

Exercícios

Exercício 1

$$\sum_{k=1}^n a^k =$$

$$S_n = \sum_{k=1}^n a^k$$

$$\begin{aligned} S_{n+1} &= S_n + a^{n+1} = \sum_{k=1}^{n+1} a^k \\ &= a^1 + \sum_{k=2}^{n+1} a^k \text{ (Separação)} \\ &= a + \sum_{k=1}^n a^{k+1} \text{ (Reindexação)} \\ &= a + \sum_{k=1}^n a^k \cdot a^1 \\ &= a + a \cdot \sum_{k=1}^n a^k \text{ (Distributividade)} \end{aligned}$$

$$S_n + a^{n+1} = a + a \cdot S_n$$

$$S_n - a \cdot S_n = a - a^{n+1}$$

$$S_n(1 - a) = a - a^{n+1}$$

$$S_n = (a - a^{n+1}) / (1 - a) \rightarrow \text{resposta final}$$

Exercício 2

$$\sum_{i=1}^n \frac{2^{i+1}}{5^i}$$

$$S_{n+1} = S_n + \frac{(2^{n+2})}{5^{n+1}} = \sum_{i=1}^{n+1} \frac{2^{i+1}}{5^i}$$

$$S_n + \frac{(2^{n+2})}{5^{n+1}} = \frac{4}{5} + \sum_{i=2}^{n+1} \frac{2^{i+1}}{5^i}$$

$$S_n + \frac{(2^{n+2})}{5^{n+1}} = \frac{4}{5} + \sum_{i=1}^n \frac{2^{i+1+1}}{5^{i+1}}$$

$$S_n + \frac{(2^{n+2})}{5^{n+1}} = \frac{4}{5} + \sum_{i=1}^n \frac{2^{i+2}}{5^{i+1}}$$

$$S_n + \frac{(2^{n+2})}{5^{n+1}} = \frac{4}{5} + \sum_{i=1}^n \frac{2^{i+1}}{5^i} \cdot \frac{2^1}{5^1}$$

$$S_n + \frac{(2^{n+2})}{5^{n+1}} = \frac{4}{5} + \frac{2}{5} \sum_{i=1}^n \frac{2^{i+1}}{5^i}$$

$$S_n + \frac{(2^{n+2})}{5^{n+1}} = \frac{4}{5} + \frac{2}{5} S_n$$

$$\frac{(2^{n+2})}{5^{n+1}} - \frac{4}{5} = \frac{2}{5} S_n - S_n$$

$$\frac{(2^{n+2})}{5^{n+1}} - \frac{4}{5} = \frac{2}{5} S_n - S_n$$

$$\frac{-3}{5} S_n = \frac{2^{n+2}}{5} n + 1 - \frac{4}{5}$$

$$S_n = \frac{\frac{2^{n+2}}{5^{n+1}} - \frac{4}{5}}{\frac{-3}{5}}$$

$$S_n = \left(\frac{2^{n+2}}{5^{n+1}} - \frac{4}{5} \right) \cdot \frac{-5}{3}$$

$$S_n = \frac{-5}{3} \left(\frac{2^{n+2}}{5^{n+1}} \right) + \frac{4}{3} \rightarrow \text{resposta final}$$

Exercício 3

$$\sum_{i=1}^n 7i - 3$$

$$S_n = \sum_{i=1}^n 7i - 3$$

$$S_n = \sum_{i=1}^n 7i - \sum_{i=1}^n 3$$

$$S_n = 7 \left(\sum_{i=1}^n i \right) - 3n$$

$$S_n = 7 \frac{(n(n+1))}{2} - 3n$$

$$S_n = \frac{7n^2 + 7n}{2} - 3n$$

$$S_n = \frac{7n^2 + 7n - 6n}{2}$$

$$S_n = \frac{7n^2 + n}{2} \rightarrow \text{resposta final}$$