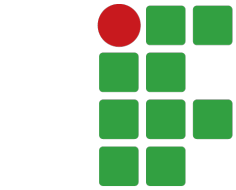


INSTITUTO FEDERAL
Santa Catarina

Câmpus
São José

Prova 2

Processos Estocásticos

Gabriel Luiz Espindola Pedro

19 de Setembro de 2023

Sumário

1. Questão **3**
2. Desenvolvimento **4**
2.1. PMF conjunta de X e de Y 4
2.2. PMFs marginais de X e Y 5
2.2.1. Esboços das PMFs marginais de X e Y 5
2.3. PMFs de X dado que $Y = y$ 6
2.4. PMFs de Y dado que $X = x$ 7
3. Resultados **7**

1. Questão

Sejam $U_1, U_2, U_3 \backsim \text{Unif}(\{0, 1, 2\})$ variáveis aleatórias sorteadas independentemente. Sejam

$$\begin{aligned} X &= U_1 + U_2 + U_3 \\ Y &= U_1 + U_2 - U_3 \end{aligned} \tag{1}$$

(a) Determine a PMF conjunta de X e Y.
(b) Determine e esboce as PMFs marginais de X e Y.
(c) Determine e esboce as PMFs de X dado que $Y = y$, para dois valores de $y \in S_Y$ à sua escolha.

2. Desenvolvimento

2.1. PMF conjunta de X e de Y

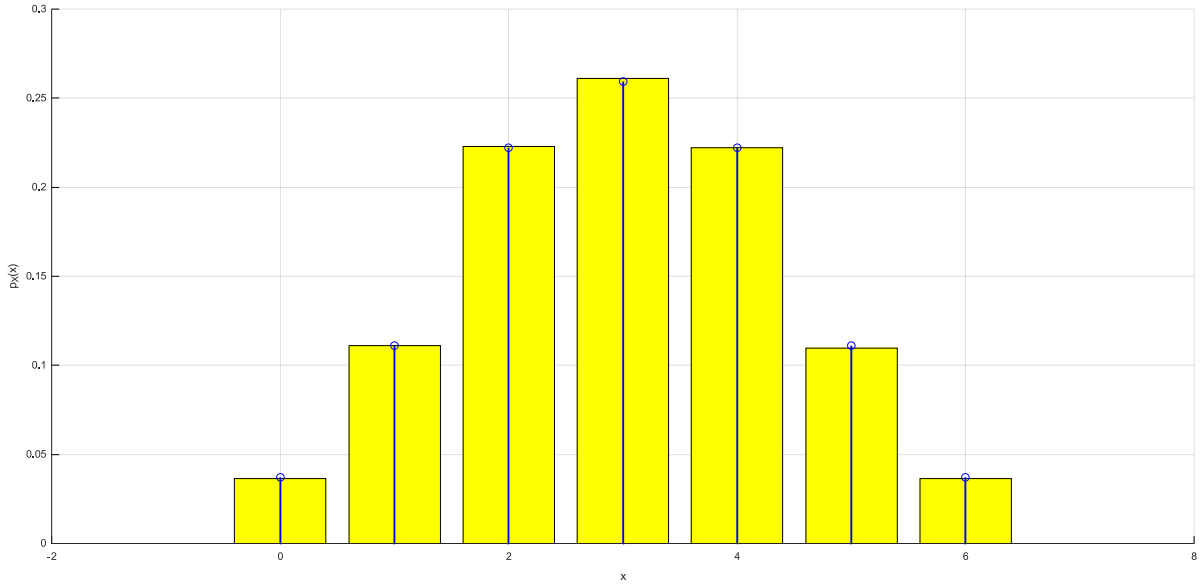
U_1	U_2	U_3	$\Pr[U_1 \wedge U_2 \wedge U_3]$	$X = U_1 + U_2 + U_3$	$Y = U_1 + U_2 - U_3$
0	0	0	$\frac{1}{27}$	0	0
0	0	1	$\frac{1}{27}$	1	-1
0	0	2	$\frac{1}{27}$	2	-2
0	1	0	$\frac{1}{27}$	1	1
0	1	1	$\frac{1}{27}$	2	0
0	1	2	$\frac{1}{27}$	3	-1
0	2	0	$\frac{1}{27}$	2	2
0	2	1	$\frac{1}{27}$	3	1
0	2	2	$\frac{1}{27}$	4	0
1	0	0	$\frac{1}{27}$	1	1
1	0	1	$\frac{1}{27}$	2	0
1	0	2	$\frac{1}{27}$	3	-1
1	1	0	$\frac{1}{27}$	2	2
1	1	1	$\frac{1}{27}$	3	1
1	1	2	$\frac{1}{27}$	4	0
1	2	0	$\frac{1}{27}$	3	3
1	2	1	$\frac{1}{27}$	4	2
1	2	2	$\frac{1}{27}$	5	1
2	0	0	$\frac{1}{27}$	2	2
2	0	1	$\frac{1}{27}$	3	1
2	0	2	$\frac{1}{27}$	4	0

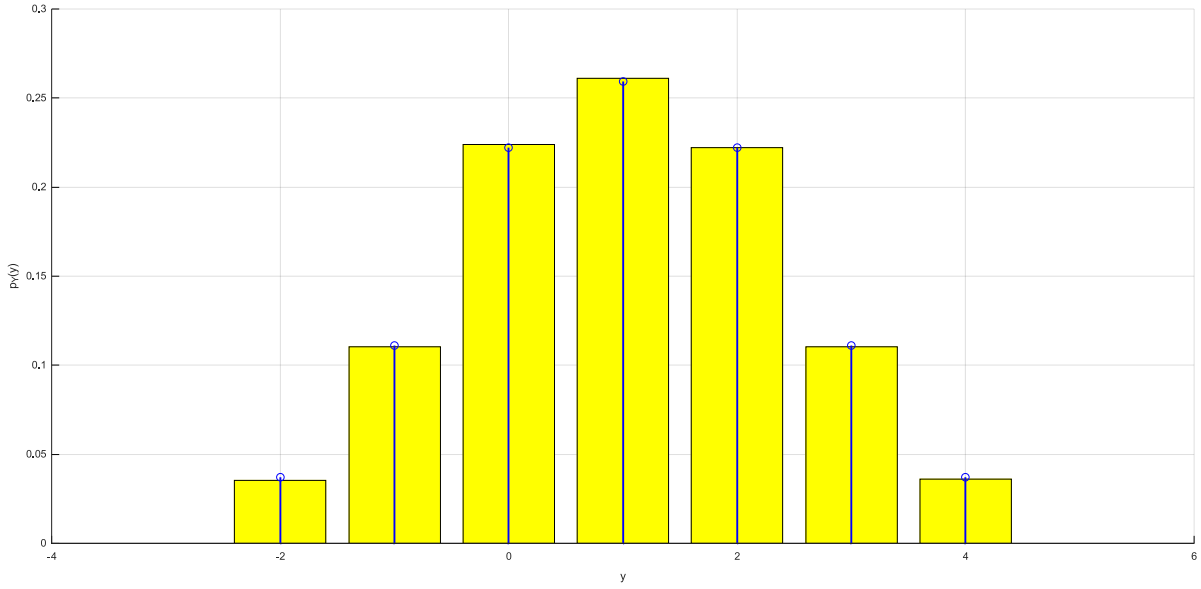
2	1	0	$\frac{1}{27}$	3	3
2	1	1	$\frac{1}{27}$	4	2
2	1	2	$\frac{1}{27}$	5	1
2	2	0	$\frac{1}{27}$	4	4
2	2	1	$\frac{1}{27}$	5	3
2	2	2	$\frac{1}{27}$	6	2

2.2. PMFs marginais de X e Y

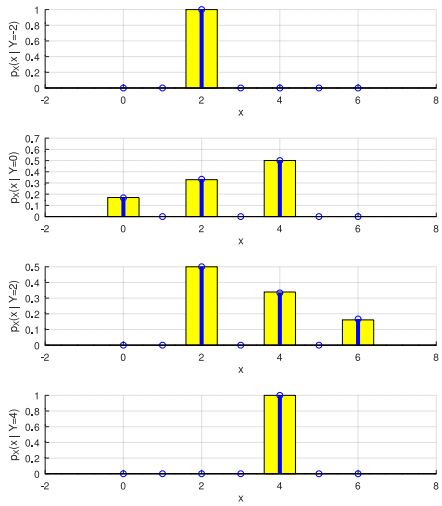
	$y = -2$	$y = -1$	$y = 0$	$y = 1$	$y = 2$	$y = 3$	$y = 4$	$p_X(x)$
$x = 0$			$\frac{1}{27}$					$\frac{1}{27}$
$x = 1$		$\frac{1}{27}$		$\frac{2}{27}$				$\frac{3}{27}$
$x = 2$	$\frac{1}{27}$		$\frac{2}{27}$		$\frac{3}{27}$			$\frac{6}{27}$
$x = 3$		$\frac{2}{27}$		$\frac{3}{27}$		$\frac{2}{27}$		$\frac{7}{27}$
$x = 4$			$\frac{3}{27}$		$\frac{2}{27}$		$\frac{1}{27}$	$\frac{6}{27}$
$x = 5$				$\frac{2}{27}$		$\frac{1}{27}$		$\frac{3}{27}$
$x = 6$					$\frac{1}{27}$			$\frac{1}{27}$
$p_Y(y)$	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{6}{27}$	$\frac{7}{27}$	$\frac{6}{27}$	$\frac{3}{27}$	$\frac{1}{27}$	1

2.2.1. Esboços das PMFs marginais de X e Y

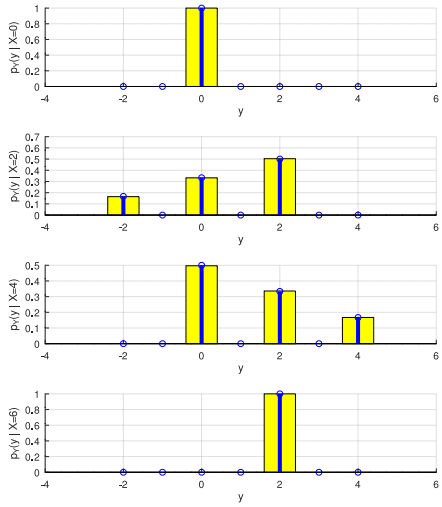




2.3. PMFs de X dado que $Y = y$



2.4. PMFs de Y dado que $X = x$



3. Resultados

Ao final do desenvolvimento obtivemos a partir do software de computação numérica Octave os seguintes resultados após desenvolvido o seguinte script:

```
close all; clear all; clc;
pkg load statistics;

N = 1e5;

U1 = randi([0 2], 1, N);
U2 = randi([0 2], 1, N);
U3 = randi([0 2], 1, N);

X = U1 + U2 + U3;
Y = U1 + U2 - U3;
```

```
x = [0 1 2 3 4 5 6];
y = [-2 -1 0 1 2 3 4];


% Conjunta

pmfXY_sim = hist3([X' Y'], {x, y}) / N
pmfXY_teo = [   0      0   1/27      0      0      0      0;
                0   1/27      0   2/27      0      0      0;
             1/27      0   2/27      0   3/27      0      0;
                0   2/27      0   3/27      0   2/27      0;
                0      0   3/27      0   2/27      0   1/27;
                0      0      0   2/27      0   1/27      0;
                0      0      0      0   1/27      0      0]

% Marginais

pmfX_sim = hist(X, x) / N
pmfX_teo = [1/27 3/27 6/27 7/27 6/27 3/27 1/27]

pmfY_sim = hist(Y, y) / N
pmfY_teo = [1/27 3/27 6/27 7/27 6/27 3/27 1/27]

% Condicionais

pmfX_condY_sim = zeros(length(x), length(y));
for i = 1:length(y)
  XcondY = X(Y == y(i));
  pmfX_condY_sim(i, :) = hist(XcondY, x) / sum(Y == y(i));
end

pmfX_condY_sim
pmfX_condY_teo = [  0   0   1   0   0   0   0;
                    0 1/3   0 2/3   0   0   0;
                  1/6   0 2/6   0 3/6   0   0;
                    0 2/7   0 3/7   0 2/7   0;
                    0   0 3/6   0 2/6   0 1/6;
                    0   0   0 2/3   0 1/3   0;
                    0   0   0   0   1   0   0]

pmfY_condX_sim = zeros(length(y), length(x));
for i = 1:length(x)
  YcondX = Y(X == x(i));
  pmfY_condX_sim(i, :) = hist(YcondX, y) / sum(X == x(i));
end

pmfY_condX_sim
pmfY_condX_teo = [  0   0   1   0   0   0   0;
                    0 1/3   0 2/3   0   0   0;
                  1/6   0 2/6   0 3/6   0   0;
                    0 2/7   0 3/7   0 2/7   0;
                    0   0 3/6   0 2/6   0 1/6;
```

```
                    0   0   0 2/3   0 1/3   0;
                    0   0   0   0   1   0   0]

figure; hold on; grid on;
bar(x, pmfX_sim, 'y');
stem(x, pmfX_teo, 'b', 'LineWidth', 3);
xlabel('x'); ylabel('p_X(x)');

figure;
for i = 1 : length(y)
  subplot(4, 2, i); hold on; grid on;
  bar(x, pmfX_condY_sim(i, :), 'y');
  stem(x, pmfX_condY_teo(i, :), 'b', 'LineWidth', 4);
  xlabel('x'); ylabel(sprintf('p_X(x | Y=%d)', y(i)));
end

figure; hold on; grid on;
bar(y, pmfY_sim, 'y');
stem(y, pmfY_teo, 'b', 'LineWidth', 3);
xlabel('y'); ylabel('p_Y(y)');

figure;
for i = 1 : length(x)
  subplot(4, 2, i); hold on; grid on;
  bar(y, pmfY_condX_sim(i, :), 'y');
  stem(y, pmfY_condX_teo(i, :), 'b', 'LineWidth', 4);
  xlabel('y'); ylabel(sprintf('p_Y(y | X=%d)', x(i)));
end
```