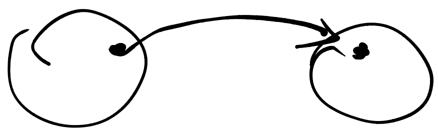
16/10/2024

- FUNCTIONS

URM MACHINES → MODELS

functions → TOTAL → Defined $\forall n \in \mathbb{N}$

- DOMAIN (INPUT) → W_x
- IMAGE (OUTPUT) → E_x



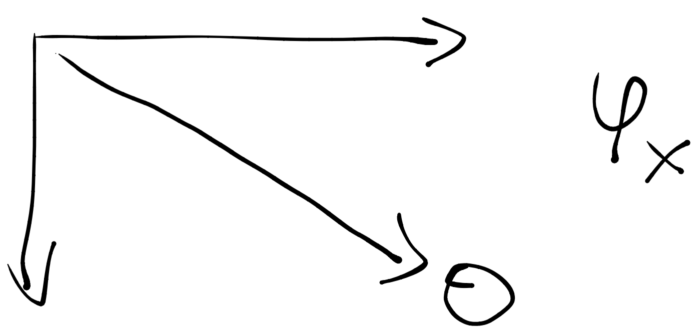
DOMAIN CODOMAIN

SUBTRACTION (MONUS)

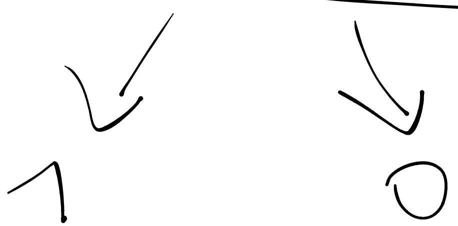
$$x, y \in \mathbb{N} \rightarrow x \dot{-} y = \begin{cases} x - y & x \geq y \\ 0 & \text{otherwise} \end{cases}$$

NON-COMPUTABLE

DIAGONALIZATION



NON-COMPUTABLE



PARTIAL → SOMETIMES TOTAL

$$f(x) = \begin{cases} 1 & \text{IT STOPS} \\ 0 & (\downarrow)\ (\uparrow)\ \text{IT DOES NOT STOP} \end{cases}$$

– DEFINED BY CASES

DIAGONALIZATION

$$f(i) = \begin{cases} \uparrow & f(i)\downarrow \\ 0 & f_i(i)\uparrow \end{cases} \qquad f : \mathbb{N} \to \mathbb{N}$$

$i \in \mathbb{N}$

- if $f_i(i)\downarrow \Rightarrow f_i(i)\uparrow$
- if $f_i(i)\uparrow \Rightarrow f_i(i)\downarrow$

$\exists$ NON-COMPUTABLE f

$f : \mathbb{N} \to \mathbb{N}$ s.t.

$img(f) = \mathbb{N} \setminus \{0\}$?

φ_X = FUNCTION DEFINED BY TERMS OF ITSELF

→ COMPUTABLE

$$f(x) = \begin{cases} \varphi_x(x) & \text{if } f(x)\downarrow \\ \uparrow & \text{otherwise} \end{cases}$$

→ total (defined $\forall n \in \mathbb{N}$)

→ not computable

URM - MACHINES

$Z(n)$ / $S(n)$ / $T(m, n)$

→ MODEL → steps in a number of steps

URM - COMPUTABLE

P(steps) = ↓

⇒ computes f

PROOF AHEAD

URM $\longrightarrow$ URM_VARIANT
$\longleftarrow$

URM (1.4)

- URM programs $\mathcal{C}(\mathcal{C}) \to \mathcal{C}$

if i-th instruction

is a jump $\to$ J(m, n, t)

$\mathcal{C} \leq \mathcal{C}'$ $\mathcal{C}'$ = VARIANT

$\Longrightarrow$ $\mathcal{C}' \to$ "n times" $\mathcal{C}$

$\mathcal{C} \to$ trivial $\to$ variant once of URM $\to \mathcal{C}'$

INDUCTION STEP

$\ell(p)$ = # of steps
(NUMBER)

$(i = 0) \rightarrow$ TRIVIAL STEP $\rightarrow$ 1st INSTRUCTION IS A JUMP

$(i = n+1)$

$Z(0)$

FOR ($i = 0$ TO N)

. . . .

JUMP OUT

I_J = J-TH INSTRUCTION WHICH MAY NOT BE A JUMP

FOR ($i = 0$ TO N)

IF JUMP

IF NOT JUMP

$$\left[f_p^{(k)} = f_{p'}^{(k)} \right]$$

$\downarrow$

FIRST PROGRAM (e) $\downarrow$

$\Downarrow$

SECOND PROGRAM $\downarrow$

- CASE 1 $\rightarrow$ 1_3 is NOT JUMP

$$\ell(p) = k$$

$$\text{for } i := 1 \text{ to } n \qquad n \leq k$$

- CASE 2 $\Rightarrow$ $S(m, n, 1)$

$$i \rightarrow i+1 \in \mathbb{N}$$

URM - MACHINE

$(\quad \dots\dots \quad) \rightarrow 0 \leq i \leq l(p)$

COMPUTATION

JUMP $\rightarrow$ TERMINATES
INSIDE
OF EXECUTION

URM
MACHINE $\rightarrow i = 0$ (DEFINED)

$\dots$

$i = m \quad (m < k) \downarrow$

DECIDABLE PREDICATES

Q = PREDICATE

$$\chi_Q = \begin{cases} 1 & \text{if } Q(\vec{x}) \\ 0 & \text{otherwise} \end{cases}$$

(CHARACTERISTIC FUNCTION

→ STRUCTURE PROJECTION

$\boxed{\text{COMPUTABILITY ON OTHER DOMAINS}}$

$$f : \alpha \circ \alpha^{-1}$$

(Θ)

THETA = SUBFUNCTION

PR → class of primitive recursive functions

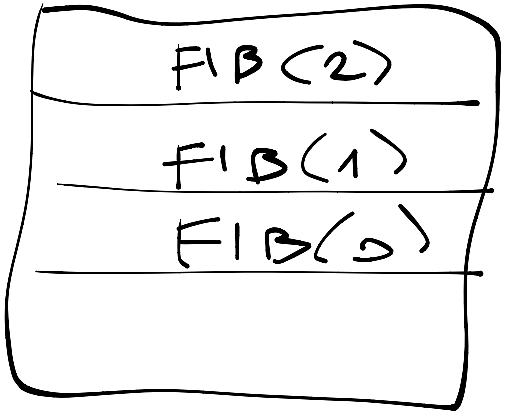
$\mathcal{C}$ → 0 (ZERO) $\cancel{0}(x) = 0$

→ S (SUCCESSOR) $S(x) = x+1$

→ U_j^k (PROJECTION)

$U_j^k(x_1, \ldots, x_k) = x_j \quad \forall x_1 \ldots x_k \in \mathbb{N}^k$

↓



$\rightarrow FIB(2) = \underbrace{FIB(0) \cdot FIB(1)}_{\text{MEANING OF COMPOSITION}}$

Δg / $\overline{\Delta g}$ / MM (NOTABLE FUNCTIONS)

- DEFINE PR
- $f : \mathbb{N} \to \mathbb{N}$ $\rightarrow$ $f(x) = \begin{cases} 1 & x \text{ even} \\ \emptyset & x \text{ odd} \end{cases}$

$$\begin{cases} f(0) = 1 = succ(0) \\ f(y+1) = succ(0) \dot{-} f(y) \end{cases}$$

RIGHT, BUT
WE WILL BOUND
THAT

$$f(y+1) = \overline{sg}(y)$$

$$\begin{cases} \overline{sg}(0) = 1 = succ(0) \\ \overline{sg}(y+1) = 0 \end{cases}$$

$f : \mathbb{N}^2 \to \mathbb{N}$

Define PR and prove it

$$f(x,y) = x^y \cdot x$$

$$\begin{cases} f(x,0) = x^0 \cdot x = x \\ f(x,y+1) = \end{cases}$$

SUCCESSION

$succ(0) \cdot succ(x-1)$

$\rightarrow succ(y) = (x^y \cdot x)$

$\rightarrow succ(f(y)) \cdot x$

CORRECT!