ELEC353 Lecture Notes Set 14

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

Homework #8: Do homework #8 by March 15, 2019.
Homework #9: Do homework #9 by March 22, 2019.
Homework #10: Do homework #10 by March 29, 2019.

Tutorial Workshop #9: Friday March 15, 2019.
Tutorial Workshop #10: Friday March 22, 2019.
Tutorial Workshop #11: Friday March 29, 2019.

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

Topics to be Covered

Transmission Lines (TLs)

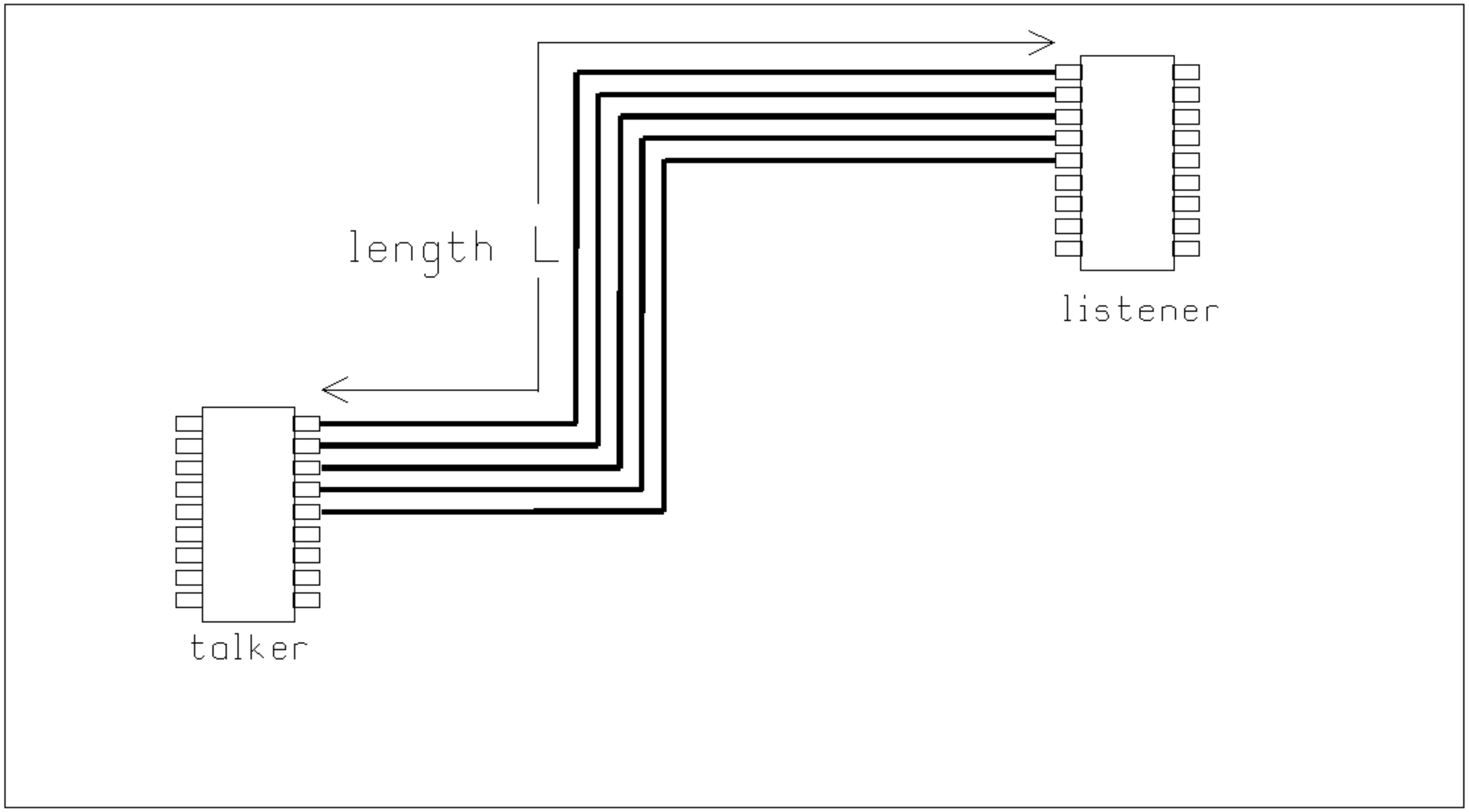
- Wave Equation and Solution - done
- Solving a TL Circuit - done
- Standing Wave Patterns - done
- Impedance Matching – done
- Bandwidth of Digital Signal - today's class

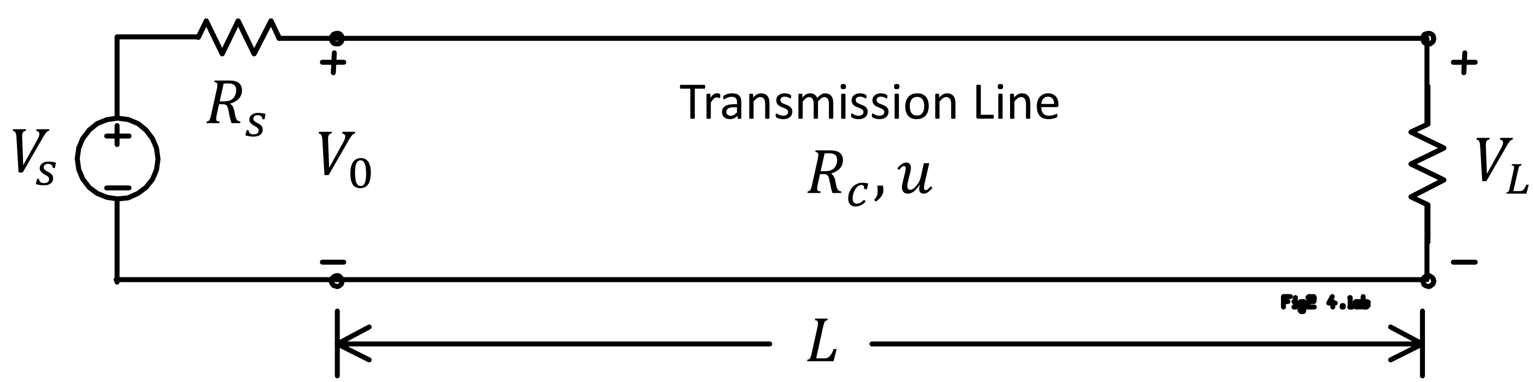
Plane Waves

- Maxwell's Equations and the Wave Equation
- Plane waves
- Material Boundaries
- Transmission Through a Wall

Antennas

Equivalent Circuit for an Interconnection

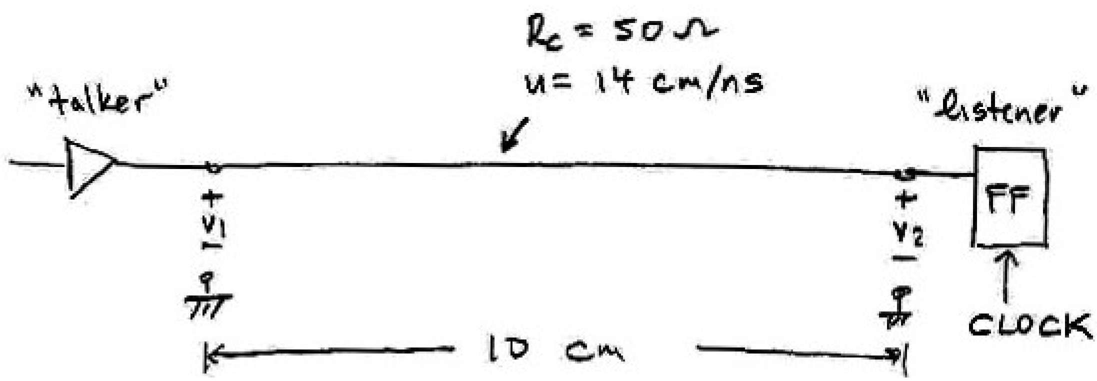


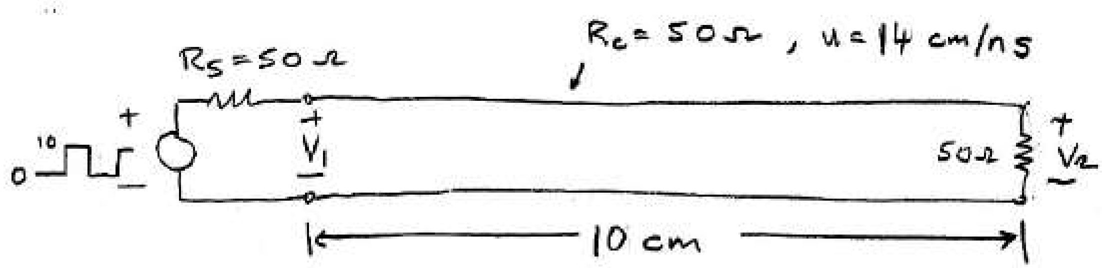


Review:

An Ideal Transmission Path

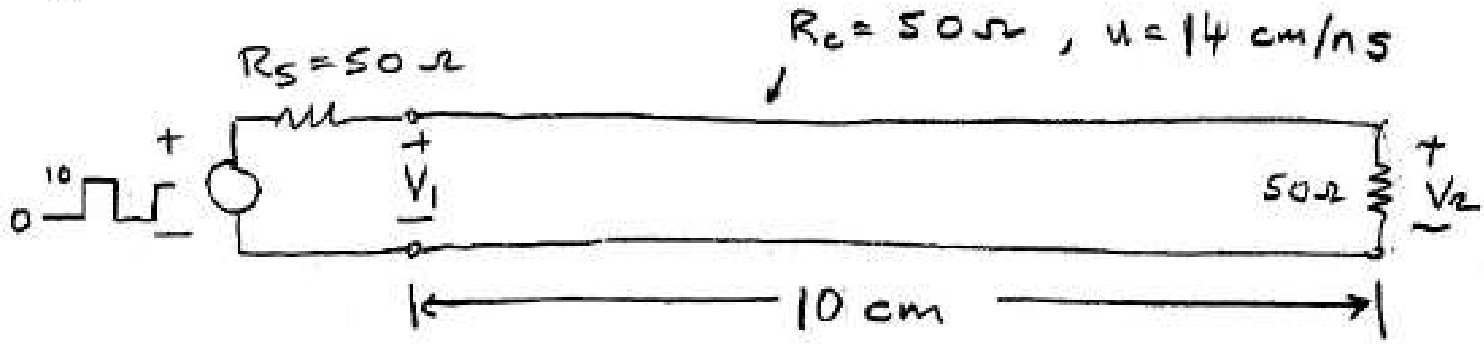
- No mismatch
- No reflections
- The only difference between the "input" and the "output" is the propagation delay of $T_d = \frac{L}{u}$.



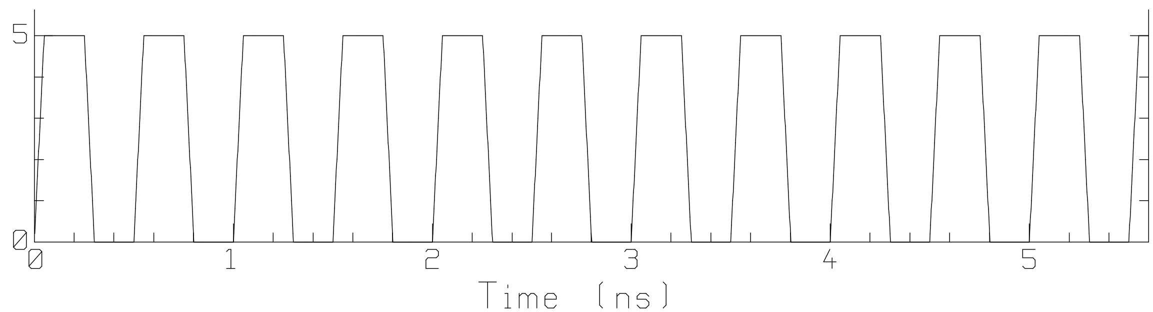


The transit time on the 10 cm line is $T = \frac{L}{u} = \frac{10}{14}$ =0.714 ns

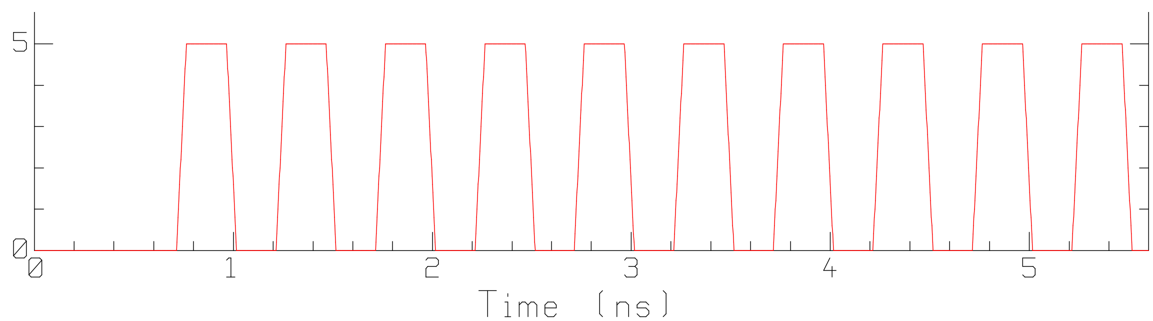
Review:



Input:

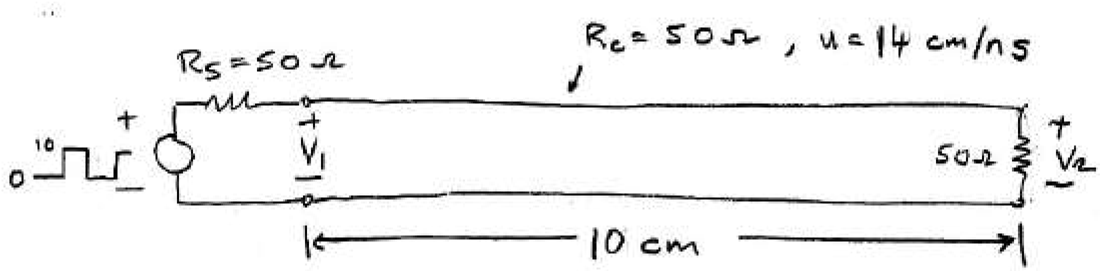


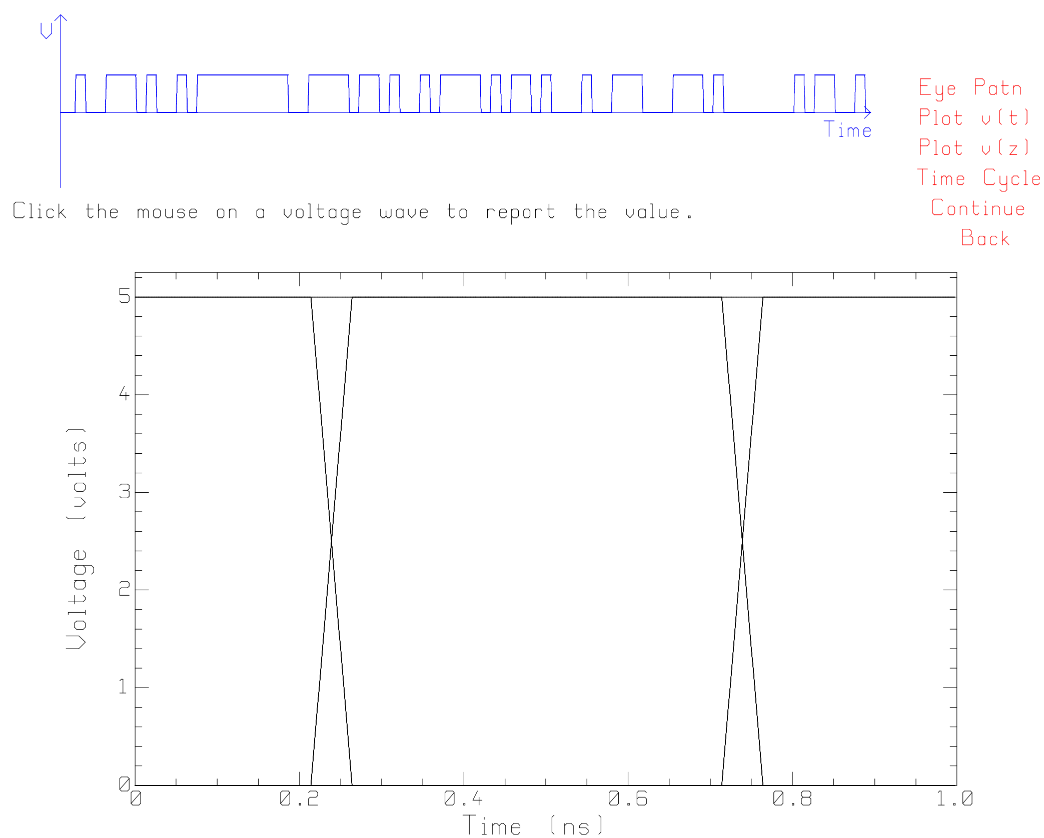
Output: the time delay is $T = 0.714$ ns



Review:

Eye Pattern for the Ideal Communication Channel



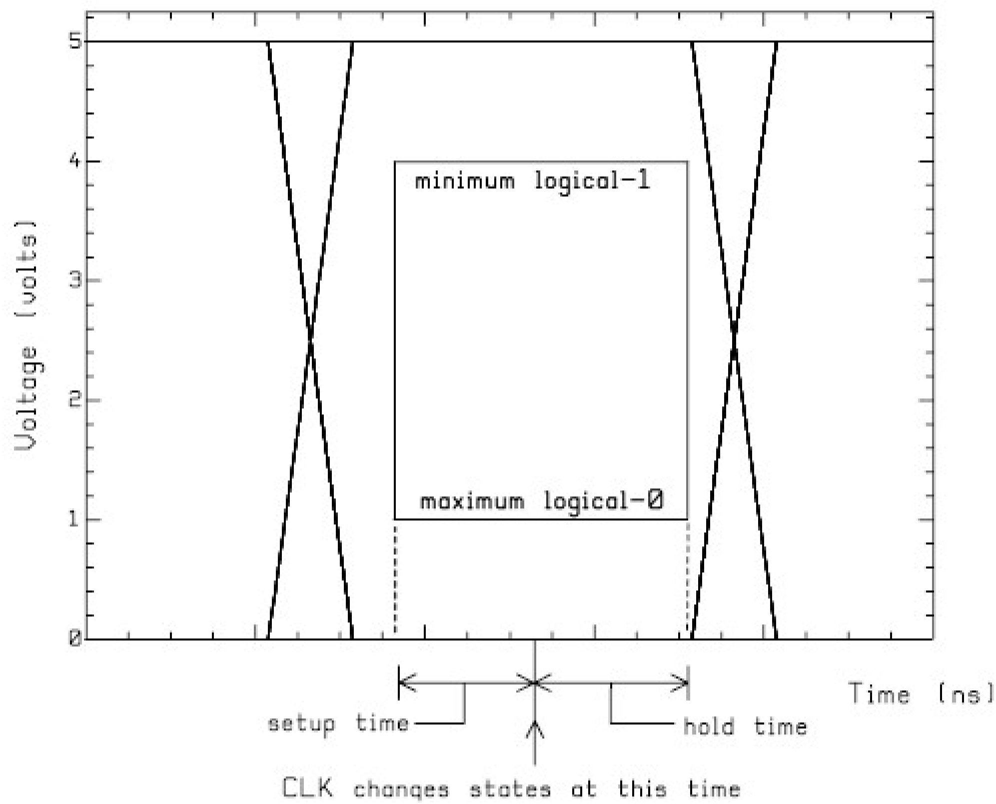


Review:

Interpreting Eye Patterns

Suppose for our logic family, the specs are:

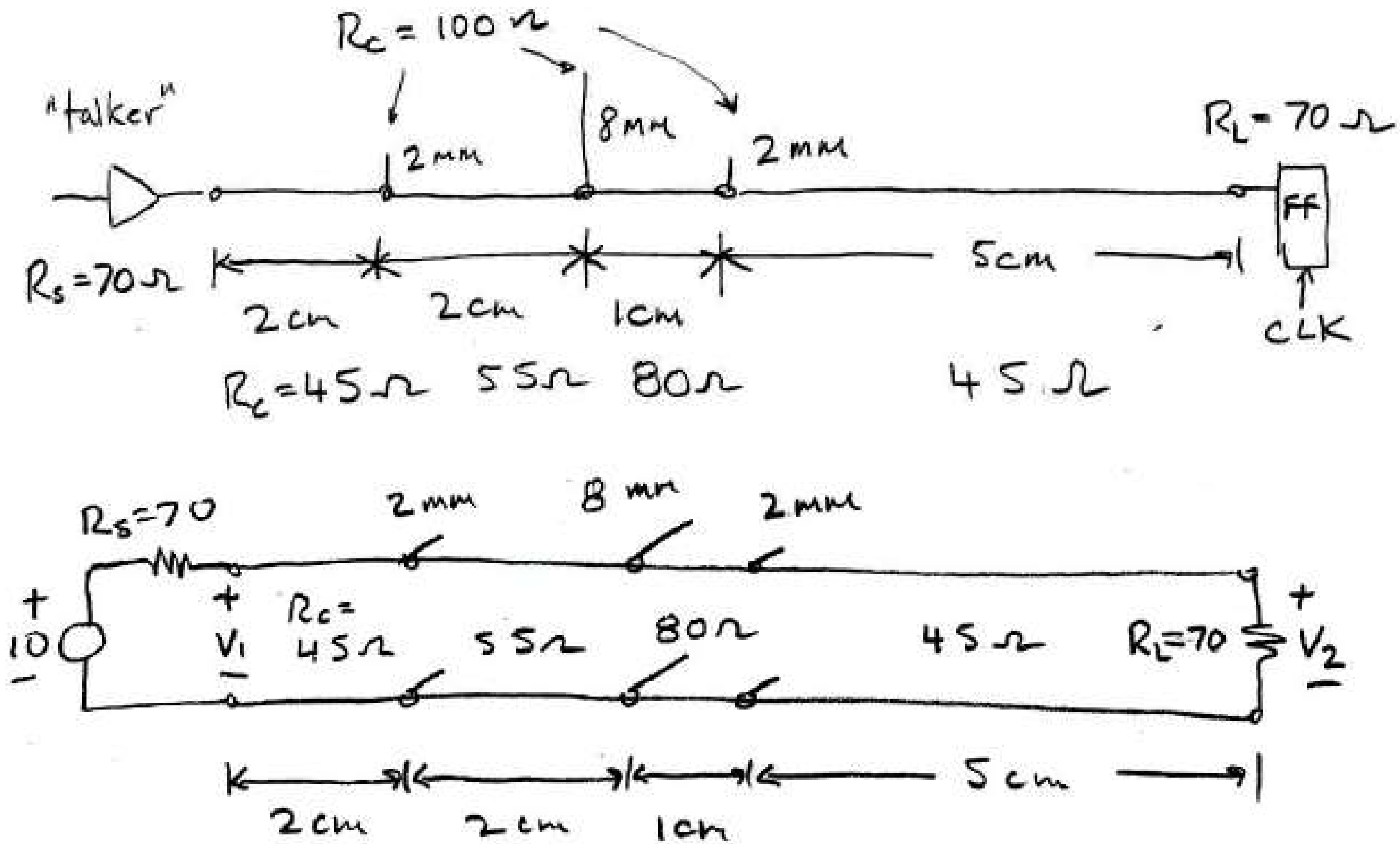
- 0 to 5 volt logic
- The minimum voltage for "logical 1" is 4 volts
- The maximum voltage for "logical 0" is 1 volt



- The "timing margin" is the amount of time between the instant when the data is "valid" and the instant when the setup time starts.
- Note that the "timing margin" is about 1/3 of the setup time in this example.

Review:

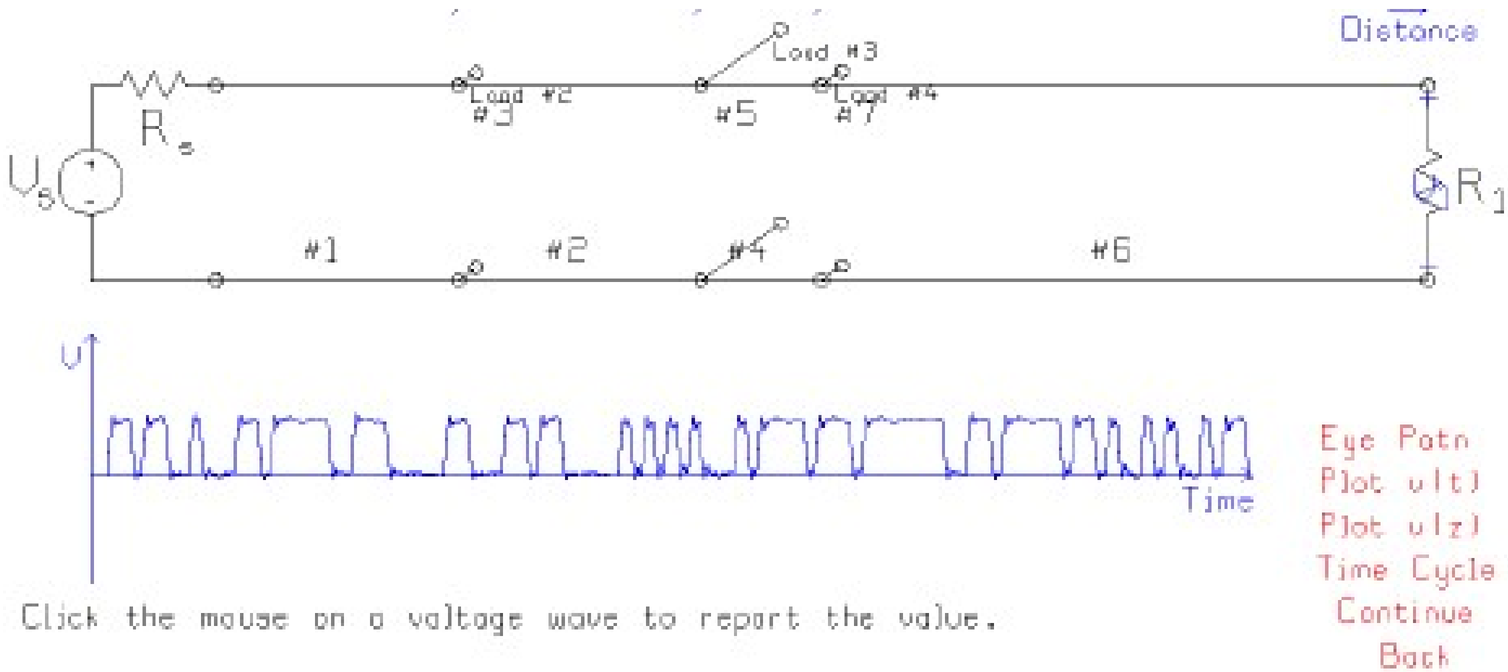
Realistic Communication Path



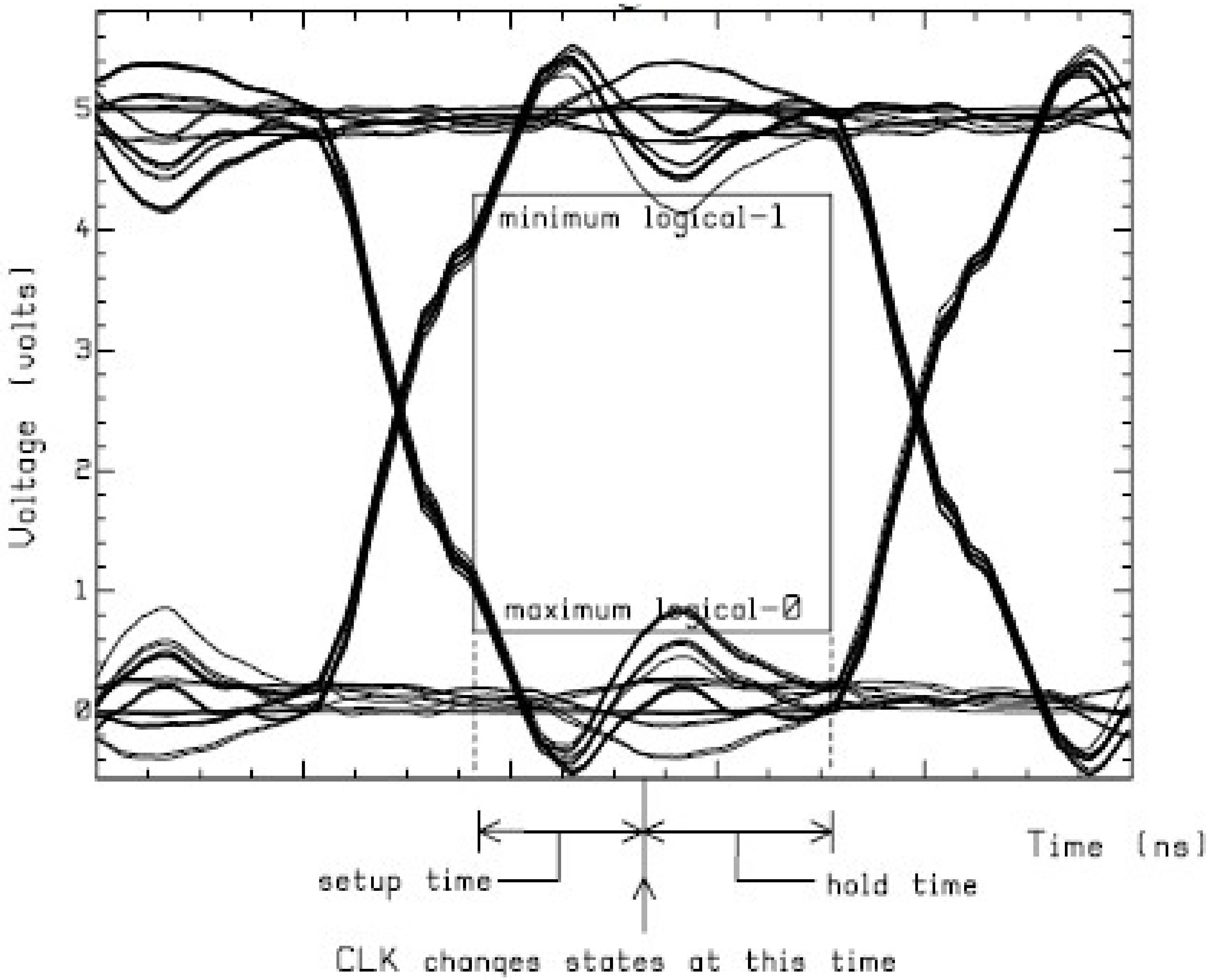
Review:

What does the eye pattern look like?

Let the generator run for a long time to have a large sample of the output of the circuit:



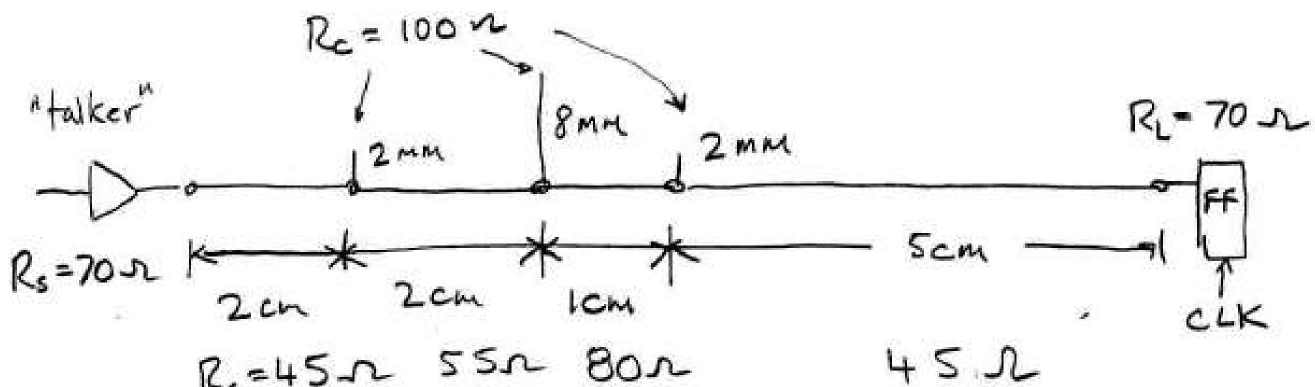
Review:

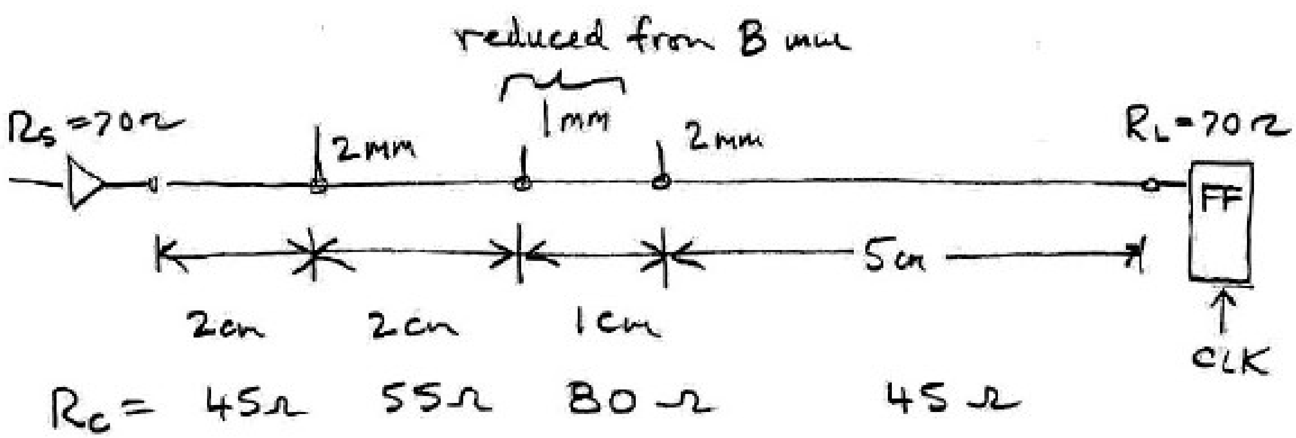


5
4
3
2
1
0
Voltage (volts)
minimum logical-1
maximum logical-0
setup time
hold time
Time (ns)
CLK changes states at this time

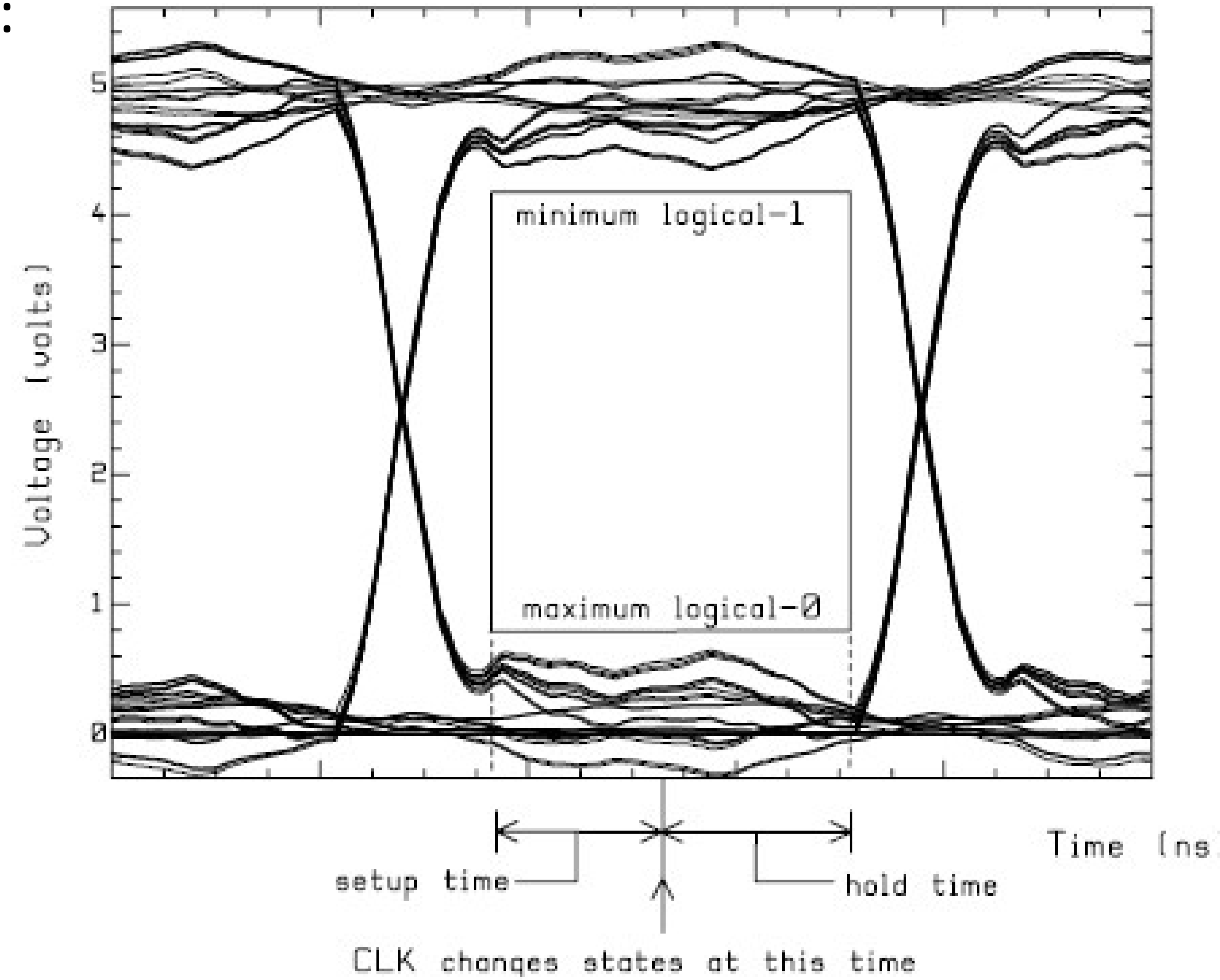
Review:

How can we "clean up" the eye pattern?





Review:



Why does the 8 mm branch spoil the eye pattern?

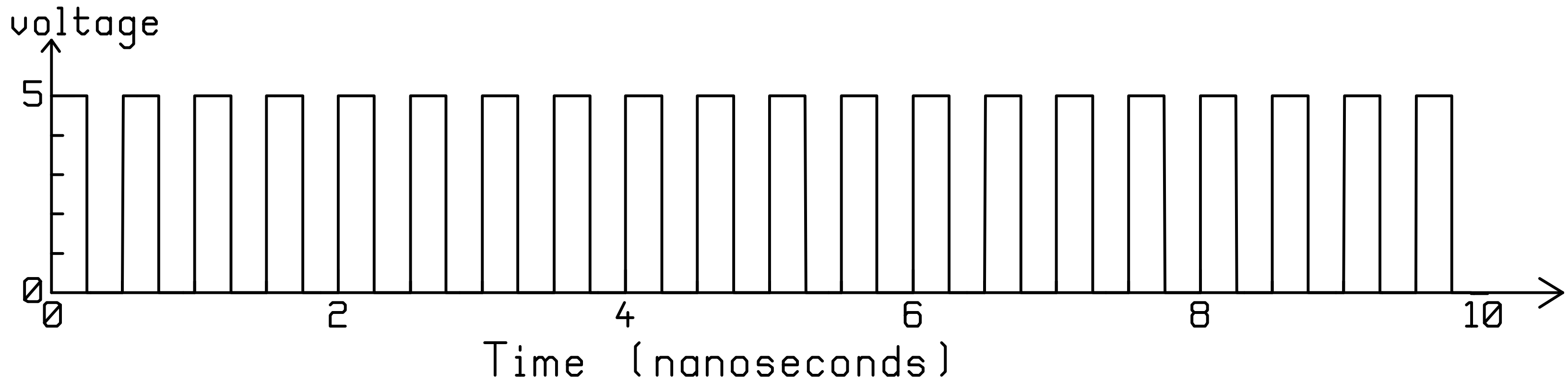
Frequency Domain Analysis of the "Communication Channel"

Clock Frequency, Rise Time and Bandwidth

Good reference:

H. Johnson and M. Graham, "High-Speed Digital Design – A Handbook of Black Magic", Prentice-Hall, 1993.

What is the bandwidth of a square wave?



Amplitude: A =5 volts
Frequency f_o =2 GHz (so $\omega_o = 2\pi f_o$)
Period T =0.5 ns
Half a period or one “bit” τ =0.25 ns
Zero Rise time.

What are the Fourier Series coefficients?

$$v(t) = \sum_{-\infty}^{\infty} c_k e^{jk\ \ {}_o t}$$

$$c_k = \frac{1}{T}\int_0^T v(t) e^{-jk\omega_o t} dt$$

We need to evaluate the coefficients for the square wave.

Evaluate the Fourier Coefficients for the square wave:

The square wave is $x(t) = A$ from $t = 0$ to $t = \tau$, and is $x(t) = 0$ from $t = \tau$ to $t = T$:

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt = \frac{A}{T}\int_0^\tau e^{-jk\omega_o t}dt = \frac{A}{T}\frac{e^{-jk\omega_o t}}{-jk\omega_o}\Big|_0^\tau$$

Note that the integration is no good for *k*=0 because the integral amounts to 1 over 0!

So for *k* not equal to zero:

$$c_k = \frac{A}{T}\frac{1}{-jk\omega_o}\left(e^{-jk\omega_o\tau} - 1\right)$$

$$c_k = \frac{jA}{k\omega_o T}\left(e^{-jk\omega_o\tau} - 1\right)$$

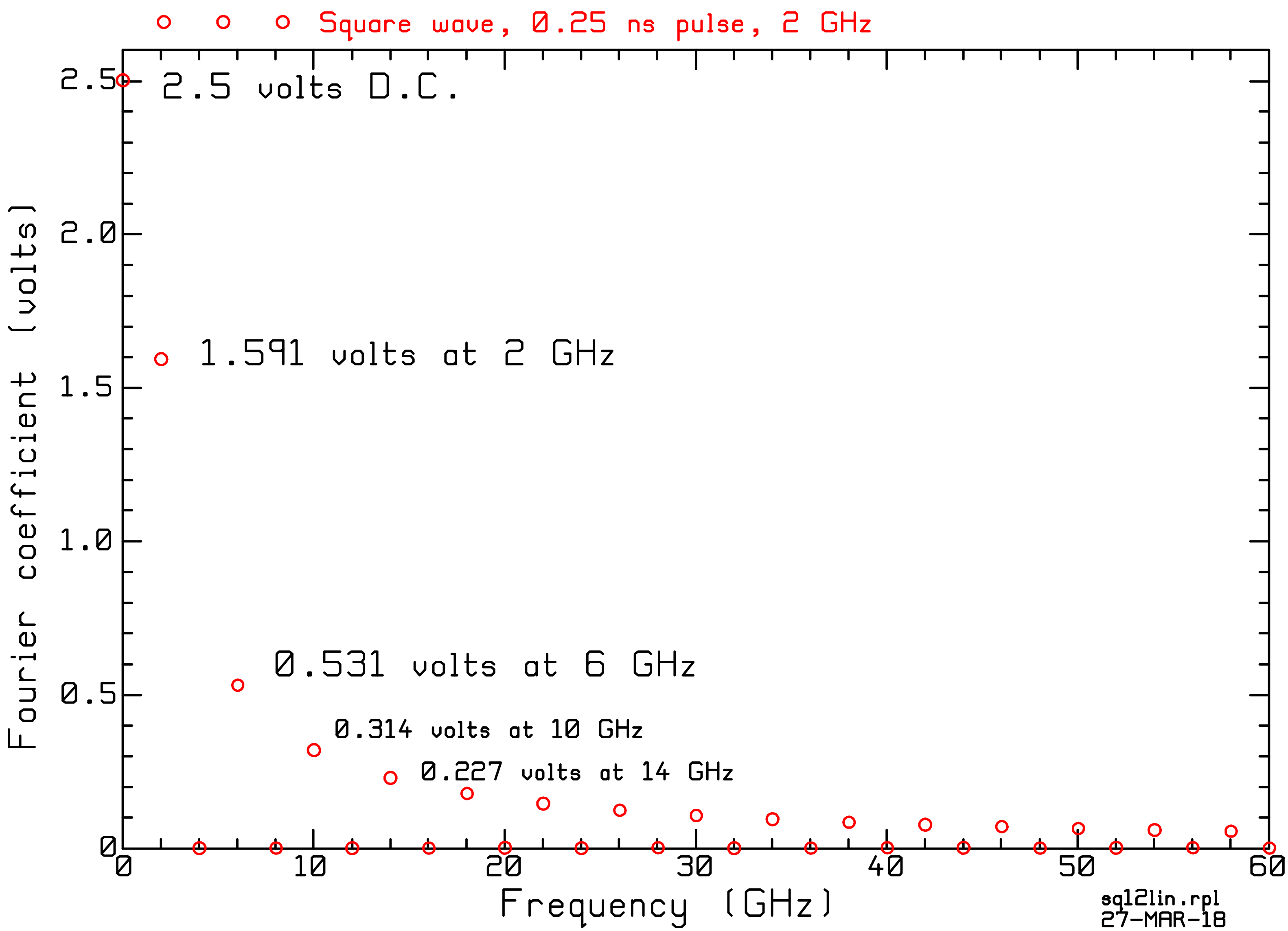
For k=0, we have the D.C. Component:

$$c_0 = \frac{A}{T}\int_0^\tau dt = \frac{A}{T}t\Big|_0^\tau = \frac{A}{T}\tau$$

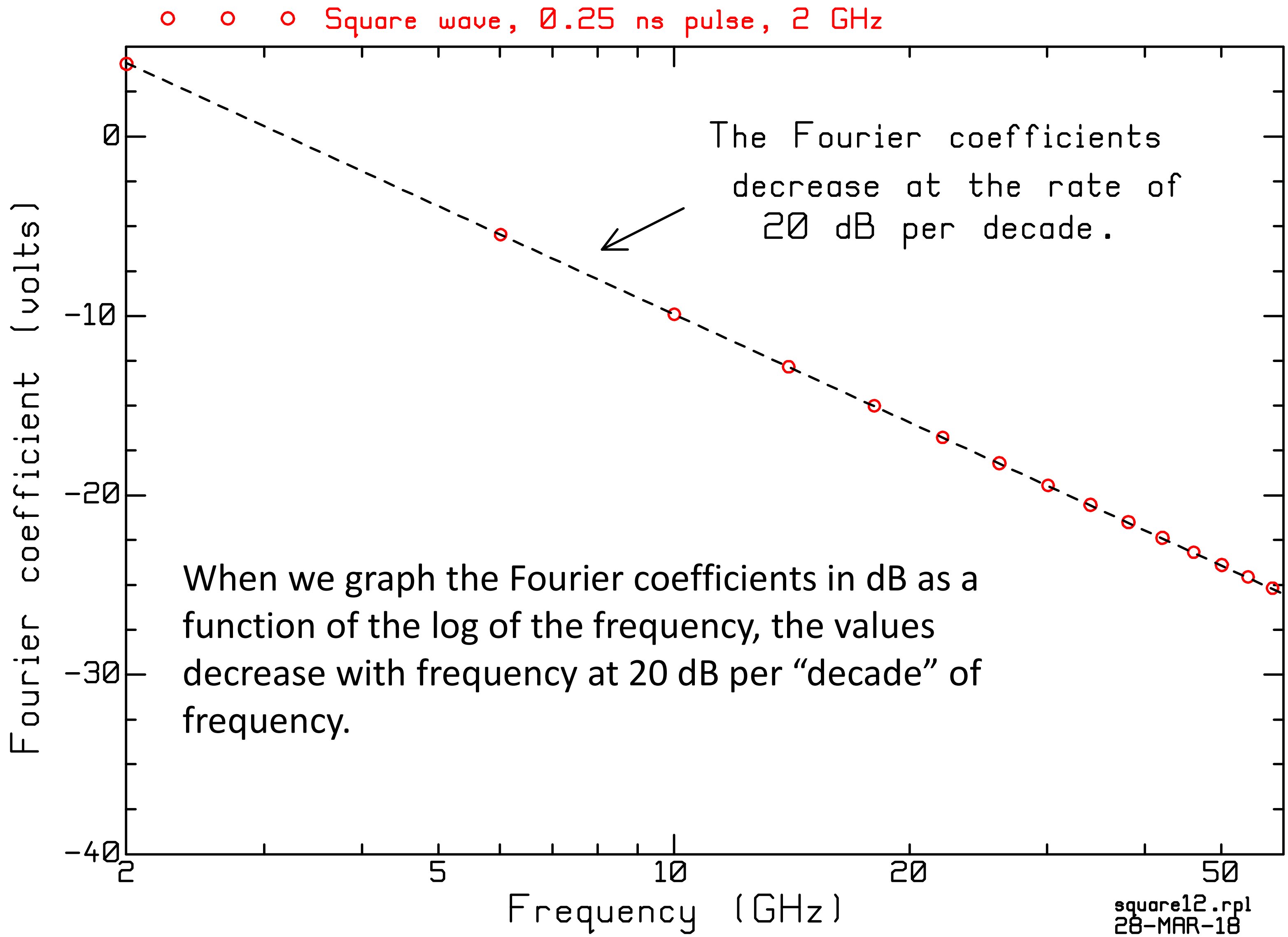
For $\tau = \frac{T}{2}$, the D.C. component is

$$c_0 = \frac{A}{T}\frac{T}{2} = \frac{A}{2} = \frac{5}{2} = 2.5 \text{ volts}$$

Fourier Coefficients of a Square Wave



Fourier Coefficients on a log-dB scale



A “decade” of frequency is a factor of 10, say from 2 GHz to 20 GHz.

Write the Fourier series as a sine series:

$$v(t) = \sum_{-\infty}^{\infty} c_k e^{jk \quad _o t}$$

Consider the terms at *k=n* and at *k=–n*. Add them to find the net amplitude at frequency nf_o:

$$v_n(t) = c_{-n} e^{-jn\omega_o t} + c_n e^{jn\omega_o t}$$

where

$$c_k = \frac{jA}{k\omega_o T}\left(e^{-jk\omega_o \tau} - 1\right)$$

After about a page of algebra, we can show that for $n = 1,3,5, \ldots$

$$v_n(t) = \frac{10}{\pi n}\sin(n\omega_o t)$$

So the Fourier Series is:

$$v(t) = \frac{A}{2} + \sum_{n=1,3,5,\ldots} \frac{10}{\pi n}\sin(n\omega_o t)$$

You can find this in any engineering mathematics textbook.

What is the bandwidth of the square wave?

Principle: the bandwidth is the range of frequencies from D.C. to f_{max} which includes most of the power in the square wave.

What is the power delivered to a 50 ohm load?

Overall power:

The power delivered to a 50-ohm resistor from 0 to $\frac{T}{2}$ is $\frac{A^2}{R}$ so the energy is $\frac{A^2}{R}\frac{T}{2}$. The energy delivered from $\frac{T}{2}$ until T is zero, so the total energy is $\frac{A^2}{R}\frac{T}{2}$. The average power is then energy/time=$\frac{A^2}{R}\frac{T}{2}\frac{1}{T} = \frac{A^2}{2R} = \frac{5^2}{2x50} = \frac{25}{100} = 250$ mW

D.C. Component: $A = 5$ volts, $T = 0.5$ ns, $\tau = \frac{T}{2} = 0.25$ ns

$c_0 = \frac{A}{2} = 2.5$ volts

If a DC voltage of 2.5 volts is applied to a 50-ohm resistor, the power delivered is

$\frac{V^2}{R} = \frac{2.5^2}{50} = 125 \text{ mW}$

What is the power delivered to a 50 ohm load at frequency nf_o?

$$v_n(t) = \frac{10}{\pi n} \sin n\omega_o t$$

The power delivered to a 50-ohm load at frequency nf_o is

$$P_{av} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{10}{\sqrt{2}\pi n}\right)^2}{50} = \frac{0.1013}{n^2} \text{ watts}$$

For $n = 1$ the power at 2 GHz is 101.3 mW

For $n = 3$ the power at 6 GHz is 11.2 mW

For $n = 5$ the power at 10 GHz is 4.05 mW

…

The power at D.C. is 125 mW.

The total power in the square wave is 250 mW.

What is the bandwidth?

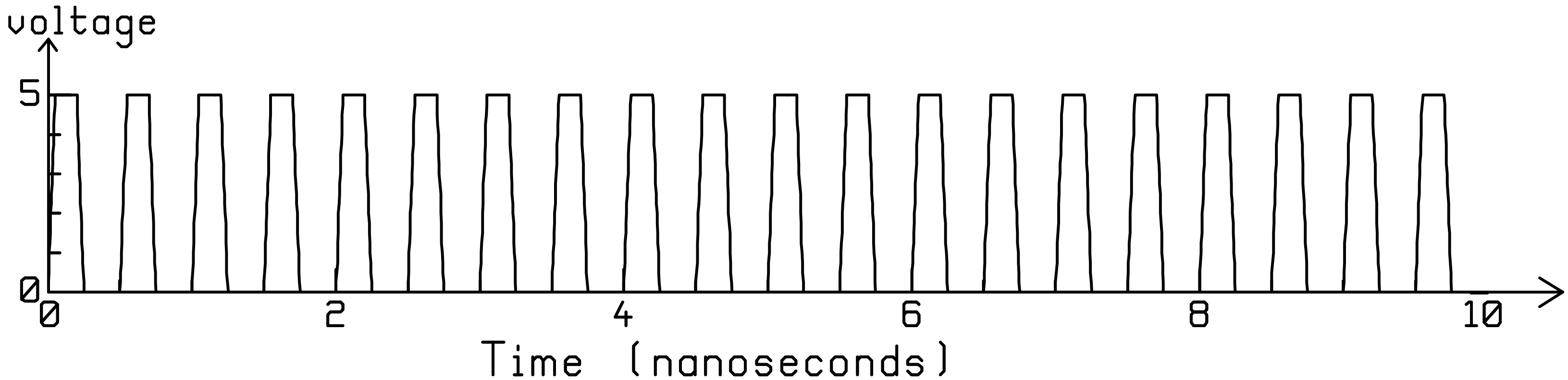
Index n	Frequency	Amplitude	Power	Accumulated power	Percent of total power
0	D.C.	2.5	125 mW	125 mW	50%
1	2	$\frac{10}{\pi}$=3.183	101.3	226.3	90.5
3	6	$\frac{10}{3\pi}$=1.061	$\frac{101.3}{3^2} = 11.2$	237.6	95.0
5	10	0.6362	$\frac{101.3}{5^2} = 4.05$	241.6	96.6
7	14	0.4547	2.07	243.7	97.5
9	18	0.3537	1.25	244.9	98.0
11	22	0.2894	0.84	245.7	98.3

To capture 95.0% of the power we need a bandwidth of 6 GHz.

To capture 96.6% of the power we need a bandwidth of 10 GHz.

A reasonable estimate is that the bandwidth is between 6 and 10 GHz.

What is the bandwidth accounting for rise time?



Amplitude: A =5 volts
Frequency f_o =2 GHz (so $\omega_o = 2\pi f_o$)
Period 0.5 ns
Half a period or one "bit" 0.25 ns
Rise time 0.05 ns.

What are the Fourier Series coefficients?

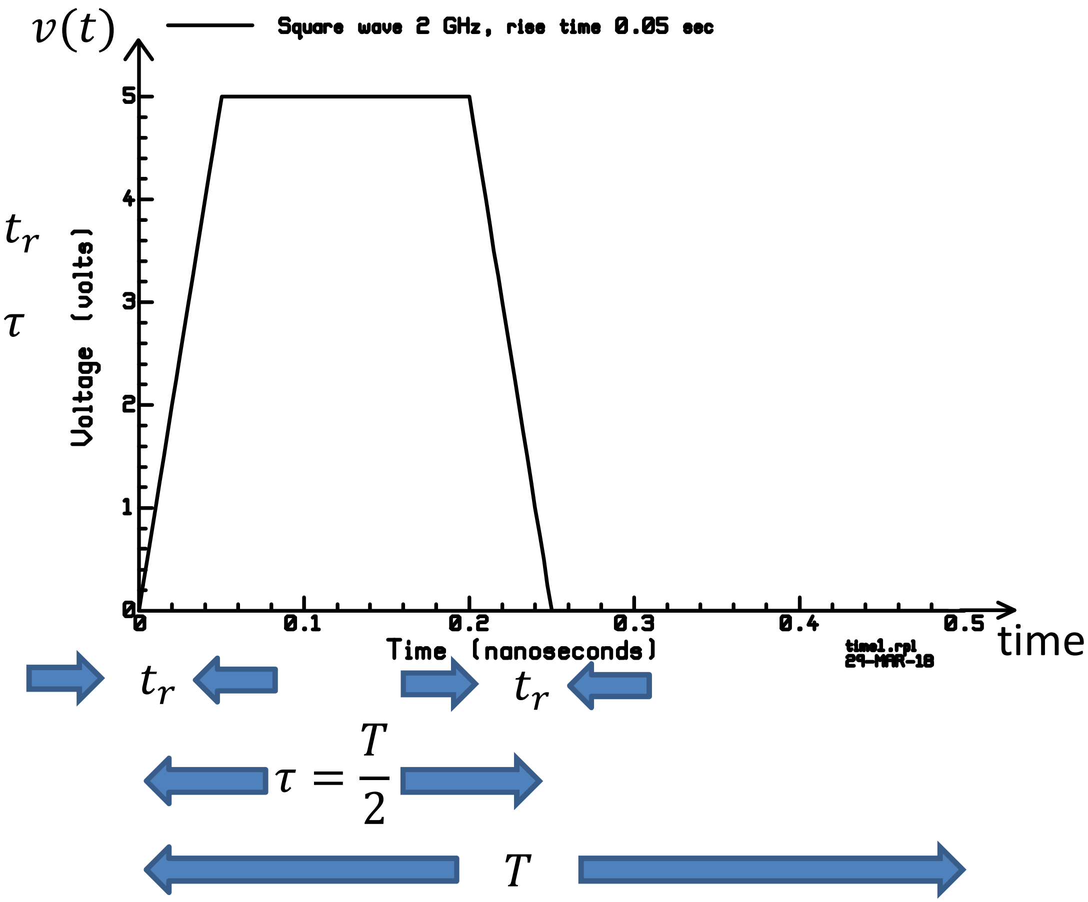
$$v(t) = \sum_{-\infty}^{\infty} c_k e^{jk\omega_o t}$$

$$c_k = \frac{1}{T}\int_0^T v(t)e^{-jk\ \ _o t}dt$$

We need to evaluate the coefficients for the square wave with rise time.

Fourier Coefficients of the Square Wave with Rise Time

$$v(t) = \begin{cases} \dfrac{A}{t_r} t & \text{for } 0 < t < t_r \\ A & \text{for } t_r < t < \tau - t_r \\ \dfrac{A}{t_r}(\tau - t) & \text{for } \tau - t_r < t < \tau \end{cases}$$



$$c_k = \frac{1}{T}\int_0^T v(t) e^{-jk\omega_o t} dt$$

$$c_k = \frac{1}{T}\left[\int_0^{t_r} \frac{A}{t_r} t e^{-jk\omega_o t} dt + \int_{t_r}^{\tau - t_r} A e^{-jk\omega_o t} dt + \int_{\tau - t_r}^{\tau} \frac{A}{t_r}(\tau - t) e^{-jk\omega_o t} dt\right]$$

After considerable manipulation:

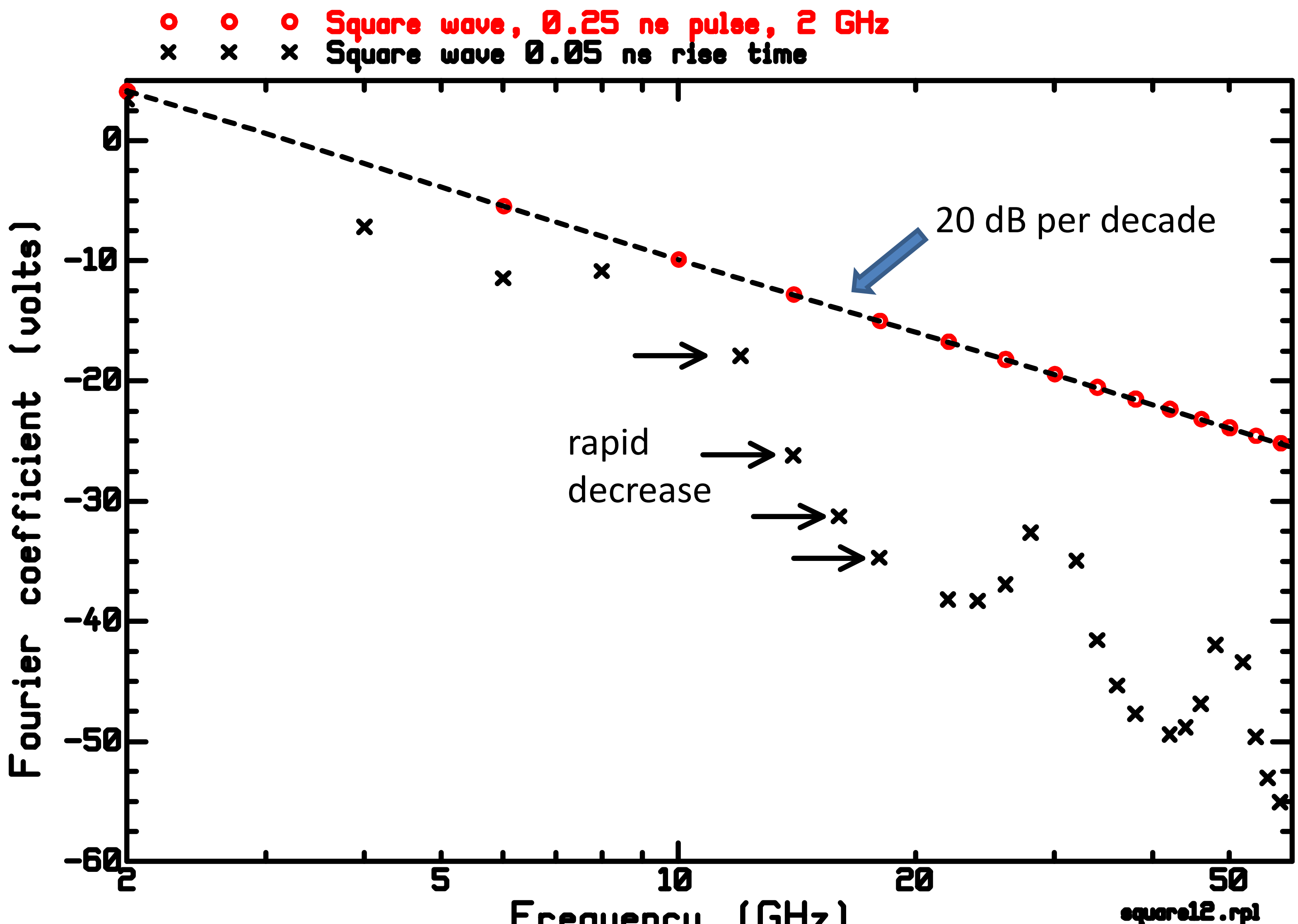
$$c_k = \frac{A}{k^2 t_r \omega_o^2 T}\left(e^{-jk\omega_o t_r} - 1 + e^{-jk\omega_o \tau} e^{jk\omega_o t_r} - e^{-jk\omega_o \tau}\right)$$

Amplitude: A =5 volts
Frequency f_o =2 GHz (so $\omega_o = 2\pi f_o$)
Period 0.5 ns
Half a period or one “bit” 0.25 ns
Rise time 0.05 ns.

How does the spectrum of the square wave with rise time compare to that of the simple square wave?

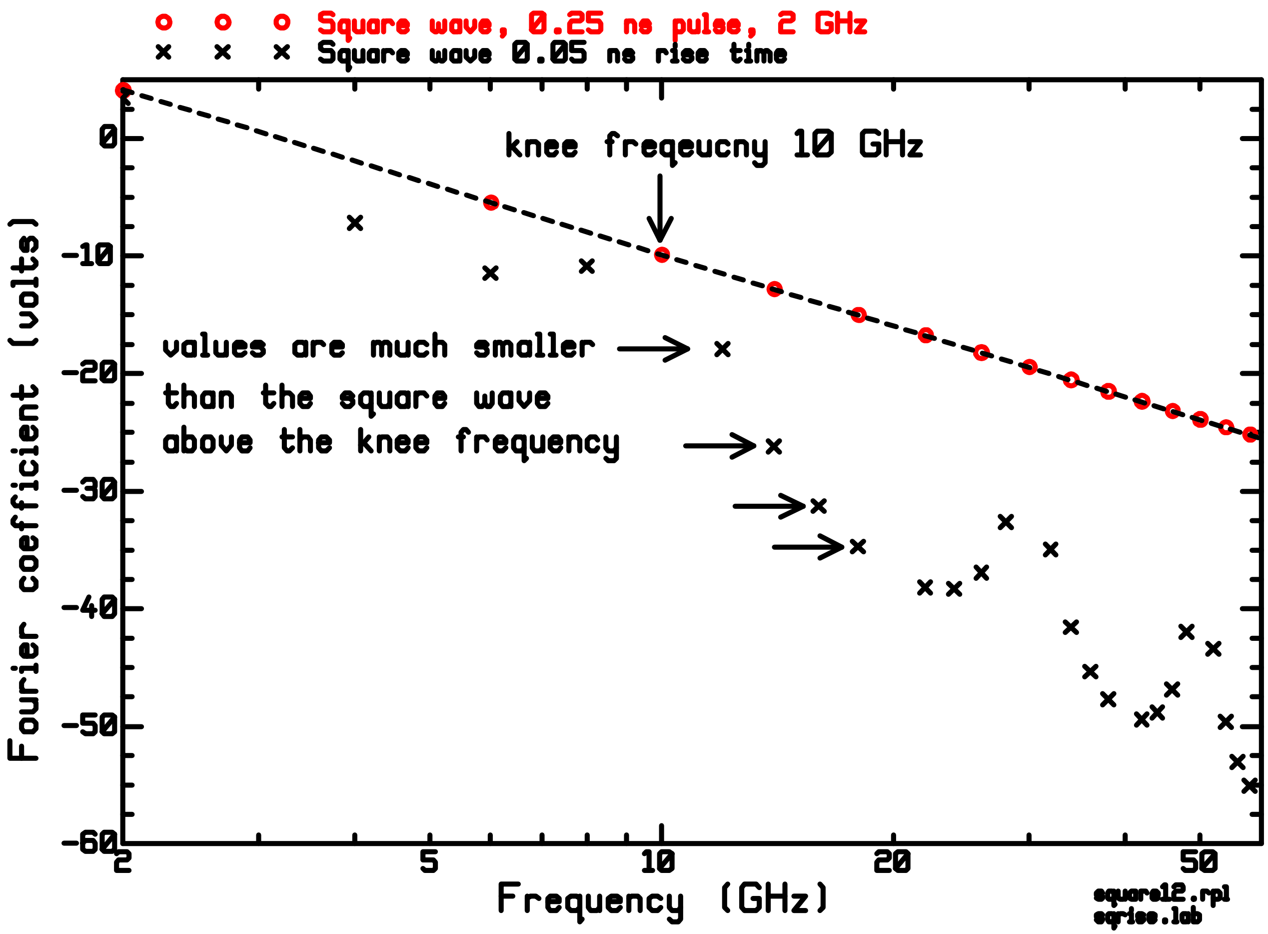
Is the bandwidth the same?

Fourier Coefficients of a Square Wave with Rise Time



1. Even harmonics appear at 4, 8, 12, 16, … GHz.
2. The coefficients rapidly decrease in value above 10 GHz.

Knee Frequency



The “knee frequency” is the frequency at which the Fourier coefficients of the square wave with rise time start to decrease rapidly with frequency.
The knee frequency is estimated as

$$f_{knee} = \frac{0.5}{t_r}$$

Hence for our parameters, $f_{knee} = \frac{0.5}{t_r} = \frac{0.5}{0.05} = 10\ \text{GHz}$

What is the bandwidth?

The bandwidth of a digital signal depends on the rise time

The bandwidth is from D.C. to the knee frequency of $f_{knee} = \frac{0.5}{t_r}$.

What is the spectrum of a digital signal?

- We can model a digital signal as a random sequence of 0s and 1s.
- What is the spectrum without accounting for rise time?
- What is the spectrum including the effect of rise time?

We can calculate the Fourier Transform to learn about the spectrum of a digital signal.

The F.T. of a digital signal $v(t)$ is given by

$$V(j\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt$$

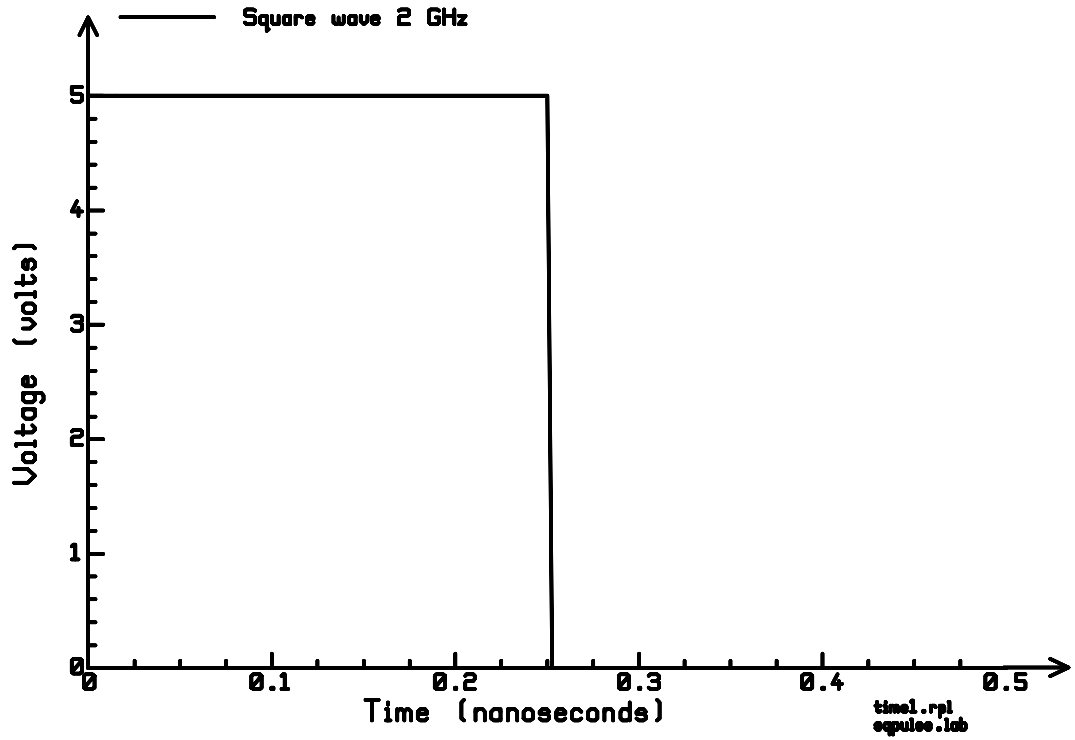
What is the Fourier Transform of a single bit?

Parameters:
Amplitude: A =5 volts
Length of one bit:
$\tau = 0.25$ ns

$$V(j\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt$$

Hence

$$V(j\omega) = \int_{0}^{\tau} A e^{-j\omega t} dt$$



τ

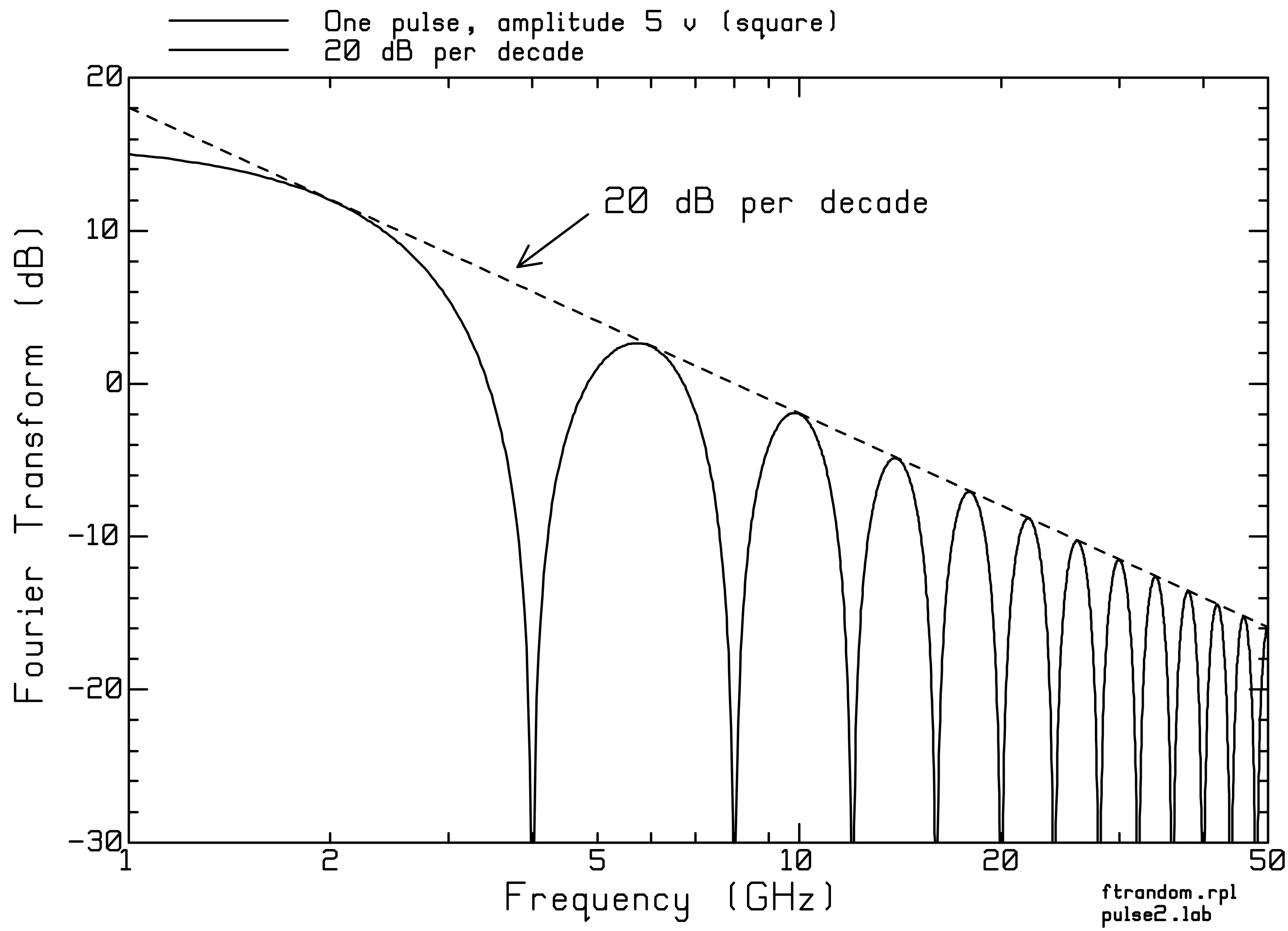
It may be shown that

$$\mathrm{sinc}(f) = \frac{\sin \pi f}{\pi f}$$

$$V(j\omega) = \frac{A}{-j\omega}\left(e^{-j\omega\tau} - 1\right)$$

$$V(j\omega) = A\tau e^{-j\frac{\omega\tau}{2}} \frac{\sin(\pi f \tau)}{\pi f \tau} = A\tau e^{-j\frac{\omega\tau}{2}} \mathrm{sinc}(f\tau)$$

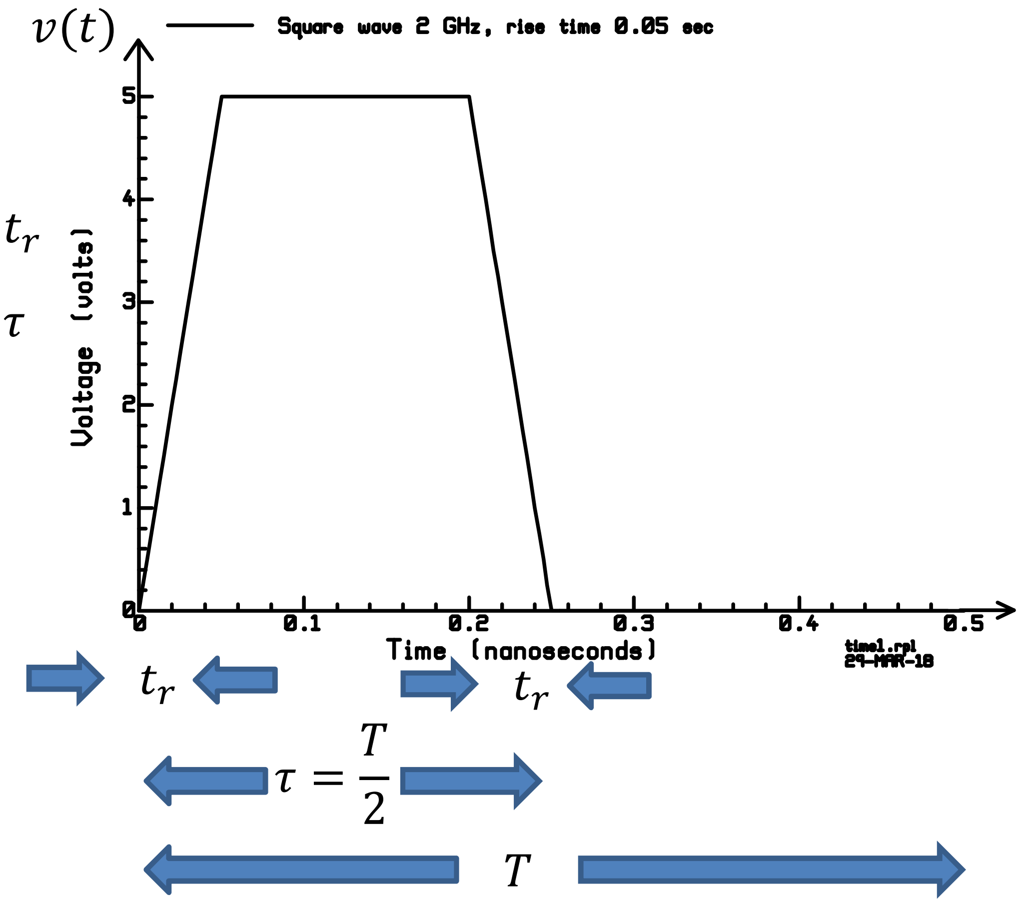
Fourier Transform of One Square Pulse



The F.T. has broad peaks at the frequencies of the Fourier Coefficients: 2, 6, 10, 14, 18, …

F.T. of a Pulse with Rise Time

$$v(t) = \begin{cases} \dfrac{A}{t_r} t & \text{for } 0 < t < t_r \\ A & \text{for } t_r < t < \tau - t_r \\ \dfrac{A}{t_r}(\tau - \mathrm{t}) & \text{for } \tau - t_r < t < \tau \end{cases}$$

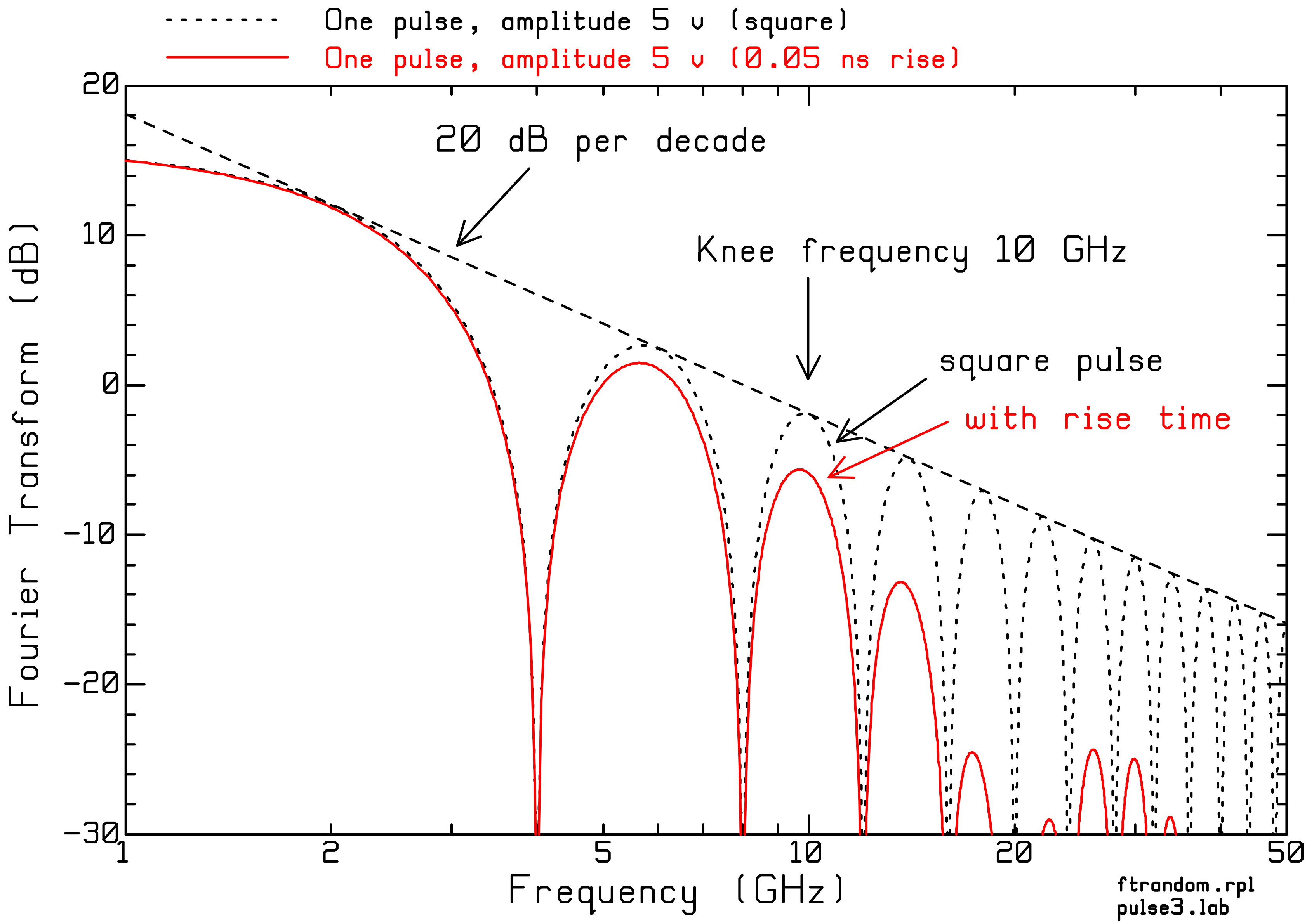


$$V(j\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt$$

It may be shown that

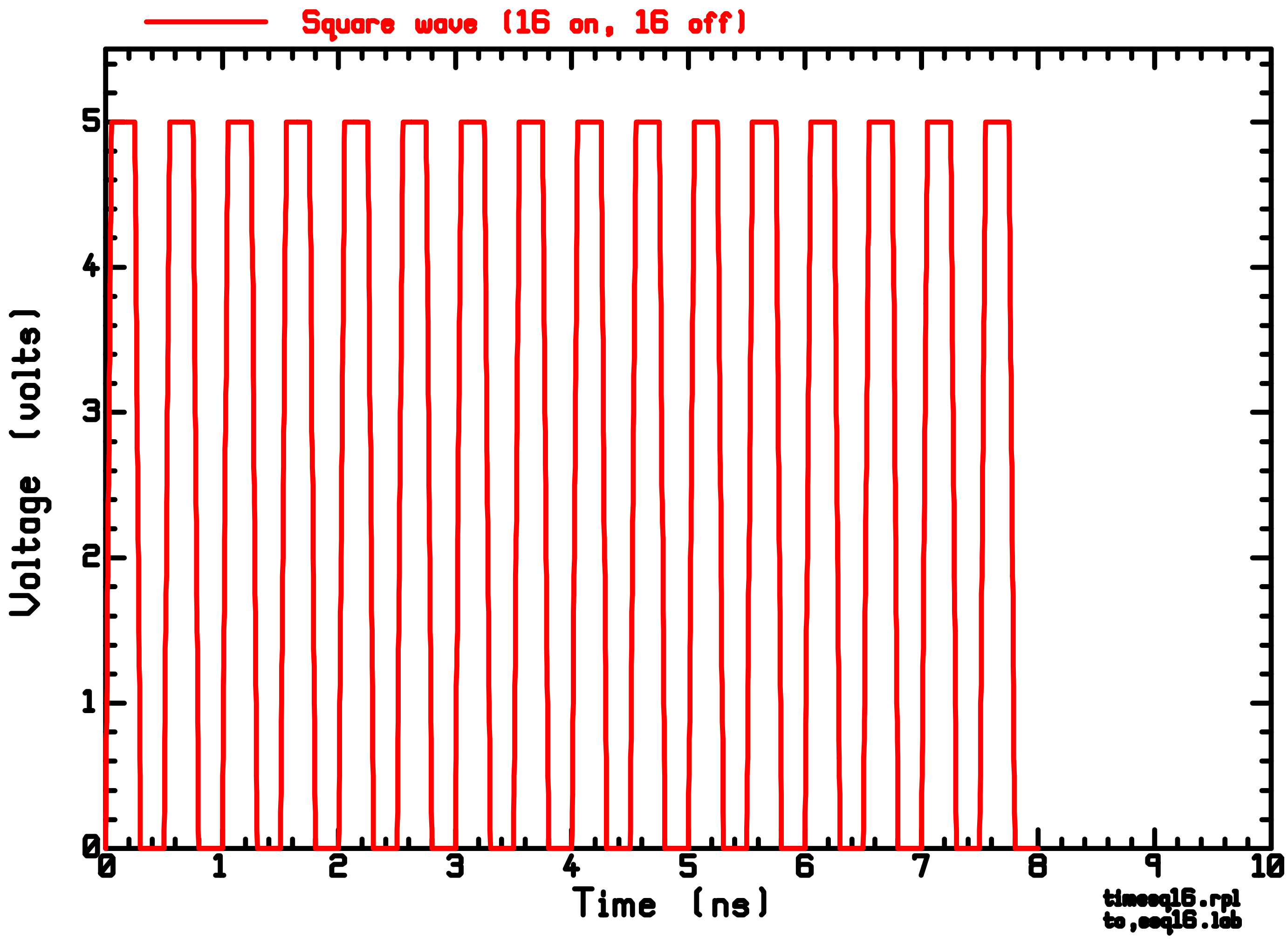
$$V(j\omega) = \frac{A}{t_r \omega^2}\left(-1 + e^{-j\omega t_r} - e^{-j\omega\tau} + e^{-j\omega\tau} e^{j\omega\ \ r}\right)$$

Fourier Transform of One Pulse with Rise Time



With rise time the F.T. decreases rapidly above the knee frequency.

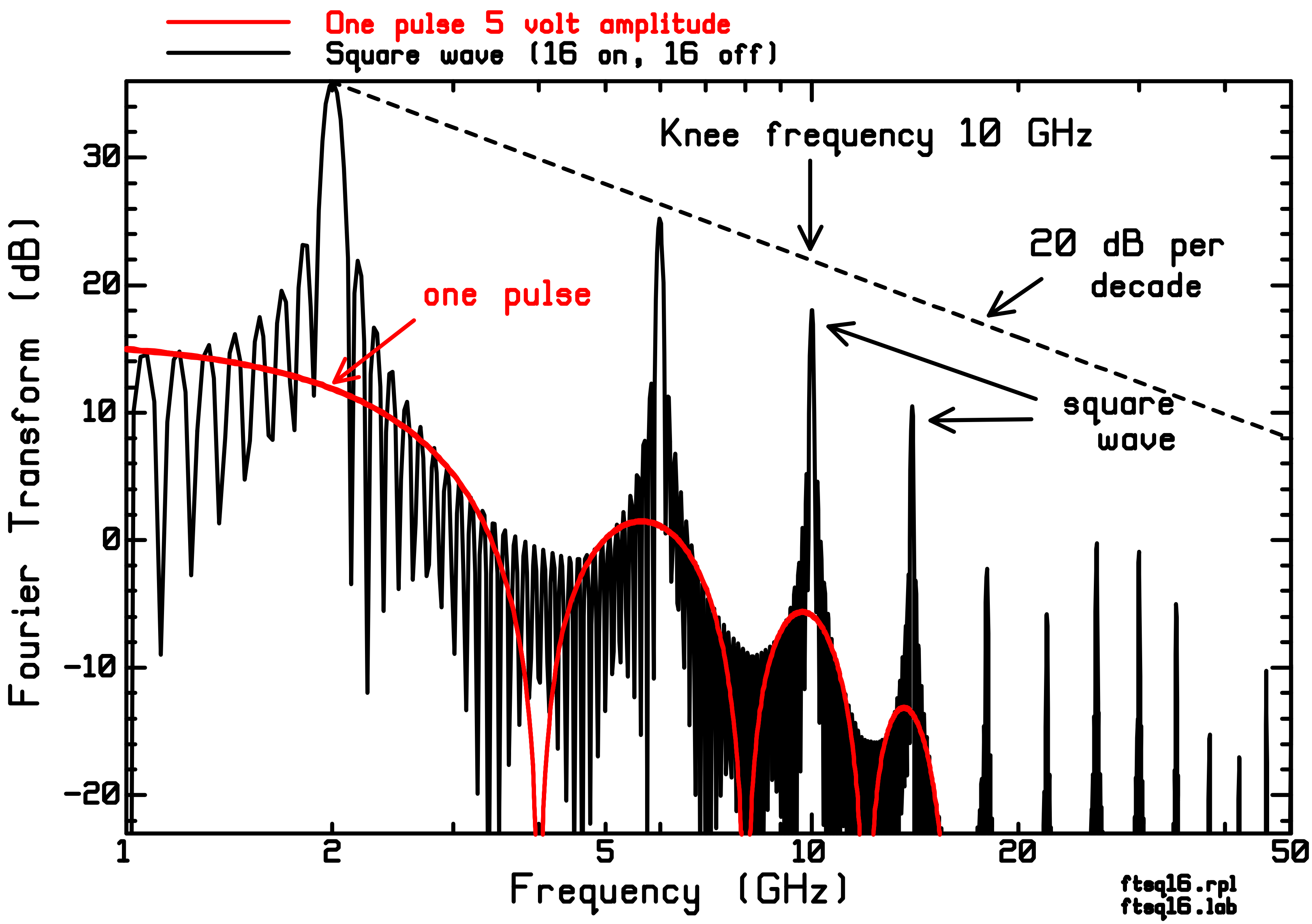
Fourier Transform of a Windowed Square Wave



Each bit lasts 0.25 ns.
The rise time is 0.05 ns.
The knee frequency is 0.5/0.05 = 10 GHz.

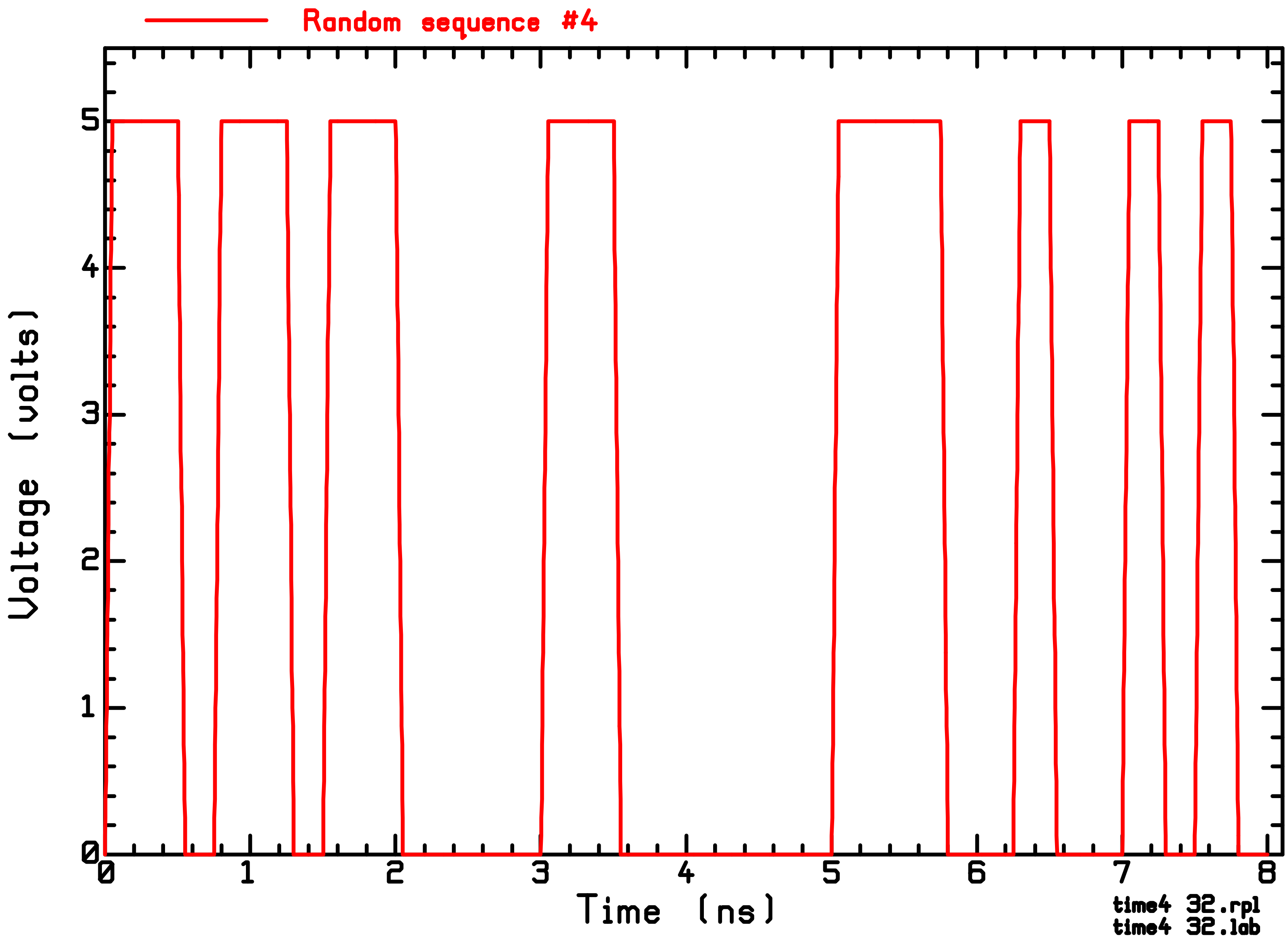
Square wave lasting 32 bits of 1,0,1,0,1,0,.....
This is like a “time window” that includes 16 cycles of a square wave.

Fourier Transform of a Windowed Square Wave



The F.T. of the "windowed" square wave looks like the Fourier Coefficients.

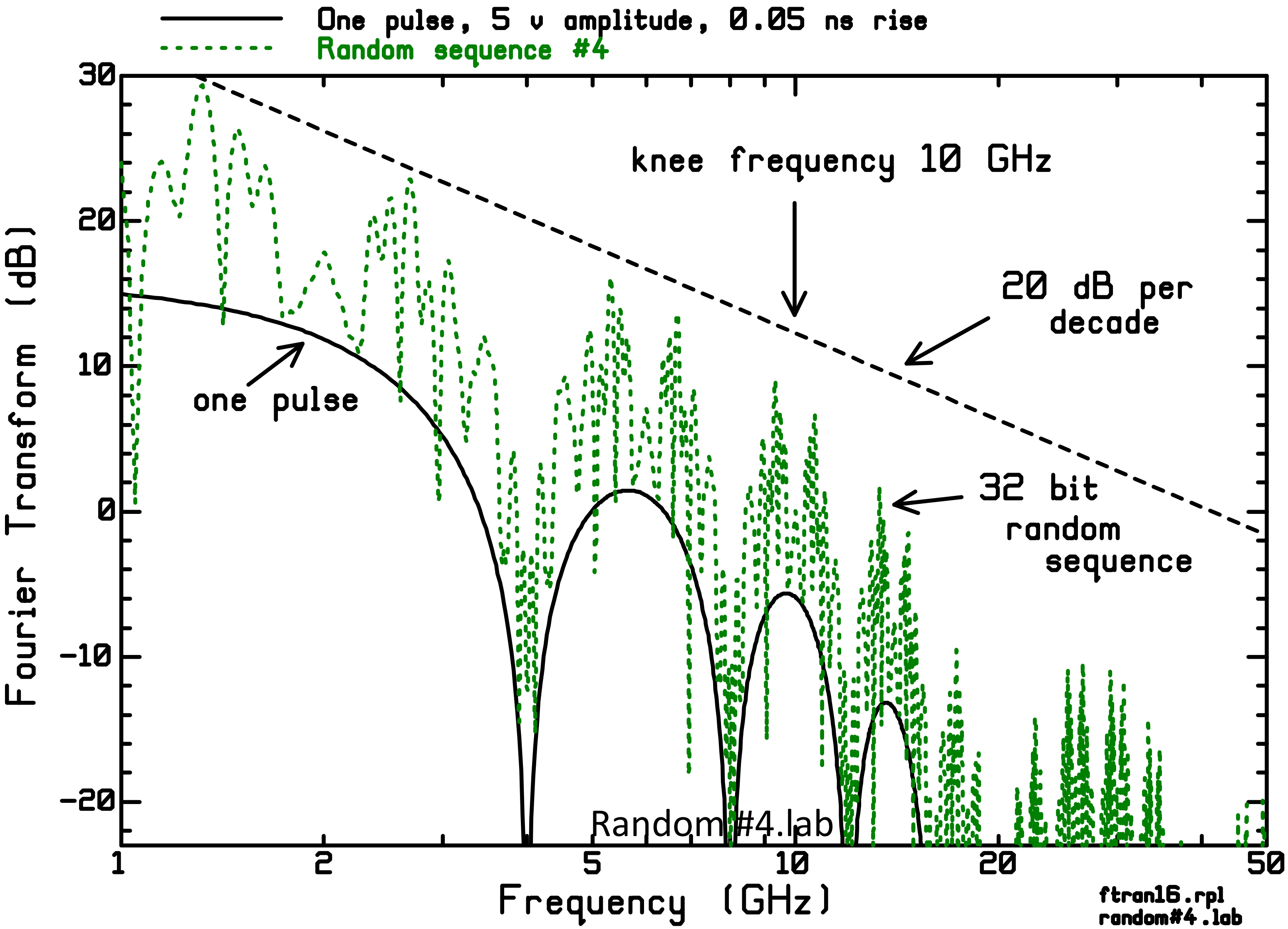
Random Sequence of 1s and 0s



Each bit lasts 0.25 ns.
The rise time is 0.05 ns.
The knee frequency is 0.5/0.05 = 10 GHz.

Random sequence #4, 32 bits long: 1101 1011 0000 1100 0000 1110 0100 1010

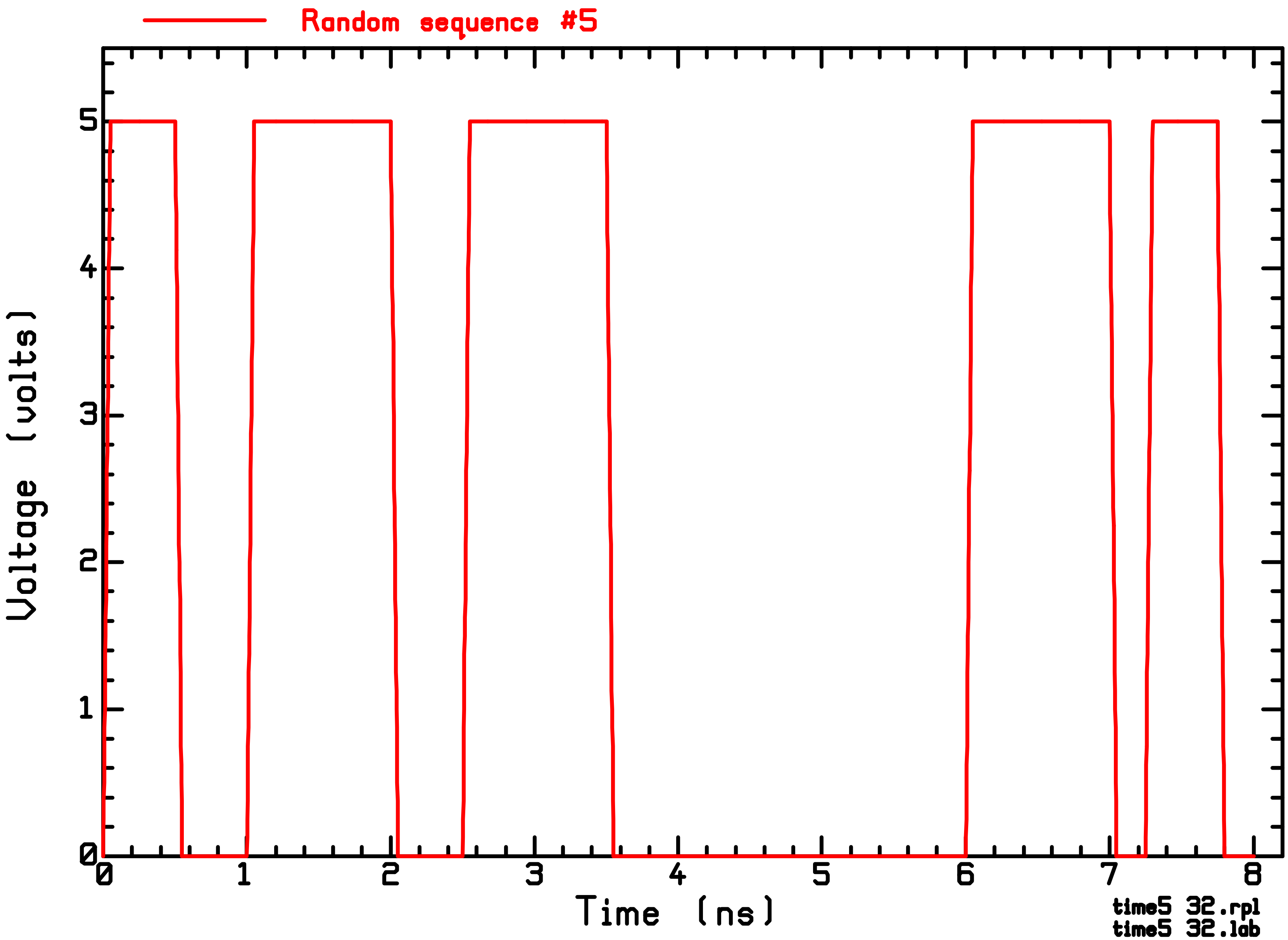
Fourier Transform of Random Sequence #4



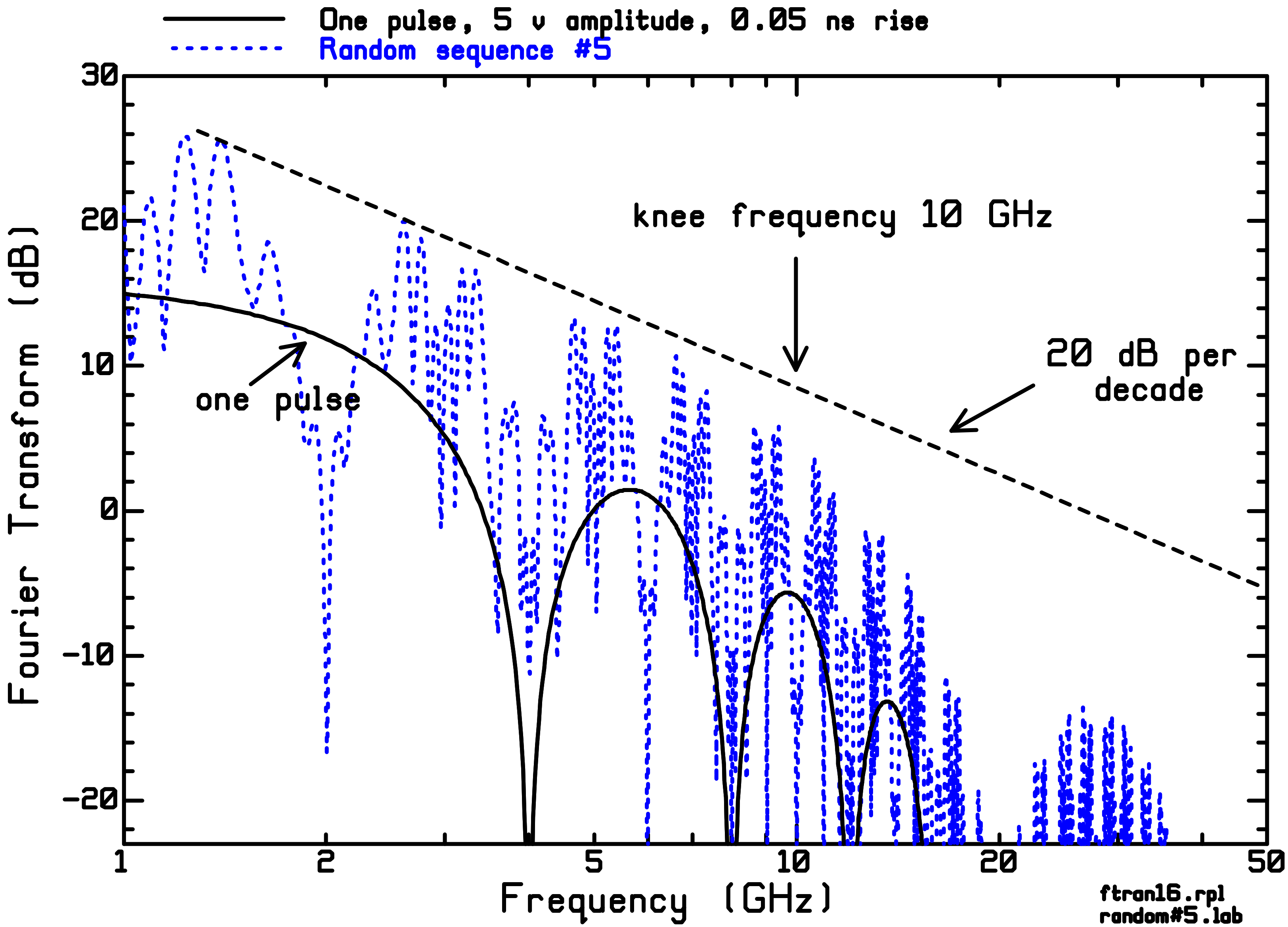
Each bit lasts 0.25 ns.
The rise time is 0.05 ns.
The knee frequency is 0.5/0.05 = 10 GHz.

The graph compares a single pulse with a sequence of pulses, which contains a lot more power. The shape of the FT of the random sequence follows the shape of the FT of the single pulse.

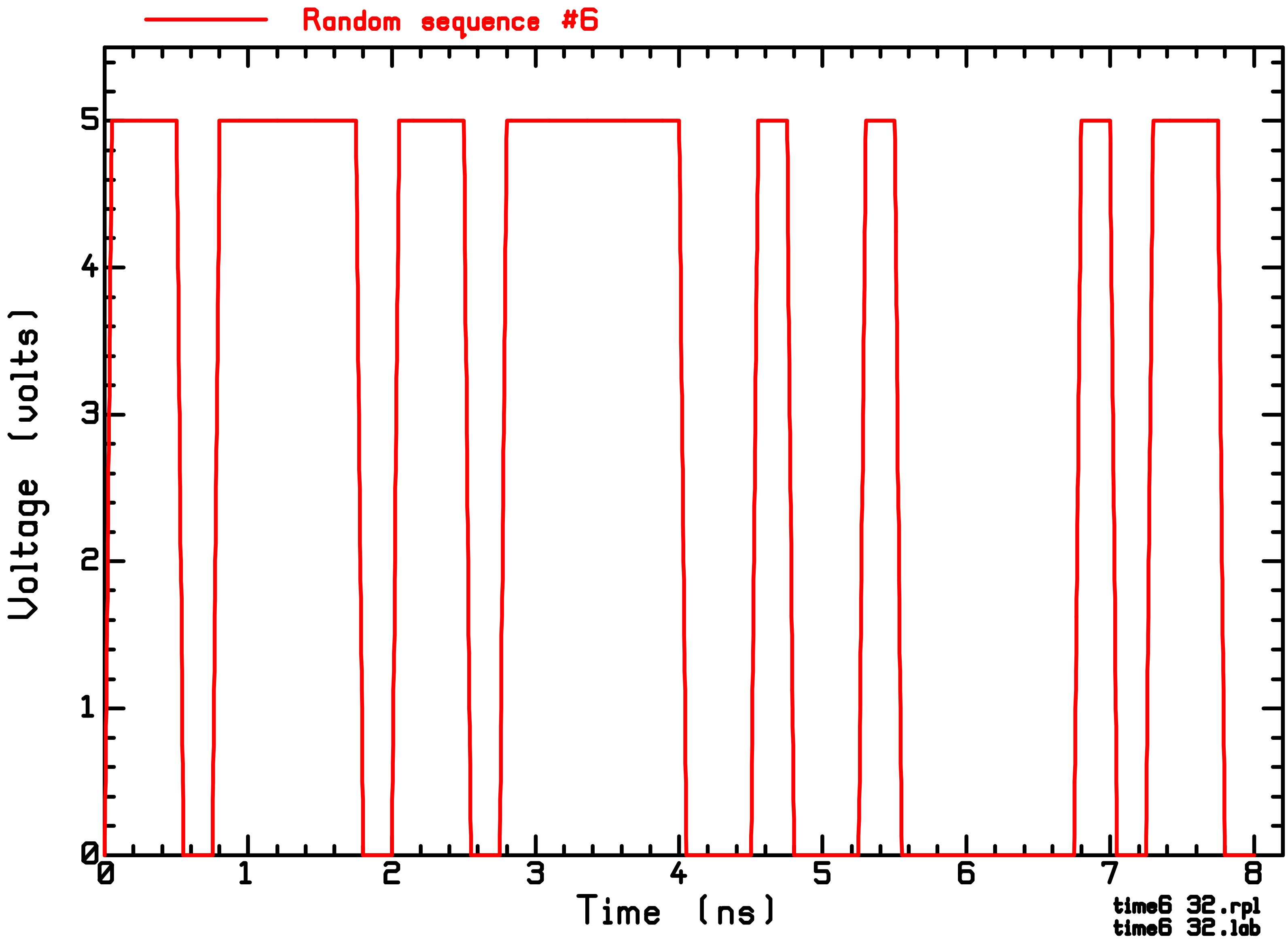
Random Sequence #5



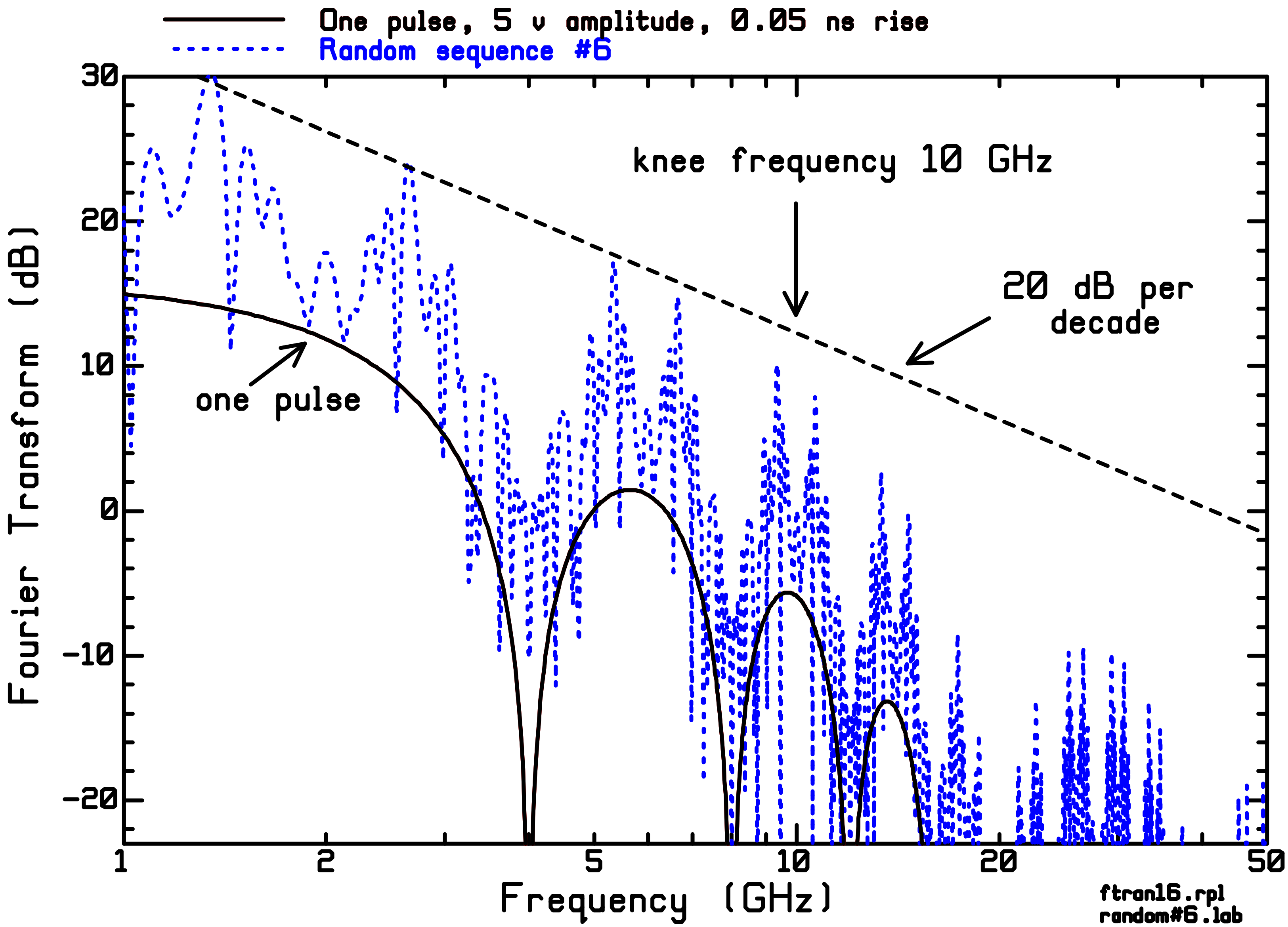
Fourier Transform of Random Sequence #5



Random Sequence #6



Fourier Transform of Random Sequence #6



Bandwidth of a Digital Signal

H. Johnson and M. Graham, “High-Speed Digital Design – A Handbook of Black Magic”, Prentice-Hall, 1993.

Example:

Clock rate $f = 1$ GHz

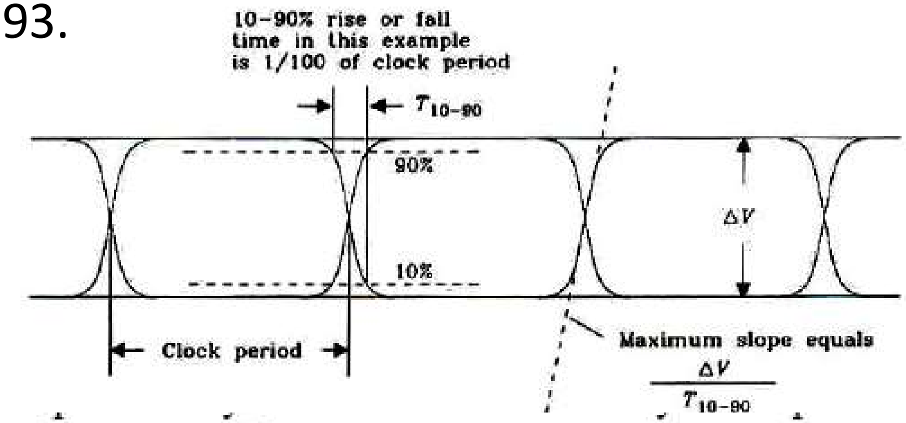
Period $T = \frac{1}{f} = \frac{1}{1\ GHz} = 1\ ns$

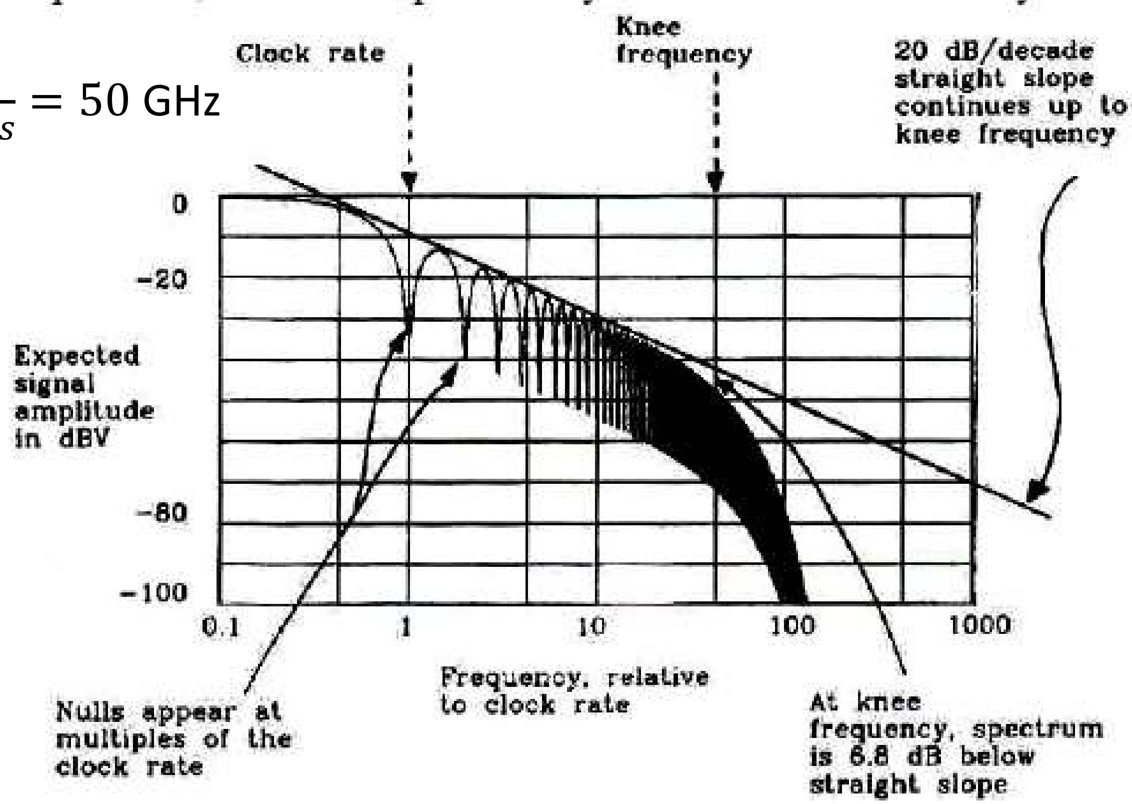
Rise time: 1/100 th of the period

$T_r = \frac{T}{100} = 0.01$ ns

Knee Frequency

$f_{knee} = \frac{0.5}{T_r} = \frac{0.5}{0.01\ ns} = 50$ GHz





- Johnson and Graham estimate the knee frequency as

$$f_{knee} = \frac{0.5}{T_r}$$

- McMorrow, Bell and Ferry in “A Solution for the Design, Simulation and Validation of Board-to-Board Interconnects”, High Frequency Electronics, January 2005, estimate the knee frequency as

$$f_{knee} = \frac{0.35}{T_r}$$

- This is narrower than Johnson and Graham’s estimate.

Principle

The bandwidth of a digital signal is from D.C. to the knee frequency.

A communication channel having a flat frequency response up to the knee frequency will propagate a digital signal without distortion.

Example
What is the bandwidth of a digital signal with a bit rate of 4 Gbs and a rise time of 0.05 ns?

Solution

The frequency is 2 GHz and the period is $\frac{1}{2} = 0.5$ ns, allowing a time of 0.25 ns for each bit.

We are given the rise time as 0.05 ns, so the estimated bandwidth is:

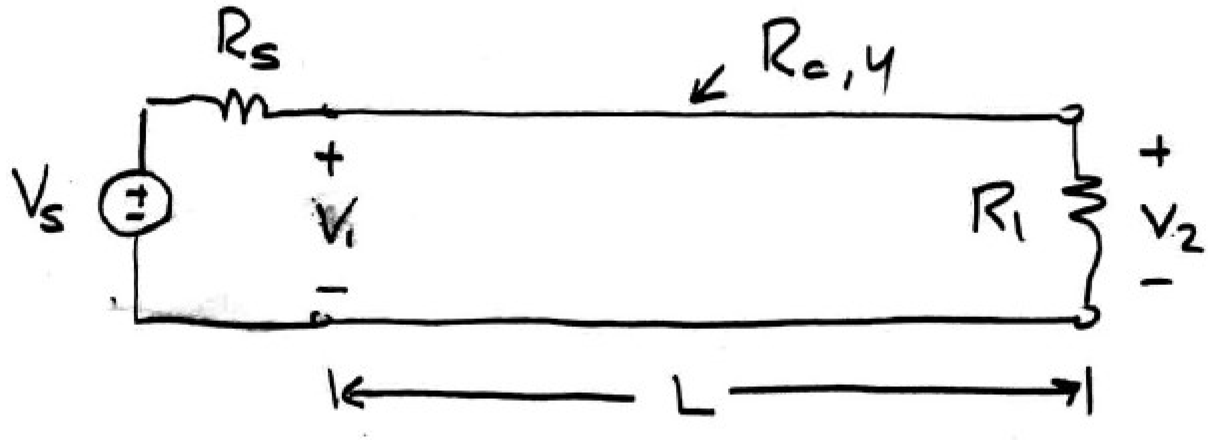
McMorrow, Bell, and Ferry's bandwidth is $f_{\max} = \frac{0.35}{T_r}. = \frac{0.35}{0.05} = 7$ GHz

Johnson and Graham's knee frequency is $f_{knee} = \frac{0.5}{T_r}$=10 GHz

Evidently a channel bandwidth between 7 and 10 GHz is needed. (At least 7 but not more than 10?)

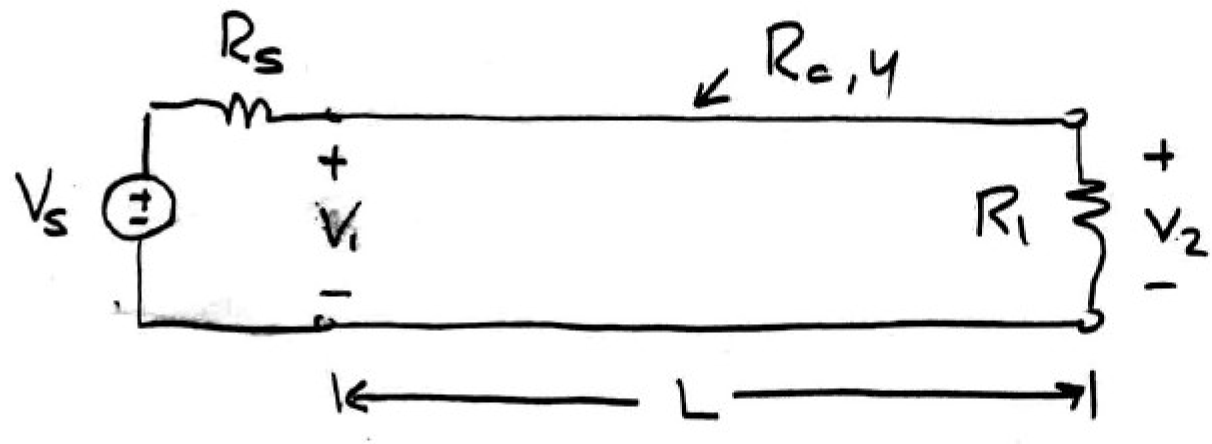
This is consistent with the square wave of frequency 2 GHz and rise time 0.05 ns, where we found that the bandwidth was estimated as 6 to 10 GHz, depending on whether we included the 3^{rd} or the 5^{th} harmonic.

Transfer Function of an "Ideal" Communication Channel

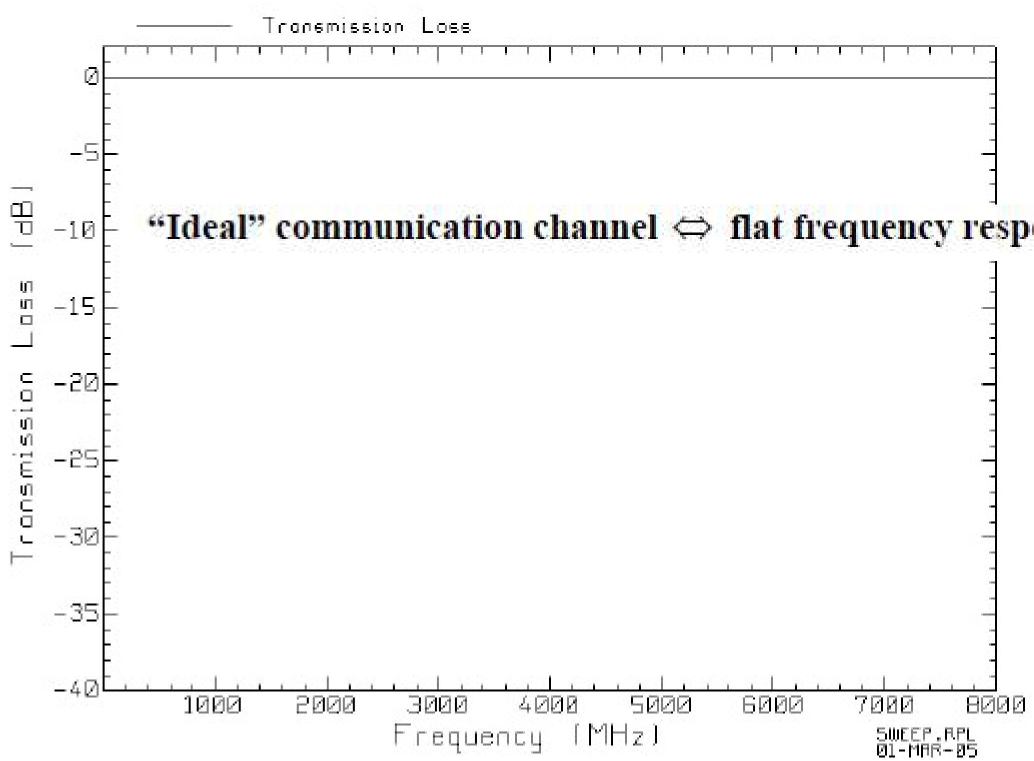


$$H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)}$$

The ideal communication channel has $|H(j\omega)| = 1$



- The transmission line has $R_c = 50$ ohms and $u = 20$ cm/ns and length 12 cm.
- The generator is "matched" with $R_s = 50$ ohms.
- The load is "matched" with $R_1 = 50$ ohms.



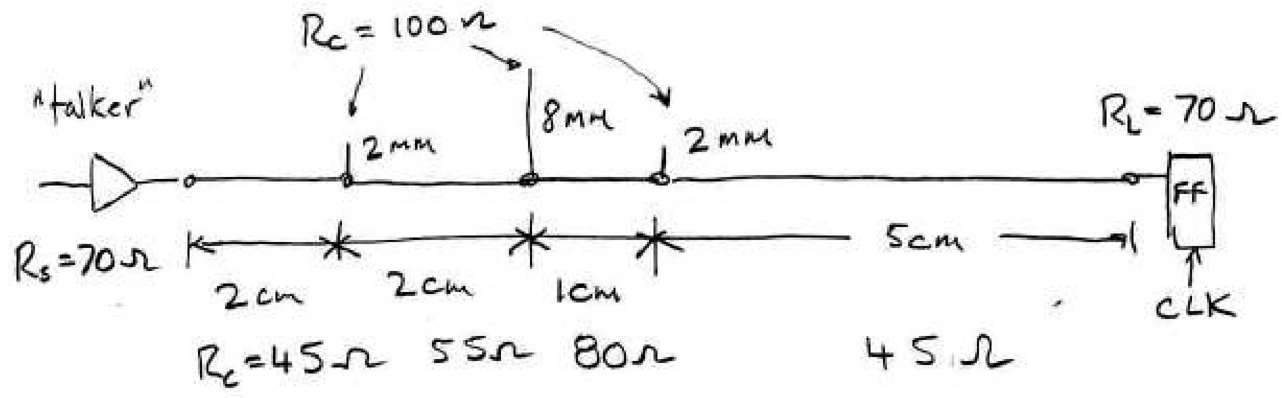
"Ideal" communication channel ⇔ flat frequency response

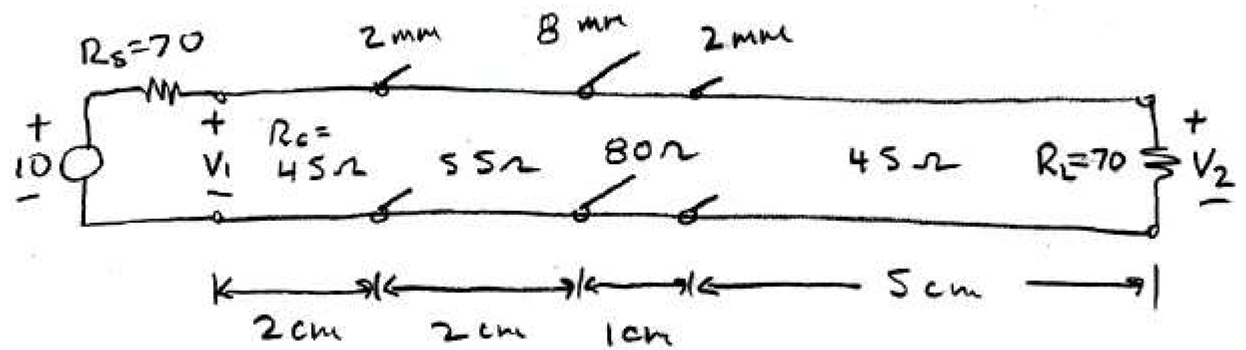
"Transmission loss" is defined as

$$TL = 20\log\left(\frac{|V_2|}{|V_1|}\right) = 20\log(|H(f)|)$$

Principle: The frequency response must be flat from DC to the knee frequency.

Transfer Function of a "Messy" Communication Channel





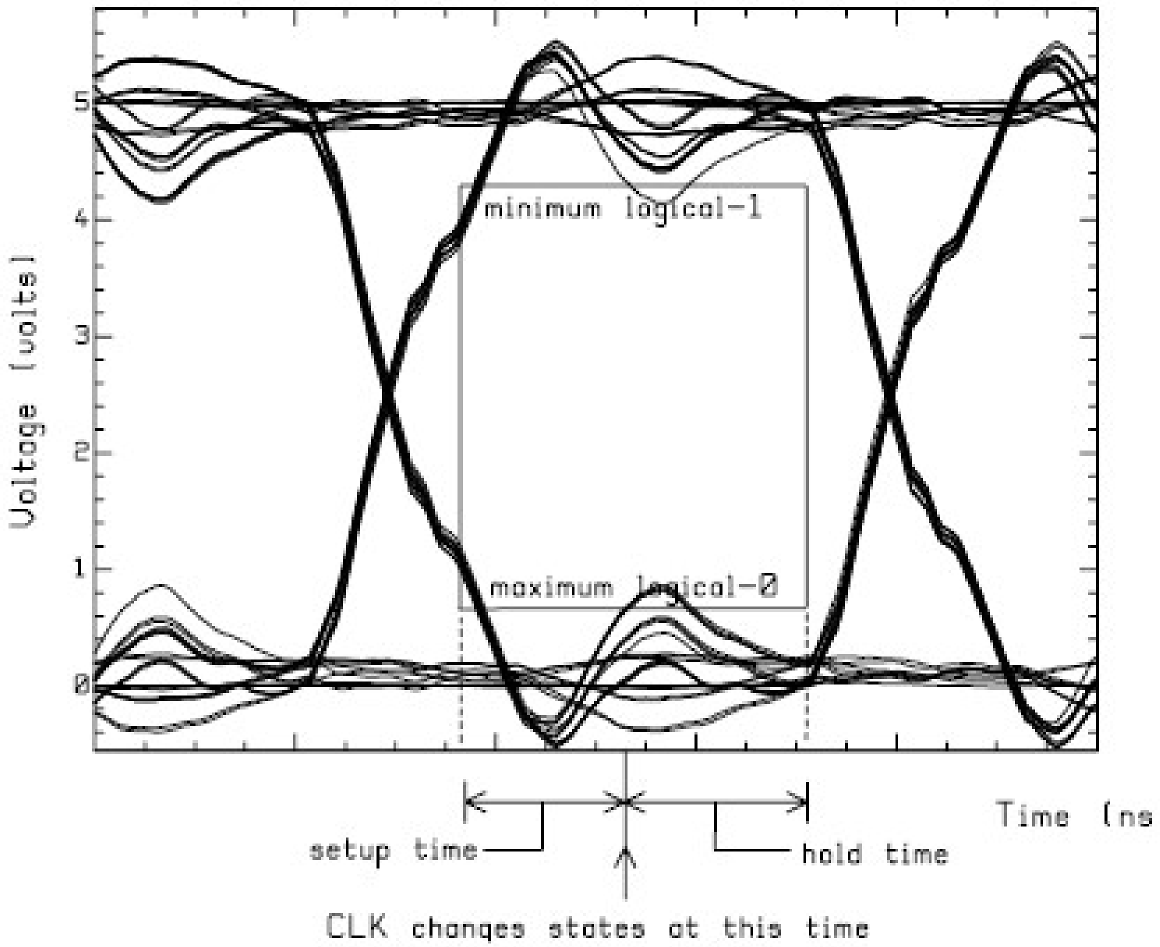
The wave speed is $u = 14$ cm/ns.
The clock frequency is 2 GHz
The rise time is 0.05 ns.

The knee frequency is between 7 and 10 GHz.

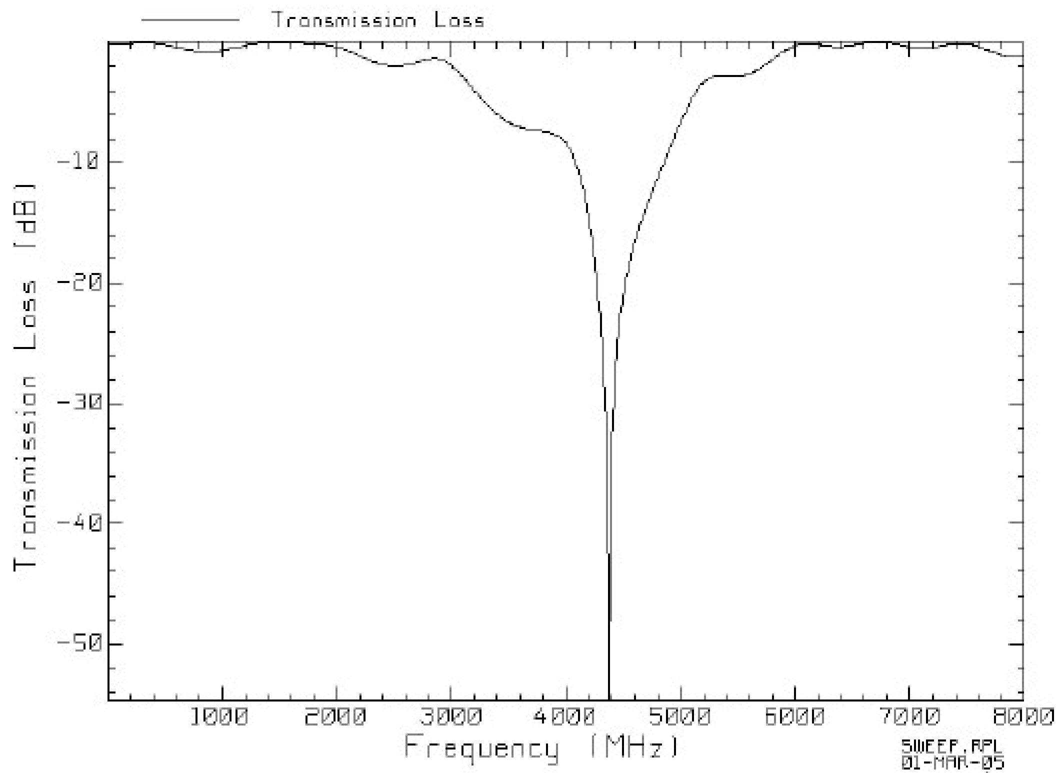
We need a channel bandwidth of about 7 to 10 GHz to propagate this signal.

Eye Pattern

From Set 8 of these lecture notes.

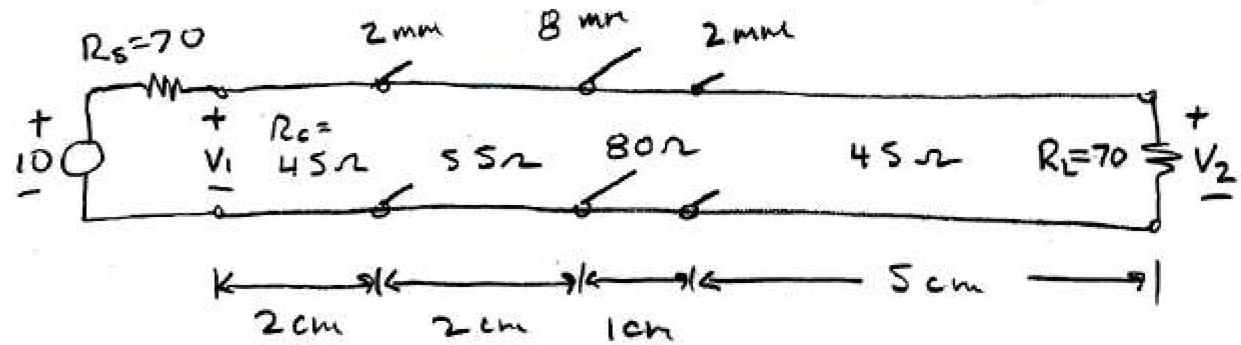


Transfer Function

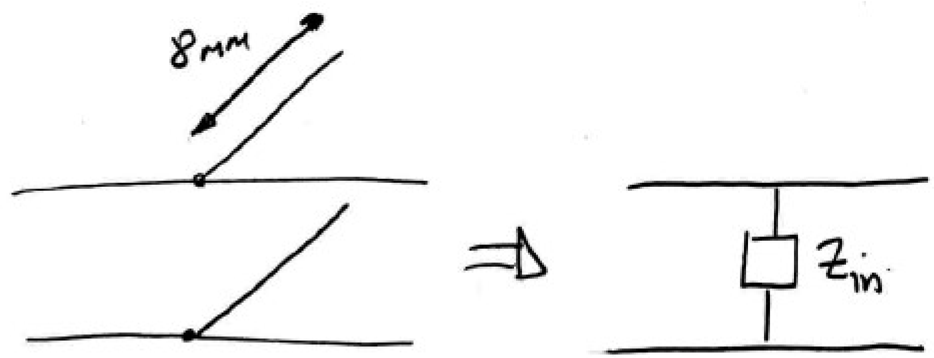


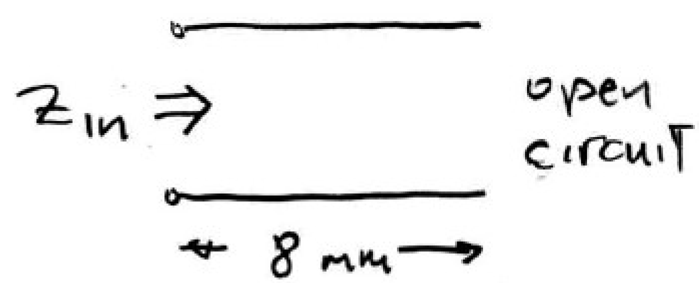
We need a channel bandwidth of about 7 to 10 GHz to propagate this signal.

Homework: Use TRLINE or LTspice to calculate this transfer function.

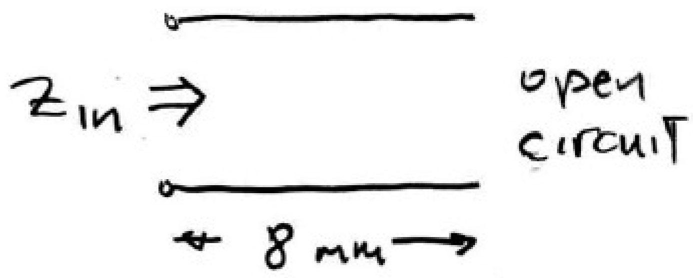


The circuit includes an 8-mm branch terminated in an open circuit.





Open-circuited branch:



$$Z_{in} = R_c \frac{Z_L + jR_c \tan \beta L}{R_c + jZ_L \tan \beta L}$$

$$Z_{in} = \lim_{Z_L \to \infty} R_c \frac{Z_L + jR_c \tan \beta L}{R_c + jZ_L \tan \beta L}$$

$$Z_{in} = R_c \frac{Z_L}{jZ_L \tan \beta L} = -jR_c \cot \beta L$$

When is Z_{in}=0?

$\cot \beta L = 0$ when $\beta L = \frac{\pi}{2}$ The branch becomes a short circuit when $\beta L = \frac{\pi}{2}$.

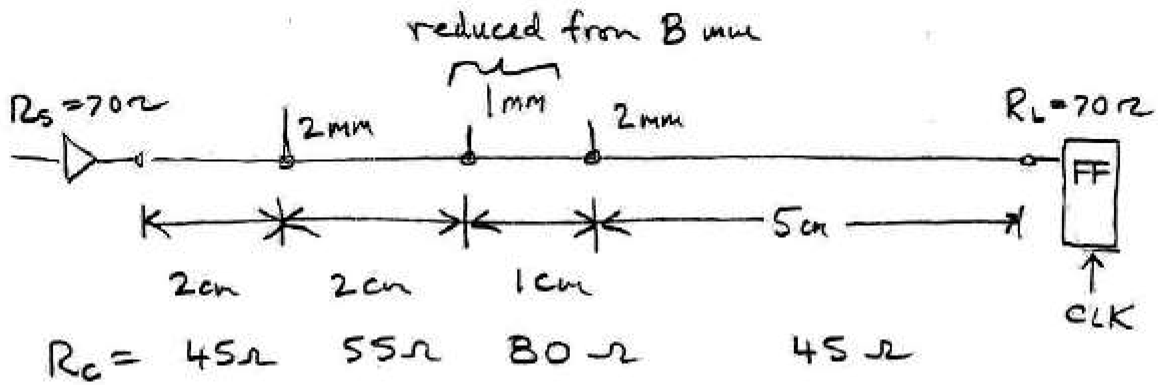
and since $\beta = \frac{2\pi}{\lambda}$, we have $\frac{2\pi}{\lambda} L = \frac{\pi}{2}$ so $L = \frac{\lambda}{4}$ so the input impedance is zero when $\lambda = 4L$

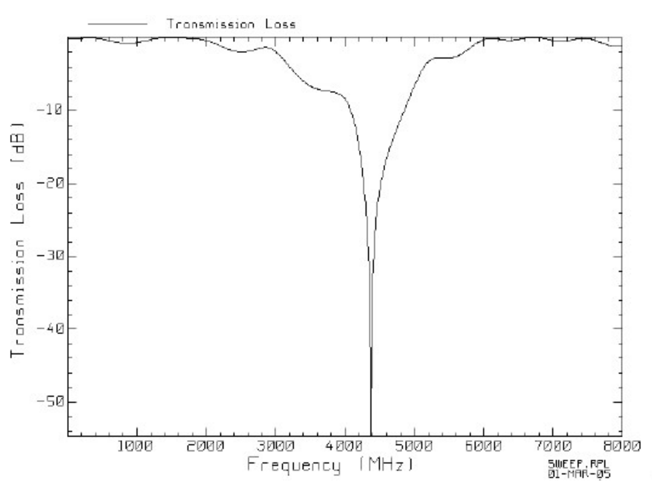
or $f = \frac{u}{\lambda} = \frac{u}{4L} = \frac{14}{4x0.8} = 4.375$ GHz.

So at 4.375 GHz the 8-mm branch "shorts out" the communication channel and there is a drop-out in the frequency response.

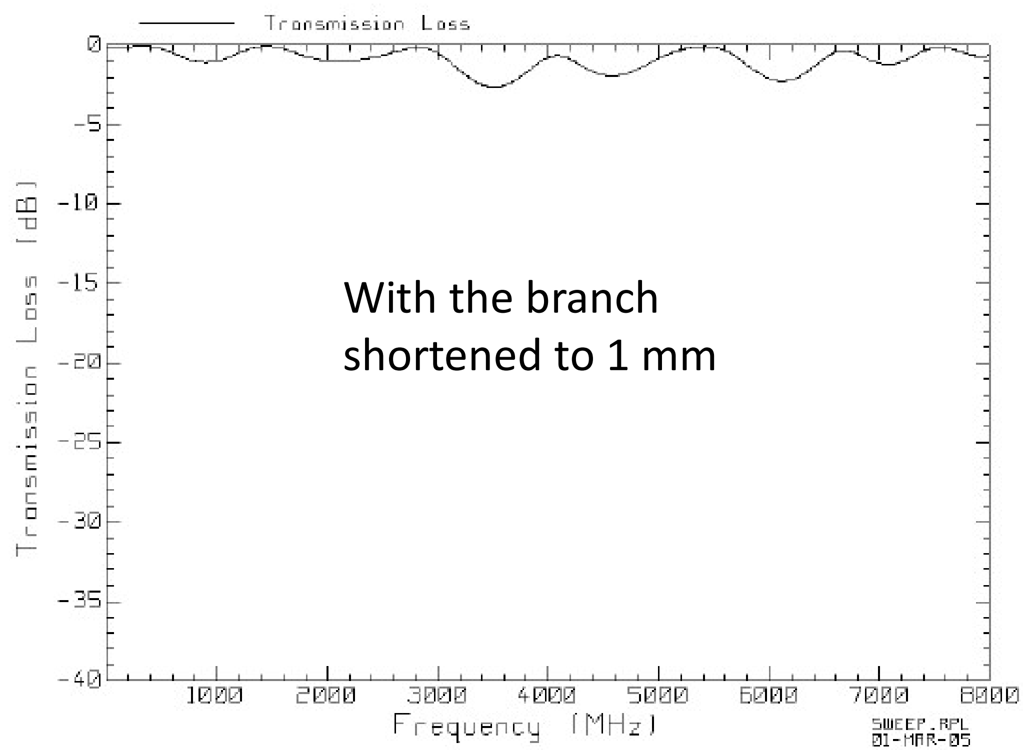
Fixing the "Messy" Communication Channel

Step 1
Reduce the 8 mm branch to 1 mm

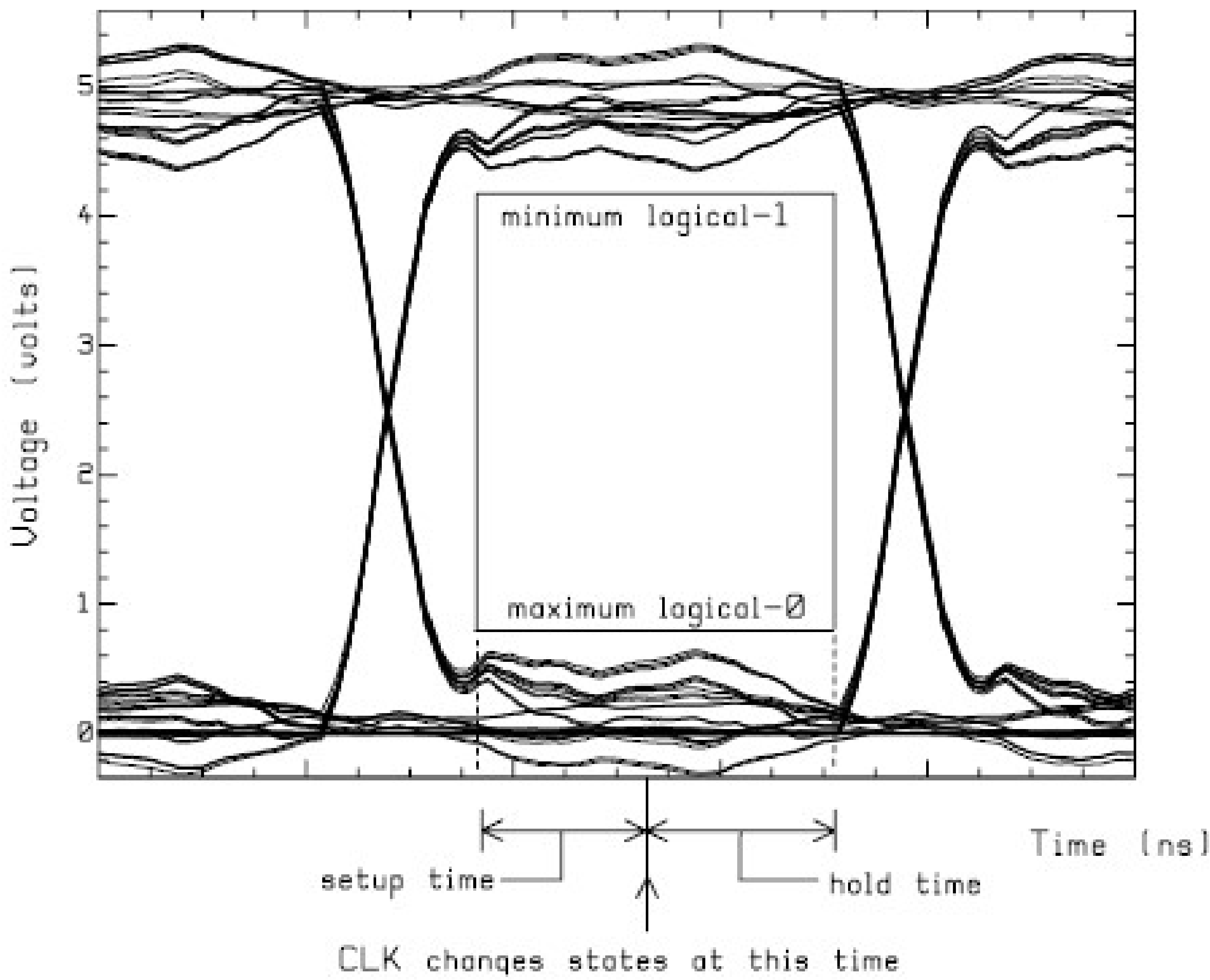




With the 8 mm branch

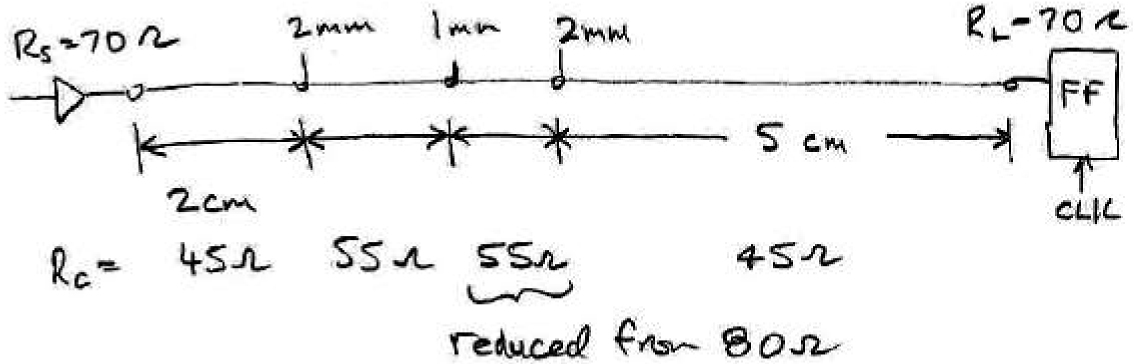


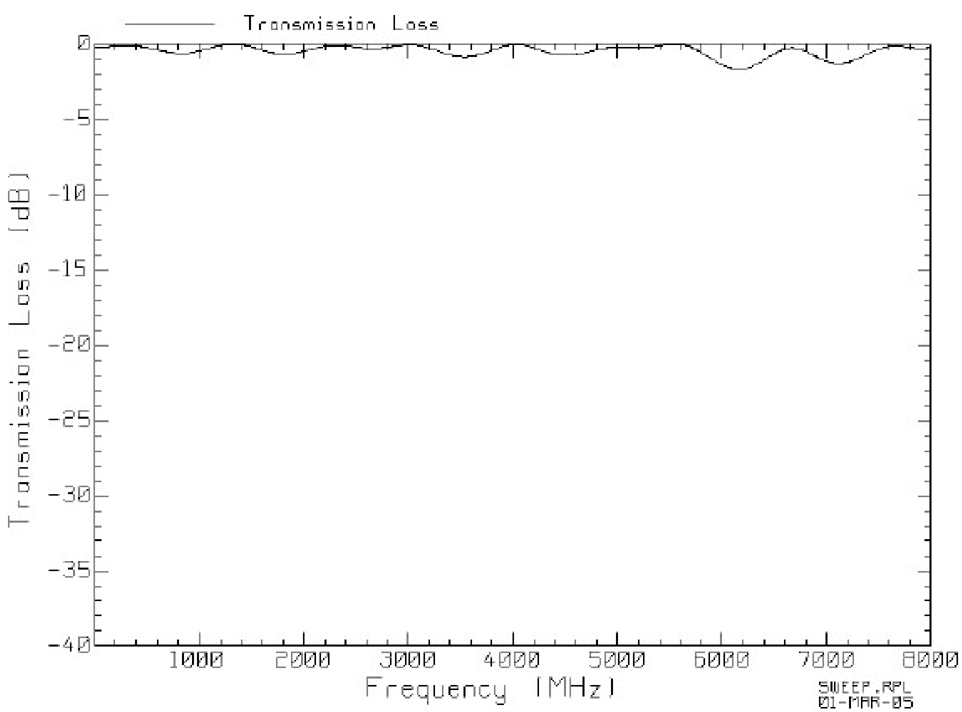
With the branch shortened to 1 mm

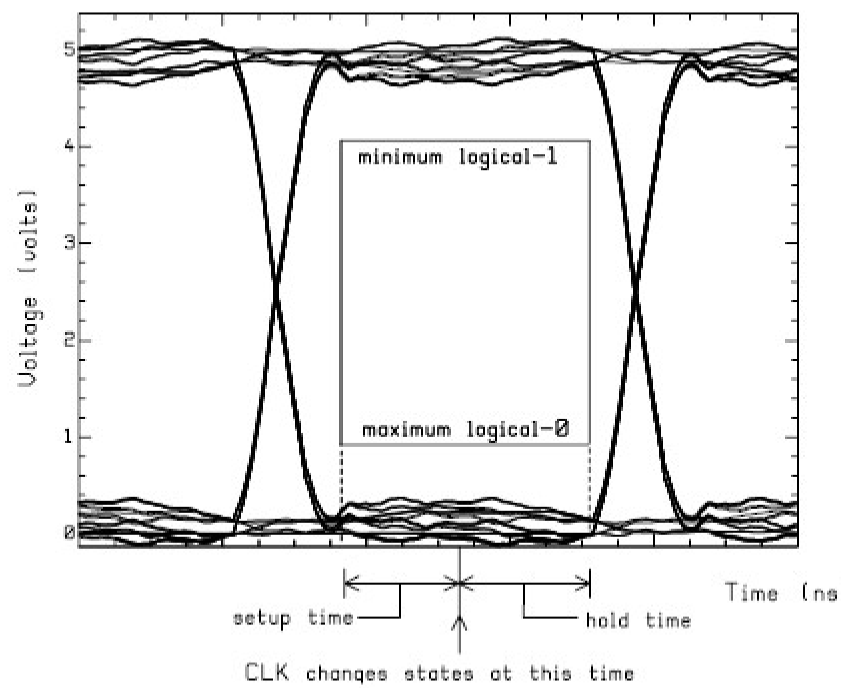


Voltage (volts)
5
4
3
2
1
0
minimum logical-1
maximum logical-0
Time (ns)
setup time
hold time
CLK changes states at this time

Step 2
Reduce the mismatch by changing the short 80-Ω line to a 55-Ω line.







Conclusions

1. The bandwidth $f_{\max}$ of a digital signal at a clock frequency f and with a rise time of T_r ns depends ONLY on the rise time, and can be estimated as

$$f_{\max} = \frac{0.35}{T_r} \text{ GHz}$$

2. A circuit interconnect must have a reasonably flat frequency response from DC to $f_{\max}$.

3. If the bandwidth is flat from DC to $f_{\max}$, then we can expect an "open" eye pattern.

4. Open-circuited branches in the transmission line can cause "dropouts" in the frequency response, and these close the "eye pattern".

5. The frequency response or "transfer function" $H(j\omega)$ is a useful tool for understanding and fixing the behaviour of circuit interconnects.