BUILDING A SPAM FILTER USING NAÏVE BAYES

Review: Bayes' Rule & Diagnosis

Likelihood *Prior*

$$P(a|b) = \frac{P(b|a) * P(a)}{P(b)}$$

Posterior

Normalization

- **Useful for assessing diagnostic probability from causal probability:**

$$P(Cause|Effect) = \frac{P(Effect|Cause) * P(Cause)}{P(Effect)}$$

Review: Bayes' Rule For Diagnosis II

$$P(Disease \mid Symptom) = \frac{P(Symptom \mid Disease) * P(Disease)}{P(Symptom)}$$

Imagine:

- **disease = TB, symptom = coughing**
- ***P(disease | symptom)* is different in TB-indicated country vs. USA**
- ***P(symptom | disease)* should be the same**
  - It is more widely useful to learn *P(symptom | disease)*

- **What about P(symptom)?**
  - Last time: Use *conditioning*
  - For determining, e.g., the *most likely* disease given the symptom, we can just ignore P(symptom)!!! (Coming up: Slide 11)

Review: Naïve Bayes I

C: Cavity
T:Toothache
X:Xray

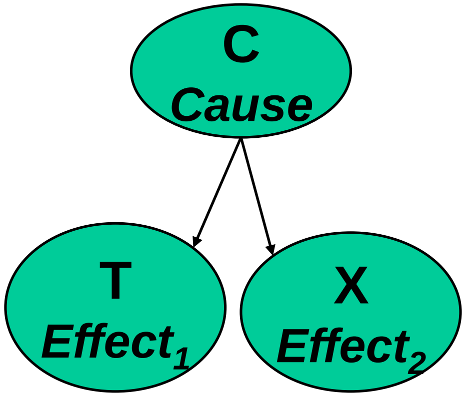
By Bayes Rule $P(C|T,X) = \frac{P(T,X|C)P(C)}{P(T,X)}$

If T and X are *conditionally independent given C*:

$$P(C|T,X) = \frac{P(T|C)P(X|C)P(C)}{P(T,X)}$$

This is a *Naïve Bayes Model:*

All effects assumed conditionally independent given Cause

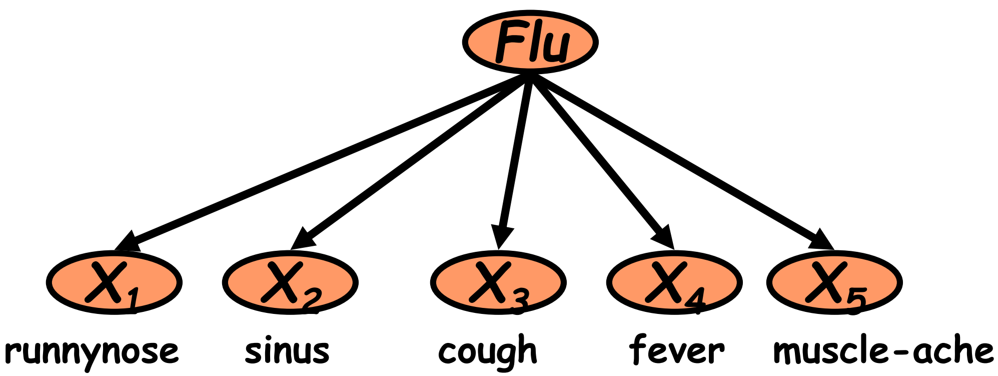


Review: Bayes' Rule II

- **More generally, if *$Effect_i$* are conditionally independent given *Cause:***

$$P(Cause, Effect_1, \ldots, Effect_{n)} = P(Cause)\prod_i P(Effect_i \mid Cause)$$

- **And total number of parameters is *linear* in *n***



Spam or not Spam: that is the question.

From: "" <takworlld@hotmail.com>

Subject: real estate is the only way... gem oalvgkay

Anyone can buy real estate with no money down

Stop paying rent TODAY !

There is no need to spend hundreds or even thousands for similar courses

I am 22 years old and I have already purchased 6 properties using the methods outlined in this truly INCREDIBLE ebook.

Change your life NOW !

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Categorization/Classification Problems

- **Given:**
  - A *description* of an instance, $x \in X$, where X is the *instance language* or *instance space*.
    - —*(Important Issue: how do we represent text documents?)*
  - *A fixed set of categories:*

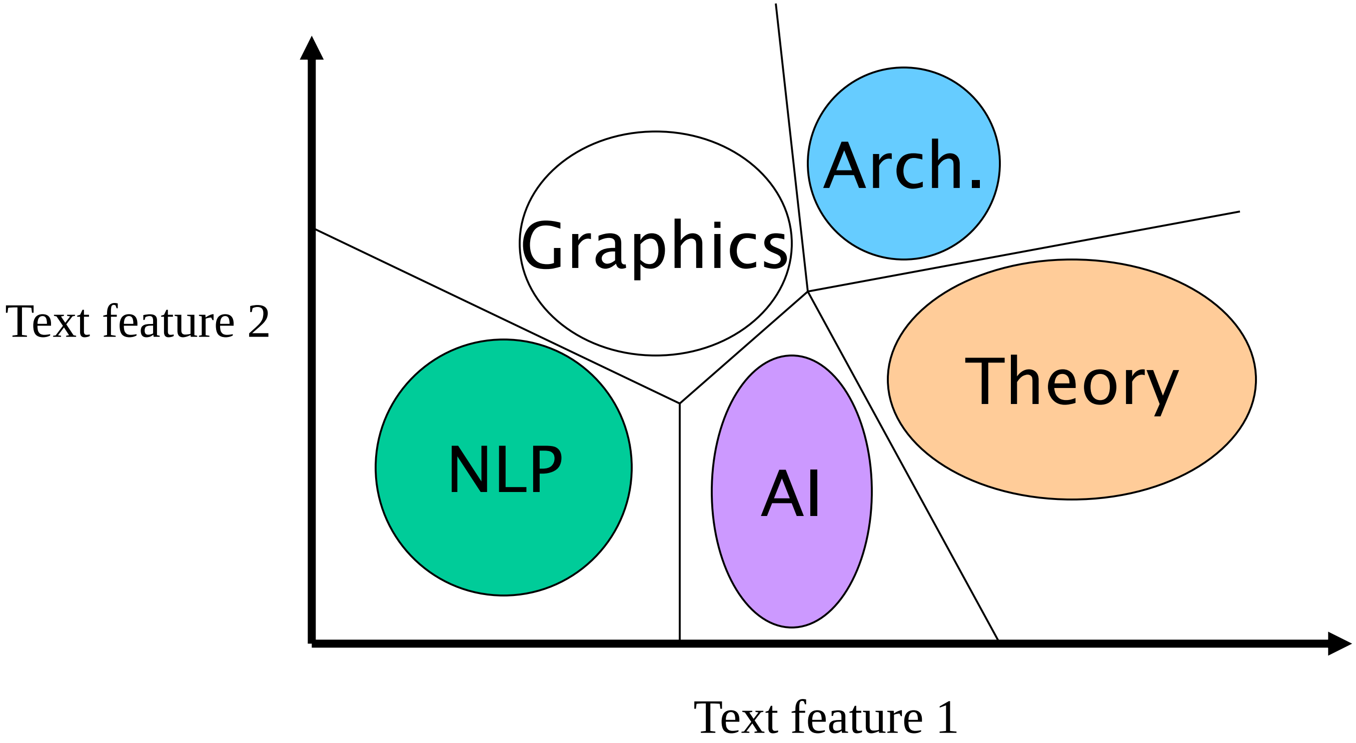
    $C = \{c_1, c_2, \ldots, c_n\}$

- **To determine:**
  - The category of x: $c(x) \in C$, where $c(x)$ is a categorization function whose domain is X and whose range is C.
    - —*We want to automatically build categorization functions ("classifiers").*

EXAMPLES OF TEXT CATEGORIZATION

- **Categories = SPAM?**
  - “spam” / “not spam”
- **Categories = TOPICS**
  - “finance” / “sports” / “asia”
- **Categories = OPINION**
  - “like” / “hate” / “neutral”
- **Categories = AUTHOR**
  - “Shakespeare” / “Marlowe” / “Ben Jonson”
  - The Federalist papers

A Graphical View of Text Classification



Bayesian Methods for Classification

- **Uses *Bayes theorem* to build a *generative model* that approximates how data is produced.**
- **First step:**

$$P(C \mid X) = \frac{P(X \mid C)P(C)}{P(X)}$$

Where C: Categories, X: Instance to be classified

- **Uses *prior probability* of each category given *no* information about an item.**
- **Categorization produces a *posterior probability* distribution over the possible categories given a description of each instance.**

Maximum a posteriori (MAP) Hypothesis

- **Let c_{MAP} be the most probable category. Then goodbye to that nasty normalization!!**

$$c_{MAP} \equiv \underset{c \in C}{\operatorname{argmax}} P(c \mid X)$$

$$= \underset{c \in C}{\operatorname{argmax}} \frac{P(X \mid c) P(c)}{P(X)}$$

No need to compute *P(X)!!!!*

$$= \underset{c \in C}{\operatorname{argmax}} P(X \mid c) P(c)$$

As *P(X)* is constant

Maximum likelihood **Hypothesis**

If all hypotheses are *a priori* equally likely,

to find the maximally likely category c_{ML},

we only need to consider the $P(X|c)$ term:

$$c_{ML} \equiv \underset{c \in C}{\operatorname{argmax}} P(X \mid c)$$

Maximum
Likelihood
Estimate
(“MLE”)

Naïve Bayes Classifiers: Step 1

Assume that instance X described by n-dimensional vector of attributes $X = \langle x_1, x_2, \ldots, x_n \rangle$

then

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}}\ P(c \mid x_1, x_2, \ldots, x_n)$$

$$= \underset{c \in C}{\operatorname{argmax}}\ \frac{P(x_1, x_2, \ldots, x_n \mid c)P(c)}{P(x_1, x_2, \ldots, x_n)}$$

$$= \underset{c \in C}{\operatorname{argmax}}\ P(x_1, x_2, \ldots, x_n \mid c)P(c)$$

Naïve Bayes Classifier: Step 2

To estimate: $$c_{MAP} = \underset{c_j \in C}{\operatorname{argmax}} P(x_1, x_2, \ldots, x_n \mid c_j) P(c_j)$$

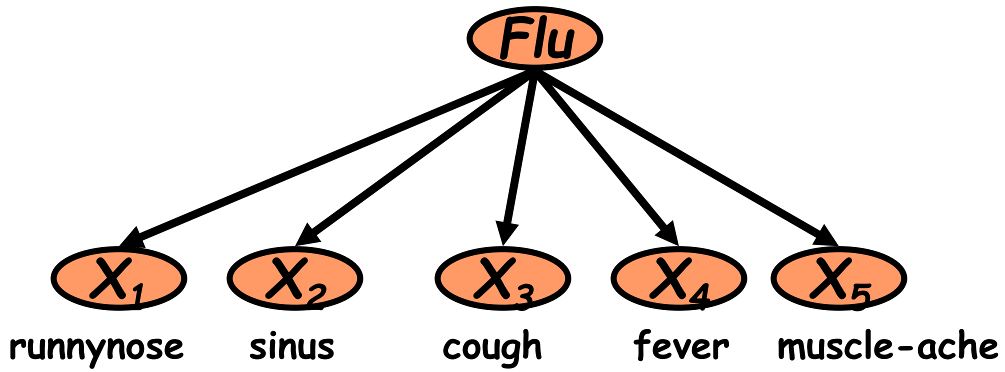
- $\boldsymbol{P(c_j)}$**:** Can be estimated from the frequency of classes in the training examples.
- $\boldsymbol{P(x_1,x_2,\ldots,x_n|c_j)}$**:** *Problem!!*
  - O(|*X*|^n•|*C*|) parameters required to estimate full joint prob. distribution

Solution:

Naïve Bayes Conditional Independence Assumption:

$$P(x_i, x_2, \ldots, x_n \mid c_j) = \prod_i P(x_i \mid c_j)$$

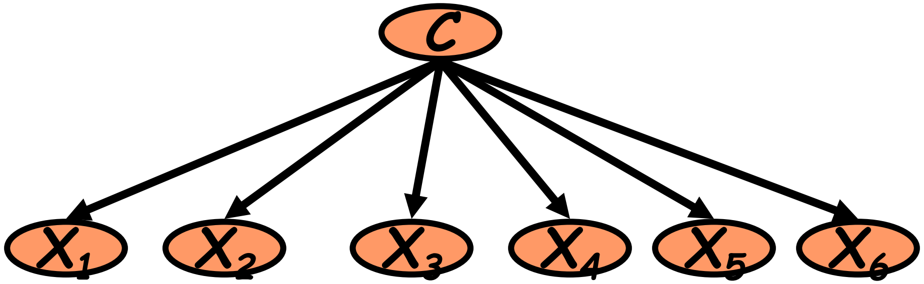
Naïve Bayes Classifier for *Boolean* variables



- Conditional Independence Assumption: **features are independent of each other given the class:**

$$P(X_1, \ldots, X_5 \mid C) = P(X_1 \mid C) \bullet P(X_2 \mid C) \bullet \cdots \bullet P(X_5 \mid C)$$

Learning the Model

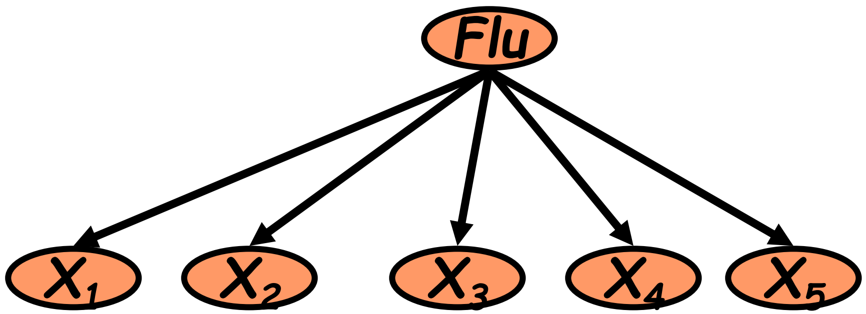


- **First attempt: *maximum likelihood* estimates**
  - Given training data for N individuals, where $count(X=x)$ is the number of those individuals for which $X=x$, $e.g\ Flu=true$
  - For each category c and each value x for a variable X

$$\hat{P}(c) = \frac{count(C=c)}{|N|}$$

$$\hat{P}(x \mid c) = \frac{count(X=x, C=c)}{count\ (C=c)}$$

Problem with Max Likelihood for Naïve Bayes



$$P(X_1,\ldots,X_5 \mid Flu) = P(X_1 \mid Flu) \bullet P(X_2 \mid Flu) \bullet \cdots \bullet P(X_5 \mid Flu)$$

- **What if no training cases where patient with flu had a cough?**

$$\hat{P}(X_3 = t \mid flu) = \frac{count(X_3 = t, flu)}{count(flu)} = 0$$

$$\text{So if } X_3 = t,\ \ P(X_1,\ldots,X_3 \mid flu) = 0$$

Zero probabilities overwhelm any other evidence!

"Add-1" Laplace Smoothing to Avoid Overfitting

$$\hat{P}(x \mid c) = \frac{count(X = x, C = c) + 1}{count(C = c) + |X|}$$

# of values of X_i, *here 2*

- **Slightly better version**

$$\hat{P}(x \mid c) = \frac{count(X = x, C = c) + \alpha}{count(C = c) + \alpha |X|}$$

extent of "smoothing"

Using Naive Bayes Classifiers to Classify Text: Basic method for *Multinomial Variables*

- **As a generative model:**
  1. Randomly pick a category c according to $P(c)$
  2. For a document of length *N, for each word$_i$*:
     1. Generate $word_i$ according to $P(w|c)$

$$P(c, D = < w_1, w_2, \ldots, w_n >) = P(c) \prod_{i=1}^{N} P(w_i \mid c)$$

- **This is a Naïve Bayes classifier for *multinomial* variables.**
- ***Note that word order is assumed irrelevant here***
  - Uses same parameters for each position
  - Result is *bag of words* model
    - —Views document not as an ordered list of words, but as a *multiset*

Naïve Bayes: Learning (First attempt)

- **From training corpus, extract *Vocabulary***

- Calculate required estimates of $P(c_j)$ and $P(w_i | c_j)$ terms,
  - For each c_j in C do

$$P(c_j) \leftarrow \frac{count_{docs}(C = c_j)}{|docs|}$$

where $count_{docs}(x)$ *is the number of documents for which* x *is true.*

  - For each word $w_i \in Vocabulary$ *and* $c_j \in C$, where $count_{doctokens}(x)$ is the number of tokens over *all* documents for which x is true of that document and that token…

$$P(w_i | c) \leftarrow \frac{count_{doctokens}(W = w_i, C = c)}{count_{doctokens}(C = c)}$$

Naïve Bayes: Learning (Second attempt)

- **Laplace smoothing must be done over the vocabulary items.**
  - We can assume we have at least one instance of each *category*, so we don't need to smooth these.
- **Assume a single new word *UNK,* that occurs nowhere within the training document set.**
- **Map all unknown words in documents to be classified (*test documents)* to UNK.**
- **with $0 \leq \alpha \leq 1$**

$$P(w_i \mid c) \leftarrow \frac{count_{doctokens}(W = w_i, C = c) + \alpha}{count_{doctokens}(C = c) + \alpha(|V| + 1)}$$

Naïve Bayes: Classifying

- **Compute c_{NB} using *either***

$$c_{NB} = \arg\max_{c} P(c) \prod_{i=1}^{N} P(w_i \mid c)$$

$$c_{NB} = \arg\max_{c} P(c) \prod_{w \in V} P(w \mid c)^{count(w)}$$

where $count(w)$: the number of times word w occurs in doc

(The two are equivalent..)

PANTEL AND LIN: SPAMCOP

- **Uses a Naïve Bayes classifier**
- **M is spam if $P(Spam|M) > P(NonSpam|M)$**
- **Method**
  - Tokenize message using Porter Stemmer
  - Estimate $P(x_k|C)$ using m-estimate (a form of smoothing)
  - Remove words that do not satisfy certain conditions
  - Train: 160 spams, 466 non-spams
  - Test: 277 spams, 346 non-spams
- **Results: ERROR RATE of 4.33%**
  - Worse results using trigrams

Naive Bayes is (was) Not So Naive

- **Naïve Bayes: First and Second place in KDD-CUP 97 competition, among 16 (then) state of the art algorithms**

  Goal: Financial services industry direct mail response prediction model: Predict if the recipient of mail will actually respond to the advertisement – 750,000 records.
- **A good dependable baseline for text classification**
  - But not the best *by itself*!
- **Optimal if the Independence Assumptions hold:**
  - If assumed independence is correct, then it is the Bayes Optimal Classifier for problem
- **Very Fast:**
  - Learning with one pass over the data;
  - Testing linear in the number of attributes, and document collection size
- **Low Storage requirements**

Engineering: Underflow Prevention

- **Multiplying lots of probabilities, which are between 0 and 1 by definition, can result in floating-point underflow.**
- **Since** $log(xy) = log(x) + log(y)$**, it is better to perform all computations by summing logs of probabilities rather than multiplying probabilities.**
- **Class with highest final un-normalized log probability score is still the most probable.**

$$c_{NB} = \underset{c_j \in C}{\operatorname{argmax}} \log P(c_j) + \sum_{i \in positions} \log P(w_i \mid c_j)$$

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