Trabalho de *TFTD*

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1. Usando a definição, obtenha a TFTD para os sinais a seguir.

   a) $x[n] = (0.1)^n u[n-1]$.

$$
\begin{aligned}
X[\Omega] &= \sum_{n=-\infty}^{\infty} x[n] e^{j\Omega n} \\
&= \sum_{n=-\infty}^{\infty} (0.1)^n u[n-1] \cdot e^{j\Omega n} \\
&= \sum_{n=1}^{\infty} (0.1)^n e^{j\Omega n} \\
&= \sum_{n=1}^{\infty} \left(0.1 \cdot e^{j\Omega}\right)^n \\
&= \boxed{0.1 \cdot \frac{e^{j\Omega}}{1 - 0.1 \cdot e^{j\Omega}}}
\end{aligned}
$$

   b) $x[n] = (10)^n u[-(n+1)]$.

$$
\begin{aligned}
X[\Omega] &= \sum_{n=-\infty}^{\infty} x[n] e^{j\Omega n} \\
&= \sum_{n=-\infty}^{\infty} (10)^n u[-(n+1)] \cdot e^{j\Omega n} \\
&= \sum_{n=-\infty}^{1} (10)^n \cdot e^{j\Omega n} \quad ; m = -n \\
&= \sum_{m=1}^{\infty} (10)^{-m} \cdot e^{j\Omega \cdot (-m)} \\
&= \sum_{m=1}^{\infty} \left(10^{-1} \cdot e^{j\Omega \cdot (-1)}\right)^m \\
&= \sum_{m=1}^{\infty} \left(0.1 \cdot e^{-j\Omega}\right)^m \\
&= \boxed{0.1 \frac{e^{-j\Omega}}{1 - 0.1 \cdot e^{-j\Omega}}}
\end{aligned}
$$

2. Usando a propriedade do deslocamento e sabendo que a *TFTD* de $a^n u[n]$ é $\frac{e^{j\Omega}}{e^{j\Omega}-a}$ obtenha a tranformada de Fourier dos sinais a seguir.

$$n \cdot x[n] \Longleftrightarrow j \cdot \frac{\mathrm{d}}{\mathrm{d}\Omega}(X[\Omega])$$

a) $x[n] = na^n u[n]$.

$$\begin{aligned} X[\Omega] &= j \cdot \frac{\mathrm{d}}{\mathrm{d}\Omega}(X[\Omega]) \\ &= j \cdot \frac{\mathrm{d}}{\mathrm{d}\Omega}\left(\frac{e^{j\Omega}}{e^{j\Omega}-a}\right) \\ &= j \cdot \frac{\mathrm{d}}{\mathrm{d}\Omega}\left(\frac{e^{j\Omega}}{e^{j\Omega}-a}\right) \end{aligned}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\begin{aligned} f &= e^{j\Omega} \quad ; \quad f' = je^{j\Omega} \\ g &= e^{j\Omega} - a \quad ; \quad g' = je^{j\Omega} \end{aligned}$$

$$\begin{aligned} &= j \cdot \frac{je^{j\Omega} \cdot \left(e^{j\Omega} - a\right) - e^{j\Omega} \cdot je^{j\Omega}}{\left(e^{j\Omega} - a\right)^2} \\ &= j^2 \cdot \frac{e^{j\Omega} \cdot \left(e^{j\Omega} - a\right) - e^{j\Omega} \cdot e^{j\Omega}}{\left(e^{j\Omega} - a\right)^2} \\ &= (-1) \cdot \frac{e^{j2\Omega} - a \cdot e^{j\Omega} - e^{j2\Omega}}{\left(e^{j\Omega} - a\right)^2} \\ &= (-1) \cdot \frac{-a \cdot e^{j\Omega}}{\left(e^{j\Omega} - a\right)^2} \\ &= \boxed{\frac{a \cdot e^{j\Omega}}{\left(e^{j\Omega} - a\right)^2}} \end{aligned}$$

b) $x[n] = (n-1)a^{2n}u[n-4]$.

$$
\begin{aligned}
x[n] &= (n-1)a^{2n}u[n-4] \\
&= (n-1)\left(a^2\right)^n u[n-4] \quad ; a^2 = b \\
&= (n-1-3+3)(b)^{n-4+4}u[n-4] \\
&= (n-4+3)(b)^{(n-4)+4}u[n-4] \\
&= (n-4+3)(b)^{n-4}(b)^4 u[n-4] \\
&= \left(b^4\right)(n-4+3)(b)^{n-4}u[n-4] \\
&= \left(b^4\right)\left[(n-4)(b)^{n-4}u[n-4] + 3(b)^{n-4}u[n-4]\right] \\
&\Updownarrow \\
X[\Omega] &= b^4\left[\frac{b \cdot e^{j\Omega}}{\left(e^{j\Omega}-b\right)^2} + 3 \cdot \frac{e^{j\Omega}}{e^{j\Omega}-b}\right] \\
&= b^4 \cdot \frac{b \cdot e^{j\Omega}}{\left(e^{j\Omega}-b\right)^2} + 3 \cdot b^4 \cdot \frac{e^{j\Omega}}{e^{j\Omega}-b} \quad ; b = a^2 \\
&= \left(a^2\right)^4 \cdot \frac{a^2 \cdot e^{j\Omega}}{\left(e^{j\Omega}-a^2\right)^2} + 3 \cdot \left(a^2\right)^4 \cdot \frac{e^{j\Omega}}{e^{j\Omega}-b} \\
&= a^8 \cdot \frac{a^2 \cdot e^{j\Omega}}{\left(e^{j\Omega}-a^2\right)^2} + 3 \cdot a^8 \cdot \frac{e^{j\Omega}}{e^{j\Omega}-a^2} \\
&= \boxed{a^{10} \cdot \frac{e^{j\Omega}}{\left(e^{j\Omega}-a^2\right)^2} + 3 \cdot a^8 \cdot \frac{e^{j\Omega}}{e^{j\Omega}-a^2}}
\end{aligned}
$$

3. Considere um sistema caracterizado pela equação de diferença

$$y[n] + 0.8y[n-1] + 0.12y[n-2] = 2x[n]$$

a) Determine a resposta em frequência de $H(\Omega)$.

$$H[\Omega] = \frac{Y[\Omega]}{X[\Omega]}$$

$$y[n] + 0.8y[n-1] + 0.12y[n-2] = 2x[n]$$

$$\Updownarrow$$

$$Y[\Omega] + 0.8Y[\Omega-1] + 0.12Y[\Omega-2] = 2X[\Omega]$$

$$
\begin{aligned}
Y[\Omega]\left(1 + 0.8e^{-j\Omega} + 0.12e^{-j2\Omega}\right) &= 2X[\Omega] \\
\frac{Y[\Omega]}{X[\Omega]} &= \frac{2}{1 + 0.8e^{-j\Omega} + 0.12e^{-j2\Omega}} \\
\frac{Y[\Omega]}{X[\Omega]} &= \frac{2}{1 + 0.8e^{-j\Omega} + 0.12e^{-j2\Omega}} \cdot \frac{e^{j2\Omega}}{e^{j2\Omega}} \\
H[\Omega] = \frac{Y[\Omega]}{X[\Omega]} &= \boxed{\frac{2 \cdot e^{j2\Omega}}{e^{j2\Omega} + 0.8e^{j\Omega} + 0.12}}
\end{aligned}
$$

b) Determine $y[n]$ para $x[n] = (0.1)^n u[n]$.

$$x[n] = (0.1)^n u[n] \Longleftrightarrow X[\Omega] = \frac{e^{j\Omega}}{e^{j\Omega} - 0.1}$$

$$\begin{aligned}
Y[\Omega] &= H[\Omega] \cdot X[\Omega] \\
&= \frac{2 \cdot e^{j2\Omega}}{e^{j2\Omega} + 0.8e^{j\Omega} + 0.12} \cdot \frac{e^{j\Omega}}{e^{j\Omega} - 0.1} \\
\frac{Y[\Omega]}{e^{j\Omega}} &= \frac{2 \cdot e^{j2\Omega}}{e^{j2\Omega} + 0.8e^{j\Omega} + 0.12} \cdot \frac{1}{e^{j\Omega} - 0.1} \\
\frac{Y[\Omega]}{e^{j\Omega}} &= \frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.2)(e^{j\Omega} + 0.6)} \cdot \frac{1}{e^{j\Omega} - 0.1} \\
&= \frac{A}{e^{j\Omega} + 0.2} + \frac{C}{e^{j\Omega} + 0.6} + \frac{C}{e^{j\Omega} - 0.1}
\end{aligned}$$

$$\begin{aligned}
A &= \left[\left(e^{j\Omega} + 0.2\right) \cdot \frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.2)(e^{j\Omega} + 0.6)} \cdot \frac{1}{e^{j\Omega} - 0.1} \right]_{e^{j\Omega} = -0.2} \\
&= \left[\frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.6)} \cdot \frac{1}{e^{j\Omega} - 0.1} \right]_{e^{j\Omega} = -0.2} \\
&= \frac{2 \cdot (-0.2)^2}{(-0.2 + 0.6)} \cdot \frac{1}{-0.2 - 0.1} \\
&= \frac{2 \cdot 0.04}{0.4} \cdot \frac{1}{-0.3} \\
&= \frac{0.08}{0.4} \cdot \frac{1}{-\frac{3}{10}} \\
&= \frac{\frac{8}{100}}{\frac{40}{100}} \cdot \left(-\frac{10}{3}\right) \\
&= \frac{8}{40} \cdot \left(-\frac{10}{3}\right) \\
&= 0,2 \cdot \left(-\frac{10}{3}\right) \\
&= -\frac{2}{3}
\end{aligned}$$

$$
\begin{aligned}
B &= \left[\left(e^{j\Omega} + 0.6\right) \cdot \frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.2)(e^{j\Omega} + 0.6)} \cdot \frac{1}{e^{j\Omega} - 0.1} \right]_{e^{j\Omega} = -0.6} \\
&= \left[\frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.2)} \cdot \frac{1}{e^{j\Omega} - 0.1} \right]_{e^{j\Omega} = -0.6} \\
&= \frac{2 \cdot (-0.6)^2}{((-0.6) + 0.2)} \cdot \frac{1}{(-0.6) - 0.1} \\
&= \frac{2 \cdot (0.36)}{((-0.6) + 0.2)} \cdot \frac{1}{(-0.6) - 0.1} \\
&= \frac{0.72}{-0.4} \cdot \frac{1}{-0.7} \\
&= \frac{\frac{72}{100}}{-\frac{40}{100}} \cdot \frac{1}{-\frac{7}{10}} \\
&= \frac{72}{-40} \cdot \left(-\frac{10}{7}\right) \\
&= \frac{72}{4} \cdot \frac{1}{7} \\
&= \frac{72}{28} \\
&= \frac{36}{14} \\
&= \frac{18}{7}
\end{aligned}
$$

$$
\begin{aligned}
C &= \left[\left(e^{j\Omega} - 0.1\right) \cdot \frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.2)(e^{j\Omega} + 0.6)} \cdot \frac{1}{e^{j\Omega} - 0.1} \right]_{e^{j\Omega} = 0.1} \\
&= \left[\frac{2 \cdot e^{j2\Omega}}{(e^{j\Omega} + 0.2)(e^{j\Omega} + 0.6)} \right]_{e^{j\Omega} = 0.1} \\
&= \frac{2 \cdot (0.1)^2}{((0.1) + 0.2)((0.1) + 0.6)} \\
&= \frac{2 \cdot 0.01}{(0.3)(0.7)} \\
&= \frac{0.02}{0.21} \\
&= \frac{\frac{2}{100}}{\frac{21}{100}} \\
&= \frac{2}{21}
\end{aligned}
$$

$$
\begin{aligned}
\frac{Y[\Omega]}{e^{j\Omega}} &= -\frac{2}{3}\cdot\frac{1}{e^{j\Omega}+0.2}+\frac{18}{7}\cdot\frac{1}{e^{j\Omega}+0.6}+\frac{2}{21}\cdot\frac{1}{e^{j\Omega}-0.1} \\
Y[\Omega] &= -\frac{2}{3}\cdot\frac{e^{j\Omega}}{e^{j\Omega}+0.2}+\frac{18}{7}\cdot\frac{e^{j\Omega}}{e^{j\Omega}+0.6}+\frac{2}{21}\cdot\frac{e^{j\Omega}}{e^{j\Omega}-0.1} \\
&\Updownarrow \\
y[n] &= \boxed{-\frac{2}{3}\cdot(-0.2)^n u[n]+\frac{18}{7}(-0.6)^n u[n]+\frac{2}{21}\cdot(0.1)^n u[n]}
\end{aligned}
$$

4. Considere um sistema linear invariante ao deslocamento com resposta à amostra unitária $h[n] = \delta[n]+\delta[n-1]$. Encontre a saída do sistema quando a entrada é $x[n] = 1+\cos(\pi n/10)$.

$$
\begin{aligned}
y[n] &= h[n]\star x[n] \\
&= (\delta[n]+\delta[n-1])\star(1+\cos(\pi n/10)) \\
&= (1+\cos(\pi n/10)+(1+\cos(\pi(n-1)/10)) \\
&= \boxed{2+\cos(\pi n/10)+\cos(\pi(n-1)/10)}
\end{aligned}
$$

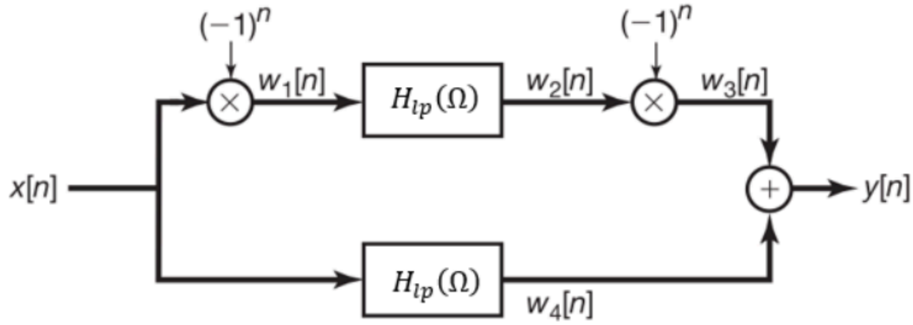
5. Sabendo que a resposta ao impulso de um sistema linear invariante ao deslocamento é $h[n] = a^n u[n]$ com $|a|<1$. Encontre a resposta à entrada $x[n]=1$.

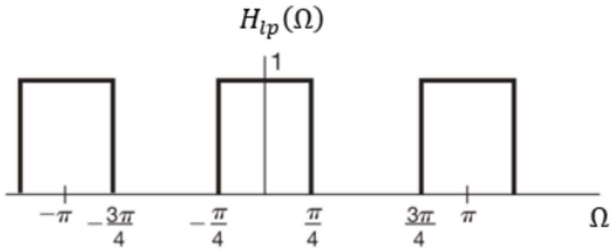
$$
\begin{aligned}
h[n] &\Longleftrightarrow H[\Omega] \\
a^n u[n] &\Longleftrightarrow \frac{e^{j\Omega}}{e^{j\Omega}-a}
\end{aligned}
$$

$$
\begin{aligned}
x[n] &\Longleftrightarrow X[\Omega] \\
a &\Longleftrightarrow a\cdot\pi 2\delta[\Omega] \\
1 &\Longleftrightarrow 1\cdot\pi 2\delta[\Omega]
\end{aligned}
$$

$$
\begin{aligned}
Y[\Omega] &= H[\Omega]\cdot X[\Omega] \\
&= \frac{e^{j\Omega}}{e^{j\Omega}-a}\cdot 1\cdot\pi 2\delta[\Omega] \\
&\Updownarrow \\
&= \boxed{a^n u[n]}
\end{aligned}
$$

6. Considere o sistema da figura a seguir com entrada $x[n]$ e saída $y[n]$. Os sistemas *LIT* com resposta $H_{lp}(e^{j\omega})$ são filtros passa-baixas ideais, com frequências de corte $\pi/4$ e ganho unitário na banda de passagem. Determine a resposta em frequência do sistema. Qual o tipo de filtro?





$$
\begin{aligned}
w_1(e^{j\Omega}) &= X(e^{j(\Omega-\pi)}) \\
w_2(e^{j\Omega}) &= H_{lp}(e^{j(\Omega-\pi)})X(e^{j(\Omega-\pi)}) \\
w_3(e^{j\Omega}) &= H_{lp}(e^{j(\Omega-\pi)})X(e^{j(\Omega)}) \\
w_4(e^{j\Omega}) &= H_{lp}(e^{j\Omega})X(e^{j\Omega}) \\
Y(\Omega) &= w_3(e^{j\Omega}) + w_4(e^{j\Omega}) \\
&= H_{lp}(e^{j(\Omega-\pi)})X(e^{j(\Omega)}) + H_{lp}(e^{j\Omega})X(e^{j\Omega}) \\
&= \left[H_{lp}(e^{j(\Omega-\pi)}) + H_{lp}(e^{j\Omega})\right]X(e^{j\Omega}) \\
H(e^{j\Omega}) &= \boxed{H_{lp}(e^{j(\Omega-\pi)}) + H_{lp}(e^{j\Omega})} \therefore \text{ Filtro passa faixa}
\end{aligned}
$$