Aux 2 - Pauta

P1 $P(\text{hijo h-cr} \mid \text{sólo papá}) = a$

$P(\text{hijo h-cr} \mid \text{sólo mamá}) = b$

$P(\text{hijo h-cr} \mid \text{papá y mamá}) = 1$

$P(\text{papá h-cr}) = P(\text{mamá h-cr}) = p$

a) $$P(\text{solo mamá} \mid \text{hijo h-cr}) = \frac{P(\text{hijo h-cr} \mid \text{solo mamá})\,P(\text{solo mamá})}{P(\text{hijo h-cr})}$$

$$P(\text{solo mamá}) = P(\text{mamá h-cr} \cap \text{papá } \underline{\text{no}} \text{ h-cr})$$
$$= P(\text{mamá h-cr})\,P(\text{papá no h-cr}) = p \cdot (1-p)$$

$$P(\text{hijo h-cr}) = P(\text{hijo} \mid \text{solo mamá})\,P(\text{solo mamá}) + P(\text{hijo} \mid \text{solo papá})\,P(\text{solo papá}) + P(\text{hijo} \mid \text{papá y mamá})\,P(\text{papá y mamá})$$

$$= b \cdot p(1-p) + a \cdot p(1-p) + 1 \cdot p \cdot p = p(1-p)(a+b) + p^2$$

$$\Rightarrow P(\text{solo mamá} \mid \text{hijo h-c}) = \frac{b \cdot p(1-p)}{p(1-p)(a+b) + p^2}$$

b) $$P(\text{hijo } 2 \mid \text{hijo } 1) = \frac{P(\text{hijo } 1 \cap \text{hijo } 2)}{P(\text{hijo } 1)}$$

$$\Rightarrow P(h_1 \cap h_2) = P(h_1 \cap h_2 \cap \text{solo papá}) + P(h_1 \cap h_2 \cap \text{solo mamá}) + P(h_1 \cap h_2 \cap \text{ambos})$$

$\Rightarrow P(\text{solo papá} \cap h_1 \cap h_2) = P(\text{solo papá}) P(h_1 | \text{solo papá})$

$P(h_2 | h_1 \cap \text{solo papá}) = p(1-p) \cdot a \cdot a$

$\Rightarrow P(\text{solo mamá} \cap h_1 \cap h_2) = p(1-p) \cdot b \cdot b$

$\Rightarrow P(\text{ambos} \cap h_1 \cap h_2) = p^2 \cdot 1 \cdot 1$

$$\Rightarrow P(h_1 \cap h_2) = p(1-p)a^2 + p(1-p)b^2 + p^2$$
$$= p(1-p)(a^2+b^2) + p^2$$

$$\Rightarrow P(\text{hijo } 2 | \text{hijo } 1) = \frac{p(1-p)(a^2+b^2) + p^2}{p(1-p)(a+b) + p^2}$$

P2) $P(\text{sacar blanca urna } n) = P_n = P(\text{sacar blanca} | n-1 \text{ blanca}) P_{n-1}$
$+ P(\text{sacar blanca} | n-1 \text{ negra}) \cdot \overline{P_{n-1}}$

$$= \frac{\alpha+1}{\alpha+\beta+1} \cdot P_{n-1} + \frac{\alpha}{\alpha+\beta+1}(1-P_{n-1}) = \frac{\alpha}{\alpha+\beta+1} + \frac{P_{n-1}}{\alpha+\beta+1}$$

$$\Rightarrow P_{n-1} = \frac{\alpha}{\alpha+\beta+1} + \frac{P_{n-2}}{\alpha+\beta+1} \Rightarrow P_n = \frac{\alpha}{\alpha+\beta+1} + \frac{\alpha}{(\alpha+\beta+1)^2} + \frac{P_{n-2}}{(\alpha+\beta+1)^2}$$

$$\Rightarrow P_n = \frac{\alpha}{\alpha+\beta+1} + \frac{\alpha}{(\alpha+\beta+1)^2} + \dots + \frac{\alpha}{(\alpha+\beta+1)^{n-1}} + \frac{P_2}{(\alpha+\beta+1)^{n-1}}$$

$$\Rightarrow P_1 = 1 \Rightarrow P_n = \sum_{i=1}^{n-1} \frac{\alpha}{(\alpha+\beta+1)^i} + \frac{1}{(\alpha+\beta+1)^{n-1}}$$

$¿n \to \infty? \Rightarrow P_n = \alpha \sum_{i=1}^{n-1} \left(\frac{1}{\alpha+\beta+1}\right)^i + \left(\frac{1}{\alpha+\beta+1}\right)^{n-1}$

$= \alpha \sum_{i=1}^{n-1} r^i + r^{n-1} = \alpha \cdot \frac{(r^n - r)}{r-1} + r^{n-1}$

$n \to \infty \Rightarrow \alpha \left(\frac{\frac{1}{\alpha+\beta+1}}{1 - \frac{1}{\alpha+\beta+1}}\right) = \frac{\alpha}{\alpha+\beta}$ //

P3 | a.1 | $P(A) = \sum_{i=1}^{n} P(A|E_i) P(E_i) = \sum_{i=1}^{n} \alpha_i p_i$

a.2 | $P(E_1|\bar{A}) = \frac{P(\bar{A}|E_1) P(E_1)}{P(\bar{A})}$

$P(\bar{A}|E_1) = 1 - P(A|E_1) = 1 - \alpha_1$

Dem: $(P(A|B))^c = 1 - P(A|B) = 1 - \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$\frac{P(B \cap (A \cap B)^c)}{P(B)} \underset{\text{De M.}}{=} \frac{P(B \cap (A^c \cup B^c))}{P(B)} = \frac{P((B \cap A^c) \cup (B \cap B^c))}{P(B)}$

$= \frac{P(A^c \cap B)}{P(B)} = P(A^c|B) = [P(A|C)]^c$

$P(E_1) = P_1$; $P(\bar{A}) = 1 - P(A) = 1 - \sum_{i=1}^{n} \alpha_i p_i$

$\Rightarrow P(E_1|\bar{A}) = \frac{(1-\alpha_1) P_1}{1 - \sum_i \alpha_i P_i}$

$¿\alpha_i = \alpha \ \forall i? \Rightarrow P(E_1|\bar{A}) = \dfrac{(1-\alpha)P_1}{1-\alpha\sum_i P_i} = \dfrac{(1-\alpha)P_1}{(1-\alpha)} = P_1$

b) B: se buscó en j y no se encontró

$$P(B) = P(\bar{A}|E_j)P(E_j) + P(\bar{A}|\bar{E_j})P(\bar{E_j})$$
$$= (1-\alpha_j)P_j + 1\cdot(1-P_j) = 1-\alpha_jP_j = P(\bar{A}|E_j)$$

$$\Rightarrow P(E_i|B) = \frac{P(B|E_i)P(E_i)}{P(B)} = \frac{1\cdot P_i}{1-\alpha_jP_j} = \frac{P_i}{1-\alpha_jP_j}$$

P4

$P(+|enf.) = 0{,}99 \Rightarrow P(-|enf) = 0{,}01$

$P(-|sano) = 0{,}8 \qquad P(+|sano) = 0{,}2$

$P(enf.) = 0{,}6 \Rightarrow P(sano) = 0{,}4$

a) $P(-|enf.) = 0{,}01 = 1\%$

b) $P(error) = P(-|enf)P(enf) + P(+|sano)P(+)$

$$= 0{,}01\cdot 0{,}6 + 0{,}2\cdot 0{,}4 = 0{,}086 = 8{,}6\%$$

c) $P(enf|+) = \dfrac{P(+|enf)P(enf)}{P(+)}$

$\Rightarrow P(+) = P(+|enf.)P(enf.) + P(+|sano)P(sano)$

$$= 0{,}99\cdot 0{,}6 + 0{,}2\cdot 0{,}4 = 0{,}674 = 67{,}4\%$$

$$P(enf|+) = \frac{0{,}99\cdot 0{,}6}{0{,}674} = 0{,}881 = 88{,}1\%$$