$\boxed{12.2}$

STATEMENT: Show that $F^n = \begin{pmatrix} fib\,(n-1) & fib\, n \\ fib\, n & fib\,(n+1) \end{pmatrix}$, for $n \geq 1$, where

$F = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $fib\, n$ is the n^{th} term of the Fibonacci sequence, given by the relation $fib\, n = fib\,(n-1) + fib\,(n-2)$, for $n \geq 2$ and $fib\, 0 = 0$, $fib\, 1 = 1$.

We will prove it with induction on n.

BASE CASE: $P(1)$: $F^1 = \begin{pmatrix} fib\, 0 & fib\, 1 \\ fib\, 1 & fib\, 2 \end{pmatrix}$

$$F^1 = F = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} fib\, 0 & fib\, 1 \\ fib\, 1 & fib\, 1 + fib\, 0 \end{pmatrix} = \begin{pmatrix} fib\, 0 & fib\, 1 \\ fib\, 1 & fib\, 2 \end{pmatrix} \quad \text{TRUE}$$

INDUCTIVE STEP:

INDUCTION HYPOTHESIS: We know that $F^n = \begin{pmatrix} fib\,(n-1) & fib\, n \\ fib\, n & fib\,(n+1) \end{pmatrix}$ (which is $P(n)$) and we will prove that

$P(n+1)$: $F^{n+1} = \begin{pmatrix} fib\, n & fib\,(n+1) \\ fib\,(n+1) & fib\,(n+2) \end{pmatrix}$ is also true.

$$F^{n+1} = F^n \cdot F \overset{IH}{=} \begin{pmatrix} fib\,(n-1) & fib\, n \\ fib\, n & fib\,(n+1) \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} fib\, n & fib\,(n-1) + fib\, n \\ fib\,(n+1) & fib\, n + fib\,(n+1) \end{pmatrix} \overset{\text{recurrence relation}}{=}$$

$$= \begin{pmatrix} fib\, n & fib\,(n+1) \\ fib\,(n+1) & fib\,(n+2) \end{pmatrix} \Rightarrow P(n+1) \text{ is also true.}$$

Therefore, the statement is true, so $F^n = \begin{pmatrix} fib\,(n-1) & fib\, n \\ fib\, n & fib\,(n+1) \end{pmatrix}$, for all $n \geq 1$.