[31-10]

- GENERALIZATION OF COMPUTABLE FUNCTIONS

$\mathcal{C}$ → CALLIGRAPHIC COMPUT. FUNCTIONS

[$\mathcal{Q}(\vec{x}, y)$ $\quad \vec{x}$ = INPUT

= OUTPUT]

[ZERO / PROJ. / SUCC.] → PR

BOUNDED
→ MINIMALIZATION
UNBOUNDED

BOUNDED → FOR

UNBOUNDED → WHILE

(μ - OPERATOR)

$$Pr(x) = \begin{cases} 1 & x \text{ is prime} \\ 0 & \text{otherwise} \end{cases}$$

$$= \mu \,|\, x.\text{isPrime} \,|$$

$$= sg(Pr(x))$$

$$\to sg\,||D(x) - 1|$$

PR → Definition of class

$f : \mathbb{N} \to \mathbb{N}$ $\left[f(x) = 2y + 1 \right]$

$f(x) \in PR$

$$\begin{cases} f(0) = \text{double}(0) = 0 \\ f(y+1) = \text{double}(y) \end{cases}$$

$$\begin{cases} double(0) = 0 \\ double(y+1) = \end{cases}$$

$$double(y) + 2 =$$

$$(double(y) + 1) + 1$$

μ - OPERATOR

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \text{ is a root} \\ \uparrow & \text{otherwise} \end{cases}$$

$= \mu \mid \sqrt{x}$ is a root $\mid = \mu \mid y \cdot y = x \mid$

$$\sqrt{x} \begin{cases} 0 \\ y+1 \end{cases}$$

$\rightarrow$ NOT WITH μ

$\mu \mid x$ is a root $) = \mu \mid x^2 = \varphi A(x) \mid$

$\downarrow$

$$\left[\begin{array}{l} \neg Q \\ Q_1 \wedge Q_2 \\ Q_1 \vee Q_2 \end{array}\right] \rightarrow \left[\begin{array}{l} H(\vec{x}, y) \\ S(\vec{x}, y) \end{array}\right]$$

$[gcd] \rightarrow$ GREATEST COMMON DIVISOR

$gcd(x, y) = \max z$ s.t.

- z divisor of x

$\wedge$

- z divisor of y

$$\left\{\begin{array}{l} \gcd(0, y) \\ \gcd(0, y-1) \end{array}\right. \Rightarrow \text{NOT VIABLE}$$

$$\rightarrow \left.\begin{array}{l} \text{mm}(x, z) = 0 \\ \text{mm}(z, y) = 0 \end{array}\right\} \text{MAX}$$

$$\max z \leq \min(x, z) + \min(z, y) = 0$$

max

$$z \leq (\min(z, x) + \min(z, y)) = 0$$

$$z = \min(x, y) - w$$

LARGEST

LEAST

$$\mu w \leq (\min(x, y) - w) \wedge (\min(z, y) = 0))$$

w = "FAKE" CONSTANT TO MAKE MINIMIZATION WORK

URM- MACHINES

- $A(m, n) \rightarrow$ WRITES IN "m" THE SUM OF THE TWO REGISTERS m AND n

- $C(M) \rightarrow \overline{Dg}(M) \rightarrow$

  $M_n \rightarrow \overline{}(\Pi_n)$

$[\mathcal{C} \subseteq \mathcal{C}_{VAR}]$

$\Rightarrow$ $\mathcal{C}$ (NORMAL)

$\mathcal{C}_{VAR}$ (SUM & $\overline{SG}$)

$\exists\, p$ s.t. $f_p^{(k)} = f_{p'}^{(k)}$

k = # of steps $\to \ell(p)$

p = a program

$n = q$

J(n, q, ZERO) // empty memory

Z(n)

J(1, 1, J+1)

ZERO: S(n) → SUM

J(1, 1, J+1)

. . . .

CREATE A CYCLE

$$\left[\begin{array}{l} 1 \to I_1 \\ \ldots \\ j \to A(m, n) \\ \ldots \\ \ell(p) \to I_k \end{array}\right]$$

(SUB OR BEFORE)

$\exists \, (m, n, c)$

$c\,(m, n)$

$\downarrow$

$SUB(\ldots)$ — $\exists\,(n, m, K)$

P''

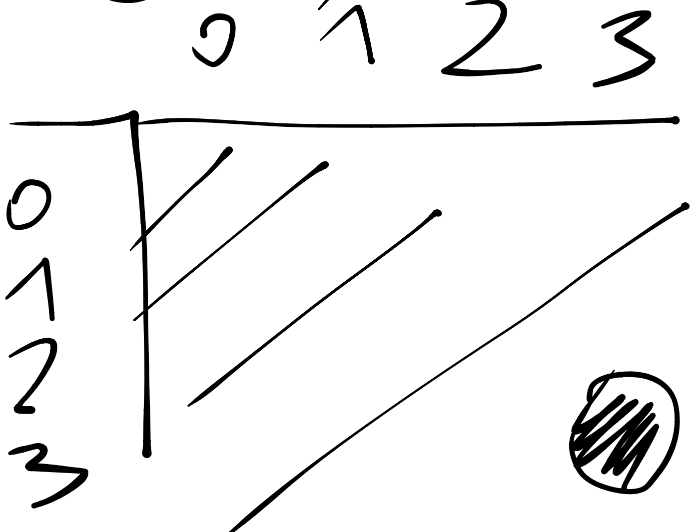
$\Leftarrow$ CONTRARY

= TRIVIAL

INVERSE FUNCTION

$$f^{-1}(y) = \begin{cases} x & \exists x \text{ s.t. } f(x) = y \\ [\top & \text{otherwise}] \end{cases}$$

$$f^{-1}(y) = [\mu x . |f(x) - y|]$$



[find x s.t. $f(x) = y$]

$$\left[\begin{array}{l} sum_k : \mathbb{N}^k \to \mathbb{N} \\ \quad \downarrow \\ sum_k \in PR \\ sum_k(x_1 \dots x_k) = \sum_{i=1}^{k} x_i \end{array}\right]$$

$$\begin{cases} sum_2(x_1, 0) = \underline{U_1^2(x_1, 0)} \\ sum_2(x_1, y+1) = S(sum_2(x_1, y)) \end{cases}$$

$$\mathbb{N}^2 \to \mathbb{N}$$

$$sum_2(x, 0) = \overline{sg}(x)$$

$$\overline{sg}(x) = \begin{cases} 0 & \text{if } x \\ 1 & \text{otherwise} \end{cases}$$