INSTITUTO FEDERAL
Santa Catarina

Câmpus
São José

Série de Fourier de Tempo Discreto

Sinais e Sistemas I

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Sumário

1. Formulário

Série de Fourier de tempo discreto

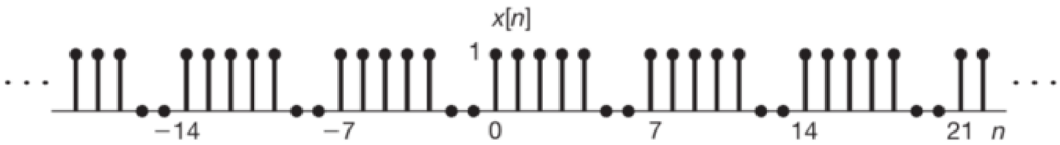
$$
\begin{aligned}
x[n] &= \sum_{k=<N>} a_k e^{jk\omega_0 n} \\
a_k &= \frac{1}{N} \sum_{k=<N>} x[n] e^{-jk\omega_0 n} \\
\omega_0 &= \frac{2\pi}{N}
\end{aligned}
\tag{1}
$$

Progressão geométrica finita

$$
S_n = \sum_{k=0}^{n} ar^k = a\frac{1 - r^{n+1}}{1 - r}
\tag{2}
$$

2. Questões

2.1. a



(a)

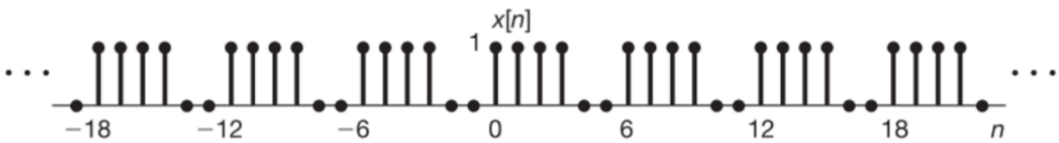
,

$$
\begin{aligned}
N &= 7 \\
\omega_0 &= \frac{2\pi}{N} = \frac{2\pi}{7}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
a_k &= \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{7} \sum_{n=0}^{6} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{7} \sum_{n=0}^{4} e^{-jk\omega_0 n} \\
&= \frac{1}{7} \sum_{n=0}^{4} \left(e^{-jk\omega_0}\right)^n \quad ; a = 1; r = e^{-jk\omega_0}; n = 4 \\
&= \frac{1}{7} \left(\frac{1 - r^{n+1}}{1 - r} \right) \\
&= \frac{1}{7} \left(\frac{1 - e^{-j5k\omega_0}}{1 - e^{-jk\omega_0}} \right)
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
x[n] &= \sum_{k=<N>} a_k e^{jk\omega_0 n} \\
&= \sum_{k=<N>} \left(\frac{1}{7} \left(\frac{1 - e^{-j5k\omega_0}}{1 - e^{-jk\omega_0}} \right) \right) e^{jk\omega_0 n}
\end{aligned} \tag{5}
$$

2.2. b



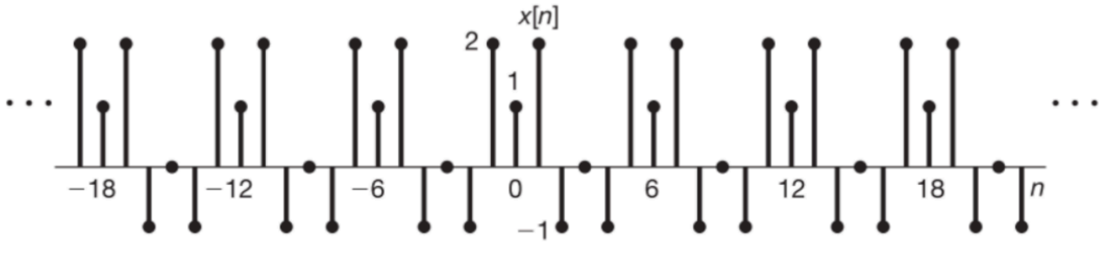
(b)

$$
\begin{aligned}
N &= 6 \\
\omega_0 &= \frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
a_k &= \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\omega_0 n} \\
&= \frac{1}{6}\sum_{n=0}^{5} x[n]e^{-jk\omega_0 n} \\
&= \frac{1}{6}\sum_{n=0}^{3} e^{-jk\omega_0 n} \\
&= \frac{1}{6}\sum_{n=0}^{3} \left(e^{-jk\omega_0}\right)^n \quad ; a = 1; r = e^{-jk\omega_0}; n = 3 \\
&= \frac{1}{6}\left(\frac{1-r^{n+1}}{1-r}\right) \\
&= \frac{1}{6}\left(\frac{1-e^{-j4k\omega_0}}{1-e^{-jk\omega_0}}\right)
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
x[n] &= \sum_{k=<N>} a_k e^{jk\omega_0 n} \\
&= \sum_{k=0}^{6} \left(\frac{1}{6}\left(\frac{1-e^{-j4k\omega_0}}{1-e^{-jk\omega_0}}\right)\right) e^{jk\omega_0 n}
\end{aligned} \tag{8}
$$

2.3. c



(c)

$$
\begin{aligned}
N &= 6 \\
\omega_0 &= \frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}
\end{aligned} \tag{9}
$$

n	...	−4	−3	−2	−1	0	1	2	3	4	...
x[n]	...	−1	0	−1	2	1	2	−1	0	−1	...

(10)

$$
\begin{aligned}
a_k &= \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{6} \sum_{n=0}^{5} x[n] e^{-jk\omega_0 n} \\
&= \frac{1}{6} \big(x[0] + x[1] e^{-jk\omega_0} + x[2] e^{-j2k\omega_0} \\
&\quad + x[3] e^{-j3k\omega_0} + x[4] e^{-j4k\omega_0} + x[5] e^{-j5k\omega_0} \big) \\
&= \frac{1}{6} \big(1 + 2e^{-jk\omega_0} - e^{-j2k\omega_0} - e^{-j4k\omega_0} + 2e^{-j5k\omega_0} \big)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
x[n] &= \sum_{k=<N>} a_k e^{jk\omega_0 n} \\
&= \sum_{k=0}^{6} \left(\frac{1}{6} \big(1 + 2e^{-jk\omega_0} - e^{-j2k\omega_0} - e^{-j4k\omega_0} + 2e^{-j5k\omega_0} \big) \right) e^{jk\omega_0 n}
\end{aligned}
\tag{12}
$$