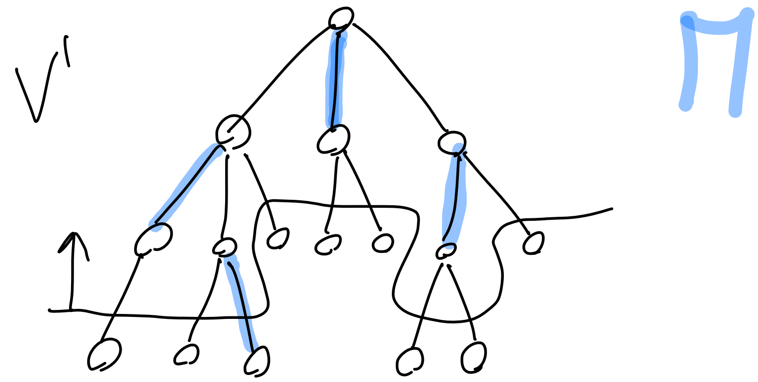
Exercise: an alternative 2-approx for vertex cover

Solution: idea: show a large enough matching in the DFS tree



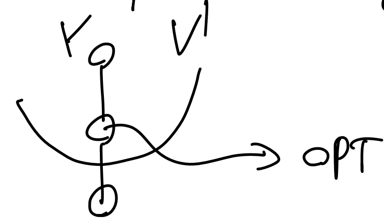
Let r be the root, choose one child v and add (r,v) to the matching Π. $\forall$ level $i \geq 1$ consider all vertices v not touched by any edge of Π; choose a child u and add (v,u) to Π. Repeat up to the leaves.

$$|V'| \leq 2|\Pi|$$

$\forall$ matching Π' of G, $|V^*| \geq |\Pi'|$

$$|V'| \leq \cancel{2|\Pi|} \leq 2|V^*|$$

Show that the 2 factor is tight :



Exercise: NP-hardness of Metric TSP

1) $w'(u,v) \overset{?}{\leq} w'(u,w) + w'(w,v)$

$w(u,v) + W \overset{?}{\leq} w(u,w) + w(w,v) + 2W$

$w(u,v) \overset{?}{\leq} w(u,w) + w(w,v) + W$

$\underbrace{w(u,w)}_{\geq 0} + \underbrace{w(w,v)}_{\geq 0} + \underbrace{W - w(u,v)}_{\geq 0} \overset{?}{\geq} 0$

2) $\Rightarrow$: $\exists$ Ham. circuit of cost k in G
$\Rightarrow$ the same circuit introduces a $+W$ $\forall$ edge $\Rightarrow$ in G' its cost is $k + nW$

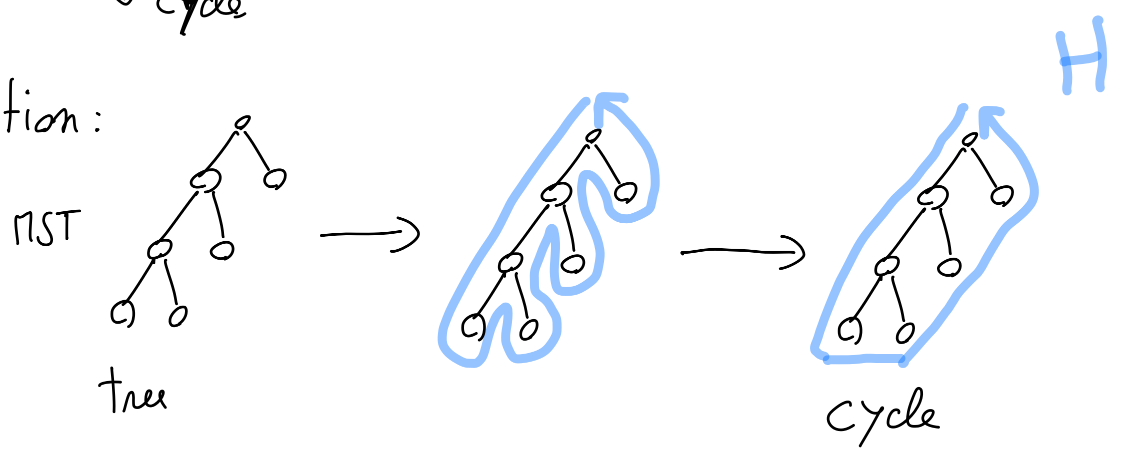
$\Leftarrow$: just remove the $+W$ $\forall$ edge to obtain a Ham. circuit of cost k in G

Metric TSP : a 2-approximation algorithm

V.C. $\rightsquigarrow$ matching

Metric TSP $\rightsquigarrow$ minimum spanning tree
$\downarrow$ cycle

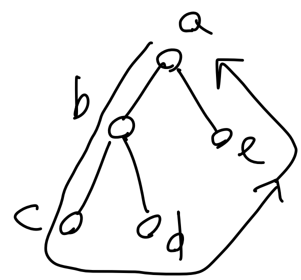
intuition:



tree $\longrightarrow$ cycle?

```
PREORDER(T, v)
    print(v)
    if internal(v) do
        for each u ∈ children(v) do
            PREORDER(u)
    return
```

Example:



T

a, b, c, d, e

idea: add to the preorder list the root $\Rightarrow$ Ham. cycle of the original graph.

APPROX_METRIC_TSP (G)

$V = \{v_1, v_2, \dots, v_n\}$

$r \leftarrow v_1$ // root, from which PRIM is run

$T^* \leftarrow \text{PRIM}(G, r)$

$\langle v_{i_1}, v_{i_2}, \dots, v_{i_n} \rangle = H' \leftarrow \text{PREORDER}(T^*, r)$

return $\langle H', v_{i_1} \rangle = H$

Analysis of the cost of H

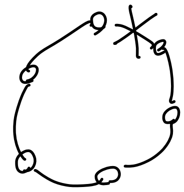
intuition: 1) cost of T^* is "low" (actually, the lowest)

2) triangle ineq. $\Rightarrow$ "shortcuts" do not increase the cost

1) lower bound to the cost of H^* (= optimal tour)

(for v.c.: $|V^*| \geqslant |A|$)

$w(H^*) \geqslant ?$

H* 

$w(T^*)$

2) upper bound to the cost of H

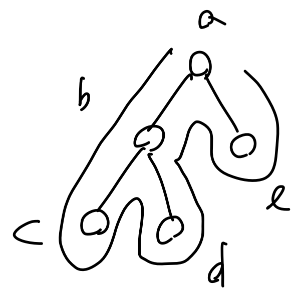
$$w(H) \leqslant \cancel{\alpha w(T^*)} \leqslant \alpha w(H^*)$$

$$\alpha = 2$$

$$w(H) \leqslant 2 w(T^*) \ ?$$

Definition: given a tree, a <u>full preorder chain</u> is a list with repetitions of the vertices of the tree which identifies the vertices reached from the recursive calls of PREORDER(T, v).

Example :



$$\text{f.p.c.}: \ a, b, c, b, d, b, a, e, a$$

Property: $w(\text{f.p.c.}) = 2w(T^*)$

$$2w(T^*) = w(\langle a,b,c,b,d,b,a,e,a\rangle)$$

$$\geq w(\langle a,b,c,d,b,a,e,a\rangle)$$

$$\geq w(\langle a,b,c,d,e,a\rangle)$$

triangle ineq.

shortcut

$$\Rightarrow 2w(T^*) \geq w(H)$$

Putting pieces together :

1) $w(H^*) \geq w(T^*)$

2) $2w(T^*) \geq w(H)$

$$2w(H^*) \geq \cancel{2w(T^*)} \geq w(H)$$

$$\Rightarrow \frac{w(H)}{w(H^*)} \leq 2$$

Exercise: show that the above analysis is tight by giving an example of a graph where APPROX_METRIC_TSP returns a solution of cost $2 \cdot H^*$