

QUESTION 1

(a) + (b) - it can do both of the subtasks

```
def findAma2(a: Array[Int], b: Array[Int]) : (Int, Boolean) = {
```

```
  var N = a.size
```

```
  var M = b.size
```

```
  var i = 0
```

```
  var j = 0
```

```
  var found = false // have we found a common element yet?
```

```
  // res is going to be the smallest value common to a and b, if there is one
```

```
  // Invariant i: a[0..i) < res ≤ a[i..N) ∧ b[0..j) < res ≤ b[j..M) ∧ 0 ≤ i < N ∧
```

```
0 ≤ j < M ∧ found = whether or not we have found a common element in the two arrays, or
```

```
found = a(i) == b(j) (after the iteration of the while-loop)
```

```
  while ((i < N) ∧ (j < M))
```

```
  { if (a(i) == b(j)) { found = true; return (a(i), found) }
```

```
    else if (a(i) < b(j)) i += 1 // res has to be bigger than a(i), otherwise we would have
```

```
found a(i) at an earlier step in b
```

```
    else if (a(i) > b(j)) j += 1 // same reasoning
```

```
  }
```

```
  // Because of the invariant, we cannot get out of bounds in the loop
```

```
  // If we get out of loop, that means we have not found a common element, so we return
```

```
false
```

```
  (0, found)
```

```
}
```

(c)

```
def findAma3(a: Array[Int], b: Array[Int], c: Array[Int]) : Int = {
```

```
  var N = a.size
```

```
  var M = b.size
```

```
  var L = c.size
```

```
  var i = 0
```

```
  var j = 0
```

```
  var k = 0
```

```
  // Invariant i: a[0..i) < res ≤ a[i..N) ∧ b[0..j) < res ≤ b[j..M) ∧ c[0..k) < res ≤ c[k..L)
```

```
  ∧ 0 ≤ i < N ∧ 0 ≤ j < M ∧ 0 ≤ k < L
```

```
while ((i < N) && (j < M) && (k < L))
```

```
{
  if (a(i) == b(j)) {
    if (a(i) == c(k)) return a(i)
    else if (a(i) < c(k)) {i += 1; j += 1}
    else if (a(i) < b(j)) {
      if (a(i) < c(k)) i += 1
      else if (a(i) == c(k)) {i += 1; k += 1}
      else k += 1
    }
  }
  else {
    if (b(j) < c(k)) j += 1
    else if (b(j) == c(k)) {j += 1; k += 1}
    else k += 1
  }
}
```

o // We will never get here as we are guaranteed to have a result

```
}
```

(d)

```
def findAways (as: Array [Array [Int]]): (Int, Boolean) = {
```

```
  var N = as.size
```

```
  var m = new Array [Int] (N)
```

```
  for (i <- 0 until N) m(i) = as(i).size
```

```
  var index = new Array [Int] (N)
```

```
  for (i <- 0 until N) index(i) = 0
```

```
  var ok = true
```

// Invariant i : as(i)[0..index(i)] <= as(i)[index(i)..m(i)] && 0 ≤ index(i) < m(i)

for all i in [0..N) && ok = whether there might be a common element in the N arrays or not

```
while (ok)
```

```
{
  var minIndex = -1
  var min = as(0)(index(0))
  for (i <- 1 until N) if (min > as(i)(index(i))) {min = as(i)(index(i)); minIndex = i}
  var maxIndex = -1
  var max = as(0)(index(0))
  for (i <- 1 until N) if (max < as(i)(index(i))) {max = as(i)(index(i)); maxIndex = i}
  if (as(minIndex)(index(minIndex)) == as(maxIndex)(index(maxIndex)))
  {
    ok = false // that means that all the values are equal, so we are done
    return (as(minIndex)(index(minIndex)), true)
  }
}
```

else

```
{ index(minIndex) += 1 // we need to go to the next element in this array  
  if (index(minIndex) >= n(minIndex)) ok = false // we got out of bounds, so we stop  
}
```

```
}
```

(0, false)

// This happens when there is no common element

```
}
```