Amigos do Beto - ICPC Library

Contents

1 Data Structures

1.1 BIT 2D Comprimida

```cpp
template<class T = int>
struct Bit2D {
public:
  // send updated points
  Bit2D(vector<pair<T, T>> pts) {
    sort(pts.begin(), pts.end());
    for(auto a : pts) {
      if(ord.empty() || a.first != ord.back()) {
        ord.push_back(a.first);
      }
    }
    fw.resize(ord.size() + 1);
    coord.resize(fw.size());
    for(auto &a : pts) {
      swap(a.first, a.second);
    }
    sort(pts.begin(), pts.end());
    for(auto &a : pts) {
      swap(a.first, a.second);
      for(int on = upper_bound(ord.begin(), ord.end(), a.first) - ord.begin(); on < fw
          .size(); on += on & -on) {
        if(coord[on].empty() || coord[on].back() != a.second) {
          coord[on].push_back(a.second);
        }
      }
    }
    for(int i = 0; i < fw.size(); i++) {
      fw[i].assign(coord[i].size() + 1, 0);
    }
  }
  void upd(T x, T y, T v) {
    for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.begin(); xx < fw.size();
        xx += xx & -xx) {
      for(int yy = upper_bound(coord[xx].begin(), coord[xx].end(), y) - coord[xx].
          begin(); yy < fw[xx].size(); yy += yy & -yy) {
        fw[xx][yy] += v;
      }
```

```
    }
  }
  T qry(T x, T y) {
    T ans = 0;
    for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.begin(); xx > 0; xx -=
        xx & -xx) {
      for(int yy = upper_bound(coord[xx].begin(), coord[xx].end(), y) - coord[xx].
          begin(); yy > 0; yy -= yy & -yy) {
        ans += fw[xx][yy];
      }
    }
    return ans;
  }
  T qry(T x1, T y1, T x2, T y2) {
    return qry(x2, y2) - qry(x2, y1 - 1) - qry(x1 - 1, y2) + qry(x1 - 1, y1 - 1);
  }
  void upd(T x1, T y1, T x2, T y2, T v) {
    upd(x1, y1, v);
    upd(x1, y2 + 1, -v);
    upd(x2 + 1, y1, -v);
    upd(x2 + 1, y2 + 1, v);
  }
private:
  vector<T> ord;
  vector<vector<T>> fw, coord;
};
```

1.2 Segment Tree with Lazy Propagation

```
struct LazyContext {
  LazyContext() { }
  void reset() { }
  void operator += (LazyContext o) { }
};
struct Node {
  Node() { }
  Node() { }
  Node(Node l, Node r) { }
  bool canBreak(LazyContext lazy) { }
  bool canApply(LazyContext lazy) { }
  void apply(LazyContext &lazy) { }
};
template <class i_t, class e_t, class lazy_cont>
class SegmentTree {
public:
  void init(std::vector<e_t> base) {
    n = base.size();
    h = 0;
    while((1 << h) < n) h++;
    tree.resize(2 * n);
    dirty.assign(n, false);
    lazy.resize(n);
    for(int i = 0; i < n; i++) {
      tree[i + n] = i_t(base[i]);
    }
    for(int i = n - 1; i > 0; i--) {
      tree[i] = i_t(tree[i + i], tree[i + i + 1]);
      lazy[i].reset();
    }
  }
  i_t qry(int l, int r) {
    if(l >= r) return i_t();
    l += n, r += n;
    push(l);
    push(r - 1);
    i_t lp, rp;
    for(; l < r; l /= 2, r /= 2) {
      if(l & 1) lp = i_t(lp, tree[l++]);
      if(r & 1) rp = i_t(tree[--r], rp);
    }
    return i_t(lp, rp);
  }
  void upd(int l, int r, lazy_cont lc) {
    if(l >= r) return;
    l += n, r += n;
    push(l);
    push(r - 1);
    int l0 = l, r0 = r;
    for(; l < r; l /= 2, r /= 2) {
      if(l & 1) downUpd(l++, lc);
      if(r & 1) downUpd(--r, lc);
    }
    build(l0);
    build(r0 - 1);
  }
  void upd(int pos, e_t v) {
    pos += n;
    push(pos);
    tree[pos] = i_t(v);
    build(pos);
  }
private:
  int n, h;
  std::vector<bool> dirty;
  std::vector<i_t> tree;
  std::vector<lazy_cont> lazy;
  void apply(int p, lazy_cont lc) {
    tree[p].apply(lc);
    if(p < n) {
      dirty[p] = true;
      lazy[p] += lc;
    }
  }
  void pushSingle(int p) {
    if(dirty[p]) {
      downUpd(p + p, lazy[p]);
      downUpd(p + p + 1, lazy[p]);
      lazy[p].reset();
      dirty[p] = false;
    }
  }
  void push(int p) {
    for(int s = h; s > 0; s--) {
      pushSingle(p >> s);
    }
  }
  void downUpd(int p, lazy_cont lc) {
    if(tree[p].canBreak(lc)) {
      return;
    } else if(tree[p].canApply(lc)) {
      apply(p, lc);
    } else {
      pushSingle(p);
      downUpd(p + p, lc);
      downUpd(p + p + 1, lc);
      tree[p] = i_t(tree[p + p], tree[p + p + 1]);
    }
  }
  void build(int p) {
    for(p /= 2; p > 0; p /= 2) {
      tree[p] = i_t(tree[p + p], tree[p + p + 1]);
      if(dirty[p]) {
        tree[p].apply(lazy[p]);
      }
    }
  }
};
```

1.3 Treap

```
mt19937 rng ((int) chrono::steady_clock::now().time_since_epoch().count());
typedef int Value;
typedef struct item * pitem;
struct item {
  item () {}
  item (Value v) { // add key if not implicit
    value = v;
    prio = uniform_int_distribution<int>() (rng);
    cnt = 1;
    rev = 0;
    l = r = 0;
  }
  pitem l, r;
  Value value;
  int prio, cnt;
  bool rev;
};
int cnt (pitem it) {
  return it ? it->cnt : 0;
}
void fix (pitem it) {
```

```
  if (it)
    it->cnt = cnt(it->l) + cnt(it->r) + 1;
}
void pushLazy (pitem it) {
  if (it && it->rev) {
    it->rev = false;
    swap(it->l, it->r);
    if (it->l)  it->l->rev ^= true;
    if (it->r)  it->r->rev ^= true;
  }
}
void insert (pitem & t, pitem it) {
  if (!t)
    t = it;
  else if (it->prio > t->prio)
    split (t, it->key, it->l, it->r),  t = it;
  else
    insert (t->key <= it->key ? t->r : t->l, it);
}
void merge (pitem & t, pitem l, pitem r) {
  pushLazy (l); pushLazy (r);
  if (!l || !r) t = l ? l : r;
  else if (l->prio > r->prio)
    merge (l->r, l->r, r),  t = l;
  else
    merge (r->l, l, r->l),  t = r;
  fix (t);
}
void erase (pitem & t, int key) {
  if (t->key == key) {
    pitem th = t;
    merge (t, t->l, t->r);
    delete th;
  }
  else
    erase (key < t->key ? t->l : t->r, key);
}
void split (pitem t, pitem & l, pitem & r, int key) {
  if (!t) return void( l = r = 0 );
  pushLazy (t);
  int cur_key = cnt(t->l); // t->key if not implicit
  if (key <= cur_key)
    split (t->l, l, t->l, key),  r = t;
  else
    split (t->r, t->r, r, key - (1 + cnt(t->l))),  l = t;
  fix (t);
}
void reverse (pitem t, int l, int r) {
  pitem t1, t2, t3;
  split (t, t1, t2, l);
  split (t2, t2, t3, r-l+1);
  t2->rev ^= true;
  merge (t, t1, t2);
  merge (t, t, t3);
}
void unite (pitem & t, pitem l, pitem r) {
  if (!l || !r) return void ( t = l ? l : r );
  if (l->prio < r->prio)  swap (l, r);
  pitem lt, rt;
  split (r, lt, rt, l->key);
  unite (l->l, l->l, lt);
  unite (l-> r, l->r, rt);
  t = l;
}
pitem kth_element(pitem t, int k) {
        if(!t) return NULL;
        if(t->l) {
                if(t->l->size >= k) return kth_element(t->l, k);
                else k -= t->l->cnt;
        }
        return (k == 1) ? t : kth_element(t->r, k - 1);
}
int countLeft(pitem t, int key) {
        if(!t) {
                return 0;
        } else if(t->key < key) {
                return 1 + (t->l ? t->l->size : 0) + countLeft(t->r, key);
        } else {
                return countLeft(t->l, key);
        }
}
```

1.4 KD-Tree

```
int d;
long long getValue(const PT &a) {return (d & 1) == 0 ? a.x : a.y; }
bool comp(const PT &a, const PT &b) {
  if((d & 1) == 0) { return a.x < b.x; }
  else { return a.y < b.y; }
}
long long sqrDist(PT a, PT b) { return (a - b) * (a - b); }
class KD_Tree {
public:
  struct Node {
    PT point;
    Node *left, *right;
  };
  void init(std::vector<PT> pts) {
    if(pts.size() == 0) {
      return;
    }
    int n = 0;
    tree.resize(2 * pts.size());
    build(pts.begin(), pts.end(), n);
  }
  long long nearestNeighbor(PT point) {
    long long ans = (long long) 1e18;
    nearestNeighbor(&tree[0], point, 0, ans);
    return ans;
  }
private:
  std::vector<Node> tree;
  Node* build(std::vector<PT>::iterator l, std::vector<PT>::iterator r, int &n, int h
      = 0) {
    int id = n++;
    if(r - l == 1) {
      tree[id].left = tree[id].right = NULL;
      tree[id].point = *l;
    } else if(r - l > 1) {
      std::vector<PT>::iterator mid = l + ((r - l) / 2);
      d = h;
      std::nth_element(l, mid - 1, r, comp);
      tree[id].point = *(mid - 1);
      // BE CAREFUL!
      // DO EVERYTHING BEFORE BUILDING THE LOWER PART!
      tree[id].left = build(l, mid, n, h^1);
      tree[id].right = build(mid, r, n, h^1);
    }
    return &tree[id];
  }

  void nearestNeighbor(Node* node, PT point, int h, long long &ans) {
    if(!node) {
      return;
    }
    if(point != node->point) {
      // THIS WAS FOR A PROBLEM
      // THAT YOU DON'T CONSIDER THE DISTANCE TO ITSELF!
      ans = std::min(ans, sqrDist(point, node->point));
    }
    d = h;
    long long delta = getValue(point) - getValue(node->point);
    if(delta <= 0) {
      nearestNeighbor(node->left, point, h^1, ans);
      if(ans > delta * delta) {
        nearestNeighbor(node->right, point, h^1, ans);
      }
    } else {
      nearestNeighbor(node->right, point, h^1, ans);
      if(ans > delta * delta) {
        nearestNeighbor(node->left, point, h^1, ans);
      }
    }
  }
};
```

1.5 Sparse Table

```
vector<vector<int>> table;
```

```
vector<int> lg2;
void build(int n, vector<int> v) {
  lg2.resize(n + 1);
  lg2[1] = 0;
  for (int i = 2; i <= n; i++) {
    lg2[i] = lg2[i >> 1] + 1;
  }
  table.resize(lg2[n] + 1);
  for (int i = 0; i < lg2[n] + 1; i++) {
    table[i].resize(n + 1);
  }
  for (int i = 0; i < n; i++) {
    table[0][i] = v[i];
  }
  for (int i = 0; i < lg2[n]; i++) {
    for (int j = 0; j < n; j++) {
      if (j + (1 << i) >= n) break;
      table[i + 1][j] = min(table[i][j], table[i][j + (1 << i)]);
    }
  }
}
int get(int l, int r) {
  int k = lg2[r - l + 1];
  return min(table[k][l], table[k][r - (1 << k) + 1]);
}
```

1.6 Max Queue

```
template <class T, class C = less<T>>
struct MaxQueue {
  MaxQueue() { clear(); }
  void clear() {
    id = 0;
    q.clear();
  }
  void push(T x) {
    pair<int, T> nxt(1, x);
    while(q.size() > id && cmp(q.back().second, x)) {
      nxt.first += q.back().first;
      q.pop_back();
    }
    q.push_back(nxt);
  }
  T qry() { return q[id].second;}
  void pop() {
    q[id].first--;
    if(q[id].first == 0) { id++; }
  }
private:
  vector<std::pair<int, T>> q;
  int id;
  C cmp;
};
```

1.7 Policy Based Structures

```
#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
ordered_set X;
X.insert(1); X.find_by_order(0);
X.order_of_key(-5); end(X), begin(X);
```

1.8 Color Updates Structure

```
struct range {
  int l, r;
  int v;
  range(int l = 0, int r = 0, int v = 0) : l(l), r(r), v(v) {}
  bool operator < (const range &a) const {
    return l < a.l;
  }
};
set<range> ranges;
vector<range> update(int l, int r, int v) { // [l, r)
  vector<range> ans;
  if(l >= r) return ans;
  auto it = ranges.lower_bound(l);
  if(it != ranges.begin()) {
    it--;
    if(it->r > l) {
      auto cur = *it;
      ranges.erase(it);
      ranges.insert(range(cur.l, l, cur.v));
      ranges.insert(range(l, cur.r, cur.v));
    }
  }
  it = ranges.lower_bound(r);
  if(it != ranges.begin()) {
    it--;
    if(it->r > r) {
      auto cur = *it;
      ranges.erase(it);
      ranges.insert(range(cur.l, r, cur.v));
      ranges.insert(range(r, cur.r, cur.v));
    }
  }
  for(it = ranges.lower_bound(l); it != ranges.end() && it->l < r; it++) {
    ans.push_back(*it);
  }
  ranges.erase(ranges.lower_bound(l), ranges.lower_bound(r));
  ranges.insert(range(l, r, v));
  return ans;
}
int query(int v) { // Substituir -1 por flag para quando nao houver resposta
  auto it = ranges.upper_bound(v);
  if(it == ranges.begin()) {
    return -1;
  }
  it--;
  return it->r >= v ? it->v : -1;
}
```

2 Graph Algorithms

2.1 Simple Disjoint Set

```
struct dsu {
  vector<int> hist, par, sz;
  vector<ii> changes;
  int n;
  dsu (int n) : n(n) {
    hist.assign(n, 1e9);
    par.resize(n);
    iota(par.begin(), par.end(), 0);
    sz.assign(n, 1);
  }

  int root (int x, int t) {
    if(hist[x] > t) return x;
    return root(par[x], t);
  }

  void join (int a, int b, int t) {
    a = root(a, t);
    b = root(b, t);
    if (a == b) { changes.emplace_back(-1, -1); return; }
    if (sz[a] > sz[b]) swap(a, b);
    par[a] = b;
    sz[b] += sz[a];
    hist[a] = t;
    changes.emplace_back(a, b);
    n--;
  }

  bool same (int a, int b, int t) {
```

```
    return root(a, t) == root(b, t);
  }

  void undo () {
    int a, b;
    tie(a, b) = changes.back();
    changes.pop_back();
    if (a == -1) return;
    sz[b] -= sz[a];
    par[a] = a;
    hist[a] = 1e9;
    n++;
  }

  int when (int a, int b) {
    while (1) {
      if (hist[a] > hist[b]) swap(a, b);
      if (par[a] == b) return hist[a];
      if (hist[a] == 1e9) return 1e9;
      a = par[a];
    }
  }
};
```

2.2 Blossom

```
#define MAXN 110
#define MAXM MAXN*MAXN
int n, m;
int mate[MAXN], first[MAXN], label[MAXN];
int adj[MAXN][MAXN], nadj[MAXN], from[MAXM], to[MAXM];
queue<int> q;
#define OUTER(x) (label[x] >= 0)
void L(int x, int y, int nxy) {
  int join, v, r = first[x], s = first[y];
  if (r == s) { return; }
  nxy += n + 1;
  label[r] = label[s] = -nxy;
  while (1) {
    if (s != 0) { swap(r, s); }
    r = first[label[mate[r]]];
    if (label[r] != -nxy) { label[r] = -nxy; }
    else {
      join = r;
      break;
    }
  }
  v = first[x];
  while (v != join) {
    if (!OUTER(v)) { q.push(v); }
    label[v] = nxy;
    first[v] = join;
    v = first[label[mate[v]]];
  }
  v = first[y];
  while (v != join) {
    if (!OUTER(v)) { q.push(v); }
    label[v] = nxy;
    first[v] = join;
    v = first[label[mate[v]]];
  }
  for (int i = 0; i <= n; i++) {
    if (OUTER(i) && OUTER(first[i])) { first[i] = join; }
  }
}

void R(int v, int w) {
  int t = mate[v];
  mate[v] = w;
  if (mate[t] != v) { return; }
  if (label[v] >= 1 && label[v] <= n) {
    mate[t] = label[v];
    R(label[v], t);
    return;
  }
  int x = from[label[v] - n - 1], y = to[label[v] - n - 1];
  R(x, y);
  R(y, x);
}
```

```
int E() {
  memset(mate, 0, sizeof(mate));
  int r = 0;
  bool e7;
  for (int u = 1; u <= n; u++) {
    memset(label, -1, sizeof(label));
    while (!q.empty()) { q.pop(); }
    if (mate[u]) { continue; }
    label[u] = first[u] = 0;
    q.push(u);
    e7 = false;
    while (!q.empty() && !e7) {
      int x = q.front();
      q.pop();
      for (int i = 0; i < nadj[x]; i++) {
        int y = from[adj[x][i]];
        if (y == x) { y = to[adj[x][i]]; }
        if (!mate[y] && y != u) {
          mate[y] = x;
          R(x, y);
          r++;
          e7 = true;
          break;
        } else if (OUTER(y)) { L(x, y, adj[x][i]); }
        else {
          int v = mate[y];
          if (!OUTER(v)) {
            label[v] = x;
            first[v] = y;
            q.push(v);
          }
        }
      }
    }
    label[0] = -1;
  }
  return r;
}
/*Exemplo simples de uso*/
memset(nadj, 0, sizeof nadj);
for (int i = 0; i < m; ++i) {  // arestas
  scanf("%d%d", &a, &b);
  a++, b++;  // nao utilizar o vertice 0
  adj[a][nadj[a]++] = i;
  adj[b][nadj[b]++] = i;
  from[i] = a;
  to[i] = b;
}
printf("O emparelhamento tem tamanho %d\n", E());
for (int i = 1; i <= n; i++) {
  if (mate[i] > i) { printf("%d com %d\n", i - 1, mate[i] - 1); }
}
```

2.3 Boruvka

```
struct edge {
  int u, v;
  int w;
  int id;
  edge () {};
  edge (int u, int v, int w = 0, int id = 0) : u(u), v(v), w(w), id(id) {};
  bool operator < (edge &other) const { return w < other.w; };
};
vector<edge> boruvka (vector<edge> &edges, int n) {
  vector<edge> mst;
  vector<edge> best(n);
  initDSU(n);
  bool f = 1;
  while (f) {
    f = 0;
    for (int i = 0; i < n; i++) best[i] = edge(i, i, inf);
    for (auto e : edges) {
      int pu = root(e.u), pv = root(e.v);
      if (pu == pv) continue;
      if (e < best[pu]) best[pu] = e;
      if (e < best[pv]) best[pv] = e;
    }
    for (int i = 0; i < n; i++) {
      edge e = best[root(i)];
      if (e.w != inf) {
```

```
          join(e.u, e.v);
          mst.push_back(e);
          f = 1;
        }
      }
    }
    return mst;
  }
```

2.4 Dinic Max Flow

```
const int ms = 1e3; // vertices
const int me = 1e5; // arestas
int adj[ms], to[me], ant[me], wt[me], z, n;
int copy_adj[ms], fila[ms], level[ms];
void clear() { // Lembrar de chamar no main
  memset(adj, -1, sizeof adj);
  z = 0;
}
void add(int u, int v, int k) {
  to[z] = v;
  ant[z] = adj[u];
  wt[z] = k;
  adj[u] = z++;
  swap(u, v);
  to[z] = v;
  ant[z] = adj[u];
  wt[z] = 0; // Lembrar de colocar = 0
  adj[u] = z++;
}
int bfs(int source, int sink) {
  memset(level, -1, sizeof level);
  level[source] = 0;
  int front = 0, size = 0, v;
  fila[size++] = source;
  while(front < size) {
    v = fila[front++];
    for(int i = adj[v]; i != -1; i = ant[i]) {
      if(wt[i] && level[to[i]] == -1) {
        level[to[i]] = level[v] + 1;
        fila[size++] = to[i];
      }
    }
  }
  return level[sink] != -1;
}
int dfs(int v, int sink, int flow) {
  if(v == sink) return flow;
  int f;
  for(int &i = copy_adj[v]; i != -1; i = ant[i]) {
    if(wt[i] && level[to[i]] == level[v] + 1 &&
      (f = dfs(to[i], sink, min(flow, wt[i])))) {
      wt[i] -= f;
      wt[i ^ 1] += f;
      return f;
    }
  }
  return 0;
}
int maxflow(int source, int sink) {
  int ret = 0, flow;
  while(bfs(source, sink)) {
    memcpy(copy_adj, adj, sizeof adj);
    while((flow = dfs(source, sink, 1 << 30))) {
      ret += flow;
    }
  }
  return ret;
}
```

2.5 Min Cost Max Flow

```
template <class T = int>
class MCMF {
public:
  struct Edge {
    Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
    int to;
    T cap, cost;
  };
  MCMF(int size) {
    n = size;
    edges.resize(n);
    pot.assign(n, 0);
    dist.resize(n);
    visit.assign(n, false);
  }
  pair<T, T> mcmf(int src, int sink) {
    pair<T, T> ans(0, 0);
    if(!SPFA(src, sink)) return ans;
    fixPot();
    // can use dijkstra to speed up depending on the graph
    while(SPFA(src, sink)) {
      auto flow = augment(src, sink);
      ans.first += flow.first;
      ans.second += flow.first * flow.second;
      fixPot();
    }
    return ans;
  }
  void addEdge(int from, int to, T cap, T cost) {
    edges[from].push_back(list.size());
    list.push_back(Edge(to, cap, cost));
    edges[to].push_back(list.size());
    list.push_back(Edge(from, 0, -cost));
  }
private:
  int n;
  vector<vector<int>> edges;
  vector<Edge> list;
  vector<int> from;
  vector<T> dist, pot;
  vector<bool> visit;
  pair<T, T> augment(int src, int sink) {
    pair<T, T> flow = {list[from[sink]].cap, 0};
    for(int v = sink; v != src; v = list[from[v]^1].to) {
      flow.first = min(flow.first, list[from[v]].cap);
      flow.second += list[from[v]].cost;
    }
    for(int v = sink; v != src; v = list[from[v]^1].to) {
      list[from[v]].cap -= flow.first;
      list[from[v]^1].cap += flow.first;
    }
    return flow;
  }
  queue<int> q;
  bool SPFA(int src, int sink) {
    T INF = numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    q.push(src);
    dist[src] = 0;
    while(!q.empty()) {
      int on = q.front();
      q.pop();
      visit[on] = false;
      for(auto e : edges[on]) {
        auto ed = list[e];
        if(ed.cap == 0) continue;
        T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
        if(toDist < dist[ed.to]) {
          dist[ed.to] = toDist;
          from[ed.to] = e;
          if(!visit[ed.to]) {
            visit[ed.to] = true;
            q.push(ed.to);
          }
        }
      }
    }
    return dist[sink] < INF;
  }
  void fixPot() {
    T INF = numeric_limits<T>::max();
    for(int i = 0; i < n; i++) {
      if(dist[i] < INF) pot[i] += dist[i];
    }
  }
```

```
};
```

2.6 Euler Path and Circuit

```
int pathV[me], szV, del[me], pathE, szE;
int adj[ms], to[me], ant[me], wt[me], z, n;
// Funcao de add e clear no dinic
void eulerPath(int u) {
  for(int i = adj[u]; ~i; i = ant[u]) if(!del[i]) {
    del[i] = del[i^1] = 1;
    eulerPath(to[i]);
    pathE[szE++] = i;
  }
  pathV[szV++] = u;
}
```

2.7 Articulation Points/Bridges/Biconnected Components

```
int adj[ms], to[me], ant[me], z;
int num[ms], low[ms], timer;
bool art[ms], bridge[me], f[me];
int bc[ms], nbc;
stack<int> st, stk;
vector<vector<int>> comps;

void clear() { // Lembrar de chamar no main
  memset(adj, -1, sizeof adj);
  z = 0;
}

void add(int u, int v) {
  to[z] = v;
  ant[z] = adj[u];
  adj[u] = z++;
}

void generateBc (int v) {
  while (!st.empty()) {
    int u = st.top();
    st.pop();
    bc[u] = nbc;
    if (v == u) break;
  }
  ++nbc;
}

void dfs (int v, int p) {
  st.push(v), stk.push(v);
  low[v] = num[v] = ++timer;
  for (int i = adj[v]; i != -1; i = ant[i]) {
    if (f[i] || f[i^1]) continue;
    f[i] = 1;
    int u = to[i];
    if (num[u] == -1) {
      dfs(u, v);
      low[v] = min(low[v], low[u]);
      if (low[u] > num[v]) bridge[i] = bridge[i^1] = 1;
      if (low[u] >= num[v]) {
        art[v] = (num[v] > 1 || num[u] > 2);
        comps.push_back({v});
        while (comps.back().back() != u)
          comps.back().push_back(stk.top()), stk.pop();
      }
    } else {
      low[v] = min(low[v], num[u]);
    }
  }
  if (low[v] == num[v]) generateBc(v);
}

void biCon (int n) {
  nbc = 0, timer = 0;
  memset(num, -1, sizeof num);
  memset(bc, -1, sizeof bc);
  memset(bridge, 0, sizeof bridge);
  memset(art, 0, sizeof art);
  memset(f, 0, sizeof f);
  for (int i = 0; i < n; i++) {
    if (num[i] == -1) {
      timer = 0;
      dfs(i, 0);
    }
  }
}

vector<int> g[ms];
int id[ms];
void buildBlockCut (int n) {
  int z = 0;
  for (int u = 0; u < n; ++u) {
    if (art[u]) id[u] = z++;
  }
  for (auto &comp : comps) {
    int node = z++;
    for (int u : comp) {
      if (!art[u]) id[u] = node;
      else {
        g[node].push_back(id[u]);
        g[id[u]].push_back(node);
      }
    }
  }
}
```

2.8 SCC - Strongly Connected Components / 2SAT

```
const int ms = 212345;
vector<int> g[ms];
int idx[ms], low[ms], z, comp[ms], ncomp;
stack<int> st;
int dfs(int u) {
  if(~idx[u]) return idx[u] ? idx[u] : z;
  low[u] = idx[u] = z++;
  st.push(u);
  for(int v : g[u]) {
    low[u] = min(low[u], dfs(v));
  }
  if(low[u] == idx[u]) {
    while(st.top() != u) {
      int v = st.top();
      idx[v] = 0;
      low[v] = low[u];
      comp[v] = ncomp;
      st.pop();
    }
    idx[st.top()] = 0;
    st.pop();
    comp[u] = ncomp++;
  }
  return low[u];
}
bool solveSat(int n) {
  memset(idx, -1, sizeof idx);
  z = 1; ncomp = 0;
  for(int i = 0; i < 2*n; i++) dfs(i);
  for(int i = 0; i < 2*n; i++) if(comp[i] == comp[i^1]) return false;
  return true;
}
int trad(int v) { return v < 0 ?(~v)*2^1 : v * 2; }
void add(int a, int b) { g[trad(a)].push_back(trad(b)); }
void addOr(int a, int b) { add(~a, b); add(~b, a); }
void addImp(int a, int b) { addOr(~a, b); }
void addEqual(int a, int b) { addOr(a, ~b); addOr(~a, b); }
void addDiff(int a, int b) { addEqual(a, ~b); }
// value[i] = comp[trad(i)] < comp[trad(~id)];
```

2.9 LCA - Lowest Common Ancestor

```
int par[ms][mlg+1], lvl[ms];
void dfs(int v, int p, int l = 0) { // chamar como dfs(root, root)
```

```
    lvl[v] = l;
    par[v][0] = p;
    for(int k = 1; k <= mlg; k++) {
      par[v][k] = par[par[v][k-1]][k-1];
    }
    for(int u : g[v]) {
      if(u != p) dfs(u, v, l + 1);
    }
  }
  int lca(int a, int b) {
    if(lvl[b] > lvl[a]) swap(a, b);
    for(int i = mlg; i >= 0; i--) {
      if(lvl[a] - (1 << i) >= lvl[b]) a = par[a][i];
    }
    if(a == b) return a;
    for(int i = mlg; i >= 0; i--) {
      if(par[a][i] != par[b][i]) a = par[a][i], b = par[b][i];
    }
    return par[a][0];
  }
```

2.10 LCA O(1)

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;

  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= (int)size(V); pw *= 2, ++k) {
      jmp.emplace_back(size(V) - pw * 2 + 1);
      for (int j = 0; j < (int)size(jmp[k]); ++j)
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};

struct LCA {
  int T = 0;
  vector<int> time, path, ret;
  RMQ<int> rmq;

  LCA(vector<vector<int>>& C) : time(size(C)), rmq((dfs(C,0,-1), ret)) {}
  void dfs(vector<vector<int>>& C, int v, int par) {
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.push_back(v), ret.push_back(time[v]);
      dfs(C, y, v);
    }
  }
  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];
  }
};
```

2.11 Heavy Light Decomposition

```
class HLD {
public:
  void init(int n) { /* resize everything */ }
  void addEdge(int u, int v) {
    edges[u].push_back(v);
    edges[v].push_back(u);
  }
  void setRoot(int r) {
    t = 0;
    p[r] = r;
    h[r] = 0;
    prep(r, r);
    nxt[r] = r;
    hld(r);
  }
  int getLCA(int u, int v) {
    while(!inSubtree(nxt[u], v)) u = p[nxt[u]];
    while(!inSubtree(nxt[v], u)) v = p[nxt[v]];
    return in[u] < in[v] ? u : v;
  }
  // is v in the subtree of u?
  bool inSubtree(int u, int v) {
    return in[u] <= in[v] && in[v] < out[u];
  }
  // returns ranges [l, r) that the path has
  vector<pair<int, int>> getPath(int u, int anc) {
    vector<std::pair<int, int>> ans;
    //assert(inSubtree(anc, u));
    while(nxt[u] != nxt[anc]) {
      ans.emplace_back(in[nxt[u]], in[u] + 1);
      u = p[nxt[u]];
    }
    // this includes the ancestor! care
    ans.emplace_back(in[anc], in[u] + 1);
    return ans;
  }
private:
  vector<int> in, out, p, rin, sz, nxt, h;
  vector<vector<int>> edges;
  int t;
  void prep(int on, int par) {
    sz[on] = 1;
    p[on] = par;
    for(int i = 0; i < (int) edges[on].size(); i++) {
      int &u = edges[on][i];
      if(u == par) {
        swap(u, edges[on].back());
        edges[on].pop_back();
        i--;
      } else {
        h[u] = 1 + h[on];
        prep(u, on);
        sz[on] += sz[u];
        if(sz[u] > sz[edges[on][0]]) {
          swap(edges[on][0], u);
        }
      }
    }
  }
  void hld(int on) {
    in[on] = t++;
    rin[in[on]] = on;
    for(auto u : edges[on]) {
      nxt[u] = (u == edges[on][0] ? nxt[on] : u);
      hld(u);
    }
    out[on] = t;
  }
};
```

2.12 Centroid Decomposition

```
template<typename T>
struct CentroidDecomposition {
  vector<int> sz, h, dad;
  vector<vector<pair<int, T>>> adj;
  vector<vector<T>> dis;
  vector<bool> removed;
  CentroidDecomposition (int n) {
    sz.resize(n);
    h.resize(n);
    dis.resize(n, vector<T>(30, 0));
    adj.resize(n);
    removed.resize(n, 0);
    dad.resize(n);
  }
  void add (int a, int b, T w = 1) {
    adj[a].push_back({b, w});
    adj[b].push_back({a, w});
  }
  void dfsSize (int v, int par){
    sz[v] = 1;
```

```
    for (auto u : adj[v]){
      if (u.x == par || removed[u.x]) continue;
      dfsSize(u.x, v);
      sz[v] += sz[u.x];
    }
  }
  int getCentroid (int v, int par, int tam){
    for (auto u : adj[v]) {
      if (u.x == par || removed[u.x]) continue;
      if ((sz[u.x]<<1) > tam) return getCentroid(u.x, v, tam);
    }
    return v;
  }
  void setDis (int v, int par, int nv){
    for (auto u : adj[v]) {
      if (u.x == par || removed[u.x]) continue;
      dis[u.x][nv] = dis[v][nv]+u.y;
      setDis(u.x, v, nv);
    }
  }
  void decompose (int v, int par = -1, int nv = 0){
    dfsSize(v, par);
    int c = getCentroid(v, par, sz[v]);
    dad[c] = par;
    removed[c] = 1;
    h[c] = nv;
    setDis(c, par, nv);
    for (auto u : adj[c]){
      if (!removed[u.x]){
        decompose(u.x, c, nv + 1);
      }
    }
  }
  int operator [] (const int idx) const {
    return dad[idx];
  }
  T dist (int u, int v) {
    if (h[u] < h[v]) swap(u, v);
    return dis[u][h[v]];
  }
};
```

2.13 Sack

```
void dfs(int v, int par = -1, bool keep = 0) {
    int big = -1;
    for (int u : adj[v]) {
        if (u == par) continue;
        if (big == -1 || sz[u] > sz[big]) {
            big = u;
        }
    }
    for (int u : adj[v]) {
        if (u == par || u == big) {
            continue;
        }
        dfs(u, v, 0);
    }
    if (big != -1) {
        dfs(big, v, 1);
    }
    for (int u : adj[v]) {
        if (u == par || u == big) {
            continue;
        }
        put(u, v);
    }
    if (!keep) {

    }
}
```

2.14 Hungarian Algorithm - Maximum Cost Matching

```
int u[ms], v[ms], p[ms], way[ms], minv[ms];
bool used[ms];
pair<vector<int>, int> solve(const vector<vector<int>> &matrix) {
  int n = matrix.size();
  if(n == 0) return {vector<int>(), 0};
  int m = matrix[0].size();
  assert(n <= m);
  memset(u, 0, (n+1)*sizeof(int));
  memset(v, 0, (m+1)*sizeof(int));
  memset(p, 0, (m+1)*sizeof(int));
  for(int i = 1; i <= n; i++) {
    memset(minv, 0x3f, (m+1)*sizeof(int));
    memset(way, 0, (m+1)*sizeof(int));
    for(int j = 0; j <= m; j++) used[j] = 0;
    p[0] = i;
    int k0 = 0;
    do {
      used[k0] = 1;
      int i0 = p[k0], delta = inf, k1;
      for(int j = 1; j <= m; j++) {
        if(!used[j]) {
          int cur = matrix[i0-1][j-1] - u[i0] - v[j];
          if (cur < minv[j]) {
            minv[j] = cur;
            way[j] = k0;
          }
          if(minv[j] < delta) {
            delta = minv[j];
            k1 = j;
          }
        }
      }
      for(int j = 0; j <= m; j++) {
        if(used[j]) {
          u[p[j]] += delta;
          v[j] -= delta;
        } else {
          minv[j] -= delta;
        }
      }
      k0 = k1;
    } while(p[k0]);
    do {
      int k1 = way[k0];
      p[k0] = p[k1];
      k0 = k1;
    } while(k0);
  }
  vector<int> ans(n, -1);
  for(int j = 1; j <= m; j++) {
    if(!p[j]) continue;
    ans[p[j] - 1] = j - 1;
  }
  return {ans, -v[0]};
}
```

2.15 Burunduk

```
struct edge {
  int a, b, l, r;
};
typedef vector <edge> List;
int cnt[N + 1], ans[N], u[N], color[N], deg[N];
vi g[N];
void add (int a, int b) {
  g[a].pb(b), g[b].pb(a);
}
void dfs (int v, int value) {
  u[v] = 1, color[v] = value;
  forn(i, sz(g[v]))
    if (!u[g[v][i]])
      dfs(g[v][i], value);
}
int compress (List &v1, int vn, int &add_vn) {
  int vn1 = 0;
  forn (i, vn) u[i] = 0;
  forn (i, vn) {
    if (!u[i]) deg[vn1] = 0, dfs(i, vn1++);
  }
  forn (i, sz(v1)) {
```

```
    v1[i].a = color[v1[i].a];
    v1[i].b = color[v1[i].b];
    if (v1[i].a != v1[i].b)
      deg[v1[i].a]++, deg[v1[i].b]++;
  }
  vn = vn1, vn1 = 0;
  forn (i, vn) {
    u[i] = vn1, vn1 += (deg[i] > 0), add_vn += !deg[i];
  }
  forn (i, sz(v1)) {
    v1[i].a = u[v1[i].a];
    v1[i].b = u[v1[i].b];
  }
  return vn1;
}
void go (int l, int r, const List &v, int vn, int add_vn) {
  if (cnt[l] == cnt[r]) return;
  if (!sz(v)){
    while (l < r)
      ans[l++] = vn + add_vn;
    return;
  }
  List v1;
  forn (i, vn) {
    g[i].clear();
  }
  forn (i, sz(v)) {
    if (v[i].a != v[i].b){
      if (v[i].l <= l && v[i].r >= r)
        add(v[i].a, v[i].b);
      else if (l < v[i].r && r > v[i].l)
        v1.pb(v[i]);
    }
  }
  int vn1 = compress(v1, vn, add_vn);
  int m = (l + r) / 2;
  go(l, m, v1, vn1, add_vn);
  go(m, r, v1, vn1, add_vn);
}
```

2.16 Minimum Arborescence

```
// uncommented O(V^2) arborescence
struct Edges {
  //set<pair<long long, int>> cost; O(Elog^2)
  long long cost[ms];
  // possible optimization, use vector of size n
  // instead of ms
  long long sum = 0;
  Edges() {
    memset(cost, 0x3f, sizeof cost);
  }
  void addEdge(int u, long long c) {
    // cost.insert({c - sum, u}); O(Elog^2)
    cost[u] = min(cost[u], c - sum);
  }
  pair<long long, int> getMin() {
    //return *cost.begin(); O(E*log^2)
    pair<long long, int> ans(cost[0], 0);
    // in this loop can change ms to n to make it faster for many cases
    for(int i = 1; i < ms; i++) {
      if(cost[i] < ans.first) {
        ans = pair<long long, int>(cost[i], i);
      }
    }
    return ans;
  }
  void unite(Edges &e) {
    /*
    O(E*log^2E)
    if(e.cost.size() > cost.size()) {
      cost.swap(e.cost);
      swap(sum, e.sum);
    }
    for(auto i : e.cost) {
      addEdge(i.second, i.first + e.sum);
    }
    e.cost.clear();
    */
    // O(V^2)
    // can change ms to n
    for(int i = 0; i < ms; i++) {
      cost[i] = min(cost[i], e.cost[i] + e.sum - sum);
    }
  }
};
typedef vector<vector<pair<long long, int>>> Graph;
Edges ed[ms];
int par[ms];
long long best[ms];
int col[ms];
int getPar(int x) { return par[x] < 0 ? x : par[x] = getPar(par[x]); }
void makeUnion(int a, int b) {
  a = getPar(a);
  b = getPar(b);
  if(a == b) return;
  ed[a].unite(ed[b]);
  par[b] = a;
}
long long arborescence(Graph edges) {
  // root is 0
  // edges has transposed adjacency list (cost, from)
  // edge from i to j cost c is
  // edge[j].emplace_back(c, i)
  int n = (int) edges.size();
  long long ans = 0;
  for(int i = 0; i < n; i++) {
    ed[i] = Edges();
    par[i] = -1;
    for(auto j : edges[i]) {
      ed[i].addEdge(j.second, j.first);
    }
    col[i] = 0;
  }
  // to change the root you can simply change this next line to
  // col[root] = 2;
  col[0] = 2;
  for(int i = 0; i < n; i++) {
    if(col[getPar(i)] == 2) {
      continue;
    }
    int on = getPar(i);
    vector<int> st;
    while(col[on] != 2) {
      assert(getPar(on) == on);
      if(col[on] == 1) {
        int v = on;
        vector<int> cycle;
        //cout << "found cycle\n";
        while(st.back() != v) {
          //cout << st.back() << endl;
          cycle.push_back(st.back());
          st.pop_back();
        }
        for(auto u : cycle) { // compress cycle
          makeUnion(v, u);
        }
        v = getPar(v);
        col[v] = 0;
        on = v;
      } else {
        // still no cycle
        // while best is in compressed cycle, remove
        // THIS IS TO MAKE O(E*log^2) ALGORITHM!!
        // while(!ed[on].cost.empty() && getPar(on) == getPar(ed[on].getMin().second)) {
        //   ed[on].cost.erase(ed[on].cost.begin());
        // }
        // O(V^2)
        for(int x = 0; x < n; x++) {
          if(on == getPar(x)) {
            ed[on].cost[x] = 0x3f3f3f3f3f3f3f3fLL;
          }
        }
        // best edge
        auto e = ed[on].getMin();
        // O(E*log^2) assert(!ed[on].cost.empty()) if every vertex appears in the arborescence
        // O(V^2)
        assert(e.first < 0x3f3f3f3f3f3f3f3fLL);
        int v = getPar(e.second);
        //cout << "found not cycle to " << v << " of cost " << e.first + ed[on].sum << '\n';
```

```
        assert(v != on);
        best[on] = e.first + ed[on].sum;
        ans += best[on];
        // compress edges
        ed[on].sum = -(e.first);
        st.push_back(on);
        col[on] = 1;
        on = v;
      }
    }
    // make everything 2
    for(auto u : st) {
      assert(getPar(u) == u);
      col[u] = 2;
    }
  }
  return ans;
}
```

2.17 Dominator Tree

```
struct dominator_tree {
  vector<basic_string<int>> g, rg, bucket;
  vector<int> arr, par, rev, sdom, dom, dsu, label;
  int n, t;
  dominator_tree(int n) : g(n), rg(n), bucket(n), arr(n, -1),
    par(n), rev(n), sdom(n), dom(n), dsu(n), label(n), n(n), t(0) {}
  void add_edge(int u, int v) { g[u] += v; }
  void dfs(int u) {
    arr[u] = t;
    rev[t] = u;
    label[t] = sdom[t] = dsu[t] = t;
    t++;
    for (int w : g[u]) {
      if (arr[w] == -1) {
        dfs(w);
        par[arr[w]] = arr[u];
      }
      rg[arr[w]] += arr[u];
    }
  }
  int find(int u, int x=0) {
    if (u == dsu[u])
      return x ? -1 : u;
    int v = find(dsu[u], x+1);
    if (v < 0)
      return u;
    if (sdom[label[dsu[u]]] < sdom[label[u]])
      label[u] = label[dsu[u]];
    dsu[u] = v;
    return x ? v : label[u];
  }
  vector<int> run(int root) {
    dfs(root);
    iota(dom.begin(), dom.end(), 0);
    for (int i=t-1; i>=0; i--) {
      for (int w : rg[i])
        sdom[i] = min(sdom[i], sdom[find(w)]);
      if (i)
        bucket[sdom[i]] += i;
      for (int w : bucket[i]) {
        int v = find(w);
        if (sdom[v] == sdom[w])
          dom[w] = sdom[w];
        else
          dom[w] = v;
      }
      if (i > 1)
        dsu[i] = par[i];
    }
    for (int i=1; i<t; i++) {
      if (dom[i] != sdom[i])
        dom[i] = dom[dom[i]];
    }
    vector<int> outside_dom(n);
    iota(begin(outside_dom), end(outside_dom), 0);
    for (int i=0; i<n; i++)
      outside_dom[rev[i]] = rev[dom[i]];
    return outside_dom;
  }
};
```

3 Dynamic Programming

3.1 Line Container

```
bool Q;
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const {
    return Q ? p < o.p : k < o.k;
  }
};
struct LineContainer : multiset<Line> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
    return l.k * x + l.m;
  }
};
```

3.2 Li Chao Tree

```
// by luucasv
typedef long long T;
const T INF = 1e18, EPS = 1;
const int BUFFER_SIZE = 1e4;
struct Line {
  T m, b;

  Line(T m = 0, T b = INF): m(m), b(b){}
  T apply(T x) { return x * m + b; }
};
struct Node {
  Node *left, *right;
  Line line;
  Node(): left(NULL), right(NULL) {}
};
struct LiChaoTree {
  Node *root, buffer[BUFFER_SIZE];
  T min_value, max_value;
  int buffer_pointer;
  LiChaoTree(T min_value, T max_value): min_value(min_value), max_value(max_value + 1)
      { clear(); }
  void clear() { buffer_pointer = 0; root = newNode(); }
  void insert_line(T m, T b) { update(root, min_value, max_value, Line(m, b)); }
  T eval(T x) { return query(root, min_value, max_value, x); }
  void update(Node *cur, T l, T r, Line line) {
    T m = l + (r - l) / 2;
    bool left = line.apply(l) < cur->line.apply(l);
    bool mid = line.apply(m) < cur->line.apply(m);
    bool right = line.apply(r) < cur->line.apply(r);
    if (mid) {
      swap(cur->line, line);
    }
    if (r - l <= EPS) return;
    if (left == right) return;
    if (mid != left) {
      if (cur->left == NULL) cur->left = newNode();
```

```
      update(cur->left, l, m, line);
    } else {
      if (cur->right == NULL) cur->right = newNode();
      update(cur->right, m, r, line);
    }
  }
  T query(Node *cur, T l, T r, T x) {
    if (cur == NULL) return INF;
    if (r - l <= EPS) {
      return cur->line.apply(x);
    }
    T m = l + (r - l) / 2;
    T ans;
    if (x < m) {
      ans = query(cur->left, l, m, x);
    } else {
      ans = query(cur->right, m, r, x);
    }
    return min(ans, cur->line.apply(x));
  }
  Node* newNode() {
      buffer[buffer_pointer] = Node();
      return &buffer[buffer_pointer++];
  }
};
```

3.3 Divide and Conquer Optimization

```
int n, k;
ll dpold[ms], dp[ms], c[ms][ms]; // c(i, j) pode ser funcao
void compute(int l, int r, int optl, int optr) {
    if(l>r) return;
    int mid = (l+r)/2;
    pair<ll, int> best = {inf, -1}; // long long inf
    for(int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {dpold[k-1] + c[k][mid], k});
    }
    dp[mid] = best.first;
    int opt = best.second;
    compute(l, mid-1, optl, opt);
    compute(mid+1, r, opt, optr);
}
ll solve() {
    dp[0] = 0;
    for(int i = 1; i <= n; i++) dp[i] = inf; // initialize row 0 of the dp
    for(int i = 1; i <= k; i++) {
        swap(dpold, dp);
        compute(0, n, 0, n); // solve row i of the dp
    }
    return dp[n]; // return dp[k][n]
}
```

3.4 Knuth Optimization

```
int n, m, mid[ms][ms];
ll dp[ms][ms];
void knuth() {
  for(int i = n; i >= 0; i--) { // limites entre 0 e n
    dp[i][i+1] = 0; mid[i][i+1] = i; // caso base
    for(int j = i+2; j <= n; j++) {
      dp[i][j] = inf; // long long inf
      for(int k = mid[i][j-1]; k <= mid[i+1][j]; k++) {
        if(dp[i][j] > dp[i][k] + dp[k][j]) {
          dp[i][j] = dp[i][k] + dp[k][j];
          mid[i][j] = k;
        }
      }
      dp[i][j] += c(i, j); // custo associado ao intervalo
    }
  }
}
```

4 Math

4.1 Chinese Remainder Theorem

```
long long modinverse(long long a, long long b, long long s0 = 1, long long s1 = 0){
  if(!b) return s0;
  else return modinverse(b, a % b, s1, s0 - s1 * (a / b));
}

long long gcd(long long a, long long b){
  if(!b) return a;
  else return gcd(b, a % b);
}

ll mul(ll a, ll b, ll m) {
  ll q = (long double) a * (long double) b / (long double) m;
  ll r = a * b - q * m;
  return (r + 5 * m) % m;
}

long long safemod(long long a, long long m){
  return (a % m + m) % m;
}

struct equation{
  equation(long long a, long long m){mod = m, ans = a, valid = true;}
  equation(){valid = false;}
  equation(equation a, equation b) {
    if(!a.valid || !b.valid) {
      valid = false;
      return;
    }
    long long g = gcd(a.mod, b.mod);
    if((a.ans - b.ans) % g != 0) {
      valid = false;
      return;
    }
    valid = true;
    mod = a.mod * (b.mod / g);
    ans = a.ans +
    mul(
      mul(a.mod, modinverse(a.mod, b.mod), mod),
      (b.ans - a.ans) / g
      , mod);
    ans = safemod(ans, mod);
  }
  long long mod, ans;
  bool valid;

  void print()
  {
    if(!valid)
      std::cout << "equation is not valid\n";
    else
      std::cout << "equation is " << ans << " mod " << mod << '\n';
  }
};
```

4.2 Diophantine Equations

```
int gcd_ext(int a, int b, int& x, int &y) {
  if (b == 0) {
    x = 1, y = 0;
    return a;
  }
  int nx, ny;
  int gc = gcd_ext(b, a % b, nx, ny);
  x = ny;
  y = nx - (a / b) * ny;
  return gc;
}

vector<int> diophantine(int D, vector<int> l) {
  int n = l.size();
  vector<int> gc(n), ans(n);
  gc[n - 1] = l[n - 1];
  for (int i = n - 2; i >= 0; i--) {
    int x, y;
    gc[i] = gcd_ext(l[i], gc[i + 1], x, y);
```

```
  }
  if (D % gc[0] != 0) {
    return vector<int>();
  }
  for (int i = 0; i < n; i++) {
    if (i == n - 1) {
      ans[i] = D / l[i];
      D -= l[i] * ans[i];
      continue;
    }
    int x, y;
    gcd_ext(l[i] / gc[i], gc[i + 1] / gc[i], x, y);
    ans[i] = (long long int) D / gc[i] * x % (gc[i + 1] / gc[i]);
    if (D < 0 && ans[i] > 0) {
      ans[i] -= (gc[i + 1] / gc[i]);
    }
    if (D > 0 && ans[i] < 0) {
      ans[i] += (gc[i + 1] / gc[i]);
    }
    D -= l[i] * ans[i];
  }
  return ans;
}
```

4.3 Discrete Logarithm

```
ll discreteLog (ll a, ll b, ll m) {
  a %= m; b %= m;
  ll n = (ll) sqrt (m + .0) + 1, an = 1;
  for (ll i = 0; i < n; i++) {
    an = (an * a) % m;
  }
  map<ll, ll> vals;
  for (ll i = 1, cur = an; i <= n; i++) {
    if (!vals.count(cur)) vals[cur] = i;
    cur = (cur * an) % m;
  }
  ll ans = 1e18; //inf
  for (ll i = 0, cur = b; i <= n; i++) {
    if (vals.count(cur)) {
      ans = min(ans, vals[cur] * n - i);
    }
    cur = (cur * a) % m;
  }
  return ans;
}
```

4.4 Discrete Root

```
//x^k = a % mod
ll discreteRoot(ll k, ll a, ll mod) {
  ll g = primitiveRoot(mod);
  ll y = discreteLog(fexp(g, k, mod), a, mod);
  if (y == -1) {
    return y;
  }
  return fexp(g, y, mod);
}
```

4.5 Division Trick

```
for(int l = 1, r; l <= n; l = r + 1) {
  r = n / (n / l);
  // n / i has the same value for l <= i <= r
}
```

4.6 Modular Sum

```
//calcula (sum(0 <= i <= n) P(i)) % mod,
//onde P(i) eh uma PA modular (com outro modulo)
namespace sum_pa_mod{
  ll calc(ll a, ll b, ll n, ll mod){
    assert(a&&b);
    if(a >= b){
      ll ret = ((n*(n+1)/2)%mod)*(a/b);
      if(a%b) ret = (ret + calc(a%b,b,n,mod))%mod;
      else ret = (ret+n+1)%mod;
      return ret;
    }
    return ((n+1)*(((n*a)/b+1)%mod) - calc(b,a,(n*a)/b,mod) + mod + n/b + 1)%mod;
  }
//P(i) = a*i mod m
  ll solve(ll a, ll n, ll m, ll mod){
    a = (a%m + m)%m;
    if(!a) return 0;
    ll ret = (n*(n+1)/2)%mod;
    ret = (ret*a)%mod;
    ll g = __gcd(a,m);
    ret -= m*(calc(a/g,m/g,n,mod)-n-1);
    return (ret%mod + mod)%mod;
  }
//P(i) = a + r*i mod m
  ll solve(ll a, ll r, ll n, ll m, ll mod){
    a = (a%m + m)%m;
    r = (r%m + m)%m;
    if(!r) return (a*(n+1))%mod;
    if(!a) return solve(r, n, m, mod);
    ll g, x, y;
    g = gcdExtended(r, m, x, y);
    x = (x%m + m)%m;
    ll d = a - (a/g)*g;
    a -= d;
    x = (x*(a/g))%m;
    return (solve(r, n+x, m, mod) - solve(r, x-1, m, mod) + mod + d*(n+1))%mod;
  }
};
```

4.7 Primitive Root

```
//is n primitive root of p ?
bool test(long long x, long long p) {
        long long m = p - 1;
        for(int i = 2; i * i <= m; ++i) if(!(m % i)) {
                if(fexp(x, i, p) == 1) return false;
                if(fexp(x, m / i, p) == 1) return false;
        }
        return true;
}
//find the smallest primitive root for p
int search(int p) {
        for(int i = 2; i < p; i++) if(test(i, p)) return i;
        return -1;
}
```

4.8 Linear Sieve

```
//check long long
vector <int> prime;
bool is_composite[MAXN];
int cnt[MAXN];
long long primePow[MAXN];
long long func[MAXN];

long long getFunction(int i, int p) {
  return cnt[i] + 1;
}

void sieve (int n) {
  fill(is_composite, is_composite + n, false);
  func[1] = 1;
  for (int i = 2; i < n; ++i) {
    if (!is_composite[i]) {
      prime.push_back (i);
      func[i] = 1; // base case
      cnt[i] = 1; primePow[i] = i;
    }
```

```cpp
      for (int j = 0; j < prime.size () && i * prime[j] < n; ++j) {
        is_composite[i * prime[j]] = true;
        if (i % prime[j] == 0) {
          func[i * prime[j]] = func[i / primePow[i]] * getFunction(i, prime[j]); // f(ip
              ) = f(i / primePow[i]) * f(primePow[i] * prime[j])
          cnt[i * prime[j]] = cnt[i] + 1;
          primePow[i * prime[j]] = primePow[i] * prime[j];
          break;
        } else {
          func[i * prime[j]] = func[i] * func[prime[j]]; // f(ip) = f(i) * f(p)
          cnt[i * prime[j]] = 1;
          primePow[i * prime[j]] = prime[j];
        }
      }
    }
  }
```

4.9 Extended Euclides

```cpp
// euclides estendido: acha u e v da equacao:
// u * x + v * y = gcd(x, y);
// u eh inverso modular de x no modulo y
// v eh inverso modular de y no modulo x

pair<ll, ll> euclides(ll a, ll b) {
  ll u = 0, oldu = 1, v = 1, oldv = 0;
  while(b) {
    ll q = a / b;
    oldv = oldv - v * q;
    oldu = oldu - u * q;
    a = a - b * q;
    swap(a, b);
    swap(u, oldu);
    swap(v, oldv);
  }
  return make_pair(oldu, oldv);
}
```

4.10 Matrix

```cpp
const ll mod = 1e9+7;
const int m = 2; // size of matrix

struct Matrix {
  ll mat[m][m];
  Matrix operator * (const Matrix &p) {
    Matrix ans;
    for(int i = 0; i < m; i++)
      for(int j = 0; j < m; j++)
        for(int k = ans.mat[i][j] = 0; k < m; k++)
          ans.mat[i][j] = (ans.mat[i][j] + mat[i][k] * p.mat[k][j]) % mod;
    return ans;
  }
};
```

4.11 FFT - Fast Fourier Transform

```cpp
typedef double ld;
const ld PI = acos(-1);
struct Complex {
  ld real, imag;
  Complex conj() { return Complex(real, -imag); }
  Complex(ld a = 0, ld b = 0) : real(a), imag(b) {}
  Complex operator + (const Complex &o) const { return Complex(real + o.real, imag + o
      .imag); }
  Complex operator - (const Complex &o) const { return Complex(real - o.real, imag - o
      .imag); }
  Complex operator * (const Complex &o) const { return Complex(real * o.real - imag *
      o.imag, real * o.imag + imag * o.real); }
  Complex operator / (ld o) const { return Complex(real / o, imag / o); }
  void operator *= (Complex o) { *this = *this * o; }
  void operator /= (ld o) { real /= o, imag /= o; }
};
typedef std::vector<Complex> CVector;
const int ms = 1 << 22;
int bits[ms];
Complex root[ms];
void initFFT() {
  root[1] = Complex(1);
  for(int len = 2; len < ms; len += len) {
    Complex z(cos(PI / len), sin(PI / len));
    for(int i = len / 2; i < len; i++) {
      root[2 * i] = root[i];
      root[2 * i + 1] = root[i] * z;
    }
  }
}
void pre(int n) {
  int LOG = 0;
  while(1 << (LOG + 1) < n) {
    LOG++;
  }
  for(int i = 1; i < n; i++) {
    bits[i] = (bits[i >> 1] >> 1) | ((i & 1) << LOG);
  }
}
CVector fft(CVector a, bool inv = false) {
  int n = a.size();
  pre(n);
  if(inv) {
    std::reverse(a.begin() + 1, a.end());
  }
  for(int i = 0; i < n; i++) {
    int to = bits[i];
    if(to > i) {
      std::swap(a[to], a[i]);
    }
  }
  for(int len = 1; len < n; len *= 2) {
    for(int i = 0; i < n; i += 2 * len) {
      for(int j = 0; j < len; j++) {
        Complex u = a[i + j], v = a[i + j + len] * root[len + j];
        a[i + j] = u + v;
        a[i + j + len] = u - v;
      }
    }
  }
  if(inv) {
    for(int i = 0; i < n; i++)
      a[i] /= n;
  }
  return a;
}
void fft2in1(CVector &a, CVector &b) {
  int n = (int) a.size();
  for(int i = 0; i < n; i++) {
    a[i] = Complex(a[i].real, b[i].real);
  }
  auto c = fft(a);
  for(int i = 0; i < n; i++) {
    a[i] = (c[i] + c[(n-i) % n].conj()) * Complex(0.5, 0);
    b[i] = (c[i] - c[(n-i) % n].conj()) * Complex(0, -0.5);
  }
}
void ifft2in1(CVector &a, CVector &b) {
  int n = (int) a.size();
  for(int i = 0; i < n; i++) a[i] = a[i] + b[i] * Complex(0, 1);
  a = fft(a, true);
  for(int i = 0; i < n; i++) {
    b[i] = Complex(a[i].imag, 0);
    a[i] = Complex(a[i].real, 0);
  }
}
std::vector<long long> mod_mul(const std::vector<long long> &a, const std::vector<long
    long> &b, long long cut = 1 << 15) {
  int n = (int) a.size();
  CVector C[4];
  for(int i = 0; i < 4; i++) C[i].resize(n);
  for(int i = 0; i < n; i++) {
    C[0][i] = a[i] % cut;
    C[1][i] = a[i] / cut;
    C[2][i] = b[i] % cut;
    C[3][i] = b[i] / cut;
  }
```

```cpp
  fft2in1(C[0], C[1]);
  fft2in1(C[2], C[3]);
  for(int i = 0; i < n; i++) {
    // 00, 01, 10, 11
    Complex cur[4];
    for(int j = 0; j < 4; j++) cur[j] = C[j/2+2][i] * C[j % 2][i];
    for(int j = 0; j < 4; j++) C[j][i] = cur[j];
  }
  ifft2in1(C[0], C[1]);
  ifft2in1(C[2], C[3]);
  std::vector<long long> ans(n, 0);
  for(int i = 0; i < n; i++) {
    // if there are negative values, care with rounding
    ans[i] += (long long) (C[0][i].real + 0.5);
    ans[i] += (long long) (C[1][i].real + C[2][i].real + 0.5) * cut;
    ans[i] += (long long) (C[3][i].real + 0.5) * cut * cut;
  }
  return ans;
}
std::vector<int> mul(const std::vector<int> &a, const std::vector<int> &b) {
  int n = 1;
  while (n - 1 < (int) a.size() + (int) b.size() - 2) n += n;
  CVector poly(n);
  for(int i = 0; i < n; i++) {
    if(i < (int) a.size()) {
      poly[i].real = a[i];
    }
    if(i < (int) b.size()) {
      poly[i].imag = b[i];
    }
  }
  poly = fft(poly);
  for(int i = 0; i < n; i++) {
    poly[i] *= poly[i];
  }
  poly = fft(poly, true);
  std::vector<int> c(n, 0);
  for(int i = 0; i < n; i++) {
    c[i] = (int) (poly[i].imag / 2 + 0.5);
  }
  while (c.size() > 0 && c.back() == 0) c.pop_back();
  return c;
}
```

4.12 NTT - Number Theoretic Transform

```cpp
const int MOD = 998244353;
const int me = 15;
const int ms = 1 << me;

#define add(x, y) x+y>=MOD?x+y-MOD:x+y

const int gen = 3; // use search() from PrimitiveRoot.cpp if MOD isn't 998244353

int bits[ms], root[ms];
void initFFT() {
  root[1] = 1;
  for(int len = 2; len < ms; len += len) {
    int z = fexp(gen, (MOD - 1) / len / 2);
    for(int i = len / 2; i < len; i++) {
      root[2 * i] = root[i];
      root[2 * i + 1] = (long long) root[i] * z % MOD;
    }
  }
}
void pre(int n) {
  int LOG = 0;
  while(1 << (LOG + 1) < n) {
    LOG++;
  }
  for(int i = 1; i < n; i++) {
    bits[i] = (bits[i >> 1] >> 1) | ((i & 1) << LOG);
  }
}
vector<int> fft(vector<int> a, int mod, bool inv = false) {
  int n = (int) a.size();
  pre(n);
  if(inv) {
    reverse(a.begin() + 1, a.end());
  }
  for(int i = 0; i < n; i++) {
    int to = bits[i];
    if(i < to)
      swap(a[i], a[to]);
  }
  for(int len = 1; len < n; len *= 2) {
    for(int i = 0; i < n; i += len * 2) {
      for(int j = 0; j < len; j++) {
        int u = a[i + j], v = (ll) a[i + j + len] * root[len + j] % mod;
        a[i + j] = add(u, v);
        a[i + j + len] = add(u, mod - v);
      }
    }
  }
  if(inv) {
    int rev = fexp(n, mod-2, mod);
    for(int i = 0; i < n; i++)
      a[i] = (ll) a[i] * rev % mod;
  }
  return a;
}
std::vector<int> shift(const std::vector<int> &a, int s) {
  int n = std::max(0, s + (int) a.size());
  std::vector<int> b(n, 0);
  for(int i = std::max(-s, 0); i < (int) a.size(); i++) {
    b[i + s] = a[i];
  }
  return b;
}

std::vector<int> cut(const std::vector<int> &a, int n) {
  std::vector<int> b(n, 0);
  for(int i = 0; i < (int) a.size() && i < n; i++) {
    b[i] = a[i];
  }
  return b;
}
std::vector<int> operator +(std::vector<int> a, const std::vector<int> &b) {
  int sz = (int) std::max(a.size(), b.size());
  a.resize(sz, 0);
  for(int i = 0; i < (int) b.size(); i++) {
    a[i] = add(a[i], b[i]);
  }
  return a;
}
std::vector<int> operator -(std::vector<int> a, const std::vector<int> &b) {
  int sz = (int) std::max(a.size(), b.size());
  a.resize(sz, 0);
  for(int i = 0; i < (int) b.size(); i++) {
    a[i] = add(a[i], MOD - b[i]);
  }
  return a;
}
std::vector<int> operator *(std::vector<int> a, std::vector<int> b) {
  while(!a.empty() && a.back() == 0) a.pop_back();
  while(!b.empty() && b.back() == 0) b.pop_back();
  if(a.empty() || b.empty()) return std::vector<int>(0, 0);
  int n = 1;
  while(n-1 < (int) a.size() + (int) b.size() - 2) n += n;
  a.resize(n, 0);
  b.resize(n, 0);
  a = fft(a, false);
  b = fft(b, false);
  for(int i = 0; i < n; i++) {
    a[i] = (int) ((long long) a[i] * b[i] % MOD);
  }
  return fft(a, true);
}
std::vector<int> inverse(const std::vector<int> &a, int k) {
  assert(!a.empty() && a[0] != 0);
  if(k == 0) {
    return std::vector<int>(1, (int) fexp(a[0], MOD - 2));
  } else {
    int n = 1 << k;
    auto c = inverse(a, k-1);
    return cut(c * cut(std::vector<int>(1, 2) - cut(a, n) * c, n), n);
  }
}
std::vector<int> log(const std::vector<int> &a, int k) {
  assert(!a.empty() && a[0] != 0);
  int n = 1 << k;
```

```
    std::vector<int> b(n, 0);
    for(int i = 0; i+1 < (int) a.size() && i < n; i++) {
      b[i] = (int)((i + 1LL) * a[i+1] % MOD);
    }
    b = cut(b * inverse(a, k), n);
    assert((int) b.size() == n);
    for(int i = n - 1; i > 0; i--) {
      b[i] = (int) (b[i-1] * fexp(i, MOD - 2) % MOD);
    }
    b[0] = 0;
    return b;
  }
  std::vector<int> exp(const std::vector<int> &a, int k) {
    assert(!a.empty() && a[0] == 0);
    if(k == 0) {
      return std::vector<int>(1, 1);
    } else {
      auto b = exp(a, k-1);
      int n = 1 << k;
      return cut(b * cut(std::vector<int>(1, 1) + cut(a, n) - log(b, k), n), n);
    }
  }
```

4.13 Fast Walsh Hadamard Transform

```
vector<ll> FWHT(char oper, vector<ll> a, const bool inv = false) {
  int n = (int) a.size();
  for(int len = 1; len < n; len += len) {
    for(int i = 0; i < n; i += 2 * len) {
      for(int j = 0; j < len; j++) {
        auto u = a[i + j] % mod, v = a[i + j + len] % mod;
        if(oper == '^') {
          a[i + j] = (u + v) % mod;
          a[i + j + len] = (u - v + mod) % mod;
        }
        if(oper == '|') {
          if(!inv) {
            a[i + j + len] = (u + v) % mod;
          } else {
            a[i + j + len] = (v - u + mod) % mod;
          }
        }
        if(oper == '&') {
          if(!inv) {
            a[i + j] = (u + v) % mod;
          } else {
            a[i + j] = (u - v + mod) % mod;
          }
        }
      }
    }
  }
  if(oper == '^' && inv) {
    ll rev = fexp(n, mod - 2);
    for(int i = 0; i < n; i++) {
      a[i] = a[i] * rev % mod;
    }
  }
  return a;
}

vector<ll> multiply(char oper, vector<ll> a, vector<ll> b) {
  int n = 1;
  while (n < (int) max(a.size(), b.size())) {
    n <<= 1;
  }
  vector<ll> ans(n);
  while (a.size() < ans.size()) a.push_back(0);
  while (b.size() < ans.size()) b.push_back(0);
  a = FWHT(oper, a);
  b = FWHT(oper, b);
  for (int i = 0; i < n; i++) {
    ans[i] = a[i] * b[i] % mod;
  }
  ans = FWHT(oper, ans, true);
  return ans;
}

const int mxlog = 17;
vector<ll> subset_multiply(vector<ll> a, vector<ll> b) {
  int n = 1;
  while (n < (int) max(a.size(), b.size())) {
    n <<= 1;
  }
  vector<ll> ans(n);
  while (a.size() < ans.size()) a.push_back(0);
  while (b.size() < ans.size()) b.push_back(0);
  vector<vector<ll>> A(mxlog + 1, vector<ll>(a.size())), B(mxlog + 1, vector<ll>(b.
       size()));
  for (int i = 0; i < n; i++) {
    A[__builtin_popcount(i)][i] = a[i];
    B[__builtin_popcount(i)][i] = b[i];
  }
  for (int i = 0; i <= mxlog; i++) {
    A[i] = FWHT('|', A[i]);
    B[i] = FWHT('|', B[i]);
  }
  for (int i = 0; i <= mxlog; i++) {
    vector<ll> C(n);
    for (int x = 0; x <= i; x++) {
      int y = i - x;
      for (int j = 0; j < n; j++) {
        C[j] = (C[j] + A[x][j] * B[y][j] % mod) % mod;
      }
    }
    C = FWHT('|', C, true);
    for (int j = 0; j < n; j++) {
      if (__builtin_popcount(j) == i) {
        ans[j] = (ans[j] + C[j]) % mod;
      }
    }
  }
  return ans;
}
```

4.14 Miller and Rho

```
//miller_rabin
typedef unsigned long long ull;
typedef long double ld;

ull fmul(ull a, ull b, ull m) {
  ull q = (ld) a * (ld) b / (ld) m;
  ull r = a * b - q * m;
  return (r + m) % m;
}
bool miller(ull p, ull a) {
  ull s = p - 1;
  while(s % 2 == 0) s >>= 1;
  while(a >= p) a >>= 1;
  ull mod = fexp(a, s, p);
  while(s != p - 1 && mod != 1 && mod != p - 1) {
    mod = fmul(mod, mod, p);
    s <<= 1;
  }
  if(mod != p - 1 && s % 2 == 0)return false;
  else return true;
}
bool prime(ull p) {
  if(p <= 3)
    return true;
  if(p % 2 == 0)
    return false;
  return miller(p, 2) && miller(p, 3)
    && miller(p, 5) && miller(p, 7)
    && miller(p, 11) && miller(p, 13)
    && miller(p, 17) && miller(p, 19)
    && miller(p, 23) && miller(p, 29)
    && miller(p, 31) && miller(p, 37);
}
//pollard_rho
ull func(ull x, ull c, ull n) {
  return (fmul(x, x, n) + c) % n;
}
ull gcd(ull a, ull b) {
  if(!b) return a;
  else return gcd(b, a % b);
}
ull rho(ull n) {
```

```
  if(n % 2 == 0) return 2;
  if(prime(n)) return n;
  while(1) {
    ull c;
    do {
      c = rand() % n;
    } while(c == 0 || (c + 2) % n == 0);
    ull x = 2, y = 2, d = 1;
    ull pot = 1, lam = 1;
    do {
      if(pot == lam) {
        x = y;
        pot <<= 1;
        lam = 0;
      }
      y = func(y, c, n);
      lam++;
      d = gcd(x >= y ? x - y : y - x, n);
    } while(d == 1);
    if(d != n) return d;
  }
}
vector<ull> factors(ull n) {
  vector<ull> ans, rest, times;
  if(n == 1) return ans;
  rest.push_back(n);
  times.push_back(1);
  while(!rest.empty()) {
    ull x = rho(rest.back());
    if(x == rest.back()) {
      int freq = 0;
      for(int i = 0; i < rest.size(); i++) {
        int cur_freq = 0;
        while(rest[i] % x == 0) {
          rest[i] /= x;
          cur_freq++;
        }
        freq += cur_freq * times[i];
        if(rest[i] == 1) {
          swap(rest[i], rest.back());
          swap(times[i], times.back());
          rest.pop_back();
          times.pop_back();
          i--;
        }
      }
      while(freq--) {
        ans.push_back(x);
      }
      continue;
    }
    ull e = 0;
    while(rest.back() % x == 0) {
      rest.back() /= x;
      e++;
    }
    e *= times.back();
    if(rest.back() == 1) {
      rest.pop_back();
      times.pop_back();
    }
    rest.push_back(x);
    times.push_back(e);
  }
  return ans;
}
```

4.15 Determinant using Mod

```
// by zchao1995
// Determinante com coordenadas inteiras usando Mod

ll mat[ms][ms];

ll det (int n) {
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      mat[i][j] %= mod;
    }
  }
  ll res = 1;
  for (int i = 0; i < n; i++) {
    if (!mat[i][i]) {
      bool flag = false;
      for (int j = i + 1; j < n; j++) {
        if (mat[j][i]) {
          flag = true;
          for (int k = i; k < n; k++) {
            swap (mat[i][k], mat[j][k]);
          }
          res = -res;
          break;
        }
      }
      if (!flag) {
        return 0;
      }
    }
    for (int j = i + 1; j < n; j++) {
      while (mat[j][i]) {
        ll t = mat[i][i] / mat[j][i];
        for (int k = i; k < n; k++) {
          mat[i][k] = (mat[i][k] - t * mat[j][k]) % mod;
          swap (mat[i][k], mat[j][k]);
        }
        res = -res;
      }
    }
    res = (res * mat[i][i]) % mod;
  }
  return (res + mod) % mod;
}
```

4.16 Lagrange Interpolation

```
class LagrangePoly {
public:
  LagrangePoly(std::vector<long long> _a) {
    //f(i) = _a[i]
    //interpola o vetor em um polinomio de grau y.size() - 1
    y = _a;
    den.resize(y.size());
    int n = (int) y.size();
    for(int i = 0; i < n; i++) {
      y[i] = (y[i] % MOD + MOD) % MOD;
      den[i] = ifat[n - i - 1] * ifat[i] % MOD;
      if((n - i - 1) % 2 == 1) {
        den[i] = (MOD - den[i]) % MOD;
      }
    }
  }

  long long getVal(long long x) {
    int n = (int) y.size();
    x %= MOD;
    if(x < n) {
      //return y[(int) x];
    }
    std::vector<long long> l, r;
    l.resize(n);
    l[0] = 1;
    for(int i = 1; i < n; i++) {
      l[i] = l[i - 1] * (x - (i - 1) + MOD) % MOD;
    }
    r.resize(n);
    r[n - 1] = 1;
    for(int i = n - 2; i >= 0; i--) {
      r[i] = r[i + 1] * (x - (i + 1) + MOD) % MOD;
    }
    long long ans = 0;
    for(int i = 0; i < n; i++) {
      long long coef = l[i] * r[i] % MOD;
      ans = (ans + coef * y[i] % MOD * den[i]) % MOD;
    }
    return ans;
  }

private:
  std::vector<long long> y, den;
};

int main(){
```

```
    fat[0] = ifat[0] = 1;
    for(int i = 1; i < ms; i++) {
      fat[i] = fat[i - 1] * i % MOD;
      ifat[i] = fexp(fat[i], MOD - 2);
    }
    // Codeforces 622F
    int x, k;
    std::cin >> x >> k;
    std::vector<long long> a;
    a.push_back(0);
    for(long long i = 1; i <= k + 1; i++) {
      a.push_back((a.back() + fexp(i, k)) % MOD);
    }
    LagrangePoly f(a);
    std::cout << f.getVal(x) << '\n';
  }
```

4.17 Lagrange extracting polynomial

```
  // O(n^2), receve v {x, y} e retorna o polinomio em fracao
  vector<pair<int, int>> interpolate(vector<ii> v) {
    int n = v.size();
    vector<int> prod(n+1);
    prod[0] = 1;
    for(auto p : v) {
      for(int i = n; i > 0; i--) {
        prod[i] = prod[i-1] - p.first * prod[i];
      }
      prod[0] = -p.first * prod[0];
    }
    vector<pair<int, int>> ans(n+1);
    for(int i = 0; i <= n; i++) ans[i].second = 1;
    for(int i = 0; i < n; i++) {
      vector<int> pol(n+1); // (x - v[i].first)
      for(int j = n; j > 0; j--) {
        pol[j-1] = prod[j] + pol[j] * v[i].first;
      }
      for(int j = 0; j < n; j++) {
        pol[j] *= v[i].second;
      }
      int k = 1;
      for(int j = 0; j < n; j++) {
        if(i==j) continue;
        k *= v[i].first - v[j].first;
      }
      if(k < 0) {
        k = -k;
        for(auto &p : pol) p = -p;
      }
      for(int i = 0; i < n; i++) {
        ans[i] = {ans[i].first*k + pol[i]*ans[i].second, k*ans[i].second};
        if(ans[i].first == 0) ans[i].second = 1;
        else {
          int gc = __gcd(abs(ans[i].first), ans[i].second);
          ans[i].first /= gc;
          ans[i].second /= gc;
        }
      }
    }
    return ans;
  }
```

4.18 Prime Counting

```
  const int ms = 5001000, lim_n = 3e5, lim_p = 1e2;
  std::vector<int> primes;
  int id[ms];
  int memo[lim_n][lim_p];
  void pre() {
    std::vector<bool> isPrime(ms, true);
    for(int i = 2; i < ms; i++) {
      id[i] = (int) primes.size();
      if(!isPrime[i]) continue;
      id[i]++;
      primes.push_back(i);
      for(int j = i+i; j < ms; j += i) isPrime[j] = false;
    }
    for(int i = 1; i < lim_n; i++) {
      memo[i][0] = i;
      for(int j = 1; j < lim_p; j++) memo[i][j] = memo[i][j-1] - memo[i/primes[j-1]][j
          -1];
    }
  }
  int cbc(long long n) {
    int ans = std::max(0, (int) pow((double) n, 1.0 / 3) - 2);
    while((ll) ans * ans * ans < n) ans++;
    return ans;
  }
  long long dp(long long n, int i) {
    if(n == 0) return 0; if(i == 0) return n;
    if(primes[i-1] >= n) return 1;
    if((ll) primes[i-1] * primes[i-1] > n && n < ms) return id[n] - (i-1);
    else if(n < lim_n && i < lim_p) return memo[n][i];
    else return dp(n, i-1) - dp(n / primes[i-1], i-1);
  }
  long long primeFunction(long long n) {
    if(n < ms) return id[(int)n];
    int i = id[cbc(n)];
    long long ans = dp(n, i) + i - 1;
    while((long long) primes[i] * primes[i] <= n) {
      ans -= primeFunction(n / primes[i]) - i;
      i++;
    }
    return ans;
  }
```

5 Geometry

5.1 Geometry

```
  const double inf = 1e100, eps = 1e-12;
  const double PI = acos(-1.0L);
  int cmp (double a, double b = 0) {
    if (abs(a-b) < eps) return 0;
    return (a < b) ? -1 : +1;
  }
  struct PT {
    double x, y;
    PT(double x = 0, double y = 0) : x(x), y(y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
    bool operator <(const PT &p) const {
      if(cmp(x, p.x) != 0) return x < p.x;
      return cmp(y, p.y) < 0;
    }
    bool operator ==(const PT &p) const {return !cmp(x, p.x) && !cmp(y, p.y);}
    bool operator != (const PT &p) const {return !(p == *this);}
  };
  ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << "," << p.y << ")";
  }
  double dot (PT p, PT q) { return p.x * q.x + p.y*q.y; }
  double cross (PT p, PT q) { return p.x * q.y - p.y*q.x; }
  double dist2 (PT p, PT q = PT(0, 0)) { return dot(p-q, p-q); }
  double dist (PT p, PT q) { return hypot(p.x-q.x, p.y-q.y); }
  double norm (PT p) { return hypot(p.x, p.y); }
  PT normalize (PT p) { return p/hypot(p.x, p.y); }
  double angle (PT p, PT q) { return atan2(cross(p, q), dot(p, q)); }
  double angle (PT p) { return atan2(p.y, p.x); }
  double polarAngle (PT p) {
    double a = atan2(p.y,p.x);
    return a < 0 ? a + 2*PI : a;
  }
  PT rotateCCW90 (PT p) { return PT(-p.y, p.x); }
  PT rotateCW90 (PT p) { return PT(p.y, -p.x); }
  PT rotateCCW (PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
  }
  typedef pair<PT, int> Line;
  PT getDir (PT a, PT b) {
    if (a.x == b.x) return PT(0, 1);
    if (a.y == b.y) return PT(1, 0);
```

```
  int dx = b.x-a.x;
  int dy = b.y-a.y;
  int g = __gcd(abs(dx), abs(dy));
  if (dx < 0) g = -g;
  return PT(dx/g, dy/g);
}
Line getLine (PT a, PT b) {
  PT dir = getDir(a, b);
  return {dir, cross(dir, a)};
}
PT projPtLine (PT a, PT b, PT c) { // ponto c na linha a - b, a.b = |a| cost * |b|
  return a + (b-a) * dot(b-a, c-a)/dot(b-a, b-a);
}
PT reflectPointLine (PT a, PT b, PT c) {
  PT p = projPtLine(a, b, c);
  return p*2 - c;
}
PT projPtSeg (PT a, PT b, PT c) { // c no segmento a - b
  double r = dot(b-a, b-a);
  if (cmp(r) == 0) return a;
  r = dot(b-a, c-a)/r;
  if (cmp(r, 0) < 0) return a;
  if (cmp(r, 1) > 0) return b;
  return a + (b - a) * r;
}
double distancePointSegment (PT a, PT b, PT c) { //  ponto c e o segmento a - b
  return dist(c, projPtSeg(a, b, c));
}
bool ptInSegment (PT a, PT b, PT c) { // ponto c esta em um segmento a - b
  if (a == b) return a == c;
  a = a-c, b = b-c;
  return cmp(cross(a, b)) == 0 && cmp(dot(a, b)) <= 0;
}
bool parallel (PT a, PT b, PT c, PT d) {
  return cmp(cross(b - a, c - d)) == 0;
}
bool collinear (PT a, PT b, PT c, PT d) {
  return parallel(a, b, c, d) && cmp(cross(a - b, a - c)) == 0 && cmp(cross(c - d, c -
      a)) == 0;
}
// Calcula distancia entre o ponto (x, y, z) e o plano ax + by + cz = d
double distPtPlane(double x, double y, double z, double a, double b, double c, double
    d) {
    return abs(a * x + b * y + c * z - d) / sqrt(a * a + b * b + c * c);
}
bool segInter (PT a, PT b, PT c, PT d) {
  if (collinear(a, b, c, d)) {
    if (cmp(dist(a, c)) == 0 || cmp(dist(a, d)) == 0 || cmp(dist(b, c)) == 0 || cmp(
        dist(b, d)) == 0) return true;
    if (cmp(dot(c - a, c - b)) > 0 && cmp(dot(d - a, d - b)) > 0 && cmp(dot(c - b, d -
        b)) > 0) return false;
    return true;
  }
  if (cmp(cross(d - a, b - a) * cross(c - a, b - a)) > 0) return false;
  if (cmp(cross(a - c, d - c) * cross(b - c, d - c)) > 0) return false;
  return true;
}
// Calcula a intersecao entre as retas a - b e c - d assumindo que uma unica
    intersecao existe
// Para intersecao de segmentos, cheque primeiro se os segmentos se intersectam e que
    nao sao paralelos
PT lineLine (PT a, PT b, PT c, PT d) {
  b = b - a; d = c - d; c = c - a;
  assert(cmp(cross(b, d)) != 0);
  return a + b * cross(c, d) / cross(b, d);
}
PT circleCenter (PT a, PT b, PT c) {
  b = (a + b) / 2; // bissector
  c = (a + c) / 2; // bissector
  return lineLine(b, b + rotateCW90(a - b), c, c + rotateCW90(a - c));
}
vector<PT> circle2PtsRad (PT p1, PT p2, double r) {
  vector<PT> ret;
  double d2 = dist2(p1, p2);
  double det = r * r / d2 - 0.25;
  if (det < 0.0) return ret;
  double h = sqrt(det);
  for (int i = 0; i < 2; i++) {
    double x = (p1.x + p2.x) * 0.5 + (p1.y - p2.y) * h;
    double y = (p1.y + p2.y) * 0.5 + (p2.x - p1.x) * h;
    ret.push_back(PT(x, y));
    swap(p1, p2);
  }
  return ret;
}
bool circleLineIntersection(PT a, PT b, PT c, double r) {
    return cmp(dist(c, projPtLine(a, b, c)), r) <= 0;
}
vector<PT> circleLine (PT a, PT b, PT c, double r) {
  vector<PT> ret;
  PT p = projPtLine(a, b, c), p1;
  double h = norm(c-p);
  if (cmp(h,r) == 0) {
    ret.push_back(p);
  } else if (cmp(h,r) < 0) {
    double k = sqrt(r*r - h*h);
    p1 = p + (b-a)/(norm(b-a))*k;
    ret.push_back(p1);
    p1 = p - (b-a)/(norm(b-a))*k;
    ret.push_back(p1);
  }
  return ret;
}
bool ptInsideTriangle(PT p, PT a, PT b, PT c) {
  if(cross(b-a, c-b) < 0) swap(a, b);
  if(ptInSegment(a,b,p)) return 1;
  if(ptInSegment(b,c,p)) return 1;
  if(ptInSegment(c,a,p)) return 1;
  bool x = cross(b-a, p-b) < 0;
  bool y = cross(c-b, p-c) < 0;
  bool z = cross(a-c, p-a) < 0;
  return x == y && y == z;
}
bool pointInConvexPolygon(const vector<PT> &p, PT q) {
  if (p.size() == 1) return p.front() == q;
  int l = 1, r = p.size()-1;
  while(abs(r-l) > 1) {
    int m = (r+l)/2;
    if(cross(p[m]-p[0] , q-p[0]) < 0) r = m;
    else l = m;
  }
  return ptInsideTriangle(q, p[0], p[l], p[r]);
}
// Determina se o ponto esta num poligono possivelmente nao-convexo
// Retorna 1 para pontos estritamente dentro, 0 para pontos estritamente fora do
    poligno
// e 0 ou 1 para os pontos restantes
bool pointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for(int i = 0; i < p.size(); i++){
    int j = (i + 1) % p.size();
    if((p[i].y <= q.y && q.y < p[j].y || p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
      c = !c;
  }
  return c;
}
// Determina se o ponto esta na borda do poligno
bool pointOnPolygon(const vector<PT> &p, PT q) {
  for(int i = 0; i < p.size(); i++)
    if(cmp(dist(projPtSeg(p[i], p[(i + 1) % p.size()], q), q)) == 0)
      return true;
    return false;
}
// area / semiperimeter
double rIncircle (PT a, PT b, PT c) {
  double ab = norm(a-b), bc = norm(b-c), ca = norm(c-a);
  return abs(cross(b-a, c-a)/(ab+bc+ca));
}
vector<PT> circleCircle (PT a, double r, PT b, double R) {
  vector<PT> ret;
  double d = norm(a-b);
  if (d > r + R || d + min(r, R) < max(r, R)) return ret;
  double x = (d*d - R*R + r*r) / (2*d); // x = r*cos(R opposite angle)
  double y = sqrt(r*r - x*x);
  PT v = (b - a)/d;
  ret.push_back(a + v*x + rotateCCW90(v)*y);
  if (cmp(y) > 0)
    ret.push_back(a + v*x - rotateCCW90(v)*y);
  return ret;
}
double circularSegArea (double r, double R, double d) {
  double ang = 2 * acos((d*d - R*R + r*r) / (2*d*r)); // cos(R opposite angle) = x/r
  double tri = sin(ang) * r * r;
  double sector = ang * r * r;
  return (sector - tri) / 2;
}
double computeSignedArea (const vector<PT> &p) {
  double area = 0;
```

```
  for (int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area/2.0;
}
double computeArea(const vector<PT> &p) { return abs(computeSignedArea(p)); }
PT computeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * computeSignedArea(p);
  for(int i = 0; i < p.size(); i++){
    int j = (i + 1) % p.size();
    c = c + (p[i] + p[j]) * (p[i].x * p[j].y - p[j].x * p[i].y);
  }
  return c / scale;
}
// Testa se o poligno listada em ordem CW ou CCW eh simples (nenhuma linha se
    intersecta)
bool isSimple(const vector<PT> &p) {
  for(int i = 0; i < p.size(); i++) {
    for(int k = i + 1; k < p.size(); k++) {
      int j = (i + 1) % p.size();
      int l = (k + 1) % p.size();
      if (i == l || j == k) continue;
      if (segInter(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}
vector< pair<PT, PT> > getTangentSegs (PT c1, double r1, PT c2, double r2) {
  if (r1 < r2) swap(c1, c2), swap(r1, r2);
  vector<pair<PT, PT> > ans;
  double d = dist(c1, c2);
  if (cmp(d) <= 0) return ans;
  double dr = abs(r1 - r2), sr = r1 + r2;
  if (cmp(dr, d) >= 0) return ans;
  double u = acos(dr / d);
  PT dc1 = normalize(c2 - c1)*r1;
  PT dc2 = normalize(c2 - c1)*r2;
  ans.push_back(make_pair(c1 + rotateCCW(dc1, +u), c2 + rotateCCW(dc2, +u)));
  ans.push_back(make_pair(c1 + rotateCCW(dc1, -u), c2 + rotateCCW(dc2, -u)));
  if (cmp(sr, d) >= 0) return ans;
  double v = acos(sr / d);
  dc2 = normalize(c1 - c2)*r2;
  ans.push_back({c1 + rotateCCW(dc1, +v), c2 + rotateCCW(dc2, +v)});
  ans.push_back({c1 + rotateCCW(dc1, -v), c2 + rotateCCW(dc2, -v)});
  return ans;
}
```

5.2 Convex Hull

```
vector<PT> convexHull(vector<PT> p, bool needs = 1) {
  if(needs) sort(p.begin(), p.end());
  p.erase(unique(p.begin(), p.end()), p.end());
  int n = p.size(), k = 0;
  if(n <= 1) return p;
  vector<PT> h(n + 2); // se der wa bota n*2
  for(int i = 0; i < n; i++) {
    while(k >= 2 && cross(h[k - 1] - h[k - 2], p[i] - h[k - 2]) <= 0) k--;
    h[k++] = p[i];
  }
  for(int i = n - 2, t = k + 1; i >= 0; i--) {
    while(k >= t && cross(h[k - 1] - h[k - 2], p[i] - h[k - 2]) <= 0) k--;
    h[k++] = p[i];
  }
  h.resize(k); // n+1 points where the first is equal to the last
  return h;
}
vector<PT> splitHull(const vector<PT> &hull) {
  vector<PT> ans(hull.size());
  for(int i = 0, j = (int) hull.size()-1, k = 0; k < (int) hull.size(); k++) {
    if(hull[i] < hull[j]) {
      ans[k] = hull[i++];
    } else {
      ans[k] = hull[j--];
    }
  }
  return ans;
}
// uniao de convex hulls
vector<PT> ConvexHull(const vector<PT> &a, const vector<PT> &b) {
  auto A = splitHull(a);
  auto B = splitHull(b);
  vector<PT> C(A.size() + B.size());
  merge(A.begin(), A.end(), B.begin(), B.end(), C.begin());
  return ConvexHull(C, false);
}
int maximizeScalarProduct(const vector<PT> &hull, PT vec) {
  // this code assumes that there are no 3 colinear points
  int ans = 0;
  int n = hull.size();
  if(n < 20) {
    for(int i = 0; i < n; i++) {
      if(dot(hull[i], vec) > dot(hull[ans], vec)) {
        ans = i;
      }
    }
  } else {
    if(dot(hull[1], vec) > dot(hull[ans], vec)) {
      ans = 1;
    }
    for(int rep = 0; rep < 2; rep++) {
      int l = 2, r = n - 1;
      while(l != r) {
        int mid = (l + r + 1) / 2;
        bool flag = dot(hull[mid], vec) >= dot(hull[mid-1], vec);
        if(rep == 0) { flag = flag && dot(hull[mid], vec) >= dot(hull[0], vec); }
        else { flag = flag || dot(hull[mid-1], vec) < dot(hull[0], vec); }
        if(flag) {
          l = mid;
        } else {
          r = mid - 1;
        }
      }
      if(dot(hull[ans], vec) < dot(hull[l], vec)) {
        ans = l;
      }
    }
  }
  return ans;
}
```

5.3 Cut Polygon

```
struct Segment {
  typedef long double T;
  PT p1, p2;
  T a, b, c;

  Segment() {}

  Segment(PT st, PT en) {
    p1 = st, p2 = en;
    a = -(st.y - en.y);
    b = st.x - en.x;
    c = a * en.x + b * en.y;
  }

  T plug(T x, T y) {
    // plug >= 0 is to the right
    return a * x + b * y - c;
  }

  T plug(PT p) {
    return plug(p.x, p.y);
  }

  bool inLine(PT p) { return cross((p - p1), (p2 - p1)) == 0; }
  bool inSegment(PT p) {
    return inLine(p) && dot((p1 - p2), (p - p2)) >= 0 && dot((p2 - p1), (p - p1)) >=
        0;
  }

  PT lineIntersection(Segment s) {
    long double A = a, B = b, C = c;
    long double D = s.a, E = s.b, F = s.c;
    long double x = (long double) C * E - (long double) B * F;
    long double y = (long double) A * F - (long double) C * D;
    long double tmp = (long double) A * E - (long double) B * D;
    x /= tmp;
```

```
    y /= tmp;
    return PT(x, y);
  }

  bool polygonIntersection(const vector<PT> &poly) {
    long double l = -1e18, r = 1e18;
    for(auto p : poly) {
      long double z = plug(p);
      l = max(l, z);
      r = min(r, z);
    }
    return l - r > eps;
  }
};

vector<PT> cutPolygon(vector<PT> poly, Segment seg) {
  int n = (int) poly.size();
  vector<PT> ans;
  for(int i = 0; i < n; i++) {
    double z = seg.plug(poly[i]);
    if(z > -eps) {
      ans.push_back(poly[i]);
    }
    double z2 = seg.plug(poly[(i + 1) % n]);
    if((z > eps && z2 < -eps) || (z < -eps && z2 > eps)) {
      ans.push_back(seg.lineIntersection(Segment(poly[i], poly[(i + 1) % n])));
    }
  }
  return ans;
}
```

5.4 Smallest Enclosing Circle

```
typedef pair<PT, double> circle;
bool inCircle (circle c, PT p){
  return cmp(dist(c.first, p), c.second) <= 0;
}

PT circumcenter (PT p, PT q, PT r){
  PT a = p-r, b = q-r;
  PT c = PT(dot(a, p+r)/2, dot(b, q+r)/2);
  return PT(cross(c, PT(a.y,b.y)), cross(PT(a.x,b.x), c)) / cross(a, b);
}

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
circle spanningCircle (vector<PT> &v) {
  int n = v.size();
  shuffle(v.begin(), v.end(), rng);
  circle C(PT(), -1);
  for (int i = 0; i < n; i++) if (!inCircle(C, v[i])) {
    C = circle(v[i], 0);
    for (int j = 0; j < i; j++) if (!inCircle(C, v[j])) {
      C = circle((v[i]+v[j])/2, dist(v[i], v[j])/2);
      for(int k = 0; k < j; k++) if (!inCircle(C, v[k])){
        PT o = circumcenter(v[i], v[j], v[k]);
        C = circle(o,dist(o,v[k]));
      }
    }
  }
  return C;
}
```

5.5 Minkowski

```
bool comp(PT a, PT b){
  int hp1 = (a.x < 0 || (a.x==0 && a.y<0));
  int hp2 = (b.x < 0 || (b.x==0 && b.y<0));
  if(hp1 != hp2) return hp1 < hp2;
  long long R = cross(a, b);
  if(R) return R > 0;
  return dot(a, a) < dot(b, b);
}
 // This code assumes points are ordered in ccw and the first points in both vectors
     is the min lexicographically
vector<PT> minkowskiSum(const vector<PT> &a, const vector<PT> &b){
  if(a.empty() || b.empty()) return vector<PT>(0);
  vector<PT> ret;
  int n1 = a.size(), n2 = b.size();
  if(min(n1, n2) < 2){
    for(int i = 0; i < n1; i++) {
      for(int j = 0; j < n2; j++) {
        ret.push_back(a[i]+b[j]);
      }
    }
    return ret;
  }
  PT v1, v2, p = a[0]+b[0];
  ret.push_back(p);
  for (int i = 0, j = 0; i + j + 1 < n1+n2; ){
    v1 = a[(i+1)%n1]-a[i];
    v2 = b[(j+1)%n2]-b[j];
    if(j == n2 || (i < n1 && comp(v1, v2))) p = p + v1, i++;
    else p = p + v2, j++;
    while(ret.size() >= 2 && cmp(cross(p-ret.back(), p-ret[(int)ret.size()-2])) == 0)
        {
      // removing colinear points
      // needs the scalar product stuff it the result is a line
      ret.pop_back();
    }
    ret.push_back(p);
  }
  return ret;
}
```

5.6 Half Plane Intersection

```
struct L {
    PT a, b;
    L(){}
    L(PT a, PT b) : a(a), b(b) {}
};
double angle (L la) { return atan2(-(la.a.y - la.b.y), la.b.x - la.a.x); }
bool comp (L la, L lb) {
    if (cmp(angle(la), angle(lb)) == 0) return cross((lb.b - lb.a), (la.b - lb.a)) >
        eps;
    return cmp(angle(la), angle(lb)) < 0;
}
PT computeLineIntersection (L la, L lb) {
    return computeLineIntersection(la.a, la.b, lb.a, lb.b);
}
bool check (L la, L lb, L lc) {
    PT p = computeLineIntersection(lb, lc);
    double det = cross((la.b - la.a),  (p - la.a));
    return cmp(det) < 0;
}
vector<PT> hpi (vector<L> line) { // salvar (i, j) CCW, (j, i) CW
    sort(line.begin(), line.end(), comp);
    vector<L> pl(1, line[0]);
    for (int i = 0; i < (int)line.size(); ++i) if (cmp(angle(line[i]), angle(pl.back()
        )) != 0) pl.push_back(line[i]);
    deque<int> dq;
    dq.push_back(0);
    dq.push_back(1);
    for (int i = 2; i < (int)pl.size(); ++i) {
        while ((int)dq.size() > 1 && check(pl[i], pl[dq.back()], pl[dq[dq.size() -
            2]])) dq.pop_back();
        while ((int)dq.size() > 1 && check(pl[i], pl[dq[0]], pl[dq[1]])) dq.pop_front
            ();
        dq.push_back(i);
    }
    while ((int)dq.size() > 1 && check(pl[dq[0]], pl[dq.back()], pl[dq[dq.size() -
        2]])) dq.pop_back();
    while ((int)dq.size() > 1 && check(pl[dq.back()], pl[dq[0]], pl[dq[1]])) dq.
        pop_front();
    vector<PT> res;
    for (int i = 0; i < (int)dq.size(); ++i){
        res.emplace_back(computeLineIntersection(pl[dq[i]], pl[dq[(i + 1) % dq.size()
            ]]));
    }
    return res;
}
```

5.7 Closest Pair

```cpp
double closestPair(vector<PT> p) {
  int n = p.size(), k = 0;
  sort(p.begin(), p.end());
  double d = inf;
  set<PT> ptsInv;
  for(int i = 0; i < n; i++) {
    while(k < i && p[k].x < p[i].x - d) {
      ptsInv.erase(swapCoord(p[k++]));
    }
    for(auto it = ptsInv.lower_bound(PT(p[i].y - d, p[i].x - d));
      it != ptsInv.end() && it->x <= p[i].y + d; it++) {
      d = min(d, dist(p[i] - swapCoord(*it), PT(0, 0)));
    }
    ptsInv.insert(swapCoord(p[i]));
  }
  return d;
}
```

5.8 Voronoi

```cpp
Segment getBisector(PT a, PT b) {
  Segment ans(a, b);
  swap(ans.a, ans.b);
  ans.b *= -1;
  ans.c = ans.a * (a.x + b.x) * 0.5 + ans.b * (a.y + b.y) * 0.5;
  return ans;
}

// BE CAREFUL!
// the first point may be any point
// O(N^3)
vector<PT> getCell(vector<PT> pts, int i) {
  vector<PT> ans;
  ans.emplace_back(0, 0);
  ans.emplace_back(1e6, 0);
  ans.emplace_back(1e6, 1e6);
  ans.emplace_back(0, 1e6);
  for(int j = 0; j < (int) pts.size(); j++) {
    if(j != i) {
      ans = cutPolygon(ans, getBisector(pts[i], pts[j]));
    }
  }
  return ans;
}

// O(N^2) expected time
vector<vector<PT>> getVoronoi(vector<PT> pts) {
  // assert(pts.size() > 0);
  int n = (int) pts.size();
  vector<int> p(n, 0);
  for(int i = 0; i < n; i++) {
    p[i] = i;
  }
  shuffle(p.begin(), p.end(), rng);
  vector<vector<PT>> ans(n);
  ans[0].emplace_back(0, 0);
  ans[0].emplace_back(w, 0);
  ans[0].emplace_back(w, h);
  ans[0].emplace_back(0, h);
  for(int i = 1; i < n; i++) {
    ans[i] = ans[0];
  }
  for(auto i : p) {
    for(auto j : p) {
      if(j == i) break;
      auto bi = getBisector(pts[j], pts[i]);
      if(!bi.polygonIntersection(ans[j])) continue;
      ans[j] = cutPolygon(ans[j], getBisector(pts[j], pts[i]));
      ans[i] = cutPolygon(ans[i], getBisector(pts[i], pts[j]));
    }
  }
  return ans;
}
```

6 String Algorithms

6.1 KMP

```cpp
vector<int> getBorder(string str) {
  int n = str.size();
  vector<int> border(n, -1);
  for(int i = 1, j = -1; i < n; i++) {
    while(j >= 0 && str[i] != str[j + 1]) {
      j = border[j];
    }
    if(str[i] == str[j + 1]) {
      j++;
    }
    border[i] = j;
  }
  return border;
}
int matchPattern(const string &txt, const string &pat, const vector<int> &border) {
  int freq = 0;
  for(int i = 0, j = -1; i < txt.size(); i++) {
    while(j >= 0 && txt[i] != pat[j + 1]) {
      j = border[j];
    }
    if(pat[j + 1] == txt[i]) {
      j++;
    }
    if(j + 1 == (int) pat.size()) {
      //found occurence
      freq++;
      j = border[j];
    }
  }
  return freq;
}
```

6.2 KMP Automaton

```cpp
// trad converts a char to its index
int trad(char ch) { return (int) ch; }
// sigma should be greater then the greatest value returned by trad
vector<vector<int>> buildAutomaton(string p, int sigma=300) {
  vector<vector<int>> A(p.size() + 1, vector<int>(sigma));
  int brd = 0;
  for (int i = 0; i < sigma; i++) A[0][i] = 0;
  A[0][trad(p[0])] = 1;
  for (int i = 1; i <= p.size(); i++) {
    for (int ch = 0; ch < sigma; ch++) {
      A[i][ch] = A[brd][ch];
    }
    if (i < p.size()) {
      A[i][trad(p[i])] = i + 1;
      brd = A[brd][trad(p[i])];
    }
  }
  return A;
}
```

6.3 Aho-Corasick

```cpp
const int ms = 1e6;    // quantidade de caracteres
const int sigma = 26;  // tamanho do alfabeto
int trie[ms][sigma], fail[ms], superfail[ms], terminal[ms], qtd;
void init() {
  qtd = 1;
  memset(trie[0], -1, sizeof trie[0]);
}
void add(string &s) {
  int node = 0;
  for (char ch : s) {
    int pos = val(ch);  // no caso de alfabeto a-z: val(ch) = ch - 'a'
    if (trie[node][pos] == -1) {
      memset(trie[qtd], -1, sizeof trie[qtd]);
```

```
        terminal[qtd] = 0;
        trie[node][pos] = qtd++;
      }
      node = trie[node][pos];
    }
    ++terminal[node];  // trocar pela info que quiser
  }
  void buildFailure() {
    memset(fail, 0, sizeof(int) * qtd), memset(superfail, 0, sizeof(int) * qtd);
    queue<int> Q;
    Q.push(0);
    while (Q.size()) {
      int node = Q.front();
      Q.pop();
      for (int pos = 0; pos < sigma; ++pos) {
        int &v = trie[node][pos];
        int f = node == 0 ? 0 : trie[fail[node]][pos];
        // int sf = present[f] ? f : superfail[f];
        // present marks if that vertex is a terminal node or not
        // if summing up on terminal, doesn't work
        if (v == -1) {
          v = f;
        } else {
          fail[v] = f;
        // superfail[v] = sf;
          Q.push(v);
          // dar merge nas infos (por ex: terminal[v] += terminal[f])
        }
      }
    }
  }
  void search(string &s) {
    int node = 0;
    for (char ch : s) {
      int pos = val(ch);
      node = trie[node][pos];
      // processar infos no no atual (por ex: ocorrencias += terminal[node])
    }
  }
  // se tiver usando super fail, cuidado com o estado que voce ta, antes de subir pro sf
     , porque pode ser que o estado que ta nao seja no terminal
```

6.4 Algoritmo de Z

```
template <class T>
vector<int> ZFunc(const vector<T> &v) {
  vector<int> z(v.size(), 0);
  int n = (int) v.size(), a = 0, b = 0;
  if (!z.empty()) z[0] = n;
  for (int i = 1; i < n; i++) {
    int end = i; if (i < b) end = min(i + z[i - a], b);
    while(end < n && v[end] == v[end - i]) ++end;
    z[i] = end - i; if(end > b) a = i, b = end;
  }
  return z;
}
```

6.5 Suffix Array

```
vector<int> buildSa(const string& in) {
  int n = in.size(), c = 0;
  vector<int> temp(n), posBucket(n), bucket(n), bpos(n), out(n);
  for (int i = 0; i < n; i++) out[i] = i;
  sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });
  for (int i = 0; i < n; i++) {
    bucket[i] = c;
    if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
  }
  for (int h = 1; h < n && c < n; h <<= 1) {
    for (int i = 0; i < n; i++) posBucket[out[i]] = bucket[i];
    for (int i = n - 1; i >= 0; i--) bpos[bucket[i]] = i;
    for (int i = 0; i < n; i++) {
      if (out[i] >= n - h) temp[bpos[bucket[i]]++] = out[i];
    }
    for (int i = 0; i < n; i++) {
      if (out[i] >= h) temp[bpos[posBucket[out[i] - h]]++] = out[i] - h;
    }
    c = 0;
    for (int i = 0; i + 1 < n; i++) {
      int a = (bucket[i] != bucket[i + 1]) || (temp[i] >= n - h)
          || (posBucket[temp[i + 1] + h] != posBucket[temp[i] + h]);
      bucket[i] = c;
      c += a;
    }
    bucket[n - 1] = c++;
    temp.swap(out);
  }
  return out;
}
vector<int> buildLcp(string s, vector<int> sa) {
  int n = (int) s.size();
  vector<int> pos(n), lcp(n, 0);
  for(int i = 0; i < n; i++) {
    pos[sa[i]] = i;
  }
  int k = 0;
  for(int i = 0; i < n; i++) {
    if (pos[i] + 1 == n) {
      k = 0;
      continue;
    }
    int j = sa[pos[i] + 1];
    while(i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
    lcp[pos[i]] = k;
    k = max(k - 1, 0);
  }
  return lcp;
}
```

6.6 Suffix Tree

```
#define fpos adla
const int inf = 1e9;
const int maxn = 1e4;
char s[maxn];
map<int, int> to[maxn];
int len[maxn], fpos[maxn], link[maxn];
int node, pos;
int sz = 1, n = 0;
int make_node(int _pos, int _len)
{
    fpos[sz] = _pos;
    len [sz] = _len;
    return sz++;
}
void go_edge()
{
    while(pos > len[to[node][s[n - pos]]])
    {
        node = to[node][s[n - pos]];
        pos -= len[node];
    }
}
void add_letter(int c)
{
    s[n++] = c;
    pos++;
    int last = 0;
    while(pos > 0)
    {
        go_edge();
        int edge = s[n - pos];
        int &v = to[node][edge];
        int t = s[fpos[v] + pos - 1];
        if(v == 0)
        {
            v = make_node(n - pos, inf);
            link[last] = node;
            last = 0;
        }
        else if(t == c)
        {
            link[last] = node;
            return;
        }
```

```
        else
        {
            int u = make_node(fpos[v], pos - 1);
            to[u][c] = make_node(n - 1, inf);
            to[u][t] = v;
            fpos[v] += pos - 1;
            len [v] -= pos - 1;
            v = u;
            link[last] = u;
            last = u;
        }
        if(node == 0)
            pos--;
        else
            node = link[node];
    }
}
//len[0] = inf
```

6.7 Suffix Automaton

```
int len[ms*2], link[ms*2], nxt[ms*2][sigma];
int sz, last;
void build(string &s) {
  len[0] = 0; link[0] = -1;
  sz = 1; last = 0;
  memset(nxt[0], -1, sizeof nxt[0]);
  for(char ch : s) {
    int c = ch-'a', cur = sz++;
    len[cur] = len[last]+1;
    memset(nxt[cur], -1, sizeof nxt[cur]);
    int p = last;
    while(p != -1 && nxt[p][c] == -1) {
      nxt[p][c] = cur; p = link[p];
    }
    if(p == -1) {
      link[cur] = 0;
    } else {
      int q = nxt[p][c];
      if(len[p] + 1 == len[q]) {
        link[cur] = q;
      } else {
        len[sz] = len[p]+1; link[sz] = link[q];
        memcpy(nxt[sz], nxt[q], sizeof nxt[q]);
        while (p != -1 && nxt[p][c] == q) {
          nxt[p][c] = sz; p = link[p];
        }
        link[q] = link[cur] = sz++;
      }
    }
    last = cur;
  }
}
```

6.8 Manacher

```
std::array<std::vector<int>, 2> manacher(const std::string& s) {
        int n = (int) s.size();
        std::array<std::vector<int>, 2> p = {std::vector<int>(n+1), std::vector<int>(n
            )};
        for(int z = 0; z < 2; z++) for (int i = 0, l = 0, r = 0; i < n; i++) {
                int t = r-i+!z;
                if (i<r) p[z][i] = std::min(t, p[z][l+t]);
                int L = i-p[z][i], R = i+p[z][i]-!z;
                while (L>=1 && R+1<n && s[L-1] == s[R+1])
                        p[z][i]++, L--, R++;
                if (R>r) l=L, r=R;
        }
        return p;
} // pra cada centro o tamanho max do palindromo centrado ali, qualquer coisa printa a
     saida pra abacabaab
```

6.9 Polish Notation

```
inline bool isOp(char c) {
        return c=='+' || c=='-' || c=='*' || c=='/' || c=='^';
}

inline bool isCarac(char c) {
        return (c>='a' && c<='z') || (c>='A' && c<='Z') || (c>='0' && c<='9');
}
int paren2polish(char* paren, char* polish) {
        map<char, int> prec;
        prec['('] = 0;
        prec['+'] = prec['-'] = 1;
        prec['*'] = prec['/'] = 2;
        prec['^'] = 3;
        int len = 0;
        stack<char> op;
        for (int i = 0; paren[i]; i++) {
                if (isOp(paren[i])) {
                        while (!op.empty() && prec[op.top()] >= prec[paren[i]]) {
                                polish[len++] = op.top(); op.pop();
                        }
                        op.push(paren[i]);
                }
                else if (paren[i]=='(') op.push('(');
                else if (paren[i]==')') {
                        for (; op.top()!='('; op.pop())
                                polish[len++] = op.top();
                        op.pop();
                }
                else if (isCarac(paren[i]))
                        polish[len++] = paren[i];
        }
        for(; !op.empty(); op.pop())
                polish[len++] = op.top();
        polish[len] = 0;
        return len;
}
```

7 Miscellaneous

7.1 Ternary Search

```
// R
for(int i = 0; i < LOG; i++){
  long double m1 = (A * 2 + B) / 3.0;
  long double m2 = (A + 2 * B) / 3.0;

  if(f(m1) > f(m2))
    A = m1;
  else
    B = m2;
}
ans = f(A);

// Z
while(B - A > 4){
  int m1 = (A + B) / 2;
  int m2 = (A + B) / 2 + 1;
  if(f(m1) > f(m2))
    A = m1;
  else
    B = m2;
}
ans = inf;
for(int i = A; i <= B; i++) ans = min(ans , f(i));
```

7.2 Count Sort

```
int H[(1<<15)+1], to[mx], b[mx];
void sort(int m, int a[]) {
  memset(H, 0, sizeof H);
  for (int i = 1; i <= m; i++) H[a[i] % (1<<15)]++;
  for (int i = 1; i < 1<<15; i++) H[i] += H[i-1];
  for (int i = m; i; i--) to[i] = H[a[i] % (1 << 15)]--;
  for (int i = 1; i <= m; i++) b[to[i]] = a[i];
```

```
  memset(H, 0, sizeof H);
  for (int i = 1; i <= m; i++) H[b[i]>>15]++;
  for (int i = 1; i < 1<<15; i++) H[i] += H[i-1];
  for (int i = m; i ; i--) to[i] = H[b[i]>>15]--;
  for (int i = 1; i <= m; i++) a[to[i]] = b[i];
}
```

7.3 Random Number Generator

```
// mt19937_64 se LL
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
// Random_Shuffle
shuffle(v.begin(), v.end(), rng);
// Random number in interval
int randomInt = uniform_int_distribution(0, i)(rng);
double randomDouble = uniform_real_distribution(0, 1)(rng);
// bernoulli_distribution, binomial_distribution, geometric_distribution
// normal_distribution, poisson_distribution, exponential_distribution
```

7.4 Safe Hash

```
namespace {
  struct safe_hash {
    static uint64_t splitmix64(uint64_t x) {
      // http://xorshift.di.unimi.it/splitmix64.c
      x += 0x9e3779b97f4a7c15;
      x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
      x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
      return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
      static const uint64_t FIXED_RANDOM = std::chrono::steady_clock::now().
          time_since_epoch().count();
      return splitmix64(x + FIXED_RANDOM);
    }
  };
}
```

7.5 Unordered Map Tricks

```
// pair<int, int> hash function
struct HASH{
  size_t operator()(const pair<int,int>&x)const{
    return (size_t) x.first * 37U + (size_t) x.second;
  }
};


unordered_map<int,int>mp;
mp.reserve(1024);
mp.max_load_factor(0.25);
```

7.6 Iterate masks in bitcount order

```
for(int k = n-1; k >= 0; k--) {
  unsigned int i = (1 << k) -1;
  while(i < (1 << n)) {
    // do what you want
    unsigned int t = (i | (i - 1)) + 1;
    if(i == 0) break;
    i = t | ((((t & -t) / (i & -i)) >> 1) - 1);
  }
}
```

7.7 Submask Enumeration

```
for (int s=m; ; s=(s-1)&m) {
  ... you can use s ...
  if (s==0)  break;
}
```

7.8 Sum over Subsets DP

```
// F[i] = Sum of all A[j] where j is a submask of i
for(int i = 0; i<(1<<N); ++i)
  F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
  if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
}
```

7.9 Dates

```
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;

  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
}
```

7.10 Stable Marriage

```
std::vector<std::vector<int>> stableMarriage(std::vector<std::vector<int>> first, std
    ::vector<std::vector<int>> second, std::vector<int> cap) {
        assert(cap.size() == second.size());
        int n = (int) first.size(), m = (int) second.size();
        // if O(N * M) first in memory, use table
        std::map<std::pair<int, int>, int> prio;
        std::vector<std::set<std::pair<int, int>>> current(m);
        for(int i = 0; i < n; i++) {
                std::reverse(first[i].begin(), first[i].end());
        }
        for(int i = 0; i < m; i++) {
                for(int j = 0; j < (int) second[i].size(); j++) {
                        prio[{second[i][j], i}] = j;
                }
        }
        for(int i = 0; i < n; i++) {
                int on = i;
                while(!first[on].empty()) {
                        int to = first[on].back();
                        first[on].pop_back();
```

```cpp
                if(cap[to]) {
                        cap[to]--;
                        assert(prio.count({on, to}));
                        current[to].insert({prio[{on, to}], on});
                        break;
                }
                assert(!current[to].empty());
                auto it = current[to].end();
                it--;
                if(it->first > prio[{on, to}]) {
                        int nxt = it->second;
                        current[to].erase(it);
                        current[to].insert({prio[{on, to}], on});
                        on = nxt;
                }
            }
        }
        std::vector<std::vector<int>> ans(m);
        for(int i = 0; i < m; i++) {
                for(auto it : current[i]) {
                        ans[i].push_back(it.second);
                }
        }
        return ans;
}
```

8 Teoremas e formulas uteis

8.1 Grafos

```
Formula de Euler: V - E + F = 2 (para grafo planar)
Handshaking: Numero par de vertices tem grau impar
Kirchhoff's Theorem: Monta matriz onde Mi,i = Grau[i] e Mi,j = -1 se houver aresta i-j
      ou 0 caso contrario, remove uma linha e uma coluna qualquer e o numero de
     spanning trees nesse grafo eh o det da matriz

Grafo contem caminho hamiltoniano se:
Dirac's theorem: Se o grau de cada vertice for pelo menos n/2
Ore's theorem: Se a soma dos graus que cada par nao-adjacente de vertices for pelo
     menos n
Boruvka's: enquanto grafo nao conexo, para cada componente conexa use a aresta que sai
      de menor custo.

Trees:
Tem Catalan(N) Binary trees de N vertices
Tem Catalan(N-1) Arvores enraizadas com N vertices
Caley Formula: n^(n-2) arvores em N vertices com label
Prufer code: Cada etapa voce remove a folha com menor label e o label do vizinho eh
     adicionado ao codigo ate ter 2 vertices

Flow:
Recuperar min cut eh ver se level[u] != -1 ai eh do lado do source
Max Edge-disjoint paths: Max flow com arestas com peso 1
Max Node-disjoint paths: Faz a mesma coisa mas separa cada vertice em um com as
     arestas de chegadas e um com as arestas de saida e uma aresta de peso 1
     conectando o vertice com aresta de chegada com ele mesmo com arestas de saida
Konig's Theorem: minimum node cover = maximum matching se o grafo for bipartido,
     complemento eh o maximum independent set
Min vertex cover sao os vertices da particao do source que nao tao do lado do source
     do cut e os do sink que tao do lado do source, independent set o contrario
Min edge cover eh N - match, pega as arestas do match mais qualquer aresta dos outros
     vertices
Min Node disjoint path cover: formar grafo bipartido de vertices duplicados, onde
     aresta sai do vertice tipo A e chega em tipo B, entao o path cover eh N -
     matching
Min General path cover: Mesma coisa mas colocando arestas de A pra B sempre que houver
      caminho de A pra B
Dilworth's Theorem: Min General Path cover = Max Antichain (set de vertices tal que
     nao existe caminho no grafo entre vertices desse set)
Hall's marriage: um grafo tem um matching completo do lado X se para cada subconjunto
     W de X,
    |W| <= |vizinhosW| onde |W| eh quantos vertices tem em W
feasible flow in a network with both upper and lower capacity constraints, no source
     or sink: capacities are changed to upper bound - lower bound. Add a new source
     and a sink. let M[v] = (sum of lower bounds of ingoing edges to v) - (sum of
     lower bounds of outgoing edges from v). For all v, if M[v] > 0 then add edge (S,v
     ) with capcity M, otherwise add (v,T) with capacity -M. If all outgoing edges
     from S are full, then a feasible flow exists, it is the flow plus the original
     lower_bounds
```

8.2 Math

```
Goldbach's: todo numero par n > 2 pode ser representado com n = a + b onde a e b sao
     primos
Twin prime: existem infinitos pares p, p + 2 onde ambos sao primos
Legendre's: sempre tem um primo entre n^2 e (n+1)^2
Lagrange's: todo numero inteiro pode ser inscrito como a soma de 4 quadrados
Zeckendorf's: todo numero pode ser representado pela soma de dois numeros de
     fibonnacis diferentes e nao consecutivos
Euclid's: toda tripla de pitagoras primitiva pode ser gerada com
    (n^2 - m^2, 2nm, n^2+m^2) onde n, m sao coprimos e um deles eh par
Wilson's: n eh primo quando (n-1)! mod n = n - 1
Mcnugget: Para dois coprimos x, y o maior inteiro que nao pode ser escrito como ax +
     by eh (x-1)(y-1)/2

Fermat: Se p eh primo entao a^(p-1) % p = 1
Se x e m tambem forem coprimos entao x^k % m = x^(k mod(m-1)) % m
Euler's theorem: x^(phi(m)) mod m = 1 onde phi(m) eh o totiente de euler

Chinese remainder theorem:
Para equacoes no formato x = a1 mod m1, ... , x = an mod mn onde todos os pares m1,
     ... , mn sao coprimos
Deixe Xk = m1*m2*..*mn/mk e Xk^-1 mod mk = inverso de Xk mod mk, entao
x = somatorio com k de 1 ate n de ak*Xk*(Xk,mk^-1 mod mk)
Para achar outra solucao so somar m1*m2*..*mn a solucao existente

Catalan number: exemplo expressoes de parenteses bem formadas
C0 = 1, Cn = somatorio de i=0 -> n-1 de Ci*C(n-1+1)
outra forma: Cn = (2n escolhe n)/(n+1)
Bertrand's ballot theorem: p votos tipo A e q votos tipo B com p>q, prob de em todo
     ponto ter mais As do que Bs antes dele = (p-q)/(p+q)
Se puder empates entao prob = (p+1-q)/(p+1), para achar quantidade de possibilidades
     nos dois casos basta multiplicar por (p + q escolhe q)

Propriedades de Coeficientes Binomiais:
Somatorio de k = 0 -> m de (-1)^k * (n escolhe k) = (-1)^m* (n -1 escolhe m)
(N escolhe K) = (N escolhe N-K)
(N escolhe K) = N/K * (n-1 escolhe k-1)
Somatorio de k = 0 -> n de (n escolhe k) = 2^n
Somatorio de m = 0 -> n de (m escolhe k) = (n+1 escolhe k + 1)
Somatorio de k = 0 -> m de (n+k escolhe k) = (n+m+1 escolhe m)
Somatorio de k = 0 -> n de (n escolhe k)^2 = (2n escolhe n)
Somatorio de k = 0 ou 1 -> n de k*(n escolhe k) = n * 2^(n-1)
Somatorio de k = 0 -> n de (n-k escolhe k) = Fib(n+1)

Hockey-stick: Somatorio de i = r -> n de (i escolhe r) = (n + 1 escolhe r + 1)
Vandermonde: (m+n escolhe r) = somatorio de k = 0 -> r de (m escolhe k) * (n escolhe r
      - k)

Burnside lemma: colares diferentes nao contando rotacoes quando m = cores e n =
     comprimento
(m^n + somatorio i =1 - > n-1 de m^gcd(i, n))/n

Distribuicao uniforme a,a+1, ..., b Expected[X] = (a+b)/2
Distribuicao binomial com n tentativas de probabilidade p, X = sucessos:
    P(X = x) = p^x * (1-p)^(n-x) * (n escolhe x) e E[X] = p*n
Distribuicao geometrica onde continuamos ate ter sucesso, X = tentativas:
    P(X = x) = (1-p)^(x-1) * p e E[X] = 1/p
Linearity of expectation: Tendo duas variaveis X e Y e constantes a e b, o valor
     esperado de aX + bY = a*E[X] + b*E[X]
V(X) = E((X-u)^2)
V(X) = E(X^2) - E(X^2)

PG: a1 * (q^n - 1)/(q - 1)

Mobius Inverse: Sum(d|n): mobius(d) = [n = 1] (expressao booleana)
```

8.3 Geometry

```
Formula de Euler: V - E + F = 2
```

```
Pick Theorem: Para achar pontos em coords inteiras num poligono Area = i + b/2 - 1
     onde i eh o o numero de pontos dentro do poligono e b de pontos no perimetro do
     poligono
Two ears theorem: Todo poligono simples com mais de 3 vertices tem pelo menos 2
     orelhas, vertices que podem ser removidos sem criar um crossing, remover orelhas
     repetidamente triangula o poligono
Incentro triangulo:  (a(Xa, Ya) + b(Xb, Yb) + c(Xc, Yc))/(a+b+c) onde a = lado oposto
     ao vertice a, incentro eh onde cruzam as bissetrizes, eh o centro da
     circunferencia inscrita e eh equidistante aos lados

Delaunay Triangulation: Triangulacao onde nenhum ponto esta dentro de nenhum circulo
     circunscrito nos triangulos
Eh uma triangulacao que maximiza o menor angulo e a MST euclidiana de um conjunto de
     pontos eh um subconjunto da triangulacao

Brahmagupta's formula: Area cyclic quadrilateral
s = (a+b+c+d)/2
area = sqrt((s-a)*(s-b)*(s-c)*(s-d))
d = 0 => area = sqrt((s-a)*(s-b)*(s-c)*s)
```

8.4 Dynamic Programming

```
Divide and conquer - dp[i][j] = mink < j{dp[i - 1][k] + C[k][j]}
dividir o subsegmento ate j em i segmentos com custo, valido se A[i][j] <= A[i][j+1]
Knuth - p[i][j] = mini < k < j{dp[i][k] + dp[k][j]} + C[i][j], valido se A[i, j - 1]
     <= A[i][j] <= A[i+1, j]
onde A[i][j] eh o menor k que da a resposta otima
slope trick - funcao piecewise linear convexa, descrita pelos pontos de mudanca de
     slope (multiset/heap)
lembre que existe fft, cht, aliens trick e bitset
```