

1. Problem Analysis

Step 1: Identify Requirements

- Look for patterns in what's being asked:
 - Domain control ($Wk(n)$)
 - Range control ($Ek(n)$)
 - Both domain and range control
 - Size control ($|Wk(n)|$)

Step 2: Target Properties

Write down clearly:

- What should be in $Wk(n)$?
- What should be in $Ek(n)$?
- Any special conditions (totality, size)?

2. Solution Construction

Step 1: Helper Function Design

```
# Pattern for domain control:
f(n,x) = { something  if x in desired_domain
          { ↑         otherwise

# Pattern for range control:
f(n,x) = { desired_output  if condition
          { default_value  otherwise

# Pattern for both:
f(n,x) = { desired_output  if x in desired_domain
          { ↑              otherwise
```

Step 2: Make it Computable

Common techniques:

- For divisibility: use `rm(x,n)`
- For domain control: use `μz`
- For even numbers: multiply by 2
- For size control: use bounded counters

Step 3: Apply SMN Theorem

Standard steps:

1. State the theorem
2. Show helper function is computable
3. Get $k(n)$ such that $\phi_{k(n)}(x) = f(n,x)$

3. Verification

Step 1: Check Domain ($W_k(n)$)

```
Wk(n) = {x | f(n,x)↓}
Verify this matches requirements
```

Step 2: Check Range ($E_k(n)$)

```
Ek(n) = {f(n,x) | x ∈ Wk(n)}
Verify this matches requirements
```

Step 3: Verify Special Properties

- Check totality if required
- Verify size conditions
- Confirm any other constraints

4. Common Patterns

Domain Control

```
# For x ≥ n:
f(n,x) = something + μz.(n-x)
```

```
# For  $x < n$ :  
f(n,x) = something +  $\mu_z.(x-n)$ 
```

Range Control

```
# For even numbers:  
f(n,x) = 2*something  
  
# For specific sets:  
f(n,x) = desired_value * sg(condition) + default * sg(condition)
```

Size Control

```
# For fixed size k:  
f(n,x) = { 0 if  $x < k$   
          {  $\uparrow$  otherwise
```

5. Tips and Tricks

1. For totality:
 - Include a default value
 - Make sure every case is covered
2. For range control:
 - Use multiplication for even/odd
 - Use sign functions (sg, $\overline{\text{sg}}$) for switching
3. For domain control:
 - Use minimalization (μ)
 - Use remainder function (rm)
4. For verification:
 - Write out $W_k(n)$ explicitly
 - Write out $E_k(n)$ as set builder notation
 - Show transformations step by step

Remember: The key is matching the helper function design to the requirements while ensuring computability.