Quick Recognition Patterns

"Existence" patterns → Usually R.E.

- "∃y such that..."
- "some element has property P"
- "at least one thing happens"
- "the set intersects with..."
- "there is a witness"

Examples:

- $\{x \mid x \in W_x \cup E_x\}$ → "x exists in at least one set" → **r.e.**
- $\{x \mid W_x \cap E_x \neq \emptyset\}$ → "some element exists in both" → **r.e.**
- $\{x \mid \exists y > x.\ y \in E_x\}$ → "some large y exists" → **r.e.**

"Universal" patterns → Usually NOT R.E.

- "∀y such that..."
- "every element has property P"
- "all things must satisfy..."
- "nothing bad happens"

Examples:

- $\{x \mid \forall y \in W_x.\ \varphi_x(y) > x\}$ → "every computation is large" → **not r.e.**
- $\{x \mid \varphi_x \text{ is total}\}$ → "defined for ALL inputs" → **not r.e.**

The Intuitive Rule

Can you "witness" membership by finding something finite?

- **Yes** → Probably r.e. (search until you find the witness)
- **No** → Probably not r.e. (you'd need to check infinitely many things)

For $\{x \mid x \in W_x \cup E_x\}$: You can witness membership by either:

- Finding some y,t such that $H(x,y,t)$ ($x \in W_x$), OR
- Finding some y,z,t such that $S(x,y,z,t)$ ($x \in E_x$)

This is finite searching → **r.e.**

For $\{x \mid \forall y > x.\ y \in W_x\}$: To verify membership, you'd need to check infinitely many y values → **not r.e.**

90% of the time, this heuristic works immediately.