***Seminar 7: Frequency Response of FIR Filters  
[solutions]***

1. Suppose the input signal to an FIR system is: . If we define a new signal to be the first difference , it is possible to express in the form . Determine the numerical values of A, , and .  
   **ANSWER**:

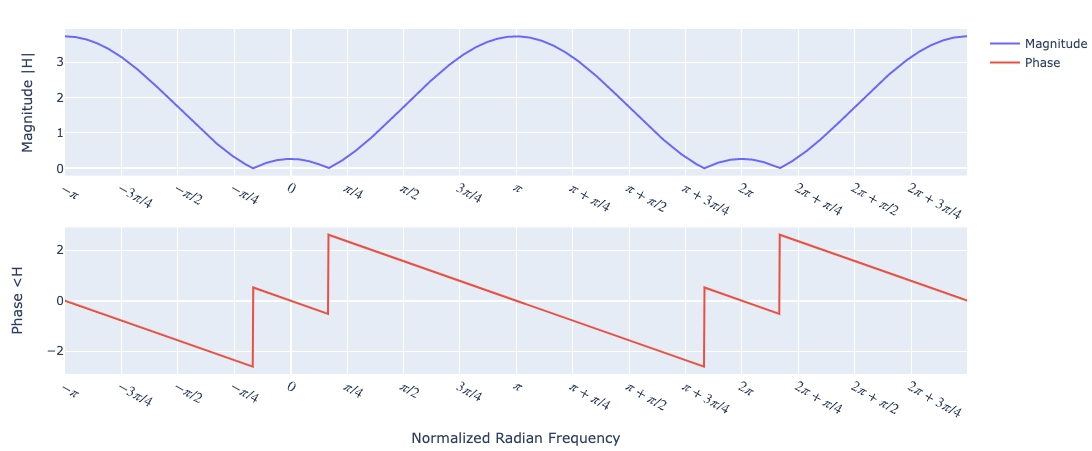
1. A linear time-invariant system is described by the difference equation
   1. Find the frequency response ; then express it as a mathematical formula, in polar form (magnitude and phase).  
      **ANSWER**:

Magnitude:

Phase:

* 1. is a periodic function of ; determine the period.

**ANSWER**: period is

* 1. Plot the magnitude and phase of as a function of for . Do it by hand and then check the Python (package scipy.signal) function freqz.  
     **ANSWER**:  
     
  2. Find all the frequencies , for which the output response to the input is zero.  
     **ANSWER**: The magnitude response is zero when , thus when
  3. When the input to the system is , determine the output signal and express it in the form .  
     **ANSWER**: since

1. For each of the following frequency responses determine the corresponding impulse response
   1. = 1+  
      **ANSWER**:
   2. = 2  
      **ANSWER**:
   3. =1+ + 3  
      **ANSWER**:
2. The frequency response of a linear time-invariant filter is given by the formula
   1. Write the difference equation that gives the relation between the input x[n] and the output y[n].  
      **ANSWER**:
   2. What is the output if the input is ?  
      **ANSWER**:
   3. If the input is of the form =, for what values of will y[n]=0 for all n?  
      **ANSWER**:  
      whenever

or not in

or not in   
The only such is

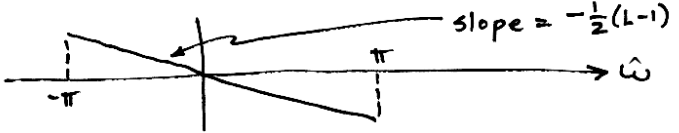
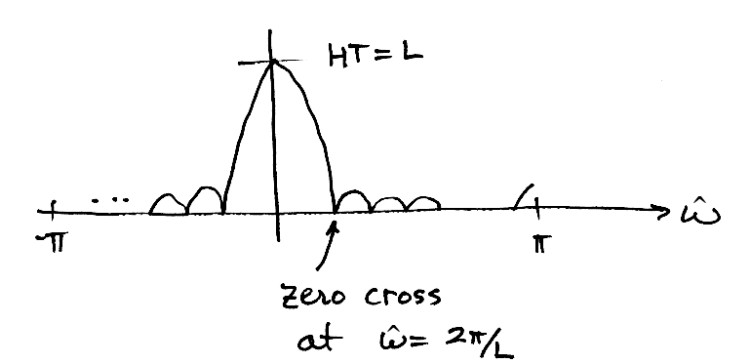
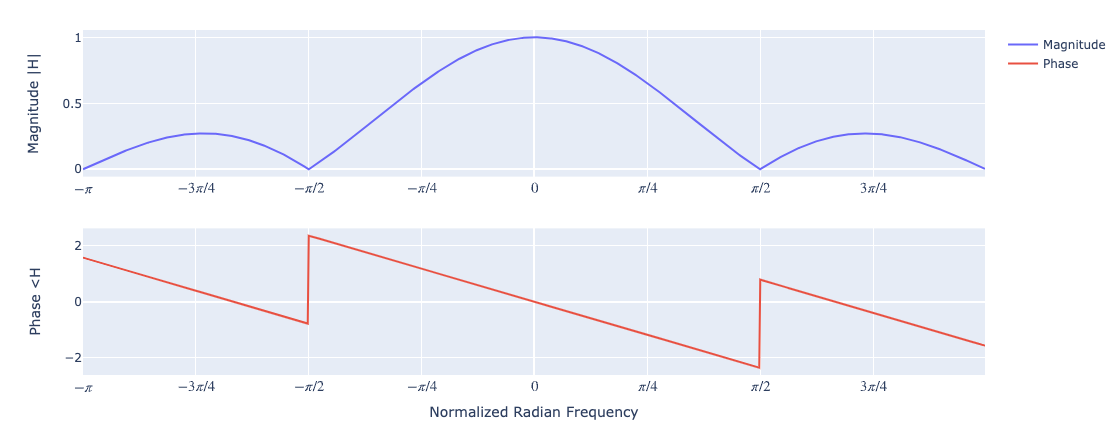
1. Suppose that three systems are connected in cascade; i.e., the output of is the input to and the output of is the input to. The three systems as specified as follows:

where the output of is and its input is .

* 1. Determine the equivalent system that is a single operation from the input x[n] (into ) to the output y[n], which is the output of . Thus, x[n] is and y[n] is .  
     **ANSWER**:

* 1. Use the frequency response to write one difference equation that defines the overall system in terms of x[n] and y[n] only.

**ANSWER**:   
  
  
   
  
which matches previous results

1. The complex-values frequency response of an L-point moving average filter is  
   1. Derive a formula for the phase of and make a plot.  
      **ANSWER**:   
        
      thus the phase is   
      
   2. Derive a formula for the magnitude of and make a plot.  
      **ANSWER**:   
      
2. An LTI filter is described by the difference equation
   1. What is the impulse response of this system?  
      **ANSWER**:
   2. Obtain an expression for the frequency response of this system.  
      **ANSWER**:   
         
       where D is called the Dirichlet function.
   3. Sketch the frequency response (magnitude and phase) as a function of frequency.  
      **ANSWER**:  
      
   4. Suppose that the input is for . Obtain an expression for the output in the form .  
      **ANSWER**: the signal x[n] has 3 frequencies: , thus

cos(n+<)+0  
 cos(

* 1. Suppose that the input is , where u[n] is the unit-step sequence. For what values of n will the output be equal to the output y[n] in (d)?  
     **ANSWER**: h[n] has 4 points, thus ther order of the filter, M=3, thus for we have

1. The general frequency response of an FIR filter of order 2 is . If we have a representation of an FIR filter of order 2 of the form: :
   1. Compute the coefficients   
      **ANSWER**:

, thus

* 1. What is its magnitude, ?  
     **ANSWER**: magnitude is
  2. What is its phase, ?  
     **ANSWER**: phase is