

QUESTION 1

(a)

$$> (\cdot) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$$

$$> f \cdot g = \lambda x \rightarrow f (g x)$$

$$> \text{zipWith} :: (a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$$

$$> \text{zipWith} \_ [] \_ = []$$

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$$> \text{zipWith } f (x:xs) (y:ys) = f x y : \text{zipWith } f xs ys$$

$$> \text{uncurry} :: (a \rightarrow b \rightarrow c) \rightarrow (a, b) \rightarrow c$$

$$> \text{uncurry } f (x, y) = f x y$$

(b)

$$\text{filter } p \cdot \text{map } f = \text{map } f \cdot \text{filter } (p \cdot f)$$

$$\text{filter } p \cdot \text{concat} = \text{concat} \cdot \text{map } (\text{filter } p)$$

$$\text{zipWith } f = \text{map } (\text{uncurry } f) \cdot \text{zip}$$

(c)

$$> \text{pascal} :: [[\text{Integer}]]$$

$$> \text{pascal} = \text{iterate } (\lambda xs \rightarrow \text{zipWith } (+) (0:xs) (xs \# [0])) [1]$$

$$(d) > f g h x y = g \cdot h x \cdot g y$$

From the definition of  $(\cdot)$ , we know that

$$(h x) :: b \rightarrow c \Rightarrow h :: x \rightarrow b \rightarrow c$$

$$(g y) :: a \rightarrow b \Rightarrow g :: y \rightarrow a \rightarrow b$$

$$(h x) \cdot (g y) :: a \rightarrow c$$

From the same definition, we have

$$g :: c \rightarrow d$$

$$\Rightarrow y = c \text{ and } d = a \rightarrow b$$

$$g \cdot (h x) \cdot (g y) :: a \rightarrow d \Rightarrow f g h x y :: a \rightarrow d$$

Therefore, we have

$$f :: (c \rightarrow a \rightarrow b) \rightarrow (x \rightarrow b \rightarrow c) \rightarrow x \rightarrow c \rightarrow a \rightarrow a \rightarrow b$$