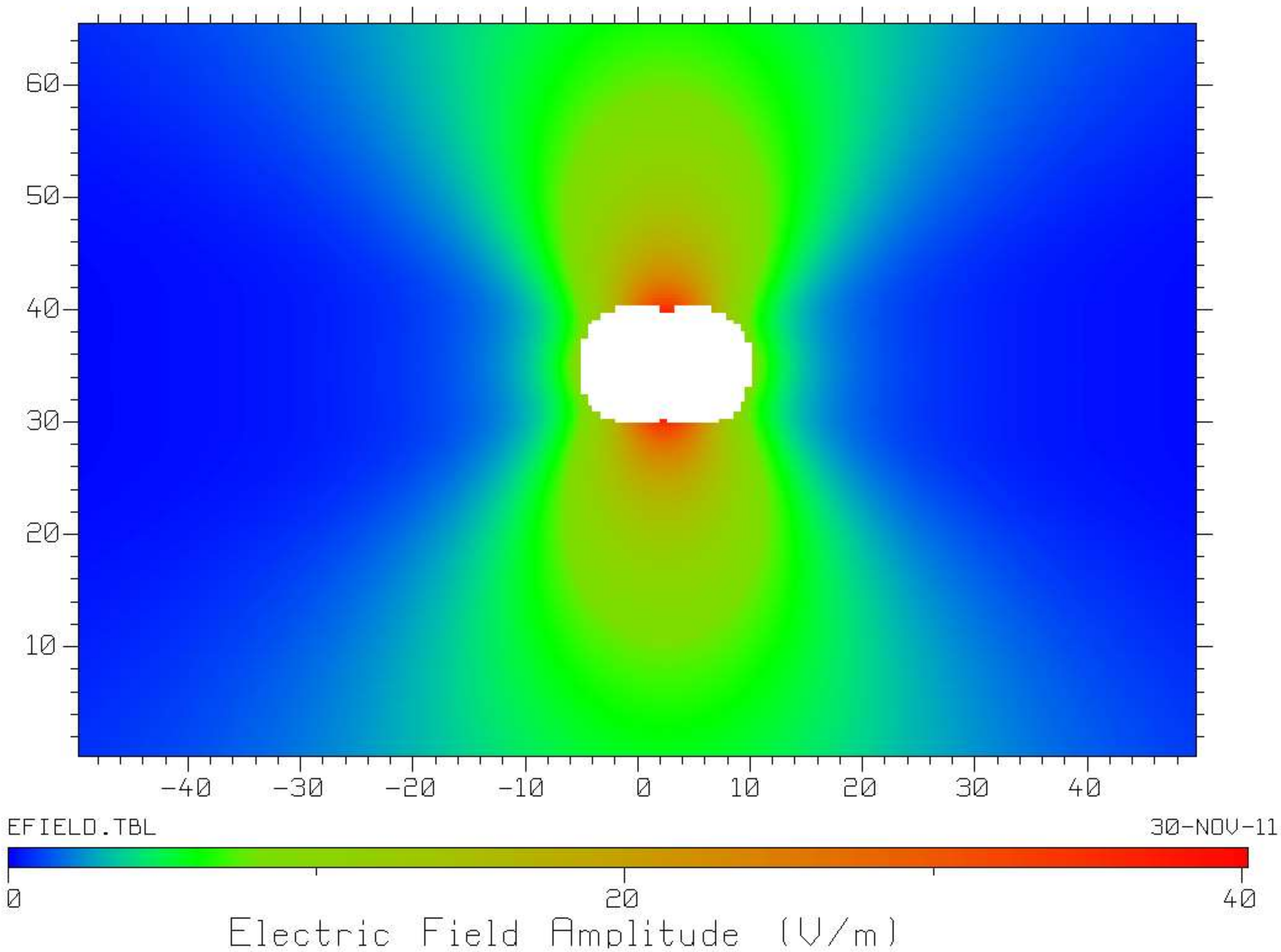
$$|E_\theta(0)| = \frac{CI_o}{r}\left|1 + e^{j\pi\cos 180}\right| = \frac{CI_o}{r}\left|1 + e^{-j\pi}\right| = \frac{CI_o}{r}|1 - 1| = 0$$

The field strength is zero in the $-x$ direction.

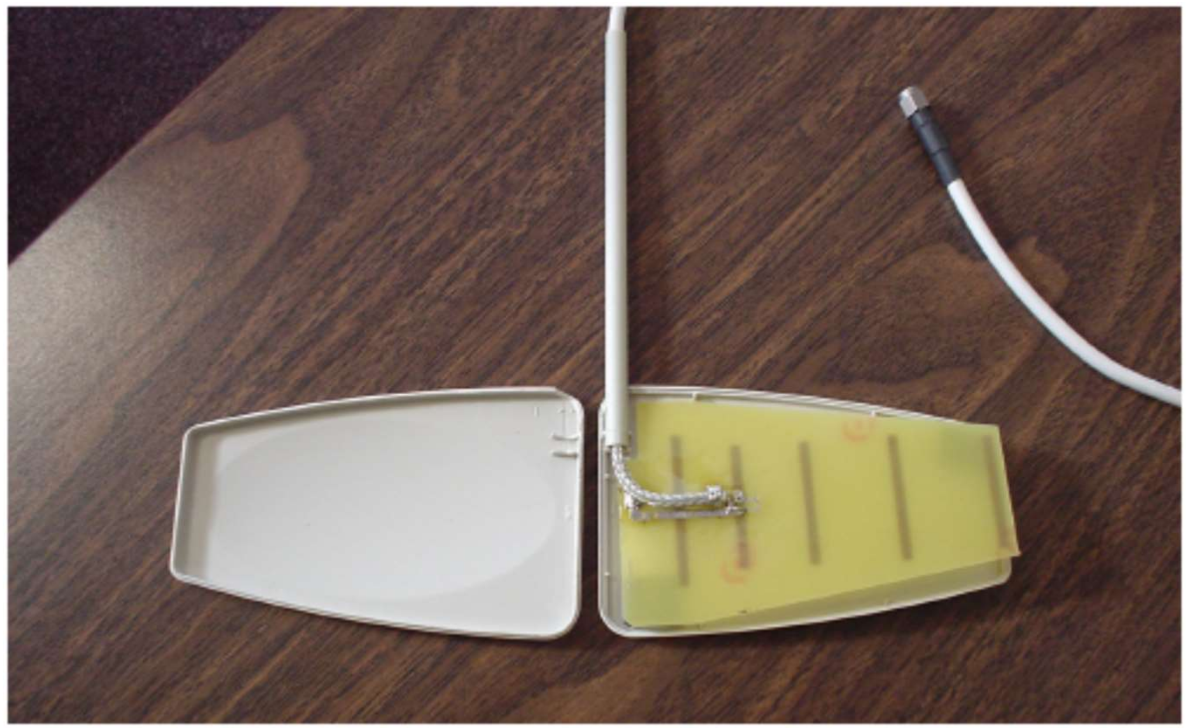
# Radiation Pattern



# Amplitude vs. Position

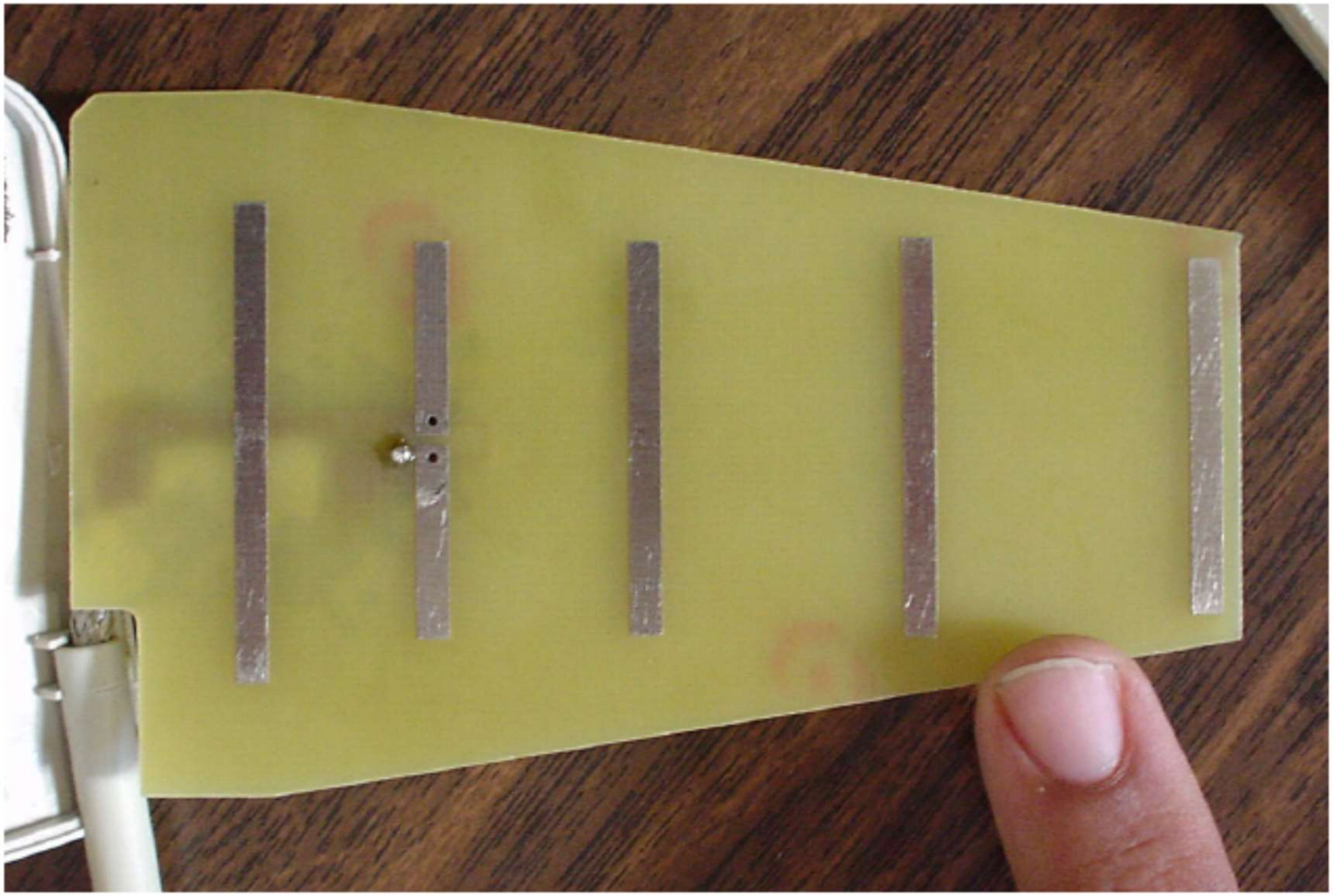


# Directional Access-Point Antenna

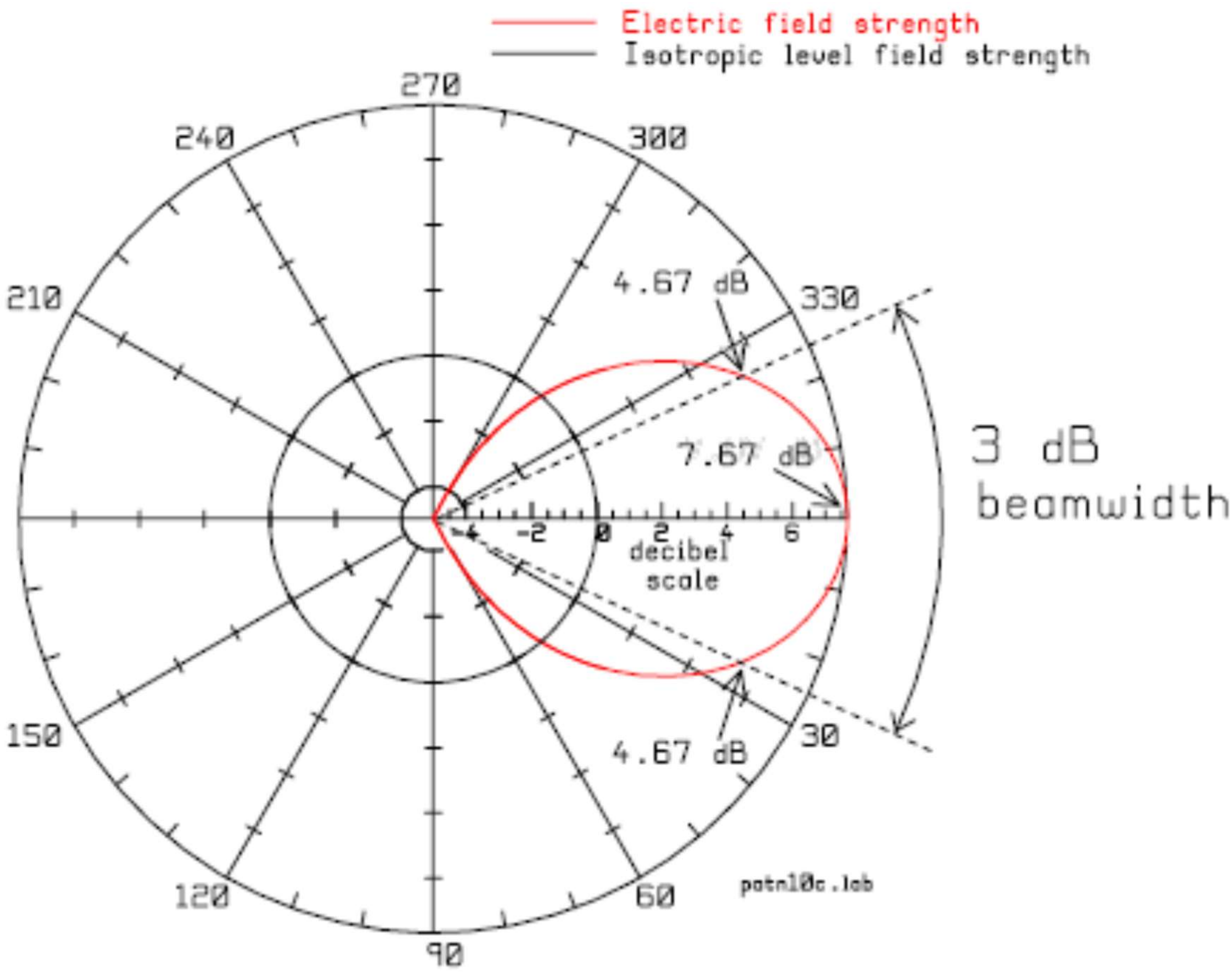


Circuit board material

FR4 epoxy
epsr = 4.4,
tan $\delta$ = 0.02



# Radiation Pattern of the Directional AP Antenna



# ELEC353 Lecture Notes Set 19

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

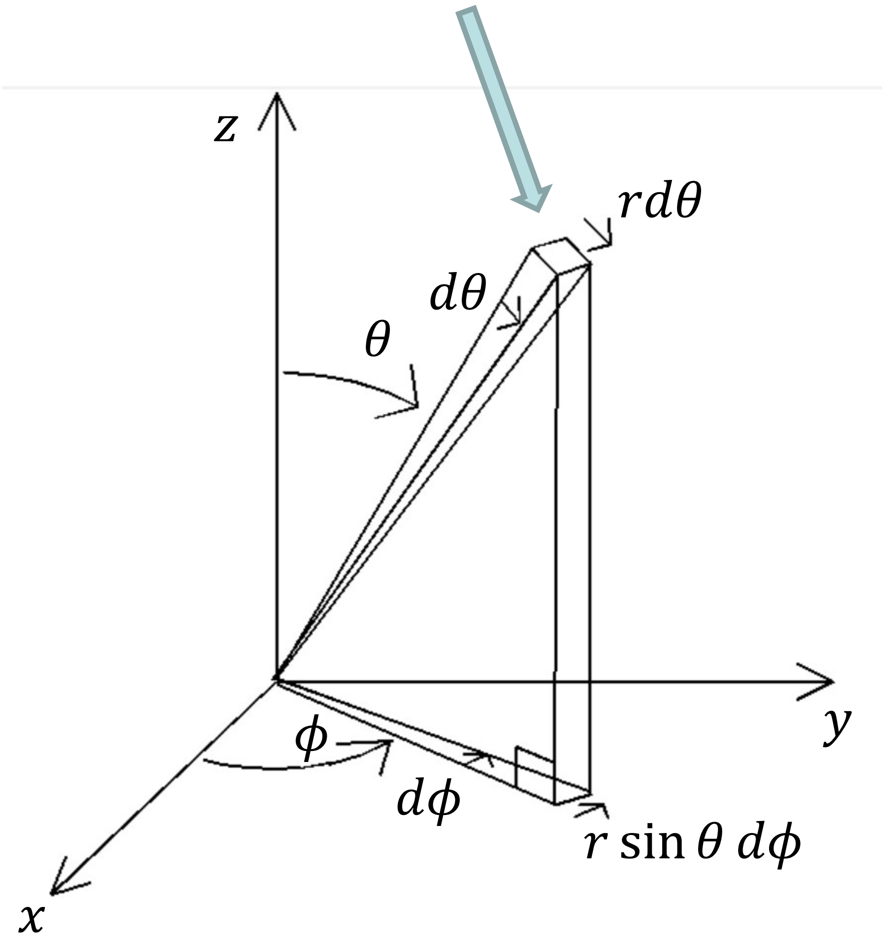
Homework #11: Do homework #11 by April 5, 2019.
Homework #12: Do homework #12 by April 12, 2019.

Tutorial Workshop #12: Friday April 5, 2019.
Tutorial Workshop #13: Friday April 12, 2019

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

# The Radiated Power




Enclose the antenna in a sphere of radius $r$. The power density flowing through the surface of the sphere is $S_{av}(\theta,\phi)$ W/m$^2$. The power flowing through a patch of the surface of the sphere of area $ds = r^2\sin\theta d\theta d\phi$ is

$$dP_{rad} = S_{av}(\theta,\phi)ds$$

To trace out a full sphere, we must let $\theta$ increase from 0 to $\pi$, and for each $\theta$ we must let $\phi$ trace out a full circle from 0 to $2\pi$. Then the total amount of power flowing through the sphere is

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi)ds$$

$$ds = r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi) r^2 \sin\theta d\theta d\phi$$

$$\bar{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right]$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{r^2}\left(\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right) r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left(\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right) \sin\theta d\theta d\phi$$

# The Isotropic Power Density

Suppose that we have an antenna that radiates the same power density $S_{iso}$ uniformly in all directions.

$$S_{av}(r,\theta,\phi) = S_{iso}(r)$$

$S_{iso}$ is not a function of $\theta$ or $\phi$

In general:

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_{av}(\theta,\phi) r^2 \sin\theta d\theta d\phi$$

For the isotropic antenna:

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_{iso} r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = S_{iso} r^2 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta d\theta d\phi$$

$$P_{rad} = 4\pi r^2 S_{iso}$$

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

"isotropic power density"

# The Isotropic Level Field Strength

“isotropic power density”

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

In general

$$S_{av} = \frac{1}{r^2}\left[\frac{\left|e_\theta\right|^2}{2\eta} + \frac{\left|e_\phi\right|^2}{2\eta}\right]$$

Suppose $e_\theta(\theta,\phi) = e_{iso}$ and $e_\phi(\theta,\phi) = 0$

$$S_{av} = \frac{1}{r^2}\left[\frac{e_{iso}^2}{2\eta}\right] \qquad = \frac{P_{rad}}{4\pi r^2}$$

$$\frac{e_{iso}^2}{2\eta} = \frac{P_{rad}}{4\pi}$$

$$e_{iso} = \sqrt{\frac{\eta P_{rad}}{2\pi}}$$

“isotropic level”
field strength

# System-Level Antenna Definitions

Directive Gain

$$D(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{S_{iso}}$$ = power density function / isotropic power density

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

$$D(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{P_{rad}/4\pi r^2}$$

$P_{rad}$ = the radiated power

Directivity = the maximum value of the directive gain:

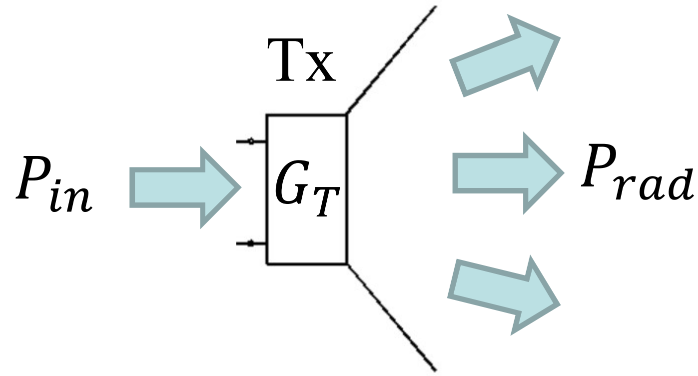
$$D_{\max} = \max\left[ D(\theta,\phi)\Big|_{\substack{0\le\theta\le\pi \\ 0\le\phi\le 2\pi}} \right]$$

In practice:

$$S_{av}(\theta,\phi) = S_{iso} D(\theta,\phi)$$

$$S_{av}(\theta,\phi) = \frac{P_{rad}}{4\pi r^2} D(\theta,\phi)$$

# Lossy Antennas and Gain




Efficiency: $e = \frac{P_{rad}}{P_{in}} \leq 1$

$P_{rad}$ = the **radiated** power

$P_{in}$ = the **input** power

Power Gain: $G(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{P_{in}/4\pi r^2}$

Relation of the Power Gain to the Directive Gain:

$$G(\theta,\phi) = \frac{S_{av}}{P_{in}/4\pi r^2} = \frac{S_{av}}{\frac{1}{e}P_{rad}/4\pi r^2} = e\frac{S_{av}}{P_{rad}/4\pi r^2} = eD(\theta,\phi)$$

$$G(\theta,\phi) = eD(\theta,\phi).$$

# The "Gain" of an Antenna

The "gain" of an antenna is the maximum value of the power gain:

$$G_{max} = \max_{\substack{0 \le \theta < \pi \\ 0 \le \phi < 2\pi}} G(\theta,\phi)$$

$$G_{max} = \max_{\substack{0 \le \theta < \pi \\ 0 \le \phi < 2\pi}} \left[eD(\theta,\phi)\right] = eD_{max}$$

# Directivity of a Half-Wave Dipole

$$D(\theta,\phi)=\frac{S_{av}(\theta,\phi)}{P_{rad}/4\pi r^2}$$

$$S_{av}=\frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta_0}+\frac{|e_\phi|^2}{2\eta_0}\right]$$

$$S_{av}=\frac{1}{2\eta_0 r^2}\left(|e_\theta|^2+|e_\phi|^2\right)$$

$$e_\theta(\theta,\phi)=\frac{jI_0\eta_0}{2\pi}F(\theta)$$

$$F(\theta)=\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$S_{av}=\frac{1}{2\eta_0 r^2}\left(\frac{I_0\eta_0}{2\pi}\right)^2\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2$$

$$S_{av}=\frac{I_0^2\eta_0}{8\pi^2 r^2}\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2$$

$$S_{av} = \frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi) r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2 r^2 \sin\theta d\theta d\phi$$

This can be integrated numerically to obtain

$$P_{rad} = 36.5 I_0^2$$

Half-wave Dipole:

$$S_{av} = \frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2 \Rightarrow P_{rad} = 36.5 I_0^2$$

directive gain

$$D(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{P_{rad}/4\pi r^2}$$

so

$$D(\theta,\phi) = \frac{\frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2}{36.5 I_0^2 / 4\pi r^2}$$

From the previous slide:

$$D(\theta,\phi)=\frac{\dfrac{I_0^2\eta_0}{8\pi^2r^2}\left|\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2}{36.5I_0^2\big/4\pi r^2}$$

$$D(\theta,\phi)=\frac{\eta_0}{36.5(2\pi)}\left|\frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2$$

This is the directive gain of the half wave dipole antenna.

The "directivity" is the maximum value of the directive gain.

As angle $\theta$ varies the maximum value of $\frac{\cos\left(\frac{\pi}{2}\cos\ \right)}{\sin\theta}$ is unity for $\theta = 90$ degrees.

For $\theta = 90$ degrees, $D(\theta,\phi) = D_{max}$ and

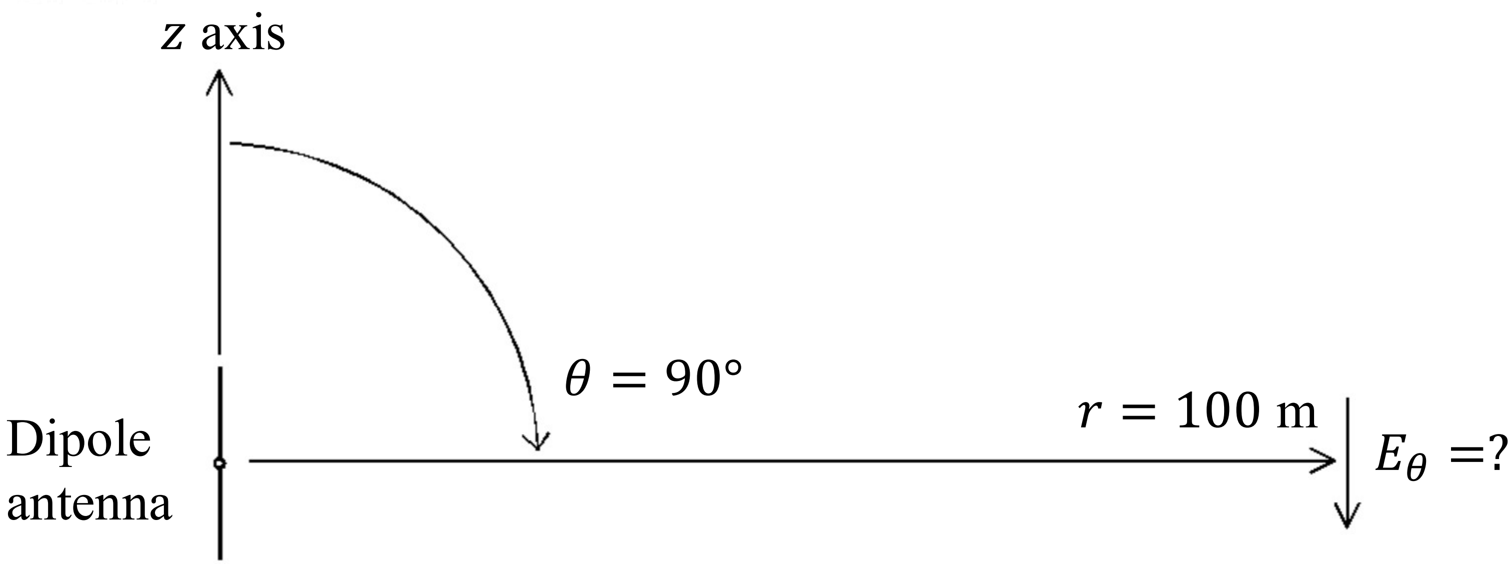
$$D_{\max}=\frac{\eta_0}{36.5(2\pi)}=1.64$$

The directivity of a half-wave dipole antenna is 1.64.

**Example**

What is the electric field strength 100 m from a half-wave dipole which radiates 600 mW at 850 MHz? Assume that the observer is in a plane perpendicular to the axis of the dipole.

Solution




- An isotropic antenna that radiates $P_{rad}$ W has a power density of

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

- Since $D(\theta,\phi) = \dfrac{S_{av}(\theta,\phi)}{S_{iso}}$, the power density of the dipole is larger according to the directive gain of the dipole $S_{av}(\theta) = D(\theta) S_{iso}$

$$S_{iso} = \frac{P_{rad}}{4\pi r^2} \qquad\qquad S_{av}(\theta) = D(\theta) S_{iso}$$

Since the observer is on a line perpendicular to the axis of the dipole, we have $\theta = 90$ degrees, and we can use the directivity of the dipole.

$D_{max} = D(90) = 1.64$

$$S_{av}(90) = D(90) S_{iso} = 1.64 \frac{P_{rad}}{4\pi r^2}$$

The power density radiated by a dipole antenna is $S_{av} = \frac{E_\theta^2}{2\eta}$ so $E_\theta = \sqrt{2\eta S_{av}}$

$$E_\theta(r) = \sqrt{2\eta S_{av}} = \sqrt{2\eta\left(1.64 \frac{P_{rad}}{4\pi r^2}\right)}$$

$\eta_0 \approx 120\pi$

$$E_\theta = \sqrt{2 \cdot 120\pi\left(1.64 \frac{P_{rad}}{4\pi r^2}\right)} = \frac{9.92}{r}\sqrt{P_{rad}}$$

$$E_\theta = \frac{9.92}{100}\sqrt{0.6} = 76.8 \text{ mV/m}$$ Volts/meter (amplitude)

# EMC due to a cell phone

- Electromagnetic compatibility or EMC deals with the ability of electrical devices to work together without interfering with one another.
- The "immunity" of a device is the largest electric field strength that a device can tolerate and still function correctly.

Example #1

In the 1990s medical devices had an immunity of 3 V/m rms. Analog cell phones operated at 850 MHz and radiated 600 mW of power. Assuming that the antenna behaves as a vertical, half-wave dipole, how close can the cell phone be to a medical device without exceeding the immunity field strength?

Example #2

A modern digital cell phone radiates 100 mW at 1900 MHz. Modern medical devices have immunity 20 V/m rms. How close can a digital cell phone come to a medical device and still be safe?

Example #1

In the 1990s medical devices had an immunity of 3 V/m. Analog cell phones operated at 850 MHz and radiated 600 mW of power. Assuming that the antenna behaves as a vertical, half-wave dipole, how close can the cell phone be to a medical device without exceeding the immunity field strength?

Solution

The isotropic power density is

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

The directivity of the dipole is $D = 1.64$, so the actual power density is

$$S_{av} = DS_{iso} = D\frac{P_{rad}}{4\pi r^2}$$

The dipole radiates field strength $E_\theta$ V/m (rms) and so the power density is related to the field strength by

$$S_{av} = \frac{E_\theta^2}{\eta}$$

So the field strength in volts rms at distance $r$ from the dipole is

$$E_\theta = \sqrt{\eta S_{av}} = \sqrt{\eta D\frac{P_{rad}}{4\pi r^2}}$$

$$E_\theta = \sqrt{\eta D \frac{P_{rad}}{4\pi r^2}}$$

The distance at which the field strength is equal to the immunity of $E_I$ is

$$r = \frac{1}{E_I}\sqrt{\eta D \frac{P_{rad}}{4\pi}}$$

For older medical devices with an immunity of $E_I$=3 V/m, and for the analog cell phone with $P_{rad}$ =600 mW:

$$r = \frac{1}{3}\sqrt{377x1.64x\frac{0.600}{4\pi}} = 1.81 \text{ m}$$

So to be safe, your cell phone must be further than 1.8 m from medical devices.

Example #2

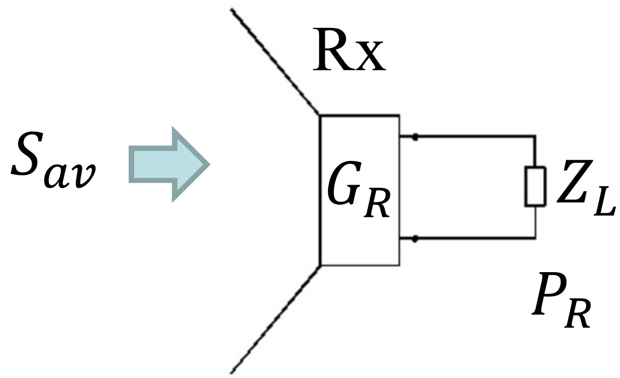
A modern digital cell phone radiates 100 mW at 1900 MHz. Modern medical devices have immunity 20 V/m. How close can a digital cell phone come to a medical device and still be safe?

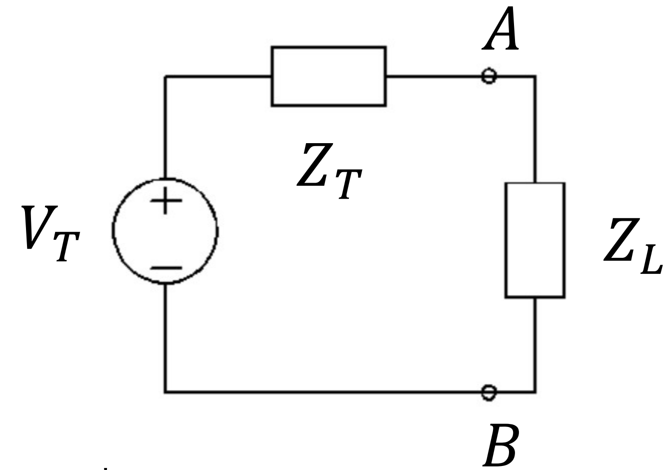
For the digital cell phone, $P_{rad}$ = 100 mW and $E_I$=20 V/m

$$r = \frac{1}{20}\sqrt{377x1.64x\frac{0.100}{4\pi}} = 0.11 \text{ m}$$

To be safe your cell phone must be further than 11 cm from medical devices.

# Receiving Antennas




$$S_{av} = \frac{E^2}{2\eta} \text{ watts/meter}^2,$$

"Matched" load: $Z_L = Z_T^*$

$$P_{rec} = A_e S_{av}$$

$A_e$ = "effective area" or "aperture" of the antenna.

The effective area is related to the gain in a simple way:

$$A_e = \frac{\lambda^2}{4\pi} G$$

Lossless antennas: gain = directivity so $A_e = \frac{\lambda^2}{4\pi} D$

## Example

What is the maximum effective area of a half-wave dipole antenna operating at 1900 MHz? If the field strength of the incident plane wave is 100 mV/m, what is the power that the dipole receives into a matched load? Assume that the dipole is a lossless antenna.

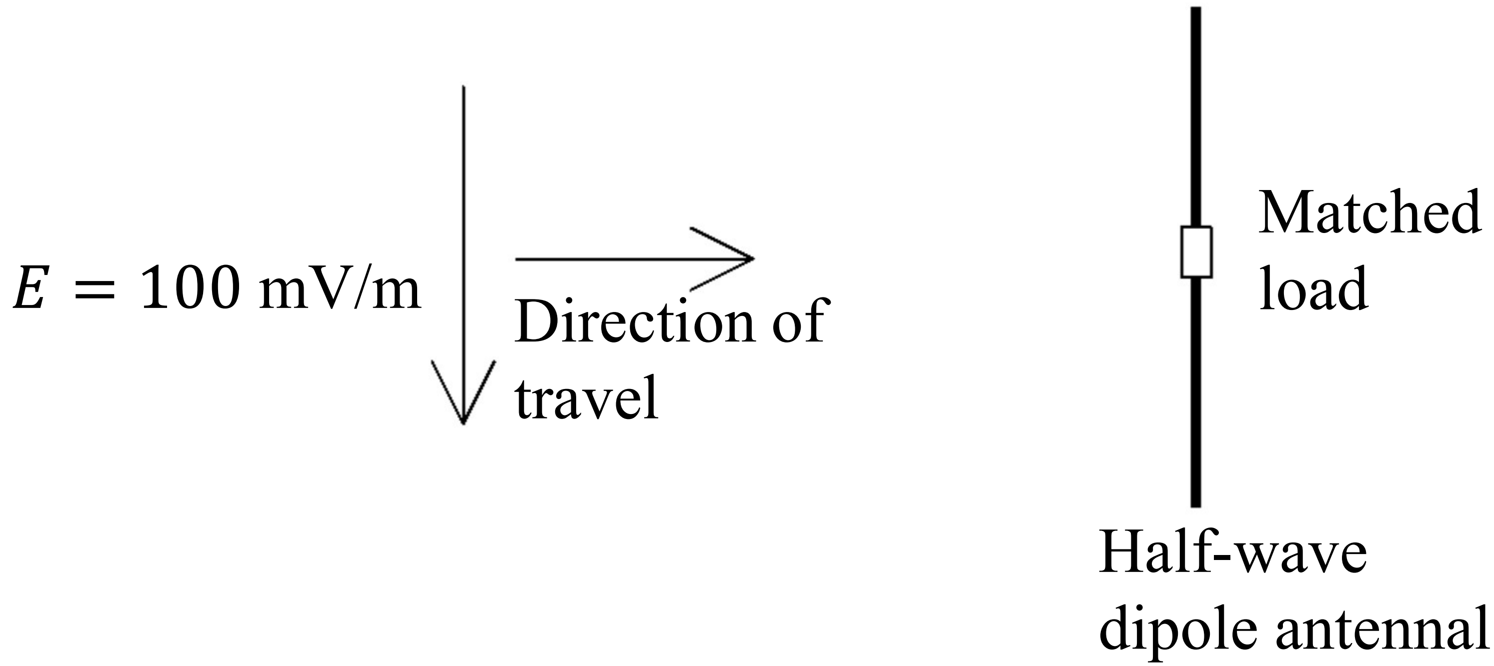
Solution

- The effective area of the dipole is $A_e = \dfrac{\lambda^2}{4\pi} G$
- The dipole is lossless so the power gain is equal to the directive gain, $G(\theta) = D(\theta)$. ("Lossless" means that the efficiency is unity, $e = 1$.)
- So the effective area is

$$A_e(\theta) = \frac{\lambda^2}{4\pi} D(\theta)$$

- The maximum directive gain of the half-wave dipole is the "directivity" and is $D_{\max} = 1.64$
- The wavelength at 1900 MHz is $\lambda = \dfrac{c}{f} = \dfrac{300}{1900} = 0.1579 \text{ m}$
- So the maximum effective area is

$$A_e = \frac{\lambda^2}{4\pi} D_{\max} = \frac{(0.1579)^2}{4\pi} x1.64 = 3.25x10^{-3} \text{ square meters}$$



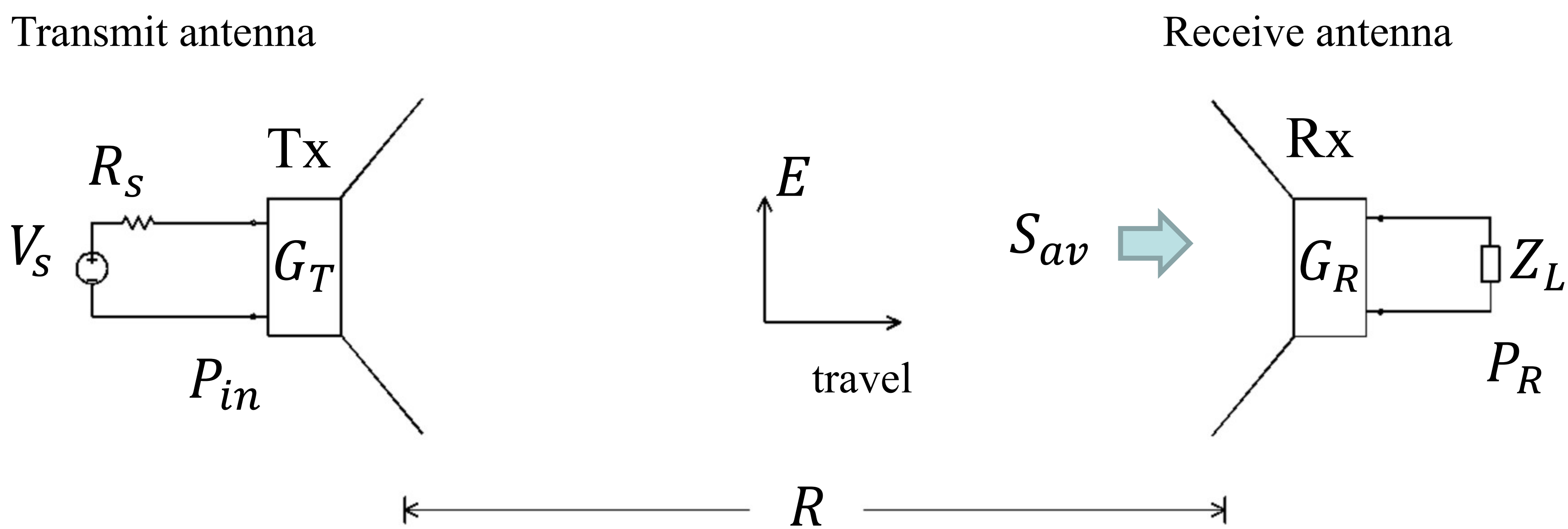

- The power density of a plane wave of field strength $E = 100$ mV/m amplitude is

  $$S_{av} = \frac{E^2}{2\eta} = \frac{(0.1)^2}{2 \cdot 377} = 13.26 \text{ microwatts per square meter}$$

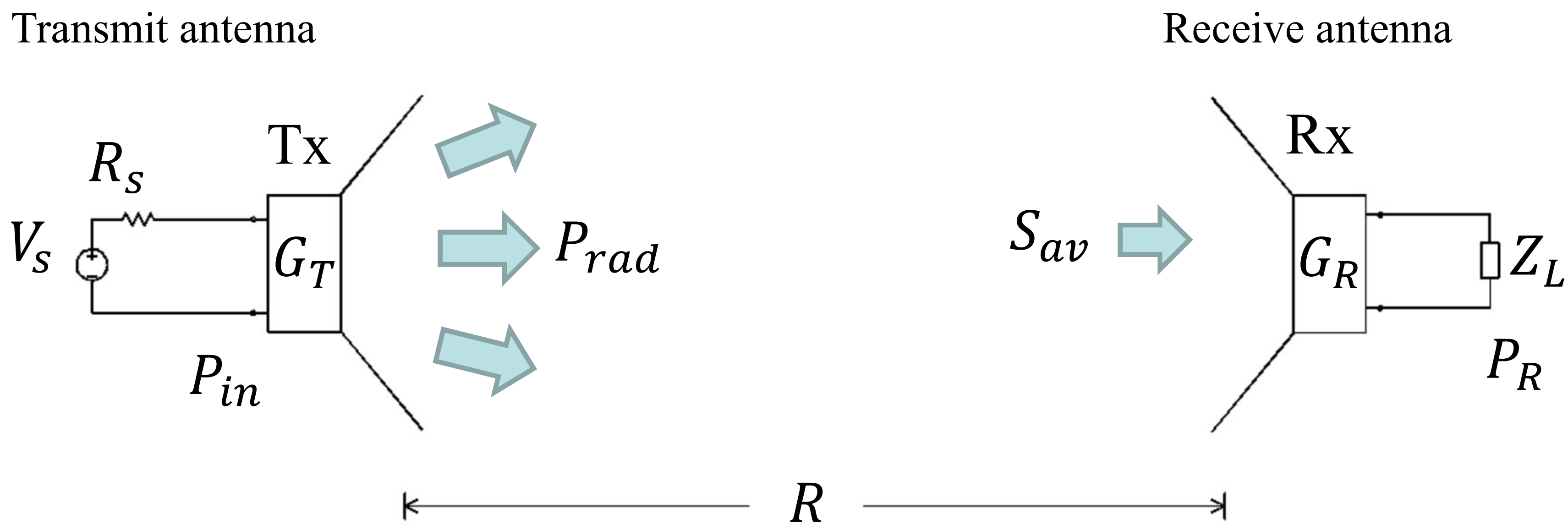
  where the approximation $\eta \approx 377$ ohms has been used.
- The power received into a matched load is

  $$P_{rec} = A_e S_{av} = (3.25x10^{-3})(13.26x10^{-6}) = 43.1 \text{ nanoWatts}$$

# The Friis Transmission Equation




- A transmit antenna (Tx) has input power $P_{in}$ and gain $G_T$.
- It is located at a distance $R$ from a receiving antenna.
- The receiving antenna (Rx) has gain $G_R$ and is terminated in a matched load.
- Find the received power $P_R$ into the matched load.



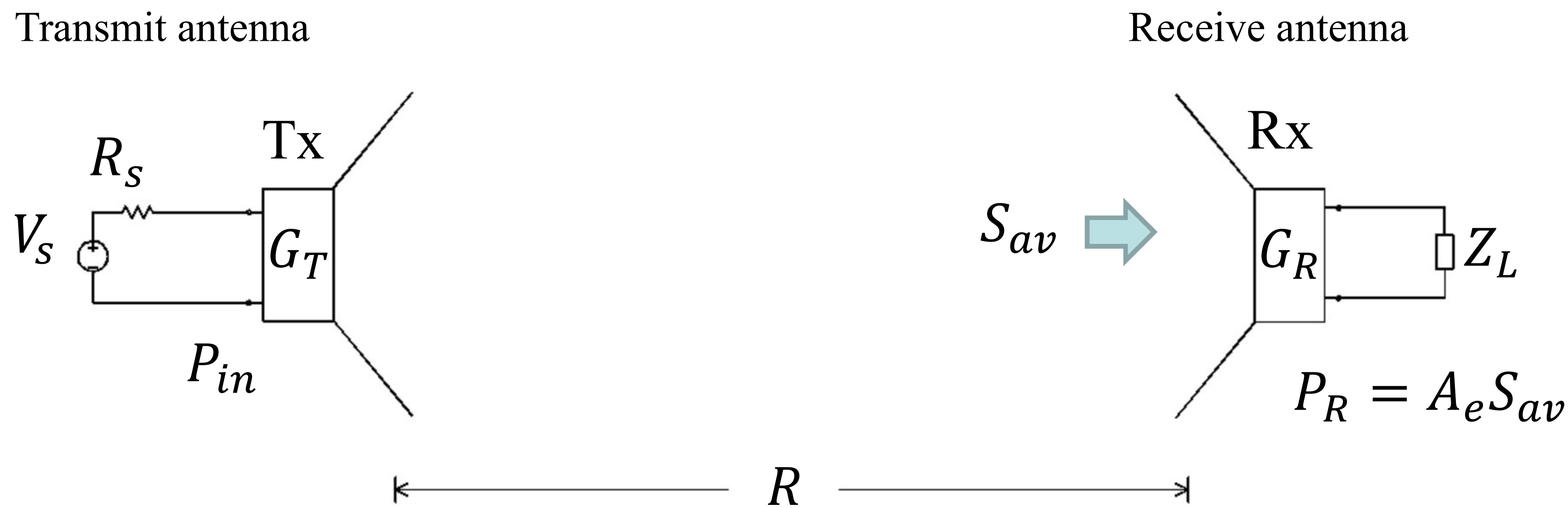

- The radiated power is $P_{rad} = eP_{in}$ where $e$ is the efficiency of the antenna.
- The isotropic power density $S_{iso}$ at a distance $R$ from the transmitter is

$$S_{iso} = \frac{P_{rad}}{4\pi R^2} = \frac{eP_{in}}{4\pi R^2}$$

- The actual power density $S_{av}$ is the directive gain times the isotropic power density

$$S_{av} = D_T S_{iso} = D_T \frac{eP_{in}}{4\pi R^2} = eD_T \frac{P_{in}}{4\pi R^2}$$

where $D_T$ is the directive gain of the transmit antenna




$$S_{av} = eD_T \frac{P_{in}}{4\pi R^2}$$

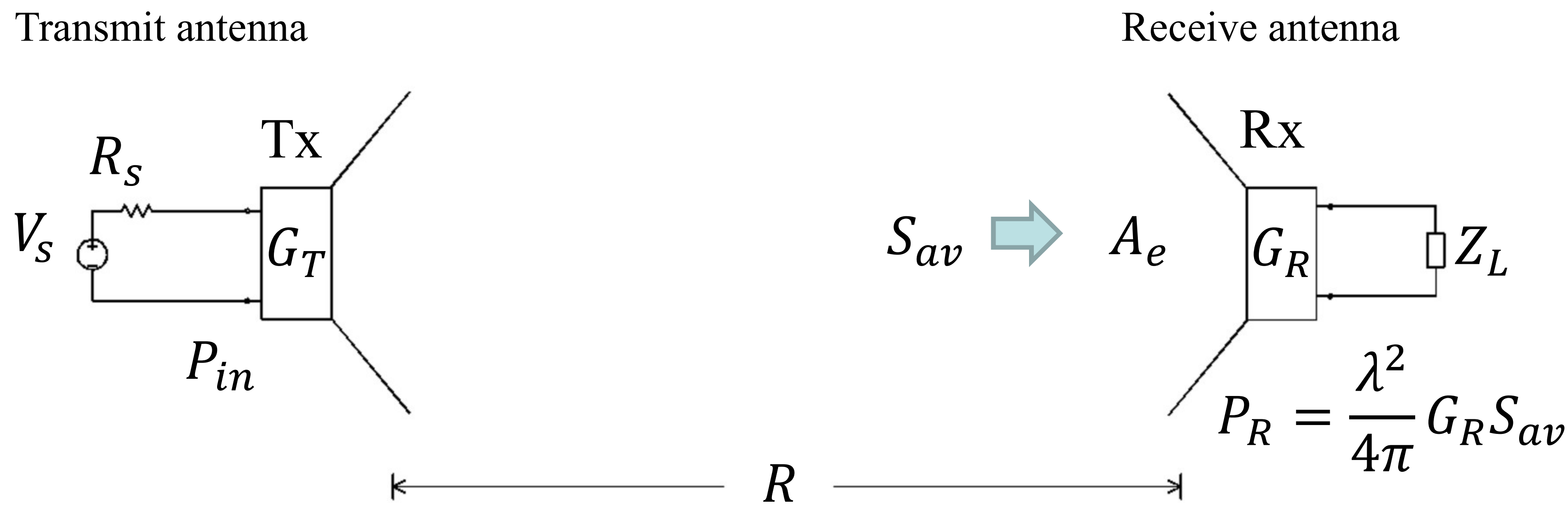
- Recognize $eD_T$ as the gain of the antenna $G_T = eD_T$ so we can write that the power density at the location of the receive antenna is

$$S_{av} = G_T \frac{P_{in}}{4\pi R^2}$$

- The receive antenna is terminated in a matched load so the power delivered to the matched load is

$$P_R = A_e S_{av}$$

where $A_e$ is the "effective area" of the receive antenna.




The effective area is related to the gain by

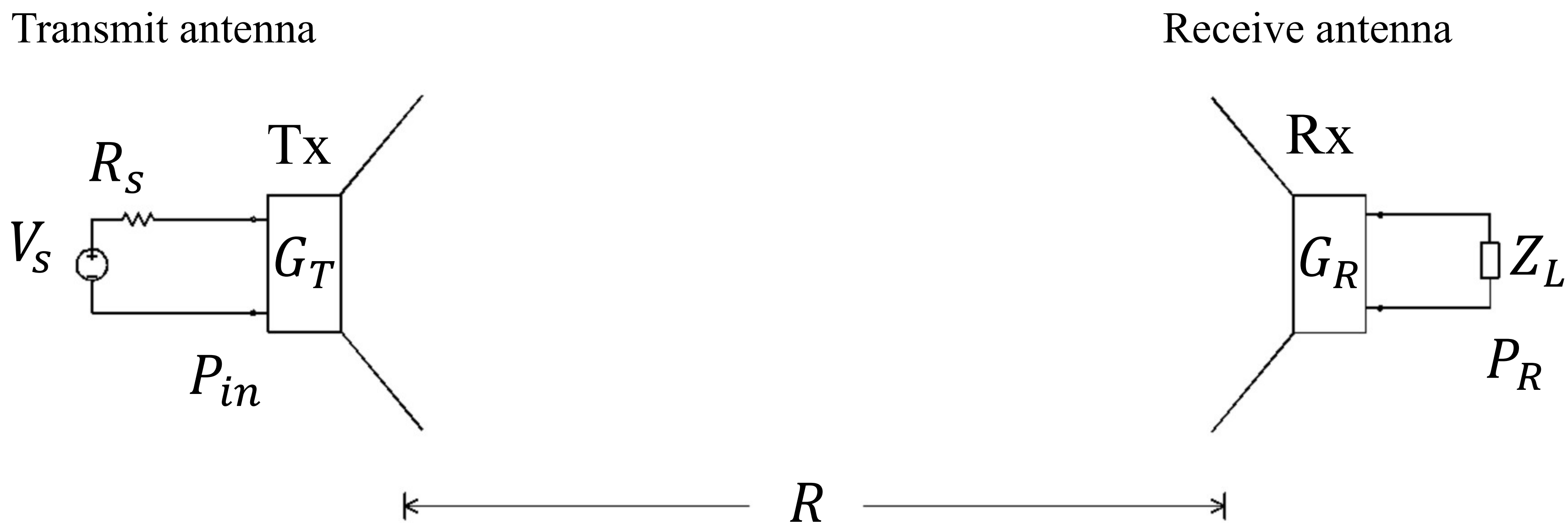
$$A_e = \frac{\lambda^2}{4\pi} G_R$$

so the received power is

$$P_R = \frac{\lambda^2}{4\pi} G_R S_{av}$$

$$S_{av} = G_T \frac{P_{in}}{4\pi R^2}$$

$$P_R = \frac{\lambda^2}{4\pi} G_R \, G_T \frac{P_{in}}{4\pi R^2}$$




$$P_R = \frac{\lambda^2}{4\pi} G_R G_T \frac{P_{in}}{4\pi R^2}$$

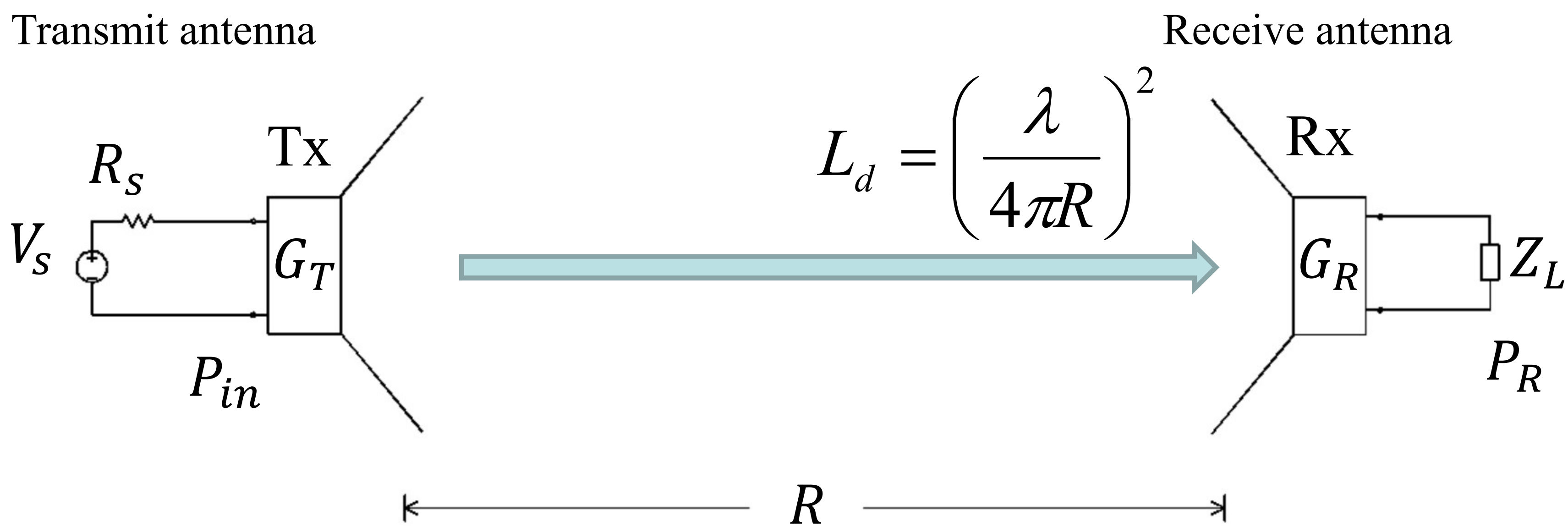
- Neaten this equation by writing it as

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in}$$

- This equation is called the "Friis Transmission Equation".
- It is the design equation for wireless links.

H. T. Friis, "A note on a simple transmission formula," *Proc. IRE*, vol.34, no.5, pp. 254-256,1946.

# Spatial Loss Factor




$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in}$$

- The term $\left(\frac{\lambda}{4\pi R}\right)^2$ is called the "spreading factor" or "spatial loss factor"

$$L_d = \left(\frac{\lambda}{4\pi R}\right)^2$$

# Exam Question on Antennas

A 20 Watt transmitter operates at a frequency of 147.06 MHz. It is connected to an antenna that has 4 dB of gain. A receiving antenna is 40 km away and has 2 dB of gain.

(1) The received power is

(a) $2.08x10^{-9}$ W
(b) $1.31x10^{-9}$ W
(c) $4.04x10^{-9}$ W
(d) $9.42x10^{-10}$ W
(e) none of these

(2) The aperture of the receive antenna is

(a) 0. 402 $m^2$
(b) 0. 118 $m^2$
(c) 0. 589 $m^2$
(d) 0. 527 $m^2$
(e) none of these

(3) The incident power density at the receive antenna is

(a) $1.11x10^{-9}$ W/ $m^2$
(b) $3.55x10^{-9}$ W/ $m^2$
(c) $6.10x10^{-9}$ W/ $m^2$
(d) $2.49x10^{-9}$ W/ $m^2$
(e) none of these

(4) The RMS electric field strength at the receive antenna is

(a) 552 $\mu V/m$
(b) 968 $\mu V/m$
(c) 777 $\mu V/m$
(d) 440 $\mu V/m$
(e) none of these

# Solution

A 20 Watt transmitter operates at a frequency of 147.06 MHz. It is connected to an antenna that has 4 dB of gain. A receiving antenna is 40 km away and has 2 dB of gain.

$$P_{in} = 20 \text{ watts}$$
$$f = 147.06 \text{ MHz}$$
$$G_{T,dB} = 4 \text{ dB}$$
$$R = 40 \text{ km}$$
$$G_{R,dB} = 2 \text{ dB}$$

**(1) The received power is?**

Evaluate the Friis Transmission Equation

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in}$$

where

$$f = 147.06 \text{ MHz so } \lambda = \frac{c}{f} = \frac{3x10^8}{147.06x10^6} = 2.04 \text{ m}$$

Recall that gain in dB is 10 log (gain on a linear scale) so

$$G_{T,dB} = 4 \text{ dB so on a linear scale } G_T = 10^{4/10} = 2.51$$

$$G_{R,dB} = 2 \text{ dB so on a linear scale } G_R = 10^{2/10} = 1.58$$

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in}$$

$\lambda = 2.04$ m

$G_T = 2.51$

$G_R = 1.58$

$R = 40$ km

Spatial loss or "spreading factor":

$$L = \left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{2.04}{4\pi x 40{,}000}\right)^2 = 1.65 x 10^{-11}$$

Friis Transmission Equation

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in} = 1.65 x 10^{-11} x 2.51 x 1.58 x 20 = 1.31 x 10^{-9} \text{ watts}$$

(a) $2.08x10^{-9}$ W

(b) $1.31x10^{-9}$ W >>> correct answer

(c) $4.04x10^{-9}$ W

(d) $9.42x10^{-10}$ W

(e) none of these

Power in dBm:

$$P_{RdBm} = 10\log\left(\frac{P_r}{0.001}\right) = -58.83 \quad \text{dBm}$$

**(2) The aperture of the receive antenna is?**

$$A_e = \frac{\lambda^2}{4\pi} G_R = \frac{2.04^2}{4\pi} x1.58 = 0.523m^2$$

(a) 0. 402 $m^2$
(b) 0. 118 $m^2$
(c) 0. 589 $m^2$
(d) 0. 527 $m^2$ >>> correct answer (within 3%)
(e) none of these

## (3) The incident power density at the receive antenna is?

- The radiated power is $P_{rad} = eP_{in}$ where $e$ is the efficiency of the antenna.
- The isotropic power density at a distance $R$ from the transmitter is

$$S_{iso} = \frac{P_{rad}}{4\pi R^2} = \frac{eP_{in}}{4\pi R^2}$$

- The actual power density $S_{av}$ is the directivity times the isotropic power density,

$$S_{av} = D_T S_{iso} = D_T \frac{eP_{in}}{4\pi R^2} = eD_T \frac{P_{in}}{4\pi R^2}$$

where $D_T$ is the directivity of the transmit antenna.

- Recognize $eD_T$ as the gain of the antenna $G_T = eD_T$ so we can write that the power density at the location of the receive antenna is

$$S_{av} = G_T \frac{P_{in}}{4\pi R^2}$$

$$S_{av} = 2.51 \frac{20}{4\pi(40{,}000)^2} = 2.50x10^{-9} \quad \text{W/m}^2$$

(a) $1.11x10^{-9}$ W/ m$^2$
(b) $3.55x10^{-9}$ W/ m$^2$
(c) $6.10x10^{-9}$ W/ m$^2$
(d) $2.49x10^{-9}$ W/ m$^2$ >>>
(e) none of these

## (4) The RMS electric field at the antenna is?

The incoming wave behaves as a plane wave so

$$S_{av} = \frac{E_{rms}^2}{\eta}$$

$$E_{rms} = \sqrt{\eta S_{av}} = \sqrt{377x2.50x10^{-9}} = 9.71x10^{-4}\ \text{V/m} = 971x10^{-6}\ \text{V/m} = 971\ \mu\text{V/m}$$

(a) 552 $\mu$V/m

(b) 968 $\mu$V/m >>> correct answer (close enough)

(c) 777 $\mu$V/m

(d) 440 $\mu$V/m

(e) none of these

# Friis Transmission Equation in dB

$$P_r = \left(\frac{\lambda}{4\pi R}\right)^2 G_T G_R P_t$$

$$L_d = \left(\frac{\lambda}{4\pi R}\right)^2$$

$$P_r = L_d G_T G_R P_t$$

$$dBm = 10\log\left(\frac{power}{1mW}\right)$$

$$\frac{P_r}{1\text{mW}} = L_d G_T G_R \frac{P_t}{1\text{mW}}$$

$$10\log\left(\frac{P_r}{1\text{mW}}\right) = 10\log\left(L_d G_T G_R \frac{P_t}{1\text{mW}}\right)$$

$$P_{rdBm} = 10\log(L_d) + 10\log(G_T) + 10\log(G_R) + P_{tdBm}$$

$$L_{dB} = -10\log\left(\frac{1}{L_d}\right)$$

$$P_{rdBm} = P_{tdBm} + G_{TdB} - L_{dB} + G_{RdB}$$

# For the example:

$P_{in} = 20$ Watts, or in dBm: $P_{indBm} = 10\log\left(\frac{20}{0.001}\right) = 43.01$ dBm

$$L = \left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{2.04}{4\pi x 40{,}000}\right)^2 = 1.65\text{x}10^{-11}$$

$$L_{dB} = 10\log\left(\frac{1}{1.65x10^{-11}}\right) = 107.84 \quad \text{dB}$$

$G_T = 4$ dB

$G_R = 2$ dB

$$P_{rdBm} = P_{tdBm} + G_{TdB} - L_{dB} + G_{RdB}$$

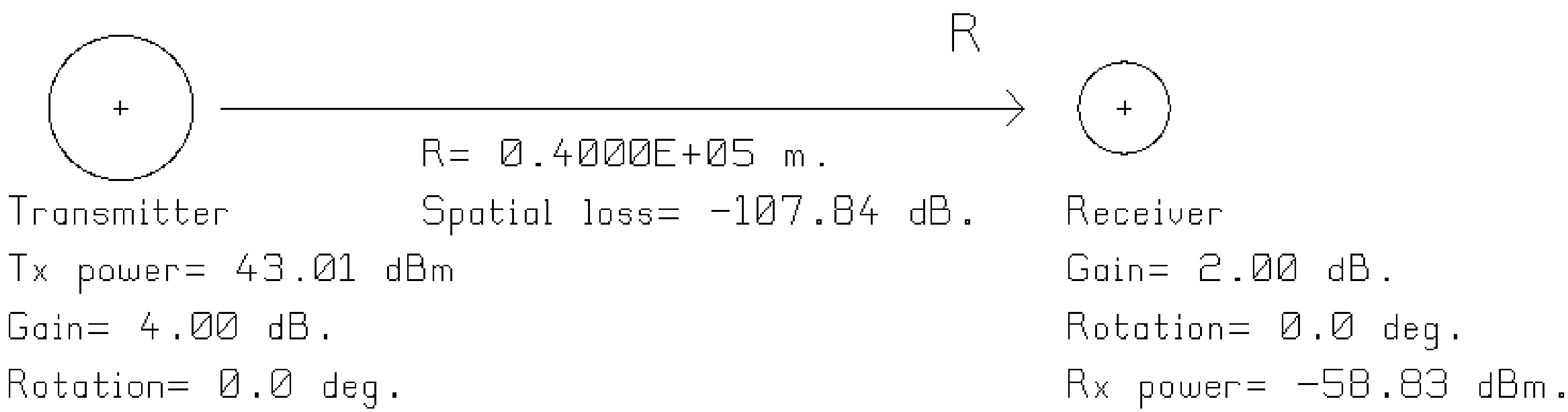
$$P_{rdBm} = 43.01 + 4 - 107.84 + 2 = -58.83 \quad \text{dBm}$$

# Friis Program

A 20 Watt transmitter operates at a frequency of 147.06 MHz. It is connected to an antenna that has 4 dB of gain. A receiving antenna is 40 km away and has 2 dB of gain.

```
*** FRIIS    ** VERSION 1A    ****    Dec. 6, 2011 ****

------ Program Friis  -------  Evaluate the Friis Transmission Equation -------

          Frequency:                   0.1471     GHz.

          Transmitted Power:            20.00     Watts.

          Transmitter Position: x=-.2000E+05  y=  0.000     m.

          Transmitter Rotation Angle: 0.000     _egrees.

          Receiver    Position: x=0.2000E+05  y=  0.000     m.

          Receiver    Rotation Angle: 0.000     degrees.

          Evaluate the Friis Transmission Equation.

          Specify the transmitter radiation pattern.

          Specify the receiver radiation pattern.

          Tabulation of the link calculation.

          Exit from the program.
```

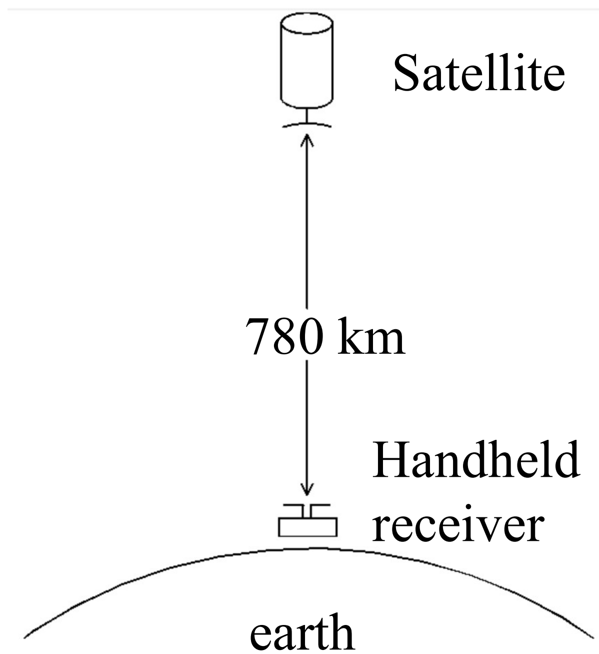
# Pictorial Representation of the Link




$$P_{rdBm} = P_{tdBm} + G_{TdB} + L_{dB} + G_{RdB}$$

$$P_{rdBm} = 43.01 + 4 - 107.84 + 2 = -58.83 \qquad \text{dBm}$$

**Example** – satellite link




A satellite in low-earth orbit (LEO) must communicate with a handheld "satellite telephone". The satellite is 780 km above the ground. The operating frequency is 1.65 GHz. The satellite transmits 14 watts of power using an antenna with a gain of 6 dB. The handheld receiver uses a half-wave dipole antenna to receive the signal. What is the received power into a matched load?

Solution

The gain of the transmitter is 6 dB so $6 = 10\log G_T$ and $G_T = 3.98$. The gain of the receiver is $G_R = 1.64$ for a half-wave dipole. The spatial loss factor for a distance of $r = 780$ km at 1.65 GHz is

$$L_d = \left(\frac{\lambda}{4\pi r}\right)^2 = 3.44x10^{-16}$$

The Friis Transmission Equation reads

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in} = L_d G_R G_T P_{in}$$

so we can calculate the received power as

$$P_R = (3.44x10^{-16})(1.65)(3.98)(14) = \quad 3.16x10^{-14} \text{ watts}$$

Remarks:

- To "design" the satellite link we need to know the minimum *signal-to-noise ratio* that must be maintained at the receiver.

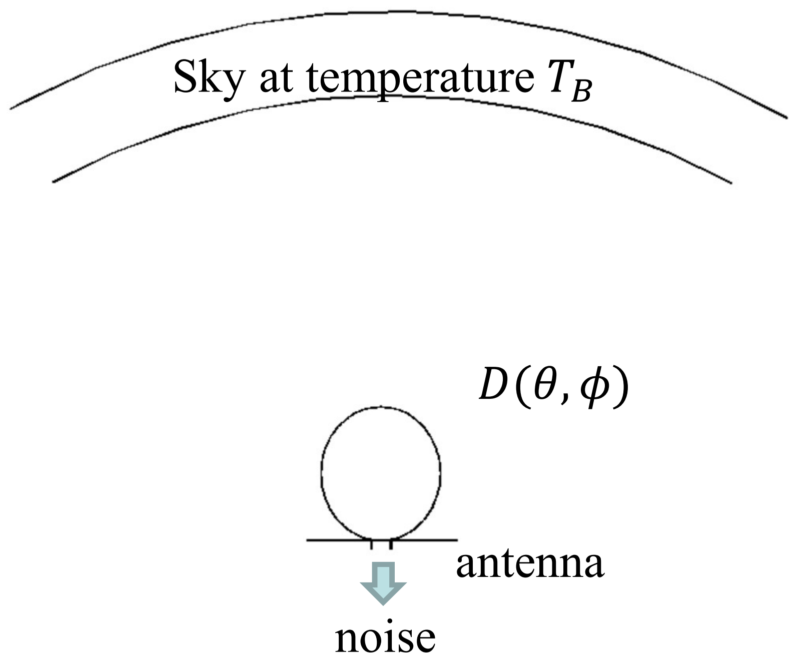
The noise power is given by

$$N = kT_r B$$

where $B$ Hz is the channel bandwidth

$k$ is Boltzmann's Constant =$1.38\text{x}10^{-23}$ Joules per Kelvin degree

$T_r$ is the "brightness temperature" of the receive antenna in degrees

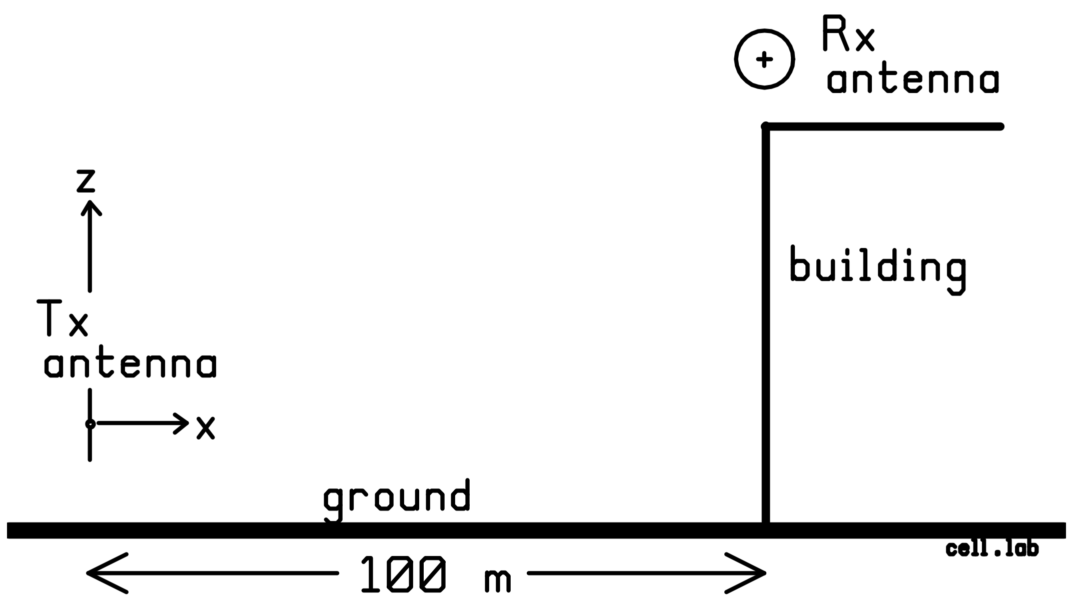



$$T_b = \frac{\iint\limits_{4\pi} T_B(\theta,\phi) D(\theta,\phi) \sin\theta d\theta d\phi}{\iint\limits_{4\pi} D(\theta,\phi) \sin\theta d\theta d\phi}$$

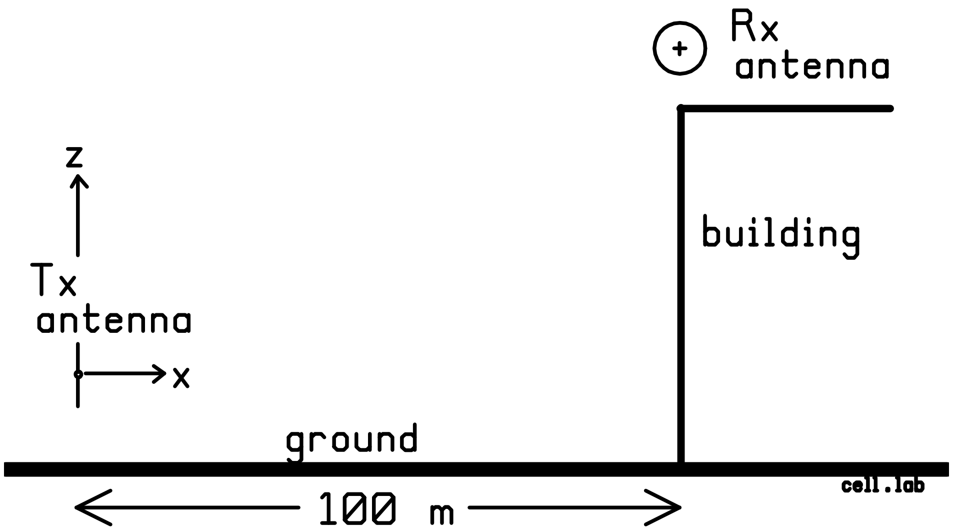
Sky: $T_B = 0$ K

Ground: $T_B = 273$ K

# Cell Phone Example




A cell phone user at Tx communicates with a base station antenna at Rx on top of a nearby building. The transmit antenna at Tx behaves as a lossless, half-wave dipole antenna oriented vertically, or in the z direction. The center of the antenna is 1.5 m above the ground and the antenna is 100 m from the base of the building. The Rx antenna is omnidirectional and is lossless. The center of the Rx antenna lies in the plane of the face of the building at an elevation of 65 m above the ground. The operating frequency is 1.9 GHz. The transmitted power is 125 mW.




Half-wave dipole formulas:

$$P_{rad} = 36.5 I_0^2$$

$$E_\theta(\theta) = \frac{jI_0\eta_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

1)What is the value of angle $\theta$ for the path from the transmitter to the receiver?

2)What is the value of $F(\theta)$ for the path from the transmitter to the receiver?

3) What is the current flowing on the dipole antenna?

4)What is the electric field strength of the dipole antenna at the location of the receiver?

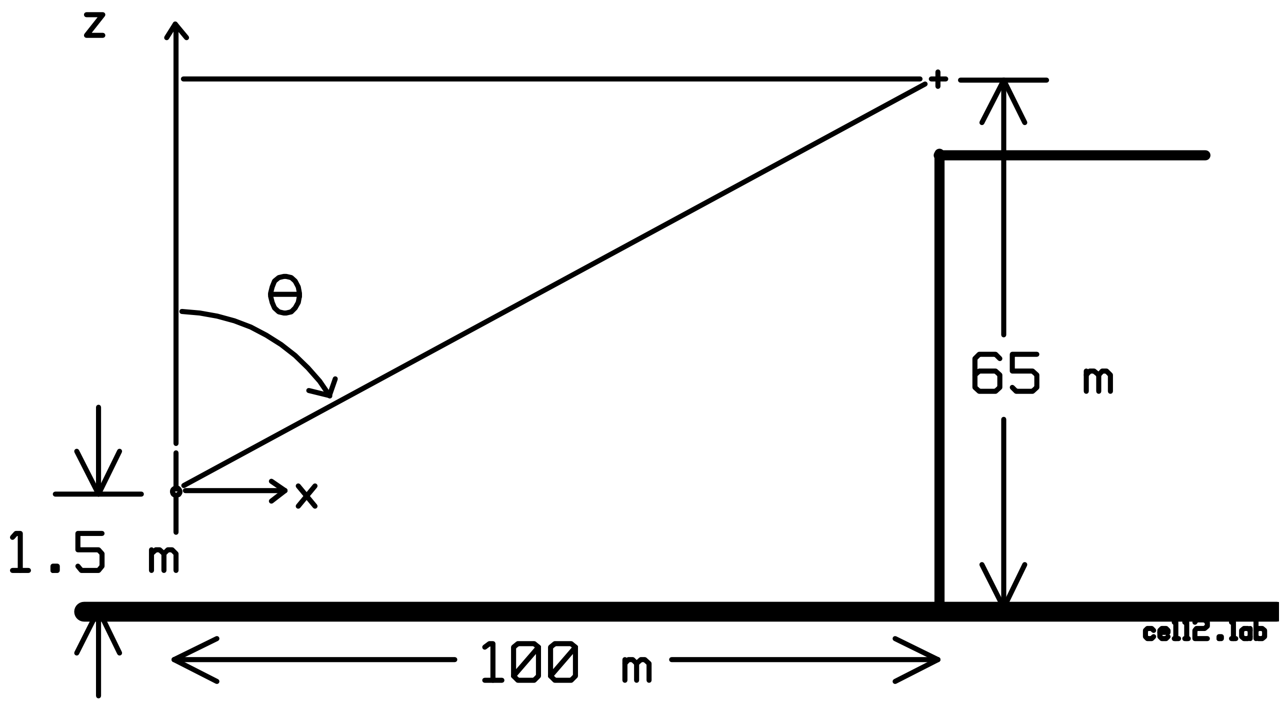
5)What is the power density due to the transmitter at the location of the receive antenna?

6)What is the effective area of the receive antenna?

7)What is the power received by the Rx antenna into a matched load?

# Solution

1.What is the value of angle $\theta$ for the path from the transmitter to the receiver?




$$\tan\theta = \frac{100}{(65-1.5)}$$

$$\theta \quad = 56.976 \text{ degrees}$$

$$r = \sqrt{100^2 + (65-1.5)^2} = 118.46$$

2.What is the value of $F(\theta)$ for the path from the transmitter to the receiver?

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$\theta$ =56.976 degrees

$$F(\theta) = \frac{\cos(90x\cos 56.976)}{\sin 56.976} \quad = 0.7817$$

3.What is the current flowing on the dipole antenna?

$$P_{rad} = 36.5 I_0^2$$

$$P_{rad} = 125 \quad \text{mW}$$

$$I_0 = \sqrt{\frac{P_{rad}}{36.5}} = \sqrt{\frac{0.125}{36.5}} = 0.05852 \quad \text{amps}$$

4. What is the electric field strength of the dipole antenna at the location of the receiver?

$$E_\theta(\theta) = \frac{jI_0\eta_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

$$\left|E_\theta\right| = \frac{I_0\eta_0}{2\pi r} F(\theta)$$

$F(\theta)$ =0.7817

$I_0 = 0.05852$ amps

$r = \sqrt{100^2 + (65-1.5)^2} = 118.46$ m

$$\left|E_\theta\right| = \frac{I_0\eta_0}{2\pi r} F(\theta) = \frac{0.05852x376.73}{2x\pi x118.46} * 0.7817 = 0.02315 \quad \text{V/m}$$

5. What is the power density due to the transmitter at the location of the receive antenna?

$$S_{av} = \frac{E^2}{2\eta} = \frac{0.02315^2}{2x376.73} = 7.113x10^{-7} = 0.7113$$ Microwatts per square meter

6. What is the effective area of the receive antenna?

Lossless antenna:
*D*=*G*

$$A_e = \frac{\lambda^2}{4\pi} G$$

$$\lambda = \frac{c}{f} = 300/1900 = 0.15789$$ m

The **directivity** of an "isotropic" antenna is **unity,** *D*=1:

$$A_e = \frac{0.15789^2}{4\pi} x1 = 1.984x10^{-3}$$ square meters

7. What is the power received by the Rx antenna into a matched load?

$$P_r = S_{av} A_e = 0.7113x10^{-6} x1.984x10^{-3} = 1.4097x10^{-9}$$ watts

# Pictorial Representation