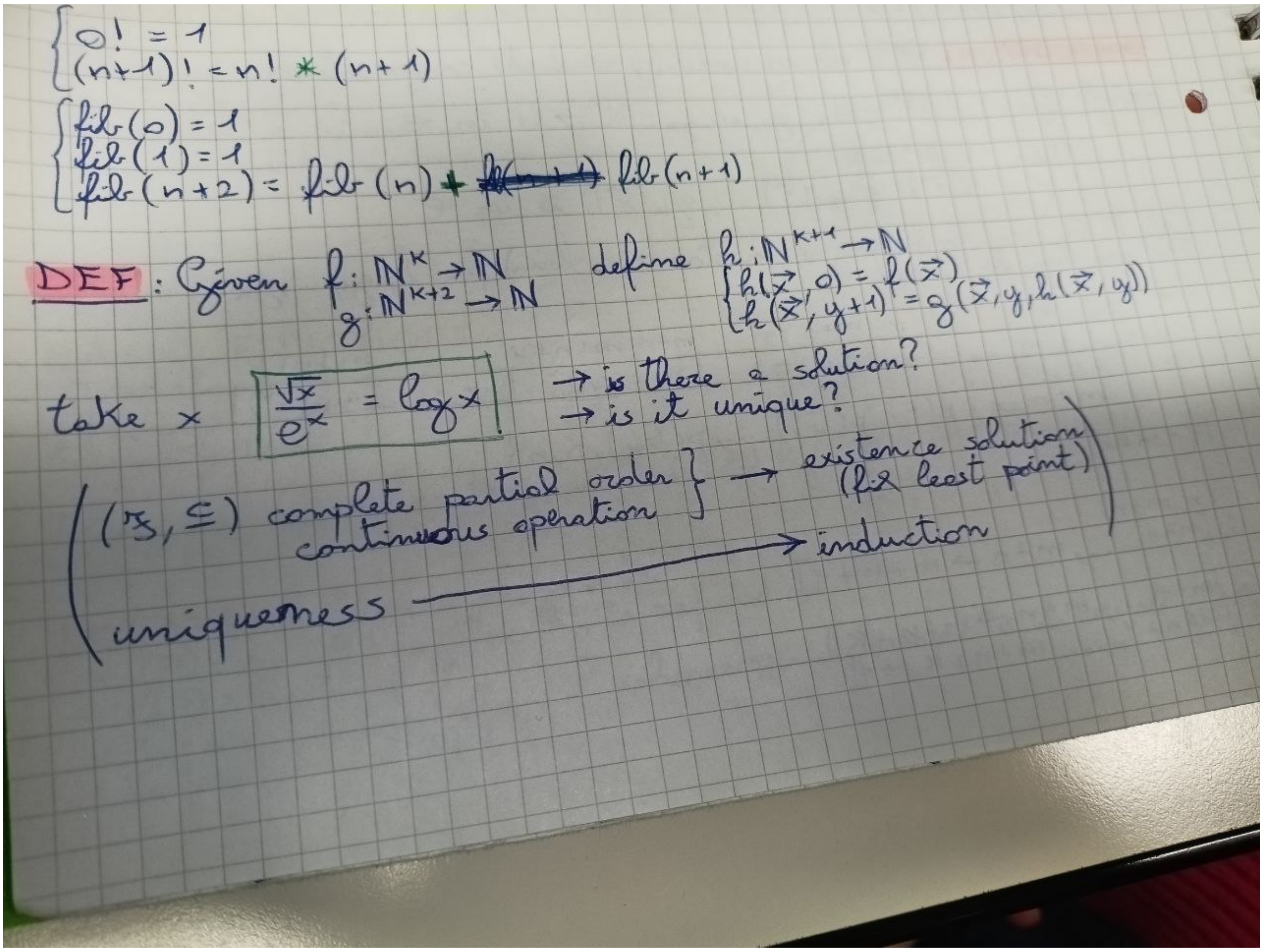

$$\begin{cases} 0! = 1 \\ (n+1)! = n! * (n+1) \end{cases}$$

$$\begin{cases} fib(0) = 1 \\ fib(1) = 1 \\ fib(n+2) = fib(n) + \cancel{fib(n+1)}\ fib(n+1) \end{cases}$$

DEF: Given $f: \mathbb{N}^k \to \mathbb{N}$, $g: \mathbb{N}^{k+2} \to \mathbb{N}$ define $h: \mathbb{N}^{k+1} \to \mathbb{N}$ $\begin{cases} h(\vec{x}, 0) = f(\vec{x}) \\ h(\vec{x}, y+1) = g(\vec{x}, y, h(\vec{x}, y)) \end{cases}$

take x $\boxed{\frac{\sqrt{x}}{e^x} = \log x}$ → is there a solution? → is it unique?

$(\mathcal{F}, \subseteq)$ complete partial order, continuous operation } → existence solution (fix least point)

uniqueness → induction

The notation F and ⊆ appears in the context of discussing complete partial orders and continuous operations. This is a fundamental concept in computability theory and domain theory.

(F,⊆) represents a complete partial order, where:

- F denotes a set (typically the set of partial functions or a function space)
- ⊆ denotes the subset relation, which in this context acts as the ordering relation

A complete partial order (CPO) has two key properties:

1. It is partially ordered, meaning the relation ⊆ is:
   - Reflexive: For all x ∈ F, x ⊆ x
   - Antisymmetric: If x ⊆ y and y ⊆ x then x = y
   - Transitive: If x ⊆ y and y ⊆ z then x ⊆ z
2. It is complete, meaning every directed subset has a least upper bound (supremum)

This structure is particularly important when dealing with recursive definitions and fixed points, as shown in the image with the recursive definition involving h(z,0) and h(z,y+1). The completeness property ensures that certain recursive definitions have solutions, which is crucial for proving the existence of computable functions defined by recursion.

The continuous operation mentioned in the image refers to functions that preserve suprema of directed sets. This property, combined with the CPO structure, allows us to prove the existence of least fixed points using techniques like induction mentioned in the image.

This mathematical framework provides the foundation for rigorously defining and reasoning about recursive functions in computability theory.