Here are some functions defined by cases and their minimized forms:

1. Threshold Function

```
f(x) = {
    0    if x ≤ 10
    x-10 otherwise
}

Minimized form:
f(x) = (x-10) * sg(10-x)
```

2. Three-Way Split

```
f(x) = {
    0   if x = 0
    1   if x is even and x > 0
    2   if x is odd
}

Minimized form:
f(x) = sg(x) * (sg(rm(x,2)) + 1) * (1 - sg(x))
```

3. Maximum with Threshold

```
f(x,y) = {
    x     if x > y
    y     if y > x
    100   if x = y
}

Minimized form:
f(x,y) = x * sg(y-x) + y * sg(x-y) + 100 * sg(|x-y|)
```

4. Modified Division

```
f(x,y) = {
    x/y   if y divides x
    0     otherwise
}

Minimized form:
f(x,y) = qt(x,y) * sg(rm(x,y))
```

5. Piecewise Growth

```
f(x) = {
    0      if x = 0
    x²     if x is even
    x+1    if x is odd
}

Minimized form:
f(x) = (x² * sg(rm(x,2)) + (x+1) * sg(rm(x,2))) * (1 - sg(x))
```

6. Binary Choice

```
f(x) = {
    2x    if x < 100
    x-50  if x ≥ 100
}

Minimized form:
f(x) = 2x * sg(100-x) + (x-50) * sg(100-x)
```

7. Three-Value Comparison

```
f(x,y) = {
    0    if x < y
    1    if x = y
    2    if x > y
}
```

```
Minimized form:
f(x,y) = sg(x-y-1) * 2 + sg(|x-y|)
```

8. Modified Remainder

```
f(x,y) = {
    rm(x,y)  if y ≠ 0
    x        if y = 0
    1        if x = y
}

Minimized form:
f(x,y) = rm(x,y) * (1 - sg(y)) + x * sg(y) + sg(|x-y|)
```

9. Step Function

```
f(x) = {
    0     if x < 10
    10    if 10 ≤ x < 20
    20    if x ≥ 20
}

Minimized form:
f(x) = 10 * sg(10-x) * sg(20-x) + 20 * sg(20-x)
```

10. Conditional Power

```
f(x) = {
    x²    if x is even
    x³    if x is odd and x < 10
    x     otherwise
}

Minimized form:
f(x) = x² * sg(rm(x,2)) + x³ * sg(rm(x,2)) * sg(10-x) + x * sg(rm(x,2)) *
sg(10-x)
```

Each of these functions shows how to:

1. Break down cases into characteristic functions using sg and $\overline{sg}$
2. Combine cases using multiplication for AND conditions
3. Use sg(|x-y|) for equality testing
4. Use $\overline{sg}$(x-y) for greater than comparisons
5. Use rm(x,y) for divisibility tests