Projective Geometry

Homography maps original image to new frame each point has some lambda. Can use vanishing points as points

$$\lambda_i x_i' = Hx_i$$

$$\lambda_1 \begin{pmatrix}-1\\-1\\-1\end{pmatrix} = H\begin{pmatrix}0\\0\\1\end{pmatrix} <--h3$$

$$\lambda_2 \begin{pmatrix}5\\0\\1\end{pmatrix} = H\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}5\lambda_2\\0\\\lambda_2\end{pmatrix}$$

ect..

$$H = \begin{pmatrix}5\lambda_2 & 0 & -1\\0 & 5\lambda_3 & -1\\1 & 1 & 1\end{pmatrix}$$

Homogenous Coordinates of line

$$L' = (H^{-1})^{-T}L$$

$$L' = H^T L$$

Homogenous form of L for line at infinity

$$l = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Vanishing Points

given an image , find two lines on image that should be parallel in real life, but intersect in the image. a and b are two points on the top line, d and e are two points on bottom line then

$$v = \left(\begin{vmatrix}a\\1\end{vmatrix} \times \begin{vmatrix}b\\1\end{vmatrix}\right) \times \left(\begin{vmatrix}d\\1\end{vmatrix} \times \begin{vmatrix}e\\1\end{vmatrix}\right)$$

CROSS RATIO DOES NOT CHANGE IN HOMOGRAPHIC TRANSFORMATION

$$\frac{||a-c||||b-v||}{||b-c||||a-v||}$$

Homography Estimation

$$x_2 = Hx_1$$

$$x_2 \times Hx_1 = 0$$

$$\begin{bmatrix}0 & -1 & v\\1 & 0 & -u\\-v & u & 0\end{bmatrix}\begin{bmatrix}x_1^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3}\\\mathbf{0}_{1x3} & x_1^T & \mathbf{0}_{1x3}\\\mathbf{0}_{1x3} & \mathbf{0}_{1x3} & x_1^T\end{bmatrix}\begin{bmatrix}h_1^T\\h_2^T\\h_3^T\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{0}_{1x3} & -x_1^T & v_2x_1^T\\x_1^T & \mathbf{0}_{1x3} & -u_2x_1^T\\-v_2x_1^T & u_2x_1^T & \mathbf{0}_{1x3}\end{bmatrix}\begin{bmatrix}h_1^T\\h_2^T\\h_3^T\end{bmatrix}$$

$$\text{This is in the form } Ax = 0H = \begin{bmatrix}h_1\\h_2\\h_3\end{bmatrix}$$

$$r_1 = K^{-1}h_1/||K^{-1}h_1||$$

$$r_2 = K^{-1}h_2/||K^{-1}h_2||$$

$$r_3 = r_1 \times r_2$$

$$t = K^{-1}h_3/||K^{-1}h_1||$$

Skew Matrix!

$$\begin{bmatrix}0 & -a_3 & a_2\\a_3 & 0 & -a_1\\-a_2 & a_1 & 0\end{bmatrix}$$

2d cross product

$$x \times y = x_1y_2 - x_2y_1$$

3d cross product

$$u \times v = (u_2v_3 - u_3v_2)\mathbf{i}$$

$$+(u_3v_1 - u_1v_3)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

as column vectors

$$\begin{bmatrix}(u_2v_3 - u_3v_2)\\(u_3v_1 - u_1v_3)\\(u_1v_2 - u_2v_1)\end{bmatrix}$$

Rotation Matricies

Rotations composed by in order by multiplying on the left example :

$$R_3 = R_2 * R_1$$

Axis Angle

rotation vector is r , given two random vector a and b and some angle between them α

$$\boldsymbol{k} = \frac{\boldsymbol{a} \times \boldsymbol{b}}{|a||b|\sin\alpha}$$

$$\boldsymbol{r} = \theta\boldsymbol{k}$$

rodrigues formula

k is unit vector for

axis of rotation

which v rotates about theta

$$\boldsymbol{v_{rot}} = \boldsymbol{v}\cos\theta + (\boldsymbol{k} \times \boldsymbol{v})\sin\theta$$

$$+\boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{v})(1-\cos\theta)$$

Axis angle to Rotation Matrix

$K =$ matrix cross product with v

$$K = \begin{pmatrix}0 & -k_3 & k_2\\k_3 & 0 & -k_1\\-k_2 & k_1 & 0\end{pmatrix}$$

$$R = I + (\sin\theta)K + (1-\cos\theta)K^2$$

Quaternions

$$ijk = -1|ij = k|jk = i|ki = j$$

$$q = \cos\frac{\theta}{2} + (u_x\mathbf{i} + u_y\mathbf{j} + u_z\mathbf{k})\sin\frac{\theta}{2}$$

$$q = \begin{bmatrix}s, & v\end{bmatrix} s\epsilon\mathbb{R}$$

$$q = \begin{bmatrix}s, & x\mathbf{i} & y\mathbf{j} & z\mathbf{k}\end{bmatrix}$$

$$q_aq_b = [s_a, \mathbf{a}][s_b, \mathbf{b}]$$

$$= [s_as_b - \mathbf{a}\cdot\mathbf{b}, s_a\mathbf{b} + s_b\mathbf{a} + \mathbf{a}\times\mathbf{b}]$$

Camera Extrinsic Paramaters

World to Camera formula

$$X_c = \begin{bmatrix}R & t\end{bmatrix}X_w$$

Cameras world position is $-R^Tt$

Rotation is R^T

Radial Distortion

$$r = ||x_{ideal}||$$

$$u_{img} = u_{ideal} +$$

$$(u_{ideal} - p_x)[k_1r^2 + k_2r^4]$$

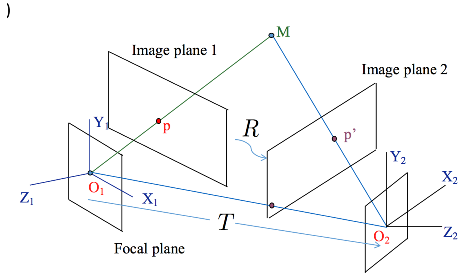
$$v_{img} = v_{ideal} +$$

$$(v_{ideal} - p_y)[k_1r^2 + k_2r^4]$$

$$\begin{bmatrix}(u_{ideal} - p_x)r^2 & (u_{ideal} - p_x)r^4\\(v_{ideal} - p_y)r^2 & (v_{ideal} - p_y)r^4\end{bmatrix}$$

$$= \begin{bmatrix}u_{img} - u_{ideal}\\v_{img} - v_{ideal}\end{bmatrix}$$

Epipolar



Essential Matrix

$$P_2^TEP_1 = 0 <--svd$$

$$E = RT$$

Fundamental Matrix

$$p_2^TFp_1 = 0$$

$$F = K_2^{-T}EK_1^{-1}$$

Epipolar Line

$$p_2 = p_0, p_0^TFp_1 = 0$$

Epipole

$$p_2^TF = 0$$

$$Fp_1 = 0$$

if we have $R_1, R_2, t_1, t_2, K_1, K_2$

$$R = R_2R_1^T$$

$$T = t_1 - (R_1R_2^Tt_2)$$

Param Estimation

$$K[R|t]\begin{bmatrix}X\\1\end{bmatrix} = P\begin{bmatrix}X\\1\end{bmatrix} u = P_1\begin{bmatrix}X\\1\end{bmatrix}$$

$$v = P_2\begin{bmatrix}X\\1\end{bmatrix}$$

$$w = P_3\begin{bmatrix}X\\1\end{bmatrix}$$

Non Linear Triangulation

$$J = \begin{bmatrix}\frac{w\frac{\partial u}{\partial X} - u\frac{\partial w}{\partial X}}{w^2}\\\frac{w\frac{\partial v}{\partial X} - v\frac{\partial w}{\partial X}}{w^2}\end{bmatrix}$$

$$\frac{\partial u}{\partial X} = \begin{bmatrix}P_{11} & P_{12} & P_{13}\end{bmatrix}$$

$$\frac{\partial v}{\partial X} = \begin{bmatrix}P_{21} & P_{22} & P_{23}\end{bmatrix}$$

$$\frac{\partial w}{\partial X} = \begin{bmatrix}P_{31} & P_{32} & P_{33}\end{bmatrix}$$

PnP

given x,X,K solve for R,t

$$\lambda \begin{bmatrix} x \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

PnP

$$[R|t] = K^{-1}P$$

Calculate Calibration

Let $Q = [R|t] \begin{bmatrix} X \\ 1 \end{bmatrix}$

$$\lambda \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} Q = 0$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \times \begin{bmatrix} f_x & s & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} Q = 0$$

$$A = \begin{bmatrix} Q_x & Q_y & Q_z & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_y & Q_z & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_z \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} A \begin{bmatrix} f_x \\ s \\ p_x \\ f_y \\ p_y \\ 1 \end{bmatrix} = 0$$