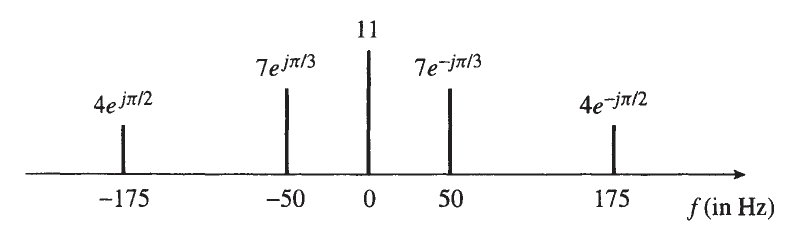
*-****Seminars 3 & 4: Spectrum Representation  
[solutions]***

1. A signal composed of sinusoids is given by the equation
   1. Sketch the spectrum of this signal, indicating the complex amplitude of each frequency component. Make separate plots for real/imaginary or magnitude/phase of the complex amplitudes at each frequency.   
      **ANSWER:** The spectral representation to be sketched is

, where each pair includes the frequency in Hz and the complex amplitude in polar representation. Then make two plots, one for the magnitude values (that will show a symmetry) and another for the phase values (that will show an anti-symmetry). The x-axis should be the frequency expressed in Hz.

* 1. Is x(t) periodic? If so, what is the period?   
     **ANSWER:** yes it is periodic with seconds.
  2. Now consider the signal y(t)=x(t)+5cos(1000πt+π/2). How has the spectrum changed? Is y(t) periodic? If so, what is the period?  
     **ANSWER:** the new components added to the spectrum are . The new seconds.

1. A signal x(t) has the following spectral representation:



* 1. Write an equation for x(t) as a sum of cosines.   
     **ANSWER:**
  2. Is x(t) periodic? If so, find its fundamental period and its fundamental frequency?  
     **ANSWER:**  seconds.
  3. Explain why the negative frequencies are needed in the spectrum.  
     **ANSWER:** They make the signal real-valued in time domain: (for every positive frequency component a negative frequency component is needed.

1. Let
   1. Determine a formula for x(t) as the real part of a sum of complex exponential signals in the form:   
      **ANSWER:**

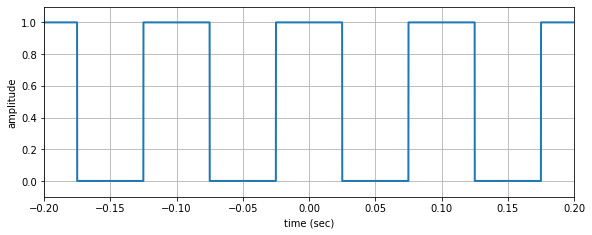
* 1. Plot the spectrum for x(t).   
     **ANSWER:** make two plots, one for the magnitude values (showing its symmetry) and another for the phase values (showing its anti-symmetry). The x-axis should be the frequency expressed in radians.
  2. What is the fundamental period of x(t) ?  
     **ANSWER:**  seconds.

1. Consider the signal x(t)=10 + 20 cos(2π(100)t+π/4) + 10 sin(2π(250)t)
   1. Using Euler's relation, the signal x(t) can be expressed as a sum of complex exponential signals using the finite Fourier synthesis summation as follows . Determine values for N, and indicating the correct index value k for each . *It is not necessary to evaluate any integrals to obtain*  .  
      **ANSWER:** x(t) can be also expressed as x(t) = 10 + 20 cos(2π(100)t+π/4) + 10 cos(2π(250)t-π/2), then using euler’s identity it can be expressed as

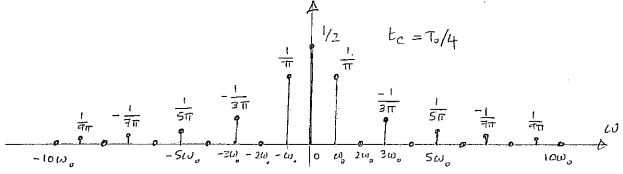
, and

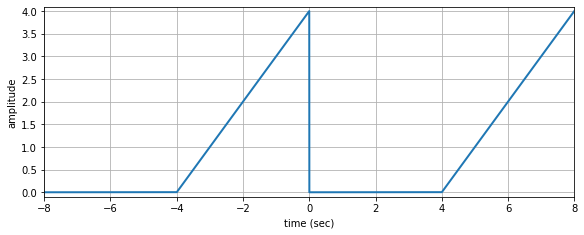
then

* 1. Is x(t) periodic? If so, what is the period?   
     **ANSWER:** Yes. sec.
  2. Plot the spectrum of the signal.  
     **ANSWER:** similar to previous exercises.

1. An amplitude-modulated (AM) cosine wave is represented by the formula x(t) = [12+7sin(πt−π/3)] cos(13πt)
   1. Use Euler's formula to show that x(t) can be expressed in the form   
      **ANSWER:**    
      thus
   2. Sketch the two-sided spectrum of this signal on a frequency axis. Be sure to label important features of the plot.  
      **ANSWER:** similar to previous exercises.
2. A periodic signal is described over one period by the equation: , where <
   1. Sketch x(t) over the time interval for the case .  
      **ANSWER**:  
      
   2. Determine the DC coefficient .  
      **ANSWER**:
   3. Determine a formula for the Fourier series coefficients . Your final result should depend on and .  
      **ANSWER**:

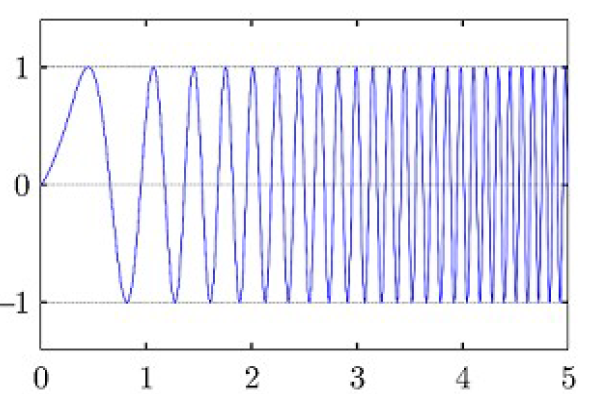
Note: The Fourier analysis integral can be taken over any period of x(t). We have chosen because it is more convenient than in this case.

* 1. Sketch the spectrum of x(t) for the case where and in the range to .  
     **ANSWER**: for , and , for   
     

1. A signal x(t) is periodic with period = 8. Therefore, it can be represented as a Fourier series of the form: . Assume that the Fourier series coefficients of the signal x(t) are given by the following integral: (\*)
   1. In the integral expression for above, the integrand and the limits define the signal x(t). Determine an equation for x(t) that is valid over one period.  
      **ANSWER**: In general, we know that , comparing this with (\*), we have: , for ; and , for
   2. Using the result from part (a), draw a plot of x(t) over the range −8 ≤ t ≤ 8 . Label your plot carefully.   
      **ANSWER**:   
      
   3. Determine the DC value of x(t).

**ANSWER**:

1. We have seen that a periodic signal x(t) can be represented by its Fourier series as follows: . It turns out that we can transform many operations on the signal into corresponding operations on the Fourier coefficients . In all the following cases, find a relationship between the Fourier series coefficients of y(t), denoted by , and the Fourier series coefficients of x(t), .
   1. **ANSWER**:
   2. y(t)=A x(t) , where A is a real-valued constant.   
      **ANSWER**:
   3. **ANSWER**:
2. A chirp signal is one where its instantaneous frequency increases or decreases in time. A chirp is linear if the instantaneous frequency varies linearly in time, i.e., f (t)=αt+β. An example of a linear chirp signal is shown below:



* 1. In the chirp signal shown in the figure above, is the frequency increasing or decreasing with time?

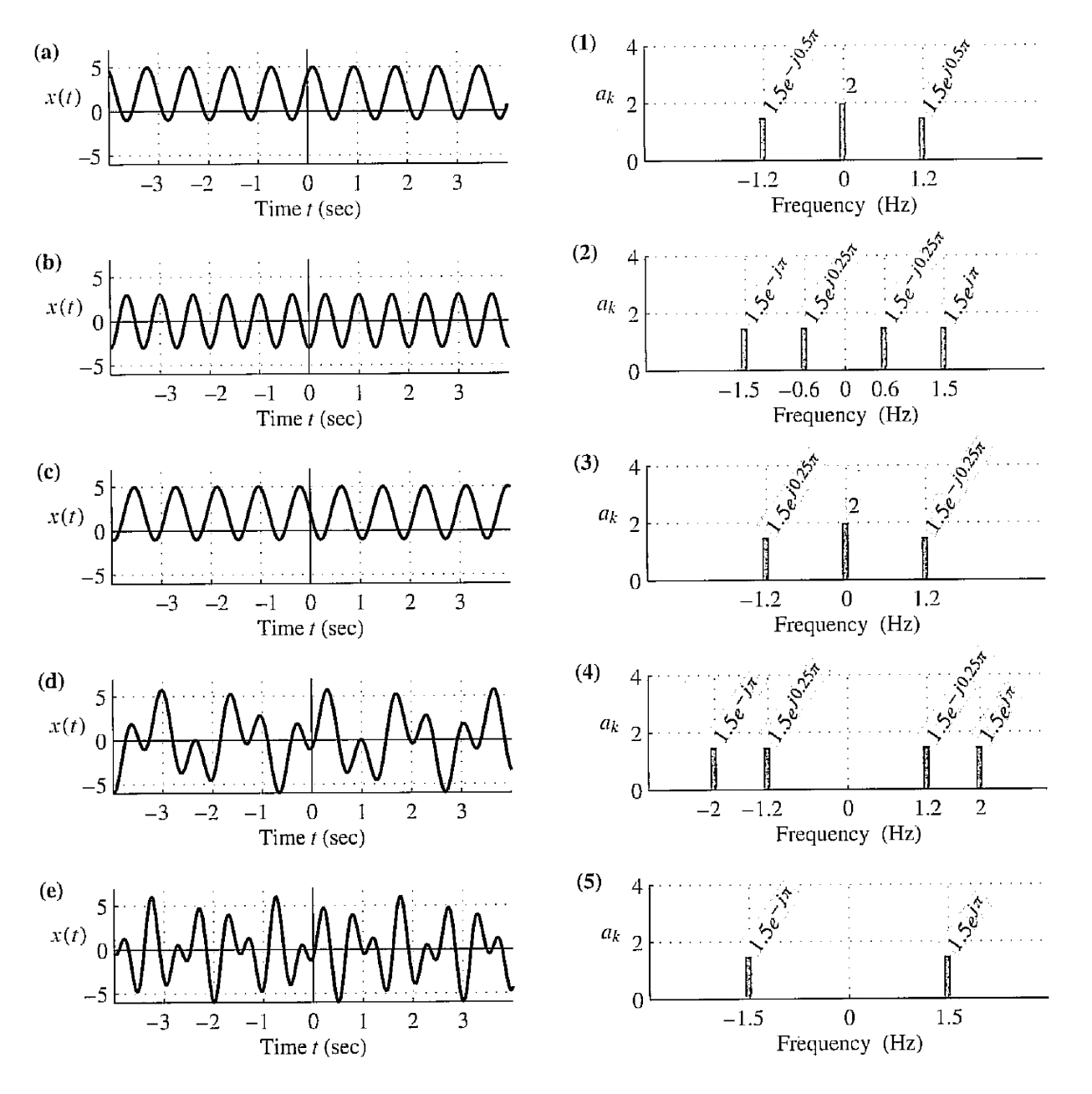
**ANSWER**: Frequency is increasing (faster oscillations)

* 1. For the chirp signal derive a formula for the instantaneous frequency in Hz. Is this chirp linear?   
     **ANSWER**:
  2. Make a plot of the instantaneous frequency (in Hz) versus time over the range 0≤t≤1 seconds.  
     **ANSWER**: …trivial!

1. To calibrate the note scales, the reference tone commonly used is the A4 which has a frequency of 440Hz. Each octave contains 12 tones and the ratio between frequencies of successive tones is constant. The 12 tones starting at A4 (note A of the 4th octave) is computed with the equation , where . The tone location is k. To compute the frequencies of other octaves, k can take values below 0 and above 12.
   1. Compute the frequencies of the 12 tones of an octave starting with C3.  
      **ANSWER:** C3 is 21 tones below A4, soC3 =

* 1. Compute the frequencies of the C major scale starting with C3. [A major scale has 7 notes in an octave, which correspond to the tones 0 (C), 2 (D), 4 (E), 5 (F), 7 (G), 9 (A), 11 (B)].  
     **ANSWER**: trivial from (a)

1. The plots in the following figure show waveforms on the left and spectra on the right.
   1. Match each waveform letter with its corresponding spectrum number.
   2. In each case, write the formula for the signal as a sum of sinusoids.



**ANSWER**:   
(1) c

(2)  
d  
(3) a

(4)   
e  
(5) b