



**INSTITUTO
FEDERAL**

Santa Catarina

Câmpus
São José

Série de Fourier de Tempo Discreto

Sinais e Sistemas I

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1. Formulário

Série de Fourier de tempo discreto

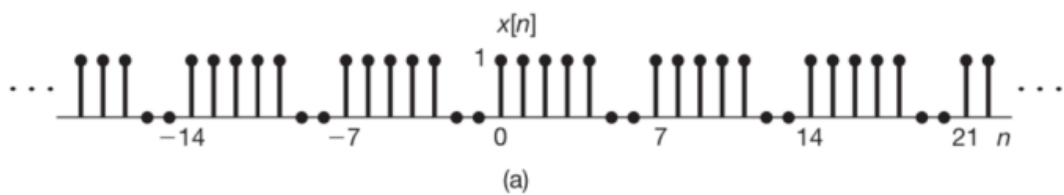
$$\begin{aligned}
 x[n] &= \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \\
 a_k &= \frac{1}{N} \sum_{k=\langle N \rangle} x[n] e^{-jk\omega_0 n} \\
 \omega_0 &= \frac{2\pi}{N}
 \end{aligned} \tag{1}$$

Progressão geométrica finita

$$S_n = \sum_{k=0}^n ar^k = a \frac{1 - r^{n+1}}{1 - r} \tag{2}$$

2. Questões

2.1. a



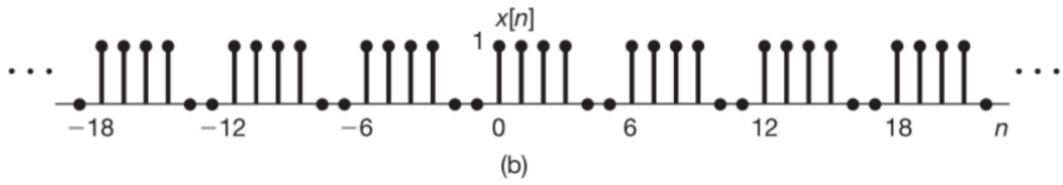
,

$$\begin{aligned}
 N &= 7 \\
 \omega_0 &= \frac{2\pi}{N} = \frac{2\pi}{7}
 \end{aligned} \tag{3}$$

$$\begin{aligned}
a_k &= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{7} \sum_{n=0}^6 x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{7} \sum_{n=0}^4 e^{-jk\omega_0 n} \\
&= \frac{1}{7} \sum_{n=0}^4 (e^{-jk\omega_0})^n \quad ; a = 1; r = e^{-jk\omega_0}; n = 4 \\
&= \frac{1}{7} \left(\frac{1 - r^{n+1}}{1 - r} \right) \\
&= \frac{1}{7} \left(\frac{1 - e^{-j5k\omega_0}}{1 - e^{-jk\omega_0}} \right)
\end{aligned} \tag{4}$$

$$\begin{aligned}
x[n] &= \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \\
&= \sum_{k=\langle N \rangle} \left(\frac{1}{7} \left(\frac{1 - e^{-j5k\omega_0}}{1 - e^{-jk\omega_0}} \right) \right) e^{jk\omega_0 n}
\end{aligned} \tag{5}$$

2.2. b

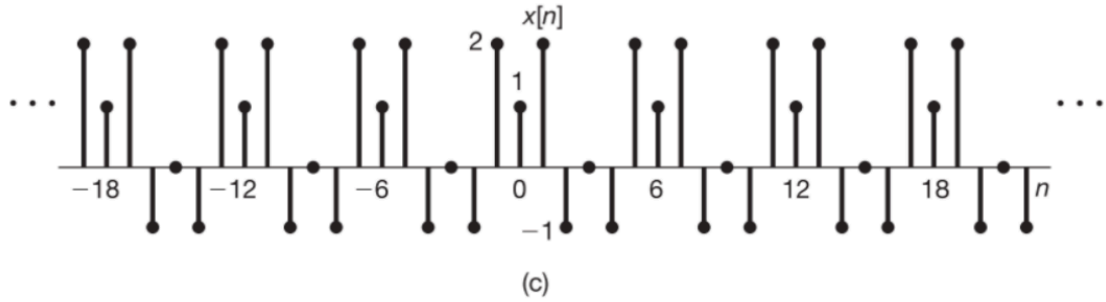


$$\begin{aligned}
N &= 6 \\
\omega_0 &= \frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}
\end{aligned} \tag{6}$$

$$\begin{aligned}
a_k &= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{6} \sum_{n=0}^5 x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{6} \sum_{n=0}^3 e^{-jk\omega_0 n} \\
&= \frac{1}{6} \sum_{n=0}^3 (e^{-jk\omega_0})^n \quad ; a = 1; r = e^{-jk\omega_0}; n = 3 \\
&= \frac{1}{6} \left(\frac{1 - r^{n+1}}{1 - r} \right) \\
&= \frac{1}{6} \left(\frac{1 - e^{-j4k\omega_0}}{1 - e^{-jk\omega_0}} \right)
\end{aligned} \tag{7}$$

$$\begin{aligned}
x[n] &= \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \\
&= \sum_{k=0}^6 \left(\frac{1}{6} \left(\frac{1 - e^{-j4k\omega_0}}{1 - e^{-jk\omega_0}} \right) \right) e^{jk\omega_0 n}
\end{aligned} \tag{8}$$

2.3. c



$$\begin{aligned}
N &= 6 \\
\omega_0 &= \frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}
\end{aligned} \tag{9}$$

n	...	-4	-3	-2	-1	0	1	2	3	4	...
x[n]	...	-1	0	-1	2	1	2	-1	0	-1	...

(10)

$$\begin{aligned}
a_k &= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{6} \sum_{n=0}^5 x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{6} (x[0] + x[1] e^{-jk\omega_0} + x[2] e^{-j2k\omega_0} \\
&\quad + x[3] e^{-j3k\omega_0} + x[4] e^{-j4k\omega_0} + x[5] e^{-j5k\omega_0}) \\
&= \frac{1}{6} (1 + 2e^{-jk\omega_0} - e^{-j2k\omega_0} - e^{-j4k\omega_0} + 2e^{-j5k\omega_0})
\end{aligned} \tag{11}$$

$$\begin{aligned}
x[n] &= \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \\
&= \sum_{k=0}^6 \left(\frac{1}{6} (1 + 2e^{-jk\omega_0} - e^{-j2k\omega_0} - e^{-j4k\omega_0} + 2e^{-j5k\omega_0}) \right) e^{jk\omega_0 n}
\end{aligned} \tag{12}$$