Digital Systems: Problem sheet 5

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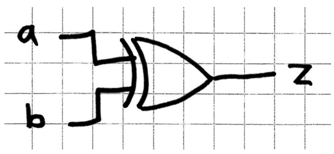
1 An XOR gate $z = a \oplus b$ has the following truth table:

a	b	z
0	0	0
0	1	1
1	0	1
1	1	0

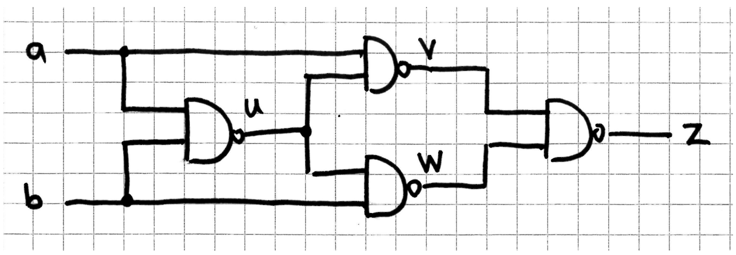


(a) Show that $\oplus$ is associative and commutative. Does it have an identity element?

(b) Show how to build an XOR gate from a 2-input OR gate, two 2-input AND gates and two inverters.

(c) Can you still build an XOR gate if one of the two AND gates is replaced by an OR gate?

(d) Show that the following circuit of four NAND gates also computes $z = a \oplus b$.



2 (a) Design a CMOS implementation of a NOR gate, with the following truth table.

a	b	z
0	0	1
0	1	0
1	0	0
1	1	0

(b) In the lecture, we designed a CMOS gate that computed the function

$$z = \neg((a \wedge b) \vee c).$$

Design a gate that computes

$$w = \neg((a \vee b) \wedge c)$$

instead.

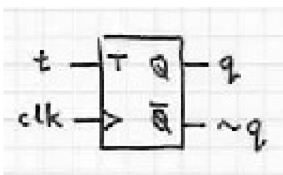
(c) What general principle underlies relates parts (a) and (b) of this question with the CMOS NAND and AND-OR-NOT gates designed in the lectures?

3 (a) Design a *clocked set/reset latch* with the following behaviour. There are two inputs a and b; if $a = 1$ at a clock edge, then the output z goes from 0 to 1. The output then remains at 1 until $b = 1$ at a clock edge, and then returns to 0. The behaviour if $a = b = 1$ at any clock edge can be whatever is easiest to implement.

(b) Enhance your design to produce an additional output w that receives a pulse for exactly one clock cycle whenever the circuit is triggered by an event with $a = 1$, but does not receive another pulse until the circuit has been reset by setting $b = 1$ at a clock edge.

4 A T-type flip-flop has a control input t, in addition to an edge-triggered clock input. If $t = 1$ at a clock edge, then the flip-flop changes state; otherwise it remains in the same state.

q_t	t	q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	0



(a) Show how to construct a T-type flip-flop from a D-type flip-flop and an XOR gate.

(b) Show how to construct a synchronous binary counter from a row of T-type flip-flops and a row of AND gates.

(c) Show how to construct a synchronous binary counter from a row of D-type flip-flops and a row of half-adders.

(d) Use your answer to part (a) to explain the connection between the circuit in parts (b) and (c).

5 Tests with an actual pull-cord light switch installed at the lecturer's home reveal that the light does not go on until the cord is released, but goes off as soon as it is pulled a second time. Modify the bathroom light-switch circuit to reproduce this behaviour.

6 In the lecture, it was shown that the set of connectives $\{\wedge, \vee, \neg\}$ is adequate to express any Boolean function, as is the singleton set {NAND}.

(a) Show that the singleton {NOR} is also adequate.

(b) Show that the set $\{\text{XOR}, \neg\}$ is *not* adequate. *Hint: find a proper subset of the set of all Boolean functions of two variables x and y that contains x, y and the two Boolean constants and is closed under* XOR *and* $\neg$.