Q1 a) $T: \mathbb{R}^5 \to \mathbb{R}^2$ DEFINIDA POR $T(x_1, x_2, x_3, x_4, x_5) =$
$(x_1 - x_2 + x_3 - 7x_5, x_1 - 9x_2 - 4x_3 - x_4 + x_5)$

PASSO 1: APLICAMOS A TL NOS VETORES DA BASE CÂNONICA
$\{(1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1)\}$
DO DOMINIO $\mathbb{R}^5$

$T(1,0,0,0,0) = (1-0+0-7\cdot0, 1-9\cdot0-4\cdot0-0+0) = (1,1)$
$T(0,1,0,0,0) = (0-1+0\cdot7\cdot0, 0-9\cdot1-4\cdot0-0+0) = (-1,-9)$
$T(0,0,1,0,0) = (0-0+1-7\cdot0, 0-9\cdot0-4\cdot1-0+0) = (1,-4)$
$T(0,0,0,1,0) = (0-0+0-7\cdot0, 0-9\cdot0-4\cdot0-1+0) = (0,-1)$
$T(0,0,0,0,1) = (0-0+0-7\cdot1, 0-9\cdot0-4\cdot0-0+1) = (-7,1)$

PASSO 2: JOGAMOS ESSES VETORES NAS COLUNAS DE UMA MATRIZ RESPEITANDO A ORDEM DA BASE:

$$\begin{bmatrix} 1 & -1 & 1 & 0 & -7 \\ 1 & -9 & -4 & -1 & 1 \end{bmatrix}$$

EIS A MATRIZ QUE REPRESENTA T

b) A PROJEÇÃO DO $\mathbb{R}^5$ NO HIPERPLANO DEFINIDO POR $x-2y-z-5w=0$,

PASSO 1: ENCONTRAR OS GERADORES DA EQUAÇÃO

$T(x,y,z,v,w) = (2y+z+5w, y, z, v, w)$
$T(x,y,z,v,w) = y(2,1,0,0,0) + z(1,0,1,0,0) + v(0,0,0,1,0) +$
$w(5,0,0,0,1)$

OS GERADORES SÃO: $\{(2,1,0,0,0), (1,0,1,0,0), (0,0,0,1,0), (5,0,0,0,1)\}$

PASSO 2: APLICAR GRAM-SCHMIDT NOS GERADORES
$\{(2,1,0,0,0), (1,0,1,0,0), (0,0,0,1,0), (5,0,0,0,1)\}$

$\frac{\sqrt{186}}{31}$

$$\left\{\left(\frac{2\sqrt5}{5}, \frac{\sqrt5}{5}, 0,0,0\right), \left(\frac{\sqrt{30}}{30}, -\frac{\sqrt{30}}{15}, \frac{\sqrt{30}}{6}, 0,0\right), (0,0,0,1,0), \left(\frac{5\sqrt{186}}{186}, -\frac{5\sqrt{186}}{93}, -\frac{5\sqrt{186}}{186}, 0, \right.\right.$$

PASSO 3: MULTIPLICA CADA UM DOS VETORES RESULTANTE ACIMA POR SUA TRANSPOSTA

$$\begin{pmatrix} \frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 0, 0, 0\right) = \begin{pmatrix} \frac{4}{5} & \frac{2}{5} & 0 & 0 & 0 \\ \frac{2}{5} & \frac{1}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\sqrt{30}}{30} \\ -\frac{\sqrt{30}}{15} \\ \frac{\sqrt{30}}{6} \\ 0 \\ 0 \end{pmatrix} \cdot \left(\frac{\sqrt{30}}{30}, \frac{-\sqrt{30}}{15}, \frac{\sqrt{30}}{6}, 0, 0\right) = \begin{pmatrix} \frac{1}{30} & \frac{-1}{15} & \frac{1}{6} & 0 & 0 \\ \frac{-1}{15} & \frac{2}{15} & \frac{-1}{3} & 0 & 0 \\ \frac{1}{6} & \frac{-1}{3} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \cdot (0, 0, 0, 1, 0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{5\sqrt{186}}{186} \\ \frac{-5\sqrt{186}}{93} \\ \frac{-5\sqrt{186}}{186} \\ 0 \\ \frac{\sqrt{186}}{31} \end{pmatrix} \cdot \left(\frac{5\sqrt{186}}{186}, \frac{-5\sqrt{186}}{93}, \frac{-5\sqrt{186}}{186}, 0, \frac{\sqrt{186}}{31}\right) = \begin{pmatrix} \frac{25}{186} & \frac{-25}{93} & \frac{-25}{186} & 0 & \frac{5}{31} \\ \frac{-25}{93} & \frac{50}{93} & \frac{25}{93} & 0 & \frac{-10}{31} \\ \frac{-25}{186} & \frac{25}{93} & \frac{25}{186} & 0 & \frac{-5}{31} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{5}{31} & \frac{-10}{31} & \frac{-5}{31} & 0 & \frac{6}{31} \end{pmatrix}$$

PASSO 4: SOMANDO TODOS OS VALORES OBTIDO ACIMA:

$$\begin{pmatrix} \frac{4}{5} & \frac{2}{5} & 0 & 0 & 0 \\ \frac{2}{5} & \frac{1}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{30} & \frac{-1}{15} & \frac{1}{6} & 0 & 0 \\ \frac{-1}{15} & \frac{2}{15} & \frac{-1}{3} & 0 & 0 \\ \frac{1}{6} & \frac{-1}{3} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & \frac{1}{6} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{-1}{3} & 0 & 0 \\ \frac{1}{6} & \frac{-1}{3} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{25}{186} & \frac{-25}{93} & \frac{-25}{186} & 0 & \frac{5}{31} \\ \frac{-25}{93} & \frac{50}{93} & \frac{25}{93} & 0 & \frac{-10}{31} \\ \frac{-25}{186} & \frac{25}{93} & \frac{25}{186} & 0 & \frac{-5}{31} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{5}{31} & \frac{-10}{31} & \frac{-5}{31} & 0 & \frac{6}{31} \end{pmatrix} = \begin{pmatrix} \frac{25}{180} & \frac{-25}{93} & \frac{-25}{180} & 0 & \frac{5}{31} \\ \frac{-25}{93} & \frac{50}{93} & \frac{25}{93} & 0 & \frac{-10}{31} \\ \frac{-25}{186} & \frac{25}{93} & \frac{25}{186} & 0 & \frac{-5}{31} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{5}{31} & \frac{-10}{31} & \frac{-5}{31} & 0 & \frac{6}{31} \end{pmatrix} +$$

EIS A MATRIZ QUE REPRESENTA TL

$$\begin{pmatrix} \frac{30}{31} & \frac{2}{31} & \frac{1}{31} & 0 & \frac{5}{31} \\ \frac{2}{31} & \frac{27}{31} & \frac{-2}{31} & 0 & \frac{-10}{31} \\ \frac{1}{31} & \frac{-2}{31} & \frac{30}{31} & 0 & \frac{-5}{31} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{5}{31} & \frac{-10}{31} & \frac{-5}{31} & 0 & \frac{6}{31} \end{pmatrix}$$

c) A reflexão do $\mathbb{R}^4$ cujo espelho é o hiperplano de equação $x - 3y - z + 2w = 0$.

Passo 1: Detectamos o vetor espelho da equação e o normalizamos

$v = (1, -3, -1, 2)$

$$u = \frac{(1, -3, -1, 2)}{\sqrt{15}} \rightarrow u = \left(\frac{1}{\sqrt{15}}, \frac{-3}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{2}{\sqrt{15}}\right)$$

Passo 2: Multiplicamos o vetor normalizado pela sua transposta

$$\begin{pmatrix} \frac{1}{\sqrt{15}} \\ \frac{-3}{\sqrt{15}} \\ \frac{-1}{\sqrt{15}} \\ \frac{2}{\sqrt{15}} \end{pmatrix} \cdot \left(\frac{1}{\sqrt{15}}, \frac{-3}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{2}{\sqrt{15}}\right) = \begin{pmatrix} \frac{1}{15} & -\frac{1}{5} & \frac{1}{15} & \frac{2}{15} \\ \frac{-1}{5} & \frac{3}{5} & \frac{-1}{5} & \frac{-2}{5} \\ \frac{1}{15} & \frac{-1}{5} & \frac{1}{15} & \frac{2}{15} \\ \frac{2}{15} & \frac{-2}{5} & \frac{2}{15} & \frac{4}{15} \end{pmatrix}$$

Passo 3: A matriz que representa a TL é o resultado da seguinte equação matricial.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 2 \cdot \begin{bmatrix} \frac{1}{15} & \frac{-1}{5} & \frac{1}{15} & \frac{2}{15} \\ \frac{-1}{5} & \frac{3}{5} & \frac{-1}{5} & \frac{-2}{5} \\ \frac{1}{15} & \frac{-1}{5} & \frac{1}{15} & \frac{2}{15} \\ \frac{2}{15} & \frac{-2}{5} & \frac{2}{5} & \frac{4}{15} \end{bmatrix} = \begin{bmatrix} \frac{13}{15} & \frac{2}{5} & \frac{-2}{15} & -\frac{4}{15} \\ \frac{2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{-2}{15} & \frac{2}{5} & \frac{13}{15} & -\frac{4}{15} \\ \frac{-4}{15} & \frac{4}{5} & \frac{4}{5} & \frac{7}{15} \end{bmatrix}$$

Eis a resposta

Q2) a) $T: \mathbb{R}^4 \to \mathbb{R}^4$ TAL QUE $T(x,y,z,w)$ É IGUAL A
$(x+y-z, 3x-4z-5w, -2x+y+3z+5w, 9x+3y-11z-10w)$.

PASSO 1: PARA ACHARMOS OS GERADORES DO NÚCLEO É SÓ IGUALARMOS A TL AO ELEMENTO NULO DO CONTRA-DOMÍNIO

$$T(x,y,z,w) = (x+y-z, 3x-4z-5w, -2x+y+3z+5w, 9x+3y-11z-10w) = (0,0,0,0)$$

$$\begin{cases} x+y-z=0 \\ 3x-4z-5w=0 \\ -2x+y+3z+5w=0 \\ 9x+3y-11z-10w=0 \end{cases}$$

PASSO 2: PODEMOS RESOLVER ESSE SISTEMA USANDO ELIMINAÇÃO GAUSSIANA PARA EM SEGUIDA SUBSTITUIR OS VALORES PARA ENCONTRAR A SOLUÇÃO GERAL:

$$\begin{cases} x-\frac{4}{3}z-\frac{5}{3}w=0 \\ y+\frac{1}{3}z+\frac{5}{3}w=0 \end{cases} \qquad x=\frac{4}{3}z+\frac{5}{3}w \quad y=-\frac{1}{3}z-\frac{5}{3}w$$

$$z=z \qquad w=w$$

PASSO 3: UM CONJUNTO DE GERADORES PARA O NÚCLEO T, É DA FORMA:

$$(x,y,z,w) = \left(\frac{4}{3}z+\frac{5}{3}w, -\frac{1}{3}z-\frac{5}{3}w, z, w\right)$$

$$(x,y,z,w) = z\left(\frac{4}{3}, -\frac{1}{3}, 1, 0\right) + w\left(\frac{5}{3}, -\frac{5}{3}, 0, 1\right)$$

$$NUC(T) = \left\langle \left(\frac{4}{3}, -\frac{1}{3}, 1, 0\right), \left(\frac{5}{3}, -\frac{5}{3}, 0, 1\right) \right\rangle$$

PASSO 4: ENCONTRAMOS UMA BASE PARA O NÚCLEO NUC(T):
ESCALONANDO DE NOVO, PODEMOS CONSTATAR QUE O CONJUNTO DE NUC(T) É L.I, E ASSIM, É UMA BASE PARA N(T), LOGO, $DIM(N(T)) = 2$.

PASSO 5: VAMOS ACHAR UM CONJUNTO DE GERADORES PARA A IMAGEM T
A TRANSFORMAÇÃO T PODE SER ESCRITA DA FORMA:

$$T(x,y,z,w) = (x+y-z,\ 3x-4z-5w,\ -2x+y+3z+5w,\ 9x+3y-11z-10w)$$

$$T(x,y,z,w) = x(1,3,-2,9) + y(1,0,1,3) + z(-1,-4,3,-11) + w(0,-5,5,-10)$$

ASSIM, $IM(T) = \langle (1,3,-2,9), (1,0,1,3), (-1,-4,3,-11), (0,-5,5,-10) \rangle$

PASSO 6: ENCONTRAMOS UMA BASE PARA A IMAGEM $IM(T)$:
ESCALONANDO DE NOVO, PODEMOS CONSTATAR QUE O CONJUNTO DE $IM(T)$ É L.I. E ASSIM É UMA BASE PARA $IM(T)$, LOGO,
$DIM(IM(T)) = 2$

PASSO 7: ENCONTRAMOS A DIMENSÃO DA IMAGEM E DO NÚCLEO:
OBSERVE QUE $DIM(NUC(T)) = 2$ E $DIM(IM(T)) = 2$,
LOGO, PELO TEOREMA DO NÚCLEO E DA IMAGEM,
$DIM(NUC(T)) + DIM(IM(T)) = DIM(R^3) = 4$

b) $T: \mathbb{R}^3 \to \mathbb{R}^4$ DEFINIDA POR

$T(x,y,z) = (x - y - z,\ 2x - 3y + z,\ -x + 2y - 2z,\ 3z - y)$

PASSO 1: PARA ACHARMOS OS GERADORES DO NÚCLEO É SÓ IGUALAMOS A TL AO ELEMENTO NULO DO CONTRA-DOMÍNIO

$T(x,y,z) = (x - y - z,\ 2x - 3y + z,\ -x + 2y - 2z,\ 3z - y) = (0,0,0,0)$

$$\begin{cases} x - y - z = 0 \\ 2x - 3y + z = 0 \\ -x + 2y - 2z = 0 \\ 3z - y = 0 \end{cases}$$

PASSO 2: PODEMOS RESOLVER ESSE SISTEMA USANDO ELIMINAÇÃO GAUSSIANA PARA EM SEGUIDA SUBSTITUIR OS VALORES PARA ENCONTRAR A SOLUÇÃO GERAL:

$$\begin{cases} x - 4z = 0 \\ y - 3z = 0 \end{cases} \qquad \begin{aligned} x &= 4z \\ y &= 3z \\ z &= z \end{aligned}$$

PASSO 3: UM CONJUNTO DE GERADORES PARA O NÚCLEO T, É DA FORMA:

$(x,y,z) = (4z, 3z, z)$

$(x,y,z) = z(4,3,1)$

$Nuc(T) = \langle (4,3,1) \rangle$

PASSO 4: ENCONTRAMOS UMA BASE PARA O NÚCLEO NUC(T): ESCALONANDO DE NOVO, PODEMOS VER QUE O CONJUNTO DE NUC(T) É L.I PORQUE TÊM UMA VARIAVEL LIVRE, E ASSIM, É UMA BASE PARA NUC(T), LOGO, $\dim(Nuc(T)) = 1$

PASSO 5: VAMOS ACHAR UM CONJUNTO DE GERADORES PARA A IMAGEM. A TRANSFORMAÇÃO T PODE SER ESCRITA DA FORMA:

$T(x,y,z) = (x-y-z, 2x-3y+z, -x+2y-2z, 3z-y)$

$T(x,y,z) = x(1,2,-1,0) + y(-1,-3,2,-1) + z(-1,1,-2,3)$

ASSIM, $Im(T) = \langle (1,2,-1,0), (-1,-3,2,-1), (-1,1,-2,3) \rangle$

PASSO 6: ENCONTRAMOS UMA BASE PARA A $Im(T)$:
ESCALONANDO DE NOVO, PODEMOS CONSTATAR QUE O CONJUNTO DE $Im(T)$ É L.I, E ASSIM É UMA BASE PARA $Im(T)$, LOGO,
$\dim(Im(T)) = 2$

PASSO 7: ENCONTRAMOS A DIMENSÃO DA IMAGEM E DO NÚCLEO.
OBSERVE QUE $\dim(Nuc(T)) = 1$ E $\dim(Im(T)) = 2$, LOGO, PELO TEOREMA DO NÚCLEO E DA IMAGEM, $\dim(Nuc(T)) + \dim(Im(T)) = \dim(\mathbb{R}^3) = 3$.

$Q_3$ DETERMINE A TRANSFORMAÇÃO $P_U: \mathbb{R}^4 \to \mathbb{R}^2$ QUE EXECUTA A PROJEÇÃO DO $\mathbb{R}^4$ SOBRE O PLANO DA EQUAÇÃO

$x - y - z = x + y - w = 0.$

PASSO 1: ACHAR OS GERADORES DAS DUAS EQUAÇÕES USANDO ELIMINAÇÃO GAUSSIANA E MONTAR UMA MATRIZ

$$\begin{cases} x - y - z = 0 \\ x + y - w = 0 \end{cases} \xrightarrow[\text{GAUSSIANA}]{\text{ELIMINAÇÃO}} \begin{cases} x - y - z = 0 \\ 2y + z - w = 0 \end{cases}$$

COM AS EQUAÇÕES OBTEREMOS AS SEGUINTES VARIÁVEIS:

$$x = \frac{1}{2}z + \frac{1}{2}w \qquad z = z$$

$$y = -\frac{1}{2}z + \frac{1}{2}w \qquad w = w$$

$$(x, y, z, w) = \left(\frac{1}{2}z + \frac{1}{2}w, -\frac{1}{2}z + \frac{1}{2}w, z, w\right)$$

$$(x, y, z, w) = z\left(\frac{1}{2}, -\frac{1}{2}, 1, 0\right) + w\left(\frac{1}{2}, \frac{1}{2}, 0, 1\right)$$

ACHADO OS GERADORES MONTAMOS UMA MATRIZ

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

PASSO 2: ESCALONAMOS A MATRIZ ACIMA E EM SEGUIDA APLICAMOS GRAM-SCHMIDT NOS VETORES DA BASE

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 \end{bmatrix} \xrightarrow[\text{GAUSSIANA}]{\text{ELIMINAÇÃO}} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

$$\begin{cases} \frac{1}{2}x - \frac{1}{2}y + z = 0 \\ y - z + w = 0 \end{cases} \to \begin{array}{l} x = -z - w \\ y = z - w \\ z = z \\ w = w \end{array}$$

$$(x, y, z, w) = (-z - w, z - w, z, w)$$

$$(x, y, z, w) = z(-1, 1, 1, 0) + w(-1, -1, 0, 1)$$

APLICAMOS O GRAM-SCHMIDT NOS GERADORES

$$\langle(-1,1,1,0),(-1,-1,0,1)\rangle$$

$$\downarrow$$

$$\left\langle\left(-\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},0\right),\left(-\frac{\sqrt{3}}{3},-\frac{\sqrt{3}}{3},0,\frac{\sqrt{3}}{3}\right)\right\rangle$$

PASSO 3: MULTIPLICAMOS O RESULTADO ACIMA POR UM VETOR $(a_1,a_2,a_3,a_4)$, QUE FICA:

$$\left\langle\left(-\frac{\sqrt{3}}{3}a_1,\frac{\sqrt{3}}{3}a_2,\frac{\sqrt{3}}{3}a_3,0\right),\left(-\frac{\sqrt{3}}{3}a_1,-\frac{\sqrt{3}}{3}a_2,0,-\frac{\sqrt{3}}{3}a_4\right)\right\rangle$$

A TRANSFORMAÇÃO $P_U: \mathbb{R}^4 \to \mathbb{R}^2$ É:

$$\left\langle\frac{\sqrt{3}}{3}(-a_1,a_2,a_3,0),\frac{\sqrt{3}}{3}(-a_1,-a_2,0,a_4)\right\rangle$$

Q5) PASSO 1: PEGAMOS OS GERADORES DAS EQUAÇÕES DOS HIPERPLANOS E NORMALIZAMOS USANDO GRAM-SCHMIDT.

$T(x,y,z,w) = w - z - x = 0$

$x = -z + w \quad y = y \quad z = z \quad w = w$

$T(x,y,z,w) = (-z+w, y, z, w)$

$T(x,y,z,w) = y(0,1,0,0) + z(-1,0,1,0) + w(1,0,0,1)$

$\langle (0,1,0,0), (-1,0,1,0), (1,0,0,1) \rangle$

$\downarrow$ GRAM-SCHMIDT

$\alpha = \left\langle (0,1,0,0), \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0\right), \left(\frac{\sqrt{6}}{6}, 0, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right) \right\rangle$

$T(x,y,z,w) = w - 2z - 3y = 0$

$x = x \quad y = y \quad z = z \quad w = 2z + 3y$

$T(x,y,z,w) = (x, y, z, 2z+3y)$

$T(x,y,z,w) = x(1,0,0,0) + y(0,1,0,3) + z(0,0,1,2)$

$\langle (1,0,0,0), (0,1,0,3), (0,0,1,2) \rangle$

$\downarrow$ GRAM-SCHMIDT

$\beta = \left\langle (1,0,0,0), \left(0, \frac{\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10}\right), \left(0, -\frac{3\sqrt{35}}{35}, \frac{\sqrt{35}}{7}, \frac{\sqrt{35}}{35}\right) \right\rangle$

PASSO 2: CALCULAMOS A MATRIZ NA BASE CANÔNICA ENTRE $\alpha$ E $\beta$ DA SEGUINTE MANEIRA:

$(1,0,0,0) = a_1(0,1,0,0) + b_1\left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0\right) + c_1\left(\frac{\sqrt{6}}{6}, 0, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right)$

$a_1 = 0,\ b_1 = -\frac{\sqrt{2}}{2},\ c_1 = \frac{\sqrt{6}}{6} \rightarrow [(1,0,0,0)]_\alpha = \begin{bmatrix} 0 \\ -\sqrt{3}/2 \\ \sqrt{6}/6 \end{bmatrix}$

$\left(0, \frac{\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10}\right) = a_2(0,1,0,0) + b_2\left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0\right) + c_2\left(\frac{\sqrt{6}}{6}, 0, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right)$

$a_2 = \frac{\sqrt{10}}{10},\ b_2 = 0,\ c_2 = \frac{\sqrt{15}}{5} \rightarrow \left[0, \frac{\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10}\right]_\alpha = \begin{bmatrix} \sqrt{10}/10 \\ 0 \\ \sqrt{15}/5 \end{bmatrix}$

$$\left(0, -\frac{3\sqrt{35}}{35}, \frac{\sqrt{35}}{7}, \frac{\sqrt{35}}{35}\right) = a_3(0,1,0,0) + b_3\left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0\right) + c_3\left(\frac{\sqrt{6}}{6}, 0, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right)$$

$$a_3 = -\frac{3\sqrt{35}}{35}, \quad b_3 = \frac{\sqrt{70}}{14}, \quad c_3 = \frac{\sqrt{210}}{30} \to \left[0, -\frac{3\sqrt{35}}{35}, \frac{\sqrt{35}}{7}, \frac{\sqrt{35}}{35}\right]_\alpha = \begin{bmatrix} -3\sqrt{35}/35 \\ \sqrt{70}/14 \\ \sqrt{210}/30 \end{bmatrix}$$

ASSIM NOSSA MATRIZ $[Id]_{\alpha \leftarrow \beta}$ FICA:

$$[Id]_{a \leftarrow b} = \begin{bmatrix} 0 & \frac{\sqrt{10}}{10} & -\frac{3\sqrt{35}}{35} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{70}}{14} \\ \frac{\sqrt{6}}{6} & \frac{\sqrt{15}}{5} & \frac{\sqrt{210}}{30} \end{bmatrix}$$

(Q6) PASSO 1: PRECISAMOS ACHAR OS GERADORES DO NÚCLEO INFORMADO E DEPOIS DETERMINAR A BASE DO NÚCLEO.

$$U(x,y,z,w) = (x - y + z - 3w) = 0$$

$$x = y - z + 3w \quad y = y \quad z = z \quad w = w$$

$$U(x,y,z,w) = (y - z + 3w,\ y,\ z,\ w)$$

$$U(x,y,z,w) = y(1,1,0,0) + z(-1,0,1,0) + w(3,0,0,1)$$

$$NUC(U) = \langle (1,1,0,0), (-1,0,1,0), (3,0,0,1) \rangle$$

PASSO 2: ESCALONAMOS ESSE NÚCLEO E O COMPLETAMOS POS A MATRIZ TÊM QUE SER 4 x 4.

$$NUC(U) = \langle (1,1,0,0), (-1,0,1,0), (3,0,0,1) \rangle$$

$$\downarrow$$

$$NUC(U) = \langle (1,1,0,0), (0,1,1,0), (0,0,3,1), (0,0,0,1) \rangle$$

PASSO 3: DETERMINAREMOS UMA TL QUALQUER QUE QUANDO APLICADO OS VETORES DO NÚCLEO ESCALONANDO, RETORNA AS IMAGENS INFORMADAS:

$$IM: \{(1,0,1,2)\}$$

$$T = \left(-\tfrac{1}{3}x + \tfrac{1}{3}y - \tfrac{1}{3}z + w,\ 0,\ -\tfrac{1}{3}x + \tfrac{1}{3}y - \tfrac{1}{3}z + w,\ -\tfrac{2}{3}x + \tfrac{2}{3}y - \tfrac{2}{3}z + 2w\right)$$

$$T(1,1,0,0)$$

$$T(0,1,1,0)$$

$$T(0,0,1,\tfrac{1}{3})$$

$$T(0,0,0,1)$$

PASSO 4: COMO A BASE DO $\mathbb{R}^4$ É $e = (e_1, e_2, e_3, e_4)$, TEMOS

$$T = \begin{bmatrix} -\frac{1}{3}x & \frac{1}{3}y & -\frac{1}{3}z & 1 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{3}x & \frac{1}{3}y & -\frac{1}{3}z & 1 \\ -\frac{2}{3}x & \frac{2}{3}y & -\frac{2}{3}z & 2 \end{bmatrix}$$

Q7) a) PARA ACHAR UMA BASE ORTOGONAL AO PLANO U, PRECISAMOS ESCOLHER DOIS VETORES ORTOGONAIS AO EIXO $\ell = (1,1,1)$

NO CASO ESCOLHEMOS:

$$v_1 = (1,-1,0) \text{ e } v_2 = (1,0,-1)$$

MAS ANTES, PRECISAMOS NORMALIZAR O EIXO

$$\frac{1}{\sqrt{3}}(1,1,1)$$

AGORA, NORMALIZAMOS OS VETORES USANDO GRAM-SCHMIDT:

$$w_1 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right) \text{ e } w_2 = \left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}\right).$$

COM ISSO FORMAMOS A BASE:

$$\beta = \{u, w_1, w_2\}$$

$$\beta = \left\{\frac{1}{\sqrt{3}}(1,1,1), \left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, 0\right), \left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}\right)\right\}$$

OU

$$\beta = \left\{\underbrace{\frac{\sqrt{3}}{3}(1,1,1)}_{U_1}, \underbrace{\frac{\sqrt{2}}{2}(1,-1,0)}_{U_2}, \underbrace{\frac{\sqrt{6}}{3}\left(\frac{1}{2}, \frac{1}{2}, -1\right)}_{U_3}\right\}$$

b) DETERMINAMOS AS PROJEÇÕES DE $V_1$ E $V_2$ SUBSTITUINDO VALORES NAS SEGUINTES FORMULAS:

$$\text{PROJ}_{V_1} = \left[\frac{\langle V_1|U_1\rangle}{\langle U_1|U_1\rangle}\right]U_1 + \left[\frac{\langle V_1|U_2\rangle}{\langle U_2|U_2\rangle}\right]U_2 + \left[\frac{\langle V_1|U_3\rangle}{\langle U_3|U_3\rangle}\right]U_3$$

$$= \left[\frac{\langle(1,0,0)|U_1\rangle}{\langle U_1|U_1\rangle}\right]U_1 + \left[\frac{\langle(1,0,0)|U_2\rangle}{\langle U_2|U_2\rangle}\right]U_2 + \left[\frac{\langle(1,0,0)|U_3\rangle}{\langle U_3|U_3\rangle}\right]U_3$$

$$= \left[\frac{\frac{\sqrt{3}}{3}}{1}\right]\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) + \left[\frac{\frac{\sqrt{2}}{2}}{1}\right]\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right) + \left[\frac{\frac{\sqrt{6}}{6}}{1}\right]\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}\right)$$

$$= \left[\frac{\sqrt{3}}{3}\right]\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) + \left[\frac{\sqrt{2}}{2}\right]\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right) + \left[\frac{\sqrt{6}}{6}\right]\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{-\sqrt{6}}{3}\right)$$

$$= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) + \left(\frac{1}{2}, -\frac{1}{2}, 0\right) + \left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3}\right)$$

$$\text{Proj } v_1 = (1, 0, 0)$$

$$\text{Proj } v_2 = \left[\frac{\langle \frac{1}{15}(9,-2,-7) \mid u_1 \rangle}{\langle u_1 \mid u_1 \rangle}\right] u_1 + \left[\frac{\langle \frac{1}{15}(9,-2,-7) \mid u_2 \rangle}{\langle u_2 \mid u_2 \rangle}\right] u_2 +$$

$$\left[\frac{\langle \frac{1}{15}(9,-2,-7) \mid u_3 \rangle}{\langle u_3 \mid u_3 \rangle}\right] u_3$$

$$= \left[\frac{0}{1}\right]\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) + \left[\frac{\frac{11\sqrt{2}}{30}}{1}\right]\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right) + \left[\frac{\frac{7\sqrt{6}}{30}}{1}\right]\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{-\sqrt{6}}{3}\right)$$

$$= (0, 0, 0) + \left[\frac{11}{30}, \frac{-11}{30}, 0\right] + \left[\frac{7}{30}, \frac{7}{30}, \frac{-7}{15}\right]$$

$$= \left[\frac{3}{5}, \frac{-2}{15}, \frac{-7}{15}\right]$$

c) Conseguimos encontrar o ângulo entre $v_1$ e $v_2$; pela seguinte fórmula:

$$\vec{v_1} \cdot \vec{v_2} = v_1 v_2 \cos\theta$$

$$\langle v_1 \mid v_2 \rangle = |v_1| \cdot |v_2| \cos\theta$$

$$\langle (1,0,0) \mid \tfrac{1}{15}(9,-2,-7) \rangle = |(1,0,0)| \cdot \left|\tfrac{1}{15}(9,-2,-7)\right| \cos\theta$$

$$\frac{3}{5} = 1 \cdot \frac{\sqrt{134}}{15} \cos\theta$$

$$45 = 5\sqrt{134}\cos(\theta)$$

$$9 = \sqrt{134}\cos(\theta)$$

$$\cos(\theta) = \frac{9}{\sqrt{134}}$$

$$\cos(\theta) = \frac{9\sqrt{134}}{134}$$

$$\theta = \arccos\left(\frac{9\sqrt{134}}{134}\right)$$

$$\theta = 39^\circ \text{ ou}$$

$$\theta = \frac{13\pi}{60}$$

d) No caso da rotação de $\rho$, usamos outra fórmula:

$$(\rho)_{\mathcal{E}} = (id)_{\mathcal{BE}} \cdot (\rho)_{\mathcal{B}} \cdot (id)_{\mathcal{BE}}^{t}$$

$$(\rho)_{\mathcal{E}} = \begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} \\ \frac{\sqrt{3}}{3} & 0 & -\frac{\sqrt{6}}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\operatorname{sen}(\theta) \\ 0 & \operatorname{sen}(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{3} \end{bmatrix}$$

$$(\rho)_{\mathcal{E}} = \begin{bmatrix} \frac{1}{3} + \frac{2\cos\theta}{3} & \frac{1}{3} - \frac{\sqrt{3}\operatorname{sen}\theta + \cos\theta}{3} & \frac{1}{3} + \frac{\sqrt{3}\operatorname{sen}\theta}{3} - \frac{\cos\theta}{3} \\ \frac{1}{3} + \frac{\sqrt{3}\operatorname{sen}\theta - \cos\theta}{3} & \frac{1}{3} + \frac{2\cos\theta}{3} & \frac{1}{3} - \frac{\sqrt{3}\operatorname{sen}\theta}{3} - \frac{\cos\theta}{3} \\ \frac{1}{3} - \frac{\sqrt{3}\operatorname{sen}\theta}{3} - \frac{\cos\theta}{3} & \frac{1}{3} + \frac{\operatorname{sen}\theta\sqrt{3}}{3} - \frac{\cos\theta}{3} & \frac{1}{3} + \frac{2\cos\theta}{3} \end{bmatrix}$$

e)