Optimizing Portfolio Strategies: A Comparative Study of Mean-Variance, Minimum-Variance, Equally Weighted, and Risk Parity Portfolios

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Abstract

This paper critically examines the effectiveness of Mean-Variance Optimization (MVO), Minimum-Variance Portfolio (MVP), Equally Weighted and Risk Parity portfolio strategies in the the long run and during volatile periods. While MVO aims to maximize expected returns while minimizing risks, it often results in concentrated portfolios vulnerable to abrupt changes. MVP, focusing on minimizing overall portfolio variance without explicit return predictions, faces issues of concentration on low-volatility assets. The study introduces Risk Parity portfolios as an alternative strategy, emphasizing risk budgeting to achieve optimal risk diversification. The study concludes that all three portfolio strategies consistently outperform the S&P 500 index, demonstrating their potential for delivering superior risk-adjusted returns across various market conditions. Notably, Risk Parity portfolios exhibit balanced risk contributions, making them resilient in downturns, and the MVP strategy stands out with promising risk-adjusted returns. The research provides valuable insights for investors seeking optimal portfolio strategies in dynamic markets.

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Introduction

The inception of Modern Portfolio Theory (MPT) by Markowitz in 1952 revolutionized portfolio optimization, emphasizing the dual goals of maximizing returns while minimizing risks. However, the application of MPT strategies such as Mean-Variance Optimization (MVO) and Minimum-Variance Portfolio (MVP) has faced criticism due to concentration issues and challenges in risk diversification. To address these concerns, Risk Parity Portfolios emerged as an alternative approach, aiming to balance risk contributions across assets. This paper investigates and compares these strategies, aiming to provide a comprehensive understanding of their efficacy and limitations in modern financial landscapes.

1. Literature Review

1.1. Mean-Variance Optimization (MVO)

The concept of *Modern Portfolio Theory* traces its origins to the pioneering work by Markowitz in 1952. This theory emphasizes the dual objectives of maximizing expected returns while minimizing risks, standing as the cornerstone in portfolio optimization (Markowits, 1952). The approach employs the fundamental principles of portfolio diversification and leverages covariance relationships. Portfolio diversification, integral to risk reduction, is crucial as portfolio investment risk (variance) is impacted by both individual asset variances and covariances in a bipartite manner. However, the risk mitigation aspect of portfolio diversification should be approached judiciously, as its primary potential lies in mitigating unsystematic risk rather than systematic risk.

The primary aim of mean-variance optimization is to minimize portfolio volatility while targeting an expected return. However, mean-variance optimization often leads to highly concentrated portfolios, making them vulnerable to abrupt changes in allocations due to minor input variations. There exists confusion between optimizing volatility and risk diversification (Bruder & Roncalli, 2012).

Despite its foundational principles, mean–variance optimization (MVO) faces scrutiny when confronted with contemporary challenges, particularly financial crises. Critics argue that it overlooks risk diversification and only takes the portfolio's overall risk into account, which results in an excessive concentration of risk in a small number of assets—as was shown during the 2008 financial crisis. This critique, additionally, questions the reliance of Modern Portfolio

Theory (MPT) on historical data as is also it is highly sensitive to parameter estimation errors , suggesting its potential irrelevance in current and future markets. Consequently, MPT's predictive capacity becomes less dependable and more susceptible to deviations from actual market behavior (Steinbach, 2001). Another area of criticism pertains to Markowitz's definition of risk, often equated with "volatility" (both upside and downside). This overlooks the perspective that investors aren't inherently risk-averse; rather, they exhibit a tendency toward aversion to losses, therefore in actuality, the variance is a poor indicator of risk because it penalizes both desired low losses and unwanted large losses. As Harold Evensky, a renowned financial planner and the founder of Evensky & Katz Foldes Wealth Management, states, *"Investors aren't risk-averse, they're loss-averse."*

1.2. Minimum-Variance Portfolio (MVP)

The Minimum-Variance Portfolio (MVP) seeks to create a portfolio with the lowest possible risk among a set of assets without emphasizing explicit return predictions, contrasting with the Mean-Variance Optimization (MVO) method.

In the MVP, the primary aim is to allocate weights to assets to minimize overall portfolio variance. However, this approach tends to concentrate on low-volatility assets, resulting in less diversified portfolios with uneven weight distributions (Lohre, Opfer & Orszag, 2014).

This portfolio is at the left-most end of the mean-variance efficient frontier possessing the unique trait of having security weights independent of expected returns on individual securities. Although all portfolios on the efficient frontier aim to minimize risk for a given return, the minimum-variance portfolio achieves this without considering expected returns directly (Clarke, De Silva & Thorley, 2013).

Despite its potential benefits, minimum-variance portfolios commonly struggle with concentration issues (Maillard, Roncalli & Teïletche, 2010). Ans across different portfolio components.

1.3. Risk Parity Portfolios

The criticism of MVO and MVP introduces alternative portfolio optimization strategies, with a particular focus on risk budgeting (or diversified risk parity strategies). This strategy is widely accepted in both academic and professional circles (Bruder & Roncalli, 2012; Choueifaty & Coignard, 2008; Lohre, Neugebauer & Zimmer, 2012; Maillard *et al.*, 2010; Meucci, 2005, 2009).

The resolution to concentrated risk in a select few assets observed in both Mean-Variance Optimization (MVO) and Minimum Variance Portfolio (MVP) strategies appears to lie in the adoption of a Risk Parity portfolio. Maillard *et al.* (2010) delves into the theoretical properties of the risk budgeting portfolio, demonstrating its volatility positioning between the minimum variance and weight budgeting portfolios Unlike the minimum variance portfolio, the Risk Parity portfolio is invested in all assets. While its volatility is greater than the minimum variance portfolio but smaller than the 1/N strategy, the Risk Parity portfolio maintains more balanced risk contributions, even though it ranks similarly to a MVP in terms of weight distribution.

In contrast to minimum-variance portfolios that equalize the marginal contributions of each asset to portfolio risk, risk parity portfolios equalize the total risk contribution, minimum-variance portfolios, positioning risk parity portfolios slightly inside the efficient frontier rather than on it (Clarke *et al.*, 2013: 40).

And unlike Mean-Variance Optimization (MVO), risk parity portfolios do not explicitly prioritize the expected return or the risk of a portfolio. Nonetheless, they do necessitate a positive expected return (Fisher, Maymin & Maymin, 2015: 42). Furthermore, the purpose of risk parity is not solely to minimize portfolio risk like standard MVO portfolios. Instead, by equalizing asset risk contributions, risk parity aims for optimal risk diversification (Costa & Kwon, 2020).

Initially, a risk parity portfolio defined weights based on asset class inverse volatility, disregarding their correlations (Clarke *et al.*, 2013: 39). Subsequently, Qian (2005) developed a more comprehensive definition that considers correlations and grounds the property in a risk budget where weights are adjusted to ensure each asset contributes equally to portfolio risk.

They allocate weights to different asset classes based on their risk measures, ensuring each asset contributes an equal risk amount to the portfolio and reducing estimation noise (Maillard *et al.*, 2010). This approach emphasizes risk allocation, typically defined as volatility, rather than capital allocation. By adjusting asset allocations to the same risk level, the risk parity portfolio can achieve a higher Sharpe ratio and better withstand market downturns compared to traditional portfolios. It ensures a well-diversified portfolio by requiring each asset to contribute the same level of risk (Merton, 1980).

Furthermore, Ardia, Bolliger, Boudt & Gagnon-Fleury (2017) demonstrate that risk parity portfolios are less susceptible to covariance misspecification compared to other risk-based investment strategies.

Nevertheless, the Risk Parity portfolio is not devoid of shortcomings. Maillard *et al.* (2010:

16) points out that even though minimum variance portfolios face constraints due to asset concentration, risk parity portfolios lack adequate risk oversight. However, implementing this approach optimizes risk distribution, acting as a balanced risk filter that prevents any single asset from dominating the portfolio, as observed in the case of the MVP.

2. Portfolio construction

This paper aligns closely with Maillard *et al.* (2010) by constructing a *Global diversified portfolio* and utilizes monthly data from 2002-02-28 to 2023-08-31 provided by Katzke (2023a), focusing on various indices including the Morgan Stanley Capital International All Country World Index ("MSCI ACWI"), the Gold Spot rate ("Gold Spot $/Oz"), the Bloomberg Global Aggregate Bond Index ("GlobalAgg Unhedged USD"), the Capped SWIX All Share Index ("J433"), and the FTSE/JSE All Bond Index ("ALBI"). The analysis aims to compare various portfolio optimization strategies. Utilizing a standard shrinkage technique, covariance matrices are estimated for all strategies (Chaves, Hsu, Li & Shakernia, 2011).

The study adopts several global assumptions: (1) full investment ($\boldsymbol{\omega}^{\mathbf{1}} 1 = 1$), (2) a long-only strategy($\boldsymbol{\omega} \geq 0$). All portfolios undergo quarterly rebalancing, with an upper limit of 25% and a lower bound of 1% applied to all asset classes. Furthermore, Bonds are capped at 25%, Equities at 60%, and Gold at 10% exposure.

The analysis commences with a naïve $\frac{1}{N}$, assigning equal weights to asset classes. Subsequently, Minimum Variance Optimization (MVO) and Minimum Variance Portfolio (MVP) techniques are implemented. Equations representing the minimum variance portfolios are detailed, encompassing both MVP and MVO, as provided by (Katzke, 2023b):

$$\frac{1}{2}\boldsymbol{\omega^T} * \boldsymbol{Dmat} * \boldsymbol{\omega} = \boldsymbol{dvec^T \omega} \tag{2.1}$$

$$s.t.\ \boldsymbol{Amat} * \boldsymbol{\omega} \geq \boldsymbol{bvec} \tag{2.2}$$

$$\tag{2.3}$$

$$\boldsymbol{\omega_{mvp}} = arg \min \boldsymbol{\omega' \Sigma \omega} \tag{2.4}$$

$$s.t. \sum_{n=1}^{N} \omega_i = 1 \; and \; \omega \geq 0 \tag{2.5}$$

$$\boldsymbol{\omega_{mvp}} = arg \min \boldsymbol{\omega' \Sigma \omega} \tag{2.6}$$

$$s.t. \sum_{n=1}^{N} \omega_i = 1 \; and \; \omega \geq 0 \tag{2.7}$$

$$\boldsymbol{\omega_{mvo}} = arg \max \quad \boldsymbol{\mu}^T \omega - \lambda \boldsymbol{\omega^T \Sigma \omega} \tag{2.8}$$

$$s.t. \sum_{n=1}^{N} \omega_i = 1 \; and \; \omega \geq 0 \tag{2.9}$$

To ensure robustness against outliers in the dataset, a covariance matrix shrinkage approach, advocated by [Ledoit & Wolf (2003), is applied. Shrinkage serves to minimize the impact of outliers, enhancing the stability of covariance estimates.

$$\hat{\Sigma}^{Shrunk} = (1 - \rho) \; * \hat{\Sigma} \; + \rho \; * T \tag{2.10}$$

$$with \; T = \frac{1}{N} Tr \times I \tag{2.11}$$

Moving to risk parity, the portfolio setup involves optimizing risk contribution through weight allocations, ensuring proportional risk distribution among assets. The process for equalizing risk contributions is delineated, highlighting the attempt to balance risks across the portfolio (Vinícius & Palomar, 2019).

The volatility of the portfolio $\sigma(\boldsymbol{w}) = \sqrt{\boldsymbol{w^t \Sigma w}}$, the risk contribution of the ith asset to the total risk is defined as:

$RC_i = \frac{w_i((\boldsymbol{\Sigma w}))i}{\sqrt{\boldsymbol{w^t \Sigma w}}}$ which satisfies $\sum_{i=1}^{N} RC_i = \sigma(\boldsymbol{w})$. The relative or marginal risk contribution (RRC) is a normalized version so that $\sum_{i=1}^{N} RRC_i = 1$.

Therefore the risk parity profile which attempts to equalize the risk contributions is: $RC_i = \frac{1}{N}\sigma(\boldsymbol{w})$

The risk budget constraint attempts to allocate the risk according to the risk profile determined by the weights $\boldsymbol{b}$ (with $\mathbf{1}^T\boldsymbol{b} = 1$ and $\boldsymbol{b} \geq 0$) so that it can be expressed as $RC_i = b_i\sigma(\boldsymbol{w})$

For the naïve diagonal formulation the constraints are $\mathbf{1}^T\boldsymbol{w} = 1$ and $\boldsymbol{w} \geq 0$) which will state that the portfolio is inversely proportional to the assets' volatilities. Mathematically,

$$\boldsymbol{w} = \frac{\boldsymbol{\sigma}^{-1}}{\mathbf{1}^T\boldsymbol{\sigma}^{-1}} \tag{2.12}$$

where $\boldsymbol{\sigma^2} = Diag(\boldsymbol{\Sigma})$. Therefore, lower weights are given to high volatility assets and higher weights to low volatility assets. This is often referred to as "equal volatility" portfolio. Creating an "equal volatility" portfolio involves selecting and assigning weights to individual assets in such a way that, when combined, each asset's volatility contributes equally to the total volatility of the portfolio. This approach aims to mitigate the impact of any single asset's volatility on the overall risk of the portfolio, potentially providing a more diversified and risk-balanced investment strategy. $sd(w_i r_i) = w_i\sigma_i = \frac{1}{N}$

Adding the additional constraints we get:

$$\min_{w} \; R(\boldsymbol{w}) + \lambda F(\boldsymbol{w}) \tag{2.13}$$

$$s.t.\ \boldsymbol{Cw} = \boldsymbol{c}, \boldsymbol{Dw} \leq \boldsymbol{d} \tag{2.14}$$

3. Results

3.1. Preliminary analysis

This section examines the risk contribution, weight allocation, and cumulative returns of each portfolio. While offering insights into expected portfolio behavior, it is essential to recognize that this analysis alone may not comprehensively assess portfolio performance. A more robust and detailed analysis follows this section.

Figure 3.1 illustrates the expected weight allocation of the Risk Parity portfolio, aligning with its relative risk contributions. In contrast, the Minimum Variance Portfolio (MVP) portfolio exhibits an inverse relationship, with a greater risk contribution corresponding to a smaller weight allocation. A similar trend is observed in the MVO portfolio, with variations. The Risk Parity Portfolio, notable for its diverse allocation, particularly allocates a significant weight to the 'ALBI' asset class, showcasing the intuitive nature of the Risk Parity strategy. Conversely, the MVP portfolio, while aggressively targeting equities and gold, overlooks 'J433' and aligns with the traditional weight distribution of minimum variance, concentrating heavily on low-risk asset classes such as bonds (Lohre *et al.*, 2014).

Figure 3.1 also provides insights into the investment risk profiles of different portfolios, revealing key exposures for each asset class. Global bonds face currency risk, while local equities are subject to market risk influenced by economic conditions, geopolitical events, and investor sentiment. Global equities are exposed to global market movements, economic conditions, currency risk, and political and regulatory risk. The Risk Parity portfolio demonstrates the highest exposure to both local and global equities, indicating sensitivity to global market dynamics. In contrast, the MVP Portfolio has a greater emphasis on global and local bonds.

A higher weight allocation to bonds in a portfolio offers several benefits. Bonds are generally less volatile than equities, contributing to a more stable portfolio value, which is suitable for risk-averse investors. Bonds also provide regular interest payments, creating a predictable income stream. Additionally, bonds are considered safer in terms of capital preservation, offering a hedge during equity market downturns. The lower correlation between bonds and equities contributes to diversification, reducing overall portfolio risk.

On the other hand, the Mean-Variance Optimization (MVO) exhibits a higher exposure to the gold spot rate and local stocks. This suggests that gold serves as a potential protective buffer, employed for hedging against inflation risk and mitigating market and economic stress.

Figure 3.2 compares cumulative returns across portfolio strategies, highlighting MVO and Risk Parity as prominent contenders.

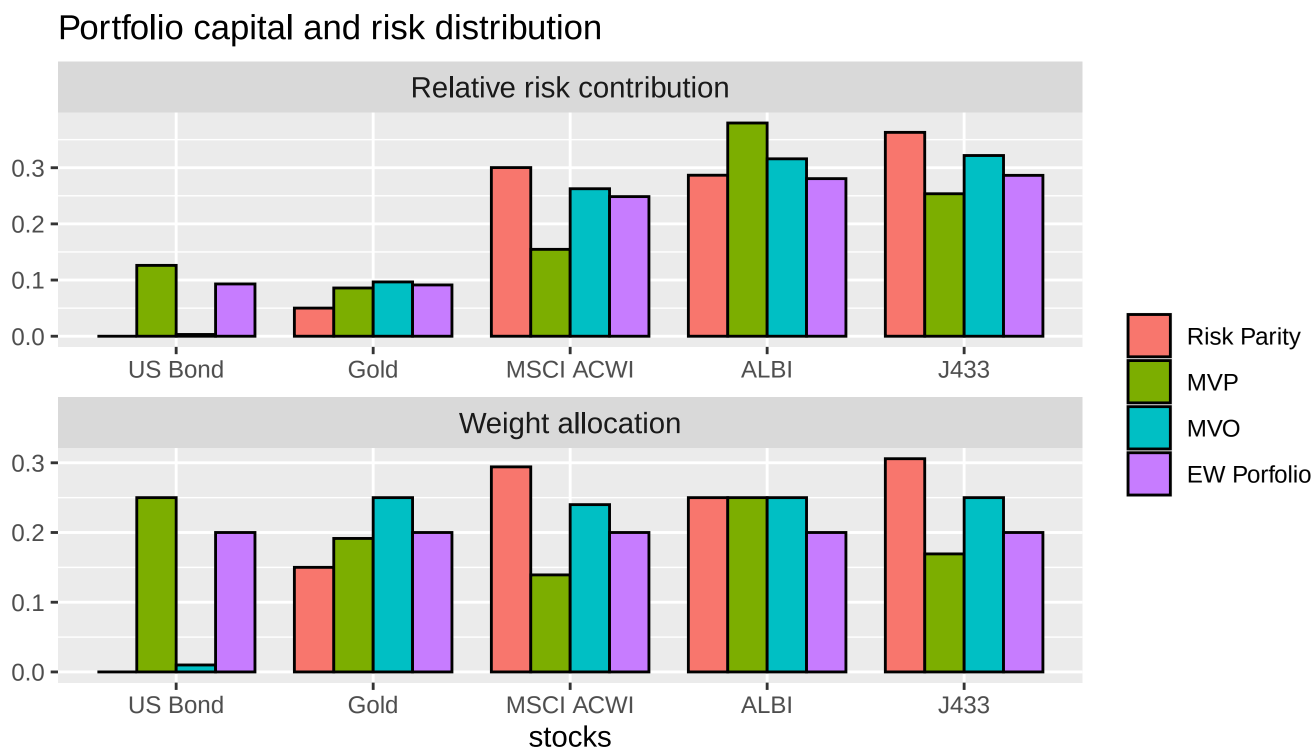

Figure 3.1: Relative risk contribution and Weights allocation of different portfolios

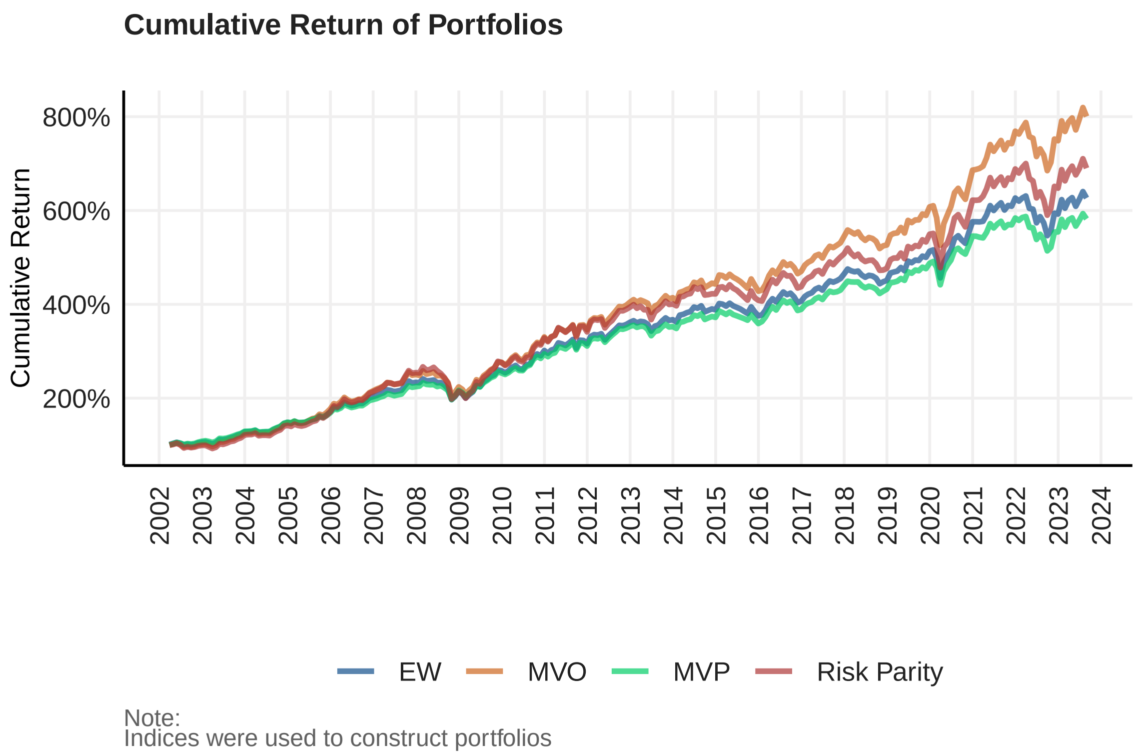

Figure 3.2: Cumulative Returns

3.2. Further Analysis

A drawdown signifies the extent to which the portfolio declines from its peak value before subsequently recovering. The portfolios were assessed in relation to the S&P 500. While the S&P 500 may not be the ideal benchmark, given that other portfolios are primarily constructed from different market indices, its broad market representation, market-capitalization weighting, diverse industry coverage, liquidity, reputation, and global influence make it the most suitable benchmark for this analysis.

In Figure 3.3, several crucial metrics stand out when dissecting this graph. These metrics include the depth, duration, frequency, and recovery periods of drawdowns observed in various portfolios relative S&P 500 index. Notably, the S&P 500 index exhibits the most substantial depth in drawdowns, coupled with the lengthiest recovery periods. This implies that the index not only experiences the most significant loss in value following its peaks but also takes an extended period to recover from these downturns. In contrast, the MVP portfolio displays lower frequency, depth, and duration of drawdowns, indicating comparatively smaller losses in portfolio value and consequently lower risk compared to other portfolios.

The Risk Parity portfolio follows a similar trend to the MVP, albeit with greater depth in drawdowns, though not as pronounced as observed in the S&P 500. This suggests that while the Risk Parity portfolio experiences more significant declines in value during drawdowns, it still maintains a risk profile that is less pronounced than that of the S&P 500. The $\frac{1}{N}$ exhibits the last amoutn of depth in drawdowns, followed by the MVo portfolio. The $\frac{1}{N}$ portfolio demonstrates the least depth in drawdowns, with the MVO portfolio following closely.

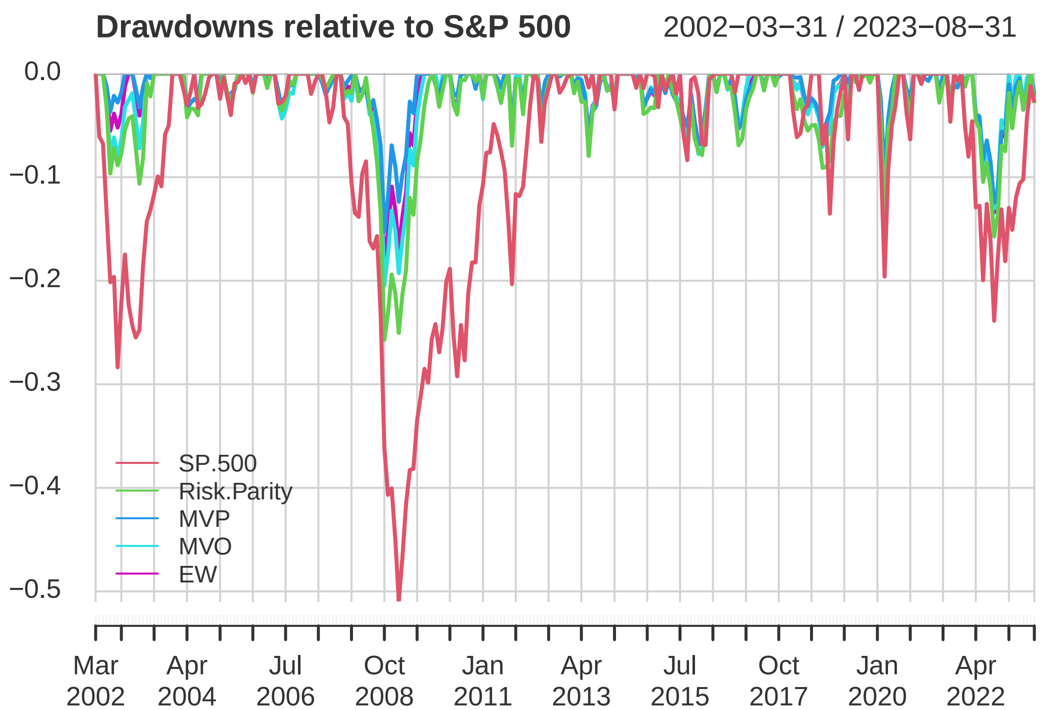

Figure 3.3: Drawdowns

Furthermore, I conducted an analysis of the annualized risk-return performance of each portfolio relative to the S&P 500 over five distinct periods. The initial period encompasses the entire dataset, while the second period spans from October 2008, marked by a significant drop in the S&P 500 after reaching its peak. The third period, December 2018, witnessed another downturn attributed to factors such as Donald Trump's trade war with China, global economic slowdown, and concerns about the Federal Reserve's rapid interest rate hikes. Additionally, the year 2020, marked by the onset of the COVID-19 pandemic, and January 2022, following a peak in the S&P 500, represent subsequent periods characterized by geopolitical tensions, the Ukraine war, extensive COVID-related lockdowns in China, interest rate hikes by the US Federal Reserve, and overall economic uncertainty in the US post-pandemic. These chosen periods were deliberately selected since it encapsulated volatile circumstances.

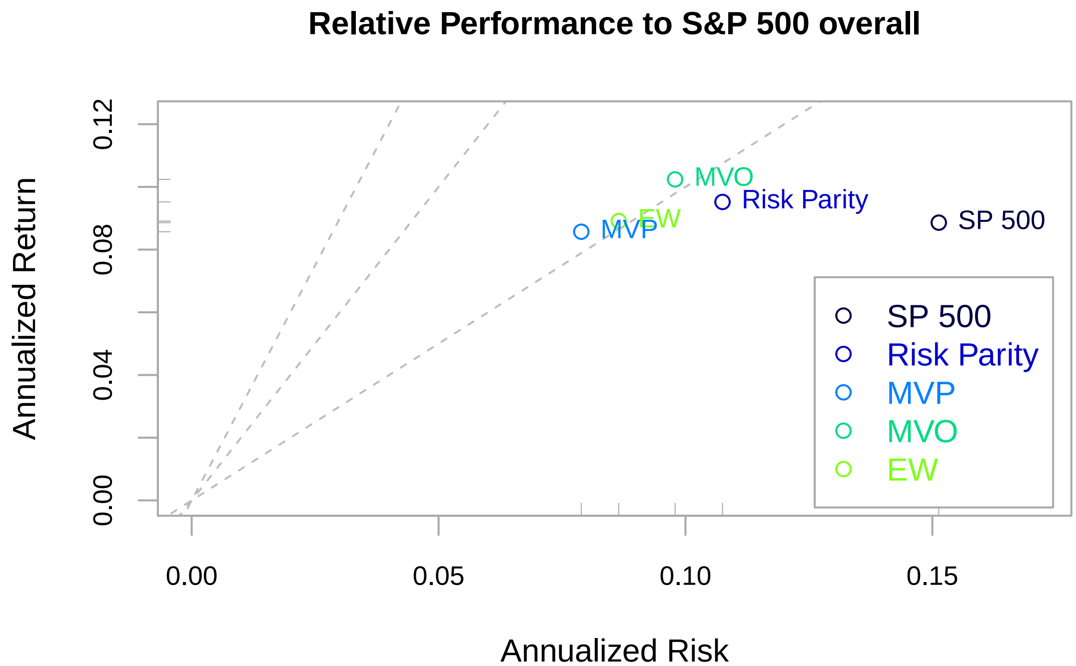

Figure 3.4: Annualized Risk-Return performance of portfolios relative to SP 500 over the entire sample period.

Figure 3.4, the Mean-Variance Optimization (MVO) portfolio emerges with the highest potential return accompanied by moderate risk. Conversely, the Minimum Variance Portfolio (MVP) strategy exhibits the lowest risk but correspondingly the lowest return. Both Equal Weight (EW) and Risk Parity strategies offer a balanced combination of moderate returns and risks, outperforming the S&P 500 to some extent.

Figures 3.5 through 3.7 illustrate that the MVO portfolio achieves the highest return but also carries the highest risk relative to the S&P 500 index. Meanwhile, the MVP portfolio features the lowest risk but at the expense of the lowest return. The Risk Parity portfolio showcases a moderate return with higher risk than the MVO portfolio.

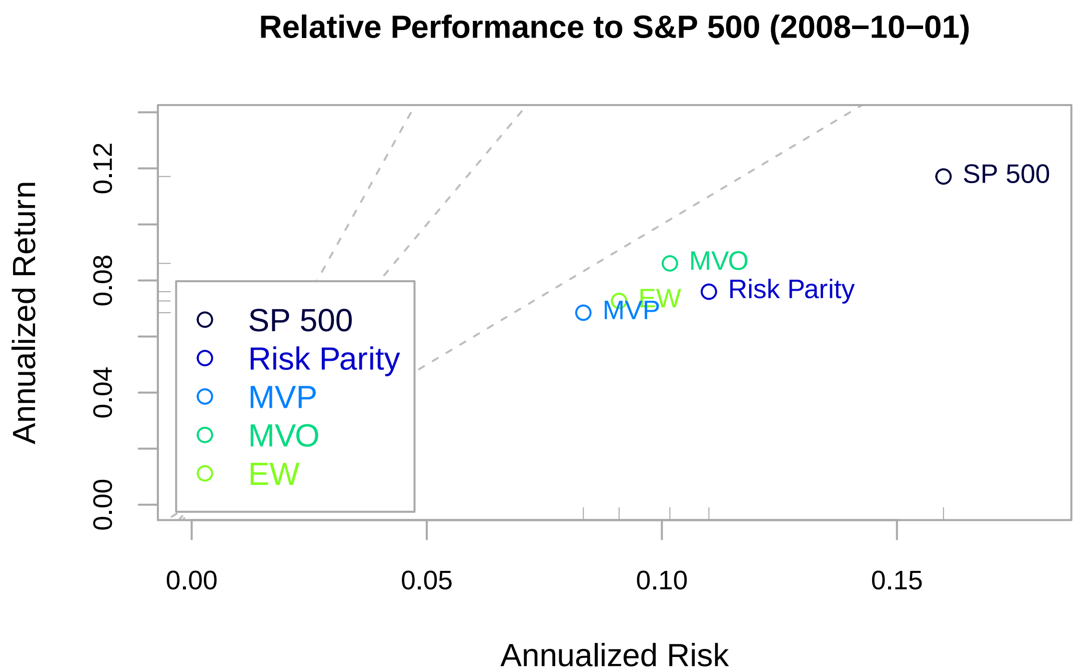

Figure 3.5: Annualized Risk-return performance of portfolios relative to SP 500 since October 2008

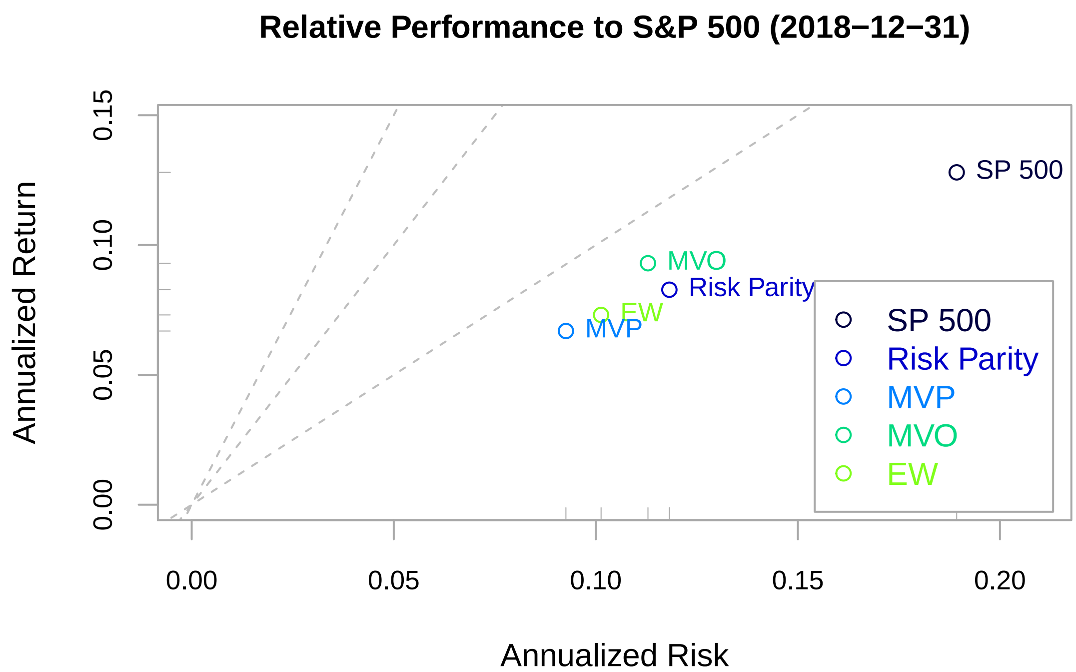

Figure 3.6: Annualized Risk-return performance of portfolios relative to SP 500 since December 2018

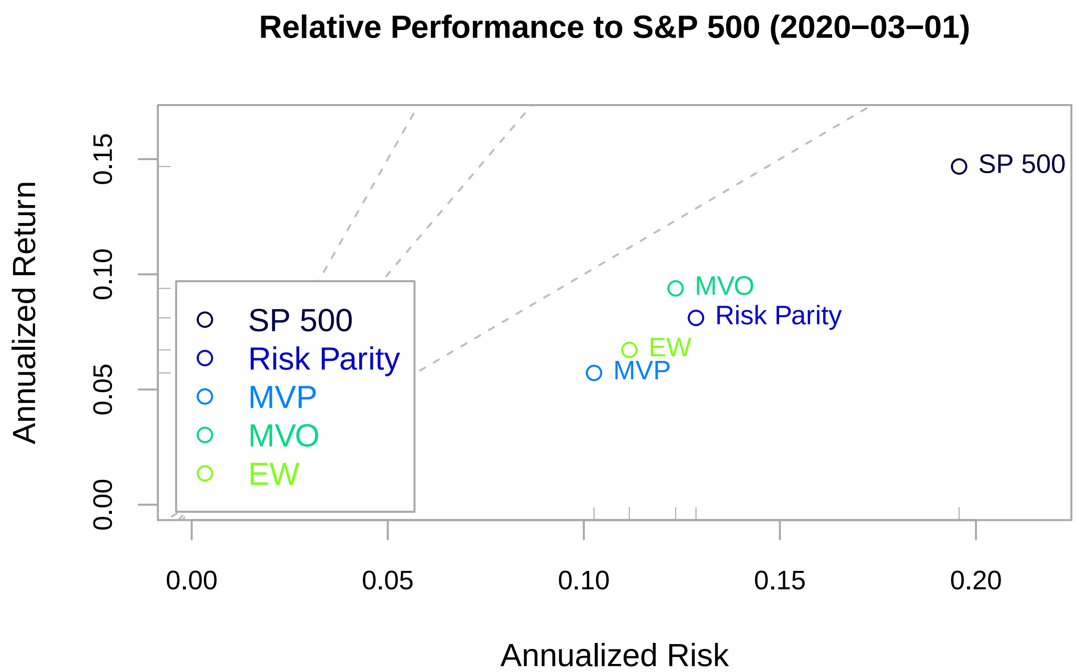

Figure 3.7: Annualized Risk-return performance of portfolios relative to SP 500 since March 2020

Examining the portfolios' performance relative to the S&P 500 during a short-term volatile period in Figure 3.8, suggests that the portfolios outperform the S&P index. Notably, the MVO portfolio exhibits the highest annualized return with risk levels comparable to those of the $\frac{1}{N}$ and Risk Parity portfolios, albeit with the latter two experiencing lower returns. In contrast, the MVP portfolio performs less favorably among the portfolios.

During a short-term volatile period depicted in Figure 3.8, all portfolios exhibit outperformance compared to the S&P 500. Particularly noteworthy is the MVO portfolio, which not only delivers the highest annualized return but also maintains risk levels similar to the $\frac{1}{N}$ and Risk Parity portfolios, albeit with the latter two experiencing lower returns. In contrast, the MVP portfolio lags behind in performance among the portfolios considered.

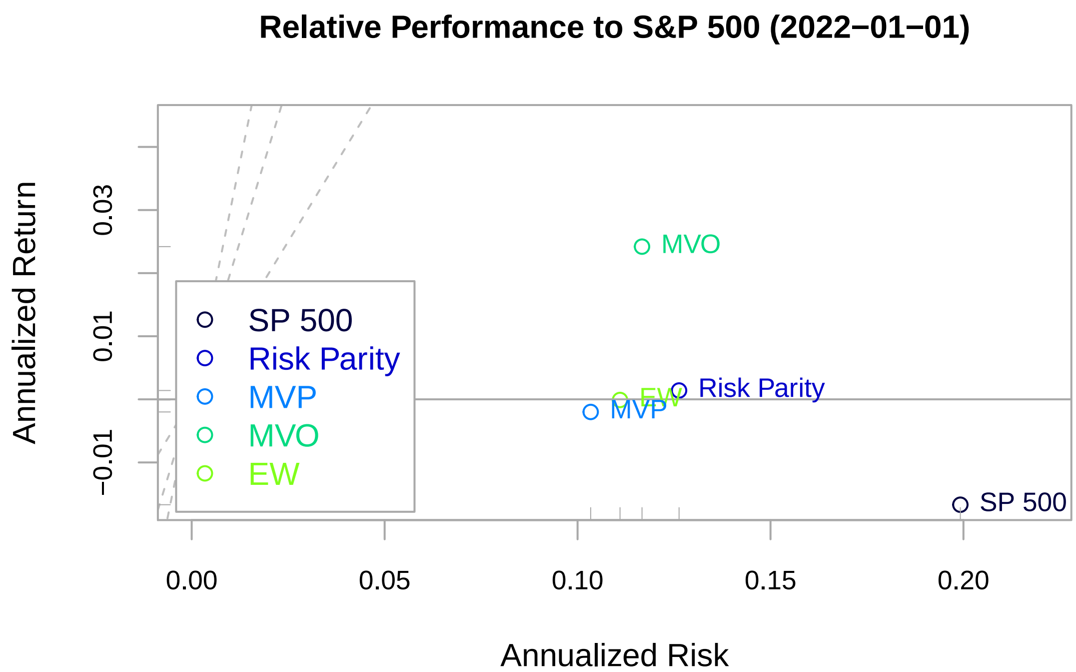

Figure 3.8: Annualized Risk-return performance of portfolios relative to SP 500 since January 2022

3.3. Index Statistics

This section comprehensively explores portfolio performance through an in-depth analysis of various metrics, including returns, volatility, expected shortfall, average drawdown, and performance during market upswings and downswings. Notably, the assessment excludes the examination of beta for the portfolios. The rationale for this omission lies in the anticipated and confirmed beta values being less than one, indicative of lower volatility compared to the benchmark due to better diversification. Despite the exclusion of beta, other statistical measures are deemed sufficient for providing a comprehensive overview of the diverse portfolio performances.

The comparison in Table 1 reveals distinctive characteristics of each strategy. The Minimum-Variance Portfolio (MVP) portfolio, while exhibiting the lowest volatility and pain index, suggesting lower risk, it also leads in terms of the adjusted Sharpe ratio compared to the other portfolios, followed by the Mean-Variance (MVO) portfolio (seen in Table 2), implying potential nefficiency in risk-adjusted returns, aligning with conclusions by Chaves *et al.* (2011).

The Risk Parity, along with the MVO, demonstrates superior annualized returns, consistent with the findings of Maillard *et al.* (2010). The Risk Parity, MVO, and EW portfolios have a positive and greater than the benchmark excess return statistic, whereas MVP has a negative excess return statistic, it implies that RP, MVO, and EW have generated higher returns than the S&P 500 index over the same period, after adjusting for the risk-free rate, whereas MVP has lagged in this regard

The S&P 500, has the highest cumulative returns, but bears the highest volatility and pain index. On the other hand, the Risk Parity portfolio, with a higher adjusted Sharpe ratio than the S&P 500, suggests potential efficiency in risk-adjusted returns and also exhibits the highest up-down ratio which indicates that the portfolio has a greater ability to capture upside potential while minimizing downside risk. In contrast the MVP has the lowest up-down ratio, followed by the equally weighted portfolio which is unsurprising since these portfolios are constructed to be more conservative. It suggests that these portfolios have a greater focus on capital preservation and downside protection, which also means it has a lower ability to capture upside potential.

The pain index, a measure of risk in losses, aligns with the drawdown graph in Figure 3.3 and the average drawdown in Table 1. It reaffirms that the S&P 500 experiences the greatest drawdown, followed by the Risk Parity portfolio, with MVP showing the lowest drawdown.

Examining tracking error and information ratio provides insights into portfolio consistency and effectiveness. The MVO strategy stands out with the lowest tracking error and the highest information ratio, indicates that the portfolio has consistently generated excess returns relative to the benchmark, taking into account the volatility of those returns..MVP exhibits the highest tracking error. Correspondingly, the information ratio suggests that MVP consistently underperforms the benchmark on a risk-adjusted basis, aligning with its lower excess returns and reaffirming the findings in Figure 3.4-3.8.

The modified Conditional Value at Risk (CVaR) assesses risk-adjusted performance, representing the expected loss beyond the 95% confidence level. Therefore in the worst 5% of the returns, your average loss of the Risk Parity portfolio is lower than the S&P 500 index. However, again as expected, given the risk-averse nature of the portfolios, MVO and MVP has a smaller expected average loss than the Risk Parity portfolio, but the Equally weighted portfolio has a smaller average loss than the MVO portfolio, and only marginally more than the MVP portfolio.

Table 1: Long-Term Index Statistics: Relative to S&P 500

Info[1]	SP 500	Risk Parity	MVP	MVO	EW
Historical					

Cum Returns	520.6%	606.0%	486.1%	713.1%	526.5%
Returns (Ann.)	8.9%	9.5%	8.6%	10.2%	8.9%
Returns Excess (Ann.)	0.0%	0.6%	−0.3%	1.3%	0.0%
SD (Ann.)	15.1%	10.8%	7.9%	9.8%	8.7%
Pain Index	0.071	0.029	0.016	0.021	0.019
Avg DD	7.1%	4.3%	2.9%	3.6%	3.1%
Tracking Error	0.0%	11.0%	12.0%	10.8%	11.1%
Information Ratio	-	0.060	-0.024	0.127	0.004
Up-Down Ratio	0.000	0.335	0.249	0.312	0.254
Modified CVaR	-0.102	-0.074	-0.050	-0.066	-0.057

[1] Utilising the US 3-Month Libor Rate as a proxy for the risk-free rate

Table 2: Adjusted Sharpe Ratio: Relative to S&P 500

Info[1]	S&P 500	Risk Parity	MVP	MVO	EW
Adj. Sharpe Ratio	0.669	0.478	0.635	0.58	0.568

[1] Utilising the US 3-Month Libor Rate as a proxy for the risk-free rate

3.4. Stratified Sample Periods

Tables 3 and 4 provide a more detailed exploration of the statistics presented in Tables 1 and 2. Specifically, the analysis stratifies high and low volatility periods concerning exchange rate movements (between the South African Rand and the US Dollar), gold movements, and Brent crude oil movements. The focus is on understanding how each portfolio reacts relative to the S&P 500 index during these different volatility conditions.

The inclusion of exchange rate movement (US Dollar to Rand) is crucial when constructing a portfolio with both local (South African) and global (US) assets. Currency fluctuations can significantly impact portfolio value, as changes in exchange rates affect the value of global assets when converted back to the local currency. Foreign exchange exposure involves the risk that fluctuations in exchange rates will influence the value of a portfolio's assets, liabilities, or cash flows. For a portfolio with both local and global assets, shifts in the exchange rate between the Rand and the US Dollar can impact the value of US assets when converted back to Rand.

In the context of gold and Brent crude oil literature, extensive research demonstrates the spillover effects of the global crude oil and gold markets on equity and bond markets[1].

Table 3 presents high volatility scenarios for the portfolios in comparison to the S&P 500 index.

In terms of risk-adjusted performance, the Risk Parity portfolio consistently exhibits a lower adjusted Sharpe ratio compared to the other portfolios. However, it still outperforms the S&P 500 index. Notably, the MVP portfolio stands out with the most promising risk-adjusted returns among the portfolios.

It is interesting to observe that all the portfolios surpass the benchmark index, aligning with the trends identified in Figures 3.4 through 3.8. This consistency in outperformance suggests the effectiveness of the selected portfolio strategies across various market conditions and reinforces their potential for delivering superior risk-adjusted returns compared to the S&P 500.

Table 3: Fund Moments Comparison

Info[1]	SP 500	Risk Parity	MVP	MVO	EW
High Volatility Currency Movements					
Returns (Ann.)	−1.3%	4.4%	7.3%	6.8%	6.4%
Returns Excess (Ann.)	1.2%	6.9%	9.9%	9.4%	9.1%
Adj. Sharpe Ratio	-0.079	0.394	0.886	0.695	0.725
Pain Index	0.198	0.064	0.030	0.036	0.035
Avg DD	15.7%	11.3%	6.2%	6.7%	6.1%
Tracking Error	3.1%	13.8%	15.0%	13.7%	14.1%
Information Ratio	Inf	0.492	0.644	0.675	0.627
Modified CVaR	-0.105	-0.060	-0.042	-0.051	-0.046
High Volatility Gold Movements					
Returns (Ann.)	2.4%	10.3%	9.7%	10.8%	9.7%
Returns Excess (Ann.)	1.3%	9.1%	8.5%	9.6%	8.5%
Adj. Sharpe Ratio	0.145	0.856	1.115	1.023	1.018

[1]For interested readers, relevant literature includes Dai & Kang (2021), Balcilar, Gupta, Wang & Wohar (2020), Morrison (2019), Kang, Ratti & Yoon (2014), Akhtaruzzaman, Boubaker, Lucey & Sensoy (2021), Cohen & Qadan (2010), Lucey & Tully (2003), Sadorsky (1999), Chiang, Lin & Huang (2013), Shahzad, Raza, Shahbaz & Ali (2017), Choudhry, Hassan & Shabi (2015), Morema & Bonga-Bonga (2020), Gomes, Chaibi & others (2014). However, these references won't be expanded upon in this analysis, as they fall outside the focus of this study.

Pain Index	0.119	0.036	0.018	0.025	0.022
Avg DD	23.8%	5.1%	3.6%	4.1%	3.7%
Tracking Error	2.7%	13.5%	14.4%	13.3%	13.5%
Information Ratio	Inf	0.686	0.599	0.731	0.638
Modified CVaR	-0.103	-0.061	-0.042	-0.052	-0.047
High Volatility Brent Movements					
Returns (Ann.)	−6.7%	4.0%	7.2%	5.7%	6.1%
Returns Excess (Ann.)	1.2%	12.8%	16.3%	14.7%	15.1%
Adj. Sharpe Ratio	-0.369	0.341	0.866	0.546	0.662
Pain Index	0.324	0.077	0.026	0.044	0.035
Avg DD	51.6%	11.4%	6.7%	9.1%	6.3%
Tracking Error	3.1%	14.8%	16.5%	14.8%	15.4%
Information Ratio	Inf	0.800	0.906	0.912	0.899
Modified CVaR	-0.111	-0.065	-0.042	-0.054	-0.047

[1] Utilising the US 3-Month Libor Rate as a proxy for the risk-free rate, except for Adjsuted Sharpe Ratio which uses a 0% risk-free rate

Compared to the previous table, the performance of the portfolios (Table 4) are generally better under the low volatility scenarios than under the high volatility scenarios.

Table 4: Fund Moments Comparison

Info[1]	SP 500	Risk Parity	MVP	MVO	EW
Low Volatility Currency Movements					
Returns (Ann.)	12.5%	9.8%	7.6%	10.0%	8.3%
Returns Excess (Ann.)	0.0%	−2.4%	−4.3%	−2.2%	−3.7%
Adj. Sharpe Ratio	0.976	1.270	1.261	1.347	1.279
Pain Index	0.019	0.018	0.018	0.017	0.016
Avg DD	4.2%	3.0%	2.5%	2.8%	2.5%
Tracking Error	0.0%	9.3%	10.4%	9.4%	9.7%
Information Ratio	-	-0.284	-0.467	-0.267	-0.430
Modified CVaR	-0.065	-0.036	-0.028	-0.035	-0.030
Low Volatility Gold Movements					
Returns (Ann.)	7.6%	7.0%	6.5%	8.5%	6.8%

Returns Excess (Ann.)	0.0%	−0.5%	−1.0%	0.8%	−0.8%
Adj. Sharpe Ratio	0.450	0.775	0.905	1.009	0.861
Pain Index	0.046	0.031	0.018	0.021	0.021
Avg DD	9.7%	3.4%	2.4%	3.0%	2.6%
Tracking Error	0.0%	11.0%	12.4%	11.2%	11.5%
Information Ratio	-	-0.052	-0.086	0.078	-0.072
Modified CVaR	-0.093	-0.047	-0.039	-0.043	-0.042
Low Volatility Brent Movements					
Returns (Ann.)	17.8%	11.0%	9.0%	10.9%	9.8%
Returns Excess (Ann.)	−0.1%	−5.9%	−7.6%	−6.0%	−6.9%
Adj. Sharpe Ratio	1.314	1.101	1.259	1.224	1.256
Pain Index	0.016	0.013	0.010	0.011	0.009
Avg DD	5.0%	2.3%	1.8%	2.6%	2.1%
Tracking Error	0.3%	9.4%	9.8%	9.1%	9.2%
Information Ratio	-Inf	-0.736	-0.918	-0.777	-0.887
Modified CVaR	-0.061	-0.051	-0.035	-0.044	-0.038

[1] Utilising the US 3-Month Libor Rate as a proxy for the risk-free rate, except for Adjsuted Sharpe Ratio which uses a 0% risk-free rate

Conclusion

The analysis different portfolio startegies can be useful when investots evaluate risk-adjusted performance that align with their risk tolerance. For risk-averse investors prioritizing capital preservation, the MVP strategy may be favored despite its negative excess return, given its lower volatility and pain index. In contrast, risk-tolerant investors seeking higher returns may lean towards MVO or EW, which exhibit higher excess return statistics and upside potential.

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