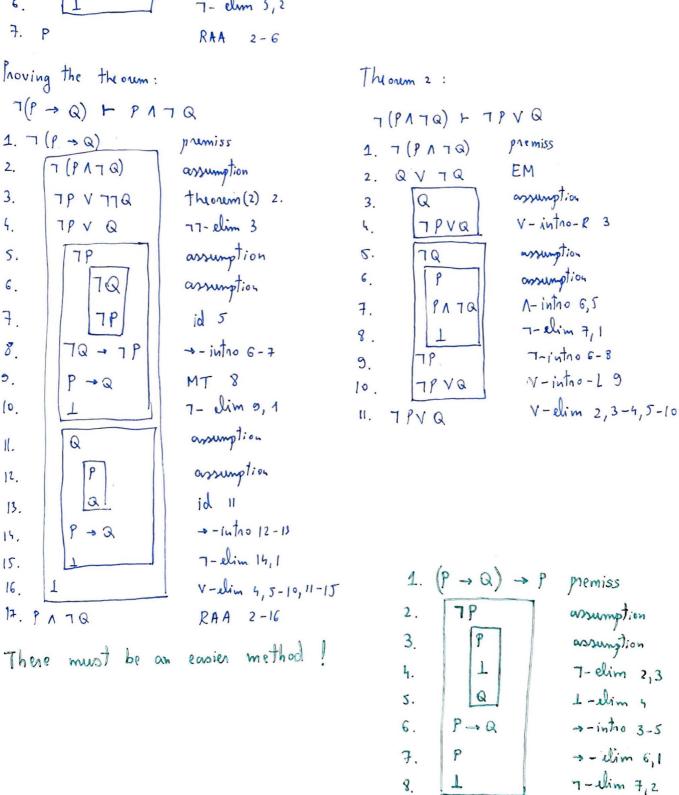
INTRODUCTION TO FORMAL PROOF 2018

QUESTION

(a) Proof by contradiction (RAA):

5.
$$1 \Rightarrow -e \lim_{n \to \infty} 5$$
. $1 - \lim_{n \to \infty} 5$

77- elim can be proven as



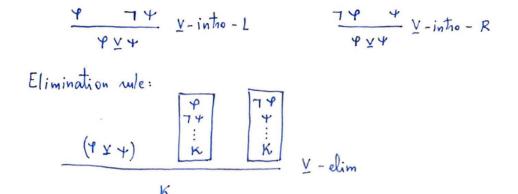
9. P

2

RAA 2-8

. (d) P exclusive on Q is PYQ, true when exactly one of P,Q is True

(i) Introduction rules:



The introduction rules are designed so that we have exclusive or when exactly one of the two inputs is true, and the elimination is similar to the V-elim rule:

- · P Y + => one of them is true, on false
- · The two boxes ensure that K happens no matter the configuration

(ii) TPYQ FPY7Q

1.	7 P Y Q	premiss
2.	79	assumption 1
3.	70	assumption 2
4.	PYTQ	V-intro-R 2,3
5.	779	assumption 1
6.	a	assumption 2
7.	P	77- elim 5
8.	77 Q	77- into 6
5.	PYTQ	¥ -intno-L 7,8
10.	Py7Q	Y-elim 1, 2-4, 5-9

QUESTION 8

- · R = unary relation symbol
- · P, Q = binary relation simbols
 - (a) A variable is fresh in a proof if it doesn't appear free in any hypothesis on in the conclusion
 - (1) (i) YX.R(x) H73Y.7R(y)
 - 1. $\forall x \cdot R(x)$

premiss

3y.7R(y) assumption

1 frush 7 R(3)

7 3 y. 7 R(y) 7 - intro 2 - 6

R(1) | Y- elim 1 (1) free don x in R(x))

J-elim (2) 3-5

- (i) 73×· R(x) + 4y· 7 R(y)
 - 1. $73 \times R(x)$ 1 fresh

premiss

R (1)

 $\exists \times \cdot R(x)$

7 R(1)

assumption

(I free of x in R(x))

7- elim 3,1

7-intro 2-4

6. 4y. 7 R(y) V-intro 2-5

- (ii) 7 7 × × × (x) × × × F Γ (iii)
 - 1. 7 3x. 7R(x) gremiss
 - 2. \(\forall \(y\) . \(\tau \) (i) thoum (b) (ii)
 - 77 dim 2 3. +y · R(y)
- (c) $\forall x \cdot \forall y \cdot P(x,y) \vdash \forall y \cdot \forall x \cdot P(x,y)$
 - 1. $\forall x \cdot \forall y \cdot P(x,y)$ frush)

premiss

fresh w \ Y y . P (w, y)

1 p (w, 1)

5. 44.4x. p(x,y)

Y-elim 1 (w fru for x in P(x,y))

4- alim 2 (i free for y in p(w,y))

Y-intro 2-3

4-intro 2-4

(d) $\forall x. \forall y. (p(x,y) \rightarrow Q(x,y)) \vdash (\exists y. \exists x. \neg Q(x,y)) \rightarrow (\exists y. \exists x. \neg p(x,y))$

1.
$$\forall x \cdot \forall y \cdot (p(x,y) \rightarrow Q(x,y))$$

2. $\exists y \cdot \exists x \cdot \neg Q(x,y)$
3. $\exists x \cdot \neg Q(x,\lambda)$
 $\forall x \cdot \neg Q(x,\lambda)$
 $\forall y \cdot (p(w,y) \rightarrow Q(w,y))$
 $\Rightarrow (w,\lambda) \rightarrow \neg P(w,\lambda)$
3. $\forall y \cdot (p(w,y) \rightarrow Q(w,y))$
 $\Rightarrow (w,\lambda) \rightarrow \neg P(w,\lambda)$
 $\Rightarrow (w,\lambda) \rightarrow \neg P(x,y)$
10. $\Rightarrow (x,y) \rightarrow Q(x,y)$
11. $\Rightarrow (x,y) \rightarrow Q(x,y)$
12. $\Rightarrow (x,y) \rightarrow Q(x,y)$

13. $(\exists y \cdot \exists x \cdot \neg \otimes (x,y)) \rightarrow (\exists y \cdot \exists x \cdot \neg P(x,y))$

assumption

assumption

MT 6.