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Questão 2

a) $G(s) = \frac{5}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} \Rightarrow \frac{5}{s(s+5)} = \frac{A(s+5)+Bs}{s(s+5)}$

$5 = As + 5A + Bs$

$5 = As + Bs + 5A$

$5 = (A+B)s + 5A$

$0 = (A+B)s \Rightarrow A+B = 0 \quad A = 1 \quad B = -1 \quad 5A = 5$

$\frac{5}{s(s+5)} = \frac{1}{s} + \frac{-1}{s+5} \Rightarrow \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+5}\right) \Rightarrow c(t) = 1 - e^{-5t}$

$-a = -5 \Rightarrow a = 5 \quad T = \frac{1}{a} = \frac{1}{5} = 0{,}2$

$T_r = \frac{2{,}2}{5} = 0{,}44s \quad T_s = \frac{4}{5} = 0{,}8s$

b) $\frac{1}{s} \cdot \frac{20}{s+20} = C(s) \Rightarrow \frac{20}{s+(s+20)} = \frac{A}{s} + \frac{B}{s+20} \Rightarrow 20 = As + 20A + Bs$

$20 = (A+B)s + 20A$

$0 = (A+B)s \Rightarrow A+B = 0 \Rightarrow 20A = 20 \Rightarrow A = 1 \quad B = -1$

$\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+20}\right) = 1 - e^{-20t} \quad a = 20$

$T = \frac{1}{a} = \frac{1}{20} = 0{,}05s \quad T_r = \frac{2{,}2}{20} = 0{,}11s \quad T_s = \frac{4}{20} = 0{,}2s$

4) $5 - 2\frac{dq}{dt} - \frac{q}{0{,}5} = 0 \quad 2q = v \Rightarrow 2\frac{dq}{dt} + 2q = 5 \Rightarrow \frac{dv}{dt} + v = 5$

$V = \frac{5}{s+1} \Rightarrow T_c = \frac{1}{1} = 1s, \quad T_r = \frac{2{,}2}{1} = 2{,}2s \quad T_s = \frac{4}{1} = 4s$

6) $M\ddot{x} + 6\dot{x} = f$

$M\dot{v} + 6v = f$

$T(s) = \frac{1}{\underbrace{Ms+6}} \rightarrow s = \frac{-6}{M}$ $T_v = \frac{2,2M}{6} = 0,366M$ $T_s = \frac{4M}{6} = \frac{2M}{3}$

8) a) $a + ke^{-2t}$

b) $a + k_1 e^{-3t} + k_2 e^{-6t}$

c) $a + k_1 e^{-10t} + k_2 e^{-20t}$

d) $a + e^{-3t}(k_1 \cos(11,6t) + k_2 \operatorname{sen}(11,6t))$

e) $a + k_1 \cos(3t) + k_2 \operatorname{sen}(3t)$

f) $a + k_1 e^{-10t} + k_2 t e^{-10t}$

10) $T(s) = C(sI - A)^{-1} B + D$, onde B é uma coluna, C é um vetor linha e $D = 0$

$A = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 0 & 1 \\ 4 & 7 & -5 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ -2 \\ +3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 7 & 1 \end{bmatrix}$, $D = 0$

Raízes = [12 -38 25]

Resposta = -7,5 4,81 0,69

12) $\frac{V - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1}{sL} + \frac{sV_1}{1/C}$

$\frac{V_1}{V} = \frac{1}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{sL} + sC\right)}$ $\frac{V_1}{V} = \frac{10}{s^2 + 20s + 500}$

Polos complexos $(-10 \pm 20i)$

forma geral: $e^{-10t}(k_1 \cos(20t) + k_2 \operatorname{sen}(20t))$

14) $F = (Ms^2 + f_v s + k_s)X \rightarrow \frac{X}{F} = \frac{1}{Ms^2 + f_v s + k_s}$

$\frac{X}{F} = \frac{1}{2s^2 + 6s + 2} \rightarrow X = \frac{1}{2s(s^2 + 3s + 1)}$

19) $\dfrac{a}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ $\quad \omega_n = 20$, $\zeta = 0{,}15$ $\quad \dfrac{a}{s^2 + 6s + 400}$

$s = 0 \rightarrow a = 400$

$T(s) = \dfrac{400}{s^2 + 6s + 400}$ $\quad C(s) = \dfrac{400}{s(s^2 + 6s + 400)} \rightarrow \dfrac{1}{s} - \dfrac{s+3}{(s+3)^2 + 391} - \dfrac{3}{(s+3)^2 + 391}$

$C(t) = 1 - e^{-3t}\left[\cos(\sqrt{391}\,t) + \frac{3}{\sqrt{391}}\,\text{sen}(\sqrt{391}\,t)\right]$

20) $T_s = \dfrac{4}{\zeta\omega_n}$, $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ e $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cdot 100$

a) $\omega_n = 4$ rad/s | $\zeta = 0{,}375$ $\quad T_s = 2{,}666\,s$, $T_p = 0{,}847\,s$, $\%OS = 28\%$

b) $\omega_n = 0{,}2$ rad/s | $\zeta = 0{,}05$ $\quad T_s = 400\,s$, $T_p = 15{,}72$ e $\%OS = 85{,}4\%$

c) $\omega_n = 3240{,}37$ rad/s | $\zeta = 0{,}247$ $\quad T_s = 5\,ms$, $T_p = 1\,ms$, $\%OS = 44{,}9\%$

25) $F - 2sx - 20x = 5s^2x$

$T(s) = \dfrac{X}{F} = \dfrac{1}{5s^2 + 2s + 20}$

b) $\omega_n = \sqrt{\dfrac{20}{5}} = 2$ | $\zeta = \dfrac{2}{5 \cdot 2 \cdot 2} = 0{,}1$ | $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cdot 100 \cong 72{,}92\%$

$T_s = \dfrac{4}{0{,}2} = 20\,s$ | $T_p = \dfrac{\pi}{2\sqrt{1 - 0{,}1^2}} = 1{,}58\,s$ | T_r $\dfrac{1{,}104}{2} = 0{,}552\,s$

$\dfrac{1}{20} = 0{,}05$ ($s \rightarrow 0$