Exam

written test $\to$ 2 hours

2 parts :

1) theory questions 3 (~4 points each)
2) problem solving 2 (~10 points each)

~32 points

$\to$ see Moodle for some examples

Exercise (approx algo)

Given a graph $G = (V, E)$, recall that a matching $M \subseteq E$ is a subset of edges that do not share vertices. We want to compute a matching of maximum size (that is, containing as many edges as possible).

$\exists$ polynomial-time algorithms, but are slow/complicated

Consider the following simple algorithm:

GREEDY-MATCHING(G)

$m = |E|$

$M = \emptyset$

let $E = \{e_1, e_2, \dots, e_m\}$

for $i=1$ to m do

if $\forall e \in M \quad e \cap e_i = \emptyset$ then

$M = M \cup \{e_i\}$

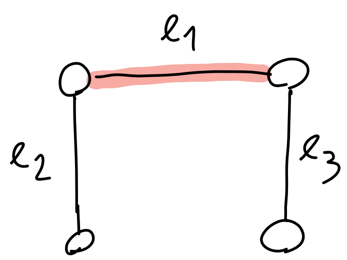
return M

maximum $\neq$

obs.: this algorithm returns a maximal matching

↳ can't be augmented

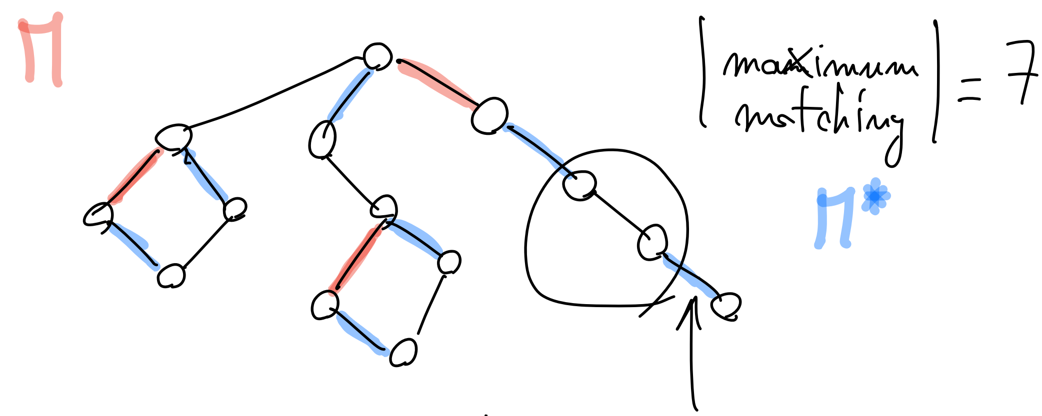
1) Give a graph G for which GREEDY-MATCHING returns a solution with half the edges of an optimal solution.



$M = \{e_1\}$

maximum matching $= \{e_2, e_3\}$

2) Prove that GREEDY-MATCHING is a 2-approximation algorithm
(Hint: reason by contradiction)



clearly, $|M| \leq |M^*|$

need to show $|M| \geq \frac{|M^*|}{2}$

suppose, by contradiction, that $|M| < \frac{|M^*|}{2}$

3 7

edges of M cover $2|M|$ vertices

$< |M^*|$

key part

$\Rightarrow \exists$ edge of M^* that does <u>not</u> cover any vertex covered by edges of M

$\Rightarrow$ that edge(s) can be added to M, obtaining a matching: contradiction, because M is a maximal matching.