

Amigos do Beto - ICPC Library

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```

template<class T = int>
struct Bit2D {
public:
    // send updated points
    Bit2D(vector<pair<T, T>> pts) {
        sort(pts.begin(), pts.end());
        for(auto a : pts) {
            if(ord.empty() || a.first != ord.back()) {
                ord.push_back(a.first);
            }
        }
        fw.resize(ord.size() + 1);
        coord.resize(fw.size());
        for(auto &a : pts) {
            swap(a.first, a.second);
        }
        sort(pts.begin(), pts.end());
        for(auto &a : pts) {
            swap(a.first, a.second);
            for(int on = upper_bound(ord.begin(), ord.end(), a.first) - ord.begin(); on < fw.size(); on += on & -on) {
                if(coord[on].empty() || coord[on].back() != a.second) {
                    coord[on].push_back(a.second);
                }
            }
        }
        for(int i = 0; i < fw.size(); i++) {
            fw[i].assign(coord[i].size() + 1, 0);
        }
    }
    void upd(T x, T y, T v) {
        for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.begin(); xx < fw.size(); xx += xx & -xx) {
            for(int yy = upper_bound(coord[xx].begin(), coord[xx].end(), y) - coord[xx].begin(); yy < fw[xx].size(); yy += yy & -yy) {
                fw[xx][yy] += v;
            }
        }
    }
    T qry(T x, T y) {
        T ans = 0;
        for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.begin(); xx > 0; xx -= xx & -xx) {
            for(int yy = upper_bound(coord[xx].begin(), coord[xx].end(), y) - coord[xx].begin(); yy > 0; yy -= yy & -yy) {

```

```

        ans += fw[xx][yy];
    }
    return ans;
}
T qry(T x1, T y1, T x2, T y2) {
    return qry(x2, y2) - qry(x2, y1 - 1) - qry(x1 - 1, y2) + qry(x1 - 1, y1 - 1);
}
void upd(T x1, T y1, T x2, T y2, T v) {
    upd(x1, y1, v);
    upd(x1, y2 + 1, -v);
    upd(x2 + 1, y1, -v);
    upd(x2 + 1, y2 + 1, v);
}
private:
vector<T> ord;
vector<vector<T>> fw, coord;
};

```

1.2 Segment Tree with Lazy Propagation

```

struct LazyContext {
    LazyContext() {}
    void reset() {}
    void operator += (LazyContext o) {}
};
struct Node {
    Node() {}
    Node() {}
    Node(Node l, Node r) {}
    bool canBreak(LazyContext lazy) {}
    bool canApply(LazyContext lazy) {}
    void apply(LazyContext &lazy) {}
};
template <class i_t, class e_t, class lazy_cont>
class SegmentTree {
public:
    void init(std::vector<e_t> base) {
        n = base.size();
        h = 0;
        while((1 << h) < n) h++;
        tree.resize(2 * n);
        dirty.assign(n, false);
        lazy.resize(n);
        for(int i = 0; i < n; i++) {
            tree[i + n] = i_t(base[i]);
        }
        for(int i = n - 1; i > 0; i--) {
            tree[i] = i_t(tree[i + 1], tree[i + i + 1]);
            lazy[i].reset();
        }
    }
    i_t qry(int l, int r) {
        if(l >= r) return i_t();
        l += n, r += n;
        push(l);
        push(r - 1);
        i_t lp, rp;
        for(; l < r; l /= 2, r /= 2) {
            if(l & 1) lp = i_t(lp, tree[l++]);
            if(r & 1) rp = i_t(tree[--r], rp);
        }
        return i_t(lp, rp);
    }
    void upd(int l, int r, lazy_cont lc) {
        if(l >= r) return;
        l += n, r += n;
        push(l);
        push(r - 1);
        int l0 = l, r0 = r;
        for(; l < r; l /= 2, r /= 2) {
            if(l & 1) downUpd(l++, lc);
            if(r & 1) downUpd(--r, lc);
        }
        build(l0);
        build(r0 - 1);
    }
    void upd(int pos, e_t v) {
        pos += n;
        push(pos);
        tree[pos] = i_t(v);
    }

```

```

        build(pos);
    }
private:
    int n, h;
    std::vector<bool> dirty;
    std::vector<i_t> tree;
    std::vector<lazy_cont> lazy;
    void apply(int p, lazy_cont lc) {
        tree[p].apply(lc);
        if(p < n) {
            dirty[p] = true;
            lazy[p] += lc;
        }
    }
    void pushSingle(int p) {
        if(dirty[p]) {
            downUpd(p + p, lazy[p]);
            downUpd(p + p + 1, lazy[p]);
            lazy[p].reset();
            dirty[p] = false;
        }
    }
    void push(int p) {
        for(int s = h; s > 0; s--) {
            pushSingle(p >> s);
        }
    }
    void downUpd(int p, lazy_cont lc) {
        if(tree[p].canBreak(lc)) {
            return;
        } else if(tree[p].canApply(lc)) {
            apply(p, lc);
        } else {
            pushSingle(p);
            downUpd(p + p, lc);
            downUpd(p + p + 1, lc);
            tree[p] = i_t(tree[p + p], tree[p + p + 1]);
        }
    }
    void build(int p) {
        for(p /= 2; p > 0; p /= 2) {
            tree[p] = i_t(tree[p + p], tree[p + p + 1]);
            if(dirty[p]) {
                tree[p].apply(lazy[p]);
            }
        }
    }
};

```

1.3 Treap

```

mt19937 rng ((int) chrono::steady_clock::now().time_since_epoch().count());
typedef int Value;
typedef struct item * pitem;
struct item {
    item() {}
    item(Value v) { // add key if not implicit
        value = v;
        prio = uniform_int_distribution<int>() (rng);
        cnt = 1;
        rev = 0;
        l = r = 0;
    }
    pitem l, r;
    Value value;
    int prio, cnt;
    bool rev;
};
int cnt (pitem it) {
    return it ? it->cnt : 0;
}
void fix (pitem it) {
    if (it)
        it->cnt = cnt(it->l) + cnt(it->r) + 1;
}
void pushLazy (pitem it) {
    if (it && it->rev) {
        it->rev = false;
        swap(it->l, it->r);
        if (it->l) it->l->rev ^= true;
    }
}

```

```

        if (it->r) it->r->rev ^= true;
    }
}

void insert (pitem & t, pitem it) {
    if (!t)
        t = it;
    else if (it->prio > t->prio)
        split (t, it->key, it->l, it->r), t = it;
    else
        insert (t->key <= it->key ? t->r : t->l, it);
}

void merge (pitem & t, pitem l, pitem r) {
    pushLazy (l); pushLazy (r);
    if (!l || !r) t = l ? l : r;
    else if (l->prio > r->prio)
        merge (l->r, l->r, r), t = l;
    else
        merge (r->l, l, r->l), t = r;
    fix (t);
}

void erase (pitem & t, int key) {
    if (t->key == key) {
        pitem th = t;
        merge (t, t->l, t->r);
        delete th;
    }
    else
        erase (key < t->key ? t->l : t->r, key);
}

void split (pitem t, pitem & l, pitem & r, int key) {
    if (!t) return void( l = r = 0 );
    pushLazy (t);
    int cur_key = cnt(t->l); // t->key if not implicit
    if (key <= cur_key)
        split (t->l, l, t->l, key), r = t;
    else
        split (t->r, t->r, r, key - (1 + cnt(t->l))), l = t;
    fix (t);
}

void reverse (pitem t, int l, int r) {
    pitem t1, t2, t3;
    split (t, t1, t2, l);
    split (t2, t2, t3, r-1+1);
    t2->rev ^= true;
    merge (t, t1, t2);
    merge (t, t, t3);
}

void unite (pitem & t, pitem l, pitem r) {
    if (!l || !r) return void ( t = l ? l : r );
    if (l->prio < r->prio) swap (l, r);
    pitem lt, rt;
    split (r, lt, rt, l->key);
    unite (l->l, l->l, lt);
    unite (l->r, l->r, rt);
    t = l;
}

pitem kth_element(pitem t, int k) {
    if(!t) return NULL;
    if(t->l) {
        if(t->l->size >= k) return kth_element(t->l, k);
        else k -= t->l->cnt;
    }
    return (k == 1) ? t : kth_element(t->r, k - 1);
}

int countLeft(pitem t, int key) {
    if(!t) return 0;
    else if(t->key < key) {
        return 1 + (t->l ? t->l->size : 0) + countLeft(t->r, key);
    } else {
        return countLeft(t->l, key);
    }
}
}

```

1.4 KD-Tree

```

int d;
long long getValue(const PT &a) {return (d & 1) == 0 ? a.x : a.y; }
bool comp(const PT &a, const PT &b) {
    if((d & 1) == 0) { return a.x < b.x; }
}

```

```

    else { return a.y < b.y; }
}

long long sqrDist(PT a, PT b) { return (a - b) * (a - b); }

class KD_Tree {
public:
    struct Node {
        PT point;
        Node *left, *right;
    };
    void init(std::vector<PT> pts) {
        if(pts.size() == 0) {
            return;
        }
        int n = 0;
        tree.resize(2 * pts.size());
        build(pts.begin(), pts.end(), n);
    }
    long long nearestNeighbor(PT point) {
        long long ans = (long long) 1e18;
        nearestNeighbor(&tree[0], point, 0, ans);
        return ans;
    }
private:
    std::vector<Node> tree;
    Node* build(std::vector<PT>::iterator l, std::vector<PT>::iterator r, int &n, int h
        = 0) {
        int id = n++;
        if(r - l == 1) {
            tree[id].left = tree[id].right = NULL;
            tree[id].point = *l;
        }
        else if(r - l > 1) {
            std::vector<PT>::iterator mid = l + ((r - l) / 2);
            d = h;
            std::nth_element(l, mid - 1, r, comp);
            tree[id].point = *(mid - 1);
            // BE CAREFUL!
            // DO EVERYTHING BEFORE BUILDING THE LOWER PART!
            tree[id].left = build(l, mid, n, h^1);
            tree[id].right = build(mid, r, n, h^1);
        }
        return &tree[id];
    }

    void nearestNeighbor(Node* node, PT point, int h, long long &ans) {
        if(!node) {
            return;
        }
        if(point != node->point) {
            // THIS WAS FOR A PROBLEM
            // THAT YOU DON'T CONSIDER THE DISTANCE TO ITSELF!
            ans = std::min(ans, sqrDist(point, node->point));
        }
        d = h;
        long long delta = getValue(point) - getValue(node->point);
        if(delta <= 0) {
            nearestNeighbor(node->left, point, h^1, ans);
            if(ans > delta * delta) {
                nearestNeighbor(node->right, point, h^1, ans);
            }
        }
        else {
            nearestNeighbor(node->right, point, h^1, ans);
            if(ans > delta * delta) {
                nearestNeighbor(node->left, point, h^1, ans);
            }
        }
    }
};

```

1.5 Sparse Table

```

vector<vector<int>> table;
vector<int> lg2;
void build(int n, vector<int> v) {
    lg2.resize(n + 1);
    lg2[1] = 0;
    for (int i = 2; i <= n; i++) {
        lg2[i] = lg2[i >> 1] + 1;
    }
    table.resize(lg2[n] + 1);
}

```

```

for (int i = 0; i < lg2[n] + 1; i++) {
    table[i].resize(n + 1);
}
for (int i = 0; i < n; i++) {
    table[0][i] = v[i];
}
for (int i = 0; i < lg2[n]; i++) {
    for (int j = 0; j < n; j++) {
        if (j + (1 << i) >= n) break;
        table[i + 1][j] = min(table[i][j], table[i][j + (1 << i)]);
    }
}
int get(int l, int r) {
    int k = lg2[r - l + 1];
    return min(table[k][l], table[k][r - (1 << k) + 1]);
}

```

1.6 Max Queue

```

template <class T, class C = less<T>>
struct MaxQueue {
    MaxQueue() { clear(); }
    void clear() {
        id = 0;
        q.clear();
    }
    void push(T x) {
        pair<int, T> nxt(1, x);
        while(q.size() > id && cmp(q.back().second, x)) {
            nxt.first += q.back().first;
            q.pop_back();
        }
        q.push_back(nxt);
    }
    T qry() { return q[id].second; }
    void pop() {
        q[id].first--;
        if(q[id].first == 0) { id++; }
    }
private:
    vector<std::pair<int, T>> q;
    int id;
    C cmp;
};

```

1.7 Policy Based Structures

```

#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
ordered_set X;
X.insert(1); X.find_by_order(0);
X.order_of_key(-5); end(X), begin(X);

```

1.8 Color Updates Structure

```

struct range {
    int l, r;
    int v;
    range(int l = 0, int r = 0, int v = 0) : l(l), r(r), v(v) {}
    bool operator < (const range &a) const {
        return l < a.l;
    }
};
set<range> ranges;
vector<range> update(int l, int r, int v) { // [l, r)
    vector<range> ans;
    if(l >= r) return ans;
    auto it = ranges.lower_bound(l);

```

```

if(it != ranges.begin()) {
    it--;
    if(it->r > l) {
        auto cur = *it;
        ranges.erase(it);
        ranges.insert(range(cur.l, l, cur.v));
        ranges.insert(range(l, cur.r, cur.v));
    }
}
it = ranges.lower_bound(r);
if(it != ranges.begin()) {
    it--;
    if(it->r > r) {
        auto cur = *it;
        ranges.erase(it);
        ranges.insert(range(cur.l, r, cur.v));
        ranges.insert(range(r, cur.r, cur.v));
    }
}
for(it = ranges.lower_bound(l); it != ranges.end() && it->l < r; it++) {
    ans.push_back(*it);
}
ranges.erase(ranges.lower_bound(l), ranges.lower_bound(r));
ranges.insert(range(l, r, v));
return ans;
}

int query(int v) { // Substituir -1 por flag para quando nao houver resposta
    auto it = ranges.upper_bound(v);
    if(it == ranges.begin()) {
        return -1;
    }
    it--;
    return it->r >= v ? it->v : -1;
}

```

2 Graph Algorithms

2.1 Simple Disjoint Set

```

struct dsu {
    vector<int> hist, par, sz;
    vector<ii> changes;
    int n;
    dsu(int n) : n(n) {
        hist.assign(n, 1e9);
        par.resize(n);
        iota(par.begin(), par.end(), 0);
        sz.assign(n, 1);
    }

    int root(int x, int t) {
        if(hist[x] > t) return x;
        return root(par[x], t);
    }

    void join(int a, int b, int t) {
        a = root(a, t);
        b = root(b, t);
        if(a == b) { changes.emplace_back(-1, -1); return; }
        if(sz[a] > sz[b]) swap(a, b);
        par[a] = b;
        sz[b] += sz[a];
        hist[a] = t;
        changes.emplace_back(a, b);
        n--;
    }

    bool same(int a, int b, int t) {
        return root(a, t) == root(b, t);
    }

    void undo() {
        int a, b;
        tie(a, b) = changes.back();
        changes.pop_back();
        if(a == -1) return;
        sz[b] -= sz[a];
    }
}

```

```

    par[a] = a;
    hist[a] = 1e9;
    n++;
}

int when (int a, int b) {
    while (1) {
        if (hist[a] > hist[b]) swap(a, b);
        if (par[a] == b) return hist[a];
        if (hist[a] == 1e9) return 1e9;
        a = par[a];
    }
}
};

```

2.2 Blossom

```

#define MAXN 110
#define MAXM MAXN*MAXN
int n, m;
int mate[MAXN], first[MAXN], label[MAXN];
int adj[MAXN][MAXN], nadj[MAXN], from[MAXM], to[MAXM];
queue<int> q;
#define OUTER(x) (label[x] >= 0)
void L(int x, int y, int nxy) {
    int join, v, r = first[x], s = first[y];
    if (r == s) { return; }
    nxy += n + 1;
    label[r] = label[s] = -nxy;
    while (1) {
        if (s != 0) { swap(r, s); }
        r = first[label[mate[r]]];
        if (label[r] != -nxy) { label[r] = -nxy; }
        else {
            join = r;
            break;
        }
    }
    v = first[x];
    while (v != join) {
        if (!OUTER(v)) { q.push(v); }
        label[v] = nxy;
        first[v] = join;
        v = first[label[mate[v]]];
    }
    v = first[y];
    while (v != join) {
        if (!OUTER(v)) { q.push(v); }
        label[v] = nxy;
        first[v] = join;
        v = first[label[mate[v]]];
    }
    for (int i = 0; i <= n; i++) {
        if (OUTER(i) && OUTER(first[i])) { first[i] = join; }
    }
}

void R(int v, int w) {
    int t = mate[v];
    mate[v] = w;
    if (mate[t] != v) { return; }
    if (label[v] >= 1 && label[v] <= n) {
        mate[t] = label[v];
        R(label[v], t);
        return;
    }
    int x = from[label[v] - n - 1], y = to[label[v] - n - 1];
    R(x, y);
    R(y, x);
}

int E() {
    memset(mate, 0, sizeof(mate));
    int r = 0;
    bool e7;
    for (int u = 1; u <= n; u++) {
        memset(label, -1, sizeof(label));
        while (!q.empty()) { q.pop(); }
        if (mate[u]) { continue; }

```

```

        label[u] = first[u] = 0;
        q.push(u);
        e7 = false;
        while (!q.empty() && !e7) {
            int x = q.front();
            q.pop();
            for (int i = 0; i < nadj[x]; i++) {
                int y = from[adj[x][i]];
                if (y == x) { y = to[adj[x][i]]; }
                if (!mate[y] && y != u) {
                    mate[y] = x;
                    R(x, y);
                    r++;
                    e7 = true;
                    break;
                } else if (OUTER(y)) { L(x, y, adj[x][i]); }
                else {
                    int v = mate[y];
                    if (!OUTER(v)) {
                        label[v] = x;
                        first[v] = y;
                        q.push(v);
                    }
                }
            }
        }
        label[0] = -1;
    }
    return r;
}

/*Exemplo simples de uso*/
memset(nadj, 0, sizeof nadj);
for (int i = 0; i < m; ++i) { // arestas
    scanf("%d%d", &a, &b);
    a++, b++; // nao utilizar o vertice 0
    adj[a][nadj[a]++] = i;
    adj[b][nadj[b]++] = i;
    from[i] = a;
    to[i] = b;
}

printf("O emparelhamento tem tamanho %d\n", E());
for (int i = 1; i <= n; i++) {
    if (mate[i] > 1) { printf("%d com %d\n", i - 1, mate[i] - 1); }
}

```

2.3 Boruvka

```

struct edge {
    int u, v;
    int w;
    int id;
    edge() {}
    edge(int u, int v, int w = 0, int id = 0) : u(u), v(v), w(w), id(id) {}
};
bool operator < (edge &other) const { return w < other.w; };

vector<edge> boruvka (vector<edge> &edges, int n) {
    vector<edge> mst;
    vector<edge> best(n);
    initDSU(n);
    bool f = 1;
    while (f) {
        f = 0;
        for (int i = 0; i < n; i++) best[i] = edge(i, i, inf);
        for (auto e : edges) {
            int pu = root(e.u), pv = root(e.v);
            if (pu == pv) continue;
            if (e < best[pu]) best[pu] = e;
            if (e < best[pv]) best[pv] = e;
        }
        for (int i = 0; i < n; i++) {
            edge e = best[root(i)];
            if (e.w != inf) {
                join(e.u, e.v);
                mst.push_back(e);
                f = 1;
            }
        }
    }
    return mst;
}

```

2.4 Dinic Max Flow

```
const int ms = 1e3; // vertices
const int me = 1e5; // arestas
int adj[ms], to[me], ant[me], wt[me], z, n;
int copy_adj[ms], fila[ms], level[ms];
void clear() { // Lembrar de chamar no main
    memset(adj, -1, sizeof adj);
    z = 0;
}
void add(int u, int v, int k) {
    to[z] = v;
    ant[z] = adj[u];
    wt[z] = k;
    adj[u] = z++;
    swap(u, v);
    to[z] = v;
    ant[z] = adj[u];
    wt[z] = 0; // Lembrar de colocar = 0
    adj[u] = z++;
}
int bfs(int source, int sink) {
    memset(level, -1, sizeof level);
    level[source] = 0;
    int front = 0, size = 0, v;
    fila[size++] = source;
    while(front < size) {
        v = fila[front++];
        for(int i = adj[v]; i != -1; i = ant[i]) {
            if(wt[i] && level[to[i]] == -1) {
                level[to[i]] = level[v] + 1;
                fila[size++] = to[i];
            }
        }
    }
    return level[sink] != -1;
}
int dfs(int v, int sink, int flow) {
    if(v == sink) return flow;
    int f;
    for(int &i = copy_adj[v]; i != -1; i = ant[i]) {
        if(wt[i] && level[to[i]] == level[v] + 1 &&
            (f = dfs(to[i], sink, min(flow, wt[i])))) {
            wt[i] -= f;
            wt[i ^ 1] += f;
            return f;
        }
    }
    return 0;
}
int maxflow(int source, int sink) {
    int ret = 0, flow;
    while(bfs(source, sink)) {
        memcpy(copy_adj, adj, sizeof adj);
        while((flow = dfs(source, sink, 1 << 30))) {
            ret += flow;
        }
    }
    return ret;
}
```

2.5 Minimum Vertex Cover

```
// + Dinic
vector<int> coverU, U, coverV, V; // ITA - Particao U LEFT, particao V RIGHT, 0
    indexed
bool Zu[mx], Zv[mx];
int pairU[mx], pairV[mx];
void getreach(int u) {
    if (u == -1 || Zu[u]) return;
    Zu[u] = true;
    for (int i = adj[u]; ~i; i = ant[i]) {
        int v = to[i];
        if (v == SOURCE || v == pairU[u]) continue;
        Zv[v] = true;
        getreach(pairV[v]);
    }
}
```

```

    }
}
void minimumcover () {
    memset(pairU, -1, sizeof pairU);
    memset(pairV, -1, sizeof pairV);
    for (auto i : U) {
        for (int j = adj[i]; ~j; j = ant[j]) {
            if (!(j&1) && !wt[j]) {
                pairU[i] = to[j], pairV[to[j]] = i;
            }
        }
    }
    memset(Zu, 0, sizeof Zu);
    memset(Zv, 0, sizeof Zv);
    for (auto u : U) {
        if (pairU[u] == -1) getreach(u);
    }
    coverU.clear(), coverV.clear();
    for (auto u : U) {
        if (!Zu[u]) coverU.push_back(u);
    }
    for (auto v : V) {
        if (Zv[v]) coverV.push_back(v);
    }
}
```

2.6 Min Cost Max Flow

```
template <class T = int>
class MCMF {
public:
    struct Edge {
        Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
        int to;
        T cap, cost;
    };
    MCMF(int size) {
        n = size;
        edges.resize(n);
        pot.assign(n, 0);
        dist.resize(n);
        visit.assign(n, false);
    }
    pair<T, T> mcmf(int src, int sink) {
        pair<T, T> ans(0, 0);
        if(!SPFA(src, sink)) return ans;
        fixPot();
        // can use dijkstra to speed up depending on the graph
        while(SPFA(src, sink)) {
            auto flow = augment(src, sink);
            ans.first += flow.first;
            ans.second += flow.first * flow.second;
            fixPot();
        }
        return ans;
    }
    void addEdge(int from, int to, T cap, T cost) {
        edges[from].push_back(list.size());
        list.push_back(Edge(to, cap, cost));
        edges[to].push_back(list.size());
        list.push_back(Edge(from, 0, -cost));
    }
private:
    int n;
    vector<vector<int>>> edges;
    vector<Edge> list;
    vector<int> from;
    vector<T> dist, pot;
    vector<bool> visit;
    pair<T, T> augment(int src, int sink) {
        pair<T, T> flow = {list[from[sink]].cap, 0};
        for(int v = sink; v != src; v = list[from[v]^1].to) {
            flow.first = min(flow.first, list[from[v]].cap);
            flow.second += list[from[v]].cost;
        }
        for(int v = sink; v != src; v = list[from[v]^1].to) {
            list[from[v]].cap -= flow.first;
            list[from[v]^1].cap += flow.first;
        }
    }
}
```

```

    }
    return flow;
}
queue<int> q;
bool SPFA(int src, int sink) {
    T INF = numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    q.push(src);
    dist[src] = 0;
    while(!q.empty()) {
        int on = q.front();
        q.pop();
        visit[on] = false;
        for(auto e : edges[on]) {
            auto ed = list[e];
            if(ed.cap == 0) continue;
            T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
            if(toDist < dist[ed.to]) {
                dist[ed.to] = toDist;
                from[ed.to] = e;
                if(!visit[ed.to]) {
                    visit[ed.to] = true;
                    q.push(ed.to);
                }
            }
        }
    }
    return dist[sink] < INF;
}
void fixPot() {
    T INF = numeric_limits<T>::max();
    for(int i = 0; i < n; i++) {
        if(dist[i] < INF) pot[i] += dist[i];
    }
}
};

```

2.7 Euler Path and Circuit

```

int pathV[me], szV, del[me], pathE, szE;
int adj[ms], to[me], ant[me], wt[me], z, n;
// Funcao de add e clear no dinic
void eulerPath(int u) {
    for(int i = adj[u]; ~i; i = ant[u]) if(!del[i]) {
        del[i] = del[i^1] = 1;
        eulerPath(to[i]);
        pathE[szE++] = i;
    }
    pathV[szV++] = u;
}

```

2.8 Articulation Points/Bridges/Biconnected Components

```

int adj[ms], to[me], ant[me], z;
int num[ms], low[ms], timer;
bool art[ms], bridge[me], f[me];
int bc[ms], nbc;
stack<int> st, stk;
vector<vector<int>> comps;

void clear() { // Lembrar de chamar no main
    memset(adj, -1, sizeof adj);
    z = 0;
}

void add(int u, int v) {
    to[z] = v;
    ant[z] = adj[u];
    adj[u] = z++;
}

void generateBc (int v) {
    while (!st.empty()) {
        int u = st.top();

```

```

        st.pop();
        bc[u] = nbc;
        if (v == u) break;
    }
    ++nbc;
}

void dfs (int v, int p) {
    st.push(v), stk.push(v);
    low[v] = num[v] = ++timer;
    for (int i = adj[v]; i != -1; i = ant[i]) {
        if (f[i] || f[i^1]) continue;
        f[i] = 1;
        int u = to[i];
        if (num[u] == -1) {
            dfs(u, v);
            low[v] = min(low[v], low[u]);
            if (low[u] > num[v]) bridge[i] = bridge[i^1] = 1;
            if (low[u] >= num[v]) {
                art[v] = (num[v] > 1 || num[u] > 2);
                comps.push_back({v});
                while (comps.back().back() != u)
                    comps.back().push_back(stk.top(), stk.pop());
            }
            else {
                low[v] = min(low[v], num[u]);
            }
        }
        if (low[v] == num[v]) generateBc(v);
    }
}

void biCon (int n) {
    nbc = 0, timer = 0;
    memset(num, -1, sizeof num);
    memset(bc, -1, sizeof bc);
    memset(bridge, 0, sizeof bridge);
    memset(art, 0, sizeof art);
    memset(f, 0, sizeof f);
    for (int i = 0; i < n; i++) {
        if (num[i] == -1) {
            timer = 0;
            dfs(i, 0);
        }
    }
}

vector<int> g[ms];
int id[ms];
void buildBlockCut (int n) {
    int z = 0;
    for (int u = 0; u < n; ++u) {
        if (art[u]) id[u] = z++;
    }
    for (auto &comp : comps) {
        int node = z++;
        for (int u : comp) {
            if (!art[u]) id[u] = node;
            else {
                g[node].push_back(id[u]);
                g[id[u]].push_back(node);
            }
        }
    }
}

```

2.9 SCC - Strongly Connected Components / 2SAT

```

const int ms = 212345;
vector<int> g[ms];
int idx[ms], low[ms], z, comp[ms], ncomp;
stack<int> st;
int dfs(int u) {
    if (~idx[u]) return idx[u] ? idx[u] : z;
    low[u] = idx[u] = z++;
    st.push(u);
    for(int v : g[u]) {
        low[u] = min(low[u], dfs(v));
    }
    if(low[u] == idx[u]) {

```

```

while(st.top() != u) {
    int v = st.top();
    idx[v] = 0;
    low[v] = low[u];
    comp[v] = ncomp;
    st.pop();
}
idx[st.top()] = 0;
st.pop();
comp[u] = ncomp++;
}
return low[u];
}
bool solveSat(int n) {
    memset(idx, -1, sizeof idx);
    z = 1; ncomp = 0;
    for(int i = 0; i < 2*n; i++) dfs(i);
    for(int i = 0; i < 2*n; i++) if(comp[i] == comp[i^1]) return false;
    return true;
}
int trad(int v) { return v < 0 ? (~v)*2^1 : v * 2; }
void add(int a, int b) { g[trad(a)].push_back(trad(b)); }
void addOr(int a, int b) { add(~a, b); add(~b, a); }
void addImp(int a, int b) { addOr(~a, b); }
void addEqual(int a, int b) { addOr(a, ~b); addOr(~a, b); }
void addDiff(int a, int b) { addEqual(a, ~b); }
// value[i] = comp[trad(i)] < comp[trad(~id)];

```

2.10 LCA - Lowest Common Ancestor

```

int par[ms][mlg+1], lvl[ms];
void dfs(int v, int p, int l = 0) { // chamar como dfs(root, root)
    lvl[v] = l;
    par[v][0] = p;
    for(int k = 1; k <= mlg; k++) {
        par[v][k] = par[par[v][k-1]][k-1];
    }
    for(int u : g[v]) {
        if(u != p) dfs(u, v, l + 1);
    }
}
int lca(int a, int b) {
    if(lvl[b] > lvl[a]) swap(a, b);
    for(int i = mlg; i >= 0; i--) {
        if(lvl[a] - (1 << i) >= lvl[b]) a = par[a][i];
    }
    if(a == b) return a;
    for(int i = mlg; i >= 0; i--) {
        if(par[a][i] != par[b][i]) a = par[a][i], b = par[b][i];
    }
    return par[a][0];
}

```

2.11 Heavy Light Decomposition

```

class HLD {
public:
    void init(int n) { /* resize everything */ }
    void addEdge(int u, int v) {
        edges[u].push_back(v);
        edges[v].push_back(u);
    }
    void setRoot(int r) {
        t = 0;
        p[r] = r;
        h[r] = 0;
        prep(r, r);
        nxt[r] = r;
        hld(r);
    }
    int getLCA(int u, int v) {
        while(!inSubtree(nxt[u], v)) u = p[nxt[u]];
        while(!inSubtree(nxt[v], u)) v = p[nxt[v]];
        return in[u] < in[v] ? u : v;
    }
    // is v in the subtree of u?
    bool inSubtree(int u, int v) {

```

```

        return in[u] <= in[v] && in[v] < out[u];
    }
    // returns ranges [l, r) that the path has
    vector<pair<int, int>> getPath(int u, int anc) {
        vector<std::pair<int, int>> ans;
        //assert(inSubtree(anc, u));
        while(nxt[u] != nxt[anc]) {
            ans.emplace_back(in[nxt[u]], in[u] + 1);
            u = p[nxt[u]];
        }
        // this includes the ancestor! care
        ans.emplace_back(in[anc], in[u] + 1);
        return ans;
    }
private:
    vector<int> in, out, p, rin, sz, nxt, h;
    vector<vector<int>> edges;
    int t;
    void prep(int on, int par) {
        sz[on] = 1;
        p[on] = par;
        for(int i = 0; i < (int) edges[on].size(); i++) {
            int &u = edges[on][i];
            if(u == par) {
                swap(u, edges[on].back());
                edges[on].pop_back();
            }
            i--;
        }
        h[u] = 1 + h[on];
        prep(u, on);
        sz[on] += sz[u];
        if(sz[u] > sz[edges[on][0]]) {
            swap(edges[on][0], u);
        }
    }
    void hld(int on) {
        in[on] = t++;
        rin[in[on]] = on;
        for(auto u : edges[on]) {
            nxt[u] = (u == edges[on][0] ? nxt[on] : u);
            hld(u);
        }
        out[on] = t;
    }
};

```

2.12 Centroid Decomposition

```

template<typename T>
struct CentroidDecomposition {
    vector<int> sz, h, dad;
    vector<vector<pair<int, T>>> adj;
    vector<vector<T>> dis;
    vector<bool> removed;
    CentroidDecomposition(int n) {
        sz.resize(n);
        h.resize(n);
        dis.resize(n, vector<T>(30, 0));
        adj.resize(n);
        removed.resize(n, 0);
        dad.resize(n);
    }
    void add(int a, int b, T w = 1) {
        adj[a].push_back({b, w});
        adj[b].push_back({a, w});
    }
    void dfsSize(int v, int par) {
        sz[v] = 1;
        for(auto u : adj[v]) {
            if(u.x == par || removed[u.x]) continue;
            dfsSize(u.x, v);
            sz[v] += sz[u.x];
        }
    }
    int getCentroid(int v, int par, int tam) {
        for(auto u : adj[v]) {
            if(u.x == par || removed[u.x]) continue;

```



```

    if ((sz[u.x]<<1) > tam) return getCentroid(u.x, v, tam);
}
return v;
}
void setDis (int v, int par, int nv){
    for (auto u : adj[v]) {
        if (u.x == par || removed[u.x]) continue;
        dis[u.x][nv] = dis[v][nv]+u.y;
        setDis(u.x, v, nv);
    }
}
void decompose (int v, int par = -1, int nv = 0){
    dfsSize(v, par);
    int c = getCentroid(v, par, sz[v]);
    dad[c] = par;
    removed[c] = 1;
    h[c] = nv;
    setDis(c, par, nv);
    for (auto u : adj[c]){
        if (!removed[u.x]){
            decompose(u.x, c, nv + 1);
        }
    }
}
int operator [] (const int idx) const {
    return dad[idx];
}
T dist (int u, int v) {
    if (h[u] < h[v]) swap(u, v);
    return dis[u][h[v]];
};
};

```

2.13 Sack

```

void dfs(int v, int par = -1, bool keep = 0) {
    int big = -1;
    for (int u : adj[v]) {
        if (u == par) continue;
        if (big == -1 || sz[u] > sz[big]) {
            big = u;
        }
    }
    for (int u : adj[v]) {
        if (u == par || u == big) {
            continue;
        }
        dfs(u, v, 0);
    }
    if (big != -1) {
        dfs(big, v, 1);
    }
    for (int u : adj[v]) {
        if (u == par || u == big) {
            continue;
        }
        put(u, v);
    }
    if (!keep) {
    }
}
}

```

2.14 Hungarian Algorithm - Maximum Cost Matching

```

int u[ms], v[ms], p[ms], way[ms], minv[ms];
bool used[ms];
pair<vector<int>, int> solve(const vector<vector<int>> &matrix) {
    int n = matrix.size();
    if(n == 0) return {vector<int>(), 0};
    int m = matrix[0].size();
    assert(n <= m);
    memset(u, 0, (n+1)*sizeof(int));
    memset(v, 0, (m+1)*sizeof(int));
    memset(p, 0, (m+1)*sizeof(int));
    for(int i = 1; i <= n; i++) {

```

```

        memset(minv, 0x3f, (m+1)*sizeof(int));
        memset(way, 0, (m+1)*sizeof(int));
        for(int j = 0; j <= m; j++) used[j] = 0;
        p[0] = i;
        int k0 = 0;
        do {
            used[k0] = 1;
            int i0 = p[k0], delta = inf, k1;
            for(int j = 1; j <= m; j++) {
                if(!used[j]) {
                    int cur = matrix[i0-1][j-1] - u[i0] - v[j];
                    if (cur < minv[j]) {
                        minv[j] = cur;
                        way[j] = k0;
                    }
                    if(minv[j] < delta) {
                        delta = minv[j];
                        k1 = j;
                    }
                }
            }
            for(int j = 0; j <= m; j++) {
                if(used[j]) {
                    u[p[j]] += delta;
                    v[j] -= delta;
                } else {
                    minv[j] -= delta;
                }
            }
            k0 = k1;
        } while(p[k0]);
        do {
            int k1 = way[k0];
            p[k0] = p[k1];
            k0 = k1;
        } while(k0);
    }
    vector<int> ans(n, -1);
    for(int j = 1; j <= m; j++) {
        if(!p[j]) continue;
        ans[p[j] - 1] = j - 1;
    }
    return {ans, -v[0]};
}

```

2.15 Burunduk

```

struct edge {
    int a, b, l, r;
};
typedef vector <edge> List;
int cnt[N + 1], ans[N], u[N], color[N], deg[N];
vi g[N];
void add (int a, int b) {
    g[a].pb(b), g[b].pb(a);
}
void dfs (int v, int value) {
    u[v] = 1, color[v] = value;
    forn(i, sz(g[v]))
        if (!u[g[v][i]])
            dfs(g[v][i], value);
}
int compress (List &vl, int vn, int &add_vn) {
    int vnl = 0;
    forn (i, vn) u[i] = 0;
    forn (i, vn) {
        if (!u[i]) deg[vnl] = 0, dfs(i, vnl++);
    }
    forn (i, sz(vl)) {
        vl[i].a = color[vl[i].a];
        vl[i].b = color[vl[i].b];
        if (vl[i].a != vl[i].b)
            deg[vl[i].a]++, deg[vl[i].b]++;
    }
    vn = vnl, vnl = 0;
    forn (i, vn) {
        u[i] = vnl, vnl += (deg[i] > 0), add_vn += !deg[i];
    }
    forn (i, sz(vl)) {

```

```

    vl[i].a = u[vl[i].a];
    vl[i].b = u[vl[i].b];
}
return vnl;
}
void go (int l, int r, const List &v, int vn, int add_vn) {
    if (cnt[l] == cnt[r]) return;
    if (!sz(v)) {
        while (l < r)
            ans[l++] = vn + add_vn;
        return;
    }
    List vl;
    forn (i, vn) {
        g[i].clear();
    }
    forn (i, sz(v)) {
        if (v[i].a != v[i].b) {
            if (v[i].l <= l && v[i].r >= r)
                add(v[i].a, v[i].b);
            else if (l < v[i].r && r > v[i].l)
                vl.pb(v[i]);
        }
    }
    int vnl = compress(vl, vn, add_vn);
    int m = (l + r) / 2;
    go(l, m, vl, vnl, add_vn);
    go(m, r, vl, vnl, add_vn);
}

```

2.16 Minimum Arborescence

```

// uncommented O(V^2) arborescence
struct Edges {
    //set<pair<long long, int>> cost; O(Elog^2)
    long long cost[ms];
    // possible optimization, use vector of size n
    // instead of ms
    long long sum = 0;
    Edges() {
        memset(cost, 0x3f, sizeof cost);
    }
    void addEdge(int u, long long c) {
        // cost.insert({c - sum, u}); O(Elog^2)
        cost[u] = min(cost[u], c - sum);
    }
    pair<long long, int> getMin() {
        //return *cost.begin(); O(E*log^2)
        pair<long long, int> ans(cost[0], 0);
        // in this loop can change ms to n to make it faster for many cases
        for(int i = 1; i < ms; i++) {
            if(cost[i] < ans.first) {
                ans = pair<long long, int>(cost[i], i);
            }
        }
        return ans;
    }
    void unite(Edges &e) {
        /*
        O(E*log^2E)
        if(e.cost.size() > cost.size()) {
            cost.swap(e.cost);
            swap(sum, e.sum);
        }
        for(auto i : e.cost) {
            addEdge(i.second, i.first + e.sum);
        }
        e.cost.clear();
        */
        // O(V^2)
        // can change ms to n
        for(int i = 0; i < ms; i++) {
            cost[i] = min(cost[i], e.cost[i] + e.sum - sum);
        }
    }
};
typedef vector<vector<pair<long long, int>>> Graph;
Edges ed[ms];
int par[ms];
long long best[ms];

```

```

int col[ms];
int getPar(int x) { return par[x] < 0 ? x : par[x] = getPar(par[x]); }
void makeUnion(int a, int b) {
    a = getPar(a);
    b = getPar(b);
    if(a == b) return;
    ed[a].unite(ed[b]);
    par[b] = a;
}
long long arborescence(Graph edges) {
    // root is 0
    // edges has transposed adjacency list (cost, from)
    // edge from i to j cost c is
    // edge[j].emplace_back(c, i)
    int n = (int) edges.size();
    long long ans = 0;
    for(int i = 0; i < n; i++) {
        ed[i] = Edges();
        par[i] = -1;
        for(auto j : edges[i]) {
            ed[i].addEdge(j.second, j.first);
        }
        col[i] = 0;
    }
    // to change the root you can simply change this next line to
    // col[root] = 2;
    col[0] = 2;
    for(int i = 0; i < n; i++) {
        if(col[getPar(i)] == 2) {
            continue;
        }
        int on = getPar(i);
        vector<int> st;
        while(col[on] != 2) {
            assert(getPar(on) == on);
            if(col[on] == 1) {
                int v = on;
                vector<int> cycle;
                //cout << "found cycle\n";
                while(st.back() != v) {
                    //cout << st.back() << endl;
                    cycle.push_back(st.back());
                    st.pop_back();
                }
                for(auto u : cycle) { // compress cycle
                    makeUnion(v, u);
                }
                v = getPar(v);
                col[v] = 0;
                on = v;
            } else {
                // still no cycle
                // while best is in compressed cycle, remove
                // THIS IS TO MAKE O(E*log^2) ALGORITHM!!
                // while(!ed[on].cost.empty() && getPar(on) == getPar(ed[on].getMin().second))
                // {
                //     ed[on].cost.erase(ed[on].cost.begin());
                // }
                // O(V^2)
                for(int x = 0; x < n; x++) {
                    if(on == getPar(x)) {
                        ed[on].cost[x] = 0x3f3f3f3f3f3f3fLL;
                    }
                }
                // best edge
                auto e = ed[on].getMin();
                // O(E*log^2) assert(!ed[on].cost.empty()) if every vertex appears in the
                // arborescence
                // O(V^2)
                assert(e.first < 0x3f3f3f3f3f3f3fLL);
                int v = getPar(e.second);
                //cout << "found not cycle to " << v << " of cost " << e.first + ed[on].sum <<
                // '\n';
                assert(v != on);
                best[on] = e.first + ed[on].sum;
                ans += best[on];
                // compress edges
                ed[on].sum = -(e.first);
                st.push_back(on);
                col[on] = 1;
                on = v;
            }
        }
    }
}

```

```
// make everything 2
for(auto u : st) {
    assert(getPar(u) == u);
    col[u] = 2;
}
return ans;
}
```

3 Dynamic Programming

3.1 Line Container

```
bool Q;
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};
struct LineContainer : multiset<Line> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}); y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};
```

3.2 Li Chao Tree

```
// by luucasv
typedef long long T;
const T INF = 1e18, EPS = 1;
const int BUFFER_SIZE = 1e4;
struct Line {
    T m, b;

    Line(T m = 0, T b = INF): m(m), b(b) {}
    T apply(T x) { return x * m + b; }
};
struct Node {
    Node *left, *right;
    Line line;
    Node(): left(NULL), right(NULL) {}
};
struct LiChaoTree {
    Node *root, buffer[BUFFER_SIZE];
    T min_value, max_value;
    int buffer_pointer;
    LiChaoTree(T min_value, T max_value): min_value(min_value), max_value(max_value + 1) {
        clear();
    }
    void clear() { buffer_pointer = 0; root = newNode(); }
    void insert_line(T m, T b) { update(root, min_value, max_value, Line(m, b)); }
    T eval(T x) { return query(root, min_value, max_value, x); }
    void update(Node *cur, T l, T r, Line line) {
        T m = l + (r - l) / 2;
        bool left = line.apply(l) < cur->line.apply(l);
```

```
bool mid = line.apply(m) < cur->line.apply(m);
bool right = line.apply(r) < cur->line.apply(r);
if (mid) {
    swap(cur->line, line);
}
if (r - l <= EPS) return;
if (left == right) return;
if (mid != left) {
    if (cur->left == NULL) cur->left = newNode();
    update(cur->left, l, m, line);
} else {
    if (cur->right == NULL) cur->right = newNode();
    update(cur->right, m, r, line);
}
}
T query(Node *cur, T l, T r, T x) {
    if (cur == NULL) return INF;
    if (r - l <= EPS) {
        return cur->line.apply(x);
    }
    T m = l + (r - l) / 2;
    T ans;
    if (x < m) {
        ans = query(cur->left, l, m, x);
    } else {
        ans = query(cur->right, m, r, x);
    }
    return min(ans, cur->line.apply(x));
}
Node* newNode() {
    buffer[buffer_pointer] = Node();
    return &buffer[buffer_pointer++];
}
};
```

3.3 Divide and Conquer Optimization

```
int n, k;
ll dpold[ms], dp[ms], c[ms][ms]; // c(i, j) pode ser funcao
void compute(int l, int r, int optl, int optr) {
    if (l > r) return;
    int mid = (l+r)/2;
    pair<ll, int> best = {inf, -1}; // long long inf
    for(int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {dpold[k-1] + c[k][mid], k});
    }
    dp[mid] = best.first;
    int opt = best.second;
    compute(l, mid-1, optl, opt);
    compute(mid+1, r, opt, optr);
}
ll solve() {
    dp[0] = 0;
    for(int i = 1; i <= n; i++) dp[i] = inf; // initialize row 0 of the dp
    for(int i = 1; i <= k; i++) {
        swap(dpold, dp);
        compute(0, n, 0, n); // solve row i of the dp
    }
    return dp[n]; // return dp[k][n]
}
```

3.4 Knuth Optimization

```
int n, m, mid[ms][ms];
ll dp[ms][ms];
void knuth() {
    for(int i = n; i >= 0; i--) { // limites entre 0 e n
        dp[i][i+1] = 0; mid[i][i+1] = i; // caso base
        for(int j = i+2; j <= n; j++) {
            dp[i][j] = inf; // long long inf
            for(int k = mid[i][j-1]; k <= mid[i+1][j]; k++) {
                if(dp[i][j] > dp[i][k] + dp[k][j]) {
                    dp[i][j] = dp[i][k] + dp[k][j];
                    mid[i][j] = k;
                }
            }
        }
    }
}
```

```

    }
    dp[i][j] += c(i, j); // custo associado ao intervalo
}
}

```

4 Math

4.1 Chinese Remainder Theorem

```

long long modinverse(long long a, long long b, long long s0 = 1, long long s1 = 0) {
    if(!b) return s0;
    else return modinverse(b, a % b, s1, s0 - s1 * (a / b));
}

long long gcd(long long a, long long b) {
    if(!b) return a;
    else return gcd(b, a % b);
}

ll mul(ll a, ll b, ll m) {
    ll q = (long double) a * (long double) b / (long double) m;
    ll r = a * b - q * m;
    return (r + 5 * m) % m;
}

long long safemod(long long a, long long m) {
    return (a % m + m) % m;
}

struct equation {
    equation(long long a, long long m) { mod = m, ans = a, valid = true; }
    equation() { valid = false; }
    equation(equation a, equation b) {
        if(!a.valid || !b.valid) {
            valid = false;
            return;
        }
        long long g = gcd(a.mod, b.mod);
        if((a.ans - b.ans) % g != 0) {
            valid = false;
            return;
        }
        valid = true;
        mod = a.mod * (b.mod / g);
        ans = a.ans +
            mul(
                mul(a.mod, modinverse(a.mod, b.mod), mod),
                (b.ans - a.ans) / g,
                mod);
        ans = safemod(ans, mod);
    }
    long long mod, ans;
    bool valid;

    void print() {
        if(!valid)
            std::cout << "equation is not valid\n";
        else
            std::cout << "equation is " << ans << " mod " << mod << '\n';
    }
};

```

4.2 Diophantine Equations

```

int gcd_ext(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    int nx, ny;
    int gc = gcd_ext(b, a % b, nx, ny);
    x = ny;
    y = nx - (a / b) * ny;
    return gc;
}

```

```

}

vector<int> diophantine(int D, vector<int> l) {
    int n = l.size();
    vector<int> gc(n), ans(n);
    gc[n - 1] = l[n - 1];
    for (int i = n - 2; i >= 0; i--) {
        int x, y;
        gc[i] = gcd_ext(l[i], gc[i + 1], x, y);
    }
    if (D % gc[0] != 0) {
        return vector<int>();
    }
    for (int i = 0; i < n; i++) {
        if (i == n - 1) {
            ans[i] = D / l[i];
            D -= l[i] * ans[i];
            continue;
        }
        int x, y;
        gcd_ext(l[i] / gc[i], gc[i + 1] / gc[i], x, y);
        ans[i] = (long long int) D / gc[i] * x % (gc[i + 1] / gc[i]);
        if (D < 0 && ans[i] > 0) {
            ans[i] -= (gc[i + 1] / gc[i]);
        }
        if (D > 0 && ans[i] < 0) {
            ans[i] += (gc[i + 1] / gc[i]);
        }
        D -= l[i] * ans[i];
    }
    return ans;
}

```

4.3 Discrete Logarithm

```

ll discreteLog (ll a, ll b, ll m) {
    a %= m; b %= m;
    ll n = (ll) sqrt (m + .0) + 1, an = 1;
    for (ll i = 0; i < n; i++) {
        an = (an * a) % m;
    }
    map<ll, ll> vals;
    for (ll i = 1, cur = an; i <= n; i++) {
        if (!vals.count(cur)) vals[cur] = i;
        cur = (cur * an) % m;
    }
    ll ans = 1e18; //inf
    for (ll i = 0, cur = b; i <= n; i++) {
        if (vals.count(cur)) {
            ans = min(ans, vals[cur] * n - i);
        }
        cur = (cur * a) % m;
    }
    return ans;
}

```

4.4 Discrete Root

```

//x^k = a % mod
ll discreteRoot (ll k, ll a, ll mod) {
    ll g = primitiveRoot(mod);
    ll y = discreteLog(fexp(g, k, mod), a, mod);
    if (y == -1) {
        return y;
    }
    return fexp(g, y, mod);
}

```

4.5 Division Trick

```

for(int l = 1, r; l <= n; l = r + 1) {
    r = n / (n / l);
    // n / i has the same value for l <= i <= r
}

```

4.6 Modular Sum

```
//calcula (sum(0 <= i <= n) P(i)) % mod,
//onde P(i) eh uma PA modular (com outro modulo)
namespace sum_pa_mod{
    ll calc(ll a, ll b, ll n, ll mod){
        assert(a < b);
        if(a >= b){
            ll ret = ((n*(n+1)/2)%mod)*(a/b);
            if(a%b) ret = (ret + calc(a%b,b,n,mod))%mod;
            else ret = (ret+n+1)%mod;
            return ret;
        }
        return ((n+1)*(((n*a)/b+1)%mod) - calc(b,a,(n*a)/b,mod) + mod + n/b + 1)%mod;
    }
}
//P(i) = a*i mod m
ll solve(ll a, ll n, ll m, ll mod){
    a = (a%m + m)%m;
    if(!a) return 0;
    ll ret = (n*(n+1)/2)%mod;
    ret = (ret*a)%mod;
    ll g = __gcd(a,m);
    ret -= m*(calc(a/g,m/g,n,mod)-n-1);
    return (ret%mod + mod)%mod;
}
//P(i) = a + r*i mod m
ll solve(ll a, ll r, ll n, ll m, ll mod){
    a = (a%m + m)%m;
    r = (r%m + m)%m;
    if(!r) return (a*(n+1))%mod;
    if(!a) return solve(r, n, m, mod);
    ll g, x, y;
    g = gcdExtended(r, m, x, y);
    x = (x%m + m)%m;
    ll d = a - (a/g)*g;
    a -= d;
    x = (x*(a/g))%m;
    return (solve(r, n+x, m, mod) - solve(r, x-1, m, mod) + mod + d*(n+1))%mod;
};
```

4.7 Primitive Root

```
int primitiveRoot(int p) {
    vector<int> fact;
    int phi = p - 1, n = phi;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            fact.push_back(i);
            while (n % i == 0) {
                n /= i;
            }
        }
    }
    if (n > 1) {
        fact.push_back(n);
    }
    for (int res = 2; res <= p; res++) {
        bool ok = true;
        for (auto it : fact) {
            ok &= fexp(res, phi / it, p) != 1;
            if (!ok) {
                break;
            }
        }
        if (ok) {
            return res;
        }
    }
    return -1;
}
```

4.8 Extended Euclides

```
// euclides estendido: acha u e v da equacao:
// u * x + v * y = gcd(x, y);
```

```
// u eh inverso modular de x no modulo y
// v eh inverso modular de y no modulo x
```

```
pair<ll, ll> euclides(ll a, ll b) {
    ll u = 0, oldu = 1, v = 1, oldv = 0;
    while(b) {
        ll q = a / b;
        oldv = oldv - v * q;
        oldu = oldu - u * q;
        a = a - b * q;
        swap(a, b);
        swap(u, oldu);
        swap(v, oldv);
    }
    return make_pair(oldu, oldv);
}
```

4.9 Matrix Fast Exponentiation

```
const ll mod = 1e9+7;
const int m = 2; // size of matrix

struct Matrix {
    ll mat[m][m];
    Matrix operator * (const Matrix &p) {
        Matrix ans;
        for(int i = 0; i < m; i++)
            for(int j = 0; j < m; j++)
                for(int k = 0; k < m; k++)
                    ans.mat[i][j] = (ans.mat[i][j] + mat[i][k] * p.mat[k][j]) % mod;
        return ans;
    }
};

Matrix fExp(Matrix a, ll b) {
    Matrix ans;
    for(int i = 0; i < m; i++) for(int j = 0; j < m; j++)
        ans.mat[i][j] = i == j;
    while(b) {
        if(b & 1) ans = ans * a;
        a = a * a;
        b >>= 1;
    }
    return ans;
}
```

4.10 FFT - Fast Fourier Transform

```
typedef double ld;

const ld PI = acos(-1);

struct Complex {
    ld real, imag;
    Complex conj() { return Complex(real, -imag); }
    Complex(ld a = 0, ld b = 0) : real(a), imag(b) {}
    Complex operator + (const Complex &o) const { return Complex(real + o.real, imag + o.imag); }
    Complex operator - (const Complex &o) const { return Complex(real - o.real, imag - o.imag); }
    Complex operator * (const Complex &o) const { return Complex(real * o.real - imag * o.imag, real * o.imag + imag * o.real); }
    Complex operator / (ld o) const { return Complex(real / o, imag / o); }
    void operator *= (Complex o) { *this = *this * o; }
    void operator /= (ld o) { real /= o, imag /= o; }
};

typedef std::vector<Complex> CVector;

const int ms = 1 << 22;

int bits[ms];
Complex root[ms];

void initFFT() {
    root[1] = Complex(1);
    for(int len = 2; len < ms; len += len) {
```

```

Complex z(cos(PI / len), sin(PI / len));
for(int i = len / 2; i < len; i++) {
    root[2 * i] = root[i];
    root[2 * i + 1] = root[i] * z;
}
}

void pre(int n) {
    int LOG = 0;
    while(1 << (LOG + 1) < n) {
        LOG++;
    }
    for(int i = 1; i < n; i++) {
        bits[i] = (bits[i >> 1] >> 1) | ((i & 1) << LOG);
    }
}

CVector fft(CVector a, bool inv = false) {
    int n = a.size();
    pre(n);
    if(inv) {
        std::reverse(a.begin() + 1, a.end());
    }
    for(int i = 0; i < n; i++) {
        int to = bits[i];
        if(to > i) {
            std::swap(a[to], a[i]);
        }
    }
    for(int len = 1; len < n; len *= 2) {
        for(int i = 0; i < n; i += 2 * len) {
            for(int j = 0; j < len; j++) {
                Complex u = a[i + j], v = a[i + j + len] * root[len + j];
                a[i + j] = u + v;
                a[i + j + len] = u - v;
            }
        }
    }
    if(inv) {
        for(int i = 0; i < n; i++)
            a[i] /= n;
    }
    return a;
}

void fft2inl(CVector &a, CVector &b) {
    int n = (int) a.size();
    for(int i = 0; i < n; i++) {
        a[i] = Complex(a[i].real, b[i].real);
    }
    auto c = fft(a);
    for(int i = 0; i < n; i++) {
        a[i] = (c[i] + c[(n-i) % n].conj()) * Complex(0.5, 0);
        b[i] = (c[i] - c[(n-i) % n].conj()) * Complex(0, -0.5);
    }
}

void ifft2inl(CVector &a, CVector &b) {
    int n = (int) a.size();
    for(int i = 0; i < n; i++) {
        a[i] = a[i] + b[i] * Complex(0, 1);
    }
    a = fft(a, true);
    for(int i = 0; i < n; i++) {
        b[i] = Complex(a[i].imag, 0);
        a[i] = Complex(a[i].real, 0);
    }
}

std::vector<long long> mod_mul(const std::vector<long long> &a, const std::vector<long
long> &b, long long cut = 1 << 15) {
    // TODO cut memory here by /2
    int n = (int) a.size();
    CVector C[4];
    for(int i = 0; i < 4; i++) {
        C[i].resize(n);
    }
    for(int i = 0; i < n; i++) {
        C[0][i] = a[i] % cut;
        C[1][i] = a[i] / cut;
        C[2][i] = b[i] % cut;
        C[3][i] = b[i] / cut;
    }
}

```

```

    }
    fft2inl(C[0], C[1]);
    fft2inl(C[2], C[3]);
    for(int i = 0; i < n; i++) {
        // 00, 01, 10, 11
        Complex cur[4];
        for(int j = 0; j < 4; j++) cur[j] = C[j/2+2][i] * C[j % 2][i];
        for(int j = 0; j < 4; j++) C[j][i] = cur[j];
    }
    ifft2inl(C[0], C[1]);
    ifft2inl(C[2], C[3]);
    std::vector<long long> ans(n, 0);
    for(int i = 0; i < n; i++) {
        // if there are negative values, care with rounding
        ans[i] += (long long) (C[0][i].real + 0.5);
        ans[i] += (long long) (C[1][i].real + C[2][i].real + 0.5) * cut;
        ans[i] += (long long) (C[3][i].real + 0.5) * cut * cut;
    }
    return ans;
}

std::vector<int> mul(const std::vector<int> &a, const std::vector<int> &b) {
    int n = 1;
    while (n - 1 < (int) a.size() + (int) b.size() - 2) n += n;
    CVector poly(n);
    for(int i = 0; i < n; i++) {
        if(i < (int) a.size()) {
            poly[i].real = a[i];
        }
        if(i < (int) b.size()) {
            poly[i].imag = b[i];
        }
    }
    poly = fft(poly);
    for(int i = 0; i < n; i++) {
        poly[i] *= poly[i];
    }
    poly = fft(poly, true);
    std::vector<int> c(n, 0);
    for(int i = 0; i < n; i++) {
        c[i] = (int) (poly[i].imag / 2 + 0.5);
    }
    while (c.size() > 0 && c.back() == 0) c.pop_back();
    return c;
}

```

4.11 NTT - Number Theoretic Transform

```

const int MOD = 998244353;
const int me = 15;
const int ms = 1 << me;
//is n primitive root of p ?
bool test(ll x, ll p) {
    ll m = p - 1;
    for(int i = 2; i * i <= m; ++i) if(!(m % i)) {
        if(fexp(x, i, p) == 1) return false;
        if(fexp(x, m / i, p) == 1) return false;
    }
    return true;
}

//find the largest primitive root for p
int search(int p) {
    for(int i = p - 1; i >= 2; --i) if(test(i, p)) return i;
    return -1;
}

map<int, int> roots;
int get_root(int p) {
    if(roots[p]) {
        return roots[p];
    } else {
        roots[p] = search(p);
        return roots[p];
    }
}

#define add(x, y) x+y>=MOD?x+y-MOD:x+y
const int gen = search(MOD);
int bits[ms], root[ms];
void initFFT() {
    root[1] = 1;
    for(int len = 2; len < ms; len += len) {

```

```

    int z = fexp(gen, (MOD - 1) / len / 2);
    for(int i = len / 2; i < len; i++) {
        root[2 * i] = root[i];
        root[2 * i + 1] = (long long) root[i] * z % MOD;
    }
}

void pre(int n) {
    int LOG = 0;
    while(1 << (LOG + 1) < n) {
        LOG++;
    }
    for(int i = 1; i < n; i++) {
        bits[i] = (bits[i >> 1] >> 1) | ((i & 1) << LOG);
    }
}

vector<int> fft(vector<int> a, int mod, bool inv = false) {
    int n = (int) a.size();
    pre(n);
    if(inv) {
        reverse(a.begin() + 1, a.end());
    }
    for(int i = 0; i < n; i++) {
        int to = bits[i];
        if(i < to)
            swap(a[i], a[to]);
    }
    for(int len = 1; len < n; len *= 2) {
        for(int i = 0; i < n; i += len * 2) {
            for(int j = 0; j < len; j++) {
                int u = a[i + j], v = (ll) a[i + j + len] * root[len + j] % mod;
                a[i + j] = add(u, v);
                a[i + j + len] = add(u, mod - v);
            }
        }
    }
    if(inv) {
        int rev = fexp(n, mod-2, mod);
        for(int i = 0; i < n; i++)
            a[i] = (ll) a[i] * rev % mod;
    }
    return a;
}

std::vector<int> shift(const std::vector<int> &a, int s) {
    int n = std::max(0, s + (int) a.size());
    std::vector<int> b(n, 0);
    for(int i = 0; i < (int) a.size(); i++) {
        b[i + s] = a[i];
    }
    return b;
}

std::vector<int> cut(const std::vector<int> &a, int n) {
    std::vector<int> b(n, 0);
    for(int i = 0; i < (int) a.size() && i < n; i++) {
        b[i] = a[i];
    }
    return b;
}

std::vector<int> operator+(std::vector<int> a, const std::vector<int> &b) {
    int sz = (int) std::max(a.size(), b.size());
    a.resize(sz, 0);
    for(int i = 0; i < (int) b.size(); i++) {
        a[i] = add(a[i], b[i]);
    }
    return a;
}

std::vector<int> operator-(std::vector<int> a, const std::vector<int> &b) {
    int sz = (int) std::max(a.size(), b.size());
    a.resize(sz, 0);
    for(int i = 0; i < (int) b.size(); i++) {
        a[i] = add(a[i], MOD - b[i]);
    }
    return a;
}

std::vector<int> operator*(std::vector<int> a, std::vector<int> b) {
    while(!a.empty() && a.back() == 0) a.pop_back();
    while(!b.empty() && b.back() == 0) b.pop_back();
    if(a.empty() || b.empty()) return std::vector<int>(0, 0);
    int n = 1;
    while(n-1 < (int) a.size() + (int) b.size() - 2) n += n;
    a.resize(n, 0);

```

```

    b.resize(n, 0);
    a = fft(a, false);
    b = fft(b, false);
    for(int i = 0; i < n; i++) {
        a[i] = (int) ((long long) a[i] * b[i] % MOD);
    }
    return fft(a, true);
}

std::vector<int> inverse(const std::vector<int> &a, int k) {
    assert(!a.empty() && a[0] != 0);
    if(k == 0) {
        return std::vector<int>(1, (int) fexp(a[0], MOD - 2));
    }
    else {
        int n = 1 << k;
        auto c = inverse(a, k-1);
        return cut(c * cut(std::vector<int>(1, 2) - cut(a, n) * c, n), n);
    }
}

std::vector<int> log(const std::vector<int> &a, int k) {
    assert(!a.empty() && a[0] != 0);
    int n = 1 << k;
    std::vector<int> b(n, 0);
    for(int i = 0; i+1 < (int) a.size() && i < n; i++) {
        b[i] = (int) ((i + 1LL) * a[i+1] % MOD);
    }
    b = cut(b * inverse(a, k), n);
    assert((int) b.size() == n);
    for(int i = n - 1; i > 0; i--) {
        b[i] = (int) (b[i-1] * fexp(i, MOD - 2) % MOD);
    }
    b[0] = 0;
    return b;
}

std::vector<int> exp(const std::vector<int> &a, int k) {
    assert(!a.empty() && a[0] == 0);
    if(k == 0) {
        return std::vector<int>(1, 1);
    }
    else {
        auto b = exp(a, k-1);
        int n = 1 << k;
        return cut(b * cut(std::vector<int>(1, 1) + cut(a, n) - log(b, k), n), n);
    }
}

```

4.12 Fast Walsh Hadamard Transform

```

vector<ll> FWHT(char oper, vector<ll> a, const bool inv = false) {
    int n = (int) a.size();
    for(int len = 1; len < n; len *= 2) {
        for(int i = 0; i < n; i += 2 * len) {
            for(int j = 0; j < len; j++) {
                auto u = a[i + j] % mod, v = a[i + j + len] % mod;
                if(oper == '^') {
                    a[i + j] = (u + v) % mod;
                    a[i + j + len] = (u - v + mod) % mod;
                }
                if(oper == '|'') {
                    if(!inv) {
                        a[i + j + len] = (u + v) % mod;
                    }
                    else {
                        a[i + j + len] = (v - u + mod) % mod;
                    }
                }
                if(oper == '&') {
                    if(!inv) {
                        a[i + j] = (u + v) % mod;
                    }
                    else {
                        a[i + j] = (u - v + mod) % mod;
                    }
                }
            }
        }
    }
    if(oper == '^' && inv) {
        ll rev = fexp(n, mod - 2);
        for(int i = 0; i < n; i++) {
            a[i] = a[i] * rev % mod;
        }
    }
    return a;
}

```

```

}

vector<ll> multiply(char oper, vector<ll> a, vector<ll> b) {
    int n = 1;
    while (n < (int) max(a.size(), b.size())) {
        n <<= 1;
    }
    vector<ll> ans(n);
    while (a.size() < ans.size()) a.push_back(0);
    while (b.size() < ans.size()) b.push_back(0);
    a = FWHT(oper, a);
    b = FWHT(oper, b);
    for (int i = 0; i < n; i++) {
        ans[i] = a[i] * b[i] % mod;
    }
    ans = FWHT(oper, ans, true);
    return ans;
}

const int mxlog = 17;

vector<ll> subset_multiply(vector<ll> a, vector<ll> b) {
    int n = 1;
    while (n < (int) max(a.size(), b.size())) {
        n <<= 1;
    }
    vector<ll> ans(n);
    while (a.size() < ans.size()) a.push_back(0);
    while (b.size() < ans.size()) b.push_back(0);
    vector<vector<ll>> A(mxlog + 1, vector<ll>(a.size())), B(mxlog + 1, vector<ll>(b.size()));
    for (int i = 0; i < n; i++) {
        A[__builtin_popcount(i)][i] = a[i];
        B[__builtin_popcount(i)][i] = b[i];
    }
    for (int i = 0; i <= mxlog; i++) {
        A[i] = FWHT('|', A[i]);
        B[i] = FWHT('|', B[i]);
    }
    for (int i = 0; i <= mxlog; i++) {
        vector<ll> C(n);
        for (int x = 0; x <= i; x++) {
            int y = i - x;
            for (int j = 0; j < n; j++) {
                C[j] = (C[j] + A[x][j] * B[y][j] % mod) % mod;
            }
        }
        C = FWHT('|', C, true);
        for (int j = 0; j < n; j++) {
            if (__builtin_popcount(j) == i) {
                ans[j] = (ans[j] + C[j]) % mod;
            }
        }
    }
    return ans;
}

```

4.13 Miller and Rho

```

//miller_rabin
typedef unsigned long long ull;
typedef long double ld;

ull fmul(ull a, ull b, ull m) {
    ull q = (ld) a * (ld) b / (ld) m;
    ull r = a * b - q * m;
    return (r + m) % m;
}

bool miller(ull p, ull a) {
    ull s = p - 1;
    while(s % 2 == 0) s >>= 1;
    while(a >= p) a >>= 1;
    ull mod = fexp(a, s, p);
    while(s != p - 1 && mod != 1 && mod != p - 1) {
        mod = fmul(mod, mod, p);
        s <<= 1;
    }
    if(mod != p - 1 && s % 2 == 0) return false;
    else return true;
}

bool prime(ull p) {

```

```

    if(p <= 3)
        return true;
    if(p % 2 == 0)
        return false;
    return miller(p, 2) && miller(p, 3)
        && miller(p, 5) && miller(p, 7)
        && miller(p, 11) && miller(p, 13)
        && miller(p, 17) && miller(p, 19)
        && miller(p, 23) && miller(p, 29)
        && miller(p, 31) && miller(p, 37);
}

//pollard_rho
ull func(ull x, ull c, ull n) {
    return (fmul(x, x, n) + c) % n;
}

ull gcd(ull a, ull b) {
    if(!b) return a;
    else return gcd(b, a % b);
}

ull rho(ull n) {
    if(n % 2 == 0) return 2;
    if(prime(n)) return n;
    while(1) {
        ull c;
        do {
            c = rand() % n;
        } while(c == 0 || (c + 2) % n == 0);
        ull x = 2, y = 2, d = 1;
        ull pot = 1, lam = 1;
        do {
            if(pot == lam) {
                x = y;
                pot <<= 1;
                lam = 0;
            }
            y = func(y, c, n);
            lam++;
            d = gcd(x >= y ? x - y : y - x, n);
        } while(d == 1);
        if(d != n) return d;
    }
}

vector<ull> factors(ull n) {
    vector<ull> ans, rest, times;
    if(n == 1) return ans;
    rest.push_back(n);
    times.push_back(1);
    while(!rest.empty()) {
        ull x = rho(rest.back());
        if(x == rest.back()) {
            int freq = 0;
            for(int i = 0; i < rest.size(); i++) {
                int cur_freq = 0;
                while(rest[i] % x == 0) {
                    rest[i] /= x;
                    cur_freq++;
                }
                freq += cur_freq * times[i];
                if(rest[i] == 1) {
                    swap(rest[i], rest.back());
                    swap(times[i], times.back());
                    rest.pop_back();
                    times.pop_back();
                    i--;
                }
            }
            while(freq-- > 0) {
                ans.push_back(x);
            }
            continue;
        }
        ull e = 0;
        while(rest.back() % x == 0) {
            rest.back() /= x;
            e++;
        }
        e *= times.back();
        if(rest.back() == 1) {
            rest.pop_back();
            times.pop_back();
        }
        rest.push_back(x);
        times.push_back(e);
    }
    return ans;
}

```


}

4.14 Determinant using Mod

```
// by zchao1995
// Determinante com coordenadas inteiras usando Mod

ll mat[ms][ms];

ll det (int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            mat[i][j] %= mod;
        }
    }
    ll res = 1;
    for (int i = 0; i < n; i++) {
        if (!mat[i][i]) {
            bool flag = false;
            for (int j = i + 1; j < n; j++) {
                if (mat[j][i]) {
                    flag = true;
                    for (int k = i; k < n; k++) {
                        swap (mat[i][k], mat[j][k]);
                    }
                    res = -res;
                    break;
                }
            }
            if (!flag) {
                return 0;
            }
        }
        for (int j = i + 1; j < n; j++) {
            while (mat[j][i]) {
                ll t = mat[i][i] / mat[j][i];
                for (int k = i; k < n; k++) {
                    mat[i][k] = (mat[i][k] - t * mat[j][k]) % mod;
                    swap (mat[i][k], mat[j][k]);
                }
                res = -res;
            }
        }
        res = (res * mat[i][i]) % mod;
    }
    return (res + mod) % mod;
}
```

4.15 Lagrange Interpolation

```
class LagrangePoly {
public:
    LagrangePoly(std::vector<long long> _a) {
        //f(i) = _a[i]
        //interpola o vetor em um polinomio de grau y.size() - 1
        y = _a;
        den.resize(y.size());
        int n = (int) y.size();
        for(int i = 0; i < n; i++) {
            y[i] = (y[i] % MOD + MOD) % MOD;
            den[i] = ifat[n - i - 1] * ifat[i] % MOD;
            if((n - i - 1) % 2 == 1) {
                den[i] = (MOD - den[i]) % MOD;
            }
        }
    }

    long long getVal(long long x) {
        int n = (int) y.size();
        x %= MOD;
        if(x < n) {
            //return y[(int) x];
        }
        std::vector<long long> l, r;
        l.resize(n);
        l[0] = 1;
```

```
for(int i = 1; i < n; i++) {
    l[i] = l[i - 1] * (x - (i - 1) + MOD) % MOD;
}
r.resize(n);
r[n - 1] = 1;
for(int i = n - 2; i >= 0; i--) {
    r[i] = r[i + 1] * (x - (i + 1) + MOD) % MOD;
}
long long ans = 0;
for(int i = 0; i < n; i++) {
    long long coef = l[i] * r[i] % MOD;
    ans = (ans + coef * y[i] % MOD * den[i]) % MOD;
}
return ans;
}

private:
std::vector<long long> y, den;
};

int main(){
    fat[0] = ifat[0] = 1;
    for(int i = 1; i < ms; i++) {
        fat[i] = fat[i - 1] * i % MOD;
        ifat[i] = fexp(fat[i], MOD - 2);
    }
    // Codeforces 622F
    int x, k;
    std::cin >> x >> k;
    std::vector<long long> a;
    a.push_back(0);
    for(long long i = 1; i <= k + 1; i++) {
        a.push_back((a.back() + fexp(i, k)) % MOD);
    }
    LagrangePoly f(a);
    std::cout << f.getVal(x) << '\n';
}
```

4.16 Count integer points inside triangle

```
//gcd(p, q) == 1
ll get(ll p, ll q, ll n, bool floor = true) {
    if (n == 0) {
        return 0;
    }
    if (p % q == 0) {
        return n * (n + 1) / 2 * (p / q);
    }
    if (p > q) {
        return n * (n + 1) / 2 * (p / q) + get(p % q, q, n, floor);
    }
    ll new_n = p * n / q;
    ll ans = (n + 1) * new_n - get(q, p, new_n, false);
    if (!floor) {
        ans += n - n / q;
    }
    return ans;
}
```

5 Geometry

5.1 Geometry

```
const double inf = 1e100, eps = 1e-12;
const double PI = acos(-1.0L);
int cmp (double a, double b = 0) {
    if (abs(a-b) < eps) return 0;
    return (a < b) ? -1 : +1;
}

struct PT {
    double x, y;
    PT(double x = 0, double y = 0) : x(x), y(y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
```

```

bool operator <(const PT &p) const {
    if(cmp(x, p.x) != 0) return x < p.x;
    return cmp(y, p.y) < 0;
}
bool operator ==(const PT &p) const {return !cmp(x, p.x) && !cmp(y, p.y);}
bool operator !=(const PT &p) const {return !(p == *this);}
};
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << ", " << p.y << ")";
}
double dot (PT p, PT q) { return p.x * q.x + p.y*q.y; }
double cross (PT p, PT q) { return p.x * q.y - p.y*q.x; }
double dist2 (PT p, PT q = PT(0, 0)) { return dot(p-q, p-q); }
double dist (PT p, PT q) { return hypot(p.x-q.x, p.y-q.y); }
double norm (PT p) { return hypot(p.x, p.y); }
PT normalize (PT p) { return p/hypot(p.x, p.y); }
double angle (PT p, PT q) { return atan2(cross(p, q), dot(p, q)); }
double angle (PT p) { return atan2(p.y, p.x); }
double polarAngle (PT p) {
    double a = atan2(p.y, p.x);
    return a < 0 ? a + 2*PI : a;
}
PT rotateCCW90 (PT p) { return PT(-p.y, p.x); }
PT rotateCW90 (PT p) { return PT(p.y, -p.x); }
PT rotateCCW (PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}
typedef pair<PT, int> Line;
PT getDir (PT a, PT b) {
    if (a.x == b.x) return PT(0, 1);
    if (a.y == b.y) return PT(1, 0);
    int dx = b.x-a.x;
    int dy = b.y-a.y;
    int g = __gcd(abs(dx), abs(dy));
    if (dx < 0) g = -g;
    return PT(dx/g, dy/g);
}
Line getLine (PT a, PT b) {
    PT dir = getDir(a, b);
    return {dir, cross(dir, a)};
}
PT projPtLine (PT a, PT b, PT c) { // ponto c na linha a - b, a.b = |a| cost * |b|
    return a + (b-a) * dot(b-a, c-a)/dot(b-a, b-a);
}
PT reflectPointLine (PT a, PT b, PT c) {
    PT p = projPtLine(a, b, c);
    return p*2 - c;
}
PT projPtSeg (PT a, PT b, PT c) { // c no segmento a - b
    double r = dot(b-a, b-a);
    if (cmp(r) == 0) return a;
    r = dot(b-a, c-a)/r;
    if (cmp(r, 0) < 0) return a;
    if (cmp(r, 1) > 0) return b;
    return a + (b - a) * r;
}
double distancePointSegment (PT a, PT b, PT c) { // ponto c e o segmento a - b
    return dist(c, projPtSeg(a, b, c));
}
bool ptInSegment (PT a, PT b, PT c) { // ponto c esta em um segmento a - b
    if (a == b) return a == c;
    a = a-c, b = b-c;
    return cmp(cross(a, b)) == 0 && cmp(dot(a, b)) <= 0;
}
bool parallel (PT a, PT b, PT c, PT d) {
    return cmp(cross(b - a, c - d)) == 0;
}
bool collinear (PT a, PT b, PT c, PT d) {
    return parallel(a, b, c, d) && cmp(cross(a - b, a - c)) == 0 && cmp(cross(c - d, c - a)) == 0;
}
// Calcula distancia entre o ponto (x, y, z) e o plano ax + by + cz = d
double distPtPlane(double x, double y, double z, double a, double b, double c, double d) {
    return abs(a * x + b * y + c * z - d) / sqrt(a * a + b * b + c * c);
}
bool segInter (PT a, PT b, PT c, PT d) {
    if (collinear(a, b, c, d)) {
        if (cmp(dist(a, c)) == 0 || cmp(dist(b, c)) == 0 || cmp(dist(b, d)) == 0) return true;
        if (cmp(dot(c - a, c - b)) > 0 && cmp(dot(d - a, d - b)) > 0 && cmp(dot(c - b, d - b)) > 0) return false;
        return true;
    }
    if (cmp(cross(d - a, b - a) * cross(c - a, b - a)) > 0) return false;
}

```

```

    if (cmp(cross(a - c, d - c) * cross(b - c, d - c)) > 0) return false;
    return true;
}
// Calcula a intersecao entre as retas a - b e c - d assumindo que uma unica
// intersecao existe
// Para intersecao de segmentos, cheque primeiro se os segmentos se intersectam e que
// nao sao paralelos
PT lineLine (PT a, PT b, PT c, PT d) {
    b = b - a; d = d - c; c = c - a;
    assert(cmp(cross(b, d)) != 0);
    return a + b * cross(c, d) / cross(b, d);
}
PT circleCenter (PT a, PT b, PT c) {
    b = (a + b) / 2; // bissector
    c = (a + c) / 2; // bissector
    return lineLine(b, b + rotateCW90(a - b), c, c + rotateCW90(a - c));
}
vector<PT> circle2PtsRad (PT p1, PT p2, double r) {
    vector<PT> ret;
    double d2 = dist2(p1, p2);
    double det = r * r / d2 - 0.25;
    if (det < 0.0) return ret;
    double h = sqrt(det);
    for (int i = 0; i < 2; i++) {
        double x = (p1.x + p2.x) * 0.5 + (p1.y - p2.y) * h;
        double y = (p1.y + p2.y) * 0.5 + (p2.x - p1.x) * h;
        ret.push_back(PT(x, y));
        swap(p1, p2);
    }
    return ret;
}
bool circleLineIntersection(PT a, PT b, PT c, double r) {
    return cmp(dist(c, projPtLine(a, b, c)), r) <= 0;
}
vector<PT> circleLine (PT a, PT b, PT c, double r) {
    vector<PT> ret;
    PT p = projPtLine(a, b, c), pl;
    double h = norm(c-p);
    if (cmp(h,r) == 0) {
        ret.push_back(p);
    } else if (cmp(h,r) < 0) {
        double k = sqrt(r*r - h*h);
        pl = p + (b-a)/(norm(b-a))*k;
        ret.push_back(pl);
        pl = p - (b-a)/(norm(b-a))*k;
        ret.push_back(pl);
    }
    return ret;
}
bool ptInsideTriangle(PT p, PT a, PT b, PT c) {
    if(cross(b-a, c-b) < 0) swap(a, b);
    if(ptInSegment(a,b,p)) return 1;
    if(ptInSegment(b,c,p)) return 1;
    if(ptInSegment(c,a,p)) return 1;
    bool x = cross(b-a, p-b) < 0;
    bool y = cross(c-b, p-c) < 0;
    bool z = cross(a-c, p-a) < 0;
    return x == y && y == z;
}
bool pointInConvexPolygon(const vector<PT> &p, PT q) {
    if (p.size() == 1) return p.front() == q;
    int l = 1, r = p.size()-1;
    while(abs(r-l) > 1) {
        int m = (r+l)/2;
        if(cross(p[m]-p[0], q-p[0]) < 0) r = m;
        else l = m;
    }
    return ptInsideTriangle(q, p[0], p[l], p[r]);
}
// Determina se o ponto esta num poligono possivelmente nao-convexo
// Retorna 1 para pontos estritamente dentro, 0 para pontos estritamente fora do
// poligono
// e 0 ou 1 para os pontos restantes
bool pointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for(int i = 0; i < p.size(); i++){
        int j = (i + 1) % p.size();
        if((p[i].y <= q.y && q.y < p[j].y || p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}
// Determina se o ponto esta na borda do poligono

```

5.2 Convex Hull

```

bool pointOnPolygon(const vector<PT> &p, PT q) {
    for(int i = 0; i < p.size(); i++)
        if(cmp(dist(projPtSeg(p[i], p[(i + 1) % p.size()], q), q)) == 0)
            return true;
    return false;
}

// area / semiperimeter
double rIncircle (PT a, PT b, PT c) {
    double ab = norm(a-b), bc = norm(b-c), ca = norm(c-a);
    return abs(cross(b-a, c-a) / (ab+bc+ca));
}

vector<PT> circleCircle (PT a, double r, PT b, double R) {
    vector<PT> ret;
    double d = norm(a-b);
    if (d > r + R || d + min(r, R) < max(r, R)) return ret;
    double x = (d*d - R*R + r*r) / (2*d); // x = r*cos(R opposite angle)
    double y = sqrt(r*r - x*x);
    PT v = (b - a)/d;
    ret.push_back(a + v*x + rotateCCW90(v)*y);
    if (cmp(y) > 0)
        ret.push_back(a + v*x - rotateCCW90(v)*y);
    return ret;
}

double circularSegArea (double r, double R, double d) {
    double ang = 2 * acos((d*d - R*R + r*r) / (2*d*r)); // cos(R opposite angle) = x/r
    double tri = sin(ang) * r * r;
    double sector = ang * r * r;
    return (sector - tri) / 2;
}

double computeSignedArea (const vector<PT> &p) {
    double area = 0;
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area/2.0;
}

double computeArea(const vector<PT> &p) { return abs(computeSignedArea(p)); }
PT computeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * computeSignedArea(p);
    for(int i = 0; i < p.size(); i++){
        int j = (i + 1) % p.size();
        c = c + (p[i] + p[j]) * (p[i].x * p[j].y - p[j].x * p[i].y);
    }
    return c / scale;
}

// Testa se o poligono listada em ordem CW ou CCW eh simples (nenhuma linha se
// intersecta)
bool isSimple(const vector<PT> &p) {
    for(int i = 0; i < p.size(); i++) {
        for(int k = i + 1; k < p.size(); k++) {
            int j = (i + 1) % p.size();
            int l = (k + 1) % p.size();
            if (i == l || j == k) continue;
            if (segInter(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

vector< pair<PT, PT> > getTangentSegs (PT c1, double r1, PT c2, double r2) {
    if (r1 < r2) swap(c1, c2), swap(r1, r2);
    vector<pair<PT, PT> > ans;
    double d = dist(c1, c2);
    if (cmp(d) <= 0) return ans;
    double dr = abs(r1 - r2), sr = r1 + r2;
    if (cmp(dr, d) >= 0) return ans;
    double u = acos(dr / d);
    PT dc1 = normalize(c2 - c1)*r1;
    PT dc2 = normalize(c2 - c1)*r2;
    ans.push_back(make_pair(c1 + rotateCCW(dc1, +u), c2 + rotateCCW(dc2, +u)));
    ans.push_back(make_pair(c1 + rotateCCW(dc1, -u), c2 + rotateCCW(dc2, -u)));
    if (cmp(sr, d) >= 0) return ans;
    double v = acos(sr / d);
    dc2 = normalize(c1 - c2)*r2;
    ans.push_back({c1 + rotateCCW(dc1, +v), c2 + rotateCCW(dc2, +v)});
    ans.push_back({c1 + rotateCCW(dc1, -v), c2 + rotateCCW(dc2, -v)});
    return ans;
}

```

```

vector<PT> convexHull(vector<PT> p, bool needs = 1) {
    if(needs) sort(p.begin(), p.end());
    p.erase(unique(p.begin(), p.end()), p.end());
    int n = p.size(), k = 0;
    if(n <= 1) return p;
    vector<PT> h(n + 2);
    for(int i = 0; i < n; i++) {
        while(k >= 2 && cross(h[k - 1] - h[k - 2], p[i] - h[k - 2]) <= 0) k--;
        h[k++] = p[i];
    }
    for(int i = n - 2, t = k + 1; i >= 0; i--) {
        while(k >= t && cross(h[k - 1] - h[k - 2], p[i] - h[k - 2]) <= 0) k--;
        h[k++] = p[i];
    }
    h.resize(k); // n+1 points where the first is equal to the last
    return h;
}

void sortByAngle (vector<PT>::iterator first, vector<PT>::iterator last, const PT o) {
    first = partition(first, last, [&o] (const PT &a) { return a == o; });
    auto pivot = partition(first, last, [&o] (const PT &a) {
        return !(a < o || a == o); // PT(a.y, a.x) < PT(o.y, o.x)
    });
    auto acmp = [&o] (const PT &a, const PT &b) { // C++11 only
        if (cmp(cross(a-o, b-o)) != 0) return cross(a-o, b-o) > 0;
        else return cmp(norm(a-o), norm(b-o)) < 0;
    };
    sort(first, pivot, acmp);
    sort(pivot, last, acmp);
}

vector<PT> graham (vector<PT> v) {
    sort(v.begin(), v.end());
    sortByAngle(v.begin(), v.end(), v[0]);
    vector<PT> u (v.size());
    int top = 0;
    for (int i = 0; i < v.size(); i++) {
        while (top > 1 && cmp(cross(u[top-1] - u[top-2], v[i] - u[top-2])) <= 0) top--;
        u[top++] = v[i];
    }
    u.resize(top);
    return u;
}

vector<PT> splitHull(const vector<PT> &hull) {
    vector<PT> ans(hull.size());
    for(int i = 0, j = (int) hull.size()-1, k = 0; k < (int) hull.size(); k++) {
        if(hull[i] < hull[j]) {
            ans[k] = hull[i++];
        } else {
            ans[k] = hull[j--];
        }
    }
    return ans;
}

vector<PT> ConvexHull(const vector<PT> &a, const vector<PT> &b) {
    auto A = splitHull(a);
    auto B = splitHull(b);
    vector<PT> C(A.size() + B.size());
    merge(A.begin(), A.end(), B.begin(), B.end(), C.begin());
    return ConvexHull(C, false);
}

int maximizeScalarProduct(const vector<PT> &hull, PT vec) {
    // this code assumes that there are no 3 colinear points
    int ans = 0;
    int n = hull.size();
    if(n < 20) {
        for(int i = 0; i < n; i++) {
            if(dot(hull[i], vec) > dot(hull[ans], vec)) {
                ans = i;
            }
        }
    } else {
        if(dot(hull[1], vec) > dot(hull[ans], vec)) {
            ans = 1;
        }
        for(int rep = 0; rep < 2; rep++) {
            int l = 2, r = n - 1;
            while(l != r) {
                int mid = (l + r + 1) / 2;
                bool flag = dot(hull[mid], vec) >= dot(hull[mid-1], vec);
            }
        }
    }
}

```

```

    if(rep == 0) { flag = flag && dot(hull[mid], vec) >= dot(hull[0], vec); }
    else { flag = flag || dot(hull[mid-1], vec) < dot(hull[0], vec); }
    if(flag) {
        l = mid;
    } else {
        r = mid - 1;
    }
}
if(dot(hull[ans], vec) < dot(hull[l], vec)) {
    ans = l;
}
}
return ans;
}

```

5.3 Cut Polygon

```

struct Segment {
    typedef long double T;
    PT p1, p2;
    T a, b, c;

    Segment() {}

    Segment(PT st, PT en) {
        p1 = st, p2 = en;
        a = -(st.y - en.y);
        b = st.x - en.x;
        c = a * en.x + b * en.y;
    }

    T plug(T x, T y) {
        // plug >= 0 is to the right
        return a * x + b * y - c;
    }

    T plug(PT p) {
        return plug(p.x, p.y);
    }

    bool inLine(PT p) { return cross((p - p1), (p2 - p1)) == 0; }
    bool inSegment(PT p) {
        return inLine(p) && dot((p1 - p2), (p - p2)) >= 0 && dot((p2 - p1), (p - p1)) >=
            0;
    }

    PT lineIntersection(Segment s) {
        long double A = a, B = b, C = c;
        long double D = s.a, E = s.b, F = s.c;
        long double x = (long double) C * E - (long double) B * F;
        long double y = (long double) A * F - (long double) C * D;
        long double tmp = (long double) A * E - (long double) B * D;
        x /= tmp;
        y /= tmp;
        return PT(x, y);
    }

    bool polygonIntersection(const vector<PT> &poly) {
        long double l = -1e18, r = 1e18;
        for(auto p : poly) {
            long double z = plug(p);
            l = max(l, z);
            r = min(r, z);
        }
        return l - r > eps;
    }
};

vector<PT> cutPolygon(vector<PT> poly, Segment seg) {
    int n = (int) poly.size();
    vector<PT> ans;
    for(int i = 0; i < n; i++) {
        double z = seg.plug(poly[i]);
        if(z > -eps) {
            ans.push_back(poly[i]);
        }
        double z2 = seg.plug(poly[(i + 1) % n]);
        if((z > eps && z2 < -eps) || (z < -eps && z2 > eps)) {
            ans.push_back(seg.lineIntersection(Segment(poly[i], poly[(i + 1) % n])));
        }
    }
    return ans;
}

```

5.4 Smallest Enclosing Circle

```

typedef pair<PT, double> circle;
bool inCircle(circle c, PT p) {
    return cmp(dist(c.first, p), c.second) <= 0;
}

PT circumcenter(PT p, PT q, PT r) {
    PT a = p - r, b = q - r;
    PT c = PT(dot(a, p + r) / 2, dot(b, q + r) / 2);
    return PT(cross(c, PT(a.y, b.y)), cross(PT(a.x, b.x), c)) / cross(a, b);
}

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
circle spanningCircle(vector<PT> &v) {
    int n = v.size();
    shuffle(v.begin(), v.end(), rng);
    circle C(PT(), -1);
    for(int i = 0; i < n; i++) if(!inCircle(C, v[i])) {
        C = circle(v[i], 0);
        for(int j = 0; j < i; j++) if(!inCircle(C, v[j])) {
            C = circle((v[i] + v[j]) / 2, dist(v[i], v[j]) / 2);
            for(int k = 0; k < j; k++) if(!inCircle(C, v[k])) {
                PT o = circumcenter(v[i], v[j], v[k]);
                C = circle(o, dist(o, v[k]));
            }
        }
    }
    return C;
}

```

5.5 Minkowski

```

bool comp(PT a, PT b) {
    int hp1 = (a.x < 0 || (a.x == 0 && a.y < 0));
    int hp2 = (b.x < 0 || (b.x == 0 && b.y < 0));
    if(hp1 != hp2) return hp1 < hp2;
    long long R = cross(a, b);
    if(R) return R > 0;
    return dot(a, a) < dot(b, b);
}

// This code assumes points are ordered in ccw and the first points in both vectors
// is the min lexicographically
vector<PT> minkowskiSum(const vector<PT> &a, const vector<PT> &b) {
    if(a.empty() || b.empty()) return vector<PT>(0);
    vector<PT> ret;
    int n1 = a.size(), n2 = b.size();
    if(min(n1, n2) < 2) {
        for(int i = 0; i < n1; i++) {
            for(int j = 0; j < n2; j++) {
                ret.push_back(a[i] + b[j]);
            }
        }
        return ret;
    }
    PT v1, v2, p = a[0] + b[0];
    ret.push_back(p);
    for(int i = 0, j = 0; i + j + 1 < n1 + n2; ) {
        v1 = a[(i + 1) % n1] - a[i];
        v2 = b[(j + 1) % n2] - b[j];
        if(j == n2 || (i < n1 && comp(v1, v2))) p = p + v1, i++;
        else p = p + v2, j++;
        while(ret.size() >= 2 && cmp(cross(p - ret.back(), p - ret[(int)ret.size() - 2])) == 0) {
            // removing colinear points
            // needs the scalar product stuff if the result is a line
            ret.pop_back();
        }
        ret.push_back(p);
    }
    return ret;
}

```

5.6 Half Plane Intersection

```

struct L {
    PT a, b;
    L() {}
    L(PT a, PT b) : a(a), b(b) {}
};
double angle (L la) { return atan2(-(la.a.y - la.b.y), la.b.x - la.a.x); }
bool comp (L la, L lb) {
    if (cmp(angle(la), angle(lb)) == 0) return cross((lb.b - lb.a), (la.b - lb.a)) >
        eps;
    return cmp(angle(la), angle(lb)) < 0;
}
PT computeLineIntersection (L la, L lb) {
    return computeLineIntersection(la.a, la.b, lb.a, lb.b);
}
bool check (L la, L lb, L lc) {
    PT p = computeLineIntersection(lb, lc);
    double det = cross((la.b - la.a), (p - la.a));
    return cmp(det) < 0;
}
vector<PT> hpi (vector<L> line) { // salvar (i, j) CCW, (j, i) CW
    sort(line.begin(), line.end(), comp);
    vector<L> pl(1, line[0]);
    for (int i = 0; i < (int)line.size(); ++i) if (cmp(angle(line[i]), angle(pl.back()
        )) != 0) pl.push_back(line[i]);
    deque<int> dq;
    dq.push_back(0);
    dq.push_back(1);
    for (int i = 2; i < (int)pl.size(); ++i) {
        while ((int)dq.size() > 1 && check(pl[i], pl[dq.back()], pl[dq.size() -
            2])) dq.pop_back();
        while ((int)dq.size() > 1 && check(pl[i], pl[dq[0]], pl[dq[1]])) dq.pop_front
            ();
        dq.push_back(i);
    }
    while ((int)dq.size() > 1 && check(pl[dq[0]], pl[dq.back()], pl[dq.size() -
        2])) dq.pop_back();
    while ((int)dq.size() > 1 && check(pl[dq.back()], pl[dq[0]], pl[dq[1]])) dq.
        pop_front();
    vector<PT> res;
    for (int i = 0; i < (int)dq.size(); ++i) {
        res.emplace_back(computeLineIntersection(pl[dq[i]], pl[dq[(i + 1) % dq.size()
            ]]));
    }
    return res;
}

```

5.7 Closest Pair

```

double closestPair(vector<PT> p) {
    int n = p.size(), k = 0;
    sort(p.begin(), p.end());
    double d = inf;
    set<PT> ptsInv;
    for (int i = 0; i < n; i++) {
        while (k < i && p[k].x < p[i].x - d) {
            ptsInv.erase(swapCoord(p[k++]));
        }
        for (auto it = ptsInv.lower_bound(PT(p[i].y - d, p[i].x - d));
            it != ptsInv.end() && it->x <= p[i].y + d; it++) {
            d = min(d, dist(p[i] - swapCoord(*it), PT(0, 0)));
        }
        ptsInv.insert(swapCoord(p[i]));
    }
    return d;
}

```

5.8 Voronoi

```

Segment getBisector(PT a, PT b) {
    Segment ans(a, b);
    swap(ans.a, ans.b);
    ans.b *= -1;
    ans.c = ans.a * (a.x + b.x) * 0.5 + ans.b * (a.y + b.y) * 0.5;
    return ans;
}

```

```

// BE CAREFUL!
// the first point may be any point
// O(N^3)
vector<PT> getCell(vector<PT> pts, int i) {
    vector<PT> ans;
    ans.emplace_back(0, 0);
    ans.emplace_back(1e6, 0);
    ans.emplace_back(1e6, 1e6);
    ans.emplace_back(0, 1e6);
    for (int j = 0; j < (int) pts.size(); j++) {
        if (j != i) {
            ans = cutPolygon(ans, getBisector(pts[i], pts[j]));
        }
    }
    return ans;
}

// O(N^2) expected time
vector<vector<PT>> getVoronoi(vector<PT> pts) {
    // assert(pts.size() > 0);
    int n = (int) pts.size();
    vector<int> p(n, 0);
    for (int i = 0; i < n; i++) {
        p[i] = i;
    }
    shuffle(p.begin(), p.end(), rng);
    vector<vector<PT>> ans(n);
    ans[0].emplace_back(0, 0);
    ans[0].emplace_back(w, 0);
    ans[0].emplace_back(w, h);
    ans[0].emplace_back(0, h);
    for (int i = 1; i < n; i++) {
        ans[i] = ans[0];
    }
    for (auto i : p) {
        for (auto j : p) {
            if (j == i) break;
            auto bi = getBisector(pts[j], pts[i]);
            if (!bi.polygonIntersection(ans[j])) continue;
            ans[j] = cutPolygon(ans[j], getBisector(pts[j], pts[i]));
            ans[i] = cutPolygon(ans[i], getBisector(pts[i], pts[j]));
        }
    }
    return ans;
}

```

6 String Algorithms

6.1 KMP

```

vector<int> getBorder(string str) {
    int n = str.size();
    vector<int> border(n, -1);
    for (int i = 1, j = -1; i < n; i++) {
        while (j >= 0 && str[i] != str[j + 1]) {
            j = border[j];
        }
        if (str[i] == str[j + 1]) {
            j++;
        }
        border[i] = j;
    }
    return border;
}

int matchPattern(const string &txt, const string &pat, const vector<int> &border) {
    int freq = 0;
    for (int i = 0, j = -1; i < txt.size(); i++) {
        while (j >= 0 && txt[i] != pat[j + 1]) {
            j = border[j];
        }
        if (pat[j + 1] == txt[i]) {
            j++;
        }
        if (j + 1 == (int) pat.size()) {
            //found occurrence
            freq++;
            j = border[j];
        }
    }
}

```

```

    }
}
return freq;
}

```

6.2 KMP Automaton

```

// trad converts a char to its index
int trad(char ch) { return (int) ch; }
// sigma should be greater than the greatest value returned by trad
vector<vector<int>> buildAutomaton(string p, int sigma=300) {
    vector<vector<int>> A(p.size() + 1, vector<int>(sigma));
    int brd = 0;
    for (int i = 0; i < sigma; i++) A[0][i] = 0;
    A[0][trad(p[0])] = 1;
    for (int i = 1; i <= p.size(); i++) {
        for (int ch = 0; ch < sigma; ch++) {
            A[i][ch] = A[brd][ch];
        }
        if (i < p.size()) {
            A[i][trad(p[i])] = i + 1;
            brd = A[brd][trad(p[i])];
        }
    }
    return A;
}

```

6.3 Aho-Corasick

```

const int ms = 1e6; // quantidade de caracteres
const int sigma = 26; // tamanho do alfabeto
int trie[ms][sigma], fail[ms], terminal[ms], qtd;
void init() {
    qtd = 1;
    memset(trie[0], -1, sizeof trie[0]);
}
void add(string &s) {
    int node = 0;
    for (char ch : s) {
        int pos = val(ch); // no caso de alfabeto a-z: val(ch) = ch - 'a'
        if (trie[node][pos] == -1) {
            memset(trie[qtd], -1, sizeof trie[qtd]);
            terminal[qtd] = 0;
            trie[node][pos] = qtd++;
        }
        node = trie[node][pos];
    }
    ++terminal[node]; // trocar pela info que quiser
}
void buildFailure() {
    memset(fail, 0, sizeof(int) * qtd);
    queue<int> Q;
    Q.push(0);
    while (Q.size()) {
        int node = Q.front();
        Q.pop();
        for (int pos = 0; pos < sigma; ++pos) {
            int &v = trie[node][pos];
            int f = node == 0 ? 0 : trie[fail[node]][pos];
            if (v == -1) {
                v = f;
            } else {
                fail[v] = f;
                Q.push(v);
            }
            // dar merge nas infos (por ex: terminal[v] += terminal[f])
        }
    }
}
void search(string &s) {
    int node = 0;
    for (char ch : s) {
        int pos = val(ch);
        node = trie[node][pos];
        // processar infos no no atual (por ex: ocorrencias += terminal[node])
    }
}

```

```

}

```

6.4 Algoritmo de Z

```

template <class T>
vector<int> ZFunc(const vector<T> &v) {
    vector<int> z(v.size(), 0);
    int n = (int) v.size(), a = 0, b = 0;
    if (!z.empty()) z[0] = n;
    for (int i = 1; i < n; i++) {
        int end = i; if (i < b) end = min(i + z[i - a], b);
        while(end < n && v[end] == v[end - i]) ++end;
        z[i] = end - i; if(end > b) a = i, b = end;
    }
    return z;
}

```

6.5 Suffix Array

```

vector<int> buildSa(const string& in) {
    int n = in.size(), c = 0;
    vector<int> temp(n), posBucket(n), bucket(n), bpos(n), out(n);
    for (int i = 0; i < n; i++) out[i] = i;
    sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });
    for (int i = 0; i < n; i++) {
        bucket[i] = c;
        if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
    }
    for (int h = 1; h < n && c < n; h <= 1) {
        for (int i = 0; i < n; i++) posBucket[out[i]] = bucket[i];
        for (int i = n - 1; i >= 0; i--) bpos[bucket[i]] = i;
        for (int i = 0; i < n; i++) {
            if (out[i] >= n - h) temp[bpos[bucket[i]]++] = out[i];
        }
        for (int i = 0; i < n; i++) {
            if (out[i] >= h) temp[bpos[posBucket[out[i] - h]]++] = out[i] - h;
        }
        c = 0;
        for (int i = 0; i + 1 < n; i++) {
            int a = (bucket[i] != bucket[i + 1]) || (temp[i] >= n - h)
                || (posBucket[temp[i + 1] + h] != posBucket[temp[i] + h]);
            bucket[i] = c;
            c += a;
        }
        bucket[n - 1] = c++;
        temp.swap(out);
    }
    return out;
}
vector<int> buildLcp(string s, vector<int> sa) {
    int n = (int) s.size();
    vector<int> pos(n), lcp(n, 0);
    for(int i = 0; i < n; i++) {
        pos[sa[i]] = i;
    }
    int k = 0;
    for(int i = 0; i < n; i++) {
        if (pos[i] + 1 == n) {
            k = 0;
            continue;
        }
        int j = sa[pos[i] + 1];
        while(i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
        lcp[pos[i]] = k;
        k = max(k - 1, 0);
    }
    return lcp;
}

```

6.6 Suffix Tree

```

#define fpos adla
const int inf = 1e9;
const int maxn = 1e4;
char s[maxn];
map<int, int> to[maxn];
int len[maxn], fpos[maxn], link[maxn];
int node, pos;
int sz = 1, n = 0;
int make_node(int _pos, int _len)
{
    fpos[sz] = _pos;
    len[sz] = _len;
    return sz++;
}
void go_edge()
{
    while(pos > len[to[node][s[n - pos]]])
    {
        node = to[node][s[n - pos]];
        pos -= len[node];
    }
}
void add_letter(int c)
{
    s[n++] = c;
    pos++;
    int last = 0;
    while(pos > 0)
    {
        go_edge();
        int edge = s[n - pos];
        int &v = to[node][edge];
        int t = s[fpos[v] + pos - 1];
        if(v == 0)
        {
            v = make_node(n - pos, inf);
            link[last] = node;
            last = 0;
        }
        else if(t == c)
        {
            link[last] = node;
            return;
        }
        else
        {
            int u = make_node(fpos[v], pos - 1);
            to[u][c] = make_node(n - 1, inf);
            to[u][t] = v;
            fpos[v] += pos - 1;
            len[v] -= pos - 1;
            v = u;
            link[last] = u;
            last = u;
        }
        if(node == 0)
            pos--;
        else
            node = link[node];
    }
}
//len[0] = inf

```

6.7 Suffix Automaton

```

int len[ms*2], link[ms*2], nxt[ms*2][sigma];
int sz, last;
void build(string &s) {
    len[0] = 0; link[0] = -1;
    sz = 1; last = 0;
    memset(nxt[0], -1, sizeof nxt[0]);
    for(char ch : s) {
        int c = ch-'a', cur = sz++;
        len[cur] = len[last]+1;
        memset(nxt[cur], -1, sizeof nxt[cur]);
        int p = last;
        while(p != -1 && nxt[p][c] == -1) {
            nxt[p][c] = cur; p = link[p];
        }
        if(p == -1) {
            link[cur] = 0;

```

```

    } else {
        int q = nxt[p][c];
        if(len[p] + 1 == len[q]) {
            link[cur] = q;
        } else {
            len[sz] = len[p]+1; link[sz] = link[q];
            memcpy(nxt[sz], nxt[q], sizeof nxt[q]);
            while (p != -1 && nxt[p][c] == q) {
                nxt[p][c] = sz; p = link[p];
            }
            link[q] = link[cur] = sz++;
        }
    }
    last = cur;
}
}

```

6.8 Manacher

```

void manacher(char str[], int L[]) {
    int n = strlen(str), c = 0, r = 0;
    for(int i = 0; i < n; i++) {
        if(i < r && 2*c >= i) L[i] = min(L[2*c-i], r-i);
        else L[i] = 0;
        while(i-L[i]-1 >= 0 && i+L[i]+1 < n &&
            str[i-L[i]-1] == str[i+L[i]+1]) L[i]++;
        if(i+L[i]>r) { c=i; r=i+L[i]; }
    }
}
int LPS(char T[]) {
    int n = 2*strlen(T) + 1;
    char tmp[n+1];
    for(int i = 0, k = 0; T[i]; i++) {
        tmp[k++] = '|'; tmp[k++] = T[i];
    }
    tmp[n-1] = '|'; tmp[n] = '\0';
    int L[n], ans = 1;
    manacher(tmp, L);
    for(int i=0; i<n; i++)
        ans = max(ans, L[i]);
    return ans;
}

```

6.9 Polish Notation

```

inline bool isOp(char c) {
    return c=='+' || c=='-' || c=='*' || c=='/' || c=='^';
}

inline bool isCarac(char c) {
    return (c>='a' && c<='z') || (c>='A' && c<='Z') || (c>='0' && c<='9');
}

int paren2polish(char* paren, char* polish) {
    map<char, int> prec;
    prec['('] = 0;
    prec['+'] = prec['-'] = 1;
    prec['*'] = prec['/'] = 2;
    prec['^'] = 3;
    int len = 0;
    stack<char> op;
    for (int i = 0; paren[i]; i++) {
        if (isOp(paren[i])) {
            while (!op.empty() && prec[op.top()] >= prec[paren[i]]) {
                polish[len++] = op.top(); op.pop();
            }
            op.push(paren[i]);
        }
        else if (paren[i]=='(') op.push('(');
        else if (paren[i]==')') {
            for (; op.top()!='('; op.pop())
                polish[len++] = op.top();
            op.pop();
        }
        else if (isCarac(paren[i]))
            polish[len++] = paren[i];
    }
}

```

```

    }
    for(; !op.empty(); op.pop())
        polish[len++] = op.top();
    polish[len] = 0;
    return len;
}

```

7 Miscellaneous

7.1 Ternary Search

```

// R
for(int i = 0; i < LOG; i++){
    long double m1 = (A * 2 + B) / 3.0;
    long double m2 = (A + 2 * B) / 3.0;

    if(f(m1) > f(m2))
        A = m1;
    else
        B = m2;
}
ans = f(A);

// Z
while(B - A > 4){
    int m1 = (A + B) / 2;
    int m2 = (A + B) / 2 + 1;
    if(f(m1) > f(m2))
        A = m1;
    else
        B = m2;
}
ans = inf;
for(int i = A; i <= B; i++) ans = min(ans , f(i));

```

7.2 Count Sort

```

int H[(1<<15)+1], to[mx], b[mx];
void sort(int m, int a[]) {
    memset(H, 0, sizeof H);
    for (int i = 1; i <= m; i++) H[a[i] % (1<<15)]++;
    for (int i = 1; i < 1<<15; i++) H[i] += H[i-1];
    for (int i = m; i; i--) to[i] = H[a[i] % (1 << 15)]--;
    for (int i = 1; i <= m; i++) b[to[i]] = a[i];
    memset(H, 0, sizeof H);
    for (int i = 1; i <= m; i++) H[b[i]>>15]++;
    for (int i = 1; i < 1<<15; i++) H[i] += H[i-1];
    for (int i = m; i; i--) to[i] = H[b[i]>>15]--;
    for (int i = 1; i <= m; i++) a[to[i]] = b[i];
}

```

7.3 Random Number Generator

```

// mt19937_64 se LL
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
// Random_Shuffle
shuffle(v.begin(), v.end(), rng);
// Random number in interval
int randomInt = uniform_int_distribution(0, i)(rng);
double randomDouble = uniform_real_distribution(0, 1)(rng);
// bernoulli_distribution, binomial_distribution, geometric_distribution
// normal_distribution, poisson_distribution, exponential_distribution

```

7.4 Safe Hash

```

namespace {
    struct safe_hash {
        static uint64_t splitmix64(uint64_t x) {
            // http://xorshift.di.unimi.it/splitmix64.c
            x += 0x9e3779b97f4a7c15;
            x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
            x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
            return x ^ (x >> 31);
        }

        size_t operator()(uint64_t x) const {
            static const uint64_t FIXED_RANDOM = std::chrono::steady_clock::now().
                time_since_epoch().count();
            return splitmix64(x + FIXED_RANDOM);
        }
    };
}

```

7.5 Unordered Map Tricks

```

// pair<int, int> hash function
struct HASH{
    size_t operator()(const pair<int,int>&x) const{
        return (size_t) x.first * 37U + (size_t) x.second;
    }
};

unordered_map<int,int>mp;
mp.reserve(1024);
mp.max_load_factor(0.25);

```

7.6 Submask Enumeration

```

for (int s=m; ; s=(s-1)&m) {
    ... you can use s ...
    if (s==0) break;
}

```

7.7 Sum over Subsets DP

```

// F[i] = Sum of all A[j] where j is a submask of i
for(int i = 0; i<(1<<N); ++i)
    F[i] = A[i];
for(int i = 0; i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
    if(mask & (1<<i))
        F[mask] += F[mask^(1<<i)];
}

```

7.8 Dates

```

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
    int x, n, i, j;

    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;

```



```

x -= 1461 * i / 4 - 31;
j = 80 * x / 2447;
d = x - 2447 * j / 80;
x = j / 11;
m = j + 2 - 12 * x;
y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
    return dayOfWeek[jd % 7];
}

```

8 Teoremas e formulas uteis

8.1 Grafos

Formula de Euler: $V - E + F = 2$ (para grafo planar)
 Handshaking: Numero par de vertices tem grau impar
 Kirchhoff's Theorem: Monta matriz onde $M_{i,i} = \text{Grau}[i]$ e $M_{i,j} = -1$ se houver aresta $i-j$
 ou 0 caso contrario, remove uma linha e uma coluna qualquer e o numero de
 spanning trees nesse grafo eh o det da matriz

Grafo contem caminho hamiltoniano se:
 Dirac's theorem: Se o grau de cada vertice for pelo menos $n/2$
 Ore's theorem: Se a soma dos graus que cada par nao-adjacente de vertices for pelo
 menos n

Trees:
 Tem Catalan(N) Binary trees de N vertices
 Tem Catalan(N-1) Arvores enraizadas com N vertices
 Caley Formula: $n^{(n-2)}$ arvores em N vertices com label
 Prufer code: Cada etapa voce remove a folha com menor label e o label do vizinho eh
 adicionado ao codigo ate ter 2 vertices

Flow:
 Max Edge-disjoint paths: Max flow com arestas com peso 1
 Max Node-disjoint paths: Faz a mesma coisa mas separa cada vertice em um com as
 arestas de chegadas e um com as arestas de saida e uma aresta de peso 1
 conectando o vertice com aresta de chegada com ele mesmo com arestas de saida
 Konig's Theorem: minimum node cover = maximum matching se o grafo for bipartido,
 complemento eh o maximum independent set
 Min Node disjoint path cover: formar grafo bipartido de vertices duplicados, onde
 aresta sai do vertice tipo A e chega em tipo B, entao o path cover eh $N -$
 matching
 Min General path cover: Mesma coisa mas colocando arestas de A pra B sempre que houver
 caminho de A pra B
 Dilworth's Theorem: Min General Path cover = Max Antichain (set de vertices tal que
 nao existe caminho no grafo entre vertices desse set)
 Hall's marriage: um grafo tem um matching completo do lado X se para cada subconjunto
 W de X,
 $|W| \leq |\text{vizinhos}W|$ onde $|W|$ eh quantos vertices tem em W

8.2 Math

Goldbach's: todo numero par $n > 2$ pode ser representado com $n = a + b$ onde a e b sao
 primos
 Twin prime: existem infinitos pares $p, p + 2$ onde ambos sao primos
 Legendre's: sempre tem um primo entre n^2 e $(n+1)^2$
 Lagrange's: todo numero inteiro pode ser inscrito como a soma de 4 quadrados
 Zeckendorff's: todo numero pode ser representado pela soma de dois numeros de
 fibonnacis diferentes e nao consecutivos
 Euclid's: toda tripla de pitagoras primitiva pode ser gerada com
 $(n^2 - m^2, 2nm, n^2 + m^2)$ onde n, m sao coprimos e um deles eh par
 Wilson's: n eh primo quando $(n-1)! \bmod n = n - 1$
 Mcnugget: Para dois coprimos x, y o maior inteiro que nao pode ser escrito como $ax +$
 by eh $(x-1)(y-1)/2$

Fermat: Se p eh primo entao $a^{(p-1)} \bmod p = 1$
 Se x e m tambem forem coprimos entao $x^k \bmod m = x^{(k \bmod (m-1))} \bmod m$
 Euler's theorem: $x^{(\phi(m))} \bmod m = 1$ onde $\phi(m)$ eh o totiente de euler

Chinese remainder theorem:
 Para equacoes no formato $x = a_1 \bmod m_1, \dots, x = a_n \bmod m_n$ onde todos os pares $m_1,$
 \dots, m_n sao coprimos
 Deixe $X_k = m_1 m_2 \dots m_n / m_k$ e $X_k^{-1} \bmod m_k$ = inverso de $X_k \bmod m_k$, entao
 $x = \text{somatorio com } k \text{ de } 1 \text{ ate } n \text{ de } a_k X_k (X_k m_k^{-1} \bmod m_k)$
 Para achar outra solucao so somar $m_1 m_2 \dots m_n$ a solucao existente

Catalan number: exemplo expressoes de parenteses bem formadas
 $C_0 = 1, C_n = \text{somatorio de } i=0 \rightarrow n-1 \text{ de } C_i C_{n-1-i}$
 outra forma: $C_n = (2n \text{ escolhe } n) / (n+1)$
 Bertrand's ballot theorem: p votos tipo A e q votos tipo B com $p > q$, prob de em todo
 ponto ter mais As do que Bs antes dele = $(p-q)/(p+q)$
 Se puder empatar entao prob = $(p+1-q)/(p+1)$, para achar quantidade de possibilidades
 nos dois casos basta multiplicar por $(p + q \text{ escolhe } q)$

Propriedades de Coeficientes Binomiais:
 Somatorio de $k = 0 \rightarrow m$ de $(-1)^k * (n \text{ escolhe } k) = (-1)^m * (n-1 \text{ escolhe } m)$
 $(N \text{ escolhe } K) = (N \text{ escolhe } N-K)$
 $(N \text{ escolhe } K) = N/K * (n-1 \text{ escolhe } k-1)$
 Somatorio de $k = 0 \rightarrow n$ de $(n \text{ escolhe } k) = 2^n$
 Somatorio de $m = 0 \rightarrow n$ de $(m \text{ escolhe } k) = (n+1 \text{ escolhe } k+1)$
 Somatorio de $k = 0 \rightarrow m$ de $(n+k \text{ escolhe } k) = (n+m+1 \text{ escolhe } m)$
 Somatorio de $k = 0 \rightarrow n$ de $(n \text{ escolhe } k)^2 = (2n \text{ escolhe } n)$
 Somatorio de $k = 0$ ou $1 \rightarrow n$ de $k * (n \text{ escolhe } k) = n * 2^{(n-1)}$
 Somatorio de $k = 0 \rightarrow n$ de $(n-k \text{ escolhe } k) = \text{Fib}(n+1)$

Hockey-stick: Somatorio de $i = r \rightarrow n$ de $(i \text{ escolhe } r) = (n+1 \text{ escolhe } r+1)$
 Vandermonde: $(m+n \text{ escolhe } r) = \text{somatorio de } k = 0 \rightarrow r \text{ de } (m \text{ escolhe } k) * (n \text{ escolhe } r - k)$

Burnside lemma: colares diferentes nao contando rotacoes quando $m = \text{cores}$ e $n =$
 comprimento
 $(m^n + \text{somatorio } i = 1 \rightarrow n-1 \text{ de } m^{\text{gcd}(i, n)}) / n$

Distribuicao uniforme $a, a+1, \dots, b$ Expected[X] = $(a+b)/2$
 Distribuicao binomial com n tentativas de probabilidade p , $X = \text{sucessos}$:
 $P(X = x) = p^x * (1-p)^{(n-x)} * (n \text{ escolhe } x)$ e $E[X] = p * n$
 Distribuicao geometrica onde continuamos ate ter sucesso, $X = \text{tentativas}$:
 $P(X = x) = (1-p)^{(x-1)} * p$ e $E[X] = 1/p$
 Linearity of expectation: Tendo duas variaveis X e Y e constantes a e b , o valor
 esperado de $aX + bY = a * E[X] + b * E[Y]$
 $V(X) = E[(X-u)^2]$
 $V(X) = E(X^2) - E(X)^2$

PG: $a_1 * (q^n - 1) / (q - 1)$

8.3 Geometry

Formula de Euler: $V - E + F = 2$
 Pick Theorem: Para achar pontos em coords inteiras num poligono Area = $i + b/2 - 1$
 onde i eh o numero de pontos dentro do poligono e b de pontos no perimetro do
 poligono
 Two ears theorem: Todo poligono simples com mais de 3 vertices tem pelo menos 2
 orelhas, vertices que podem ser removidos sem criar um crossing, remover orelhas
 repetidamente triangula o poligono
 Incentro triangulo: $(a(X_a, Y_a) + b(X_b, Y_b) + c(X_c, Y_c)) / (a+b+c)$ onde $a = \text{lado oposto}$
 ao vertice a , incentro eh onde cruzam as bissetrizes, eh o centro da
 circunferencia inscrita e eh equidistante aos lados

Delaunay Triangulation: Triangulacao onde nenhum ponto esta dentro de nenhum circulo
 circunscrito nos triangulos
 Eh uma triangulacao que maximiza o menor angulo e a MST euclidiana de um conjunto de
 pontos eh um subconjunto da triangulacao

Brahmagupta's formula: Area cyclic quadrilateral
 $s = (a+b+c+d)/2$
 $\text{area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
 $d = 0 \Rightarrow \text{area} = \sqrt{(s-a)(s-b)(s-c)*s}$