ELEC353 Lecture Notes Set 17

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #10: Do homework #10 by March 29, 2019.
Homework #11: Do homework #11 by April 5, 2019.
Homework #12: Do homework #12 by April 12, 2019.

Tutorial Workshop #11: Friday March 29, 2019.
Tutorial Workshop #12: Friday April 5, 2019.
Tutorial Workshop #13: Friday April 12, 2019

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

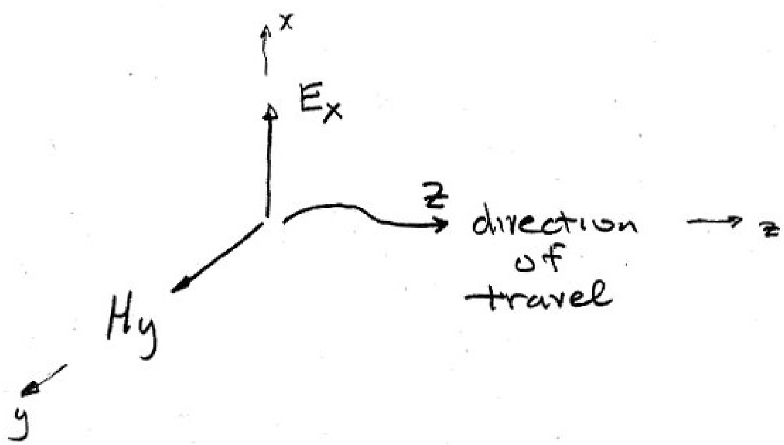
Topics to be Covered

Plane Waves

- Maxwell’s Equations and the Wave Equation - done
- Plane waves - done
- Material Boundaries - today
- Transmission Through a Wall

Antennas

Review: Plane Waves in Lossless Media



$$\overline{E} = \hat{a}_x E_x(z)$$

$$\nabla^2 \overline{E} = -\beta^2 \overline{E}$$

$$\frac{d^2 E_x}{dz^2} = -\beta^2 E_x$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$H_y(z) = \frac{1}{\eta} E_x^+ e^{-j\beta z} - \frac{1}{\eta} E_x^- e^{j\beta z}$$

Review: Plane Waves in Lossless Media

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{j\omega\mu(j\omega\varepsilon)} = j\omega\sqrt{\mu\varepsilon} = 0 + j\beta$$

- $\alpha = \text{Re}(\gamma)=0$ in lossless materials
- $\beta = \text{Im}(\gamma) = \omega\sqrt{\mu\varepsilon}$ in lossless materials

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \text{ ohms}$$

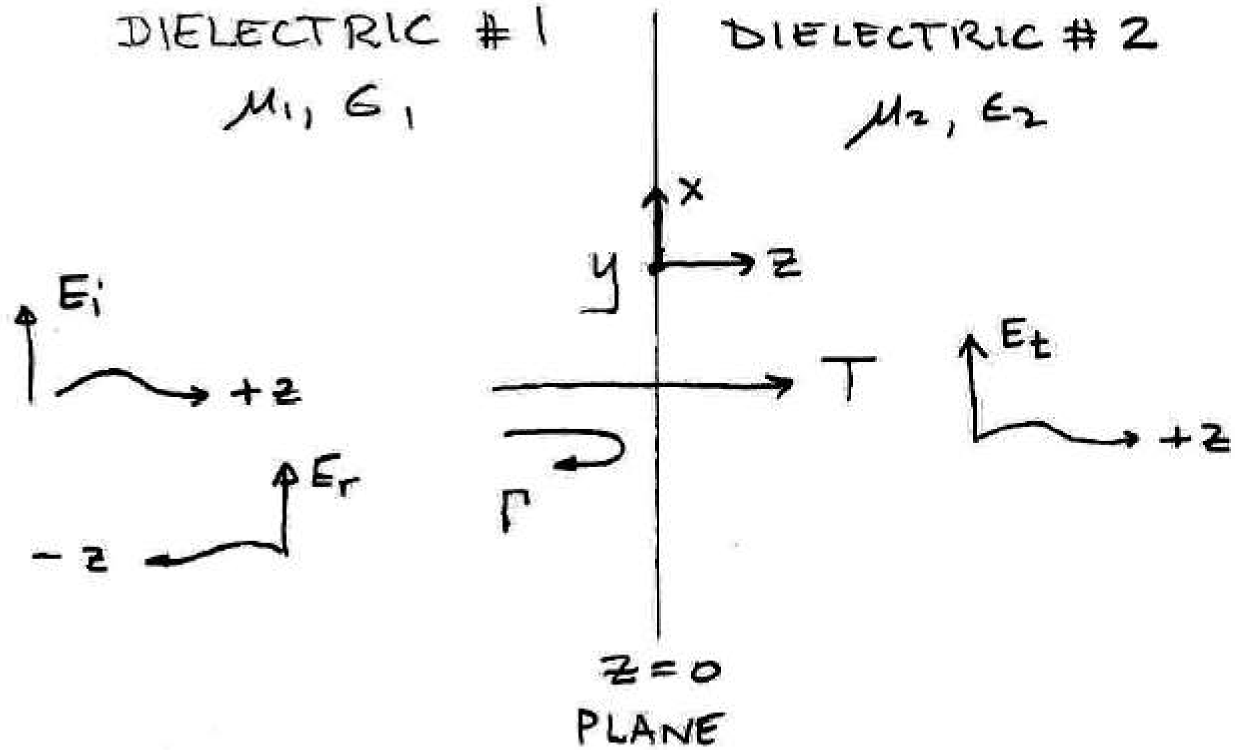
- For free space, $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.734 \approx 377$ ohms

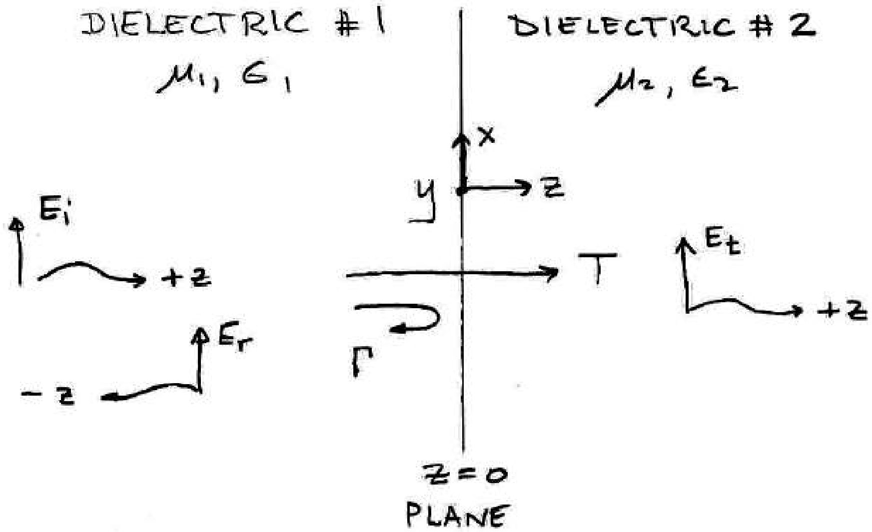
Speed of travel $u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$ m/s

- For free space, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 2.997929458x10^8 \approx 3x10^8$ m/s

Wavelength $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{1/\sqrt{\mu\varepsilon}}{\omega/(2\pi)} = \frac{u}{f}$ meters

Material Boundaries





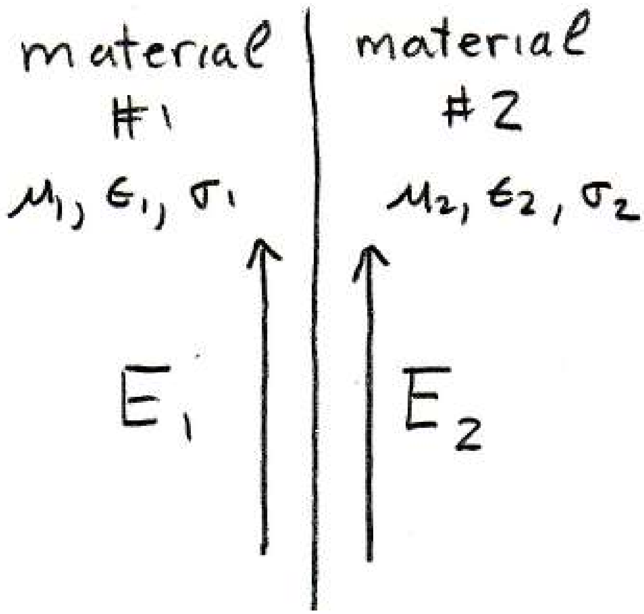
Reflection of Plane Waves from a Dielectric Half Space

(Inan and Inan page 692 and 698)

Transmission Line Analogy

Boundary Conditions

How are the fields in material #1 very close to the boundary related to the fields in material #2 very close to the boundary?



Electric Fields

Inan and Inan page 361

Faraday's Law:

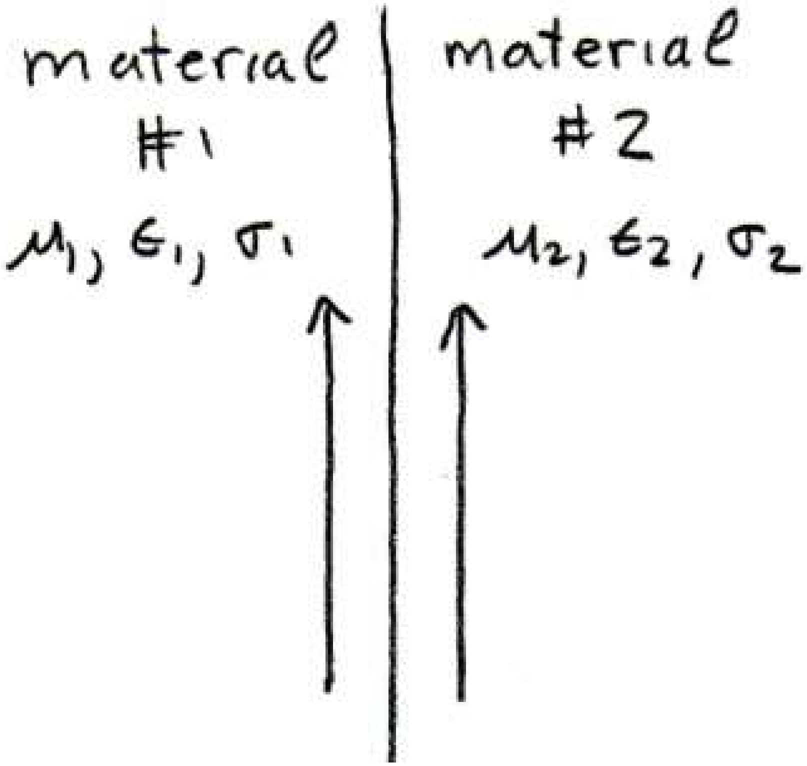
$$\oint \bar{E} \cdot \overline{dl} = -j\omega\mu \iint \bar{H} \cdot \overline{ds}$$

Magnetic Fields

Inan and Inan page 535

Ampere's Law

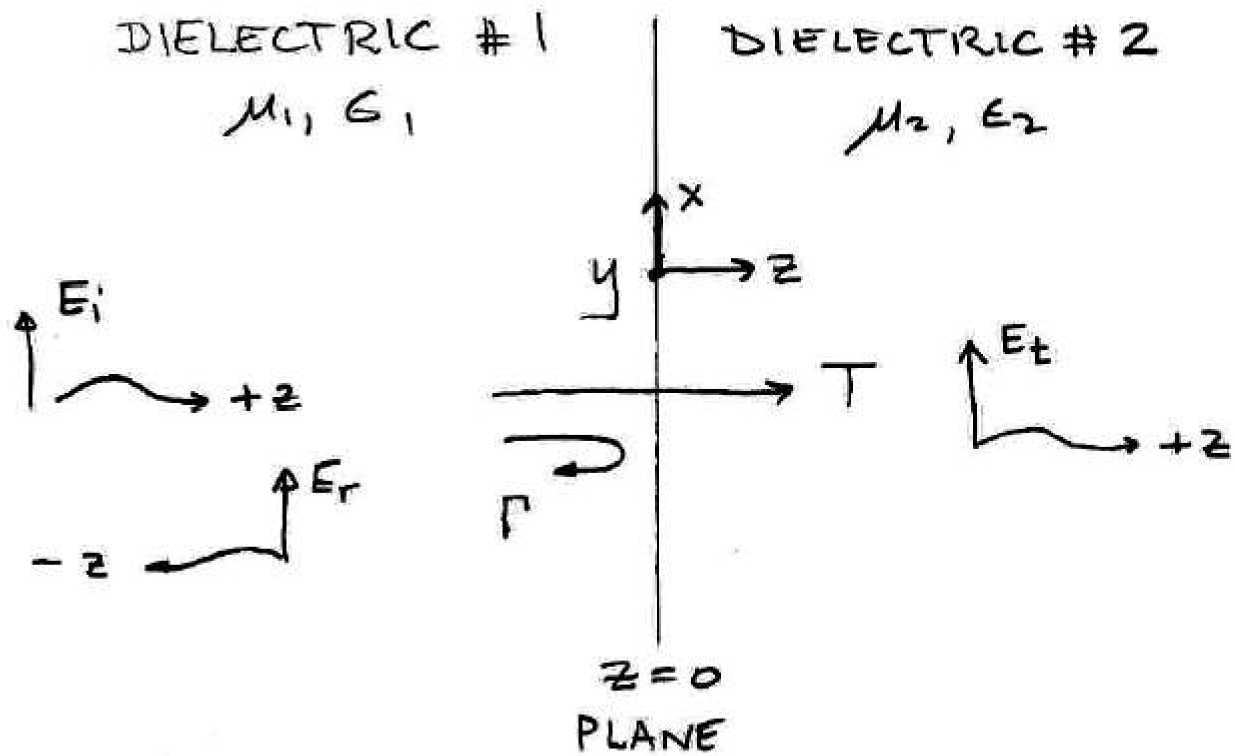
$$\oint \overline{H} \cdot \overline{dl} = \iint \overline{J} \cdot \overline{ds}$$

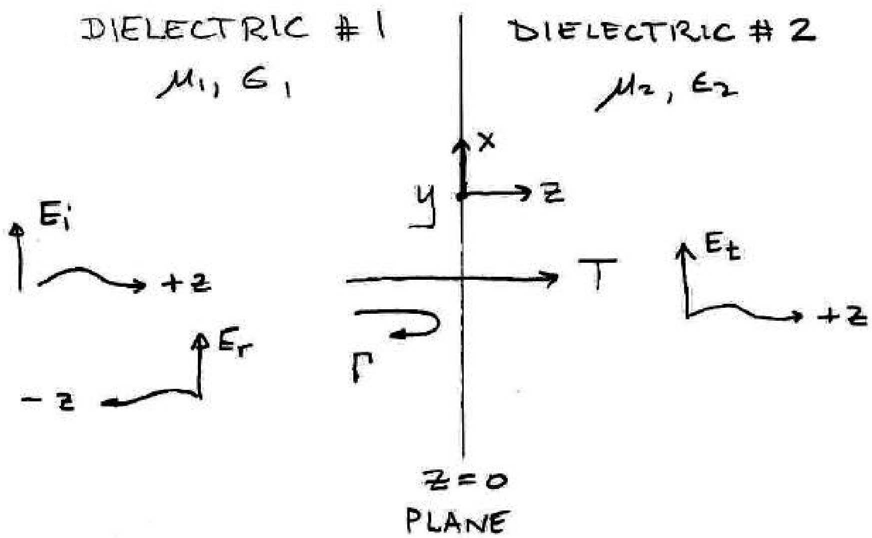


Material Boundaries

How are the fields on side #1 of the boundary related to the fields on side #2 of the boundary?

•Tangential component of E is continuous across the boundary
•Tangential component of H is continuous across the boundary

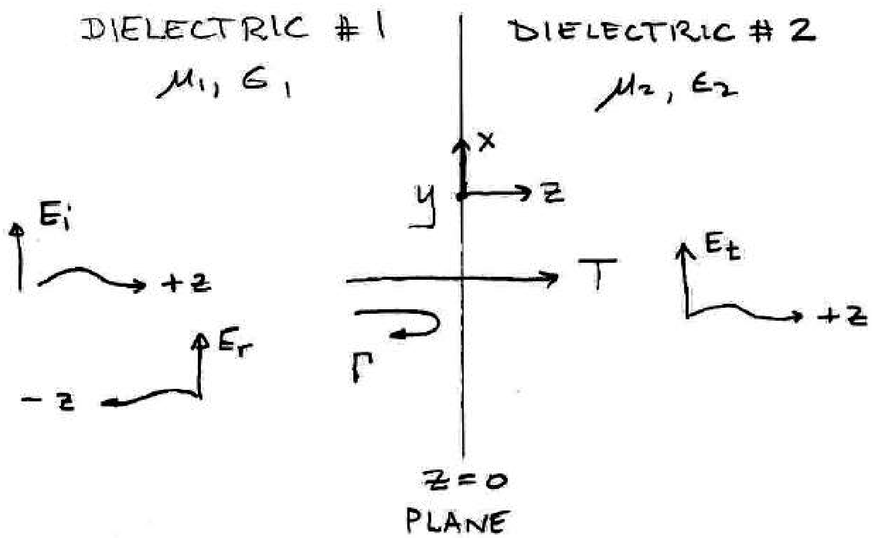




$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad\qquad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\overline{E}_1 = \text{incident} + \text{reflected} = \hat{a}_x E_i e^{-j\beta_1 z} + \hat{a}_x E_r e^{j\beta_1 z} \qquad\qquad \overline{E}_2 = \hat{a}_x E_t e^{-j\beta_2 z}$$

$$\overline{H}_1 = \hat{a}_y \frac{1}{\eta_1} E_i e^{-j\beta_1 z} - \hat{a}_y \frac{1}{\eta_1} E_r e^{j\beta_1 z} \qquad\qquad \overline{H}_2 = \hat{a}_y \frac{1}{\eta_2} E_t e^{-j\beta_2 z}$$



$$E_{1x} = E_i e^{-j\beta_1 z} + E_r e^{j\beta_1 z}$$

$$E_{2x} = E_t e^{-j\beta_2 z}$$

$$H_{1y} = \frac{1}{\eta_1} E_i e^{-j\beta_1 z} - \frac{1}{\eta_1} E_r e^{j\beta_1 z}$$

$$H_{2y} = \frac{1}{\eta_2} E_t e^{-j\beta_2 z}$$

What are the fields very near the interface between the two dielectrics?
At *z*=0:

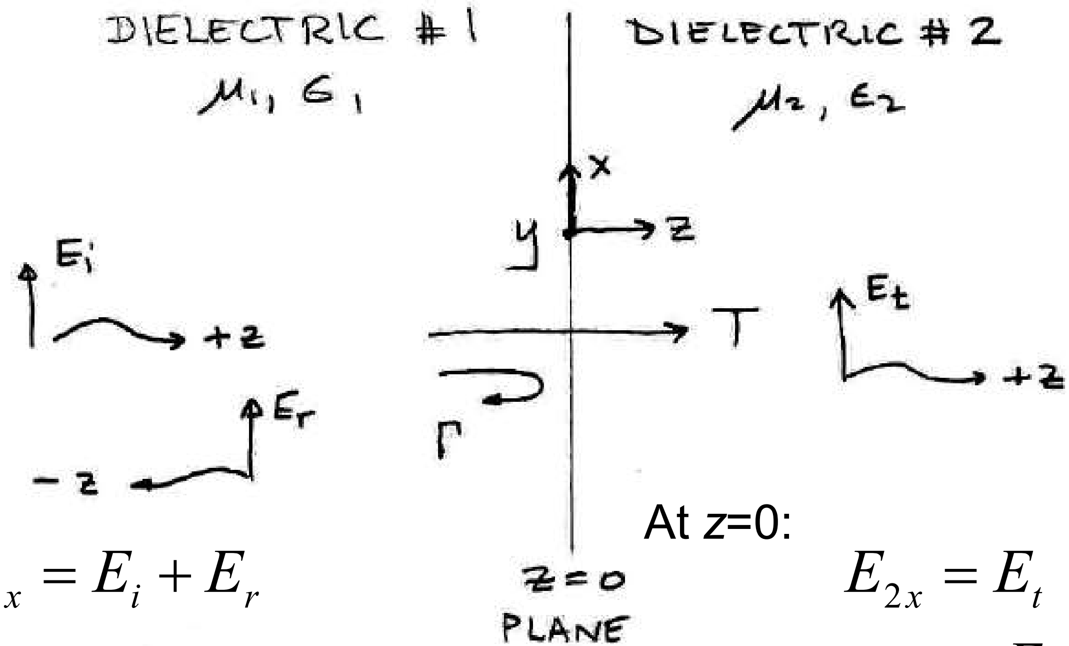
$$E_{1x} = E_i + E_r$$

$$E_{2x} = E_t$$

$$H_{1y} = \frac{1}{\eta_1} E_i - \frac{1}{\eta_1} E_r$$

$$H_{2y} = \frac{1}{\eta_2} E_t$$

Enforce the boundary conditions:



At z=0:

$$E_{1x} = E_i + E_r$$

$$H_{1y} = \frac{E_i}{\eta_1} - \frac{E_i}{\eta_1}$$

At z=0:

$$E_{2x} = E_t$$

$$H_{2y} = \frac{E_r}{\eta_2}$$

Etan must be continuous at z=0:

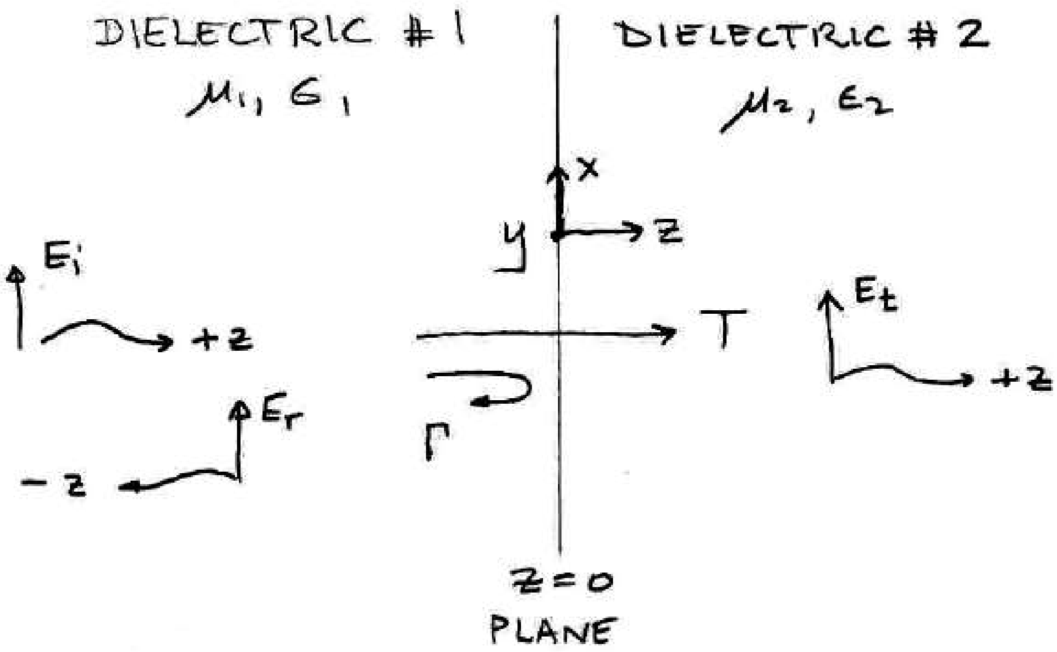
$$E_{1x} = E_{2x}$$

$$E_i + E_r = E_t$$

Htan must be continuous at z=0:

$$H_{1y} = H_{2y}$$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$



Find the Reflected Field and the Transmitted Field:

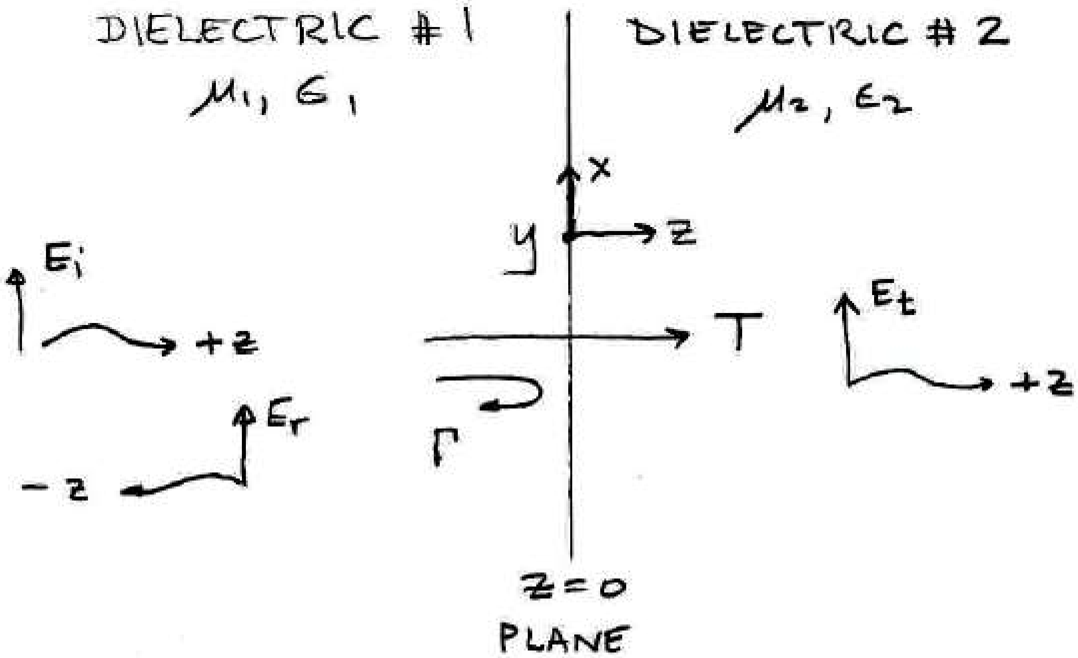
$$E_i + E_r = E_t \qquad \frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

We can solve these equations to learn that

$E_r = \Gamma E_i$ where the reflection coefficient is $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$E_t = TE_i$ where the transmission coefficient is $T = \dfrac{2\eta_2}{\eta_1 + \eta_2}$

Example



A plane wave in air has amplitude $E_i = 10$ V/m at 1.9 GHz. It is normally incident on the surface of a half-space filled with concrete, with relative permittivity $\varepsilon_r = 6.11$ and conductivity $\sigma = 153$ mS/m. Find:

1) The intrinsic impedance of air and the concrete.
2) The transmission coefficient T.
3) The complex amplitude of the transmitted field, $E_t = TE_i$
4) The propagation constant in the concrete.
5) The attenuation constant in the concrete.
6) The distance the transmitted wave must travel before its amplitude is reduced to 1 mV/m.

Solution

Remarks:

- If the problem does not mention the permeability, then assume the materials are "non-magnetic" with $\mu_r = 1$.
- "Normally incident" means that the direction of travel of the plane wave is perpendicular to the surface of the dielectric half space.

1) For air, $\eta_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 376.73$ ohms.

For the concrete:

- Evaluate the radian frequency: $\omega = 2\pi f = 2\pi \cdot 1900x10^6 = 1.1938x10^{10}$ rad/sec.
- Evaluate the intrinsic impedance:

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\eta_2 = \sqrt{\frac{j \cdot 1.1938x10^{10} \cdot 4\pi x10^{-7}}{153x10^{-3} + j \cdot 1.1938x10^{10} \cdot 6.11 \cdot 8.854x10^{-12}}}$$

$$\eta_2 = 149.33 + j17.44 \text{ ohms}$$

$\eta_0 = 376.73 \qquad \eta_2 = 149.33 + j17.44$

2) Evaluate the transmission coefficient:

$$T = \frac{2\eta_2}{\eta_2 + \eta_0} = \frac{2(149.33 + j17.44)}{149.33 + j17.44 + 376.73} = 0.5693 + j0.04749$$

3) Evaluate the transmitted field amplitude:

$$E_t = TE_i = 10\text{x}(0.5693 + j0.04749) = 5.693 + j0.4749 \quad \text{V/m}$$

So the amplitude of the transmitted wave is

$$|E_t| = |5.693 + j0.4749| = 5.713 \text{ V/m}$$

4) Evaluate the propagation constant:

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\gamma = \sqrt{(j \cdot 1.1938x10^{10} \cdot 4\pi x10^{-7})(153x10^{-3} + j \cdot 1.1938x10^{10} \cdot 6.11 \cdot 8.854x10^{-12})}$$

$$\gamma = 11.58 + j99.11$$

5) Evaluate the attenuation constant:

$$\alpha = \text{Re}(\gamma) = 11.58 \text{ (note penetration depth} = \frac{1}{\alpha} = 8.64 \text{ cm)}$$

6) Find the distance the transmitted wave must travel before its amplitude is reduced to 1 mV/m.

- The transmitted wave is

$$E_2(z) = E_t e^{-\gamma z} = E_t e^{-\alpha z} e^{-j\beta z}$$

- So the amplitude of the transmitted wave is

$$\left|E_2(z)\right| = \left|E_t\right| e^{-\alpha z}$$

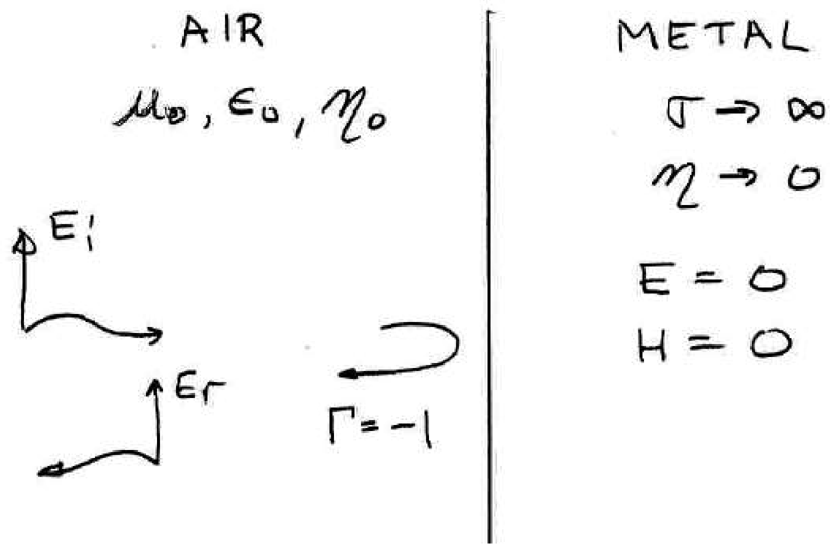
where $\left|E_t\right| = \left|5.693 + j0.4749\right|$ =5.713 V/m

- To find the required distance, set $\left|E_2(z)\right| = 0.001$ V/m and solve

$$0.001 = 5.713 e^{-11.58z}$$

$$z = \frac{1}{11.58}\ln\left(\frac{5.71}{0.001}\right) = 74.7 \text{ cm}$$

Special Case: Air to Metal Interface



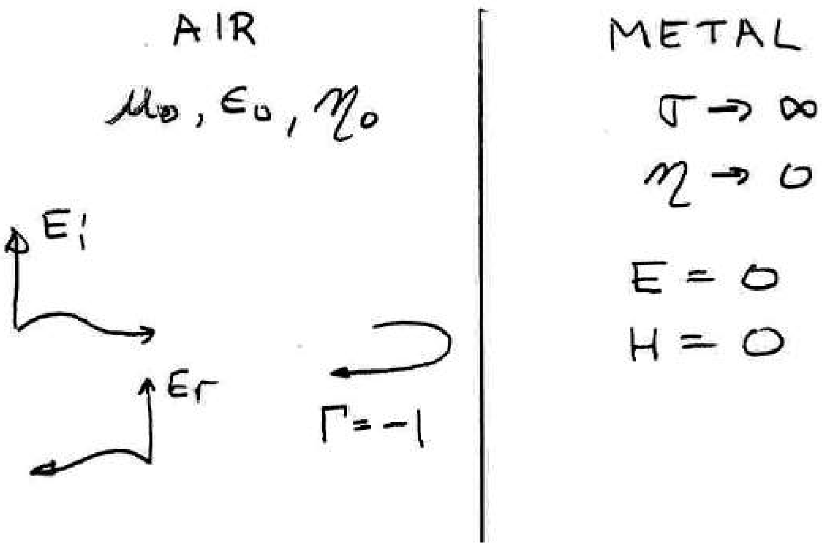
- Suppose material #1 is air and material #2 is a sheet of metal, which we will consider to be a perfect conductor, $\sigma \to \infty$.
- The intrinsic impedance of the perfect conductor is

$$\eta_2 = \lim_{\sigma \to \infty} \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = 0$$

- So the reflection coefficient is

$$\Gamma = \lim_{\eta_2 \to 0} \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) = -1$$

- So for a perfect conductor we have perfect reflection.
- The reflected field is equal and opposite to the incident field, $E_r = -E_i$.



$$\Gamma = \lim_{\eta_2 \to 0} \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) = -1$$

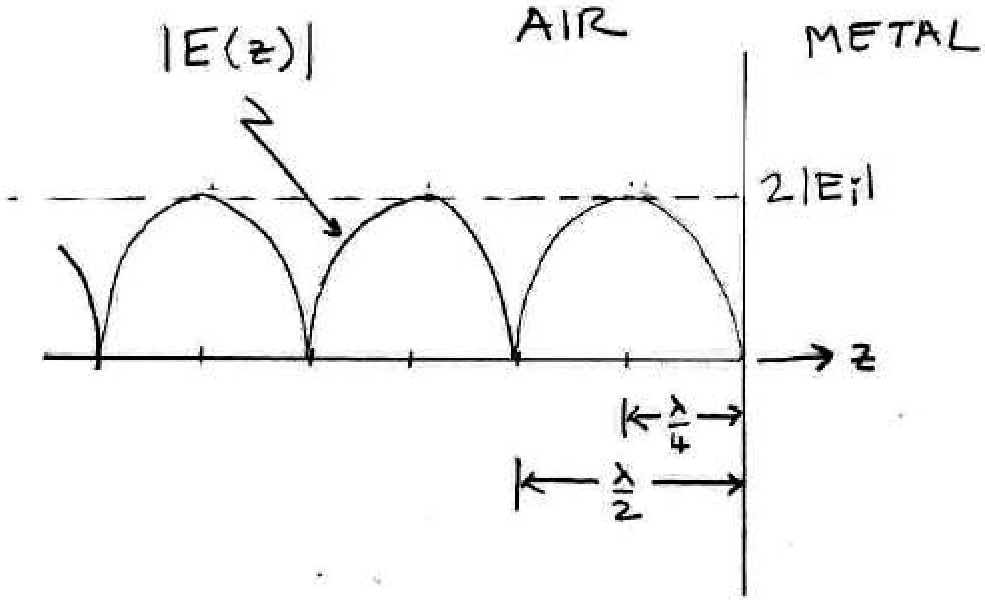
- We can write the field in the air as
$$E_{1x} = E_i \, (e^{-j\beta z} - e^{j\beta z})$$
- This will give rise to a "standing-wave pattern" similar to those we found on transmission lines.
- There will be a minimum at the surface of the metal, $z = 0$, where the field is equal to zero:
$$E_{1x}(z = 0) = E_i \, (e^{-j\beta 0} - e^{j\beta 0}) = 0$$
- Where else will there be minima and maxima?

$$E_{1x} = E_i \,(e^{-j\beta z} - e^{j\beta z})$$

$$E_{1x} = E_i \,(e^{-j\beta z} - e^{j\beta z}) = E_i\big((\cos\beta z - j\sin\beta z) - (\cos\beta z + j\sin\beta z)\big)$$

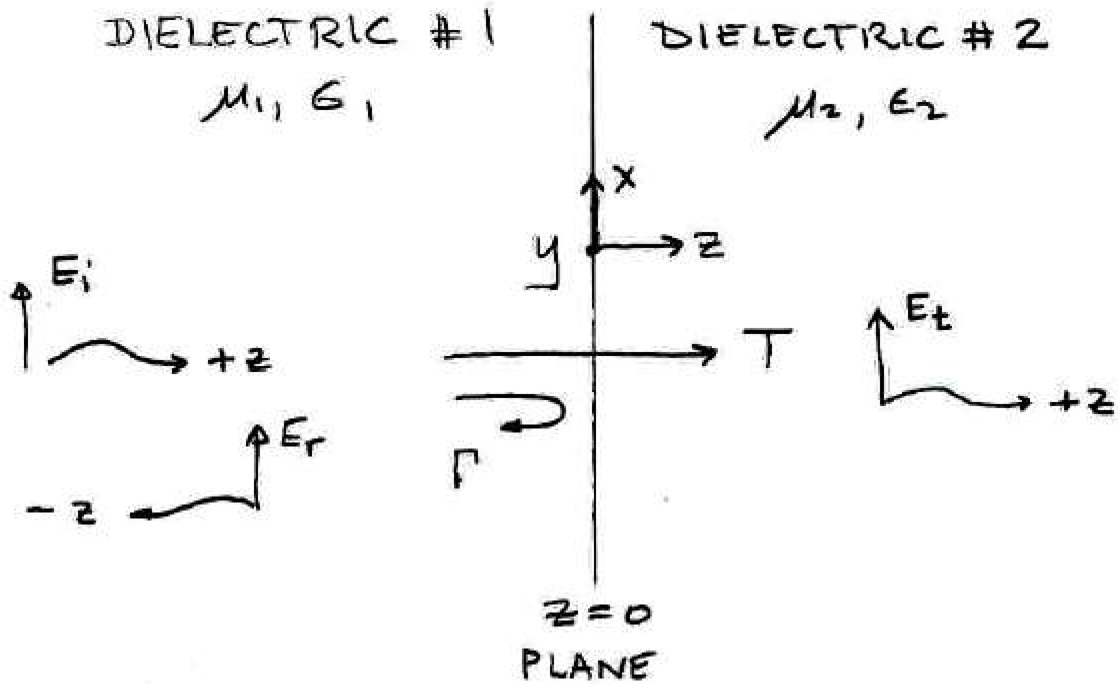
$$E_{1x} = -2jE_i \sin\beta z$$

$$|E_{1x}| = 2|E_i||\sin\beta z|$$



- The standing-wave pattern has a null at the surface of the metal ($z = 0$) and at half-wavelength intervals from the surface.
- It has a maximum a quarter-wavelength from the surface, and at half-wavelength intervals.

Standing-Wave Patterns



- For transmission line circuits we investigated "standing waves".
- It is useful to look into standing waves for fields, as well.
- The electric field in material #1 is

$$E_{1x} = E_i e^{-j\beta_1 z} + E_r e^{j\beta_1 z}$$

$$E_{1x} = E_i e^{-j\beta_1 z} + E_r e^{j\beta_1 z}$$

- At some locations z , the incident field $E_i e^{-j\beta_1 z}$ is in phase with the reflected field $E_r e^{j\beta_1 z}$ and then the incident field and the reflected field add up, and the field strength is $|E_i| + |E_r|$.
- At other locations, the incident field is 180 degrees out of phase with the reflected field, and the waves subtract, and so the field strength is $\left||E_i| - |E_r|\right|$.

$$SWR = \frac{|E_i| + |E_r|}{\left||E_i| - |E_r|\right|}$$

$$E_r = \Gamma E_i \qquad SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Position of the Maxima

$$E_{1x} = E_i e^{-j\beta_1 z} + \Gamma E_i e^{j\beta_1 z}$$

$$\Gamma = |\Gamma| e^{j\phi}$$

$$E_{1x} = E_i e^{-j\beta_1 z}(1 + |\Gamma| e^{j\phi} e^{j2\beta_1 z})$$

$$\phi + 2\beta_1 z = \pm 2n\pi \text{ for } n = 0,1,2,...$$

$$\beta_1 = \frac{2\pi}{\lambda_1}$$

$$z_{\max} = -\frac{\phi}{2\pi}\frac{\lambda}{2} \pm n\frac{\lambda}{2}$$

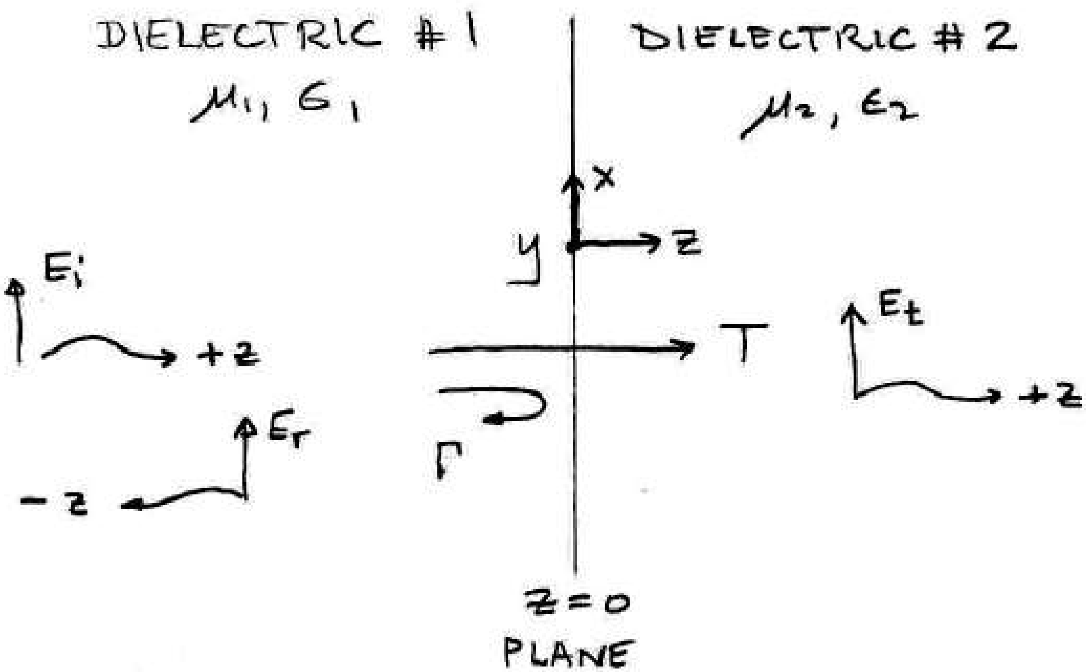
Position of the Minima

$$\phi + 2\beta_1 z = \pm 2n\pi \pm \pi = \pm(2n+1)\pi \text{ for } n = 0,1,2,...$$

$$z_{\min} = -\frac{\phi}{2\pi}\frac{\lambda}{2} \pm (2n+1)\frac{\lambda}{4}$$

Distance between the minima and the maxima:

$$z_{\max} - z_{\min} = \pm\frac{\lambda}{4}$$



Example

A plane wave in air at 2450 MHz is incident on the surface of an infinite half-space of brick material, with $\varepsilon_r = 5.1$ and zero conductivity. The amplitude of the incident electric field is 10 volts/meter.

(i) What are the reflection coefficient and the transmission coefficient?

(ii) What is the amplitude of the field transmitted into the brick material?

(iii) What is the amplitude of the reflected field?

(iv) What is the maximum value of the standing wave in the air, and how far is the maximum from the surface of the brick?

(v) What is the standing-wave ratio?

Solution

- The intrinsic impedance of the air is $\eta_0 = 376.7$ ohms.
- Evaluate the intrinsic impedance of the brick:

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{376.7}{\sqrt{\varepsilon_r}} = \frac{376.7}{\sqrt{5.1}} = 166.8 \text{ ohms}$$

- You can run WAVES to find the value as 166.82 ohms, which agrees with our "theoretical" calculation. (Use F6 in WAVES).
- Evaluate the reflection coefficient:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_2 - \eta_0}{\eta_2 + \eta_0} = \frac{166.8 - 376.7}{166.8 + 376.7} = -0.3862$$

- The WAVES program reports the value as -0.3862, exactly the same.
- Evaluate the transmission coefficient:

$$T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\eta_2}{\eta_0 + \eta_2} = \frac{2x166.8}{166.8 + 376.7} = 0.6138$$

- WAVES reports the value as 0.6138, exactly the same.

- The transmitted field is $E_t = TE_i$ where $E_i = 10$ V/m so

$$E_t = TE_i = 0.6138x10 = 6.138 \text{ V/m}$$

- The reflected field is $E_r = \Gamma E_i$ where $E_i = 10$ V/m so

$$E_r = \Gamma E_i = -0.3862x10 = -3.862 \text{ V/m}$$

- The field in the air is the sum of the incident field plus the reflected field:

$$E = 10e^{-j\beta z} - 3.862e^{j\beta z}$$

- We can write this as

$$E = 10e^{-j\beta z} + 3.862e^{j\pi}e^{j\beta z}$$

or

$$E = 10e^{-j\beta z} + 3.862e^{j(\beta z+\pi)}$$

Maximum value:

$$E = 10 + 3.862 = 13.862$$

Minimum value:

$$E = 10 - 3.862 = 6.138$$

- The standing-wave ratio is

$$\text{SWR}=\frac{E_{\max}}{E_{\min}} = \frac{13.862}{10-3.862} = \frac{13.862}{6.138} = 2.26$$

Find the location of the maximum value:

$$E = 10e^{-j\beta z} + 3.862e^{j(\beta z + \pi)}$$

- There is a maximum in the standing-wave pattern when the incident wave is in phase with the reflected wave.
- The waves are in phase when the difference between their phase angles is zero:

$$-\beta z - (\beta z + \pi) = 0$$

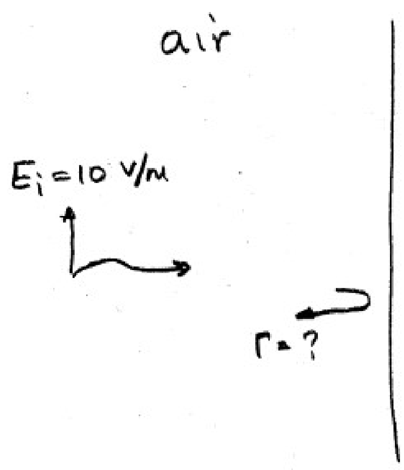
$$-2\beta z - \pi = 0$$

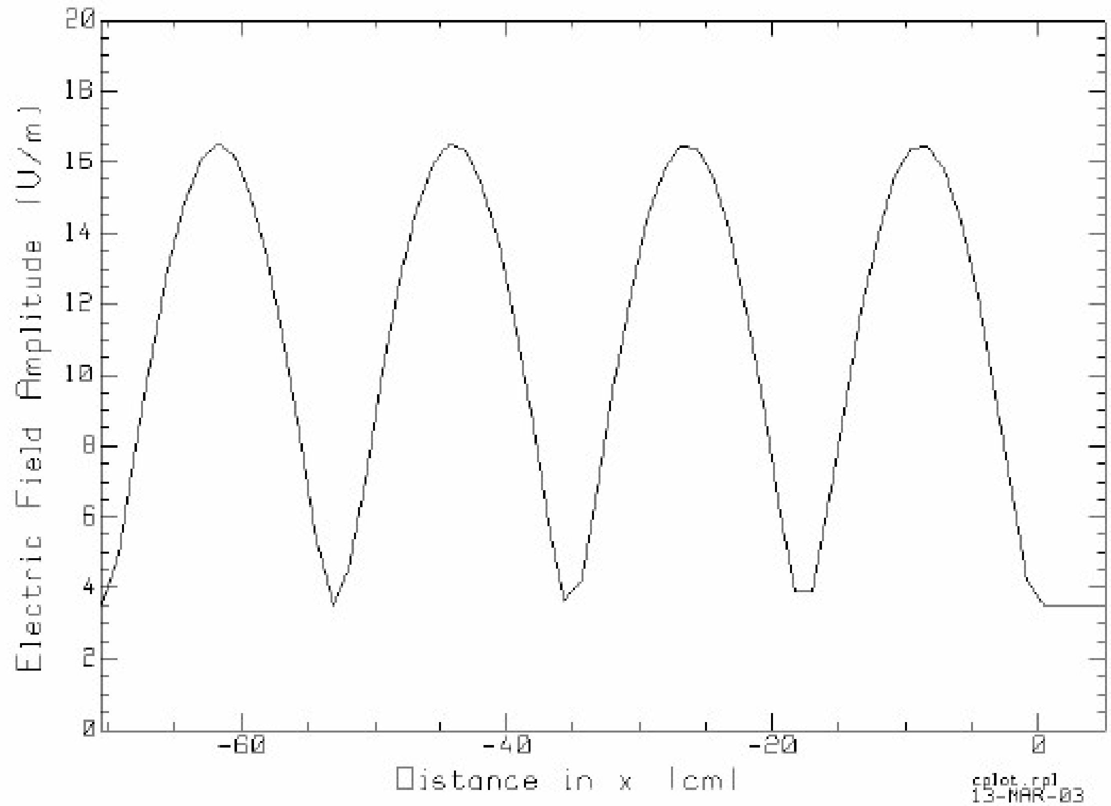
$$z = \frac{-\pi}{2\beta} = \frac{-\pi}{2\frac{2\pi}{\lambda}} = -\frac{\lambda}{4}$$

- So the field is a maximum at $z = -\lambda/4$, and at half-wavelength intervals from this location.

Example

Use a Standing-Wave Pattern to Find the Permittivity of Medium #2





A plane wave in air at 850 MHz of amplitude 10 V/m is "normally" incident on the surface of a dielectric of unknown permittivity. We would like to find the permittivity.

An engineer uses a field-strength meter to measure the standing wave pattern as shown above, where the surface of the dielectric is at x=0 cm. There is a minimum in the standing wave pattern at x=0 of value 3.67 V/m. The maximum electric field strength is 16.51 V/m. Find the permittivity of the dielectric.

Solution

- The standing-wave ratio is

$$SWR = \frac{16.51}{3.67} = 4.50$$

- Since $SWR = \dfrac{1+|\Gamma|}{1-|\Gamma|}$, we can solve for $|\Gamma|$ to find

$$|\Gamma| = \frac{SWR-1}{SWR+1} = \frac{4.50-1}{4.50+1} = 0.636$$ where $\Gamma = |\Gamma|\angle\phi = 0.636\angle\phi$

- What is the value of ϕ, the angle of the reflection coefficient?

- Since material #2 is lossless, we have $\sigma = 0$ and $\eta_2 = \sqrt{\dfrac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} = \sqrt{\dfrac{\mu_2}{\varepsilon_2}}$

  is real.

- Also, since $\varepsilon_2 = \varepsilon_r\varepsilon_0$ we have $\eta_2 = \sqrt{\dfrac{\mu_2}{\varepsilon_2}} = \dfrac{1}{\sqrt{\varepsilon_r}}\sqrt{\dfrac{\mu_0}{\varepsilon_0}} = \dfrac{\eta_0}{\sqrt{\varepsilon_r}}$

- Since $\varepsilon_r > 1$, we have $\eta_2 = \dfrac{\eta_0}{\sqrt{\varepsilon_r}} < \eta_0$
- So $\Gamma = \dfrac{\eta_2 - \eta_0}{\eta_2 + \eta_0} < 1$, meaning that Γ is negative so the angle of $\Gamma = |\Gamma| e^{j\phi}$ must be $\phi = \pi$.
- Since the magnitude of Γ is $|\Gamma| = 0.636$ and the angle of Γ is $\phi = \pi$, we have $\Gamma = 0.636 e^{j\pi} = -0.636$
- Since $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, we have

$\eta_1 = \eta_0$

$$\eta_2 = \eta_0 \frac{1+\Gamma}{1-\Gamma} = 376.7 \frac{1-0.636}{1+0.636} = 83.81 \text{ ohms}$$

- Finally, $\eta_2 = \dfrac{\eta_0}{\sqrt{\varepsilon_r}}$, so $\varepsilon_r = \dfrac{\eta_0^2}{\eta_2^2} = \left(\dfrac{376.7}{83.81}\right)^2 = 20.2$