Basic Notations

- ϕ_e - The e-th partial computable function (computed by program with code e)
- $\phi_e(x)\downarrow$ - The computation of φ_e on input x terminates/converges
- $\phi_e(x)\uparrow$ - The computation of φ_e on input x diverges
- W_e - Domain of φ_e (set of inputs where φ_e converges)
- E_e - Range/codomain of φ_e (set of outputs produced by φ_e)
- `H(e,x,t)` - "Program e halts on input x within t steps" predicate
- `S(e,x,y,t)` - "Program e outputs y on input x within t steps" predicate

Special Sets

- `K = {x | x ∈ Wₓ}` - The halting set (x halts on input x)
- $\bar{K}$ - Complement of K
- `Tot = {e | φₑ is total}` - Set of indices of total functions

Functions and Operators

- χ_a - Characteristic function of set A

```
χₐ(x) = {
  1 if x ∈ A
  0 if x ∉ A
}
```

- `sg` - Sign function

```
sg(x) = {
  0 if x = 0
  1 if x > 0
}
```

- $\bar{s}g$ - Complemented sign function

```
s̄g(x) = {
  1 if x = 0
  0 if x > 0
}
```

Function Composition and Operations

- `f ∘ g` - Function composition (f after g)
- `f ⊆ g` - f is a subfunction of g
- `f ≅ g` - f and g are extensionally equal (compute same function)
- `μy.P(y)` - Minimization operator (least y satisfying predicate P)

Special Notations

- `π` - Standard pairing function encoding two numbers as one
- `π₁, π₂` - Projection functions extracting components of pair
- `⟨x,y⟩` - Alternative notation for pair (x,y)
- `(w)ᵢ` - i-th component in coding of tuple ω

Program Composition

- Program concatenation is denoted by juxtaposition (PQ)
- $P[i_1,...,i_k \rightarrow h]$ denotes program P with:
  - Input taken from registers $i_1,...,i_k$
  - Output placed in register h
  - Other registers cleared initially

This notation is used extensively in proofs related to universal functions, smn theorem, recursion theorems, and various reducibility results.