Introducing Investment to the Basic Model

Applied International Economics.

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The Model

In this model

Objectives

- Households are able to smooth consumption over time
- The trade balance acts as a shock absorber

Assumptions

- Endowment economy
- Two periods
- Single tradable good
- Perfect capital mobility - households can borrow/lend at the international capital markets with price r

Production

Output is given by:

$$y_1 = A_1 f(k_1), \tag{1}$$

$$y_2 = A_2 f(k_2), \tag{2}$$

with $A_t, t = 1, 2$ is the total factor productivity and $f(\cdot)$ is assumed to be strictly increasing and concave ($f'(\cdot) > 0, f''(\cdot) < 0$). We assume that k_1 is given, such that households choose k_2 in the first period. Moreover, there is no depreciation of capital between periods. Formally,

$$I_1 = k_2 - (1 - \delta)k_1, \tag{3}$$

where I_1 is investment in period 1 and $\delta = 0$.

Budget constraints

For the two periods we have

$$b_1 = y_1 - c_1 - I_1, \tag{4}$$

$$0 = (1+r)b_1 + y_2 - c_2, \tag{5}$$

Combining the budget constraints yields the intertemporal budget constraint (IBC)

$$y_1 + \frac{y_2}{1+r} = c_1 + I_1 + \frac{c_2}{1+r} \tag{6}$$

Let us define the trade balance in each period as:

$$TB_1 \equiv y_1 - c_1 - I_1, \tag{7}$$

$$TB_2 \equiv y_2 - c_2. \tag{8}$$

Again, using Equation (6) it follows that

$$TB_1 + \frac{TB_2}{1+r} = 0. \tag{9}$$

Budget constraints (continued)

Now, the current account for each period is the following:

$$CA_1 \equiv y_1 - c_1 - I_1 \tag{10}$$

$$CA_2 \equiv rb_1 + y_2 - c_2 \tag{11}$$

Since savings are income minus consumption, it follows that

$$CA_1 = S_1 - I_1 \tag{12}$$

$$CA_2 = S_2 \tag{13}$$

Contrary to our previous model, now the current account in period 1 becomes the difference between savings and investment!

Households

The households' utility function has the following form:

$$W = u(c_1) + \beta u(c_2), \tag{14}$$

In this model, households choose c_1, c_2, and k_2 to maximize Equation (14) subject to Equation (6). The Lagrangian is as follows:

$$\mathcal{L} = u(c_1) + \beta u(c_2) - \lambda \left(c_1 + \frac{c_2}{1+r} + (k_2 - k_1) - A_1 f(k_1) - \frac{A_2 f(k_2)}{1+r} \right), \tag{15}$$

The first-order conditions are:

$$u'(c_1) = \lambda, \tag{16}$$

$$\beta u'(c_2) = \frac{\lambda}{1+r}, \tag{17}$$

$$A_2 f'(k_2) = 1 + r. \tag{18}$$

Equation (18) shows that at the optimum, the marginal productivity of capital must equate its marginal cost.

Households (continued)

If we combine both budget constraints we obtain our typical Euler equation

$$u'(c_1) = \beta(1 + r)u'(c_2). \tag{19}$$

Suppose we have $\beta(1 + r) = 1$, it follows from this that:

$$c_1 = c_2 = \bar{c} \tag{20}$$

Again, consumption is **fully smoothed** over time *regardless of the output/investment path*.

Reduced-form solutions

Let us give the following form to the production functions:

$$f(k_1) = k_1^{\alpha}, \tag{21}$$

$$f(k_2) = k_2^{\alpha}, \tag{22}$$

with $0 < \alpha < 1$. First-order condition (18) becomes:

$$A_2 \alpha k_2^{\alpha - 1} = 1 + r, \tag{23}$$

Solve for k_2:

$$k_2 = \left(\frac{\alpha A_2}{1 + r} \right)^{\frac{1}{1-\alpha}} . \tag{24}$$

Therefore, investment is increasing on A_2 and decreasing on r.

Reduced-form solutions (continued)

Substitute Equation (24) in Equation (2) to obtain:

$$y_2 = A_2 \left(\frac{\alpha A_2}{1+r} \right)^{\frac{\alpha}{1-\alpha}} . \tag{25}$$

Now solve for $\bar{c}$ (Equation (6)):

$$\bar{c} = \frac{1+r}{2+r} \left[A_1 f(k_1) + \frac{A_2 f(k_2)}{1+r} - (k_2 - k_1) \right] . \tag{26}$$

Since $S_1 = y_1 - \bar{c}$,

$$S_1 = A_1 f(k_1) - \frac{1+r}{2+r} \left[A_1 f(k_1) + \frac{A_2 f(k_2)}{1+r} - (k_2 - k_1) \right] ,$$

which simplifies to

$$S_1 = \frac{A_1 f(k_1) - [A_2 f(k_2) - (1+r)(k_2 - k_1)]}{2+r} \tag{27}$$

Now, savings will be determined by the difference between todays' output and the output of period 2 net of investment expenditure.

Reduced-form solutions (continued)

Now, the current account in period 1:

$$CA_1 = S_1 - I_1$$

$$CA_1 = \frac{A_1 f(k_1) - [A_2 f(k_2) - (1+r)(k_2 - k_1)]}{2+r} - (k_2 - k_1).$$

Which simplifies to:

$$CA_1 = \frac{A_1 f(k_1) - A_2 f(k_2) - (k_2 - k_1)}{2+r}. \tag{28}$$

Example I: Zero saving zero investment

Suppose $A_1 = A_2 = \bar{A}$, and $k_1 = \left(\frac{\alpha\bar{A}}{1+r}\right)^{\frac{1}{1-\alpha}}$, this implies:

$$I_1 = k_2 - k_1 = 0.$$

Output is also the same is both periods:

$$\bar{A}f(k_1) = \bar{A}f(k_2).$$

Which from Equation (27) means that:

$$S_1 = 0.$$

Which also means that $CA_1 = 0$.

Example II: Positive saving zero investment

Suppose $A_1 > A_2 = \bar{A}$, and $k_1 = \left(\frac{\alpha \bar{A}}{1+r}\right)^{\frac{1}{1-\alpha}}$, this implies:

$$I_1 = k_2 - k_1 = 0.$$

But since $A_1 > A_2$, output in period 1 is higher than period 2:

$$A_1 f(k_1) > \bar{A} f(k_2).$$

Which from Equation (27) means that:

$$S_1 = \frac{A_1 f(k_1) - \bar{A} f(k_2)}{2+r} = \frac{y_1 - y_2}{2+r} > 0.$$

Which also means that $CA_1 > 0$, as investment is zero in this example. In other words, we're back to the basic model without investment, the current account is procyclical because households smooth consumption.

Example III: Positive saving positive investment

Suppose $A_1 > A_2 > \bar{A}$, and $k_1 = \left(\frac{\alpha \bar{A}}{1+r}\right)^{\frac{1}{1-\alpha}}$, this implies:

$$I_1 = k_2 - k_1 > 0.$$

$$S_1 = \frac{A_1 f(k_1) - [A_2 f(k_2) - (1+r)(k_2 - k_1)]}{2+r}$$

By continuity, it follows that a slight positive change in A_2 will still leave S_1 positive. Yet, we're missing the behavior of S_1 when A_2 changes, for that we need:

$$\frac{\partial S_1}{\partial A_2} = -\frac{1}{2+r}\left[f(k_2) + A_2 f'(k_2)\frac{dk_2}{dA_2} - (1+r)\frac{dk_2}{dA_2}\right]$$

$$\frac{\partial S_1}{\partial A_2} = -\frac{1}{2+r}\left[f(k_2) + \frac{dk_2}{dA_2}\underbrace{(A_2 f'(k_2) - (1+r))}_{=0}\right]$$

$$\frac{\partial S_1}{\partial A_2} = -\frac{f(k_2)}{2+r} < 0$$

In summary

- When we add investment to the basic model, we can interpret the current account as the *difference between savings and investment*.
- Consistent with data, it might be the case that saving and investment go up in good times, but the investment effect dominates, which leads to current account deficits (counter-cyclical current account).
- We will see the dynamics of the model more clearly in R!