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1) a)  $f(t) = U(t) \rightarrow F(s) = \frac{1}{s}$

b)  $f(t) = tU(t) \rightarrow F(s) = \frac{1}{s^2}$

c)  $\sin \omega t U(t) \rightarrow \frac{\omega}{s^2 + \omega^2}$

d)  $\cos \omega t U(t) \rightarrow \frac{s}{s^2 + \omega^2}$

2) a)  $e^{-at} \sin \omega t U(t) \rightarrow \frac{\omega}{(s^2 + \omega^2)(s + a)}$

b)  $e^{-at} \cos \omega t U(t) \rightarrow \frac{s}{(s^2 + \omega^2)(s + a)}$

c)  $t^3 \rightarrow \frac{3!}{s^4}$

8)  $\frac{d^3 y}{dt^3} + 3\frac{d^2 y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3 x}{dt^3} + 4\frac{d^2 x}{dt^2} + 6\frac{dx}{dt} + 8x$

$\Rightarrow s^3 Y + 3s^2 Y + 5sY + Y = s^3 X + 4s^2 X + 6sX + 8X$   
 $\Rightarrow \frac{Y(s)}{X(s)} = \frac{s^3 + 3s^2 + 5s + 1}{s^3 + 4s^2 + 6s + 8}$

9)  $X(s) = \frac{7}{F(s)} \Rightarrow s^2 X(s) + 5sX(s) + 10X(s) = 7F(s)$

$F(s) = \frac{1}{s^2 + 5s + 10} \quad \mathcal{L}^{-1}\{s^2 X(s) + 5sX(s) + 10X(s) = 7F(s)\} \Rightarrow$

$\frac{d^2 x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 10x(t) = 7f(t)$

b)  $X(s) = \frac{15}{F(s)} = \frac{15}{(s+10)(s+11)} \Rightarrow \mathcal{L}^{-1}\{s^2 X(s) + 21sX(s) + 110X(s) = 15F(s)\} \Rightarrow$

$\frac{d^2 x(t)}{dt^2} + 21\frac{dx(t)}{dt} + 110x(t) = 15f(t)$

$$C) X(s) = \frac{s+3}{F(s)} \quad \rightarrow \mathcal{L}^{-1} \{ s^3 X(s) + 11s^2 X(s) + 12s X(s) + 18X(s) = sF(s) + 3F(s) \}$$

$$F(s) = \frac{s^3 + 11s^2 + 12s + 18}{s^3 + 11s^2 + 12s + 18}$$

$$\frac{d^3 x(t)}{dt^3} + 11 \frac{d^2 x(t)}{dt^2} + 12 \frac{dx(t)}{dt} + 18x(t) = \frac{d^3 f(t)}{dt^3} + 3f(t)$$

$$10) C(s) = \frac{s^5 + 2s^4 + 4s^3 + s^2 + 4}{R(s)}$$

$$R(s) = s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 5$$

$$\mathcal{L}^{-1} \{ s^5 R(s) + 2s^4 R(s) + 4s^3 R(s) + s^2 R(s) + 4R(s) = s^6 C(s) + 7s^5 C(s) + 3s^4 C(s) + 2s^3 C(s) + s^2 C(s) + 5C(s) \}$$

$$\frac{d^5 x(t)}{dt^5} + 2 \frac{d^4 x(t)}{dt^4} + 4 \frac{d^3 x(t)}{dt^3} + \frac{d^2 x(t)}{dt^2} + 4x(t) = \frac{d^6 c(t)}{dt^6} + 7 \frac{d^5 c(t)}{dt^5} + 3 \frac{d^4 c(t)}{dt^4} + \frac{d^3 c(t)}{dt^3} + \frac{d^2 c(t)}{dt^2} + 5c(t)$$

$$11) C(s) = \frac{s^4 + 3s^3 + 2s^2 + s + 1}{R(s)} \quad \rightarrow x(t) = 3t^3 \rightarrow R(s) = \frac{6}{s^4}$$

$$R(s) = s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2$$

$$\mathcal{L}^{-1} \left\{ \frac{6}{s^4} \cdot (s^4 + 3s^3 + 2s^2 + s + 1) = C(s) \cdot (s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2) \right\} =$$

$$\mathcal{L}^{-1} \left\{ 6 + \frac{18}{s} + \frac{12}{s^2} + \frac{6}{s^3} + \frac{6}{s^4} = C(s) \cdot (s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2) \right\} =$$