(13/11)

→ [DIAGONALIZATION] → NOT COMPUTABLE

→ SMN - THEOREM

FUNCTION IS NOT COMPUTABLE BY CONSTRUCTION

$R = [0,1]$

$\mathbb{N} \to \mathbb{R}$

COUNTABLE — NOT COUNTABLE

$$\begin{array}{ll} 1 & 0.3146667 \\ 2 & 0.3146678 \\ 3 & 0.315 \\ 4 & \\ 5 & \\ 6 & \\ \mathbb{N} & \end{array}$$

0.3146678

$f(1,2) + f(2,3) + \ldots f(n)$

$[\varphi_n] \neq \varphi_{n+1}$

$$f(n) = \begin{cases} \varphi_n(n) + 1 & \text{if } \varphi_n(n)\downarrow \\ 0 & \text{otherwise} \end{cases}$$

$C = \{ f : \mathbb{N} \to \mathbb{N} \mid f \text{ not computable} \}$

- if $\varphi_n(n)\downarrow \to f(n) = \varphi_n(n) + 1 \neq \varphi_n(n)$
- if $\varphi_n(n)\uparrow \to f(n) = 0 \neq \varphi_n(n)$

$\boxed{6.6}$

$f : \mathbb{N} \to \mathbb{N}$ non-computable function

s.t. $\forall g : \mathbb{N} \to \mathbb{N}$ $f+g = (f+g)(x) = f(x) + g(x)$ is computable

$+$ → QUANTIFIER → COMPUTABLE

$$f(x) = \begin{cases} \varphi_x(x)+1 & \text{if } x \in W_x \\ 0 & \text{otherwise} \end{cases} \qquad g(x) = \begin{cases} 0 & \text{if } x \in W_x \\ \uparrow & \text{otherwise} \end{cases}$$

$$h(x) = f(x) \boxed{+} g(x) = f + f = 2f$$

$f \circ f$ BECOMES COMPUTABLE

(GENERALLY) → NOT COMPUTABLE

[6.20] $f : \mathbb{N} \to \mathbb{N}$ defined

$$f(x) = \begin{cases} x+2 & \text{if } \varphi_x(x)\downarrow \\ x \dot{-} 1 & \text{otherwise} \end{cases}$$

Is f computable?

$$\underbrace{\varphi_x(x) \neq \varphi_{x+1}(x)}_{\text{NOT COMPUTABLE}} \quad = \mu w. |(x+2) \dot{-} (x \dot{-} 1)| \quad 1$$

$$\to \quad = \mu w. \, sg\, |(x+2) - (x \dot{-} 1)| \quad 0$$

NOT COMPUTABLE — DIAGONALIZATION

— K = HALTING SET

K ↑ χ_K

$$\chi_K = \begin{cases} 1 & \text{if } x \in W_x \\ \uparrow & \text{otherwise} \end{cases}$$

↑ YOU WILL SEE

SMN - THEOREM - KLEENE (PARAMETRIZATION)

e = INDEX OF A PROGRAM

$S^{n}_{m} \to S : \mathbb{N} \to \mathbb{N}$

$$\to \left[\varphi_e^{(m+n)}(\vec{x}, \vec{y}) = \varphi^{(n)}_{S(m,n)(e,\vec{x})}(\vec{y}) \right] (*)$$

$$\left[g(x,y) = \begin{cases} x & x \in W_x \\ \uparrow & \end{cases} \to S : \mathbb{N} \to \mathbb{N} \quad \varphi_{S(x)}(y) \to \ldots \right]$$

[3.3] - STATE THE SMN - THEOREM (*)

$\exists$ TOTAL COMPUTABLE FUNCTION $S : \mathbb{N} \to \mathbb{N}$

$$\text{s.t. } W_{\underbrace{S(x,y)}_{\uparrow}} = \{ z : x \cdot z = y \} \to E_{S(x,y)}$$

FIX x TO GET y

$$g(x,y,z) = \begin{cases} \emptyset & \text{if } [x \cdot z = y] \\ \uparrow & \text{otherwise} \end{cases}$$

$$= \mu w . \,\overline{sg}\,(\underbrace{\text{"} x - y \mid z \text{"}})$$ PREDICATE $\Rightarrow$ USE A COMPUT. FUNCTION (GAP)

$$= \mu w . \,\overline{sg}\,(qt(z,y))$$

$W_{S(x)} \,/\, E_{S(x)}$

$s : \mathbb{N}^2 \to \mathbb{N}$, $\exists$ computable

$\forall x, y, z \in \mathbb{N}$ s.t. $\varphi_{s(x,y)}(z) = g(x, y, z)$

$\varphi_{s(x)}(y) = g(x, y)$

LAST PART

→ CHECK IF OUTPUT WAS PARAMETRIZED CORRECTLY

- $z \in W_{s(x,y)} \mid g(x, y, z) \downarrow 1$

$W_{s(x,y)} = \{ x \cdot z = y \} = [W_z = \{ qt(z, y) \}]$

- $z \in E_{s(x,y)} = \{ z \mid x \cdot z = y \}$

$= \{ qt(z \cdot y) \}$

[2018-11-13]

DOMAIN / INPUT

$\exists$ $k : \mathbb{N} \to \mathbb{N} \mid \forall n \in \mathbb{N} \mid [\varphi_{x(n)}$ is total$]$

[TOTAL / COMP.]

and

$E_{s(x)}$ is the set of integer divisors

$g : \mathbb{N} \to \mathbb{N} \mid g(x, y)$ s.t. $\varphi_{s(x)}(y) \ldots$

$$g(x,y) = \begin{cases} [qt(z, y)] & \text{if } z \mid x \cdot z = y \\ 1 & \text{otherwise} \end{cases}$$

$\varphi_{s(x)}(y) = g(x,y) \quad \forall x,y \in \mathbb{N}$

$\rightarrow W_{s(x)}(y) = \{\mathbb{N}\} \quad x \in \mathbb{N}$ PREDICATE $Q(x)$

IF PART $E_{s(x)}(y) = \{x \mid [$ "$qt(z,y) \neq 0$" $\cup \{1\}]$

$= sg(rm(z,y)) \cup \{1\}$

NOT THE IF PART

ALTERNATIVELY

$$g(x,y) = \begin{cases} x \cdot y & x \text{ is a divisor of } y \\ 1 & \text{otherwise} \end{cases}$$

POINT ON WHICH DIVISION IS DEFINED

$E_{s(x)}$

$$g(x,y) = \underbrace{sg(rm(x,y))}_{\text{DIVISION IS DEFINED}} + \overline{sg}(rm(x,y))$$

DIVISION IS DEFINED

$W_{s(x)}(y) = \{x \text{ is the divisor?}\} = \{\mathbb{N}\}$

THE EXERCISE TELLS YOU IT'S TOTAL

$E_{s(x)}(y) = \{x \mid$ "$x \cdot y$" $?\}$

$= \{$"$rm(x,y) = 0$"$\} \cup \{1\}$

$= \{\overline{sg}(rm(x,y))\} \cup \{1\} \rightarrow x \cdot y \in \mathbb{N}$

[2017-11-20] (1) STATE THE SMN-THEOREM (4)

$g : \mathbb{N} \rightarrow \mathbb{N} \rightarrow [W_{s(x)}(y) = \{x \in \mathbb{N} \mid x \geq n\}]$

AND

$[E_{s(x)}(y) = \{y \in \mathbb{N} \mid y \text{ even } \forall n \in \mathbb{N}\}$

$$g(x,y) = \begin{cases} 2 \cdot (x \dot- n) & \text{if } x \geq n \\ \uparrow & \text{otherwise} \end{cases}$$

"DOMAIN" CASE

$$= \mu z \cdot \underbrace{(n \dot- x)}_{} + \underbrace{2 \cdot (x \dot- n)}_{}$$

n MIGHT BE GREATER THAN x

$$[\, s: \mathbb{N} \to \mathbb{N} \mid \varphi_{s(x)}(y) = g(x,y) \mid \forall x, y \in \mathbb{N} \,]$$

$$W_{s(x)}(y) = \{x \mid f(x,y)\downarrow\} = \{x \mid x \geq n\}$$

$$E_{s(x)}(y) = \{f(x,y)\downarrow \mid x \in \mathbb{N}\}$$

$$= \{2(x \dot- n) \mid x \geq n\}$$

$$= \{2(\cancel{n} + z) \cancel{\dot- n} \mid z \geq 0\} = \{2z \mid z \geq 0\}$$

SMN - EXER.

(SMN-THEOREM) $\longrightarrow$ FIXING x YOU CAN DO EVERYTHING

UNIVERSAL FUNCTION

Ψ] ← PSI (UPPERCASE)

PRACTICAL $\rightarrow \varphi_x(x) \rightarrow [\Psi_U(x,x)]$

$$\Psi_U(e, \vec{x}) = \varphi_e^{(k)}(\vec{x})$$

e = INDEX OF PROGRAM

$\vec{x}$ = COMPOSITION (WHATEVER x)

$A = \{ \text{sets} \dots \}$ → VERIFY IF PROPERTIES ARE VALID

$$\left[g(x,y) = \begin{cases} y & x \in A \\ \uparrow & x \notin A \end{cases} \right] \rightarrow \varphi_{s(x)}(y) = g(x,y)$$

- if $x \in A$ $\quad \varphi_{s(x)}(y) = g(x,y) = y$ ← $W_{s(x)} \in \mathbb{N}$
- if $x \notin A$ $\quad \varphi_{s(x)}(y) = g(x,y) = \uparrow$

↑ $E_{s(x)}(y) \mid W_{s(x)} = \emptyset$

↑ POINT ON WHICH WE USE SMN-THEOREM