Exercises: show that:

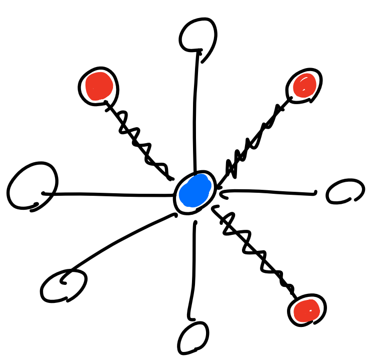
- vertex cover $\leq_p$ ind. set
- clique $\leq_p$ vertex cover

- "same" as ind. set $\leq_p$ vertex cover
- 

G has a clique of size K $\Leftrightarrow$ $\bar{G}$ has a vertex cover of size $n-K$

proof: see the book

Exercise: modify Approx-vertex-cover so as to select only one vertex of the chosen edge instead of both of them. $\rho = ?$



"star graph"

$OPT = 1$

$ALG = n-1$ $\quad \rightarrow \rho \geq n-1$

Exercise: NP-hardness of Metric TSP

1) $w'(u,v) \overset{?}{\leq} w'(u,w) + w'(w,v)$

$\qquad\qquad \| \qquad\qquad\qquad\qquad \|$

$$w(u,v) + W \overset{?}{\leq} w(u,w) + w(w,v) + 2W$$

$$w(u,v) \overset{?}{\leq} w(u,w) + w(w,v) + W$$

$$\underbrace{w(u,w)}_{\geq 0} + \underbrace{w(w,v)}_{\geq 0} + \underbrace{W - w(u,v)}_{\geq 0} \overset{?}{\geq} 0$$

2) "$\Rightarrow$" $\exists$ Ham. circuit of cost K in G $\Rightarrow$ the same circuit introduces a $+W$ $\forall$ edge $\Rightarrow$ in G' its cost is $K + nW$

"$\Leftarrow$" just remove the $+W$ $\forall$ edge to obtain a Ham. circuit of cost K in G

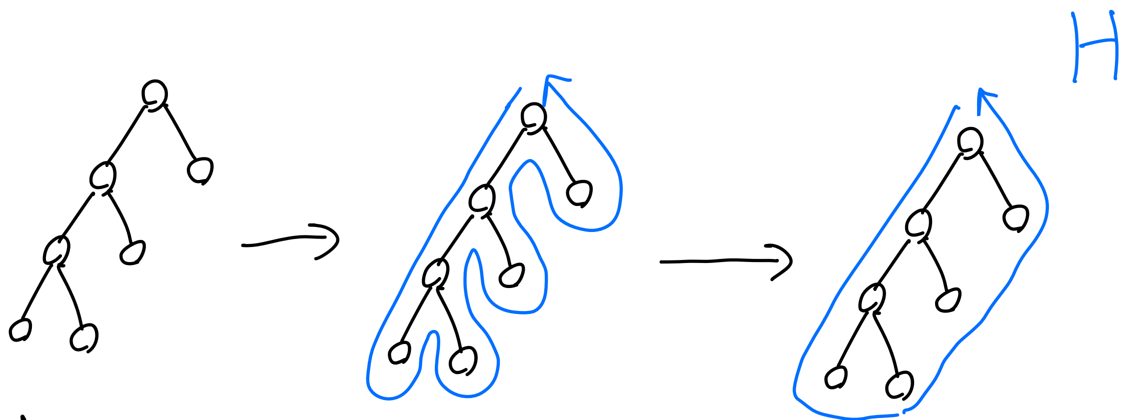
Metric TSP: a 2-approximation algorithm

vertex cover $\rightsquigarrow$ matching

metric TSP $\rightsquigarrow$ MST

intuition:

MST

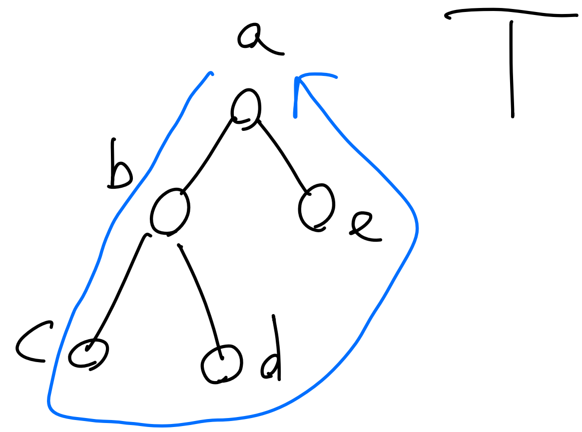


tree $\qquad\qquad\qquad\qquad$ cycle

tree $\longrightarrow$ cycle?

```
PREORDER (T, v)
    print (v)
    if internal (v) do
        for each u ∈ children (v) do
            PREORDER (u)
    return
```

Example:



$\rightarrow$ a, b, c, d, e

idea: add to the preorder list the root $\rightarrow$ Ham. cycle of the original graph

APPROX-Metric-TSP (G)

$V = \{v_1, v_2, \dots, v_n\}$

$r = v_1$ // root, from which PRIM is run

$T^* = \text{PRIM}(G, r)$

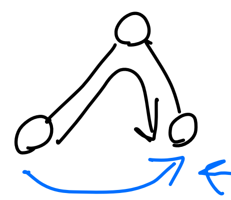
$H' = \langle v_{i_1}, v_{i_2}, \dots, v_{i_n} \rangle = \text{PREORDER}(T^*, r)$

return $H = \langle H', v_{i_1} \rangle$

Analysis of the cost of H

intuition: 1) cost of T^* is "low" (actually, the lowest)

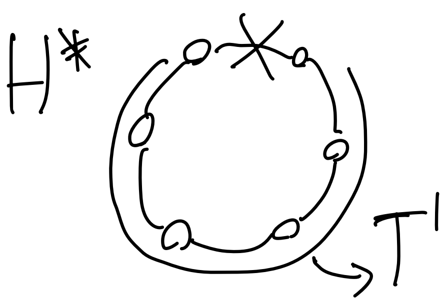
2) triangle ineq. $\Rightarrow$ "shortcuts" do not increase the cost



1) Lower bound to the cost of H^* (= optimal tour)

(for v.c.: $|V^*| \geq |A|$)

$w(H^*) \geq ?$



weights are ≥ 0

$w(H^*) \geq w(T')$

T' is a spanning tree

$\Downarrow$

$w(T') \geq w(T^*)$

$w(H^*) \geq w(T^*)$

2) Upper bound to the cost of H

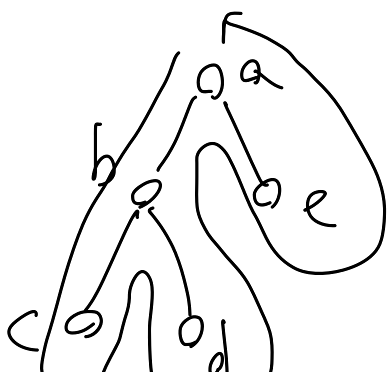
$$\underline{w(H) \leq \rho\, \cancel{w(T^*)}} \leq \rho\, w(H^*)$$

$\rho = 2$

$w(H) \leq 2\, w(T^*)$?

Definition: given a tree, a <u>full preorder chain</u> is a list with repetitions of the vertices of the tree which identifies the vertices reached from the recursive calls of PREORDER(T, v)

Example:



f.p.c. : a, b, c, b, d, b, a, e, a

Property: $w(\text{f.p.c.}) = 2\,w(T^*)$

since every edge of T^* appears twice in a f.p.c.

$$2\,w(T^*) = w(\langle a, b, c, \cancel{b}, d, b, a, e, a \rangle)$$

$$\geq w(\langle a, b, c, d, \cancel{b}, a, e, a \rangle)$$

$$\geq w(\langle a, b, c, d, \cancel{a}, e, a \rangle)$$

$$\geq w(\langle a, b, c, d, e, a \rangle)$$

triangle ineq.

shortcut

$$\Longrightarrow \quad 2\,w(T^*) \geq w(H)$$

Putting pieces together:

1) $w(H^*) \geq w(T^*)$

2) $2\,w(T^*) \geq w(H)$

$$2w(H^*) \geq \cancel{2w(T^*)} \geq w(H)$$

$$\Rightarrow \frac{w(H)}{w(H^*)} \leq 2$$

Exercise: show that the above analysis is tight by giving an example of a graph where Approx-metric-TSP returns a solution of cost $2 \cdot w(H^*)$