

DESIGN AND ANALYSIS OF ALGORITHMS — HT 2019

Problem Sheet 4

Answers for questions marked *.

DFS and connected components, cont'd

Shortest paths

Answer to question 3

- (a) n -vertex linear chain. $(v_i, v_{i+1}), 0 \leq i < n - 1$
- (b) n -vertex star graph. $(v_0, v_i), 0 \leq i < n$

Greedy algorithms

Answer to question 6

- (a) No. Take the series of denominations 40, 15, 1. The greedy algorithm yields change $(1, 1, 5)$ for the amount of 60. A better solution is $(0, 4, 0)$.
- (b) In the optimal solution, there cannot be more than $p-1$ coins of any denomination except the largest denomination p^k , as any p coins of denomination p^j can be replaced by one p^{j+1} denomination, resulting in a better solution. The greedy algorithm chooses the maximum possible number of coins of the highest denomination p^k . Any optimal solution must do the same, since otherwise the solution would have to contain p or more coins of some smaller denomination. This follows from the fact that

$$(p-1)(p^{k-1} + \dots + p + 1) < p^k.$$

The remaining value for which change has to be given needs to be given using a subset of the remaining coins, and the optimal solution will contain optimal change for this subproblem.