02-11-2020

lunedì 2 novembre 2020 08:24

LIMITI DI SUCCESSIONI

$$\lim_{n\to+\infty} \frac{n^{n+1}-(n+1)^{n+1}}{n^n} =$$

$n^{n+1} \to +\infty \qquad (n+1)^{n+1} \to +\infty$

$n^n \to +\infty$

$$= \lim_{n\to+\infty} \left(\frac{n^{n+1}}{n^n} - \frac{(n+1)^{n+1}}{n^n} \right)$$

$$= \lim_{n\to+\infty} \left(\frac{\cancel{n^n} \cdot n}{\cancel{n^n}} - \frac{(n+1)^n (n+1)}{n^n} \right)$$

$$= \lim_{n\to+\infty} \left(n - \left(\frac{n+1}{n}\right)^n (n+1) \right)$$

$$= \lim_{n\to+\infty} \left(n - \underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e} n - \underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e} \right)$$

$$= \lim_{n\to+\infty} \overset{\to+\infty}{n} \underbrace{\left[1 - \underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e} - \underbrace{\frac{1}{n}\left(1+\frac{1}{n}\right)^n}_{\to 0} \right]}_{\to 1-e}$$

$$= -\infty$$

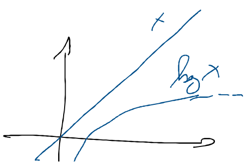
$$\lim_{n\to+\infty} n^{\frac{1}{n}} = \infty^0$$

ci mettiamo in base e

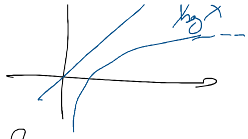
$$n^{\frac{1}{n}} = e^{\log\left(n^{\frac{1}{n}}\right)} \qquad [\log = \ln]$$

$$= e^{\frac{1}{n}\log n}$$

ora $\displaystyle \lim_{n\to+\infty} \frac{\log n}{n} = 0$



Ora $\lim_{n \to +\infty} \frac{\log n}{n} = 0$



quindi per composizione si ha

$$\lim_{n \to +\infty} n^{\frac{1}{n}} = \lim_{n \to +\infty} e^{\frac{1}{n}\log n} = e^0 = 1$$

- $\lim_{n \to +\infty} n^{\frac{1}{\log n}} = \infty^0$

$$n^{\frac{1}{\log n}} = e^{\cancel{\frac{1}{\log n}} \cdot \cancel{\log n}} = e$$

quindi $\lim_{n \to +\infty} n^{\frac{1}{n}} = \lim_{n \to +\infty} e = e$

- $\lim_{n \to +\infty} n^{\frac{1}{\log_a n}} = ?$ --

- $\lim_{n \to +\infty} \log_{e^n} n$

utilizziamo il cambio di base dei logaritmi:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

dove "c" è la nuova base scelta.

Allora $\log_{e^n} n = \frac{\log n}{\log e^n} =$

$$= \frac{\log n}{n \log e} = \frac{\log n}{n}$$

così $\lim_{n \to +\infty} \log_{e^n} n = \lim_{n \to +\infty} \frac{\log n}{n} = 0$

$\frac{1}{n}$

$$\lim_{n\to+\infty} \frac{n^{\frac{1}{n}}-1}{(\log n)(\operatorname{sen}\frac{1}{n})}$$

$n^{\frac{1}{n}} \to 1 \qquad n^{\frac{1}{n}}-1 \to 0$

$\log n \to +\infty \qquad \operatorname{sen}\frac{1}{n} \to 0$

$$\lim_{n\to+\infty} \frac{e^{\frac{1}{n}\log n}-1}{(\log n)(\operatorname{sen}\frac{1}{n})} \qquad \frac{1}{n}\log n \to 0$$

ci ricordiamo del limite notevole

$$\lim_{t\to 0} \frac{e^{t}-1}{t} = 1 \qquad t \to 0$$

allora moltiplichiamo e dividiamo per $\frac{\log n}{n}$ con

$$\lim_{n\to+\infty} \frac{e^{\frac{\log n}{n}}-1}{\frac{\log n}{n}} \cdot \frac{\cancel{\log n}}{n} \cdot \frac{1}{(\cancel{\log n})(\operatorname{sen}\frac{1}{n})}$$

$\to 1 \qquad \to 1$

perché ponendo $\frac{\log n}{n} = t$ diventa

$$\lim_{t\to 0} \frac{e^{t}-1}{t} = 1$$

inoltre $\frac{1}{n \operatorname{sen}\frac{1}{n}} = \frac{\left(\frac{1}{n}\right)}{\operatorname{sen}\left(\frac{1}{n}\right)} = 1$

perché è limite notevole

Allora il limite vale $1 \cdot 1 = 1$

$$\lim \quad n \log n^{2} - \log(\log n) \qquad (R=2)$$

$$\lim_{n \to +\infty} \frac{n \log n^2 - \log(\log n)}{\log(n^n \log n)} \qquad (\beta = 2)$$

$$\lim_{n \to +\infty} \frac{n\left(\sqrt[n]{n} - 1\right)}{\log(n+2)! - \log n!}$$

$$\lim_{n \to +\infty} \frac{n\left(e^{\frac{\log n}{n}} - 1\right)}{\underbrace{\log\left(\frac{(n+2)!}{n!}\right)}_{\log\left(\frac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}}\right)}}$$

$$\lim_{n \to +\infty} \cancel{n} \underbrace{\frac{e^{\frac{\log n}{n}} - 1}{\frac{\log n}{n}}}_{\to 1} \boxed{\frac{\log n}{\cancel{n}} \cdot \frac{1}{\log[(n+2)(n+1)]}}_{\to \frac{1}{2}} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

ora osserviamo che

$$\lim_{n \to +\infty} \frac{\log n}{\log(n+2)(n+1)} =$$

$$\lim_{n \to +\infty} \frac{\log n}{\log(n^2 + 3n + 2)} =$$

$$\lim_{n \to +\infty} \frac{\log n}{\log\left(n^2\left(1 + \frac{3}{n} + \frac{2}{n^2}\right)\right)}$$

$$\lim_{n \to +\infty} \frac{\log n}{\underbrace{\log(n^2)}_{2 \log n} + \log\left(1 + \frac{3}{n} + \frac{2}{n^2}\right)}$$

$$\lim \frac{\cancel{\log n}}{} = 1$$

$$\lim_{n\to+\infty} \frac{\cancel{\log n}}{\cancel{\log n}\left(2+\underbrace{\frac{\log\left(1+\frac{2}{n}+\frac{2}{n^2}\right)}{\log n}}_{\to 0}\right)} = \frac{1}{2}$$

$$\lim_{n\to+\infty} n\left(1-\sqrt{\left(1-\underbrace{\frac{a}{n}}_{\to 0}\right)\left(1-\underbrace{\frac{b}{n}}_{\to 0}\right)}\right) = \infty\cdot 0$$

$a \in \mathbb{R}$, $b \in \mathbb{R}$

Osserviamo che:

$$n\cdot\frac{\left(1-\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}\right)\left(1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}\right)}{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}$$

$$= n\,\frac{1^2-\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}$$

$$= n\cdot\frac{\cancel{1}-\cancel{1}+\frac{b}{n}+\frac{a}{n}-\frac{ab}{n^2}}{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}$$

$$= \frac{b+a-\underbrace{\frac{ab}{n}}_{\to 0}}{1+\sqrt{\left(1-\underbrace{\frac{a}{n}}_{\to 0}\right)\left(1-\underbrace{\frac{b}{n}}_{\to 0}\right)}} = \frac{a+b}{2}$$

$$\lim_{n\to+\infty}\left(\sqrt{n+\frac{1}{n}}-\sqrt{n-\frac{1}{n}}\right)\cdot n$$

$$\lim_{n\to+\infty}\sqrt{an+n^2}-\sqrt{n^2-bn}$$

$$\lim_{n\to+\infty}\left(\frac{an+b}{cn+d}\right)^{n} \xrightarrow{} +\infty \qquad c\neq 0,\ d\neq 0,\ a\neq 0$$

questo ricorda $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}$

osserviamo che:

$$\lim_{n\to+\infty}\frac{an+b}{cn+d}=\lim_{n\to+\infty}\frac{n\left(a+\frac{b}{n}\right)}{n\left(c+\frac{d}{n}\right)}=\frac{a}{c}$$

quindi

se $\frac{a}{c}>1$ $(a>c)$ il limite vale $+\infty$

se $\frac{a}{c}<1$ il limite vale zero.

se $\frac{a}{c}=1$ ci riconduciamo al limite notevole

$$\left(\frac{an+b}{\underset{\overset{\shortparallel}{c}}{a}n+d}\right)^{n}=\left(\frac{an+d-d+b}{an+d}\right)^{n}=$$

$$=\left(1+\frac{b-d}{an+d}\right)^{n} \qquad (b\neq d)$$

$$=\left(1+\frac{1}{\frac{an+d}{b-d}}\right)^{n} \qquad \frac{an+d}{b-d}\to+\infty$$

$$=\Bigg[\underbrace{\left(1+\frac{1}{\frac{an+d}{b-d}}\right)^{\frac{an+d}{b-d}}}_{\to e}\Bigg]^{\frac{b-d}{an+d}\cdot n} \qquad \frac{b-d}{an+d}\cdot n \to \frac{b-d}{a}$$

$\frac{b-d}{\ldots}$

$$= e^{\frac{b-d}{a}}$$

e