The Microfacet BRDF Model

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CIS 560 Spring 2015
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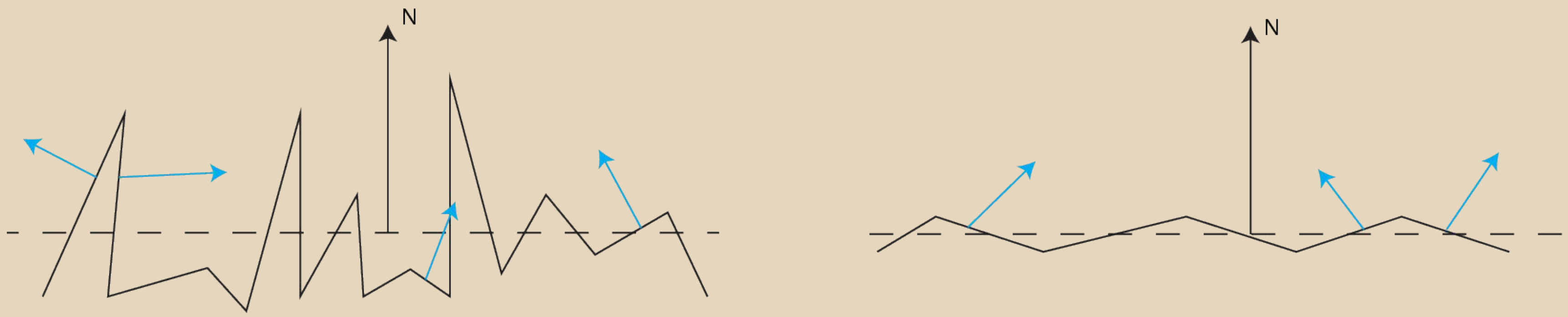
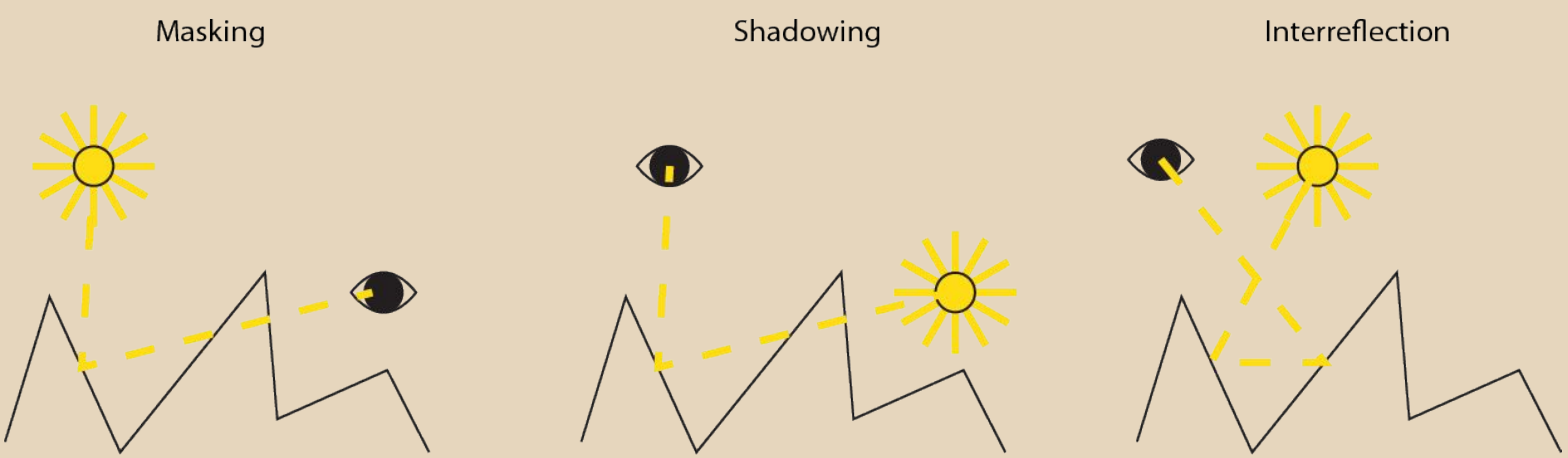
Microfacet Model of BRDFs

- A good way to approximate the reflectance of real-world surfaces is to model them as a collection of microscopic planar faces (microfacets)
  - A microfaceted surface can be thought of as a height field, where the variance in height is described by some statistical model
- Microfacet models assume that the area for which illumination is being computed is relatively large when compared to the size of the individual facets (hence the term *micro*facets)
  - This assumption allows us to use the average of many different microfacets' reflections to determine the BRDF model



Microfacet Components

- Two main components to a microfacet model
  - An expression for the distribution of facets
  - A BRDF that describes how light scatters from an individual microfacet (e.g. are the facets perfectly specular, or are they Lambertian?)
  - For the purposes of a Blinn microfacet model, facets are treated as perfectly specular
- Microfacet-level lighting effects must be considered in order to properly compute the reflection of such a model:

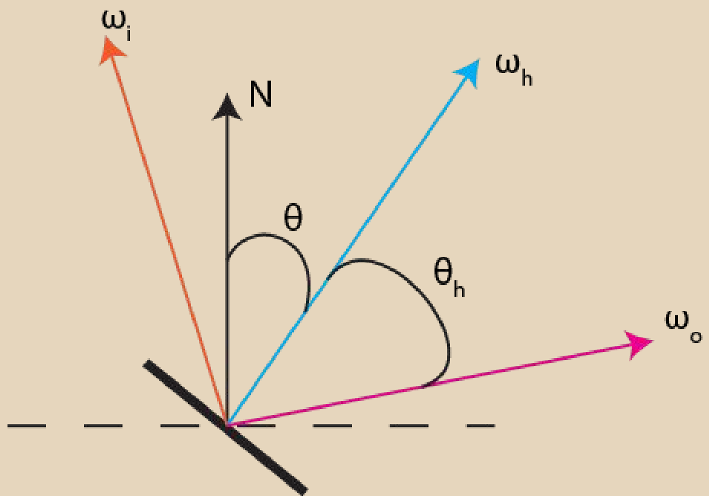


Torrance-Sparrow Microfacet Model

- Developed in 1967 as a way to represent metallic surfaces
  - Described as collections of perfectly smooth mirrored microfacets
- Surfaces described by this model have a distribution function $D(\omega_h)$ that gives the probability that a microfacet is perpendicular to a surface normal ω_h, the half-angle vector between ω_o and ω_i
  - $\omega_h = (\omega_o + \omega_i)/2$
  - Since the microfacets are perfectly specular, only those with a half-angle surface normal will provide any contribution to the light transmitted along ω_o
- $brdf(p, \omega_o, \omega_i) = (D(\omega_h)G(\omega_o,\omega_i)F_r(\omega_o))/(4\cos(\theta_o)\cos(\theta_i))$
  - G is a geometric attenuation term that accounts for effects like facet masking and facet shadowing
  - F_r is the Fresnel reflectance term
  - θ_o and θ_i are the angles between the surface normal and ω_o/ω_i
- We'll go over the derivation of the Torrance-Sparrow model in the next few slides

Torrance-Sparrow Derivation: dΦ

- The differential flux incident on the microfacets aligned with ω_h is as follows, given the definition of radiance L:
- $d\Phi_h = L_i(\omega_i)\, d\omega\, dA_{perp}(\omega_h) = L_i(\omega_i)\, d\omega \cos(\theta_h)\, dA(\omega_h)$
- We need to find the differential area of our microfacets aligned with ω_h
  - $dA(\omega_h) = D(\omega_h)\, d\omega_h\, dA$
- Now we can formulate an equation that gives us differential flux in terms of the differential area of our object:
- $d\Phi_h = L_i(\omega_i)\, d\omega\, dA_{perp}(\omega_h) = L_i(\omega_i)\, d\omega \cos(\theta_h)\, D(\omega_h)\, d\omega_h\, dA$



Torrance-Sparrow Derivation: Fresnel

- The Torrance-Sparrow model assumes that microfacets individually reflect light according to Fresnel's Law, so we can write outgoing flux as:
  - $d\Phi_o = F_r(\omega_o)\ d\Phi_h$
- Using the equation from the previous slide, this becomes:
  - $d\Phi_o = F_r(\omega_o)\ L_i(\omega_i)\ d\omega\ \cos(\theta_h)\ D(\omega_h)\ d\omega_h\ dA$
- We know the definition of radiance is:
  - $L(\omega_o) = d\Phi_o / (d\omega_o \cos(\theta_o)\ dA)$
- If we substitute $d\Phi_o$, we get:
  - $F_r(\omega_o)\ L_i(\omega_i)\ d\omega_i\ D(\omega_h)\ d\omega_h \cos(\theta_h)\ dA\ /\ (d\omega_o \cos(\theta_o)\ dA)$
  - $F_r(\omega_o)\ L_i(\omega_i)\ d\omega_i\ D(\omega_h)\ d\omega_h \cos(\theta_h)\ /\ (d\omega_o \cos(\theta_o))$
- We can simplify this equation even further by using this relationship between $d\omega_o$ and $d\omega_h$ (if you're interested in the derivation, check out page 698 of PBRT)
  - $d\omega_h = d\omega_o\ /\ 4\cos(\theta_h)$
- $F_r(\omega_o)\ L_i(\omega_i)\ d\omega_i\ D(\omega_h)\ /\ 4\cos(\theta_o)$

Torrance-Sparrow Derivation: Geometric Term

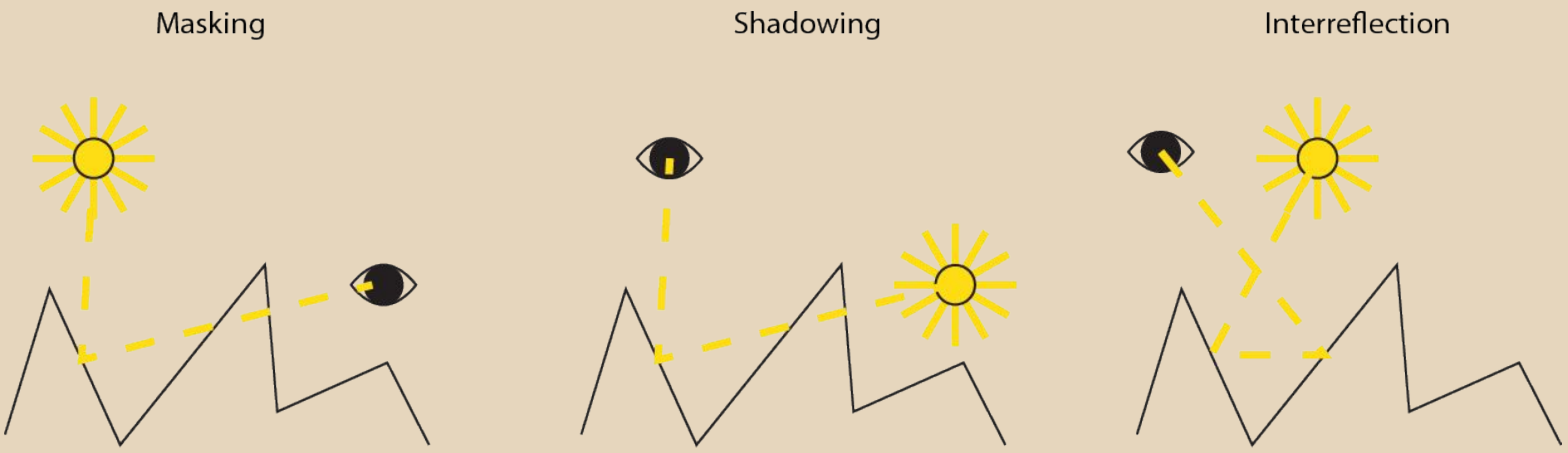
- From the previous slide, so far we have:
- $F_r(\omega_o)\ L_i(\omega_i)\ d\omega_i\ D(\omega_h)\ /\ 4\cos(\theta_o)$
- The definition of a general BRDF is:
  - $\mathrm{brdf}(p,\omega_o,\omega_i) = dL_o(p, wo)\ /\ L_i(p, wi)\ \cos(t_i)dw_i$
- Combining our T-S equation and the BRDF definition, we get:
  - $\mathrm{brdf}(p,\omega_o,\omega_i) = D(\omega_h)F_r(\omega_o)\ /\ (4\cos(\theta_o)\cos(\theta_i))$
- The full Torrance-Sparrow model also includes the geometric term used in the first slide, so our final equation is:
- $brdf(p,\ \omega_o,\ \omega_i) = (D(\omega_h)G(\omega_o,\omega_i)F_r(\omega_o))/(4\cos(\theta_o)\cos(\theta_i))$

Fresnel's Law

- For physically accurate reflection (and refraction), the reflectivity of a surface must take into account the angle between the incoming ray and the surface normal
- We'll only discuss the equations for dielectric (nonconductive) materials, such as plastic
- The Fresnel equations technically depend on the polarization of incident light, but because tracking this attribute is difficult we will use the equation for unpolarized light, which is just the combination of both polarized equations:
- $F_r = (r_L^2 + r_P^2)/2$
- $r_L = (n_t cos(\theta_i) - n_i cos(\theta_t)) \ / \ (n_t cos(\theta_i) + n_i cos(\theta_t))$
- $r_P = (n_i cos(\theta_i) - n_t cos(\theta_t)) \ / \ (n_i cos(\theta_i) + n_t cos(\theta_t))$
- n_t and n_t are the indices of refraction for the incident and transmitting media
- θ_t and θ_i are the angles between the [transmitted ray, incident ray] and the surface normal

The Geometric Term

- The geometric term is used to represent the effects of microfacets masking one another or shadowing one another
- Masking term: M = $2dot(N, \omega_h)dot(N, \omega_o) / dot(\omega_o, \omega_h)$
- Shadowing term: S = $2dot(N, \omega_h)dot(N, \omega_i) / dot(\omega_i, \omega_h)$
- Geometric term: $G(\omega_o,\omega_i) = min\{1, M, S\}$



Blinn Microfacet Distribution

- In 1977, Jim Blinn proposed a model where the distribution of microfacet normals is approximated by an exponential falloff
- The most likely facet orientation in this model is in the actual surface normal direction, falling off to **no** microfacets oriented to the normal
- For smooth (e.g. mirrored) surfaces, this falloff occurs quickly, while it is more gradual for rough surfaces
- $D(\omega_h) = dot(\omega_h, N)^e c$
- This model enforces the restriction that the distribution of microfacets is normalized, i.e. the sum of the projected area of all microfacets over some area is equal to that area:
    - $\int_H D(\omega_h) \cos(\theta_h) d\omega_h = 1$
- So, we need to compute some constant *c* that ensures this remains the case

Blinn Microfacet Distribution

- $\int_H D(\omega_h)\cos(\theta_h)d\omega_h c = \int_0^{2\pi}\int_0^{\pi/2} c(\cos(\theta_h))^{e+1}\sin(\theta_h)d\theta\, d\varphi$

  $= 2c\pi\int_0^1 u^{e+1}du$

  $= 2c\pi(u^{e+2}/(e+2))|_0^1$

  $= 2c\pi/(e+2) = 1$

- Therefore, $c = (e+2)/2\pi$
- Given this, our Blinn microfacet distribution becomes:
- $D(\omega_h) = (e+2)/2\pi\ \mathrm{dot}(\omega_h, N)^e$