University of Padova
Master's degree in Computer Science

Advanced Algorithms

Spring 2023

August 30, 2023 – 14:30–16:30

First Part: Theory Questions

Question 1 (4 points) Let $G = (V, E)$ be an undirected graph with n vertices and m edges. Given two vertices $s, t \in V$, briefly describe how to find, if it exists, a path from s to t in time $O(n + m)$.

Question 2 (4 points) Consider the following directed, weighted graph, represented by an adjacency matrix where each numerical value represents the weight of the corresponding edge, and where the symbol '–' indicates the absence of the edge between the corresponding vertices.

	s	a	b	c	d
s	-	2	4	-	-
a	-	-	-1	2	-
b	-	-	-	-	4
c	-	-	-	-	2
d	-	-	-	-	-

(a) Draw the graph.

(b) Run the Bellman-Ford algorithm on this graph, using vertex s as the source. You are to return the trace of the execution, i.e. a table with rows indexed by vertices and columns indexed by iteration indexes (starting from 0) where each entry contains the estimated distance between s and that vertex at that iteration.

Question 3 (4 points) Define the vertex cover problem and briefly describe a 2-approximation algorithm seen in class.

Second Part: Problem Solving

Exercise 1 (10 points) In the *maximum coverage* problem, the input consists of m subsets $S_1, S_2, \ldots, S_m$ of a ground set X, and a budget k; the goal is to choose k of the subsets to maximize their *coverage*, defined as the number of distinct ground set elements they contain. Prove that this problem is NP-hard.

Exercise 2 (9 points) Suppose you toss $n \gg 1$ times a coin: applying the following Chernoff bound show that the probability that you obtain more than $n/2 + \sqrt{6n \ln n}/2$ heads is at most $1/n$.[1]

Theorem 1. *Let $X_1, X_2, \ldots, X_n$ be independent indicator random variables such that $E[X_i] = p_i, 0 < p_i < 1$. Let $X = \sum_{i=1}^{n} X_i$ and $\mu = E[X]$. Then, for $0 < \delta \leq 1$,*

$$\Pr(X > (1 + \delta)\mu) \leq e^{-\mu\delta^2/3}.$$

[1] Recall that $\ln n = \log_e n$.