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INTEGRALI ELEMENTARI

$$\int e^x\,dx = e^x + c$$

$$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} + c \qquad \alpha \neq -1$$

$$\int \frac{1}{x}\,dx = \log|x| + c \qquad (\alpha = -1)$$

$$\int \operatorname{sen} x\,dx = -\cos x + c \qquad \int \frac{1}{\cos^2 x}\,dx = \operatorname{tg} x + c$$

$$\int \cos x\,dx = \operatorname{sen} x + c \qquad \int \frac{1}{\operatorname{sen}^2 x}\,dx = -\operatorname{ctg} x + c$$

$$\int \operatorname{senh} x\,dx = \cosh x + c \qquad \int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c$$

$$\int \cosh x\,dx = \operatorname{senh} x + c \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{arcsen} x + c$$

INTEGRALI IMMEDIATI

$$\int \underbrace{f(g(x))\,g'(x)}\,dx = \boxed{\underbrace{F(g(x))}} + c$$

dove $F'(t) = f(t)$

$$\int a^x\,dx = \int e^{x\log a}\,dx = \frac{1}{\log a}\int e^{x\log a}\log a\,dx$$

$$f(t) = e^t$$
$$g(x) = x\cdot\log a$$
$$g'(x) = \log a$$
$$F(t) = e^t$$

$$= \frac{1}{\log a}e^{x\log a}$$

$$= \frac{a^x}{\log a} + c$$

$$\int tg\,x\,dx = \int\frac{\operatorname{sen}x}{\cos x}\,dx = -\int\frac{1}{\cos x}(-\operatorname{sen}x)\,dx =$$

$$f(t) = \frac{1}{t} \qquad g(x) = \cos x \qquad f(g(x)) = \frac{1}{\cos x}$$
$$g'(x) = -\operatorname{sen}x \qquad F(t) = \log|t|$$

$$= -\log|\cos x| + c$$

$$\int ctg\,x\,dx = \int\frac{\cos x}{\operatorname{sen}x}\,dx = \log|\operatorname{sen}x| + c$$

$$\int tgh\,x\,dx = \int\frac{\operatorname{senh}x}{\cosh x}\,dx = \log(\cosh x) + c$$

$$\int\frac{1}{\cosh x}\,dx = \int\frac{1}{\frac{e^x+e^{-x}}{2}}\,dx =$$

$$= \int \frac{2}{e^x + \frac{1}{e^x}}\,dx = \int \frac{2e^x}{e^{2x}+1}\,dx =$$

$$= 2\int \frac{1}{(e^x)^2+1}\cdot e^x\,dx = 2\,\mathrm{arctg}(e^x) + c$$

$$f(t) = \frac{1}{1+t^2} \qquad g(x) = e^x \qquad g'(x) = e^x$$

$$f(g(x)) \qquad F(t) = \mathrm{arctg}\, t$$

$$\int \frac{1}{x\log x}\,dx = \int \frac{1}{\log x}\cdot\frac{1}{x}\,dx = \log|\log x| + c$$

$$f(t) = \frac{1}{t} \qquad g(x) = \log x \qquad g'(x) = \frac{1}{x}$$

$$F(t) = \log|t|$$

$$\int \frac{\log x}{x}\,dx = \int (\log x)^1 \frac{1}{x}\,dx = \frac{1}{2}(\log x)^2 + c$$

$$f(t) = t \qquad g(x) = \log x \qquad g'(x) = \frac{1}{x}$$

$$F(t) = \frac{1}{2}t^2$$

$$\int \log x\,dx \rightarrow \text{per parti} \ldots$$

$$\int (\operatorname{sen} x)' \cos x \, dx = \frac{(\operatorname{sen} x)^2}{2} + c$$

$f(t) = t \quad g(x) = \operatorname{sen} x \quad F(t) = \frac{t^2}{2}$

oppure

$f(t) = t \quad g(x) = \cos x \quad g'(x) = -\operatorname{sen} x$

$$\int \operatorname{sen} x \cos x \, dx = -\int (\cos x)^1 (-\operatorname{sen} x) \, dx =$$

$$= -\frac{(\cos x)^2}{2} + c$$

$$= -\frac{(1 - \operatorname{sen}^2 x)}{2} + c$$

$$= -\frac{1}{2} + \frac{\operatorname{sen}^2 x}{2} + c$$

$$= \frac{\operatorname{sen}^2 x}{2} + c$$

Oppure

$$\int \operatorname{sen} x \cos x \, dx = \int \frac{1}{2} \operatorname{sen}(2x) \, dx =$$

$$= \frac{1}{2} \frac{1}{2} \int \operatorname{sen}(2x) \, 2 \, dx = \frac{1}{4}(-\cos 2x) + c$$

$f(t) = \operatorname{sen} t \quad g(x) = 2x \quad g'(x) = 2$

$F(t) = -\cos t$

$$= -\frac{1}{4} \cos 2x + c$$

$$= -\frac{1}{4}\left(1-2\,\mathrm{sen}^2 x\right)+c$$

$$= -\frac{1}{4}+\frac{1}{2}\,\mathrm{sen}^2 x + c = \frac{(\mathrm{sen}\,x)^2}{2}+c$$

$$\int \frac{1}{\mathrm{sen}\,x}\,dx = \int \frac{1}{2\,\mathrm{sen}\frac{x}{2}\cos\frac{x}{2}}\,dx =$$

$$= \int \frac{1}{\dfrac{2\,\mathrm{sen}\frac{x}{2}}{\cos\frac{x}{2}}\ \cos^2\frac{x}{2}}\,dx$$

$$= \int \frac{1}{2\,\mathrm{tg}\frac{x}{2}\,\cos^2\frac{x}{2}}\,dx = \int \frac{1}{\mathrm{tg}\frac{x}{2}}\ \frac{1}{\cos^2\frac{x}{2}}\cdot\frac{1}{2}\,dx$$

$$f(t)=\frac{1}{t} \qquad g(x)=\mathrm{tg}\,\frac{x}{2} \qquad g'(x)=\frac{1}{\cos^2\frac{x}{2}}\cdot\frac{1}{2}$$

$$F(t) = \log|t|$$

$$= \log\left|\mathrm{tg}\,\frac{x}{2}\right| + c$$

Oppure

$$\int \frac{1}{\mathrm{sen}\,x}\,dx = \int \frac{1}{2\,\mathrm{sen}\frac{x}{2}\cos\frac{x}{2}}\,dx =$$

$$\int \frac{1}{\operatorname{sen} x}\,dx = \int \frac{1}{2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2}}\,dx =$$

$$= \int \frac{\operatorname{sen}^2\frac{x}{2} + \cos^2\frac{x}{2}}{2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2}}\,dx =$$

$$= \int \frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}\,\frac{1}{2} + \frac{\cos\frac{x}{2}}{\operatorname{sen}\frac{x}{2}}\,\frac{1}{2}\,dx$$

$$= -\int \frac{1}{\cos\frac{x}{2}}\left(-\operatorname{sen}\frac{x}{2}\,\frac{1}{2}\right)dx + \int \frac{1}{\operatorname{sen}\frac{x}{2}}\cos\frac{x}{2}\cdot\frac{1}{2}\,dx$$

$$= -\log\left|\cos\frac{x}{2}\right| + \log\left|\operatorname{sen}\frac{x}{2}\right| + c$$

$$= \log\left|\frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}\right| + c = \log\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$\int \frac{1}{\operatorname{sen} x}\,dx = \int \frac{(-\operatorname{sen} x)}{\pm\sqrt{1-\cos^2 x}}\,dx$$

$$\int \frac{1}{\cos x}\,dx = \dots = \int \frac{1}{\operatorname{sen}\left(\frac{\pi}{2}-x\right)}\,dx \;\dots$$

$$\int \frac{1}{\operatorname{sen}^2 x}\,dx = -\operatorname{ctg} x \;\dots$$

$$\int \frac{1}{1-\cos x}\,dx = \int \frac{1+\cos x}{(1-\cos x)(1+\cos x)}\,dx$$

$$= \int \frac{1+\cos x}{1-\cos^2 x}\,dx = \int \frac{1+\cos x}{\operatorname{sen}^2 x}\,dx$$

$$= \int \frac{1}{\operatorname{sen}^2 x}\,dx + \int (\operatorname{sen} x)^{-2} \cos x\,dx$$

$$= -\operatorname{ctg} x + \frac{(\operatorname{sen} x)^{-2+1}}{-2+1} + c$$

$$= -\operatorname{ctg} x - \frac{1}{\operatorname{sen} x} + c$$

$$\int \frac{1}{1+\cos x}\,dx \qquad \int \frac{1}{1+\operatorname{sen} x}\,dx \quad \ldots$$

INTEGRAZIONE PER PARTI

$$\underbrace{\int f'(x)\,g(x)\,dx} = \underline{f(x)\,g(x)} - \underbrace{\int f(x)\,g'(x)\,dx}$$

$$\int \log x\,dx = \int \underset{f'}{1} \cdot \underset{g}{\log x}\,dx =$$

$$= x \log x - \int x \frac{1}{x}\, dx$$

$$= x \log x - \int 1\, dx$$

$$= x \log x - x + c$$

$$\int \log_a x\, dx = \int \frac{\log x}{\log a}\, dx =$$

$$= \frac{1}{\log a} \int \log x\, dx = \frac{1}{\log a}\left(x \log x - x + c \right)$$

$$= x \frac{\log x}{\log a} - \frac{x}{\log a} + c$$

$$= x \log_a x - \frac{x}{\log a} + c$$

$$\int \operatorname{arctg} x\, dx = \int 1 \operatorname{arctg} x\, dx =$$

$$= x \operatorname{arctg} x - \int x \frac{1}{1+x^2}\, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$$

f'

f

$$= x \operatorname{arctg} x - \frac{1}{2} \log(1+x^2) + c$$

$$\int \arcsin x\, dx = x \arcsin x - \int x \frac{1}{\sqrt{1-x^2}}\, dx =$$

$$= x \arcsin x - \left(-\frac{1}{2}\right) \int (1-x^2)^{-\frac{1}{2}} (-2x)\, dx$$

$$= x \arcsin x + \frac{1}{\cancel{2}} \frac{(1-x^2)^{-\frac{1}{2}+1}}{\cancel{-\frac{1}{2}+1}} + c$$

$$= x \arcsin x + \sqrt{1-x^2} + c$$

$$\int \operatorname{arctgh} x\, dx \ldots$$

$$\int \arccos x\, dx \ldots$$

$$\int \operatorname{sen}^2 x\, dx = \int \underset{f'}{\operatorname{sen} x}\, \underset{g}{\operatorname{sen} x}\, dx =$$

X

$$= -\cos x \operatorname{sen} x - \int -\cos x \cos x\, dx =$$

$$= -\cos x \operatorname{sen} x + \int \cos^2 x\, dx$$

$$= -\cos x \operatorname{sen} x + \int 1 - \operatorname{sen}^2 x\, dx$$

$$= -\cos x \operatorname{sen} x + x - \int \operatorname{sen}^2 x\, dx \quad X$$

$$\Longrightarrow \int \operatorname{sen}^2 dx + \int \operatorname{sen}^2 x\, dx = -\cos x \operatorname{sen} x + x$$

$$2 \boxed{\int \operatorname{sen}^2 x\, dx} = x - \cos x \operatorname{sen} x$$ dividiamo per 2

$$\Longrightarrow \int \operatorname{sen}^2 x\, dx = \frac{x}{2} - \frac{1}{2} \cos x \operatorname{sen} x + c$$

Più facile con la bisezione:

$$\int \operatorname{sen}^2 x\, dx = \int \frac{1 - \cos 2x}{2} dx =$$

$$= \int \frac{1}{2} dx - \int \frac{\cos 2x}{2} dx$$

$$= \frac{1}{2} x - \frac{1}{4} \operatorname{sen} 2x + c$$

$$= \frac{x}{2} - \frac{1}{\cancel{4}_{2}} \cancel{2} \operatorname{sen} x \cos x + c$$

$$\int \cos^2 x\, dx = \ldots \qquad \int \operatorname{senh}^2 x\, dx = \ldots$$

$$\int \cosh^2 x\, dx = \ldots$$

$$\int \operatorname{sen}^3 x \, dx = \int \operatorname{sen}^2 x \operatorname{sen} x \, dx =$$

$$= \int (1 - \cos^2 x) \operatorname{sen} x \, dx =$$

$$= \int \operatorname{sen} x - \int \cos^2 x \operatorname{sen} x \, dx$$

$$= -\cos x + \int (\cos x)^2 (-\operatorname{sen} x) \, dx$$

$$= -\cos x + \frac{(\cos x)^{2+1}}{2+1} + c = -\cos x + \frac{1}{3}\cos^3 x + c$$

$$\int \cos^3 x \, dx \quad \ldots$$

$$\int \operatorname{senh}^3 x \, dx \quad \ldots \qquad \int \cosh^3 x \, dx \quad \ldots$$

$$\int \operatorname{sen}^4 x \, dx = \int \operatorname{sen}^2 x \operatorname{sen}^2 x \, dx =$$

$$= \int (1 - \cos^2 x) \operatorname{sen}^2 x \, dx =$$

$$= \int \operatorname{sen}^2 x \, dx - \int \overbrace{\cos^2 x \operatorname{sen}^2 x}^{(\cos x \operatorname{sen} x)^2} \, dx$$

$$= \int \frac{1 - \cos 2x}{2} \, dx - \int \frac{(\operatorname{sen} 2x)^2}{4} \, dx$$

$$= \frac{1}{2} x - \frac{1}{4} \operatorname{sen} 2x - \frac{1}{4} \int (\operatorname{sen} 2x)^2 \, dx$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen}2x - \frac{1}{4}\int (\operatorname{sen}2x)^2\,dx$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen}2x - \frac{1}{4}\int \frac{1-\cos 4x}{2}\,dx =$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen}2x - \frac{1}{8}x + \frac{1}{32}\operatorname{sen}4x + c$$

$$= \frac{3}{8}x - \frac{1}{4}\operatorname{sen}2x + \frac{1}{32}\operatorname{sen}4x + c$$

$$\int \underset{\substack{f' \\ g}}{x}\ \underset{\substack{g \\ f'}}{\underline{\operatorname{sen}x}}\,dx = \frac{x^2}{2}\operatorname{sen}x - \int \frac{x^2}{2}\,\underline{\cos x}\,dx$$

cambiamo strada...

$$\int x\operatorname{sen}x\,dx = -\cos x\cdot x - \int -\cos x\cdot 1\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \operatorname{sen}x + c$$

$$\int x^2\cos x\,dx = \dots$$

$\cosh x$

$\operatorname{senh} x$

e^x

$$\int \underset{f'}{x} \underset{g}{\log x} \, dx = \frac{x^2}{2} \log x - \int \frac{x^2}{2} \frac{1}{x} \, dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx =$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \frac{x^2}{2} + c$$

SALUTI!