# ***Seminar 9: IIR Filters [solutions]***

1. Consider the following first-order IIR filter:

and assume that the input to the system is the unit step sequence:



* 1. Calculate the output to the system for by directly using the system difference equation, assuming initial rest conditions. Write your results for each *n* as a sum of two terms in the form of   
     **ANSWER**: Initial rest condition:

* 1. Generalize the result in part (a) for a general *n*.  
     **ANSWER**:
  2. Assuming , use the formula for the sum of geometric series to simplify your result in part (b).  
     **ANSWER**:

1. Given an IIR filter defined by the difference equation:
   1. Determine the system function and its poles and zeros.  
      **ANSWER**:   
      zero at z=0; pole at z=-1/2
   2. When the input to the system is:



find the output assuming initial rest conditions.

**Hint:** Write as a sum of three impulses and use linearity to find the output as a sum of three terms. Recall that the impulse response of a first-order IIR filter with has the form .  
**ANSWER**:   
  
for we have , assuming initial rest  
Since

or

1. Given an IIR filter defined by the difference equation
   1. Find the z-transform system function for the system.  
      **ANSWER**:
   2. Find the poles of the system and plot their location on the z-plane.  
      **ANSWER**:   
      zeros at   
      poles at
2. Consider the following system:

If the input is and the system is in initial rest conditions:

* 1. Find the output of the system by iteration from the difference equation. Find the values of the output for *n* = 0,1,2 and present a formula for *n >* 2.  
     **ANSWER**:

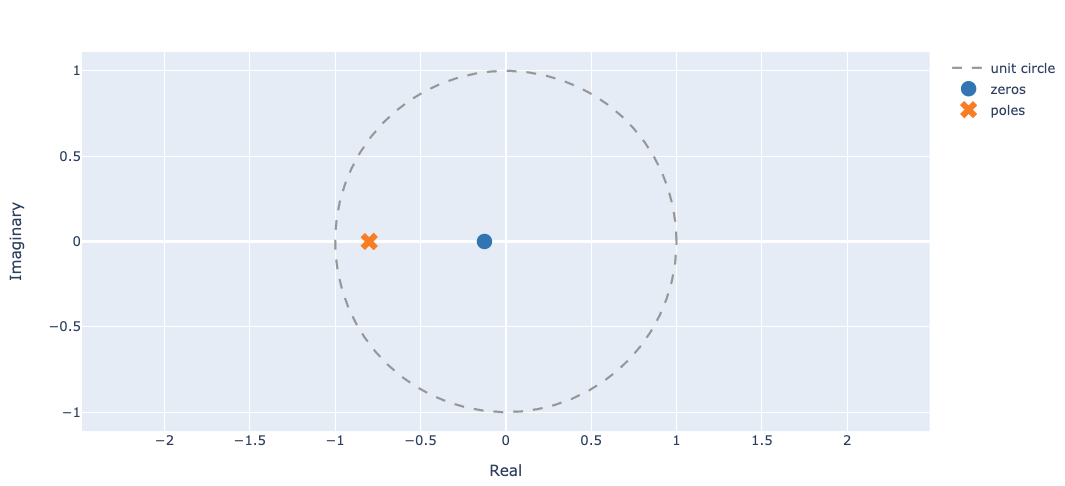
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* 1. What is the impulse response of the system? (Recall that for a first-order IIR filter with , ).  
     **ANSWER**:
  2. Use the impulse response and the linearity and time-invariance of the system to calculate the output again, this time in the form of a sum of two terms.  
     **ANSWER**:
  3. Are your results from parts (a) and (c) the same?   
     **ANSWER**:

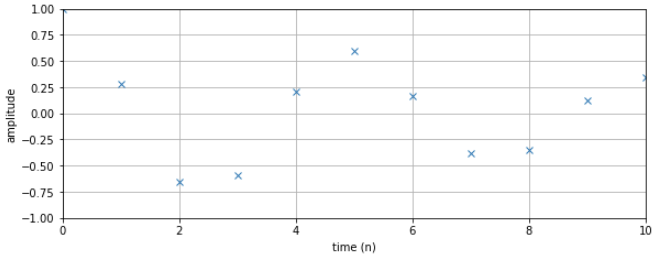
, yes they are the same

1. Find and plot the poles and zeros of the z-transform system

**ANSWER**:   
  
zero:   
pole:

1. An LTI filter is described by the difference equation
   1. Find the system function .  
      **ANSWER**:
   2. Find the poles and zeros of and plot them on the z-plane. Is the system stable?  
      **ANSWER**:   
        
      zero:   
      pole:   
      

Yes the systems is stable because the pole is inside the unit circle

1. Define a discrete-time signal
   1. Make a sketch of versus . Take the range of to be   
      **ANSWER**:  
      
   2. Design a second order feedback filter that will synthesize . Give your answer in the form of a difference equation with numerical values for the coefficients. Assume that the synthesis will be accomplished by using an impulse input to “start” the difference equation (which is at rest, i.e., has zero initial conditions).  
      **ANSWER**: Need poles at and

