1. for $r \in \{a, h\}$, $\tilde{p}_i = p_i + v_i$

where $p_r = \bar{p}_r + e_i$, $\bar{p}_a < \bar{p}_h$

and $e_i \sim N(0, \sigma^2_e)$
$v_i \sim N(0, \sigma^2_v)$
$Cov(e, v) = 0$

police stops car iff

$E[p \mid \tilde{p}, r] > p^* > \bar{p}_h$

a) given e_i is normally distributed

$p_r \sim N(\bar{p}_r, \sigma^2_e)$ and
$\tilde{p}_i = \bar{p}_r + e_i + v_i \sim N(\bar{p}_r, \sigma^2_e + \sigma^2_v)$

so from the conditional distribution of two normally distributed random variables

$$E[p_i \mid \tilde{p}_i, r] = \bar{p}_r + \frac{Cov(p_i, \tilde{p}_i)}{Var(\tilde{p}_i)} (\tilde{p}_r - \bar{p}_r)$$

$$= \bar{p}_r + \frac{Cov(p_i, p_i + n_i)}{Var(p_i + n_i)} (\tilde{p}_r - \bar{p}_u)$$

$$= \bar{p}_r + \frac{Var(p_i) + Cov(p_i, n_i)}{Var(p_i) + Var(n_i) + Cov(p_i, n_i)} (\hat{p}_i - \bar{p}_r)$$

$$= \bar{p}_r + \frac{Var(p_i)}{Var(p_i) + Var(n_i)} (\hat{p}_i - \bar{p}_r)$$

$$= \bar{p}_r + \frac{\sigma_e^2}{\sigma_e^2 + \sigma_n^2} (\tilde{p}_i - \bar{p}_r)$$

$$= \bar{p}_r \left(1 - \frac{\sigma_e^2}{\sigma_e^2 + \sigma_n^2} \right) + \tilde{p}_i \frac{\sigma_e^2}{\sigma_e^2 + \sigma_n^2}$$

$$= \bar{p}_r (1 - \gamma) + \tilde{p}_i \gamma$$

if $\tilde{p}_i = c$, since $\bar{p}_a < \bar{p}_b$

$$\begin{aligned} E[p_i \mid c, a] &= \bar{p}_a (1 - \gamma) + c\gamma \\ &< \bar{p}_b (1 - \gamma) + c\gamma \\ &= E[p_i \mid c, b] \end{aligned}$$

b) $P(E[p_i \mid \tilde{p}_i, v] > p^*)$

$= 1 - P(E[p_i \mid \tilde{p}_i, v] < p^*]$

$= 1 - P(\bar{p}_v (1-\gamma) + \tilde{p}_i \gamma < p^*)$

$= 1 - P(\tilde{p}_i \gamma < p^* - \bar{p}_v(1-\gamma))$

$= 1 - P\left(\tilde{p}_i < \dfrac{p^* - \bar{p}_v(1-\gamma)}{\gamma}\right)$

$= 1 - P\left(\tilde{p}_i - \bar{p}_v < \dfrac{p^* - \bar{p}_v(1-\gamma)}{\gamma} - \bar{p}_v\right)$

$= 1 - P\left(\tilde{p}_i - \bar{p}_v < \dfrac{p^* - \bar{p}_v + \bar{p}_v\gamma - \bar{p}_v\gamma}{\gamma}\right)$

$= 1 - P\left(\tilde{p}_i - \bar{p}_v < \dfrac{1}{\gamma}(p^* - \bar{p}_v)\right)$

$= 1 - P\left(\tilde{p}_i - \bar{p}_v < \left(\dfrac{\sigma_e^2 + \sigma_v^2}{\sigma_i^2}\right)(p^* - \bar{p}_v)\right)$

$= 1 - P\left(\dfrac{\tilde{p}_i - \bar{p}_v}{\sqrt{\sigma_e^2 + \sigma_v^2}} < \dfrac{(\sigma_e^2 + \sigma_v^2)^{1/2}}{\sigma_a^2}(p^* - \bar{p}_v)\right)$

and $\dfrac{\tilde{p}_i - \bar{p}_v}{(\sigma_e^2 + \sigma_v^2)^{1/2}} \equiv z \sim N(0,1)$

$$E[p_i \mid \hat{p}_i, r] = 1 - \Phi\left(\frac{(\sigma_e^2+\sigma_m^2)^{1/2}(p^* - \bar{p}_r)}{\sigma_e^2}\right)$$

c) $E[p_i \mid E[p_i \mid \hat{p}_i, r] > p^*, r]$

$$= E[E[p_i \mid \tilde{p}_i, r] \mid E[p_i \mid \tilde{p}_i, r] > p^*, r]$$

$$= E\left[\bar{p}_r + \frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(\tilde{p}_i - \bar{p}_r) \,\middle|\, \frac{(\tilde{p}_i - \bar{p}_r)}{(\sigma_e^2+\sigma_m^2)^{1/2}} > \frac{(\sigma_e^2+\sigma_m^2)^{1/2}}{\sigma_e^2}(p^* - \bar{p}_r), r\right]$$

$$= \bar{p}_r + \frac{\sigma_e^2}{(\sigma_e^2+\sigma_m^2)^{1/2}} E\left[\frac{(\tilde{p}_i - \bar{p}_r)}{(\sigma_e^2+\sigma_m^2)^{1/2}} \,\middle|\, \frac{(\tilde{p}_i - \bar{p}_r)}{(\sigma_e^2+\sigma_m^2)^{1/2}} > \text{"} \quad \text{"}\right]$$

$$= \bar{p}_r + \frac{\sigma_e^2}{(\sigma_e^2+\sigma_m^2)^{1/2}} \lambda\left(\frac{(\sigma_e^2+\sigma_m^2)^{1/2}}{\sigma_e^2}(p^* - \bar{p}_r)\right)$$

since $\frac{\partial \lambda(c)}{\partial \bar{p}_r} > 0$

$$\frac{\partial}{\partial \bar{p}_r} E[p_i \mid E[p_i \mid \hat{p}_i, r] > p^*, r] > 0$$

d) since type b drivers are more likely to carry drugs, the probability of making a correct call holding $\tilde{p}_i$ constant is higher

2. the utility of employer i is

$u_i = f(l_a + l_h) - R_a l_a - R_h(1+\theta_i) l_h$

where $\theta_i \sim U(a, b)$

hence, i solves

$\max_{\{l_a, l_h\}} u_i \quad$ s.t. $\quad l_a, l_h \geq 0$

$\mathcal{L} = u_i - \lambda_a(-l_a) - \lambda_h(-l_a)$

FOC$_s$:

$f' - R_a + \lambda_a = 0$

$f' - R_h(1+\theta_i) + \lambda_h = 0$

a) from KKT

(1) if $\lambda_a > 0$ and $\lambda_h = 0$, $\Rightarrow l_h > 0$

$f' = R_h(1+\theta_i)$

$\lambda_a = R_a - R_h(1+\theta_i) > 0$

$R_a > R_h(1+\theta_i)$

(2) if $\lambda_a = 0$ and $\lambda_h > 0$, $\Rightarrow l_a > 0$

$R_a < R_h(1+\theta_i)$

(3) if $\lambda_a = \lambda_h = 0$

$$\frac{R_h}{R_a} = \frac{1}{1+\theta_i}$$

let $r = \frac{R_h}{R_a}$, employers hire type h

workers when

$R_a \geq R_h(1+\theta_i)$

$1+\theta_i \leq \frac{R_a}{R_h}$

$\theta_i \leq \frac{1}{r} - 1 \equiv c^*$

hence, the proportion of employers hiring

type h workers is

$$P(\theta_i \leq c) = \frac{c-a}{b-a}$$

$$= \left(\frac{1}{r} - 1 - a\right) \div (b-a)$$

$$= \frac{1}{r(b-a)} - \frac{1+a}{(b-a)}$$

and each of these firms will hire b workers such that

$f'(l_b) = R_b(1+\theta_i)$

$l_b = f'^{-1}(R_b(1+\theta_i))$

b) $E[\theta_i] = \frac{1}{2}(a+b)$

higher expectation implies that the marginal discriminator will be more prejudiced, so the wage of type b workers will be lower

$Var(\theta_i) = \frac{1}{12}(b-a)^2$

$p(\theta_i \leq c)$ is strictly decreasing in the variance

c) lower relative supply of type b workers would imply that the marginal discriminator is less prejudiced, which imply higher wages, and similarly for a more elastic supply curve

c) there is evidence from field experiments, such as Bertrand and Mullainathan, which shows general and robust evidence for discrimination in labour markets

Charles and Guryan (2008) shows that a taste based model can explain the data, in particular with respect to the marginal discriminator and the supply curve