ELEC353 Lecture Notes Set 11

The homework assignments are posted on the course web site.
http://users.encs.concordia.ca/~trueman/web_page_353.htm

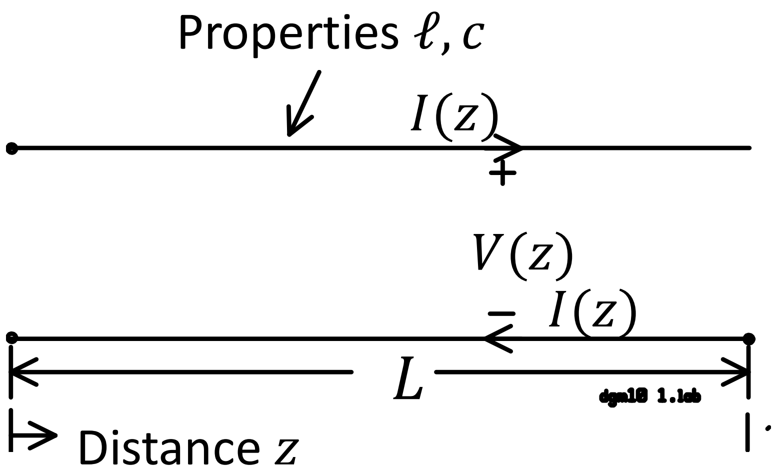
Homework #7: Do homework #7 by March 8, 2019.
Homework #8: Do homework #8 by March 15, 2019.
Homework #9: Do homework #9 by March 22, 2019.

Last Day of Classes: April 13, 2019.

Final exam date: Tuesday April 23, 2019, 9:00 to 12:00, in H531.

Review:

Summary – Lossless Transmission Lines



Lossless transmission line equations:

$$\frac{dV}{dz} = -j\omega\ell I$$

$$\frac{dI}{dz} = -j\omega c V$$

Lossless wave equation:

$$\frac{d^2V}{dt^2} = -\beta^2 V$$

where $\beta = \omega\sqrt{\ell c}$ is the phase constant.

Voltage and current:

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z) = \frac{V^+}{R_c} e^{-j\beta z} - \frac{V^-}{R_c} e^{j\beta}$$

Characteristic resistance:

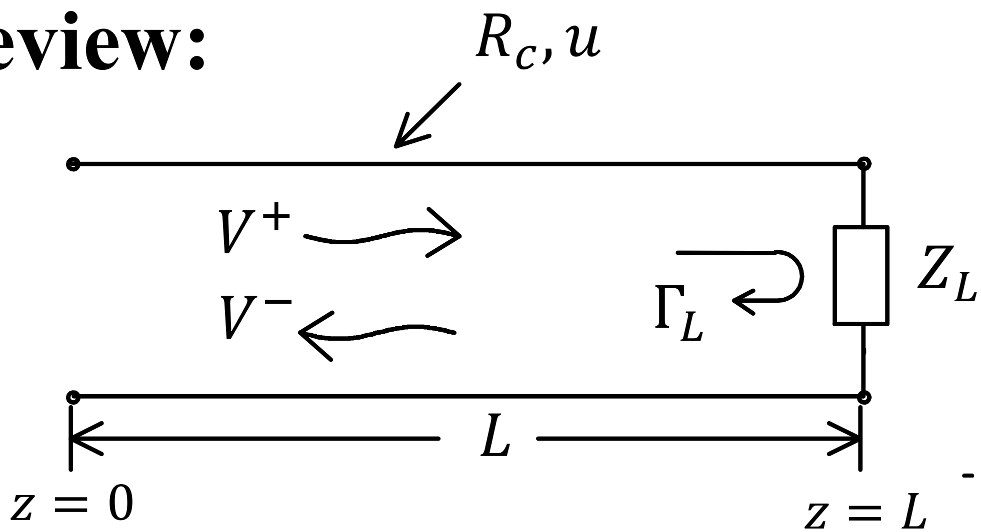
$$R_c = \sqrt{\frac{\ell}{c}}$$

In general:	Lossless case:
$u = \frac{\omega}{\beta}$	$u = \frac{1}{\sqrt{\ell c}}$
$\lambda = \frac{2\pi}{\beta}$	$\lambda = \frac{u}{f}$
	$\beta = \omega\sqrt{\ell c} = \frac{\omega}{u}$

Transmission Line Terminated with a Load

Review:

Inan and Inan Sections 3.2 and 3.3



Voltage and current:

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z) = \frac{V^+}{R_c} e^{-j\beta z} - \frac{V^-}{R_c} e^{j\beta z}$$

If we know V^+, can we find V^-?

At the load we must satisfy $V(L) = Z_L I(L)$

$$V(L) = V^+ e^{-j\beta} + V^- e^{j\beta}$$

$$I(L) = \frac{V^+}{R_c} e^{-j\beta} - \frac{V^-}{R_c} e^{j\beta}$$

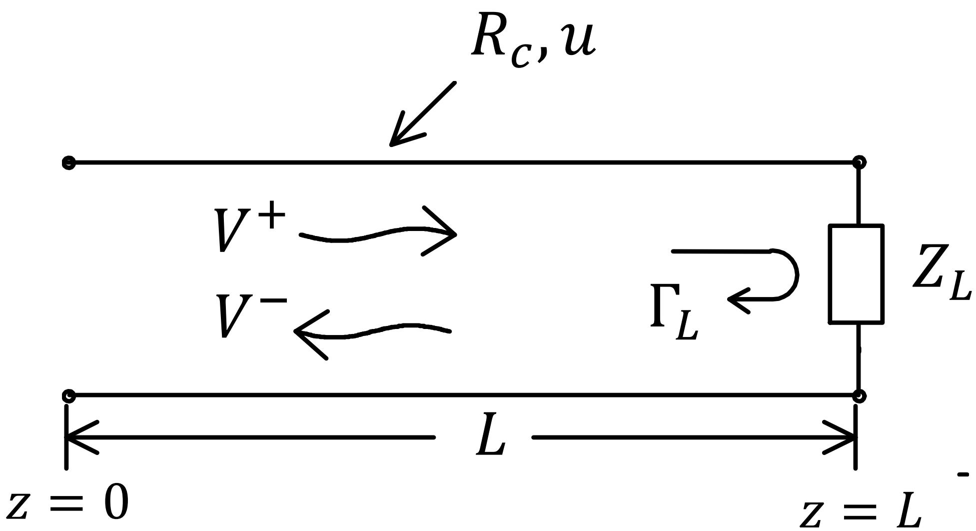
$$V(L) = Z_L I(L)$$

$$V^+ e^{-j\beta L} + V^- e^{j\beta L} = Z_L \left(\frac{V^+}{R_c} e^{-j\beta L} - \frac{V^-}{R_c} e^{j\beta L} \right)$$

$$V^- = \frac{Z_L - R_c}{Z_L + R_c} e^{-j2\beta L} V^+$$

Review:

Reflection Coefficient at the Load



$$\Gamma_L = \frac{V^- e^{j\beta L}}{V^+ e^{-j\beta L}}$$

$$V^- = V^+ e^{-2j\beta L} \frac{Z_L - R_c}{Z_L + R_c}$$

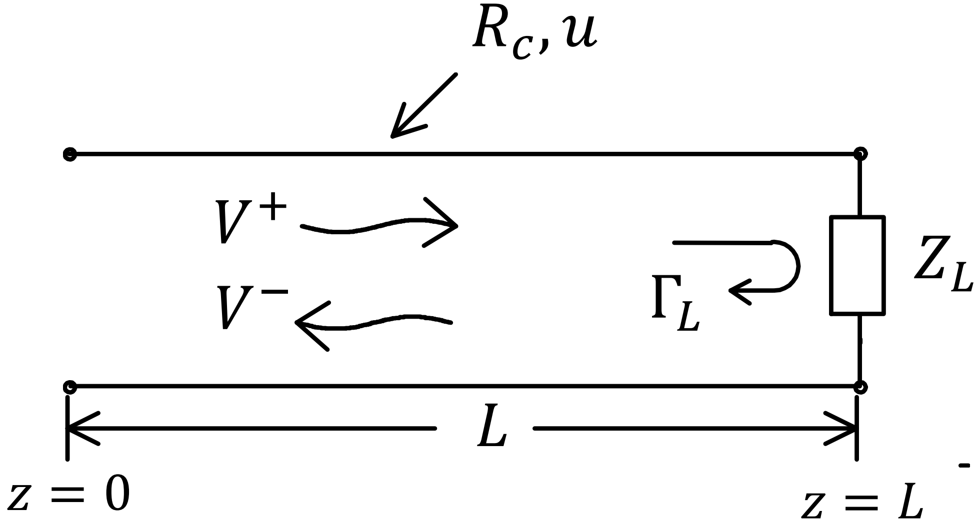
$$\Gamma_L = \frac{V^+ e^{-2j\beta L} \dfrac{Z_L - R_c}{Z_L + R_c} e^{j\beta L}}{V^+ e^{-j\beta L}}$$

$$\Gamma_L = \frac{Z_L - R_c}{Z_L + R_c}$$

$$V^- = \Gamma_L e^{-j2\beta L} V^+$$

Review:

Voltage and Current



Voltage and Current

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z) = \frac{V^+}{R_c} e^{-j\beta z} - \frac{V^-}{R_c} e^{j\beta z}$$

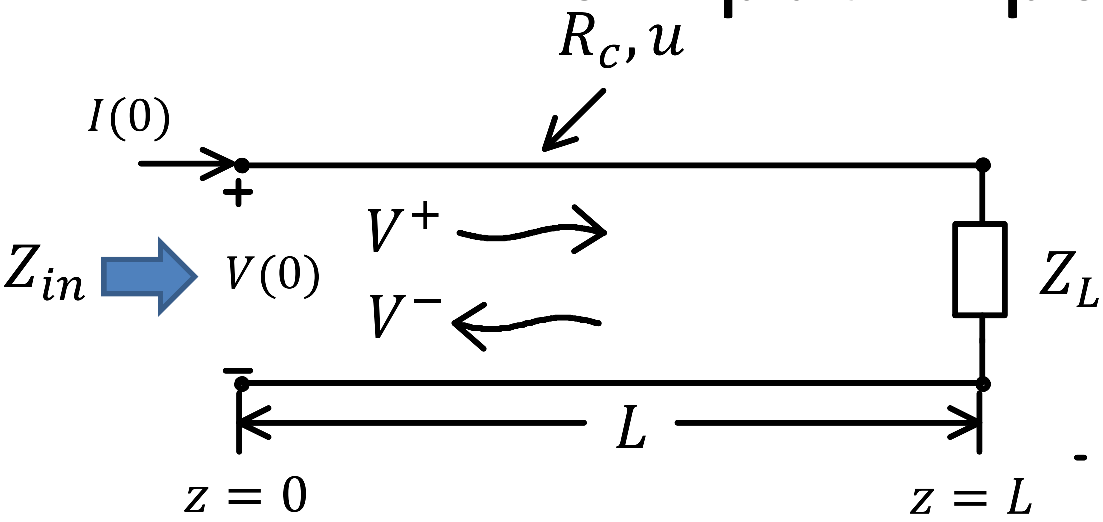
$$V^- = \Gamma_L e^{-j2\beta L} V^+$$

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta}$$
$$V(z) = V^+ e^{-j\beta z} + \Gamma_L e^{-j2\beta L} V^+ e^{j\beta z}$$
$$V(z) = V^+ (e^{-j\beta z} + \Gamma_L e^{-j2\beta L} e^{j\beta z})$$

$$V(z) = V^+ e^{-j\beta L} (e^{-j\beta(z-L)} + \Gamma_L e^{j\beta(z-L)})$$

$$I(z) = V^+ e^{-j\beta L} (\frac{1}{R_c} e^{-j\beta(z-L)} - \frac{1}{R_c} \Gamma_L e^{j\beta(z-L)})$$

Review: The Input Impedance



$$V(z) = V^+ e^{-j\beta L}\left(e^{-j\beta(z-L)} + \Gamma_L e^{j\beta(z-L)}\right)$$

$$I(z) = V^+ e^{-j\beta L}\left(\frac{1}{R_c} e^{-j\beta(z-L)} - \frac{1}{R_c}\Gamma_L e^{j\beta(z-L)}\right)$$

$$\Gamma_L = \frac{Z_L - R_c}{Z_L + R_c}$$

$$Z_{in} = \frac{V(0)}{I(0)}$$

$$V(0) = V^+ e^{-j\beta L}\left(e^{-j\beta(0-L)} + \Gamma_L e^{j\beta(0-L)}\right)$$

$$V(0) = V^+ e^{-j\beta L}\left(e^{j\beta L} + \Gamma_L e^{-j\beta L}\right)$$

$$I(0) = V^+ e^{-j\beta L}\left(\frac{1}{R_c} e^{-j\beta(0-L)} - \frac{1}{R_c}\Gamma_L e^{j\beta(0-L)}\right)$$

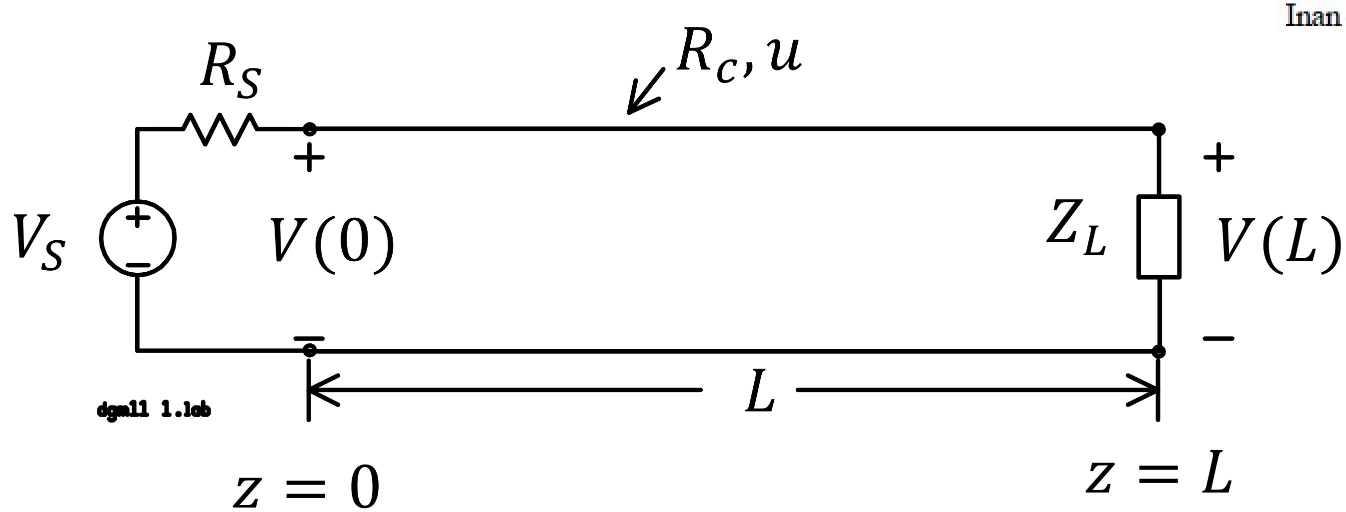
$$I(0) = V^+ e^{-j\beta L}\left(\frac{1}{R_c} e^{j\beta L} - \frac{1}{R_c}\Gamma_L e^{-j\beta L}\right)$$

$$Z_{in} = \frac{V(z=0)}{I(z=0)} = \frac{V^+ e^{-j\beta L}\left(e^{j\beta L} + \Gamma_L e^{-j\beta L}\right)}{V^+ e^{-j\beta L}\left(\frac{1}{R_c} e^{j\beta L} - \frac{1}{R_c}\Gamma_L e^{-j\beta L}\right)}$$

$$Z_{in} = R_c \frac{Z_L + jR_c \tan \beta L}{R_c + jZ_L \tan \beta L}$$

How to Solve a Transmission Line Circuit

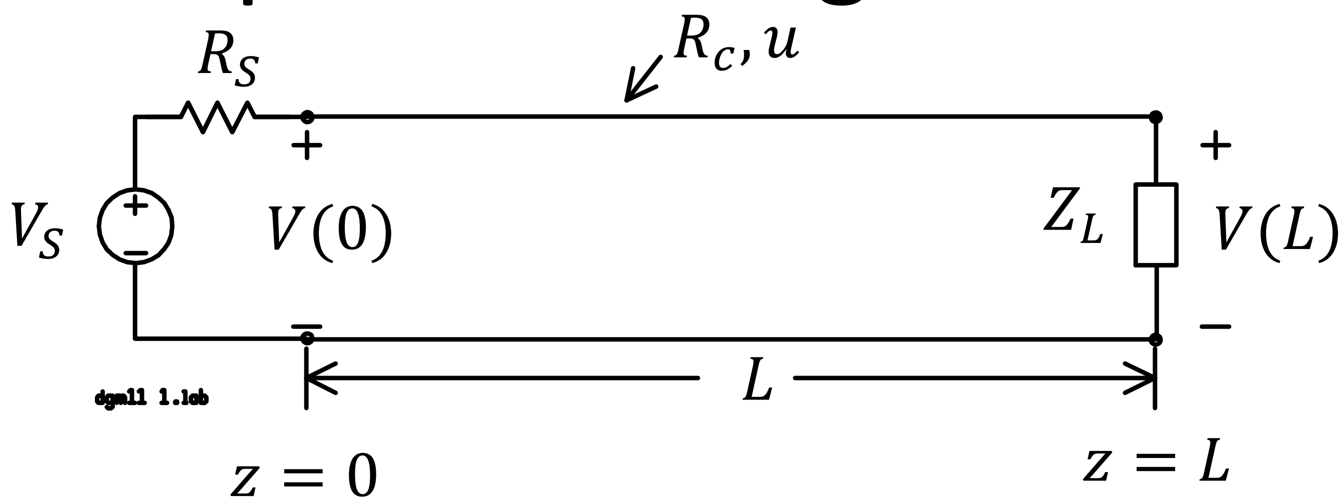
Inan and Inan Section 3.3



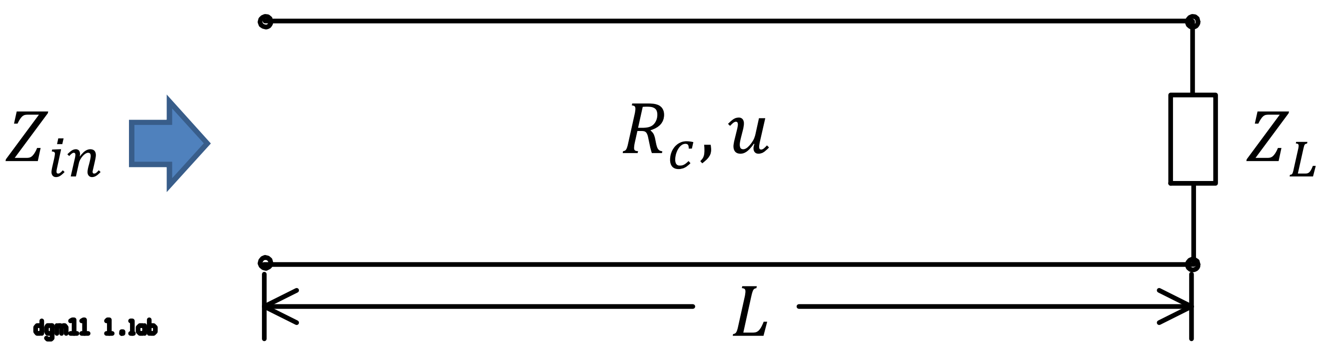
Procedure for solving a transmission-line circuit:

1. Find the input impedance.
2. Find the voltage at the input *V(0)*.
3. Find the travelling-wave amplitude V^+.
4. Find the reflected travelling-wave amplitude V^-.
5. Find the voltage at the load *V(L)*.
6. Find the power delivered to the load.

Steps for Solving the Circuit

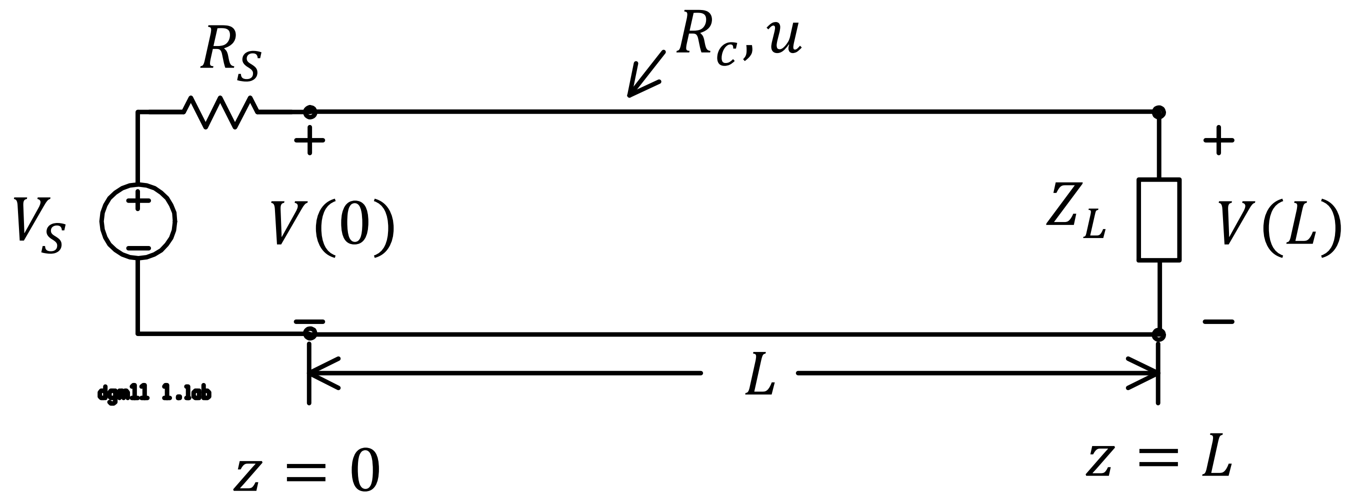


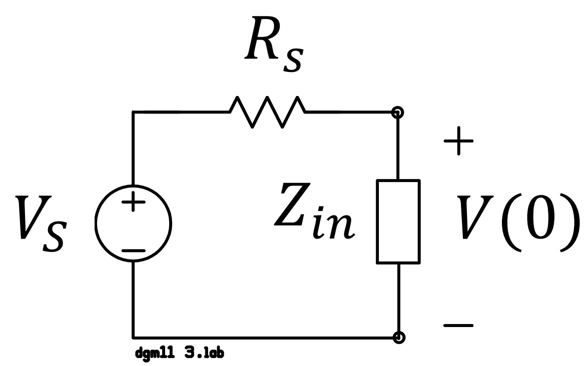
Step 1: Find the input impedance:



$$Z_{in} = R_c \frac{Z_L + jR_c \tan \beta L}{R_c + jZ_L \tan \beta L}$$

Step 2: Solve the generator circuit





$$V(0) = \frac{Z_{in} V_s}{Z_{in} + R_s}$$

Step 3: Find V^+

$$V(z) = V^+(e^{-j\beta z} + \Gamma_L e^{-2j\beta L} e^{j\beta z})$$

$$V(0) = V^+(e^{-j\beta 0} + \Gamma_L e^{-2j\beta L} e^{j\beta 0})$$

$$V(0) = V^+\left(1 + \Gamma_L e^{-j2\beta L}\right)$$

$$V^+ = \frac{V(0)}{1+\Gamma_L e^{-2j\beta L}} \qquad \text{where} \qquad \Gamma_L = \frac{Z_L - R_c}{Z_L + R_c}$$

Step 4: Find V^{-}

$$V^{-} = V^{+}\Gamma_L e^{-2j\beta L}$$

Step 5: Find $V(L)$

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z}$$

$z = L$

$$V(L) = V^{+}e^{-j\beta L} + V^{-}e^{j\beta L}$$

Step 6: Find the power

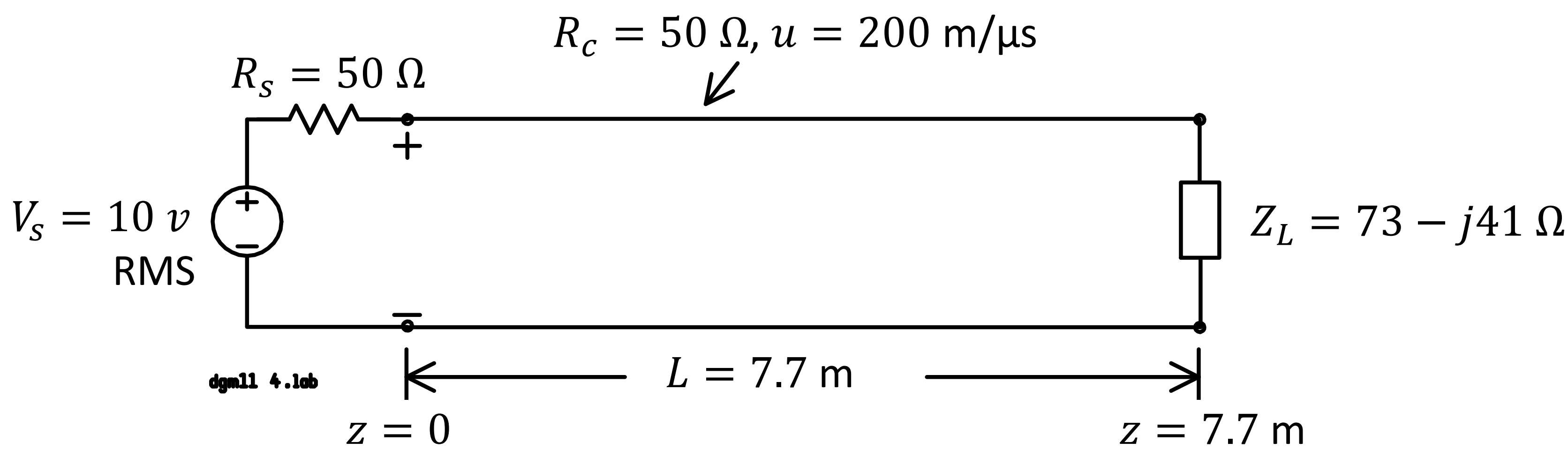
$$P_{av} = \frac{1}{2}\mathrm{Re}\left[VI^*\right]$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V(L)I^*(L)\right]$$

Note that for a lossless transmission line the INPUT power is equal to the power dissipated by the load:

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V(0)I^*(0)\right]$$

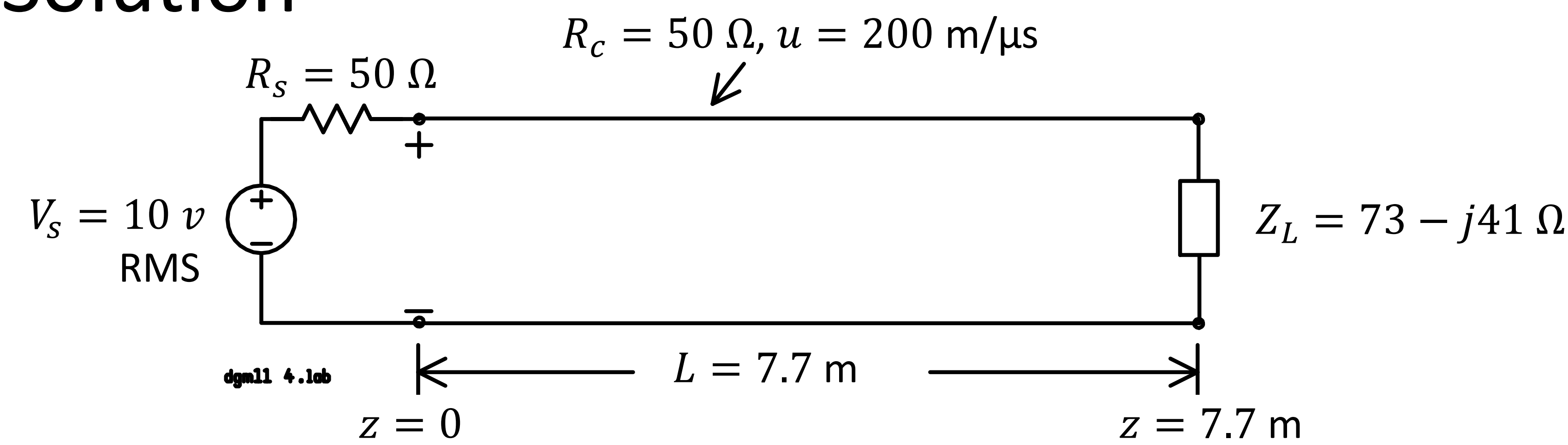
Example: Solve a Transmission-Line Circuit



A radio-frequency generator at 100 MHz produces an open-circuit voltage of 10 volts RMS and has an internal resistance of 50 ohms. It drives an antenna through a length of $L = 7.7$ m of coaxial cable with characteristic impedance $Z_0 = 50$ ohms and speed of travel $u = 20$ cm/ns. The input impedance of the antenna is $Z_L = 73 - j41$ ohms. Find the voltage across the antenna, $V(L)$, and the power delivered to the antenna.

Notation: $Z_0 = R_c$

Solution



$f = 100$ MHz and $u = 200$ meters per microsecond

$$\lambda = \frac{u}{f} = \frac{200}{100} = 2 \quad \text{meters.}$$

$$\frac{L}{\lambda} = \frac{7.7}{2} = 3.85 \quad \text{wavelengths.}$$

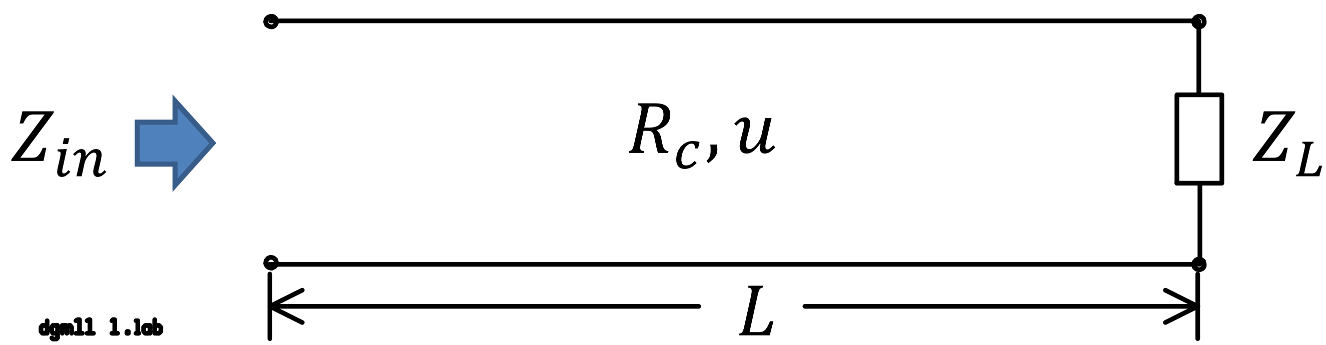
$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi \quad \text{radians per meter}$$

$$\beta L = \frac{2\pi}{\lambda} L = \frac{2\pi}{\lambda} x7.7 = 24.190 \text{ radians or 1386 degrees}$$

The "**electrical length**" of the transmission line is the length in degrees.

Remove full cycles of 360 degrees: $\beta L = 1386 - 360 - 360 - 360 - 360 = -54$ degrees

Step 1: Find the input impedance



$$\beta L = -54$$ degrees

$$\tan \beta L = \tan -54^\circ = -1.376$$

$Z_0 = R_c = 50\Omega$ and $Z_L = (73 - j41)\Omega$

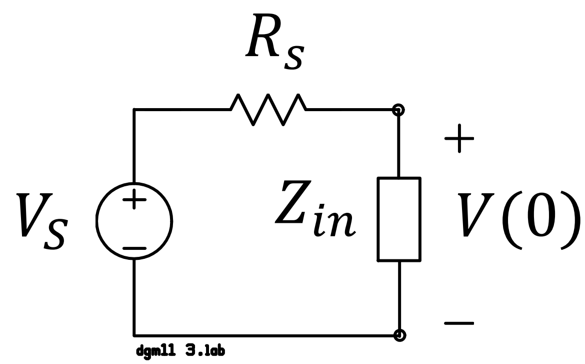
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} = 50 \frac{(73 - j41) + j50(-1.376)}{50 + j(73 - j41)(-1.376)} = 52.12+39.67\text{j Ohms}$$

Remark: My intention is that you go home and do this problem yourself from scratch. You need to practice doing the complex arithmetic.

Compare your answers with mine, and make sure that you are correct.

Step 2: Solve the Input Circuit

$V_s = 10$ Volts RMS $\quad R_s = 50\Omega$



$Z_{in} = 52.12 + j39.67$ Ohms

$$V(0) = \frac{Z_{in} V_s}{Z_{in} + R_s} = \frac{(52.12 + j39.67) \cdot 10}{(52.12 + j39.67) + 50} = 5.978\angle 16.04^\circ$$ Volts RMS

Step 3: Find V^+

$$V^+ = \frac{V(0)}{1+\Gamma_L e^{-2j\beta L}}$$

$$V(0) = 5.978\angle 16.04° = 5.79 + j1.69$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(73 - j41) - 50}{(73 - j41) + 50} = 0.2683 - 0.2439j = 0.3626\angle -42.27°$$

$\beta L = -54°$, we have $e^{-2j\beta L} = e^{j108°} = 1\angle 108°$

$$V^+ = \frac{V(0)}{1+\Gamma_L e^{-2j\beta L}} = \frac{5.79 + j1.69}{1 + (0.363\angle -42.3°)(1\angle 108°)} = 5.000\angle\left(-9.134x10^{-5}\right)°$$

and neglecting the almost-zero angle,

$V^+ = 5.000\angle 0°$ volts RMS

Remark: when the source is matched with $R_s = R_c$

then we will always have $V^+ = \frac{V_s}{2}$

Step 4: Find V^-

$$V^- = V^+\Gamma_L e^{-2j\beta L}$$

$$V^+ = 5\angle 0°$$

$$\Gamma_L = 0.3626\angle -42.27°$$

$$e^{-j2\beta L} = 1\angle 108°$$

$$V^- = V^+\Gamma_L e^{-2j\beta L} = (5.000\angle 0°)(0.3626\angle -42.27°)(1\angle 108°)$$

$$V^- = 1.813\angle 65.73° = 0.7452+1.653j \text{ volts RMS}$$

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$V(z) = (5.000\angle 0°)e^{-j\beta z} + (1.813\angle 65.73°)e^{j\beta z}$$ Volts RMS

Step 5: Find the voltage across the load.

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

so at the load, $z = L$, and

$$V(L) = V^+ e^{-j\beta L} + V^- e^{j\beta L}$$

$$V(z) = (5.000\angle 0°)e^{-j\beta z} + (1.813\angle 65.73°)e^{j\beta z}$$

$$V(z = L) = (5.000\angle 0°)e^{-j\beta L} + (1.813\angle 65.73°)e^{j\beta L}$$

Volts RMS

$$\beta L = -54°$$

$$V(L) = (5.000\angle 0°)e^{j54°} + (1.813\angle 65.73°)e^{-j54°} = 4.714 + 4.414j = 6.458\angle 43.12°$$

Volts RMS

Step 6: Find the Power Delivered to the Load

$$P_{av} = \mathrm{Re}\left[V(L)I^*(L)\right]$$

In this example, *V* and *I* are phasors relative to RMS values so we use the power formula for RMS.

$$V(L) = 6.458\angle 43.12°$$

$$Z_L = (73 - j41)\Omega$$

$$I(L) = \frac{V(L)}{Z_L} = \frac{6.458\angle 43.12°}{73 - j41} = 0.07713\angle 72.44°$$

Amps

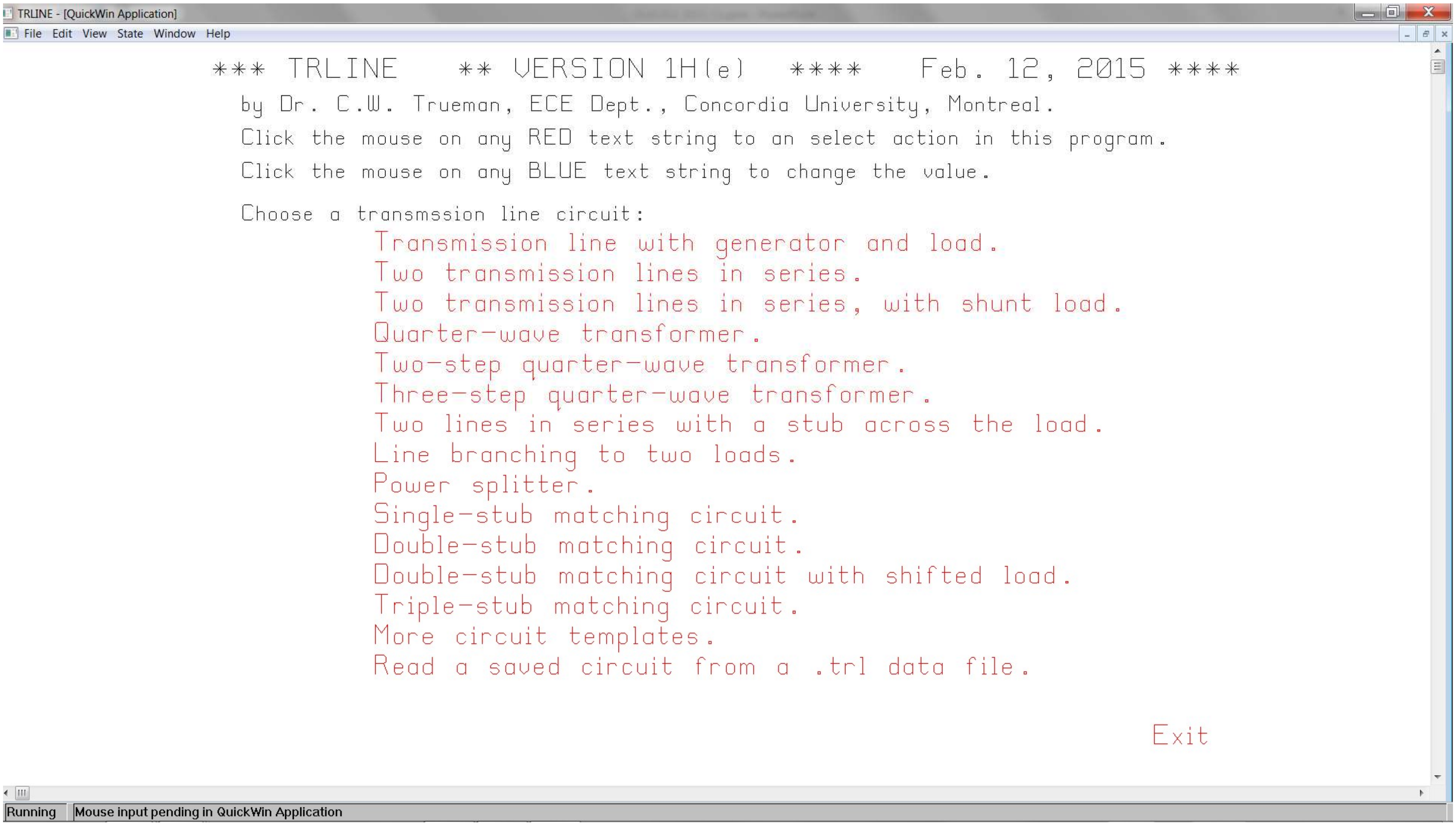
conjugate

$$\begin{aligned} P_{av} &= \mathrm{Re}\left[V(L)I^*(L)\right] = \mathrm{Re}\left[(6.458\angle 43.12)(0.07713\angle -72.44)\right] \\ &= \mathrm{Re}\left[0.4981\angle -29.32°\right] \\ &= \left[0.4981\cos(-29.32)\right] = 0.4343 \text{ watts or } 434.3 \text{ mW.} \end{aligned}$$

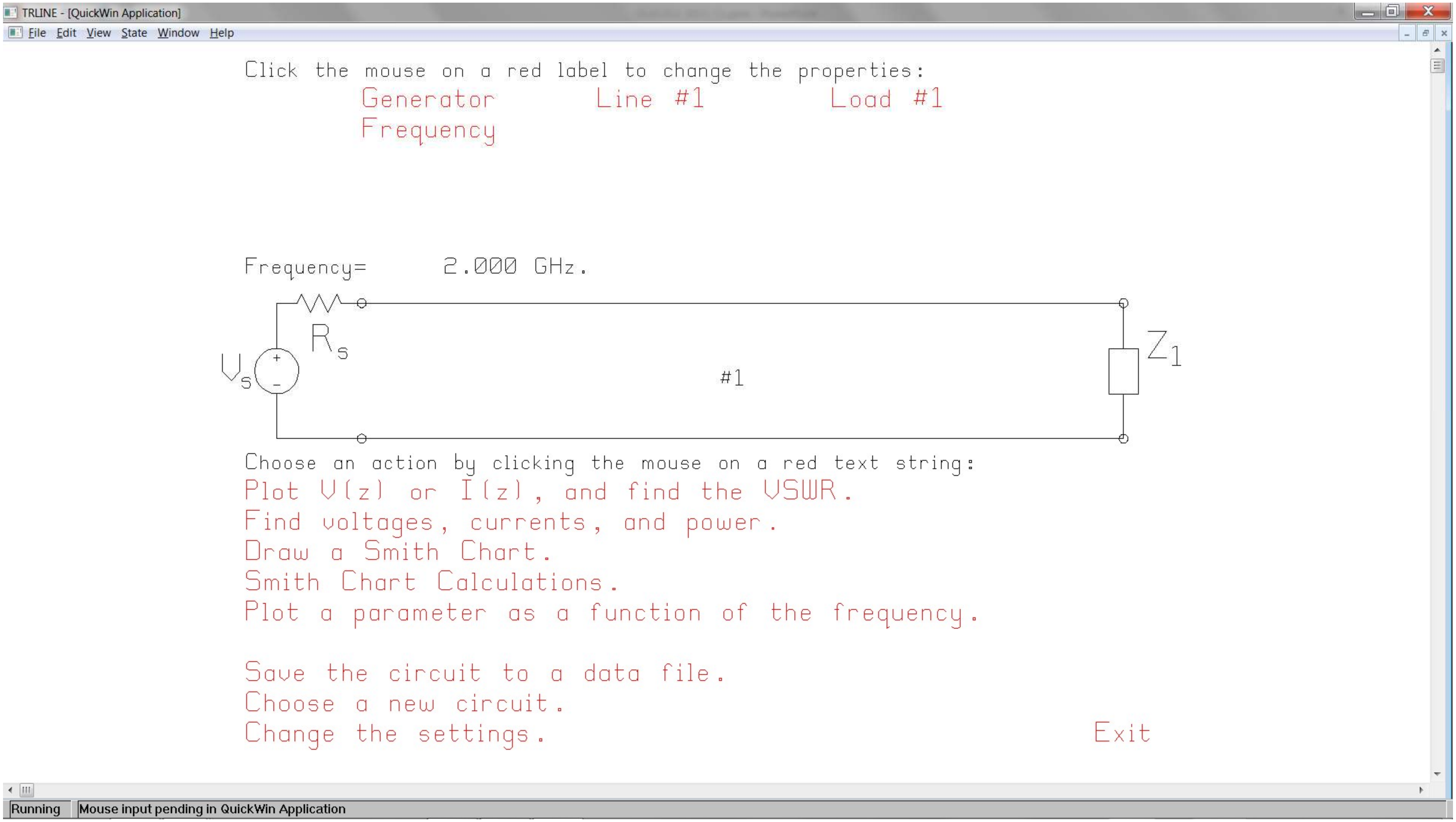
Program TRLINE

- TRLINE solves transmission-line circuits in the sinusoidal steady state.
- Use TRLINE to solve homework problems to check your answers.
- Use TRLINE to design and test single-stub and double-stub matching problems.

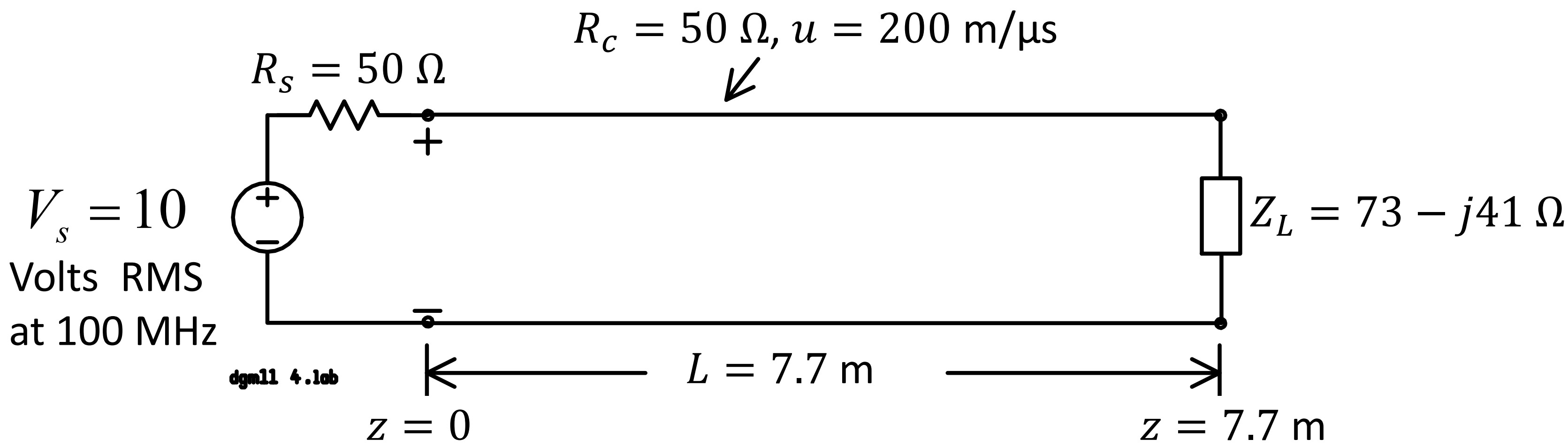
TRLINE Entry Menu



TRLINE Main Menu



Solve the example with TRLINE

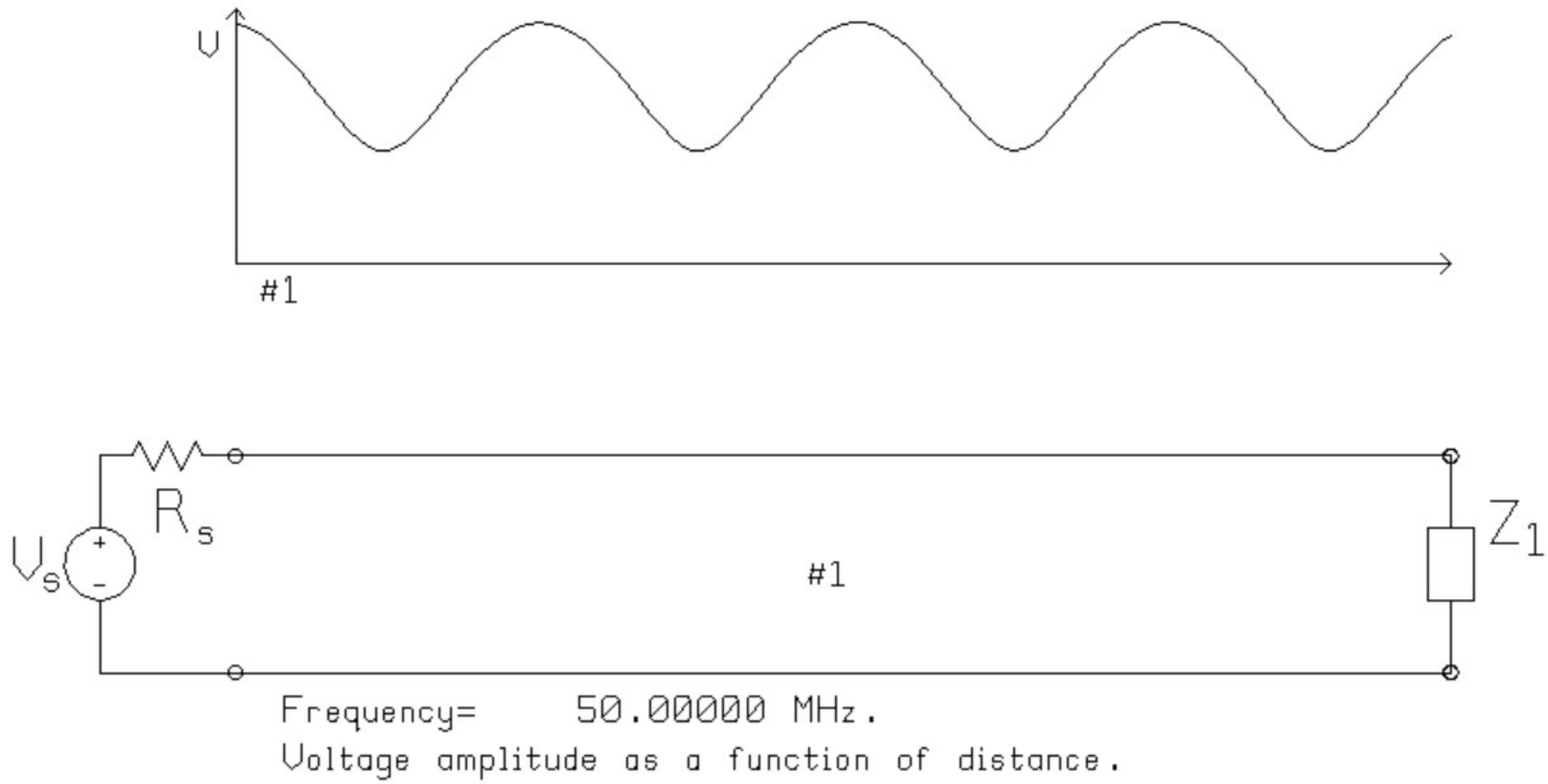


$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} = 50 \frac{(73 - j41) + j50(-1.376)}{50 + j(73 - j41)(-1.376)} = 52.12{+}39.67\text{j Ohms}$$

Verify:

- The input impedance $Z_{in} = 52.12 + j39.67$
- The voltage across the generator $V(0) = 5.978\angle 16.04°$
- The values of the travelling wave amplitudes $V^+ = 5\angle 0°$, $V^- = 1.813\angle 65.73°$
- The voltage across the load $V(L) = 6.458\angle 43.12°$
- The power delivered to the load $P_{av} = 434.3$ mW

What is the Voltage on the Transmission Line?



$$V^+ = 5\angle 0°$$
$$V^- = 1.813\angle 65.73°$$

$$V(z) = 5e^{-j\beta z} + 1.813e^{j65.73°}e^{j\beta z}$$

As $5e^{-j\beta z}$ and $1.813e^{j65.73°}e^{j\beta z}$ go in and out of phase, $V(z)$ shows a “standing-wave pattern”.