ELEC353 Lecture Notes Set 7

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #3: Do homework #3 by February 1, 2019.
Homework #4: Do homework #4 by February 8, 2019.
Homework #5: Do homework #5 by February 14, 2019.

Mid-term test: Thursday February 14, 2019.

- Includes Homework #5!

- See the course web site for sample mid-term tests with solutions.
- Study tip:
    - Download the question paper for a mid-term from a previous year.
    - Spend one hour 15 minutes solving the test with your calculator and the formula sheet, but no textbook or notes.
    - Grade your answer against the solution to the test!

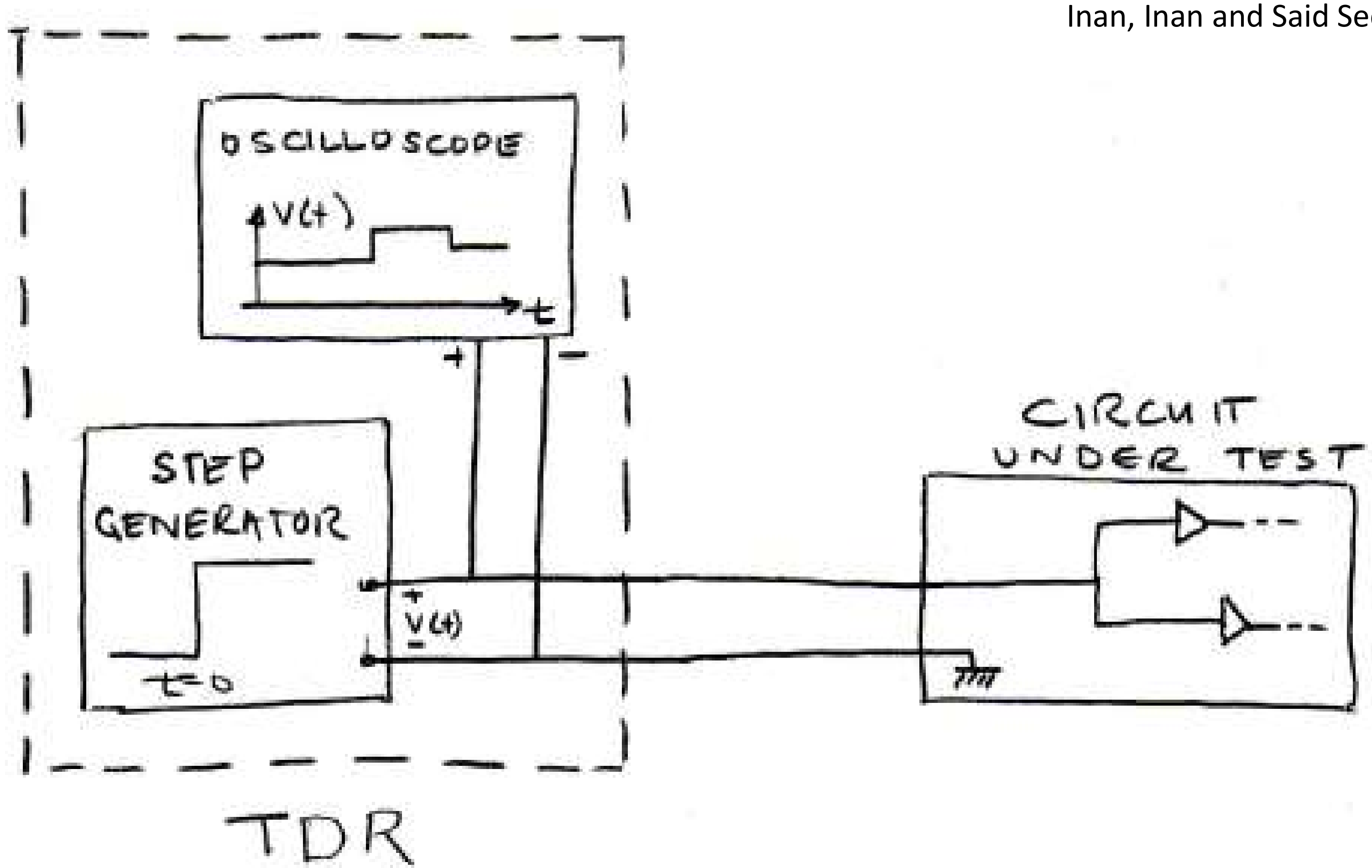
Mid-term Test: Thursday February 14, 2019

What is covered on the mid-term?
Everything done in class up to February 12.

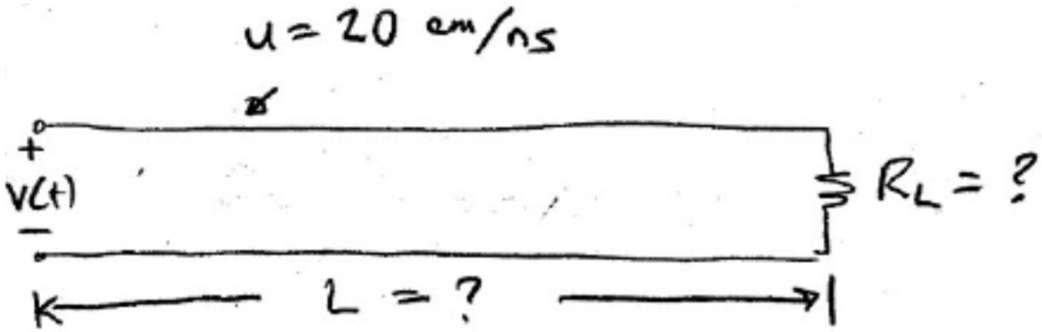
Homework assignments #1 to 5
#1 = lumped and distributed circuit analysis, RC model of a TL
#2 = introductory TL questions, bounce diagram
#3 = TL in series, TL with shunt load
#4 = branching TL, RL load, TDR
#5 = RL load, transmission lines in series, pulse generator, TDR

Time-Domain Reflectometry (TDR)

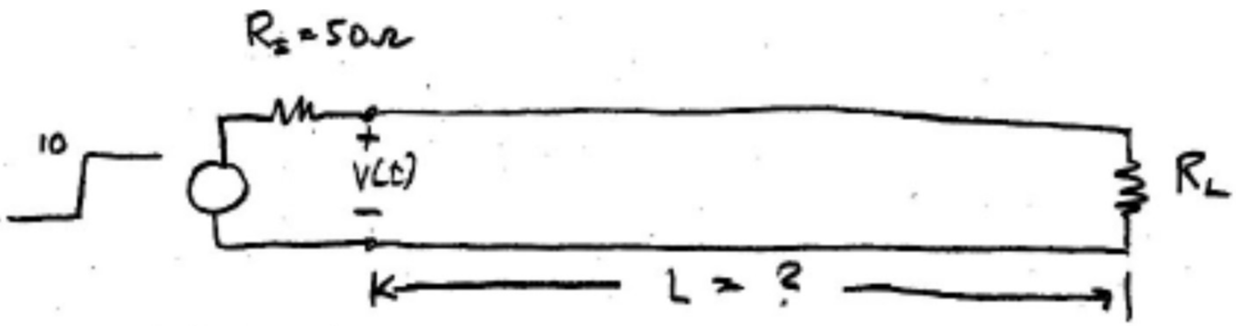
Inan, Inan and Said Section 2.6



Simple Example

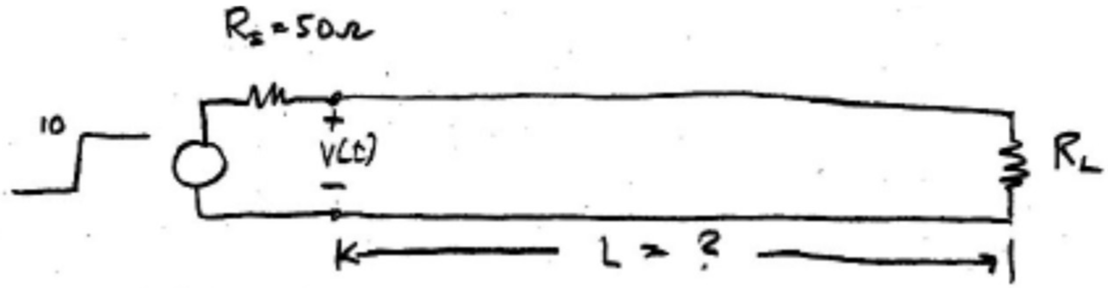


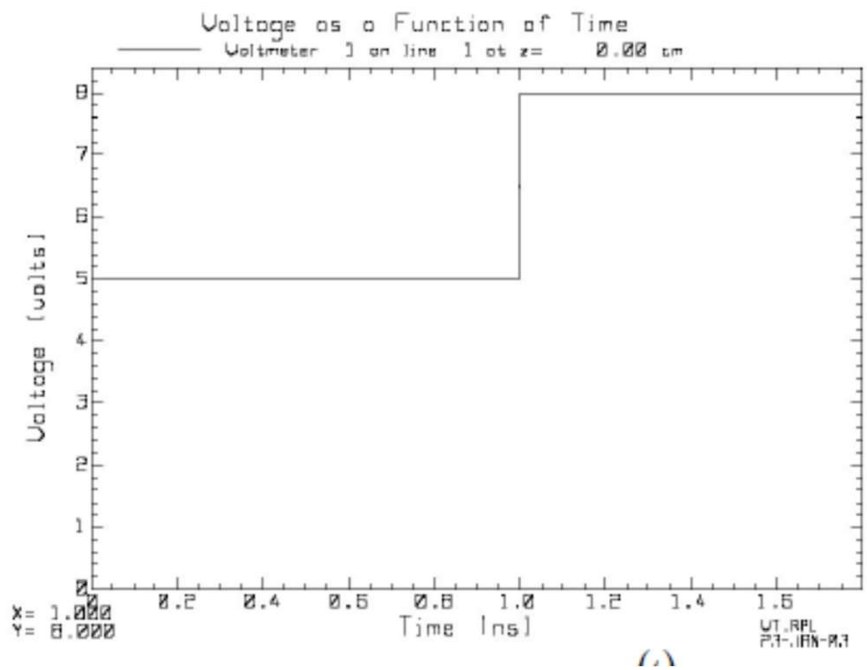
- In this circuit the circuit interconnect or "transmission line" is known to have a speed of propagation of $u = 20$ cm/ns.
- We want to find the characteristic impedance Z_0 of the transmission line, length of the transmission line L and the value of the load resistance R_L.



- A "TDR" test set is used to launch a step function voltage onto the circuit board interconnect, and to monitor the input voltage as a function of time.
- The TDR generates a 10-volt step function (open circuit) and has an internal resistance of $R_s = 50$ ohms.

TDR Measurement: Voltage at the Input





$$V^{+} = 5 \quad \text{volts}$$

$$2T_d = 1 \quad \text{ns}$$

$v(t)$

- Starts at 5 volts at t=0
- Then steps up to 8 volts at t=1 ns

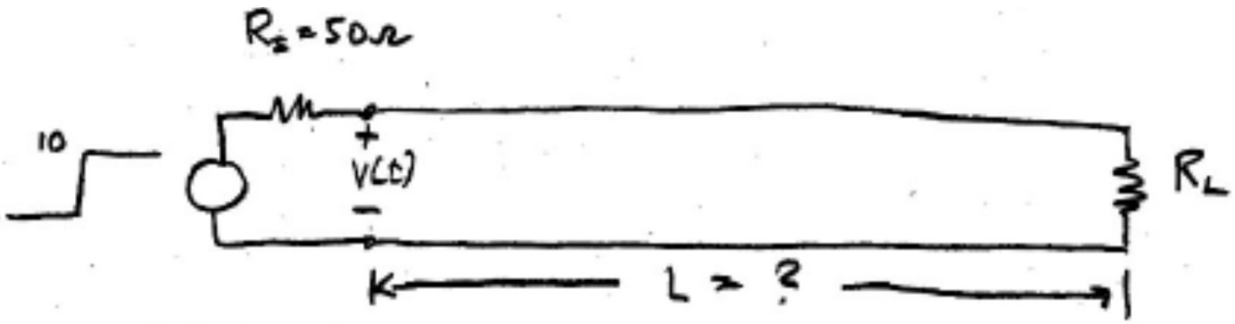
Find:

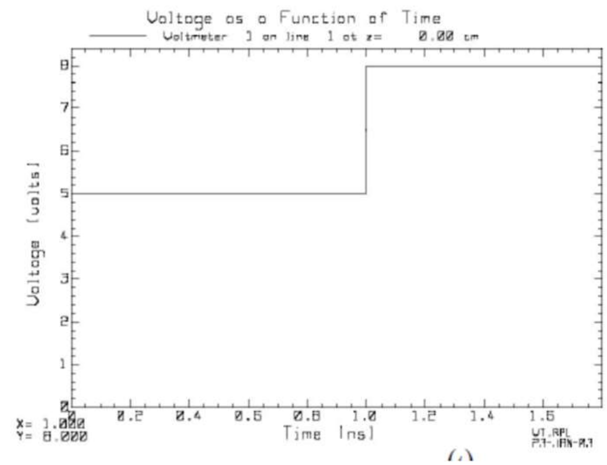
The characteristic impedance R_c

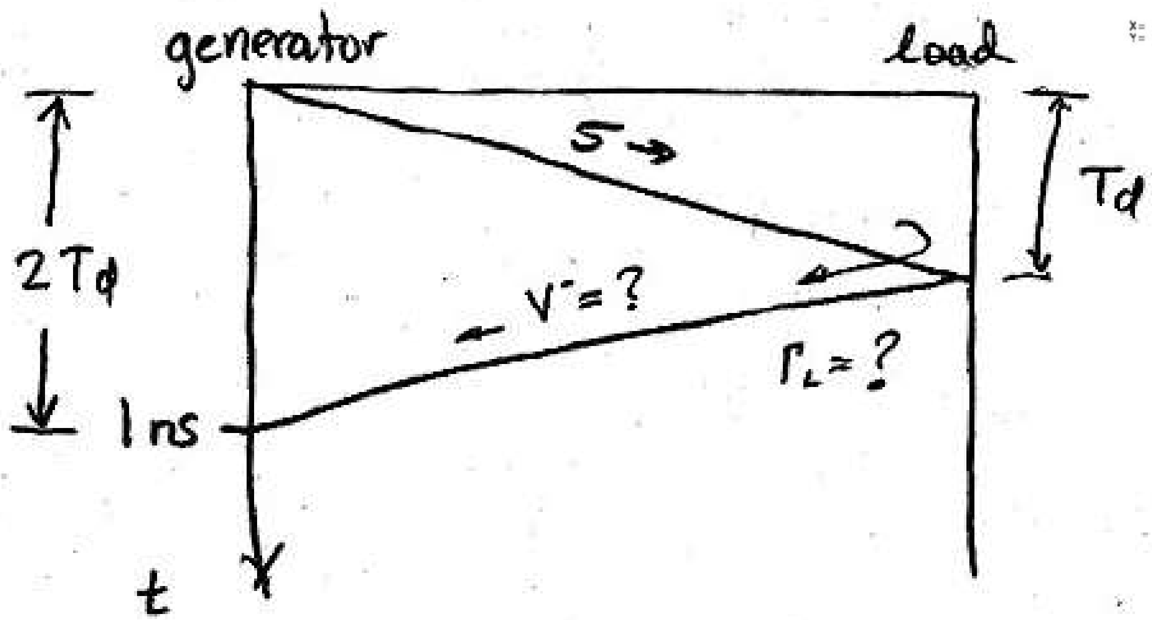
The length of the transmission line L

The load resistance R_L

Bounce Diagram







At $t = 0$ the voltage at the input steps up to five volts.

$$V^{+} = \frac{R_c V_s}{R_c + R_s}$$

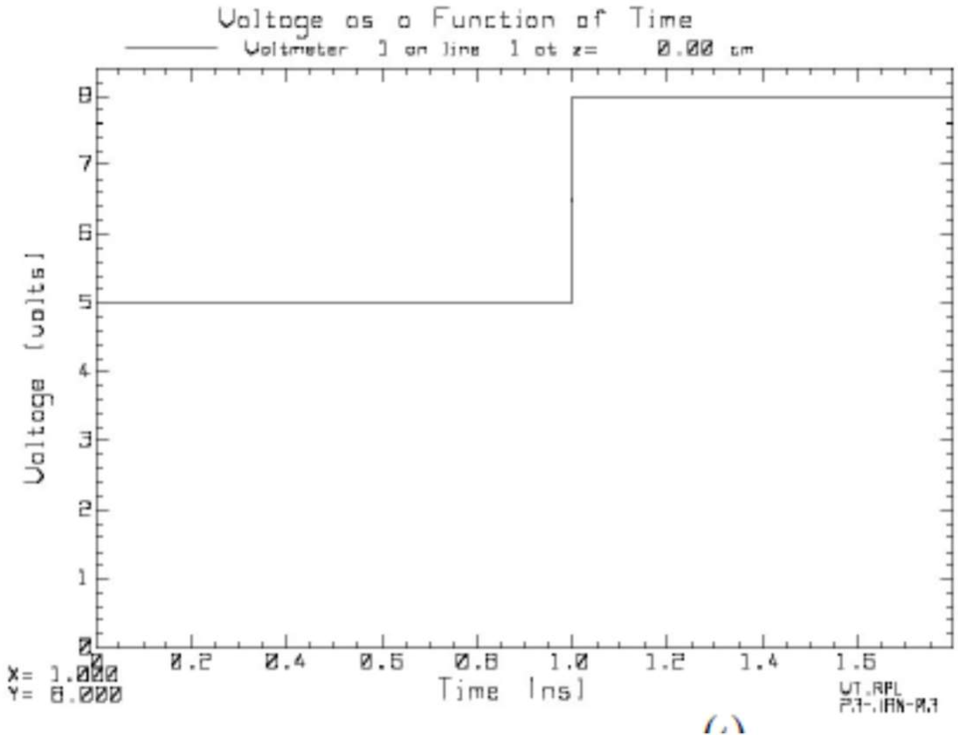
$$\frac{R_c x10}{R_c + 50} = 5$$

$$5(R_c + 50) = 10R_c$$

$$5R_c = 250$$

$$R_c = 50$$

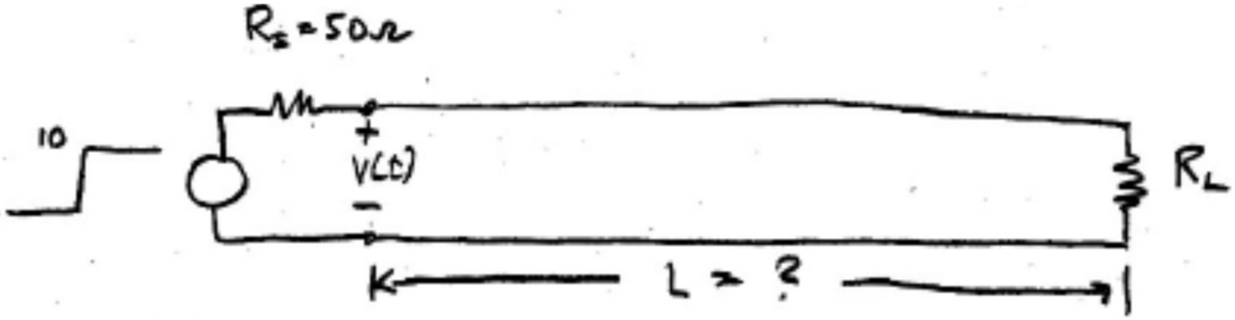
Reading the TDR Response

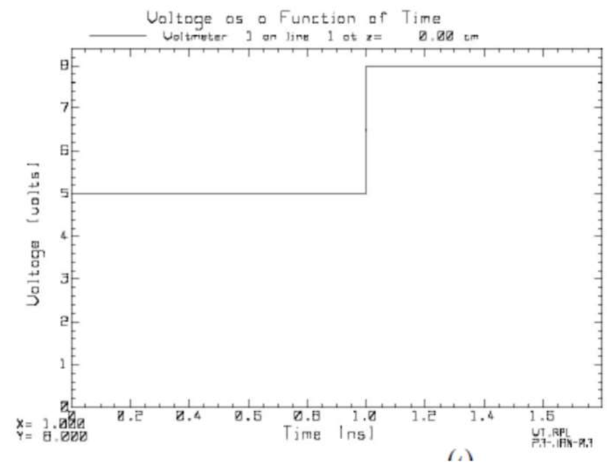


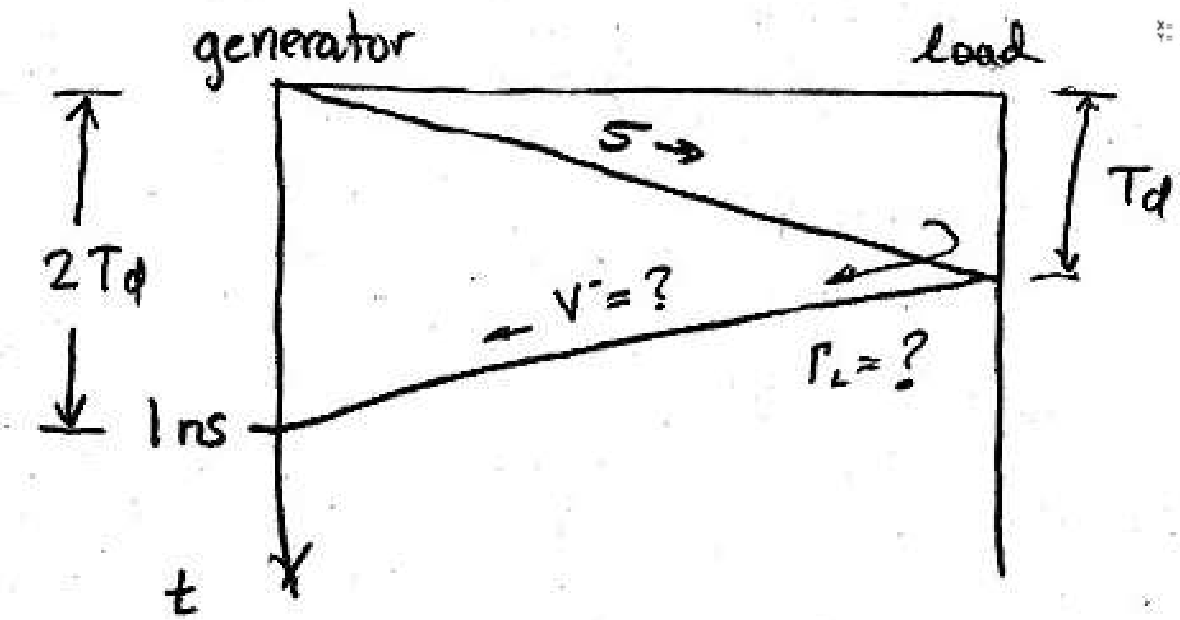
$$2T_d = 1$$ so $$2T_d = 2\frac{L}{u} = 2\frac{L}{20} = 1$$

$$L = 10$$ cm

Load Reflection Coefficient







At the generator:

- a step arrives at $t = 2T_d$ seconds
- the step height is $V(z=0) = \Gamma_L V^+$ volts

With R_c=50 ohms,

$$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = 0$$

The voltage steps up from 5 volts to 8 volts at 1 ns.
The voltage steps up by V^- volts, so
$V^- = 3$ volts.
Since $V^- = \Gamma_L V^+$ and $V^+ = 5$, we have

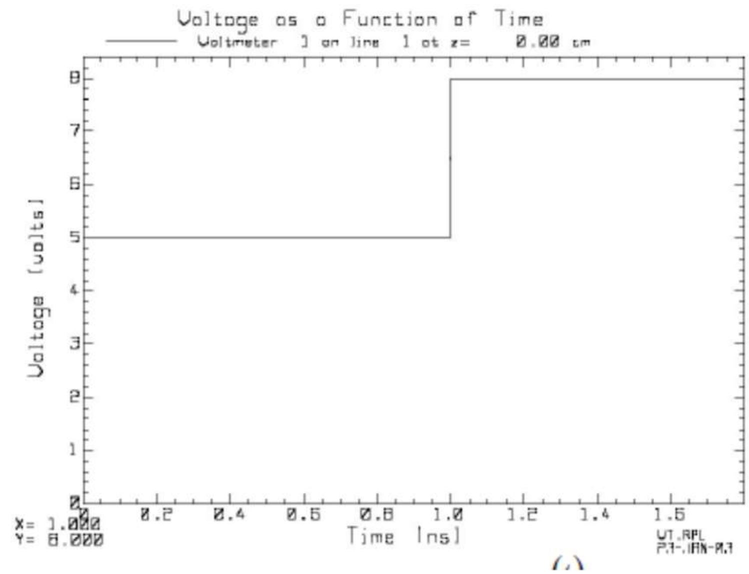
$$\Gamma_L = \frac{V^-}{V^+} = \frac{3}{5} = 0.6$$

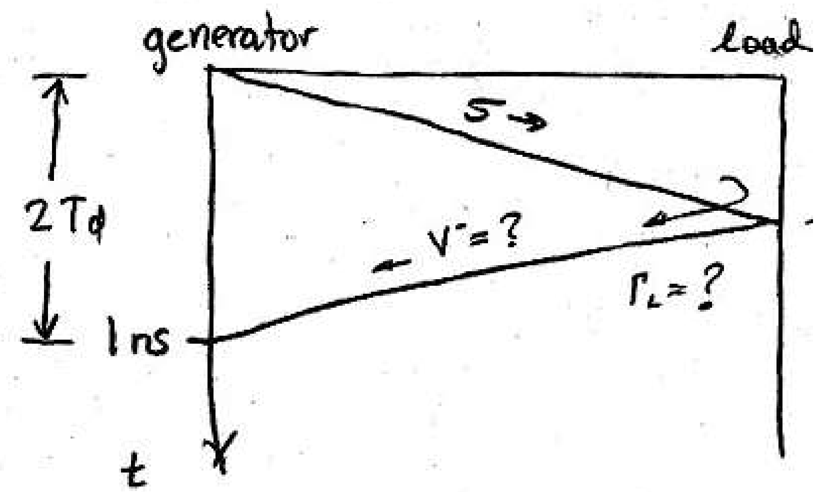
$$\Gamma_L = \frac{3}{5} = 0.6$$

$$\Gamma_L = \frac{R_L - R_c}{R_L + R_c}$$

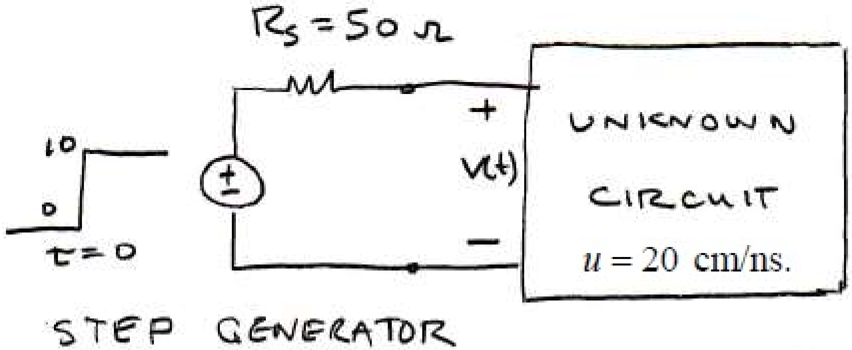
so $$R_L = R_c \frac{1+\Gamma_L}{1-\Gamma_L}$$

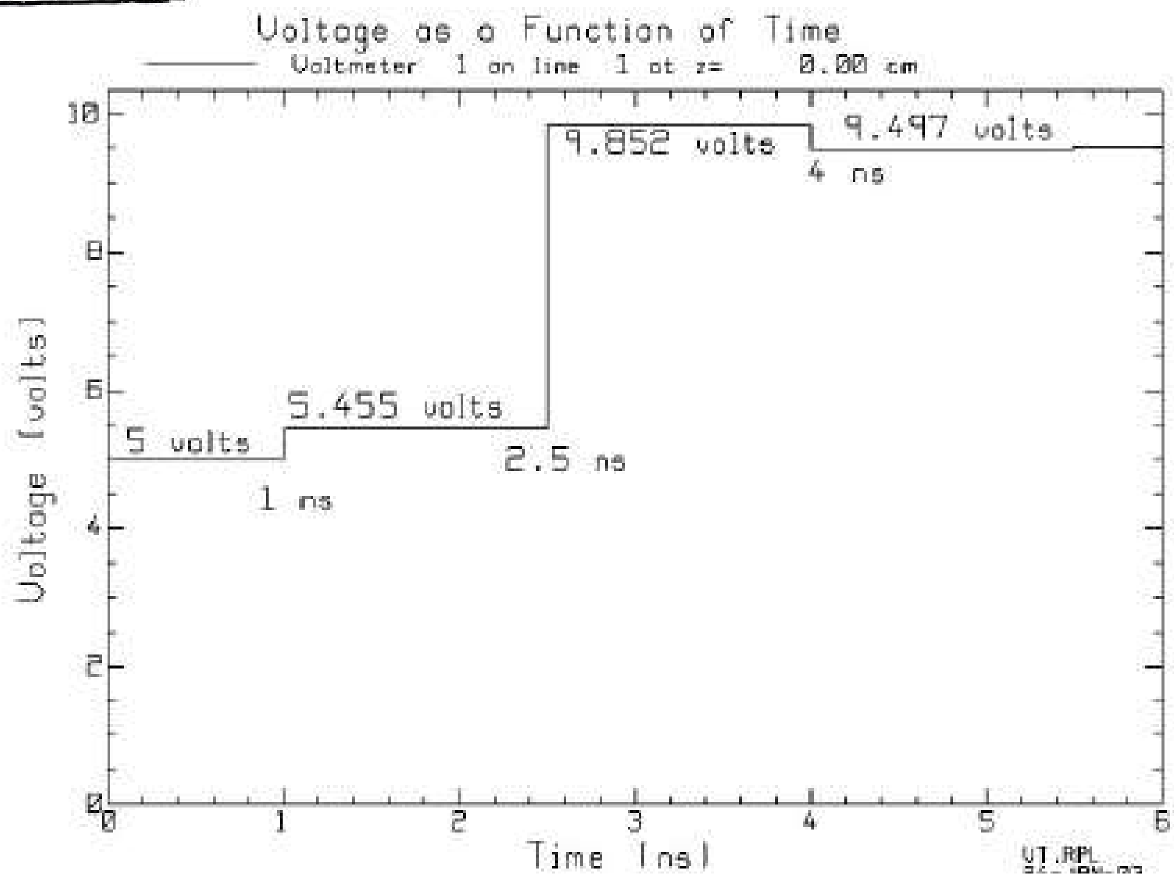
$$R_L = 50\frac{1+0.6}{1-0.6} = 200 \quad \text{Ohms}$$

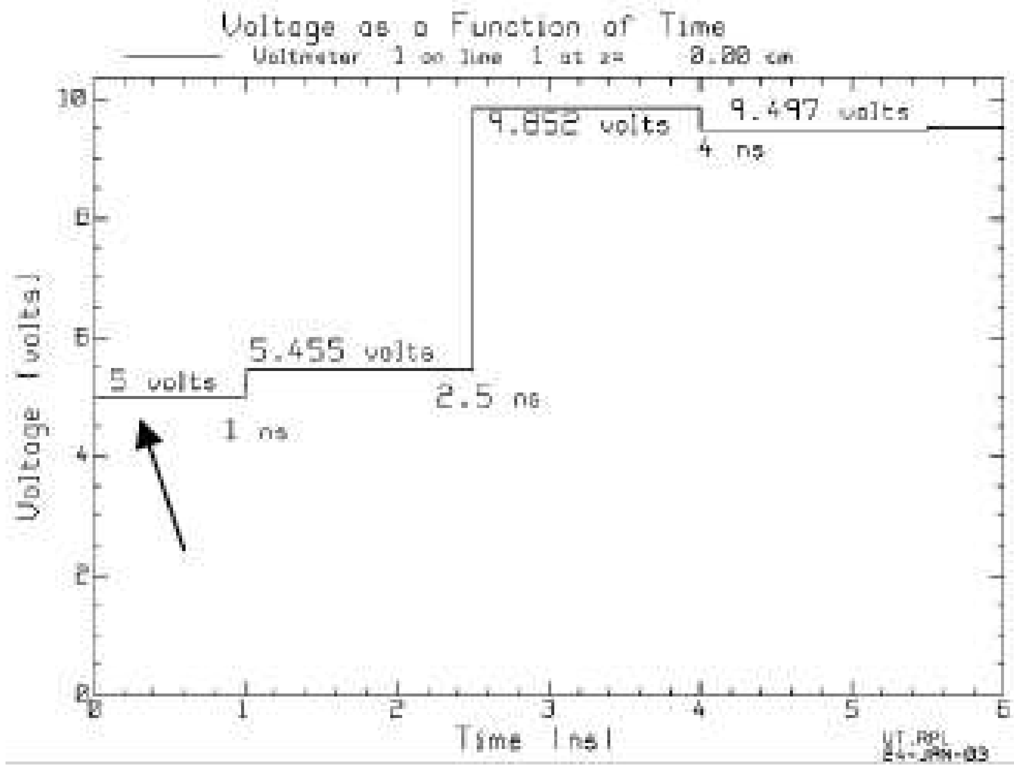




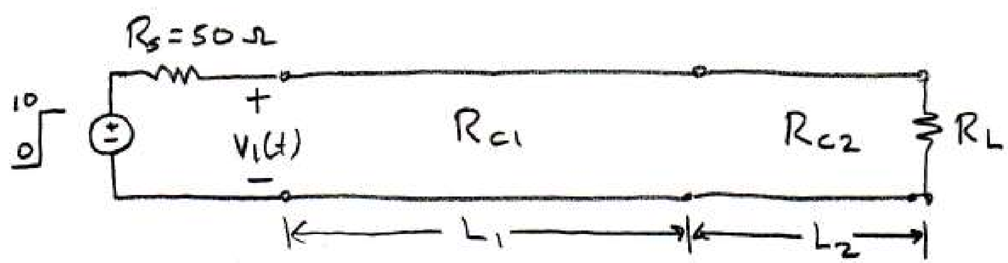
More Complicated Example

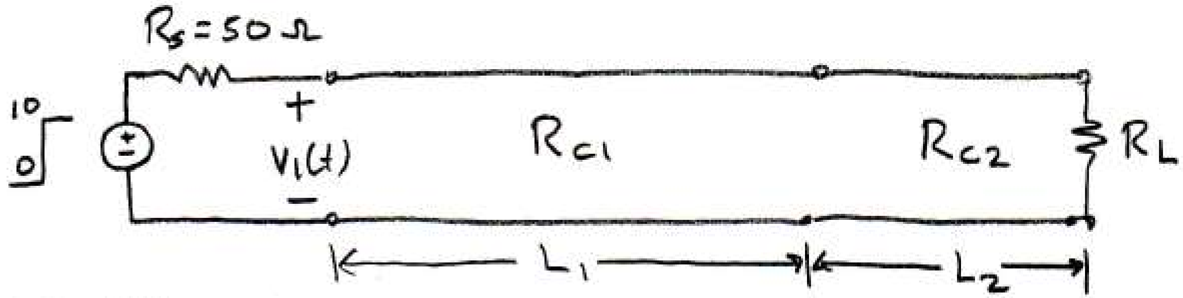


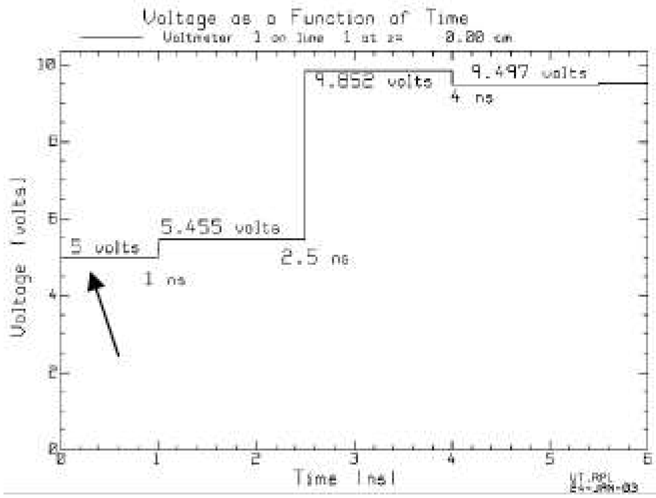


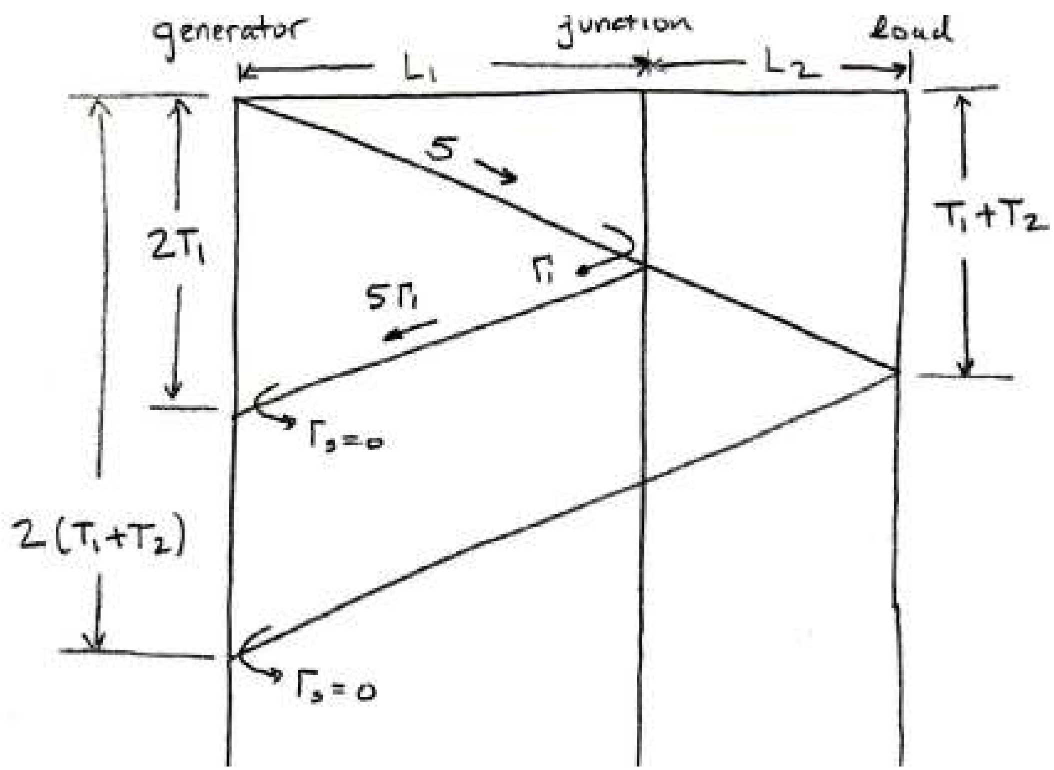


Inspection of the circuit board shows that there are two transmission lines in series, terminated by a load:









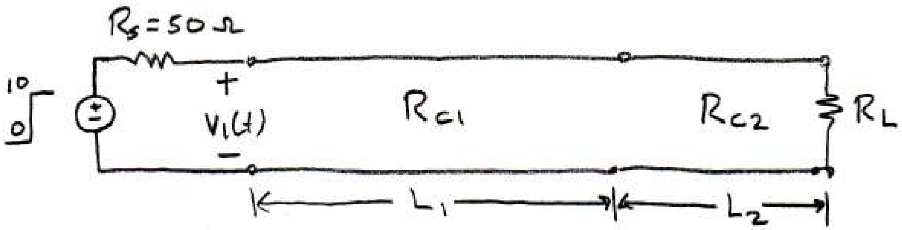
$$V(0) = 5 = \frac{R_{c1}x10}{R_{c1}+50}$$

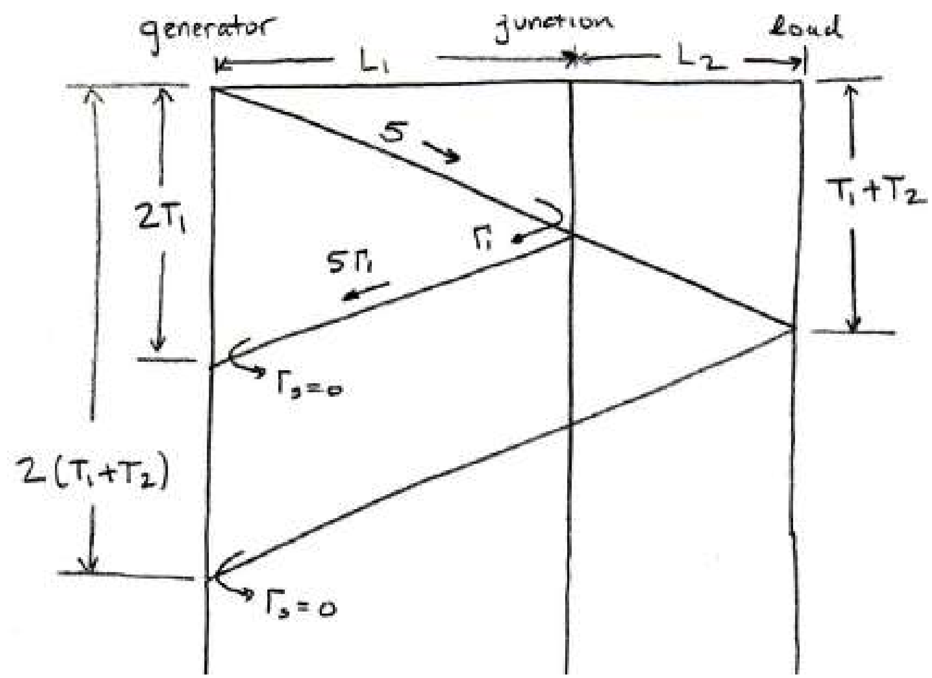
Solve this equation to find

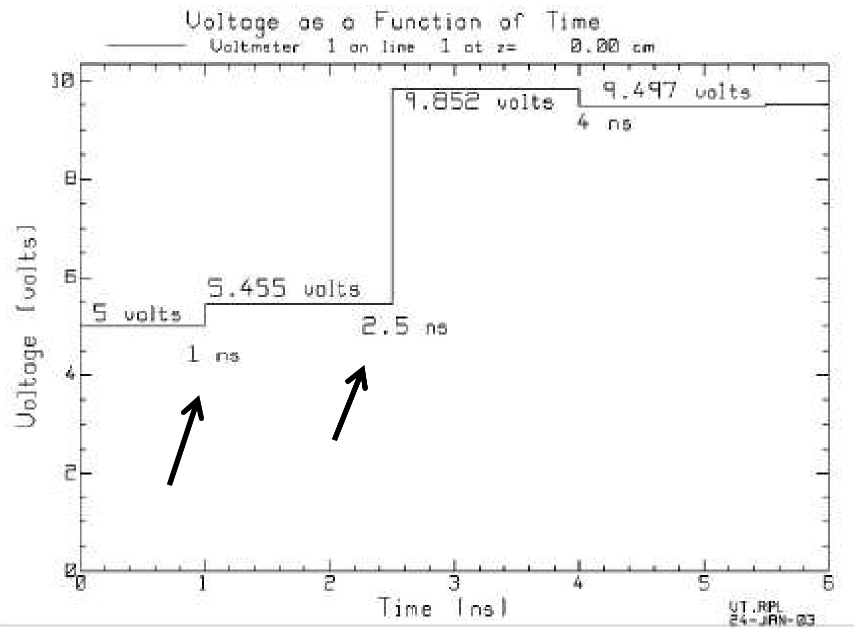
R_{c1}=50 ohms.

and

$$\Gamma_s = \frac{R_s - R_{c1}}{R_s + R_{c1}} = 0$$







The first echo:

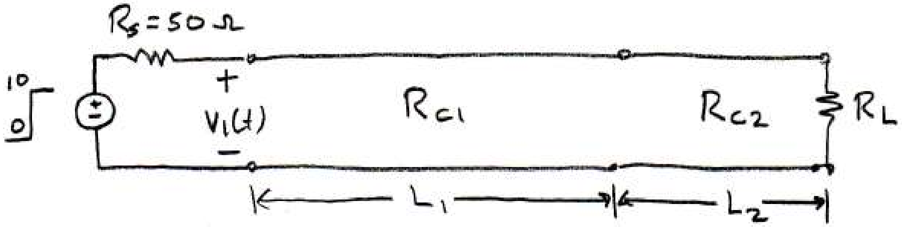
$$2T_1 = 2\frac{L_1}{u} = 1\,\text{ns, so } L_1 = \frac{1x20}{2} = 10 \text{ cm}$$

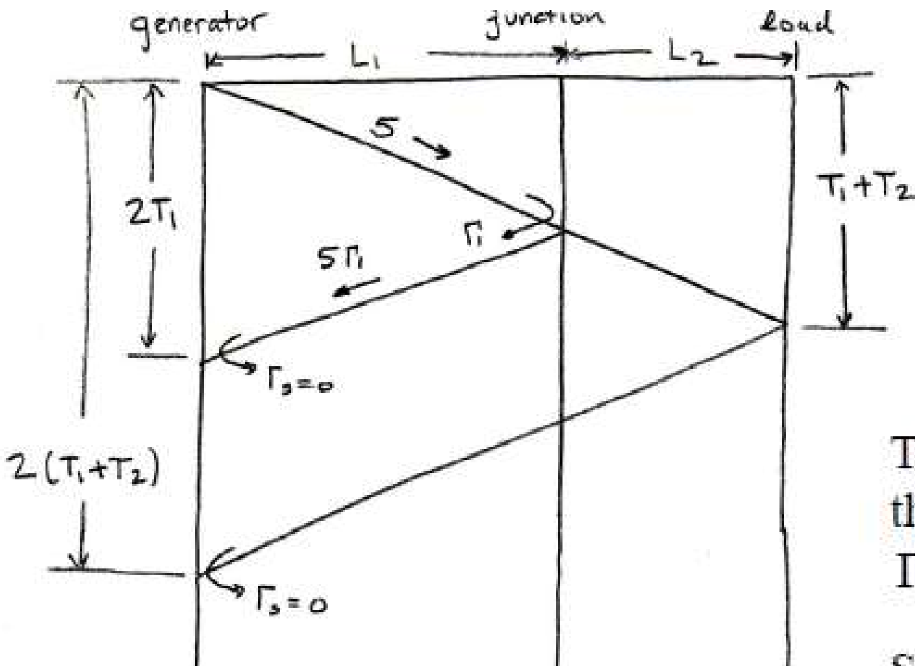
The second echo:

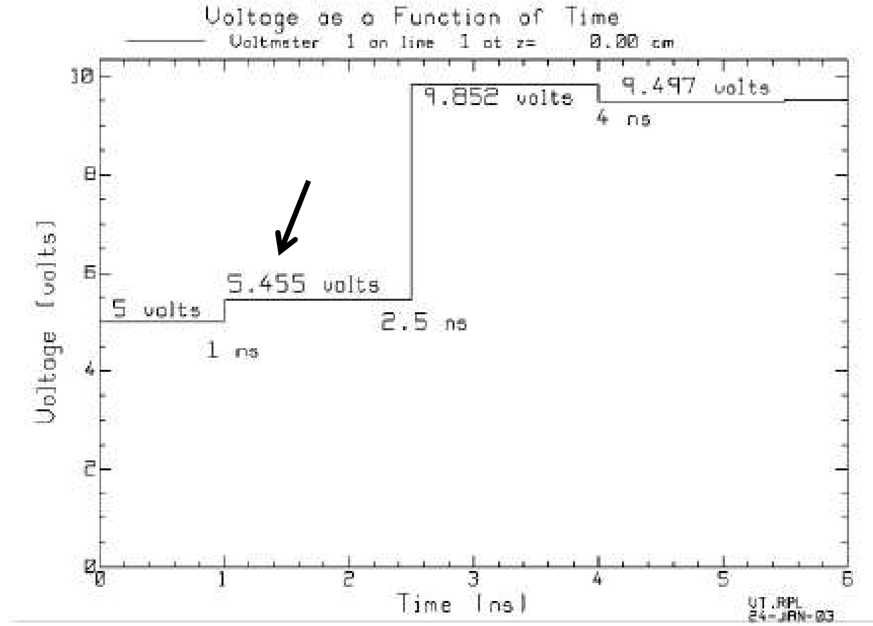
$$2(T_1 + T_2) = 2.5\,\text{ns,}$$

$$T_1 = 0.5 \text{ ns so } 2(0.5 + T_2) = 2.5$$
$$2T_2 = 2.5 - 1 = 1.5 \text{ ns}$$
$$T_2 = 0.75 \text{ ns}$$

$$L_2 = uT_2 = 20x0.75 = 15 \text{ cm}$$





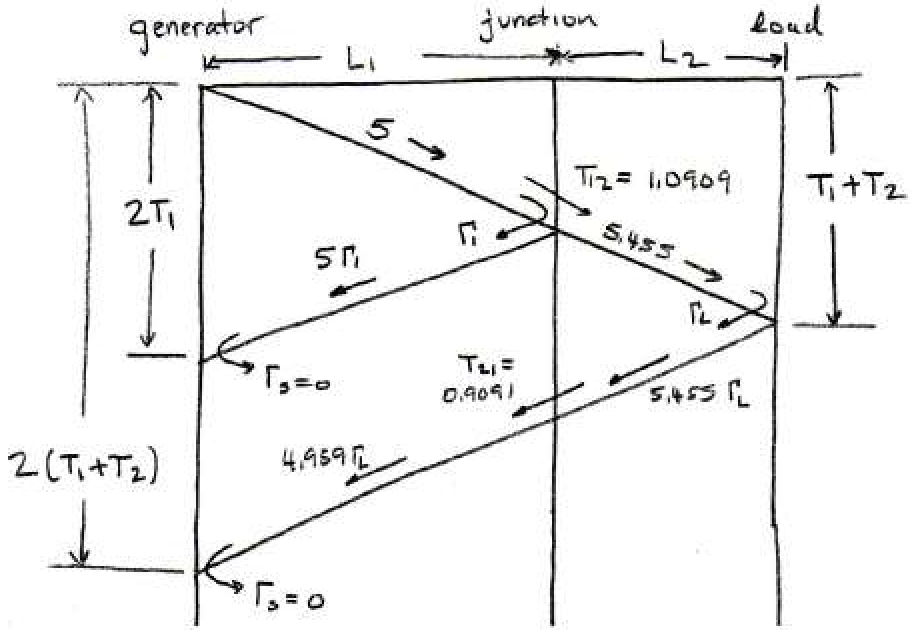


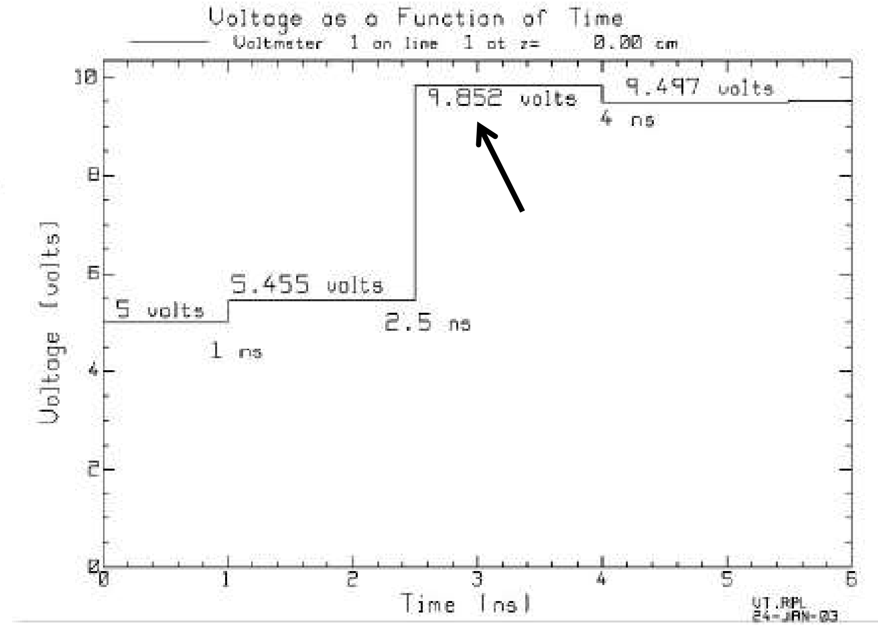
The step "up" is (5.455-5)=0.455 volts, so the reflection coefficient is

$\Gamma_1 = 0.455/5 = 0.091$

Since $\Gamma_1 = \dfrac{R_{c2} - R_{c1}}{R_{c2} + R_{c1}}$, the characteristic resistance of the 2[nd] transmission line is

$$R_{c2} = \frac{1+\Gamma_1}{1-\Gamma_1} R_{c1} = \frac{1+0.091}{1-0.091} x50 = 60.01 \quad \text{ohms}$$





$R_{c2} = 60\Omega$

$$T_{12} = \frac{2R_{c2}}{R_{c1} + R_{c2}} = \frac{2x60}{50+60} = 1.0909$$

$$T_{21} = \frac{2R_{c1}}{R_{c1} + R_{c2}} = \frac{2x50}{50+60} = 0.9091$$

$$V^- = 5T_{12}\Gamma_L T_{21}$$

$$V^- = 5x1.0909x0.9091x\Gamma_L$$

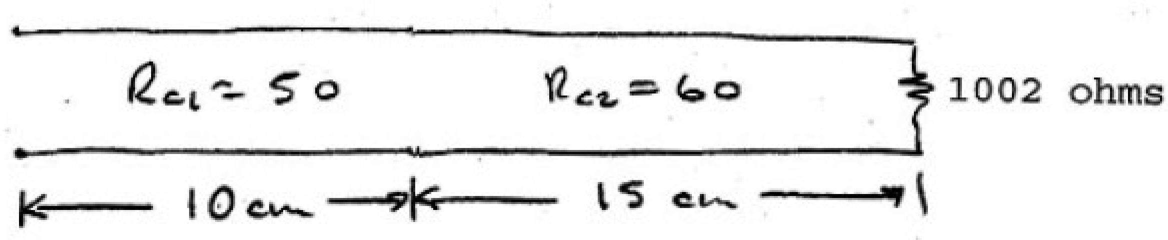
$$V^- = 4.959\Gamma_L$$

The step "up" is from 5.455 volts to 9.852 volts, of height (9.852-5.455)=4.397.

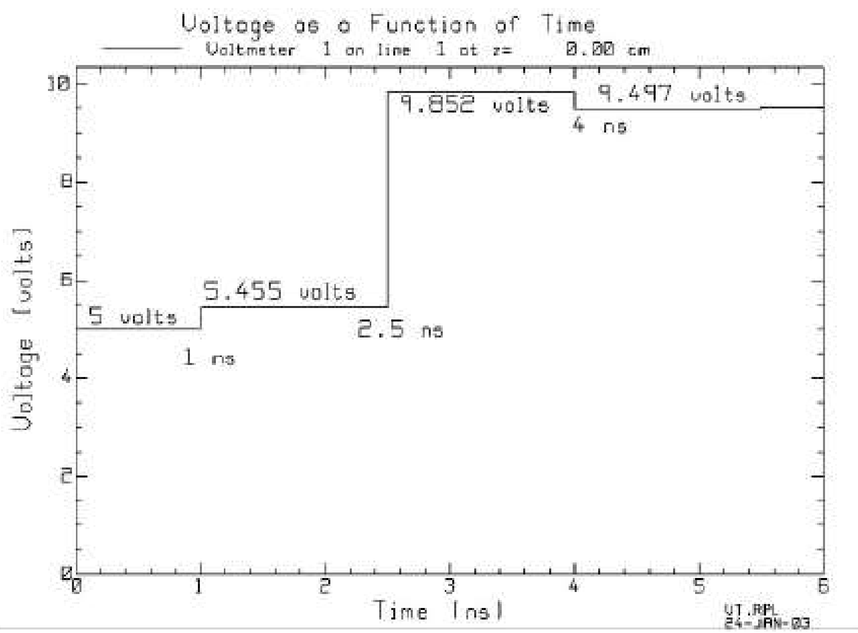
$$4.959\Gamma_L = 4.397$$
$$\Gamma_L = 0.887$$

$$\Gamma_L = \frac{R_L - R_{c2}}{R_L + R_{c2}} = +0.887$$

$$R_L = \frac{1+\Gamma_L}{1-\Gamma_L} R_{c2} = \frac{1+0.887}{1-0.887} x60 = 1002\ \ \Omega$$



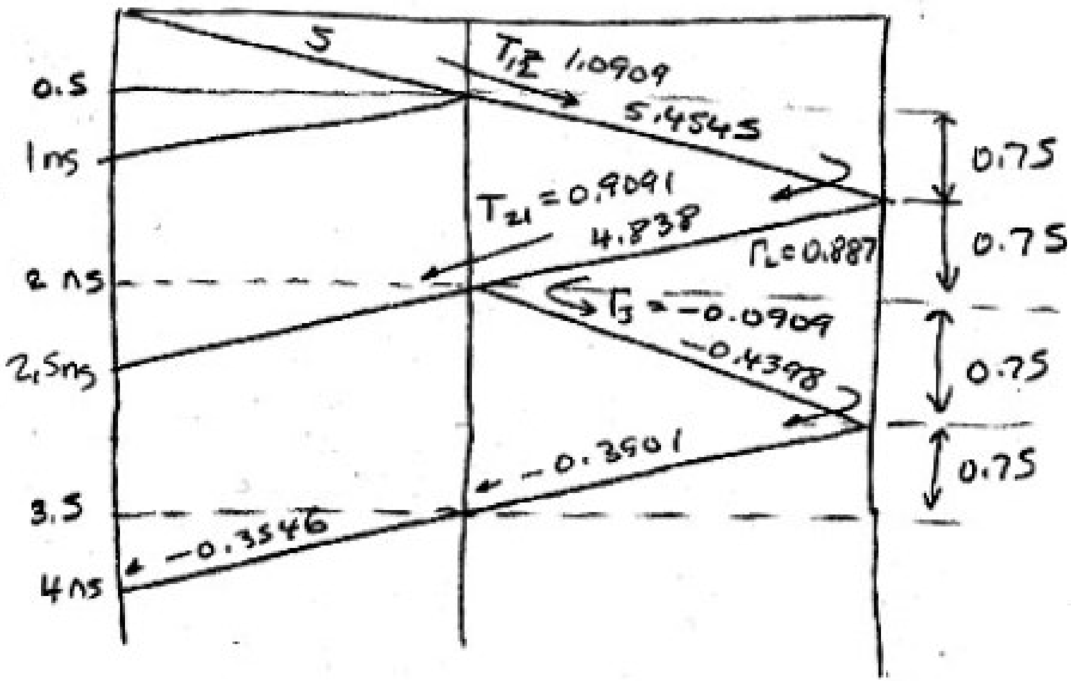
- Let's check the 3[rd] return in the TDR graph $v_1(t)$ to make sure that we are correct:

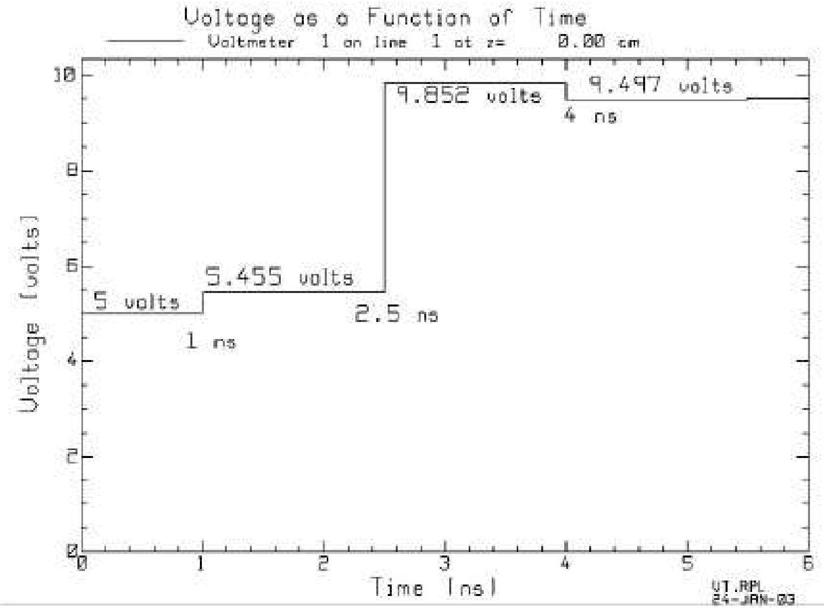


- The third "echo" arrives at 4 ns and is a step down from 9.852 to 9.497 volts, of -0.355 volts.

- The reflection coefficient at the junction "from the right" is

$$\Gamma_2 = \frac{R_{c1} - R_{c2}}{R_{c1} + R_{c2}} = \frac{50-60}{50+60} = -0.0909$$





From the bounce diagram we expect a step down of 0.3546 volts at 4 ns.

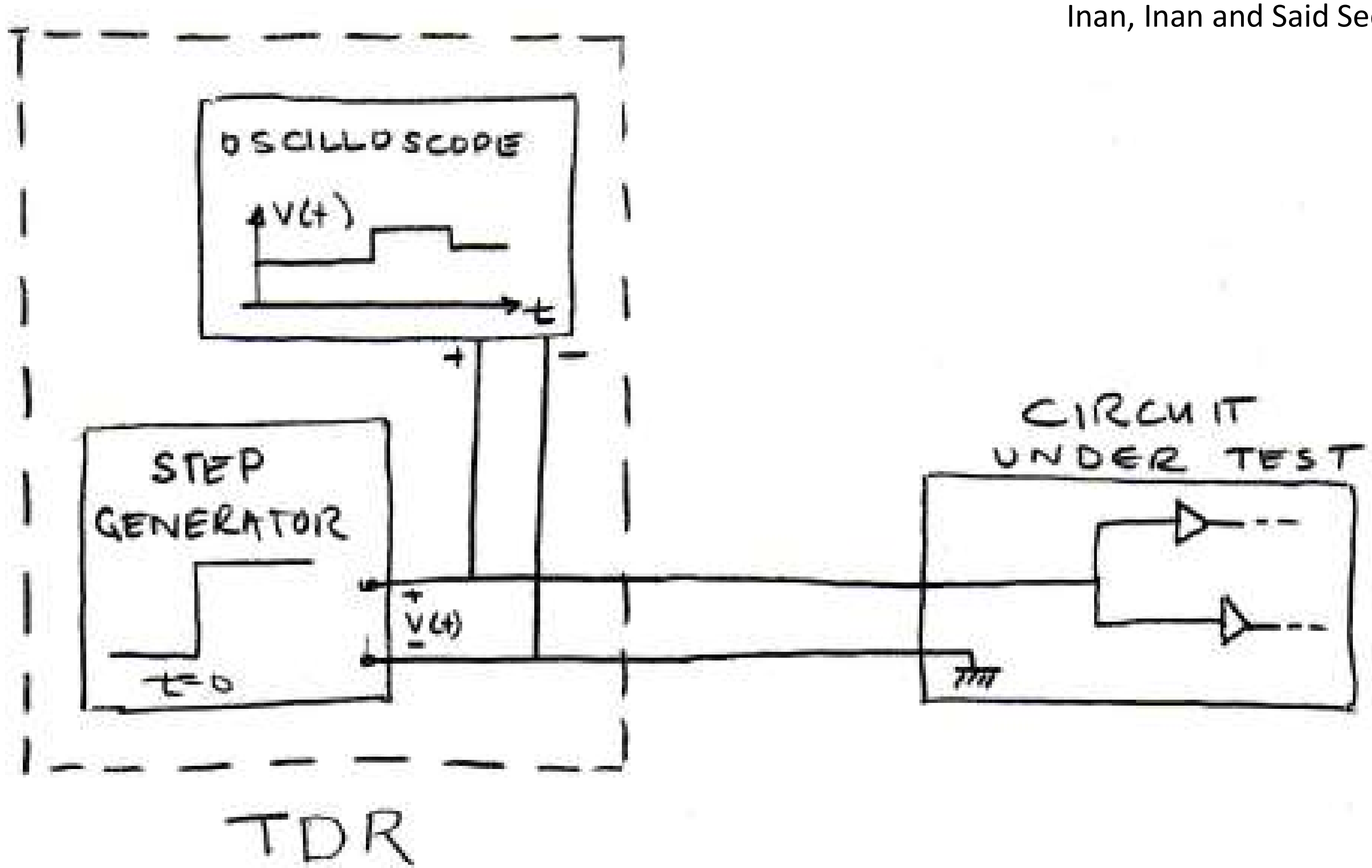
The TDR graph shows that at 4 ns the input voltage changes from 9.852 volts to 9.497 volts, which is a step down of

$$9.852-9.497= 0.355 \text{ volts}$$

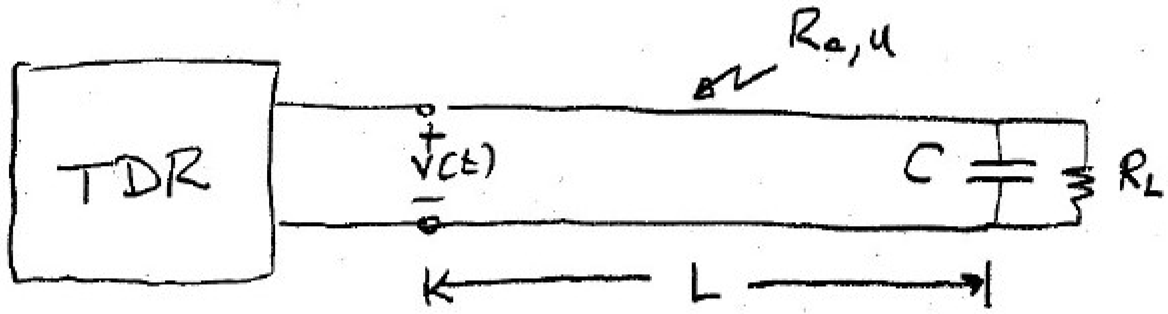
This agrees with the value we get from the bounce diagram.

Time-Domain Reflectometry (TDR)

Inan, Inan and Said Section 2.6



Time Domain Reflectometry with an RC Load

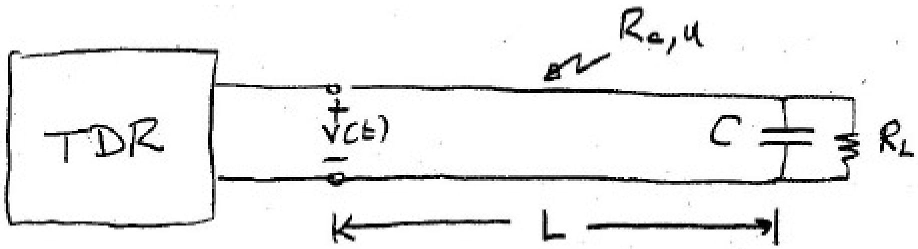


- The TDR applies a step-function voltage:
  - The step height is 1 volt
  - The source resistance is $R_s = 50\Omega$
- The capacitor is initially uncharged.
- The transmission line:
  - characteristic resistance R_c
  - Speed of travel u cm/ns

* The TDR launches a step of height $v(0) = \dfrac{R_c}{R_s + R_c} x1 \text{ volt}$

- After one time delay of $T_d = \dfrac{L}{u}$ ns, the step arrives at the load.

- The capacitor charges towards a final value of $v = (1 + \Gamma)v(0)$ where $\Gamma = \dfrac{R_L - R_c}{R_L + R_c}$



Example:

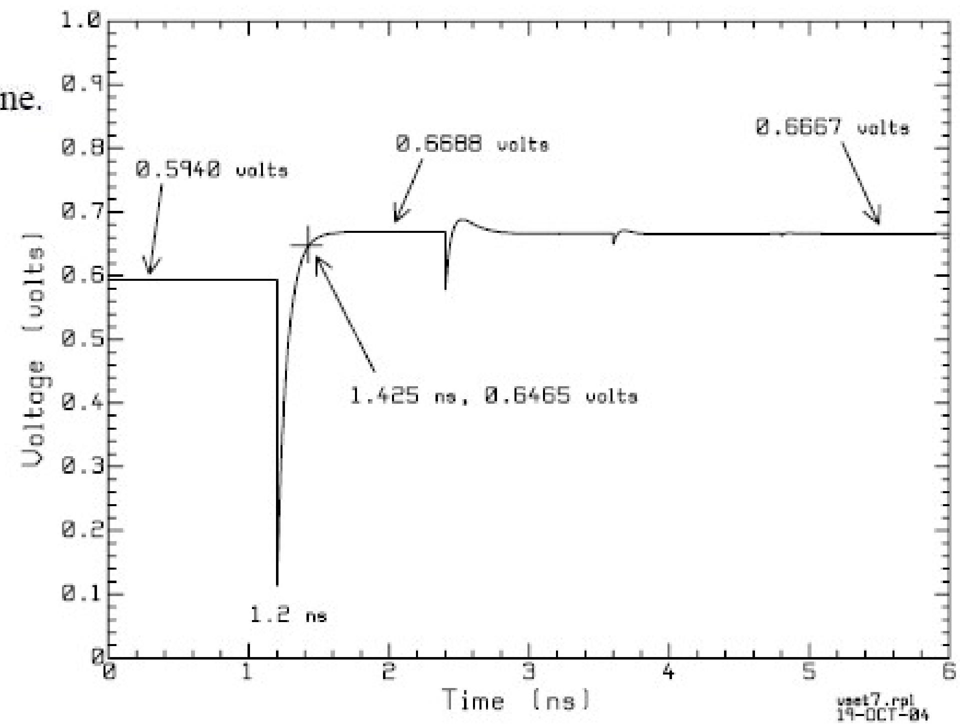
A TDR is used to measure the voltage at the input of a transmission line circuit. The TDR produces a 1-volt step function and has internal resistance $R_s = 50$ ohms. The speed of travel on the transmission line is $u = 20$ cm/ns. The characteristic resistance R_c of the transmission line is not known. The load is a resistor in parallel with a capacitor. The voltage at the input, as a function of time is shown below.

(i) Find the length of the transmission line.

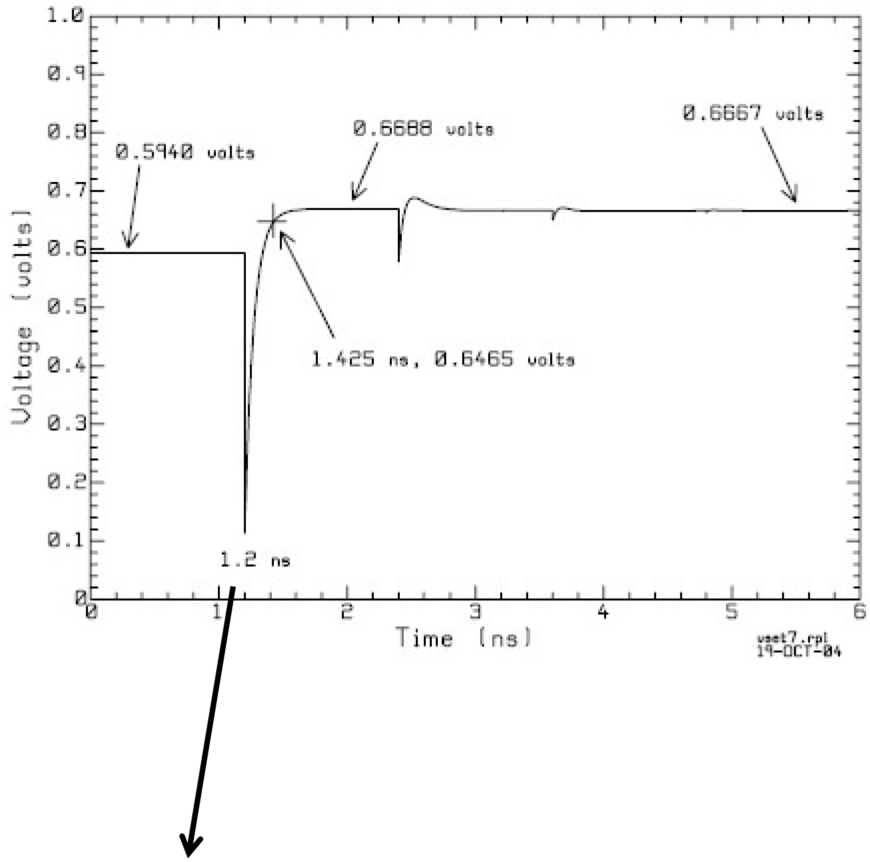
(ii) Find the characteristic resistance of the transmission line.

(iii) Find the resistance of the load.

(iv) Find the capacitance.

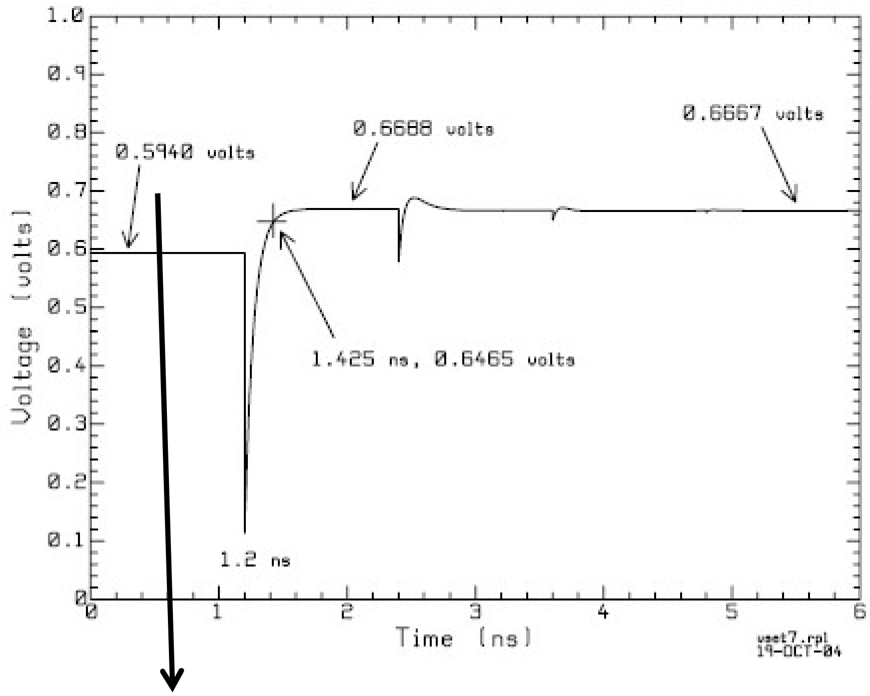


Solution:



$$2\frac{L}{u} = 1.20 \text{ ns hence } L = \frac{1.2}{2}u = 12 \text{ cm}$$

Characteristic resistance:



$$v(0) = \frac{1xR_c}{R_s + R_c} = \frac{R_c}{R_c + 50} = 0.594$$

$$R_c = 0.594R_c + 0.594x50$$

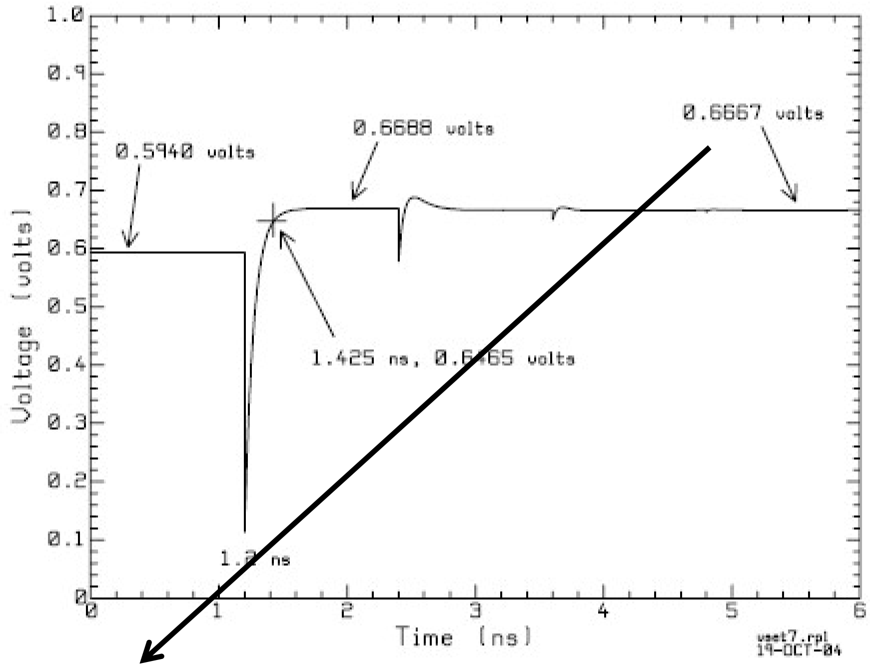
$$(1 - 0.594)R_c = 29.7$$

$$R_c = \frac{29.7}{0.406} = 73.15 \text{ ohms}$$

The reflection coefficient at the source is

$$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{50 - 73.15}{50 + 73.15} = -0.1880$$

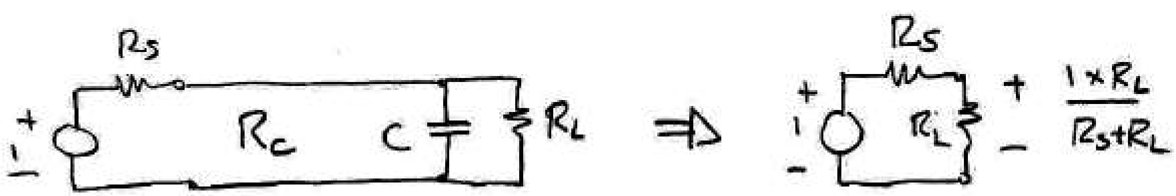
Load resistor from the final value:



What is the value of R_L?

After a long time:

- The capacitor is an open circuit.
- The voltage across the resistor stabilizes at 0.6667 volts.



$$\frac{R_L V_s}{R_s + R_L} = 0.6667$$

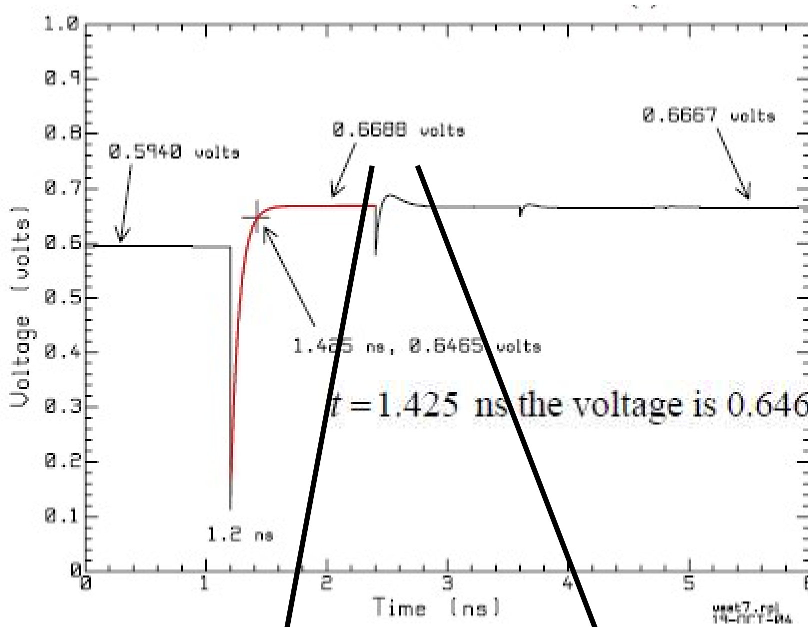
$V_s = 1$ and $R_s = 50$ so

$R_L = 0.6667x50 + 0.6667R_L$

$(1 - 0.6667)R_L = 33.34$

$R_L = \frac{33.34}{0.3333}$=100.2 ohms

Find the value of the capacitor from the exponential charging:



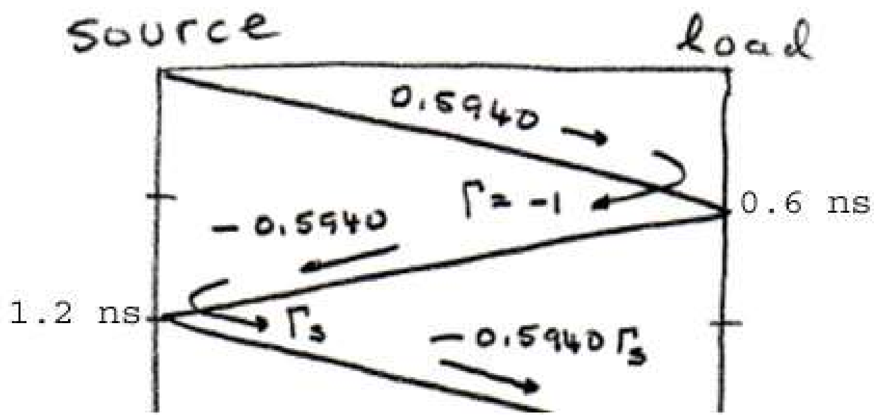
$t = 1.425$ ns the voltage is 0.6465 volts.

$$v(t) = V_{final} + (V_{initial} - V_{final})e^{-\frac{t-T_d}{\tau}}$$

$$T_d = 1.2 \text{ ns}$$

For the red exponential curve: $V_{final} = 0.6688$

What is the initial value of the red exponential curve?



Initially, the capacitor is uncharged, so is a short circuit, and

$$\Gamma_L = -1$$

The incident voltage is

$$V^+ = 0.5940$$

So the reflected voltage at the load is

$$V^- = \Gamma_L V^+ = (-1)0.5940 = -0.5940$$

The reflection coefficient at the source is

$$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{50 - 73.15}{50 + 73.15} = -0.1880$$

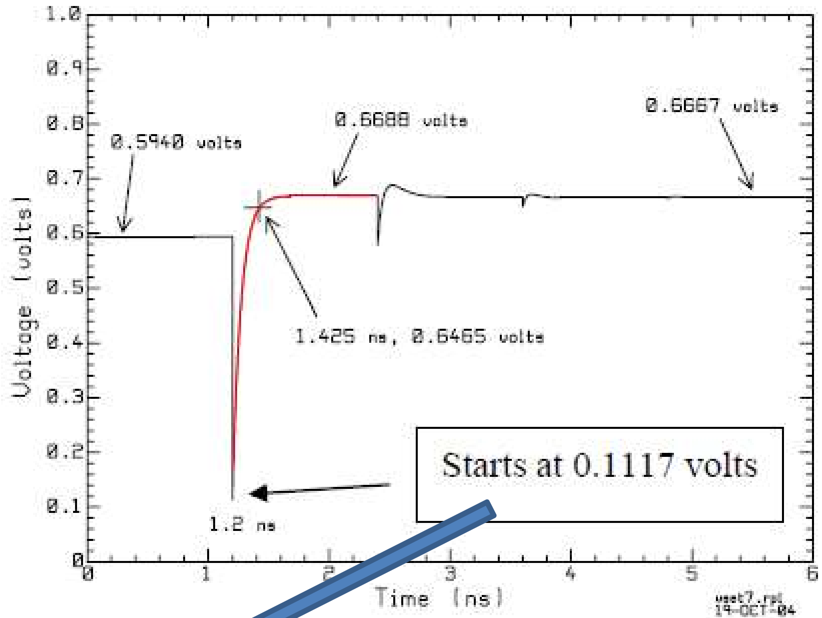
So the re-reflected voltage is

$$V_{re}^+ = \Gamma_s V^- = (-0.1880)(-0.5940) = 0.1117$$

So the source voltage is $V^+ + V^- + V_{re}^+ = 0.5940 - 0.5940 + 0.1117 = 0.1117$

So the initial value of the exponential curve is $V_{initial} = 0.1117$

$$V_{initial} = 0.1117$$



$$v(t) = V_{final} + (V_{initial} - V_{final})e^{-\frac{t-T_d}{\tau}} \qquad T_d = 1.2 \text{ ns}$$

The initial value of the red exponential curve is $V_{initial} = 0.1117$ volts.

For the red exponential curve: $V_{final} = 0.6688$

$$v(t) = 0.6688 + (0.1117 - 0.6688)e^{-\frac{t-1.2}{\tau}}$$

$$v(t) = 0.6688 - 0.5571e^{-\frac{t-1.2}{\tau}}$$

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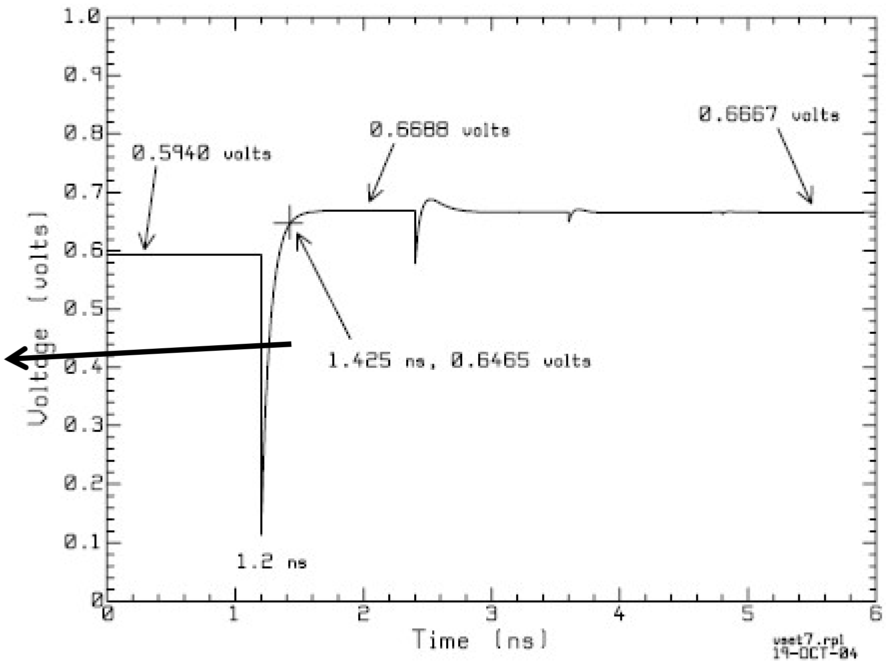


What is the time constant?

$t = 1.425$ ns the voltage is 0.6465 volts.

$$0.6465 = 0.6688 - 0.5571e^{-\frac{1.425-1.2}{\tau}}$$

Solve for τ:

$$0.6465 = 0.6688 - 0.5571e^{-\frac{1.425-1.2}{\tau}}$$

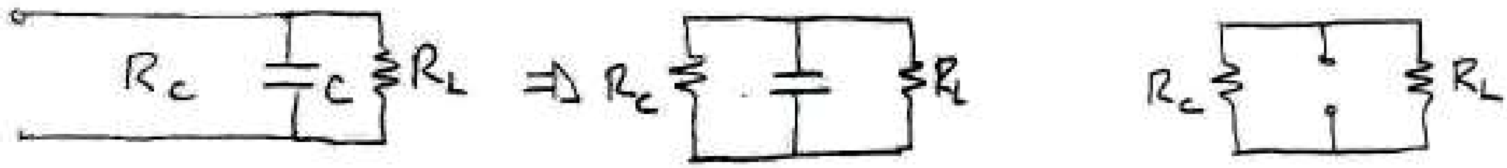
$$0.5571e^{-\frac{0.225}{\tau}} = 0.0223$$

$$e^{-\frac{0.225}{\tau}} = 0.0400$$

$$\frac{-0.225}{\tau} = \ln(0.0400) = -3.218$$

$$\tau = \frac{0.225}{3.218} = 0.0699$$

What is the value of the capacitor?



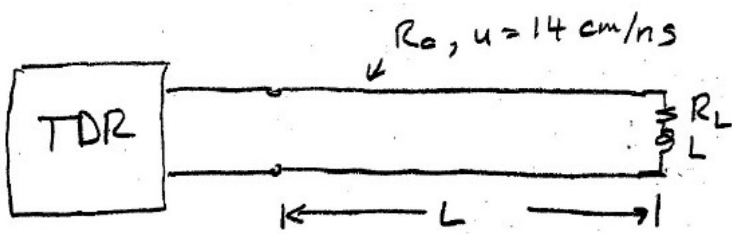
We have already found that $R_c = 73.15$ ohms and $R_L = 100$ ohms.

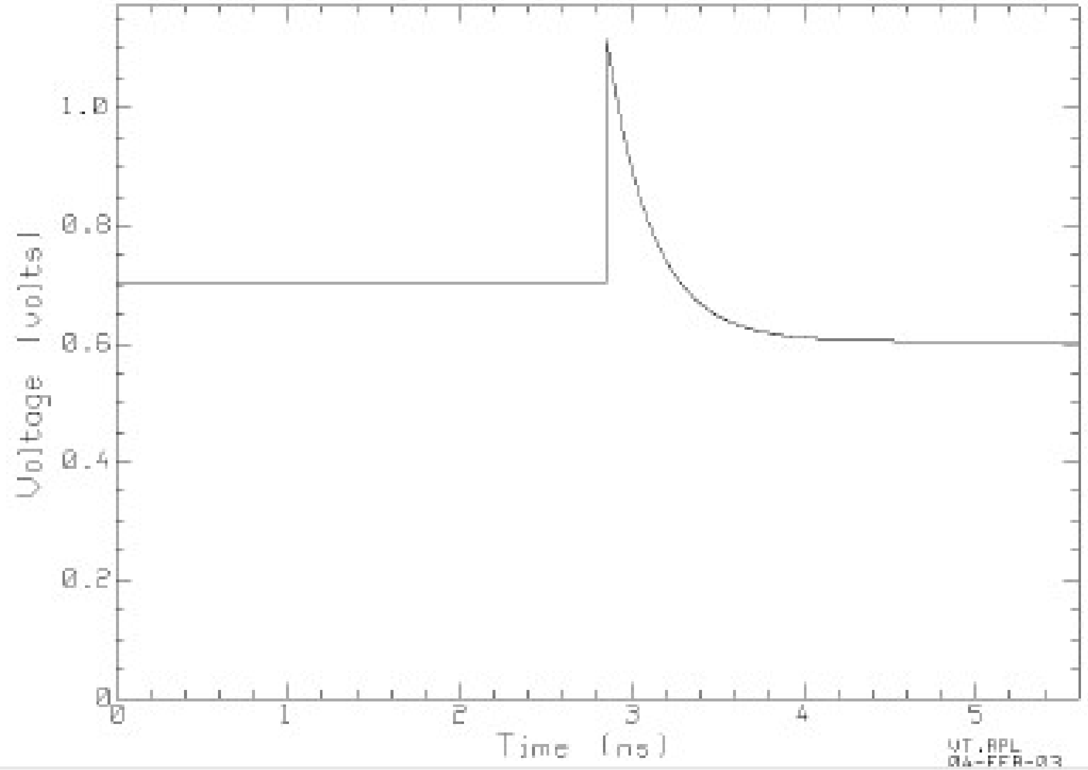
So we expect the time constant to be $\tau = R_p C$ where

$$R_p = R_c \| R_L = \frac{73.15x100}{73.15+100} = 42.25 \text{ ohms}$$

So $C = \dfrac{\tau}{R_p} = \dfrac{0.0699}{42.25} = 1.65 \text{ pF}$

Homework Example





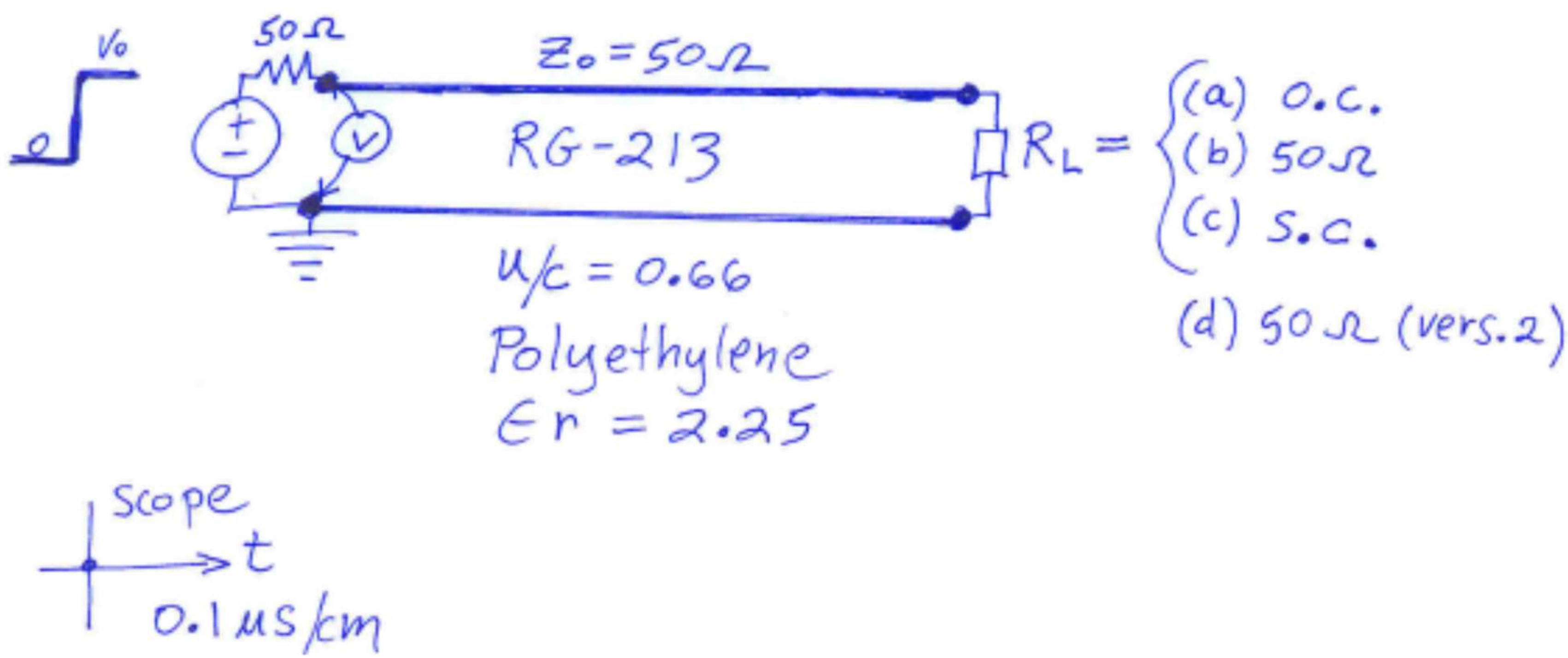
The TDR has an internal resistance of $R_s = 50$ ohms and generates a 1-volt step function into an open-circuit load.

Find:

- The characteristic resistance of the line. (Exact answer: 120 ohms)
- The length of the line. (Exact answer: 20 cm)
- The load resistance R_L . (Exact answer: 73 ohms)
- The time constant. (Exact answer: 0.259 ns)
- The value of the inductance. (Exact answer: 50 nH)

Time Domain Demonstration

Thanks to Dr. Paknys for this demonstration.



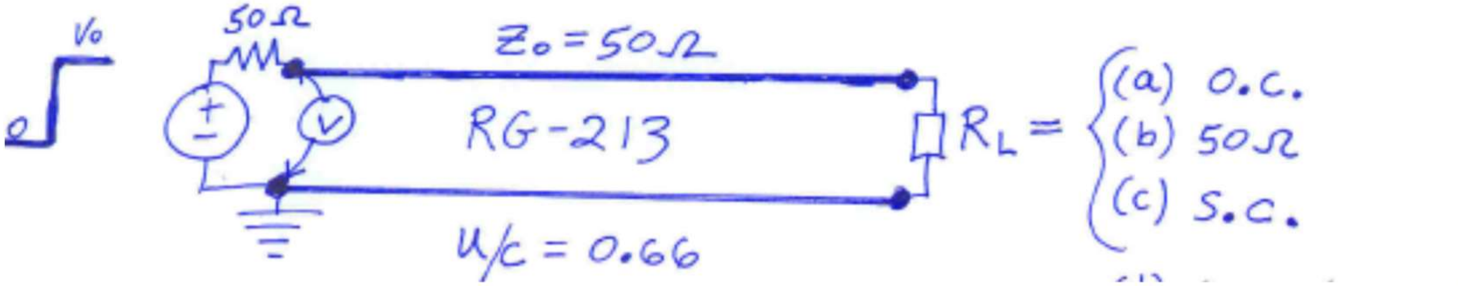
Simple Circuit:

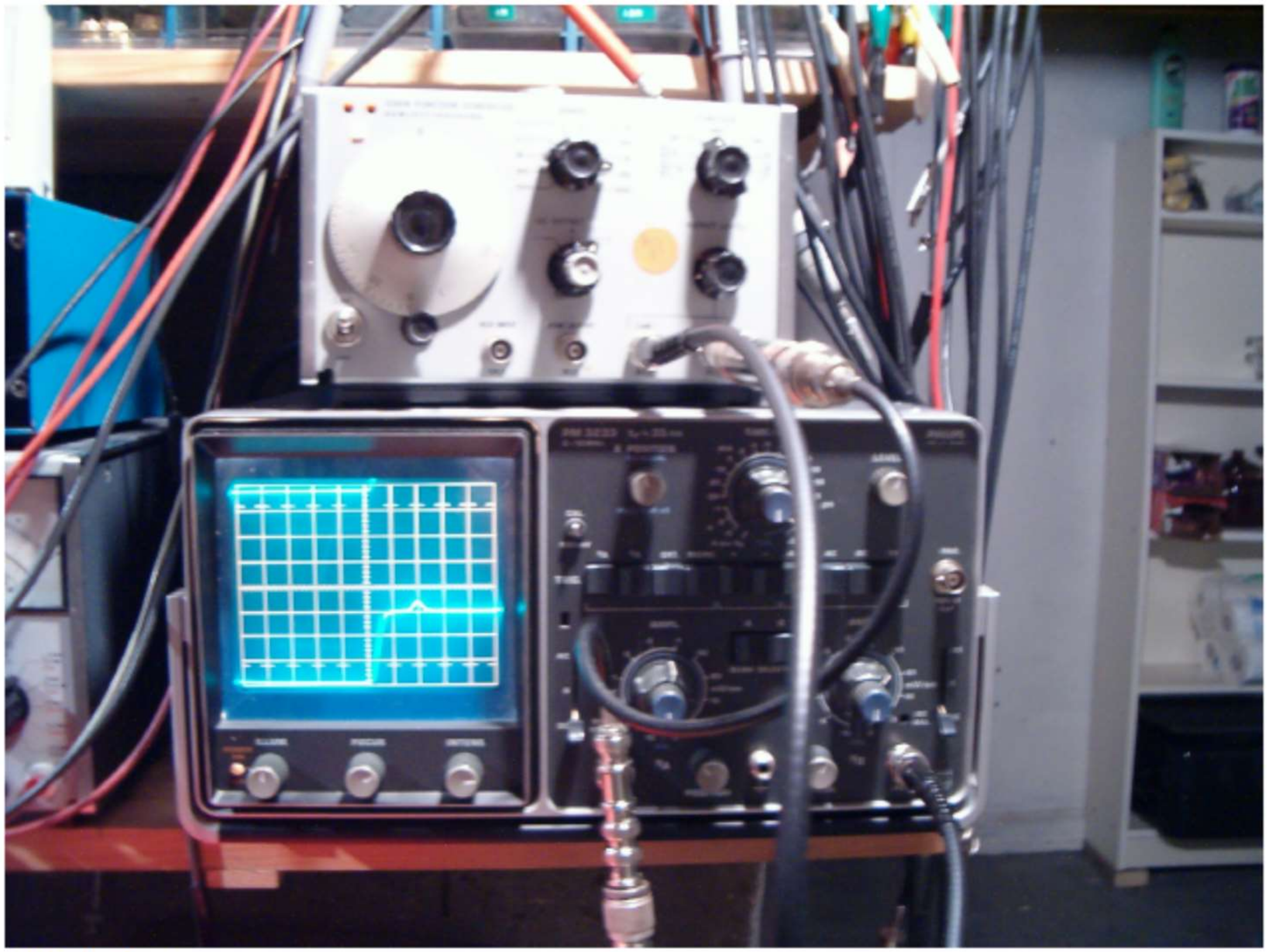
1. 50 ohm generator, step function from 0 volts to V_0 volts.
2. The line is RG-213 coaxial cable with dielectric made of polyethylene with

$\varepsilon_r = 2.25$ so $$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_r\varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = \frac{c}{\sqrt{\varepsilon_r}} = 0.66c$$

3. The characteristic impedance is $Z_0 = 50$ ohms,

4. The length of the cable is not known.
5. Four loads are considered:
   a. Open circuit, $\Gamma = 1$, reflected step has amplitude $+\frac{V_0}{2}$
   b. Matched, $\Gamma = 0$, reflected step has amplitude zero.
   c. Short circuit, $\Gamma = -1$, reflected step has amplitude $-\frac{V_0}{2}$
   d. Matched, but a different device than in part (b)





This is the test setup:

- function generator making a step function
- oscilloscope showing the step function at the line input.

This is the coaxial cable used in the test.
We see Puddles the cat, who supervised the experiment!

Coaxial cable of type RG 213/U

The characteristic impedance is $Z_o = 50\Omega$

The dielectric is polyethylene, with $\varepsilon_r = 2.25$

The speed of propagation is

$$u = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3x10^8}{\sqrt{2.25}} = 2.00x10^8 = 200$$ meters per microsecond

The “source” end of the cable: the function generator is connected to the coaxial cable.

This is how the function generator is connected to the coaxial cable.

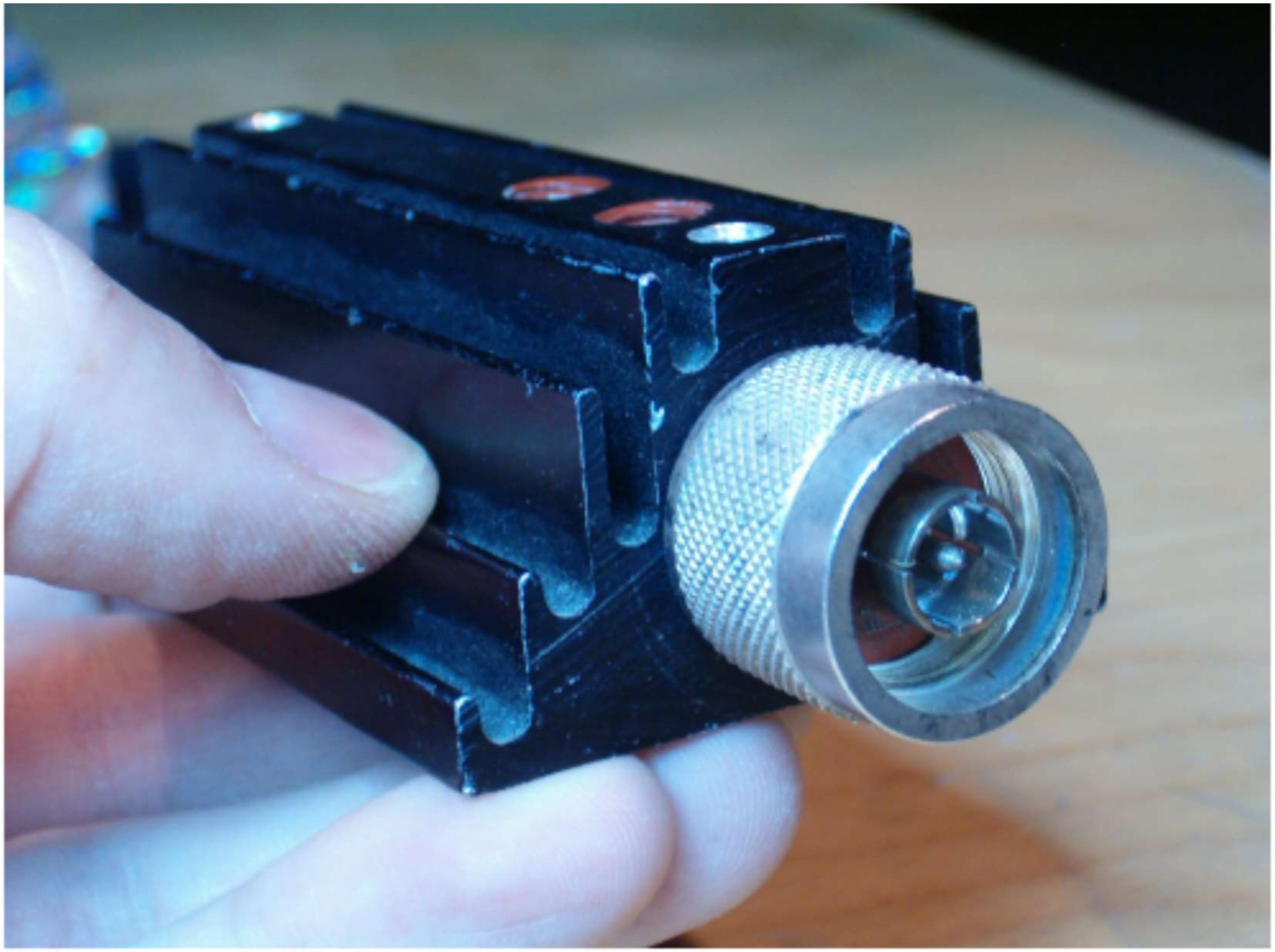
The “load” end of the coaxial cable: a type N connector.

This photo shows the type N connector where the load will be attached.

Loads: matched, short circuit, open circuit.

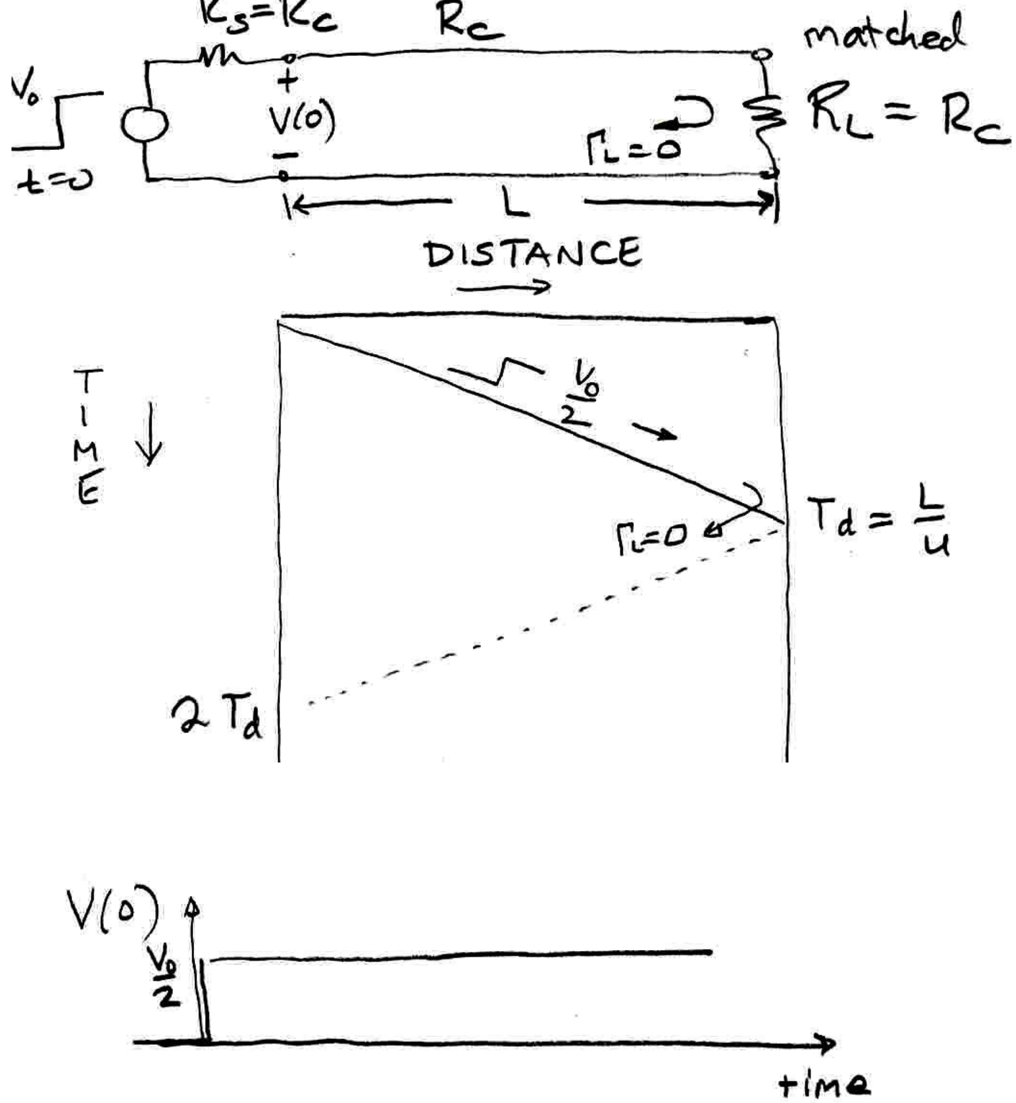
Here is low the load is connected to the end of the cable.
With no load at all we have an approximate open circuit.
The device in the center is a short circuit load, for type N connectors.
The device at left is a 'matched" load. The fins are used to dissipated the heat that is delivered by the cable to the matched load.

High-power matched load.



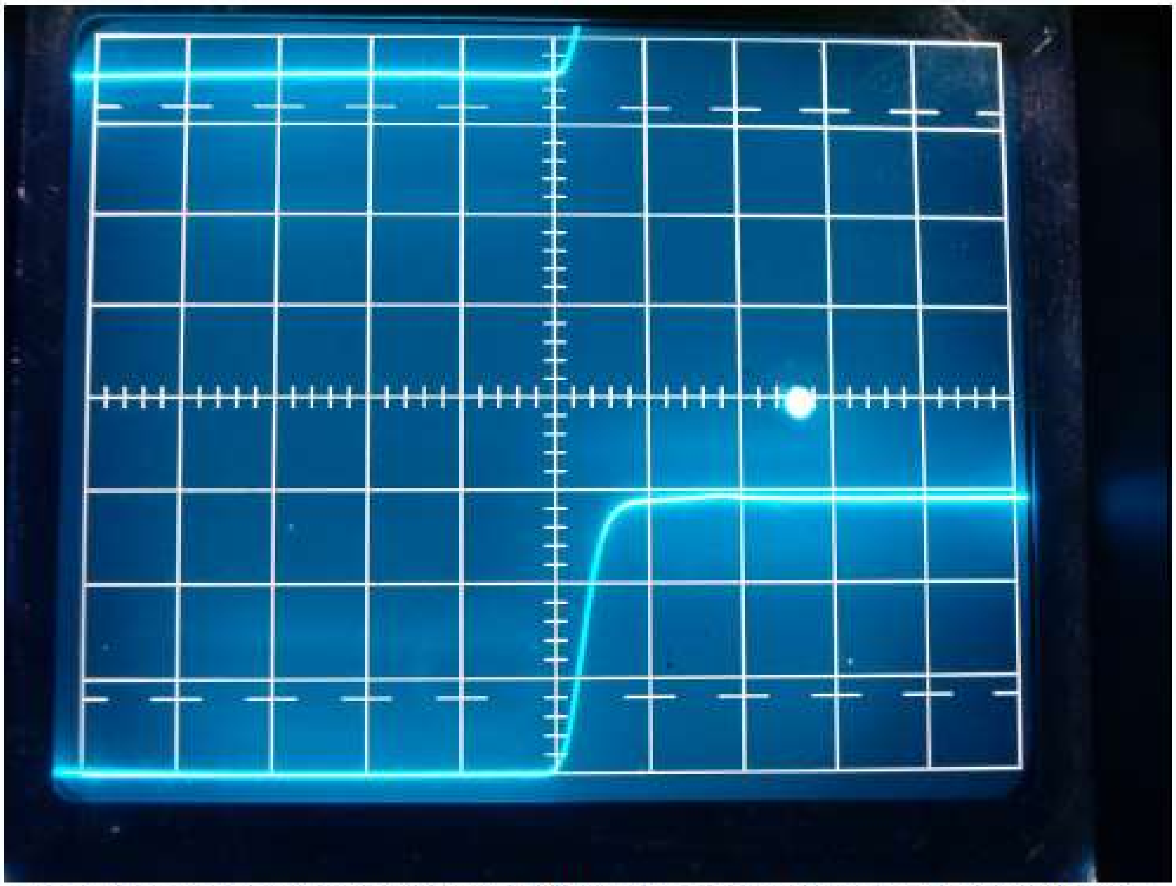
This is the matched load, with its type-N connector.

Matched Load



Matched Load

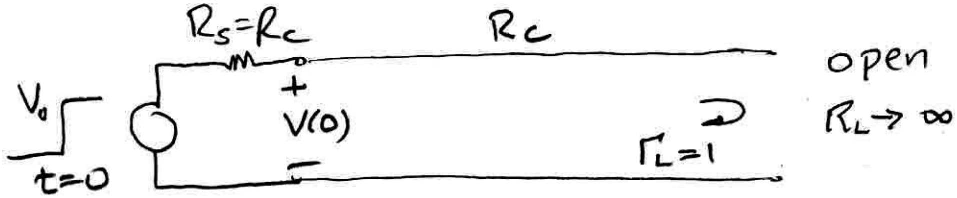
Voltage at the cable input as a function of time.

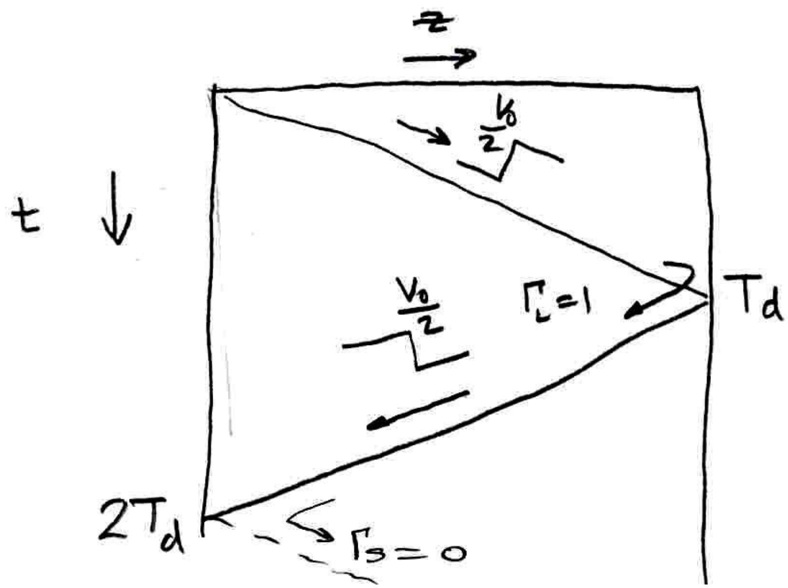


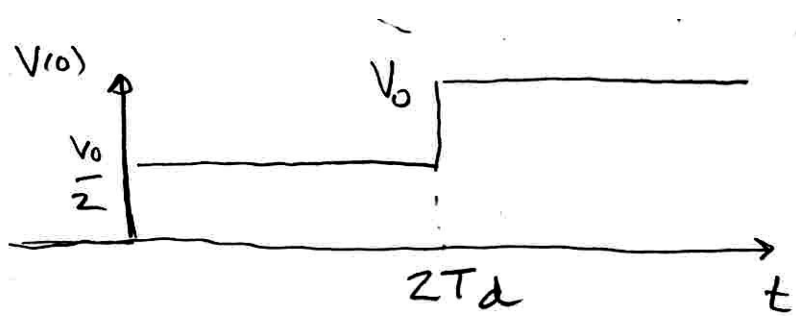
This is the voltage at the input to the cable with a matched load. There is (almost) no reflection from the matched load.

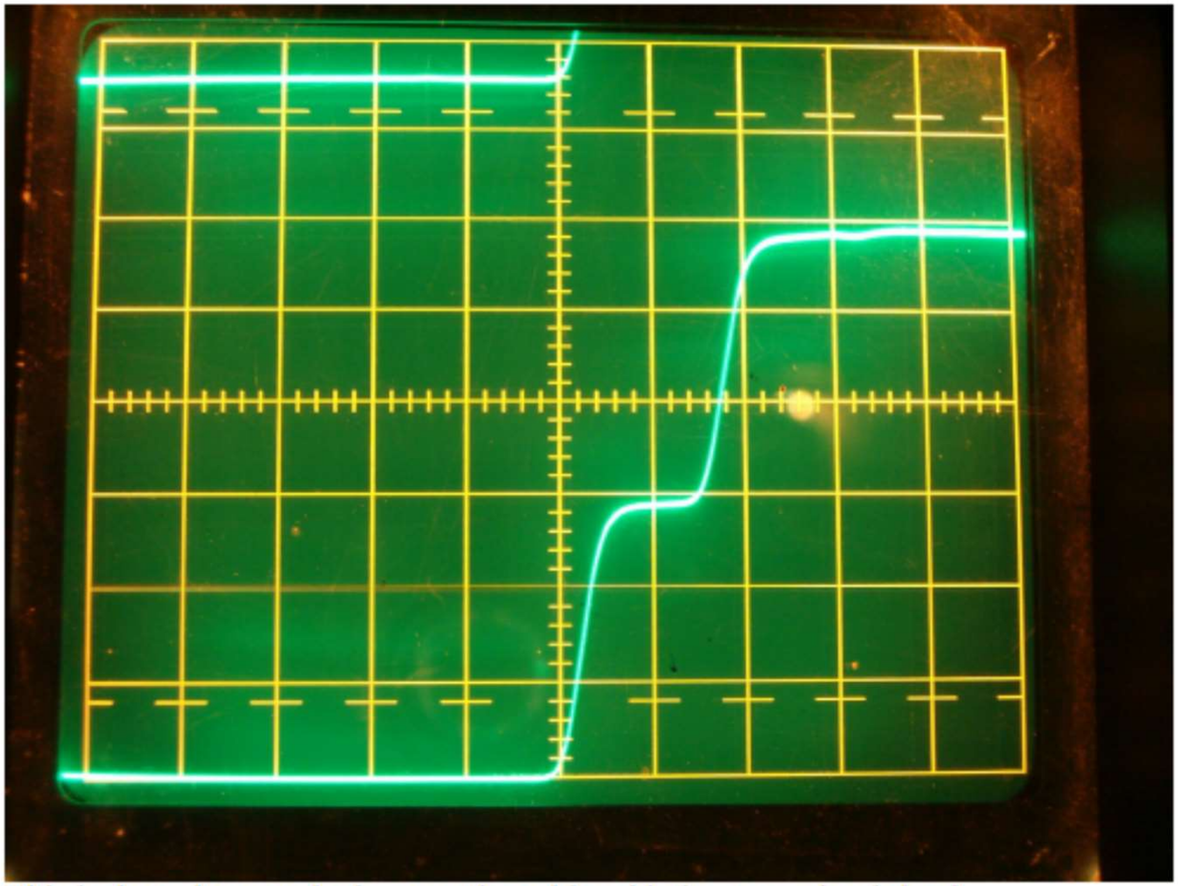
Open Circuit Load

Open Circuit Load









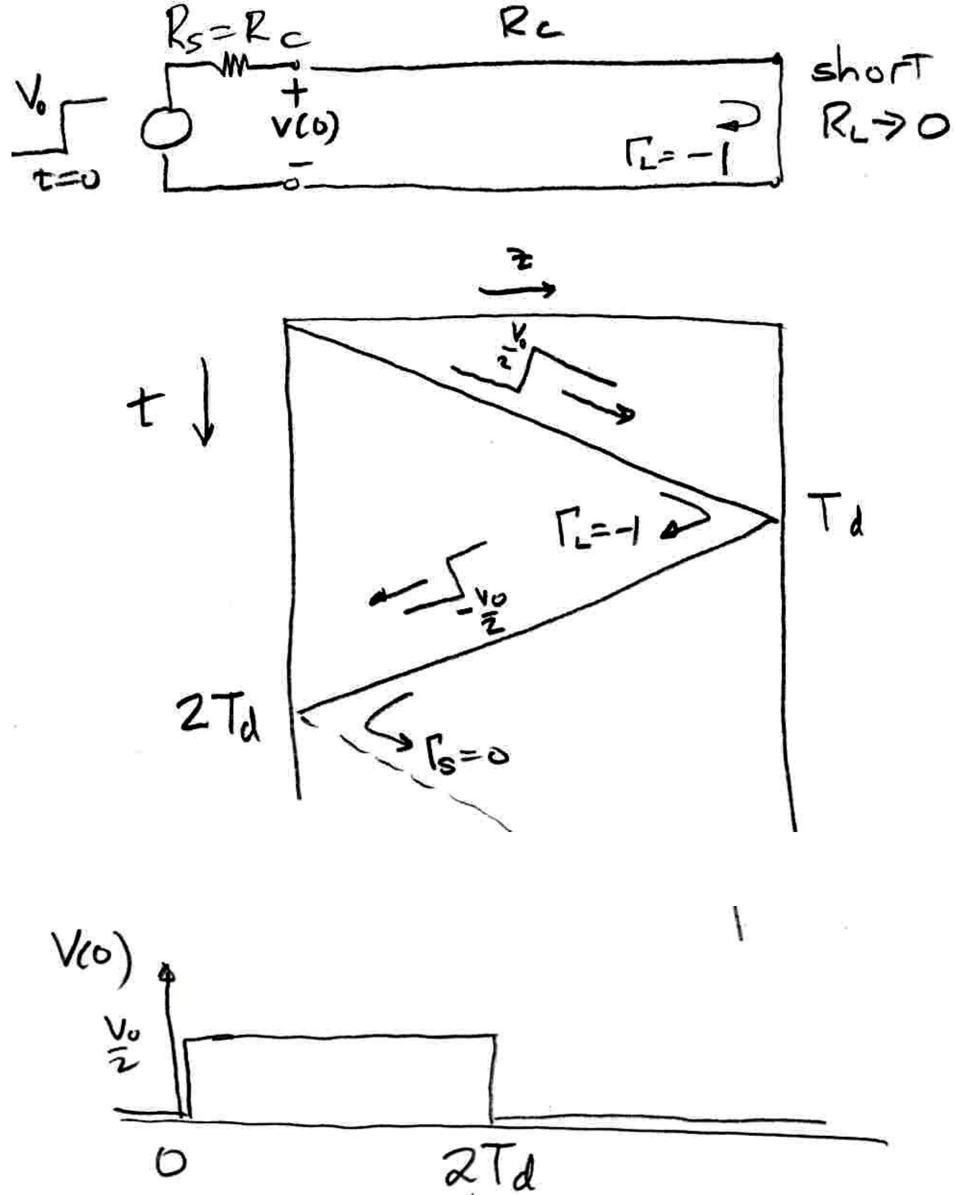
This is the voltage at the input to the cable with the open-circuit load.
We see:

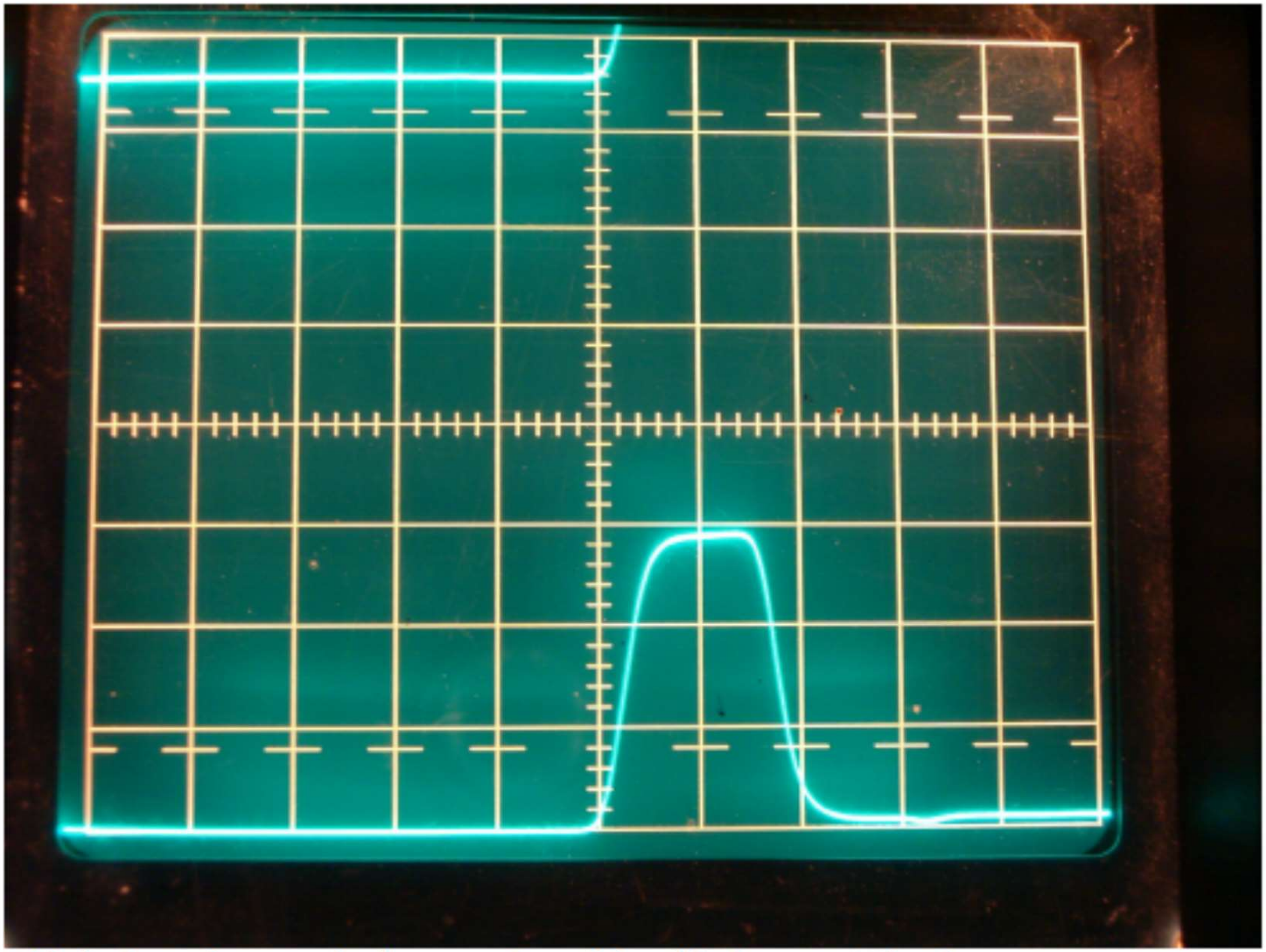
1. the initial step applied to the input of the cable
2. the reflected step from the end of the cable, with $\Gamma = +1$

Short-Circuit Load

This is the type N short circuit load.

Short Circuit Load





This is the voltage at the input to the cable with the type N load.
We see the initial step applied to the cable.
Then the reflected step arrives, with $\Gamma = -1$ and "cancels" the initial step.

Puddles the cat approved of these measurements!