ELEC353 Lecture Notes Set 12

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #7: Do homework #7 by March 8, 2019.
Homework #8: Do homework #8 by March 15, 2019.
Homework #9: Do homework #9 by March 22, 2019.

Tutorial Workshop #8: Friday March 8, 2019.
Tutorial Workshop #9: Friday March 15, 2019.
Tutorial Workshop #10: Friday March 22, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

Topics to be Covered

Transmission Lines (TLs)

- Wave Equation and Solution - done
- Solving a TL Circuit - done
- Standing Wave Patterns
- Impedance Matching
- Bandwidth of Digital Signal

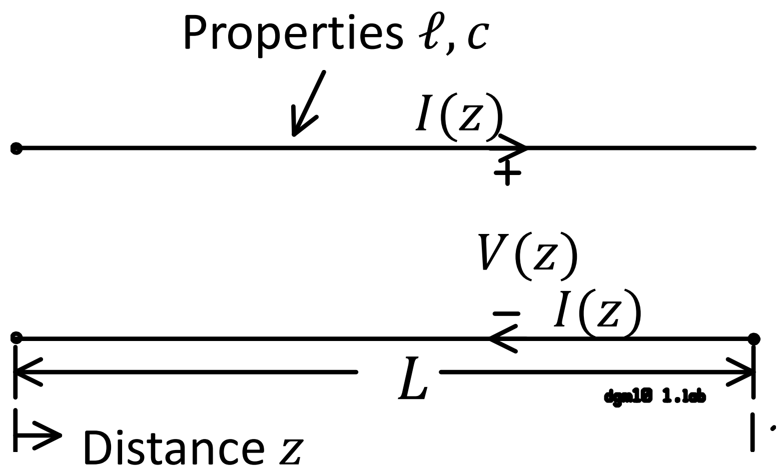
Plane Waves

- Maxwell's Equations and the Wave Equation
- Plane waves
- Material Boundaries
- Transmission Through a Wall

Antennas

Review:

Summary – Lossless Transmission Lines



Lossless transmission line equations:

$$\frac{dV}{dz} = -j\omega\ell I$$

$$\frac{dI}{dz} = -j\omega c V$$

Lossless wave equation:

$$\frac{d^2V}{dt^2} = -\beta^2 V$$

where $\beta = \omega\sqrt{\ell c}$ is the phase constant.

Voltage and current:

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z) = \frac{V^+}{R_c} e^{-j\beta z} - \frac{V^-}{R_c} e^{j\beta}$$

Characteristic resistance:

$$R_c = \sqrt{\frac{\ell}{c}}$$

In general:

$$u = \frac{\omega}{\beta}$$

$$\lambda = \frac{2\pi}{\beta}$$

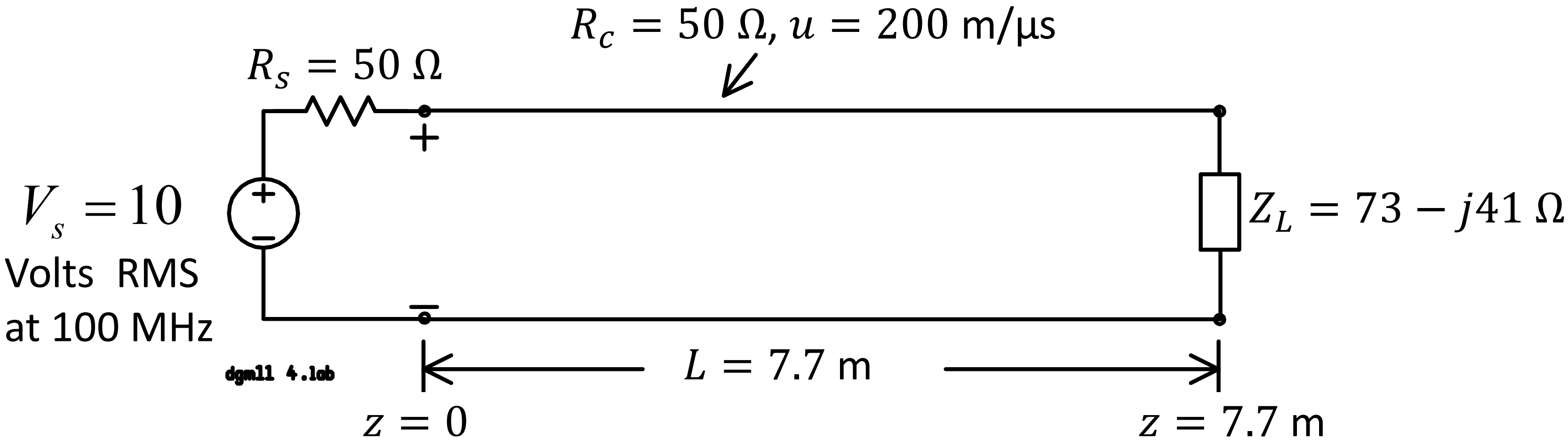
Lossless case:

$$u = \frac{1}{\sqrt{\ell c}}$$

$$\lambda = \frac{u}{f}$$

$$\beta = \omega\sqrt{\ell c} = \frac{\omega}{u}$$

Last class we solved this example:



In general, $V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$

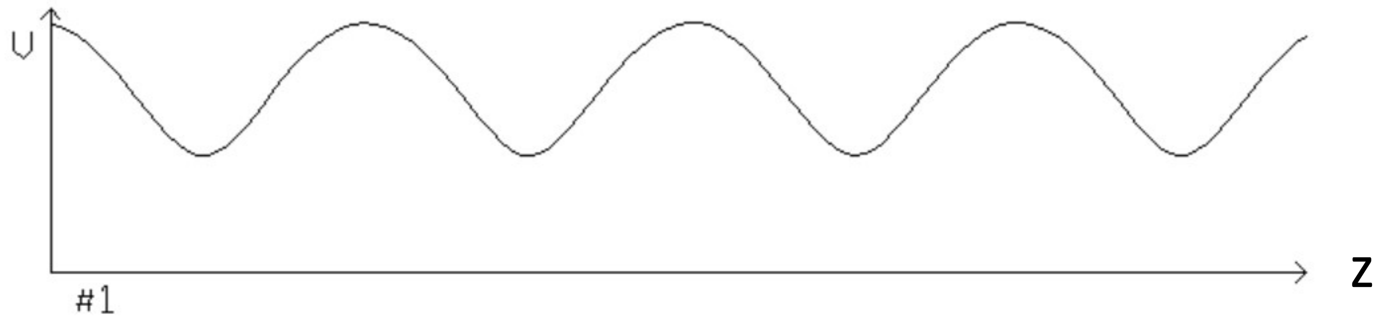
We solved the circuit to find: $V^+ = 5\angle 0°$

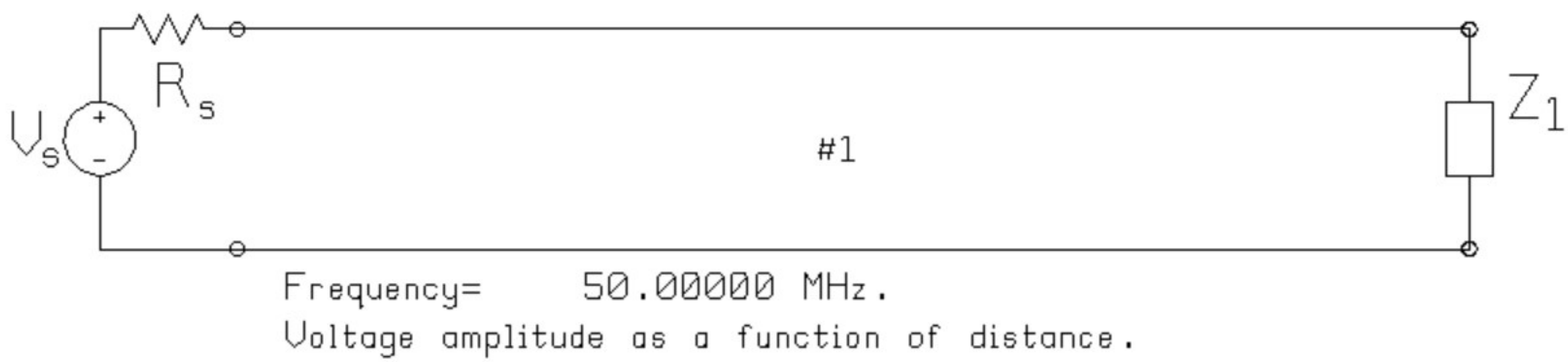
$$V^- = 1.813\angle 65.73°$$

Then the voltage on the transmission line is:

$$V(z) = 5e^{-j\beta z} + 1.813e^{j65.73°}e^{j\beta z}$$

What is the Voltage on the Transmission Line?

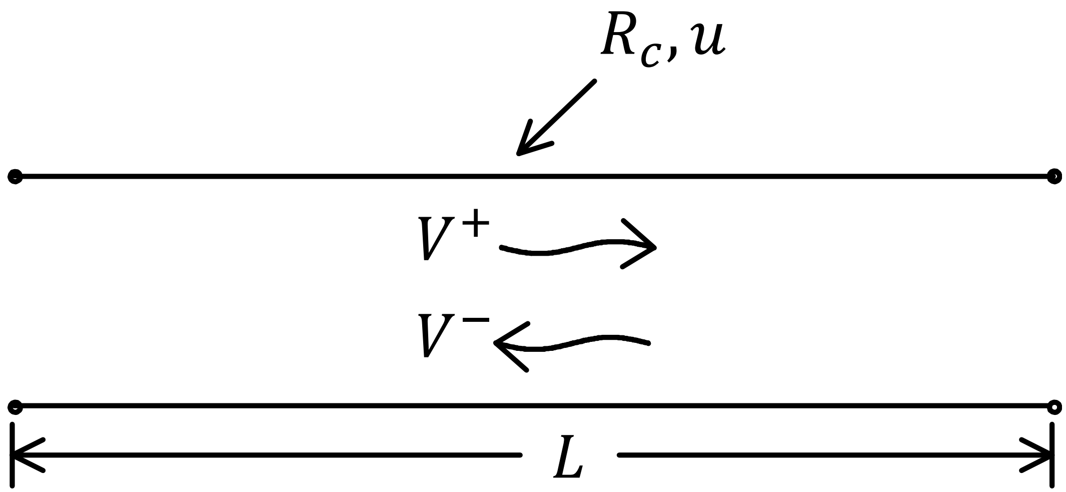




$$V(z) = 5e^{-j\beta z} + 1.813e^{j65.73^\circ}e^{j\beta z}$$

This is called an “interference pattern” or “standing-wave pattern” and is formed as $5e^{-j\beta z}$ and $1.813e^{j65.73^\circ}e^{j\beta z}$ go in and out of phase.

Standing Waves and Interference Patterns



$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$V^+ = A^+ e^{j\theta^+} \qquad V^- = A^- e^{j\theta^-}$$

$$V(z) = A^+ e^{j\theta^+} e^{-j\beta z} + A^- e^{j\theta^-} e^{j\beta z}$$

$$V(z) = A^+ e^{j(\theta^+ - \beta z)} + A^- e^{j(\theta^- + \beta z)}$$

Interference Pattern

$$V(z) = A^{+}e^{j(\theta^{+}-\beta z)} + A^{-}e^{j(\theta^{-}+\beta z)}$$

Graph the amplitude of the A.C. voltage as a function of position.

The amplitude of the voltage $A(z)$ is the magnitude of the phasor, so

$$A(z) = |V(z)| = \left| A^{+}e^{j(\theta^{+}-\beta z)} + A^{-}e^{j(\theta^{-}+\beta z)} \right|$$

Maxima

As z increases, there will be locations where $e^{j(\theta^{+}-\beta z)}$ and $A^{-}e^{j(\theta^{-}+\beta z)}$ are in phase: $(\theta^{+}-\beta z) = (\theta^{-}+\beta z) \pm 2n\pi$

Then $e^{j(\theta^{+}-\beta z)} = e^{j(\theta^{-}+\beta z)}$ and so the largest amplitude is

$$A_{max} = A^{+} + A^{-} = |V^{+}| + |V^{-}|$$

Minima

There will be locations where $e^{j(\theta^+ - \beta z)}$ and $A^- e^{j(\theta^- + \beta z)}$ are out of phase: $(\theta^+ - \beta z) = (\theta^- + \beta z) + \pi \pm 2n\pi$

Then $e^{j(\theta^+ - \beta z)} = -e^{j(\theta^- + \beta z)}$ and so

$$A_{min} = A^+ - A^- = |V^+| - |V^-|$$

Standing-Wave Ratio

The standing-wave ratio is defined as

$$SWR = \frac{A_{max}}{A_{min}}$$

$$SWR = \frac{A^+ + A^-}{A^+ - A^-} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$$

Standing-Wave Ratio

The standing-wave ratio is defined as

$$SWR = \frac{\max}{\min} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$$

For the example previously given,

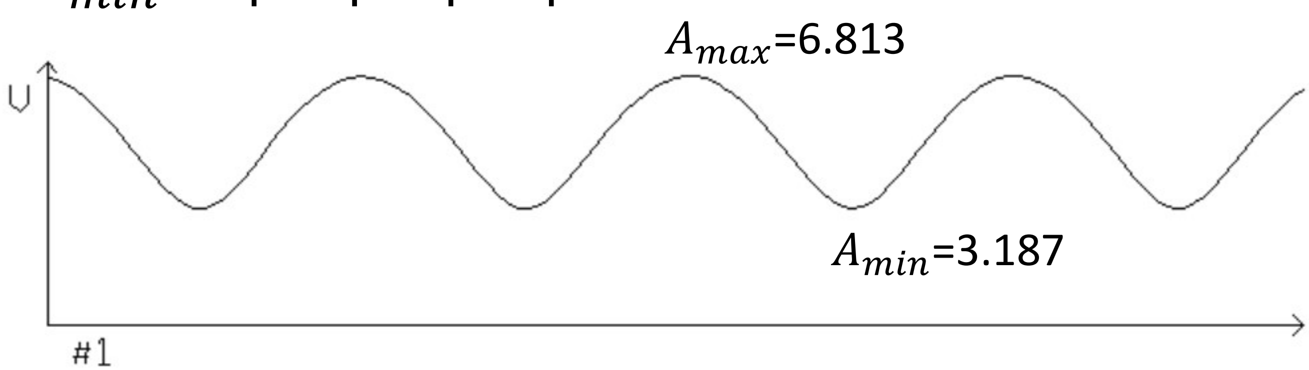
$$V(z) = 5e^{-j\beta z} + 1.813e^{j65.73^\circ}e^{j\beta z}$$

Hence the maximum value is

$$A_{max} = |V^+| + |V^-| = 5 + 1.813 = 6.813$$

And the minimum value is

$$A_{min} = |V^+| - |V^-| = 5 - 1.813 = 3.187$$



$$SWR = \frac{6.813}{3.187} = 2.14$$

z

Where are the maxima and minima?

$$V(z) = A^+ e^{j(\theta^+ - \beta z)} + A^- e^{j(\theta^- + \beta z)}$$

$$v(z,t) = A^+ \cos(\omega t + \theta^+ - \beta z) + A^- \cos(\omega t + \theta^- + \beta z)$$

There will be a maximum at locations where $A^+ \cos(\omega t + \theta^+ - \beta z)$ and

$A^- \cos(\omega t + \theta^- + \beta z)$ are in phase. This is true when:

$$(\theta^+ - \beta z) = (\theta^- + \beta z) \pm 2n\pi$$

$$2\beta z = (\theta^+ - \theta^-) \mp 2n\pi$$

$$z = \frac{(\theta^+ - \theta^-) \mp 2n\pi}{2\beta}$$

There will be a minimum at locations where $A^+ \cos(\omega t + \theta^+ - \beta z)$ and

$A^- \cos(\omega t + \theta^- + \beta z)$ are out of phase:

$$(\theta^+ - \beta z) = (\theta^- + \beta z) + \pi \pm 2n\pi$$

$$z = \frac{(\theta^+ - \theta^-) - \pi \mp 2n\pi}{2\beta}$$

$$z = \frac{(\theta^+ - \theta^-) - \pi \mp 2n\pi}{2\beta}$$

In terms of the wavelength: $\beta = \frac{2\pi}{\lambda}$

$$z = \frac{(\theta^+ - \theta^-) - \pi \mp 2n\pi}{2\frac{2\pi}{\lambda}}$$

$$z = \frac{(\theta^+ - \theta^-)\lambda - \pi\lambda \mp 2n\pi\lambda}{4\pi}$$

$$z = \frac{(\theta^+ - \theta^-)\lambda}{4\pi} - \frac{\lambda}{4} \mp n\frac{\lambda}{2}$$

Define the principal minimum as

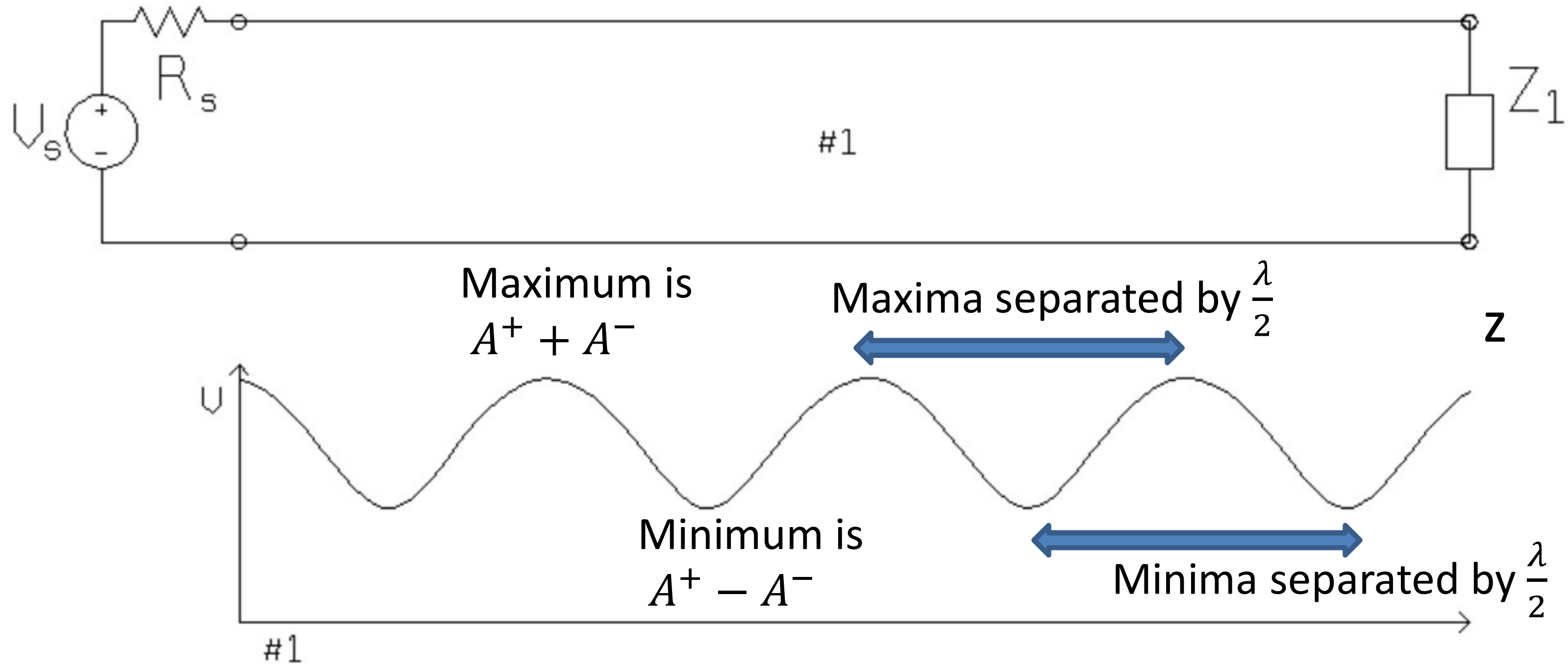
$$z_o = \frac{(\theta^+ - \theta^-)\lambda}{4\pi} - \frac{\lambda}{4}$$

Then the minima are at

$$z = z_o \pm n\frac{\lambda}{2}$$

Review:

Standing Wave Pattern

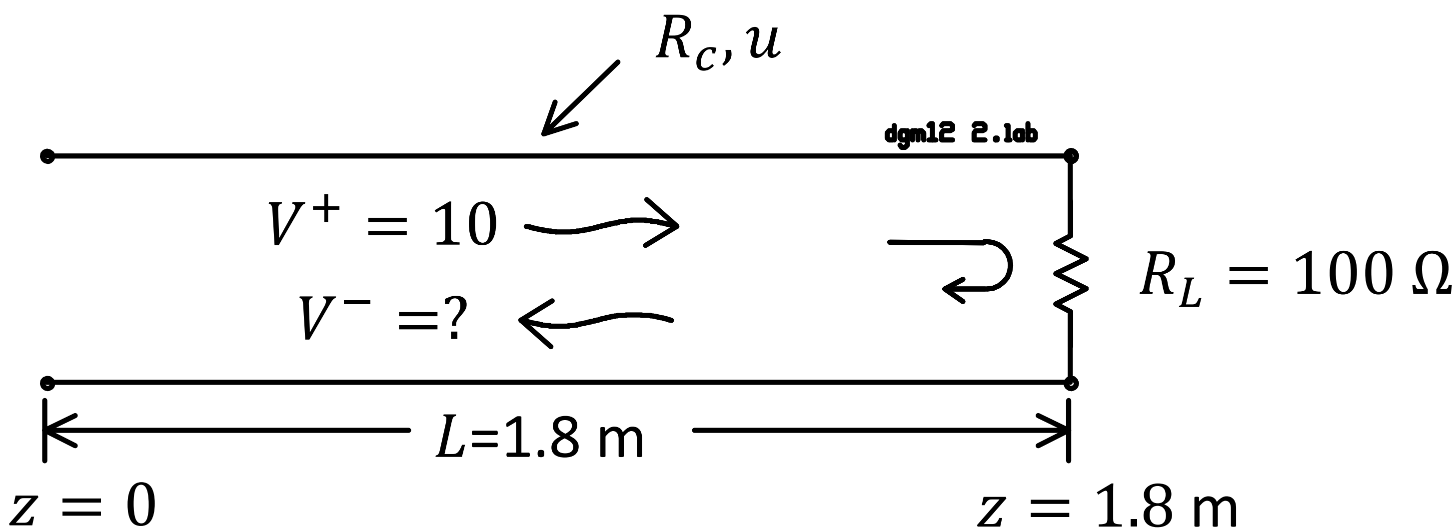


$$V(z) = \left(A^+ e^{j\theta^+}\right) e^{-j\beta z} + \left(A^- e^{j\theta^-}\right) e^{j\beta z}$$

$$SWR = \frac{\text{Maximum}}{\text{Minimum}} = \frac{A^+ + A^-}{A^+ - A^-}$$

The minima are located at $z = \frac{(\theta^+ - \theta^-)\lambda}{4\pi} - \frac{\lambda}{4} \mp 2n\frac{\lambda}{4}$

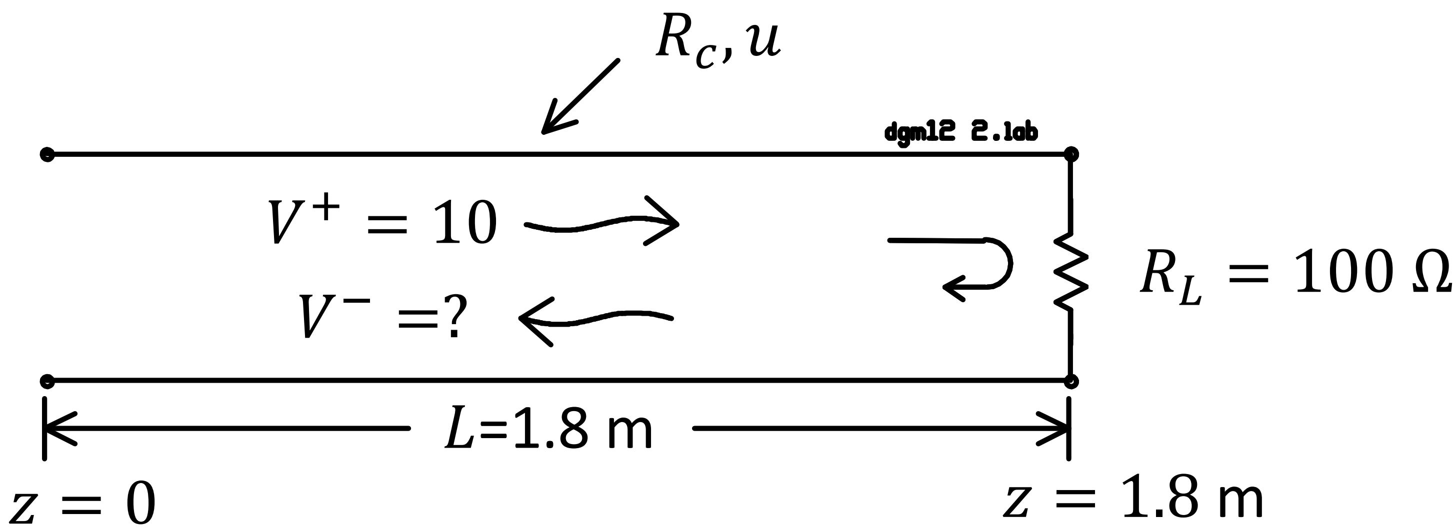
Example: Find the maxima and minima and the SWR.



A transmission line is 1.8 m long and the complex amplitude of the positive-going traveling wave is $V^+ = 10$ volts with zero phase angle. The characteristic impedance is $Z_0 = 50$ ohms and the speed of propagation is $u = 300$ meters per microsecond (=30 cm/ns). The frequency is 300 MHz. Find:

(1) The complex amplitude of the reflected wave
(2) The position of the maxima in the standing-wave pattern
(3) The position of the minima in the standing-wave pattern
(4) The largest voltage and the smallest voltage on the transmission line
(5) The "standing wave ratio" SWR=largest voltage / smallest voltage
(6) Run TRLINE to verify the SWR, and the position of the minima and maxima.

Solution



$$V^- = V^+ \Gamma_L e^{-2j\beta L} \quad \text{where } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{Z_L - R_c}{Z_L + R_c} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = 0.3333$$

$$\lambda = \frac{u}{f} = \frac{300}{300} = 1 \text{ meter}$$

$$\beta = \frac{2\pi}{\lambda} = 2\pi \text{ radians/meter or in degrees, } \beta = 360 \text{ degrees/meter}$$

$$V^- = V^+\Gamma_L e^{-2j\beta L} \quad \text{where } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$-2\beta L = -2 \cdot 360 \cdot 1.8 = -1296 \ \text{degrees}$$

- remove the full 360 degree cycles:

$$-2\beta L = -1296 + 4x360 = 144 \ \text{degrees}$$

$$V^- = V^+\Gamma_L e^{-2j\beta L} = 10 \cdot 0.3333 \cdot e^{j144^\circ} = 3.333e^{j144^\circ}$$

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z} \ \text{amounts to}$$

$$V(z) = 10e^{-j\beta z} + 3.333e^{j144^\circ} e^{j\beta z}$$

Find the Maxima

$$V(z) = 10e^{-j\beta z} + 3.333e^{j144^\circ}e^{j\beta z}$$

Where is $10e^{-j\beta z}$ in phase with $3.333e^{j(\beta z+144^\circ)}$?

$$-\beta z = (\beta z + 144^\circ) \pm 360n$$
$$2\beta z = -144^\circ \mp 360n$$
$$z = \frac{-144^\circ}{2\beta} \mp \frac{360n}{2\beta}$$

$\beta = 360$ degree/meter so $2\beta = 720$

$$z = \frac{-144}{720} \mp \frac{360n}{720}$$
$$z = -0.2 \mp 0.5n$$
$$z = +0.3, +0.8, +1.3, 1.8$$

Maxima:

$$\left|V^+\right| + \left|V^-\right| = 10 + 3.333 = 13.333$$

Find the Minima

$$V(z) = 10e^{-j\beta z} + 3.333e^{j144^\circ}e^{j\beta z}$$

Where is $10e^{-j\beta z}$ 180 degrees out of phase with $3.333e^{j144^\circ}e^{j\beta z}$?

$$-\beta z = (\beta z + 144^\circ) \pm 360n + 180^\circ$$
$$2\beta z = -144^\circ \mp 360n - 180^\circ$$
$$z = \frac{-144^\circ}{2\beta} \mp \frac{360n}{2\beta} - \frac{180}{2\beta}$$

$\beta = 360$ degrees/meter

$$z = \frac{-144}{720} \mp \frac{360n}{720} - \frac{180}{720}$$
$$z = -0.45 \mp 0.5n$$
$$z = +0.05, +0.55, +1.05, +1.55$$

Minima:

$$\left|V^+\right| - \left|V^-\right| = 10 - 3.333 = 6.667$$

Standing-Wave Ratio

The standing-wave ratio is defined as

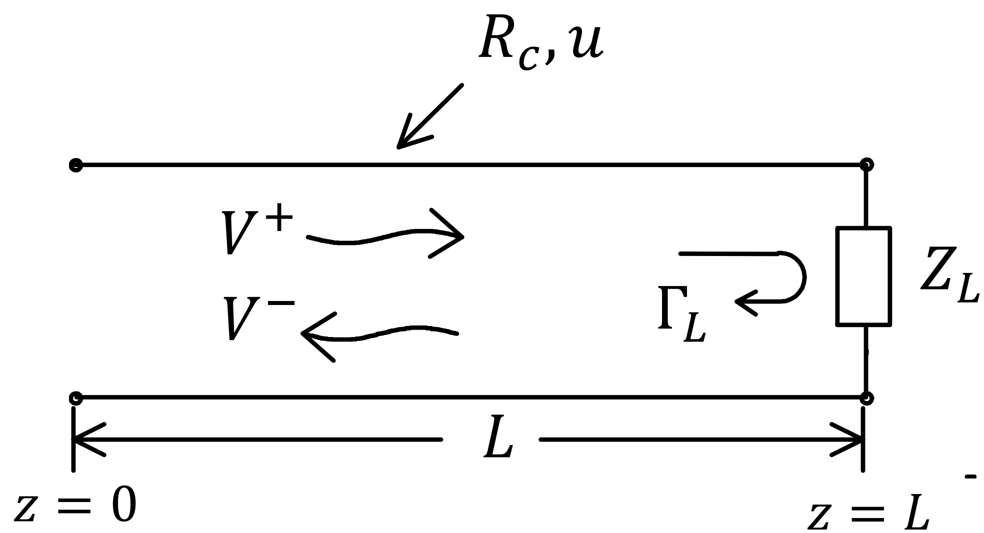
$$SWR = \frac{\max}{\min} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$$

Maxima: $\left|V^+\right| + \left|V^-\right| = 10 + 3.333 = 13.333$

Minima: $\left|V^+\right| - \left|V^-\right| = 10 - 3.333 = 6.667$

$$SWR = \frac{\max}{\min} = \frac{13.333}{6.667} = 2$$

Find a Load Impedance from a Standing-Wave Pattern



$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$V^- = V^+ \Gamma_L e^{-2j\beta L}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Gamma_L = |\Gamma_L| e^{j\phi}$ magnitude $|\Gamma_L|$ and angle ϕ

Can we find Z_L from a standing-wave pattern?

$$V(z) = V^+e^{-j\beta z} + V^-e^{j\beta z}$$

$$V^+ = A^+e^{j\theta^+}$$

$$V^- = A^-e^{j\theta^-}$$

$$V(z) = A^+e^{j\theta^+}e^{-j\beta z} + A^-e^{j\theta^-}e^{j\beta z}$$

Previously we have shown that the standing-wave ratio is

$$SWR = \frac{\max}{\min} = \frac{|V^+|+|V^-|}{|V^+|-|V^-|}$$

And that the minima in the standing wave pattern are at

$$z_n = \frac{(\theta^+ - \theta^-)\lambda}{4\pi} - \frac{\lambda}{4} \mp n\frac{\lambda}{2}$$

Can we find the load impedance Z_L from the SWR and the location of one of the minima?

What does the SWR tell us?

$$SWR = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$$

Since $V^- = V^+\Gamma_L e^{-2j\beta L}$ we can write

$$|V^-| = |V^+||\Gamma_L|\left|e^{-2j\beta L}\right|$$

$$|V^-| = |V^+||\Gamma_L|$$

So

$$SWR = \frac{|V^+| + |V^+||\Gamma_L|}{|V^+| - |V^+||\Gamma_L|}$$

$$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Hence if we know the SWR we can find $|\Gamma_L|$ as

$$|\Gamma_L| = \frac{SWR - 1}{SWR + 1}$$

What does the location of a minimum tell us?

We can find the angle of the reflection coefficient ϕ from the location of a minimum on the transmission line. How?

$$V^- = V^+ \Gamma_L e^{-2j\beta L}$$

We can expand Γ_L in terms of magnitude and angle as $\Gamma_L = |\Gamma_L| e^{j\phi}$ so

$$V^- = V^+ |\Gamma_L| e^{j\phi} e^{-2j\beta L}$$

$V^+ = A^+ e^{j\theta^+}$ and $V^- = A^- e^{j\theta^-}$

$$A^- e^{j\theta^-} = A^+ e^{j\theta^+} |\Gamma_L| e^{j\phi} e^{-2j\beta L}$$
$$A^- e^{j\theta^-} = A^+ |\Gamma_L| e^{j(\theta^+ + \phi - 2\beta L)}$$

When two complex numbers are equal, the angles must be equal so

$$\theta^- = \theta^+ + \phi - 2\beta L$$
$$\theta^+ - \theta^- = -\phi + 2\beta L$$
$$\theta^+ - \theta^- = -\phi + 2\frac{2\pi}{\lambda} L$$

$$\theta^+ - \theta^- = -\phi + \frac{4\pi}{\lambda} L$$

The minima are located at

$$z_n = \frac{(\theta^+ - \theta^-)\lambda}{4\pi} - \frac{\lambda}{4} \mp n\frac{\lambda}{2}$$

$$z_n = \frac{(-\phi + \frac{4\pi}{\lambda} L\,)\lambda}{4\pi} - \frac{\lambda}{4} \mp n\frac{\lambda}{2}$$

$$z_n = \frac{-\phi\lambda}{4\pi} + L - \frac{\lambda}{4} \mp n\frac{\lambda}{2}$$

$$L - z_n = \frac{\phi\lambda}{4\pi} + \frac{\lambda}{4} \pm n\frac{\lambda}{2}$$

Choose the minimum closest to the load

$$L - z = \frac{\phi\lambda}{4\pi} + \frac{\lambda}{4}$$

Solve for ϕ

$$\phi = \frac{4\pi}{\lambda}(L - z) - \pi$$

Knowing the distance $L - z$ from a minimum to the load tells us the angle of the reflection coefficient.

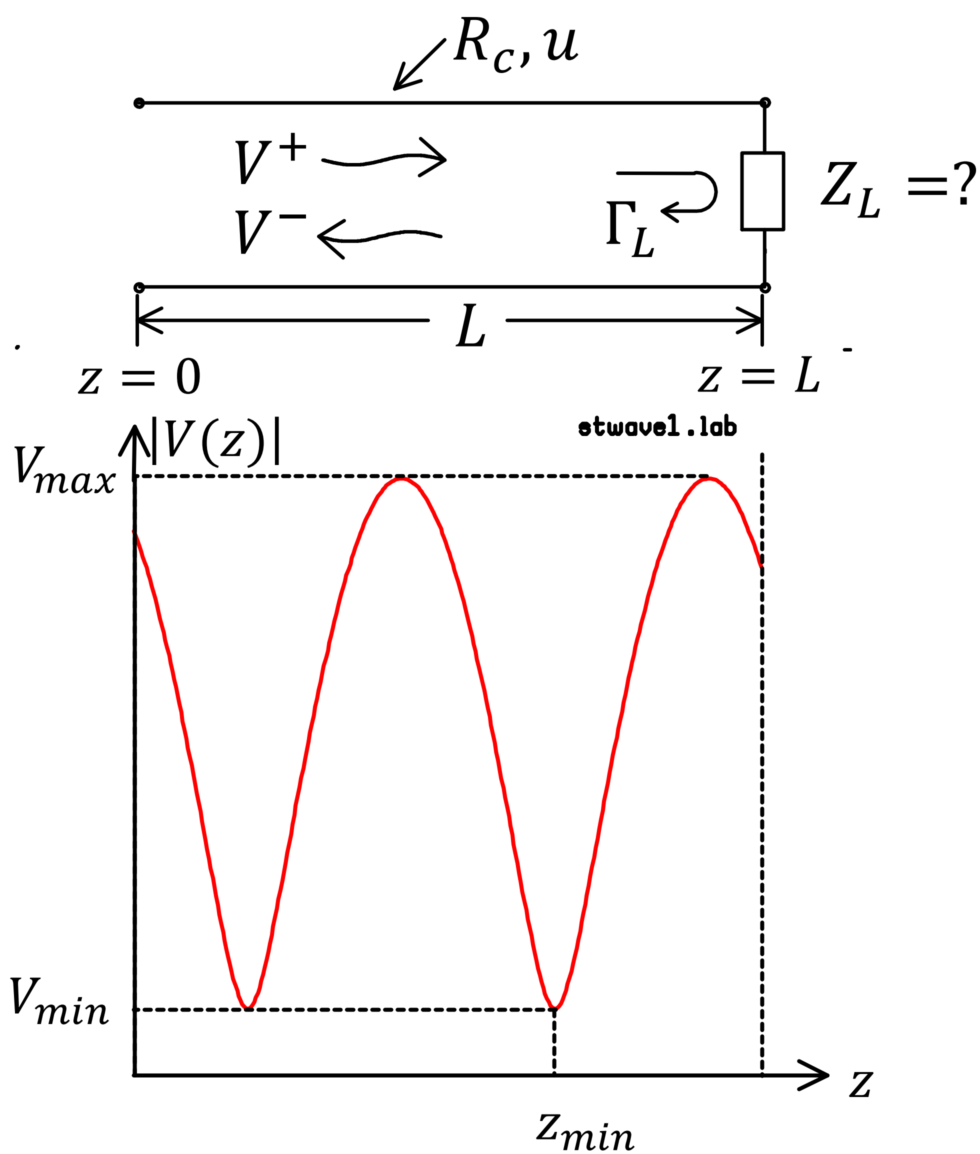
It is convenient to rewrite this formula in degrees as:

$$\phi = \frac{4\pi}{\lambda}(L - z) - \pi$$

$$\phi = 720° \frac{L - z}{\lambda} - 180°$$

Find $\Gamma_L = |\Gamma_L| e^{j\phi}$ from a Measured Standing-Wave Pattern

Measure the SWR and the location of a minimum, z_{min}.



$$SWR = \frac{V_{max}}{V_{min}}$$

$$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$|\Gamma_L| = \frac{SWR-1}{SWR+1}$$

$$\phi = 720° \frac{L-z}{\lambda} - 180°$$

then $\Gamma_L = |\Gamma_L| \angle \phi$

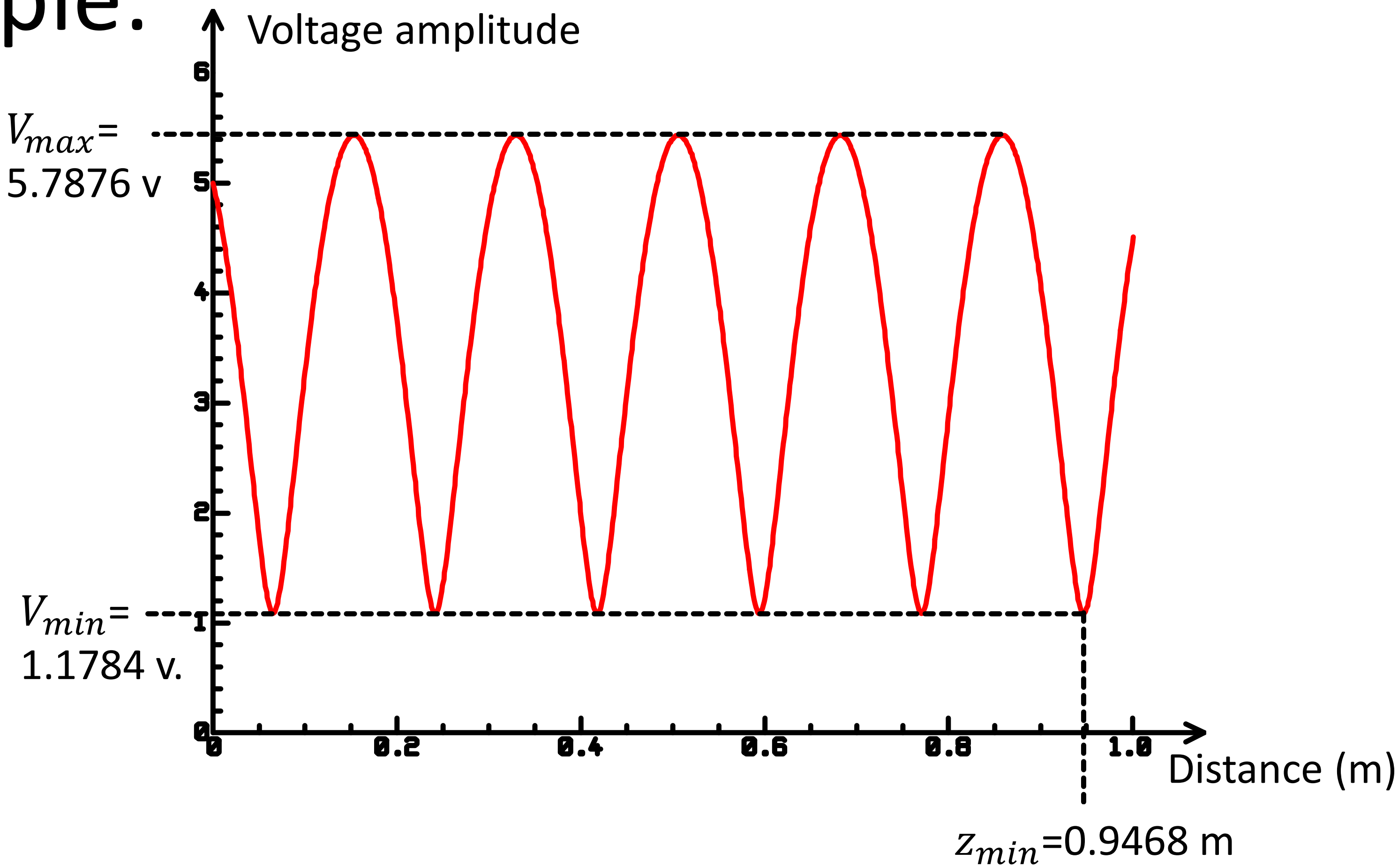
Since

$$\Gamma_L = \frac{Z_L - R_c}{Z_L + R_c}$$

then

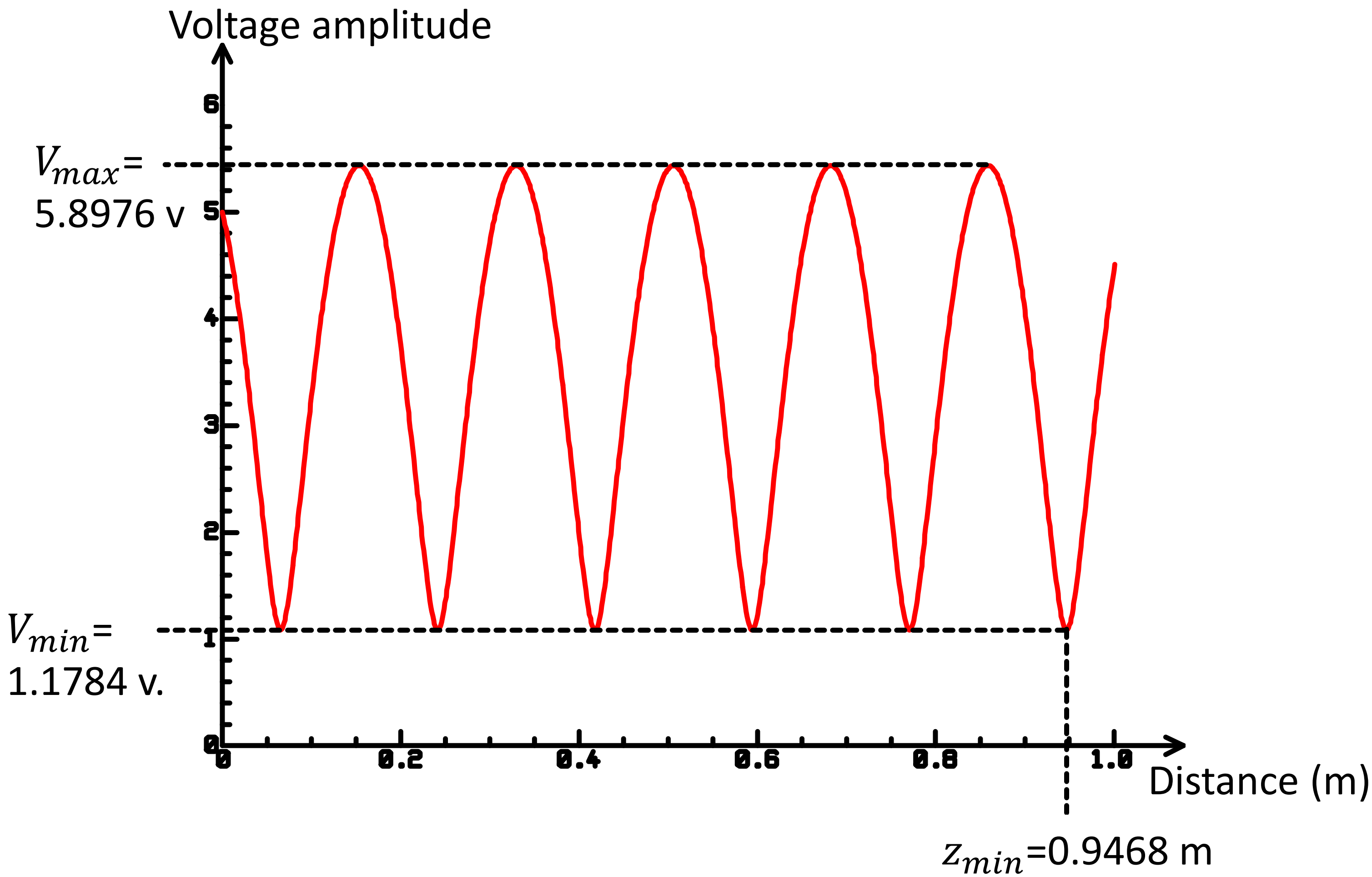
$$Z_L = \frac{1+\Gamma_L}{1-\Gamma_L} R_c$$

Example:



An engineer measures the standing-wave pattern for transmission line terminated with an unknown impedance Z_L. The transmission line has length $L = 1$ m, characteristic impedance $Z_0 = 50$ ohms and speed-of-travel $u = 300$ meters per microsecond. The frequency is 850 MHz. Find the value of the unknown impedance.

Solution:



The maximum voltage is V_{max}= 5.8976 v
The minimum voltage is V_{min}= 1.1784 v
The location of the minimum is z_{min}=0.9468 m
The load is at $\mathrm{z} = L = 1$ m.

Calculate Γ_L and Z_L:

The maximum voltage is V_{max}= 5.8976 v
The minimum voltage is V_{min}= 1.1784 v
The location of the minimum is z_{min}=0.9468 m

$$SWR = \frac{V_{\max}}{V_{\min}} = \frac{5.8974}{1.1784} = 5.005$$

$$|\Gamma_L| = \frac{SWR-1}{SWR+1} = \frac{5.005-1}{5.005+1} = 0.66694$$

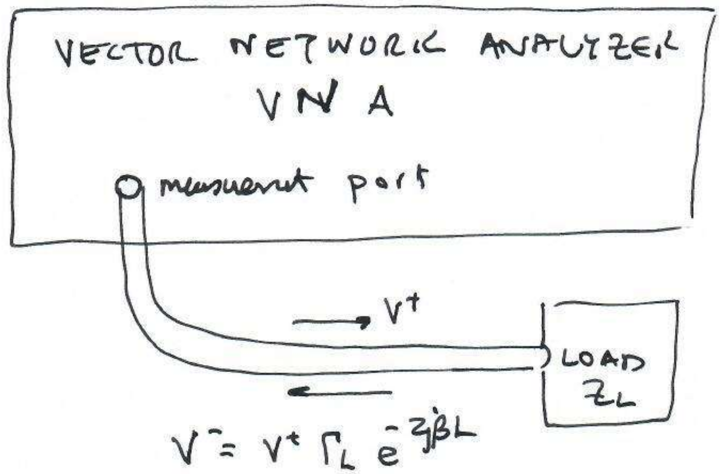
$$\lambda = \frac{u}{f} = \frac{300}{850} = 0.35294$$

$$\phi = 720° \frac{L-z}{\lambda} + 180° = 720 \frac{1-0.9468}{0.35294} - 180 = -71.2°$$

$$\Gamma_L = |\Gamma_L| e^{j\phi} = 0.66694\angle - 71.2°$$

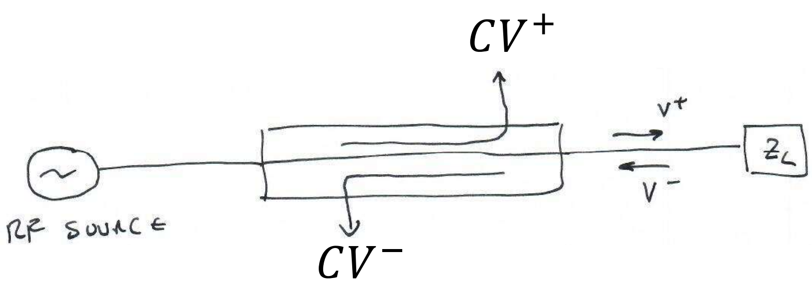
$$Z_L = \frac{1+\Gamma_L}{1-\Gamma_L} R_c = 50 \frac{1+0.66694\angle - 71.2°}{1-0.66694\angle - 71.2°} = 27.35 - j62.21\ \Omega$$

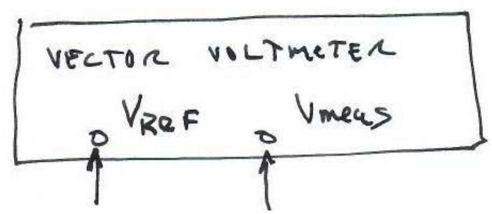
Vector Network Analyzer (VNA)



$$V^+ = A^+ \angle \theta^+$$
$$V^- = V^+ \Gamma_L e^{-2j\beta L}$$
$$= A^+ \angle \theta^+ |\Gamma_L| e^{j\phi} e^{-2j\beta L}$$
$$= A^+ |\Gamma_L| \angle (\theta^+ + \phi - 2\beta L)$$

What is in the VNA box? Answer: a directional coupler plus a vector voltmeter.





$V_{ref} = CV^+ \quad V_{meas} = CV^-$

Directional Coupler

C =coupling factor of the directional coupler

CV^+ goes to V_{ref}

CV^- goes to V_{meas}

$$\left|\frac{V_{meas}}{V_{ref}}\right| = \left|\frac{CV^-}{CV^+}\right| = \left|\frac{V^-}{V^+}\right| = |\Gamma_L|$$

$$\angle V_{meas} - \angle V_{ref} = (\theta^+ + \phi - 2\beta L) - \theta^+$$
$$= \phi - 2\beta L$$