Linear Filtering

Convolutions

Flip horizontally and vertically Gaussian Filters:
$\sum_k G(k) = 1$

Separable Filters

$I \otimes F$
$= I \otimes (A + B)$
$= I \otimes (A_1) \otimes (A_2) + I \otimes (B_1) \otimes (B_2)$

Edge Finding

Filters:

$$I(x) = G_{x,x}(x,\sigma) \otimes G(x, 3\sigma)$$
$$G_{x,x} = \begin{vmatrix} 1 & -1 \end{vmatrix}$$
$$I(y) = G_{y,y}(y,\sigma) \otimes (G)$$
$$G_{y,y} = \begin{vmatrix} 1 \\ -1 \end{vmatrix}$$
$$\theta_{edge} = \tan^{-1}(-\frac{\delta f}{\delta x} \div \frac{\delta f}{\delta y})$$

Canny Edge Detection

1. Compute image gradient
$J_x = I \otimes (\frac{\delta}{\delta x} \otimes G)$
2. Compute edge gradient magnitude $\nabla J = (J_x, J_y)$
$||\nabla J|| = \sqrt[2]{J_x^2 + J_y^2}$

Sampling

Blur then sample

Gaussian Pyramid

Reductions

pick weighting function $w_0 = a$
$w_{-1} = w_1 = 1/4$ $w_{-2} = w_2 = 1/4 - a$ $g_0 = Image$

$$g_{L,(i,j)} = \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) g_{L-1}(2i + m, 2j + n)$$
$$w(m,n) is the 2d filter$$
$$g_L = g_{L-1} \otimes w downarrow 2$$

Expansion

$$g_{L,(i,j)} = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) \dot{g}_{L,n-1}(\frac{i-m}{2}, \frac{j-n}{2})$$

Warping

Matricies

Inverse of a Matrix

$$A^{-1} = \frac{1}{det} A^T$$

cross product

$$\begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$
$$= (x_2 y_3 - x_3 y_2)i +$$
$$(x_3 y_1 - x_1 y_3)j +$$
$$(x_1 y_2 - x_2 y_1)k$$

Translation

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Scaling

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Rotation

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Skew Shear

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Affine Transformations Need 3 points Projective Transformations Need 4 points

Triangle Warping to find Homography

ONLY FOR AFFINE TRANSFORMATIONS Warp 3 source points to unit triangle ($T1^{-1}$) then warp unit triangle to destination points (T2)

$$T1 = \begin{vmatrix} A - C & B - A & A \\ 0 & 0 & 1 \end{vmatrix} \quad T2 = \begin{vmatrix} A' - C' & B' - A' & A' \\ 0 & 0 & 1 \end{vmatrix}$$
$$T1^{-1} T2 = H$$

Random Lin alg

A line can be defined as
$l = x \times x'$ (cross product)
Intersection
$x = l \times l'$
Line mapping
$l' = H^{-T} l$

Bilinear Interpolation

$$a = i + 1$$
$$b = j + 1$$
$$f(x,y) = f((1-a),(1-b)) +$$
$$f(a,(1-b)) + f(a,b) + f((1-a),b)$$

Barycentric Coordinates

if $0 \leq a \leq 1$ and $0 \leq a' \leq 1$ then x is inside the triangle

$$\begin{vmatrix} x \\ y \\ 1 \end{vmatrix} = \begin{vmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \alpha \\ \beta \\ \gamma \end{vmatrix} \qquad \begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} a'_x & b'_x & c'_x \\ a'_y & b'_y & c'_y \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \alpha' \\ \beta' \\ \gamma' \end{vmatrix}$$

TPS

$f(x) = a_1 + a_x(x_i m) + a_y(y_i m) +$
$\sum_{i=1}^{p} w_i U(||x_{ctrl_i}, y_{ctrl_i} - (x_i m, y_i m)||)$
Where p is how many mean control points there are x and y are pixel position in image $x_c trl, y_c trl$ are mean control points
$U = -r^2 log 10 r^2$ where r is the l2 norm between control points at i and the pixel w is the weighting

Blending

Alpha Blending

$I_b lend = \alpha I_f or + (1 - \alpha) I_b ack$
Where α is an alpha mask and multiplication is component wise
For multiple blending
premultiply alpha to all color channels use first equation

Two Band Blending

1. make a binary mask apply it to forground high pass image and 1 - mask to high pass background image
2. make linear mask by applying gauss filter to binary mask 3. apply linear filter to foreground and 1-linear filter to back ground
4. add all of the things

Seam Carving

Energy map
$e(Image) = |\frac{\delta}{\delta x} I| + |\frac{\delta}{\delta y} I|$
TA's just took the gradient of image and get X gradient matrix and Y gradient matrix and added them together
optimal seam:
$s* = min_s \sum_{i=1}^{n} e(I(s_i))$
Cumulative Map for vertical seam switch indices for horizontal seam
$Map(i,j) = e(i,j) + min(M(i-1,j-1), M(i-1,j), M(i-1,j+1))$
Then start from bottom of Map and select the lowest value pixel

Harris Corners

$M = \sum_{x,y} \begin{vmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{vmatrix}$
Eigen values of M λ_1 , λ_2
Corner Response
$R = detM\ TraceM$ det M = $\lambda_1 * \lambda_2$
Trace M = $\lambda_1 + \lambda_2$
edge if $\lambda_1 or \lambda_2$ are large
corner if both are large

RANSAC

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$

p = desired probability that random sample is free from outliers
$e = 1 - p$ s = number of points in sample N = number of loops in RANSAC (total samples)

Early Termination

$T = 1 - e * total number of data points$