GUIDA PASSO - PASSO EX 4 $B = \{x \in \mathbb{N} \mid \exists y \geq x . \varphi_x(y) > y\}$

① CAPIRE SE É R.E. O NON É R.E. → guardo PDF Gabriel

a) If the set is not r.e. → I use Rice-Shapiro with $f \in B$ and $\emptyset \notin B$ or viceversa

b) The set is r.e., since its characteristic function is computable.
Infact it can be expressed as:

$$sc_B(x) = 1(\mu(y,z,t). H(x,y,t) \wedge S(x,y,z,t))$$

In this case → $1(\mu(y,z,t). S(x,y,z,t) \wedge y \geq x \wedge z > y)$ ↗ Sostituisco con w_1, w_2, w_3

$$= 1(\mu w. S(x,(w)_1,(w)_2,(w)_3) \wedge (w)_1 \geq x \wedge (w)_2 > (w)_1)$$

② CAPIRE SE É RICORSIVO

B is not recursive since $K \leq_m B$. To show this

$$g(x,y) = \begin{cases} y & \text{if } x \in K \\ \uparrow & \text{otherwise} \end{cases}$$

The function is computable, given that $g(x,y) = y \cdot sc_K(x)$.
So for SMN theorem, there is a total computable function $S: \mathbb{N} \to \mathbb{N}$ s.t. $\varphi_{S(x)}(y) = g(x,y)$.
Indeed:

- If $x \in K$ then $\varphi_{S(x)}(y) = g(x,y) = y$ for each $y \in \mathbb{N}$
  $y = S(x) \geq S(x)$, we have $\varphi_{S(x)}(y) = y = S(x) \geq S(x)$. So $S(x) \in B$.
- If $x \notin K$ then $\varphi_{S(x)}(y) = g(x,y) \uparrow$ for every $y \in \mathbb{N}$
  Therefore there is no $y \geq S(x)$ s.t. $\varphi_{S(x)}(y) > y$ So $S(x) \notin B$.

③ VERIFICO ANCHE $\bar{B}$

Since B is r.e. and not recursive, $\bar{B}$ is not r.e.

④ VERIFICO SE B É SATURO → SE $e \in B \wedge E_e = E_{e'} \Rightarrow e' \in B$

$\exists y > e$ t.c. $\varphi_e(y) > y$ con $\varphi_e(y) = \varphi_{e'}(y)$

$$\Downarrow$$

$\varphi_{e'}(y) > y$

- se $y > e$ allora $e \in B$
- se $y > e'$ allora $e' \in B$

$y = e+1 \rightarrow e \in B$ perché $e+1 > e$

$e' > e$ con $\varphi_e = \varphi_{e'}$ quindi $y > e'$ ma $e' > e$ quindi $e' \notin B$

QUINDI NON SATURO