LINEAR ALGEBRA MT 2018

WEEK 4

Chapter 4: Systems of Linear Equations and Elementary Matrix Factorisations

1. (a)

$$\begin{aligned} eq1&: x+2y-3z=9 \\ eq2&: 2x-y+z=0 \\ eq3&: 4x-y+z=4 \end{aligned}$$

$$\begin{aligned} &eq1: & x+2y-3z&=9 \\ eq2 \Rightarrow\ &eq2-2eq1: & -5y+7z&=-18 \\ eq3 \Rightarrow\ &eq3-4eq1: & -9y+13z&=-32 \end{aligned}$$

$$\begin{aligned} &eq1: & x+2y-3z&=9 \\ &eq2: & -5y+7z&=-18 \\ eq3 \Rightarrow\ &eq3-\tfrac{9}{5}eq2: & \tfrac{2}{5}z&=\tfrac{2}{5} \Rightarrow \boxed{z=1} \end{aligned}$$

$\Rightarrow \boxed{y=5} \Rightarrow \boxed{x=2}$

(b)

$$\begin{aligned} eq1&: 2x_1+x_2-x_3-x_4+2x_5=3 \\ eq2&: x_2-2x_3+x_4+x_5=-1 \\ eq3&: x_3+2x_4-x_5=2 \Rightarrow \boxed{x_3=2-2x_4+x_5} \end{aligned}$$

$\Rightarrow x_2-2(2-2x_4+x_5)+x_4+x_5=-1 \Rightarrow$

$\Rightarrow x_2-4+4x_4-2x_5+x_4+x_5=-1 \Rightarrow \boxed{x_2=3-5x_4+x_5}$

eq1: $2x_1+(\cancel{3}-5x_4+\cancel{x_5})-(2-2x_4+\cancel{x_5})-x_4+2x_5=\cancel{3}$

$2x_1=2+4x_4-2x_5 \Rightarrow \boxed{x_1=1+2x_4-x_5}$

If we replace x_4 with α and x_5 with β, $\alpha,\beta\in\mathbb{R}$, we obtain the general solution

$(1+2\alpha-\beta,\ 3-5\alpha+\beta,\ 2-2\alpha+\beta,\ \alpha,\ \beta)$

(c)

$$\begin{aligned} eq1&: x_1-2x_2-x_3=-2 \\ eq2&: 2x_1+x_2+3x_3=1 \\ eq3&: -3x_1+x_2-2x_3=1 \end{aligned}$$

$$\begin{aligned} &eq1: & x_1-2x_2-x_3&=-2 \\ eq2 \Rightarrow\ &eq1-\tfrac{1}{2}eq2: & -\tfrac{5}{2}x_2-\tfrac{5}{2}x_3&=-\tfrac{5}{2} \\ eq3 \Rightarrow\ &eq1+\tfrac{1}{3}eq3: & -\tfrac{5}{3}x_2-\tfrac{5}{3}x_3&=-\tfrac{5}{3} \end{aligned}$$

$$\begin{aligned} &eq1: & x_1-2x_2-x_3&=-2 \\ &eq2: & -\tfrac{5}{2}x_2-\tfrac{5}{2}x_3&=-\tfrac{5}{2} \Rightarrow x_2+x_3=1 \Rightarrow \boxed{x_2=1-x_3} \\ eq3 \Rightarrow\ &eq2-\tfrac{3}{2}eq3: & 0&=0 \end{aligned}$$

$\Rightarrow x_1-2(1-x_3)-x_3=-2$

$\boxed{x_1=-x_3}$

If we replace x_3 with α, $\alpha \in \mathbb{R}$, we obtain the general solution
$(-\alpha, 1-\alpha, \alpha)$

2. $n \cdot (X-p) = 0$ – the normal form of the equation of a plane P
$X = p + td$, $t \in \mathbb{R}$ – the vector form of the equation of a line $\mathcal{L}$
We want to find the equation of the line where the two planes:

$$3x + 2y + z = -1$$
and
$$2x - y + 4z = 5$$
intersect.

First, we form the augmented matrix:

$$[A|b] = \left[\begin{array}{ccc|c} 3 & 2 & 1 & -1 \\ 2 & -1 & 4 & 5 \end{array}\right]$$

and perform the elementary row operations:

$$r_2 \leftarrow r_2 - \frac{2}{3} r_1 \Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & -1 \\ 0 & -\frac{7}{3} & \frac{10}{3} & \frac{17}{3} \end{array}\right]$$

$$r_1 \leftarrow r_1 + \frac{6}{7} r_2 \Rightarrow \left[\begin{array}{ccc|c} 3 & 0 & \frac{27}{7} & \frac{27}{7} \\ 0 & -\frac{7}{3} & \frac{10}{3} & \frac{17}{3} \end{array}\right]$$

$$r_1 \leftarrow 7 r_1 \Rightarrow \left[\begin{array}{ccc|c} 21 & 0 & 27 & 27 \\ 0 & -\frac{7}{3} & \frac{10}{3} & \frac{17}{3} \end{array}\right]$$

$$r_2 \leftarrow 3 r_2 \Rightarrow \left[\begin{array}{ccc|c} 21 & 0 & 27 & 27 \\ 0 & -7 & 10 & 17 \end{array}\right]$$

Here we choose z as a free variable, $z = s$, $s \in \mathbb{R}$.

$$z = s$$

$$-7y + 10s = 17 \Rightarrow 7y = 10s - 17 \Rightarrow \boxed{y = \frac{10s-17}{7}}$$

$$21x + 27s = 27 \Rightarrow 21x = 27 - 27s \Rightarrow \boxed{x = \frac{9-9s}{7}}$$

Therefore, the general solution has the form $X = \left[\frac{-9-9s}{7}, \frac{10s-17}{7}, s\right]^T$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{9}{7}s + \frac{9}{7} \\ \frac{10}{7}s - \frac{17}{7} \\ s \end{bmatrix} = \begin{bmatrix} -\frac{9}{7} \\ \frac{10}{7} \\ 1 \end{bmatrix} s + \begin{bmatrix} \frac{9}{7} \\ -\frac{17}{7} \\ 0 \end{bmatrix}$$

In conclusion, the equation of the line where the two planes intersect is

$$X = \begin{bmatrix} \frac{9}{7} \\ -\frac{17}{7} \\ 0 \end{bmatrix} + s \begin{bmatrix} -\frac{9}{7} \\ \frac{10}{7} \\ 1 \end{bmatrix}, \text{ where } p = \begin{bmatrix} \frac{9}{7} \\ -\frac{17}{7} \\ 0 \end{bmatrix}, t = s \in \mathbb{R}, d = \begin{bmatrix} -\frac{9}{7} \\ \frac{10}{7} \\ 1 \end{bmatrix}.$$

3. In $\mathbb{R}^4$ we have three planes given by

$$\begin{aligned} u + v + w + z &= 6 \\ u + \phantom{v+{}} w + z &= 4 \\ u + \phantom{v+{}} w \phantom{{}+z} &= 2 \end{aligned}$$

We want to describe the intersection of the three planes:

First, we form the augmented matrix:

$$[A|b] = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 1 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 0 & 2 \end{array}\right]$$

and perform the elementary row operations:

$$r_2 \leftarrow r_2 - r_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & 0 & 0 & -2 \\ 1 & 0 & 1 & 0 & 2 \end{array}\right]$$

$$r_3 \leftarrow r_3 - r_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & -1 & 0 & -1 & -4 \end{array}\right]$$

$$r_3 \leftarrow r_3 - r_2 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & -2 \end{array}\right]$$

$$r_1 \leftarrow r_1 + r_2 + r_3 \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & -2 \end{array}\right]$$

Now, we have $u + w = 2 \Rightarrow w = 2 - u$, where we say u is a free variable s, $s \in \mathbb{R}$

$-v = -2 \Rightarrow v = 2$

$-z = -2 \Rightarrow z = 2$

Therefore, the general solution has the form $x = [s, 2, 2-s, 2]^T$

$$\begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} s \\ 2 \\ 2-s \\ 2 \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \end{bmatrix} = td + p, \text{ where } d = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, t = s \in \mathbb{R}, p = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \end{bmatrix},$$

which is exactly the vector form of the equation of a line. So, the intersection of the three planes is a line.

By including the plane $u = -1$, we obtain $w = 3$, so the general solution has the form $x = [-1, 2, 3, 2]^T$, therefore the intersection of the four planes is a point.

A plane that would leave us with no solution is $z = 4$, as we will get $z = 4$ and $z = 2$ in the general solution, which is impossible, so we obtain no solution.

4. We want to find the nullspace of

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ -1 & 0 & 2 & -3 \\ 2 & 4 & 8 & 5 \end{bmatrix}$$

First, we'll find B, which is the Echelon form of A.

$$E_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 2 \\ -1 & 0 & 2 & -3 \\ 2 & 4 & 8 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 2 & 4 & 8 & 5 \end{bmatrix}$$

$$E_2 (E_1 A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 2 & 4 & 8 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 6 & 1 \end{bmatrix}$$

$$E_3 (E_2 E_1 A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$E_4 (E_3 E_2 E_1 A) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$E_5 (E_4 E_3 E_2 E_1 A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$E_6 (E_5 E_4 E_3 E_2 E_1 A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = B$$

Let $X \in \mathcal{N}(A) \Rightarrow AX = 0 \Rightarrow E_6 E_5 E_4 E_3 E_2 E_1 A X = 0$ (this is true because the elementary matrices have inverses) $\Rightarrow BX = 0$, where $X = [x, y, z, t]^T \Rightarrow$

$$\Rightarrow x - 2z = 0 \Rightarrow x = 2z$$
$$y + 3z = 0 \Rightarrow y = -3z$$
$$3t = 0 \Rightarrow t = 0$$

if we replace z with $s \in \mathbb{R}$, we get the general solution: $X = [2s, -3s, s, 0]^T$

$$X = \begin{bmatrix} 2s \\ -3s \\ s \\ 0 \end{bmatrix} = s \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}$$

Therefore, the nullspace of A, $\mathcal{N}(A) = \left\{ s \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} \middle| s \in \mathbb{R} \right\}$

As the basis of $\mathcal{N}(A)$ is the vector $\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \dim(\mathcal{N}(A)) = 1$

By using the Theorem 4.4.7, for $A \in \mathbb{R}^{3 \times 4}$, we have

$$\operatorname{rank}(A) + \dim(\mathcal{N}(A)) = 4 \Rightarrow \operatorname{rank}(A) = 3.$$

5.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ -2 & 1 & -3 & -2 & -4 \\ 0 & 5 & -14 & -9 & 0 \\ 2 & 10 & -28 & -18 & 4 \end{bmatrix} \qquad V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -2 \\ 1 \\ -3 \\ -2 \\ -4 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 0 \\ 5 \\ -14 \\ -9 \\ 0 \end{bmatrix}, \quad V_4 = \begin{bmatrix} 2 \\ 10 \\ -28 \\ -18 \\ 4 \end{bmatrix}.$$

V_i, $i=1,2,3,4$ are the transpose of the rows of A:

$$x_1V_1 + x_2V_2 + x_3V_3 + x_4V_4 = b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

$$\left|\begin{array}{ll} eq\,1: & x_1 - 2x_2 \quad + 2x_4 = b_1 \\ eq\,2: & x_2 + 5x_3 + 10x_4 = b_2 \\ eq\,3: & -3x_2 - 14x_3 - 28x_4 = b_3 \\ eq\,4: & -2x_2 - 9x_3 - 18x_4 = b_4 \\ eq\,5: & 2x_1 - 4x_2 \quad + 4x_4 = b_5 \end{array}\right| \xrightarrow[\substack{eq4 \leftarrow eq4 + 2eq2 \\ eq5 \leftarrow eq5 - 2eq1}]{eq3 \leftarrow eq3 + 3eq2} \left|\begin{array}{ll} eq\,1: & x_1 - 2x_2 \quad + 2x_4 = b_1 \\ eq\,2: & x_2 + 5x_3 + 10x_4 = b_2 \\ eq\,3: & x_3 + 2x_4 = 3b_2 + b_3 \\ eq\,4: & x_3 + 2x_4 = 2b_2 + b_4 \\ eq\,5: & 0 = -2b_1 + b_5 \Rightarrow \boxed{b_5 = 2b_1} \end{array}\right.$$

From eq 3 and eq 4 we get $3b_2 + b_3 = 2b_2 + b_4 \Rightarrow \boxed{b_2 + b_3 = b_4}$

Apart from those 2 conditions we don't find any other, so we can conclude that

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_2 + b_3 \\ 2b_1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ b + c \\ 2a \end{bmatrix} \in \mathbb{R}^5$$

By continuing with the Gauss-Jordan method, we get

$$\left.\begin{array}{ll} eq\,1: & x_1 - 2x_2 \quad + 2x_4 = b_1 \\ eq\,2: & x_2 + 5x_3 + 10x_4 = b_2 \\ eq\,3: & x_3 + 2x_4 = 3b_2 + b_3 \\ eq\,4: & 0 = -b_2 - b_3 + b_4 \\ eq\,5: & 0 = -2b_1 - b_5 \end{array}\right| \xrightarrow{eq2 \leftarrow eq2 - 5eq3} \left|\begin{array}{ll} eq\,1: & x_1 - 2x_2 \quad + 2x_4 = b_1 \\ eq\,2: & x_2 \quad = -2b_2 - 5b_3 \\ eq\,3: & x_3 + 2x_4 = 3b_2 + b_3 \\ eq\,4: & 0 = -b_2 - b_3 + b_4 \\ eq\,5: & 0 = -2b_1 - b_5 \end{array}\right.$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ -2 & 1 & -3 & -2 & -4 \\ 0 & 5 & -14 & -9 & 0 \\ 2 & 10 & -28 & -18 & 4 \end{bmatrix} \xrightarrow{r_2 \leftarrow r_2 + 2r_1} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 5 & -14 & -9 & 0 \\ 2 & 10 & -28 & -18 & 4 \end{bmatrix} \xrightarrow{r_4 \leftarrow r_4 - 2r_1} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 5 & -14 & -9 & 0 \\ 0 & 10 & -28 & -18 & 0 \end{bmatrix} -$$

$$\xrightarrow{r_3 \leftarrow r_3 - 5r_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 10 & -28 & -18 & 0 \end{bmatrix} \xrightarrow{r_4 \leftarrow r_4 - 10r_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 \end{bmatrix} \xrightarrow{r_4 \leftarrow r_4 - 2r_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

which is A_{REF}, so the basis vectors for $R(A)$ are $r_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$, $r_2 = \begin{bmatrix} 0 \\ 1 \\ -3 \\ -2 \\ 0 \end{bmatrix}$ and $r_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

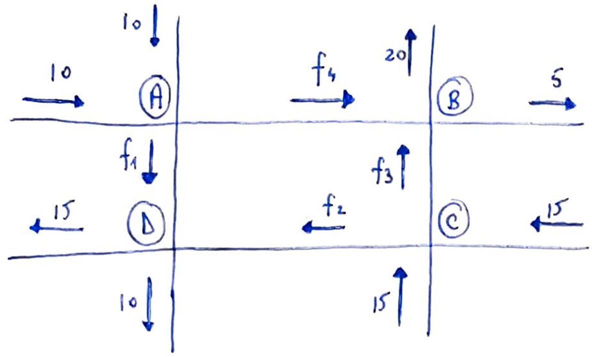
$$\mathcal{N}(A) = \{ u \in \mathbb{R}^5 \mid Au = 0 \} \Rightarrow \begin{cases} u_1 + 2u_5 = 0 \Rightarrow u_1 = -2u_5 \\ u_2 - 3u_3 - 2u_4 = 0 \\ u_3 + u_4 = 0 \Rightarrow u_4 = -u_3 \end{cases} \Bigg| \Rightarrow u_2 - 3u_3 + 2u_3 = 0 \Rightarrow u_2 = u_3 \Bigg| \Rightarrow$$

So, $u = \begin{bmatrix} -2u_5 \\ u_3 \\ u_3 \\ -u_3 \\ u_5 \end{bmatrix} = u_3 \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} + u_5 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, so $m_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ and $m_2 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ are the basis for $\mathcal{N}(A)$

A vector from $R(A)$ has the form $c_1 r_1 + c_2 r_2 + c_3 r_3 = \begin{bmatrix} c_1 \\ c_2 \\ -3c_2+c_3 \\ -2c_2+c_3 \\ 2c_1 \end{bmatrix}$, which respects the conditions on $b = \begin{bmatrix} a \\ b \\ a \\ b+c \\ 2a \end{bmatrix}$, obviously.

Applications

1.



a)

$$\begin{aligned}
&A: \quad 10+10 = f_1+f_4 \\
&B: \quad f_3+f_4 = 20+5 \\
&C: \quad 15+15 = f_2+f_3 \\
&D: \quad f_1+f_2 = 10+15
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
f_1+f_4 &= 20 \\
f_3+f_4 &= 25 \\
f_2+f_3 &= 30 \\
f_1+f_2 &= 25
\end{aligned}$$

b)

$$\begin{aligned}
f_4 &= 20 - f_1 \\
f_2 &= 25 - f_1 \\
f_3 &= 25 - f_4 = 25 - 20 + f_1 = 5 + f_1
\end{aligned}$$

$\Rightarrow$ we have the general solution $f = \begin{bmatrix} f_1 \\ 25-f_1 \\ 5+f_1 \\ 20-f_1 \end{bmatrix}$

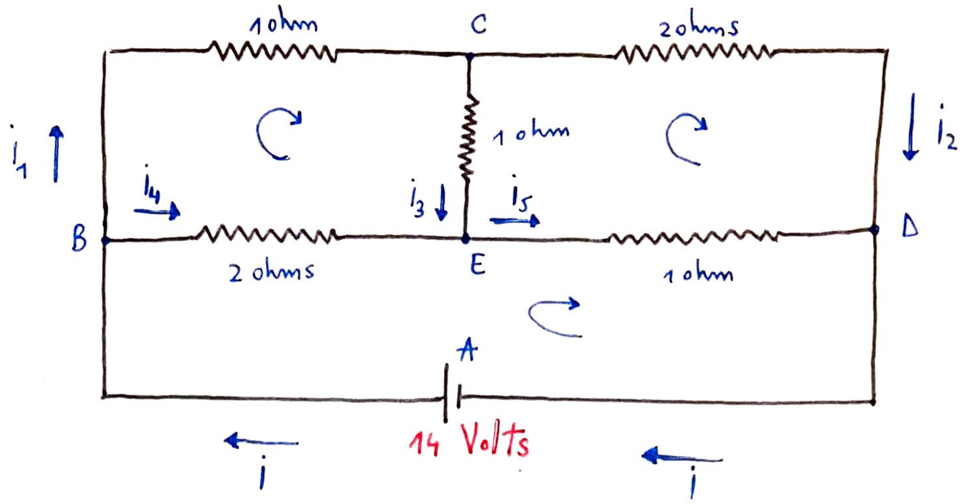
c) $\boxed{f_2 = 10} \Rightarrow 25 - f_1 = 10 \Rightarrow \boxed{f_1 = 15} \Rightarrow \boxed{f_3 = 20} \Rightarrow \boxed{f_4 = 5}$

d) $\begin{cases} \min f_1 = 0 \Rightarrow f_2 = 25, f_3 = 5, f_4 = 20 \\ \max f_1 = 20 \Rightarrow f_2 = 5, f_3 = 25, f_4 = 0 \end{cases}$ $\begin{cases} \min f_2 = 5 \Rightarrow f_1 = 20, f_3 = 25, f_4 = 15 \\ \max f_2 = 25 \Rightarrow f_1 = 0, f_3 = 5, f_4 = 20 \end{cases}$

$\begin{cases} \min f_3 = 5 \Rightarrow f_1 = 0, f_2 = 25, f_4 = 20 \\ \max f_3 = 25 \Rightarrow f_1 = 20, f_2 = 5, f_4 = 0 \end{cases}$ $\begin{cases} \min f_4 = 0 \Rightarrow f_1 = 20, f_2 = 5, f_3 = 25 \\ \max f_4 = 20 \Rightarrow f_1 = 0, f_2 = 25, f_3 = 5 \end{cases}$

These values are the extreme values so that no average is negative.

2.



a) ABEDA: $14 = 2i_4 + i_5$

BCEB: $0 = i_1 + i_3 - 2i_4$

DECD: $0 = 2i_2 - i_3 - i_5$

$\Big\}$ voltage law

B: $i = i_1 + i_4$

C: $i_1 = i_2 + i_3$

D: $i = i_2 + i_5$

E: $i_5 = i_3 + i_4$

$\Big\}$ current law

$14 = 2i_4 + i_5 \overset{E}{\Rightarrow} 14 = 2i_4 + i_3 + i_4 \Rightarrow 14 = i_3 + 3i_4 \Rightarrow i_3 = 14 - 3i_4$

$0 = i_1 + i_3 - 2i_4 \overset{C}{\Rightarrow} 0 = i_2 + i_3 + i_3 - 2i_4 \Rightarrow 0 = i_2 + 2i_3 - 2i_4 \Rightarrow 0 = i_2 + 28 - 6i_4 - 2i_4 \Rightarrow$

$0 = 2i_2 - i_3 - i_5 \overset{E}{\Rightarrow} 0 = 2i_2 - i_3 - i_3 - i_4 \Rightarrow 0 = 2i_2 - 2i_3 - i_4 \Rightarrow 0 = 2i_2 - 28 + 6i_4 - i_4 \Rightarrow$

$\Rightarrow 0 = i_2 - 8i_4 + 28 \Rightarrow i_2 - 8i_4 = -28 \;/\cdot 2 \Rightarrow 2i_2 - 16i_4 = -56$

$\Rightarrow 0 = 2i_2 + 5i_4 - 28 \Rightarrow 2i_2 + 5i_4 = 28$

$(-) \Rightarrow 21 i_4 = 84 \Rightarrow \boxed{i_4 = 4A} \Rightarrow$

$\Rightarrow \boxed{i_3 = 2A}, \boxed{i_2 = 4A} \Rightarrow \boxed{i_1 = 6A}, \boxed{i_5 = 6A} \Rightarrow \boxed{i = 10A}$

b) The effective resistance of the circuit, by applying Ohm's law for the entire circuit is $R = \frac{14}{10} \Rightarrow \boxed{R = 1{,}4\,\Omega}$