08/01/2023

THEORET. DEF.

$\Rightarrow A, B \subseteq \mathbb{N}$ recursive sets (1)

$$A \setminus B = \{ x \in \mathbb{N} \mid x \in A \wedge x \notin B \} \ (2)$$

$A \setminus B$ is recursive

$\Rightarrow \chi_A, \chi_B$ computable

$$\chi_{A \setminus B} = \chi_A(x) \cdot \overline{sg}(\chi_B(x)) \Rightarrow \underline{\text{R.S.C.}}$$

WHEN $A \to \emptyset$

THEN $B \to 1$

(3) DOES $A \setminus B$ EXTEND TO R.E SETS?

$$\underbrace{A = K}_{SC_A} \qquad \underbrace{B = \mathbb{N}}_{SC_B} \Rightarrow A \setminus B = \emptyset$$

R.E $\Rightarrow SC_x = \begin{cases} 1 \\ \uparrow \end{cases}$

SINCE Q, THIS MEANS $\exists$ SOME INPUTS ON WHICH WE DO NOT STOP

$\boxed{7.22}$ $\mathcal{A} \subseteq \mathcal{C}$ → SET OF COMPUTABLE FUNCTIONS

$f \in \mathcal{A}$ S.T. $\forall \theta \subseteq f$, $\theta \notin \mathcal{A}$

[PROVE $A = \{x \in \mathbb{N} \mid \varphi_x \in \mathcal{A}\}$ is NOT R.E.]

- NOT R.E. → $\bar{K} \leq_m A$ / RICE-SHAPIRO
- NOT RECURSIVE → $K \leq_m A$ / RICE THEOREM
- NOT SATURATED → 2 R. THEOREM

① PROVE A IS SATURATED

$A \subseteq \mathcal{A}$ $\quad A = \{\varphi_x \in \mathcal{A} \mid \exists f \in \mathcal{C}\}$

W_x $\quad$ / E_x $\quad$ $\varphi_x(y)$

dom (f) / cod (f) $\quad$ $f_x(y)$

→ RICE-SHAPIRO

$$\left[\begin{array}{l} \exists f \in \mathcal{A} \mid \forall \theta \notin \mathcal{A} \\ \exists f \notin \mathcal{A} \mid \exists \theta \in \mathcal{A} \end{array}\right]$$

NOT R.E (?)

$A = \{x \in \mathbb{N} \mid \varphi_x \in \mathcal{A}\}$

$f = ID$

$\theta = \emptyset$

$\vee$

NOT R.E → RICE ~~SHAPIRO~~

SMN - THEOREM → $g(x,y) = \begin{cases} \varphi_{e_0}(y), & x \in K \\ f(y), & x \notin K \end{cases}$

$e_0 \in \mathbb{N} \mid \varphi_{e_0}$ is computable

$\varphi_{s(x)}(y) = g(x,y), \forall x, y \in \mathbb{N}$

- if $x \in K$, $\varphi_{(s(x))} = \varphi_{e_0}(y)$, $W_{s(x)} = \emptyset$
- if $x \notin K$, $\varphi_{s(x)}(y) = f(y)$, $s(x) \in A$

$\boxed{8.2}$ → RECURSIVENESS (REC / R.E)?

$A = \{x \in \mathbb{N} : x \in W_x \wedge E_x\}$

$A / \bar{A}$ REC / R.E (?)

NOT REC

$[A = \{\varphi_x \in A ; x \in dom(f) \wedge cod(f)\}]$

A IS SATURATED

RICE - THEOREM → ① A IS SATURATED
② $[A \neq \emptyset]$ → $\varphi_x \neq \varphi_y$
③ $[A \neq \mathbb{N}]$

$A \to e_0, e_1 \in A$ ② $e_0 = ID$, $e_1 = \emptyset$
$e_0 \in A$, $e_1 \notin A$

③ $e_0 = \emptyset$, $e_1 = ID$

$e_0 \in A$, $e_1 \notin A$

[NOT RECURSIVE]

$H(x, y, A)$

$A = \{\underbrace{dom(f)}_{} \wedge \underline{cod(f)}\}$

$$SC_A = 1(\mu.(x, y, A).|H(\underbrace{x, y}_{W_x}, A) \wedge \underbrace{S(x, y, z, A)}_{E_x}|$$

$$= 1(\mu.w.|\chi_H|x, (w)_1, (w)_2| \wedge \chi_S(x, (w)_1, (w)_3, (w_2))|$$

$A \rightarrow$ NOT REC. R.E.

$\bar{A} \rightarrow$ NOT R.E. NOT REC.

$\bar{A} = \{x \in \mathbb{N} \mid x \in W_x \cup E_x\}$

8.69 $\rightarrow$ REC/R.E (?) $B / \bar{B}$

$B = \{x \in \mathbb{N} \mid \forall y \in W_x, \exists z \in W_x . (y < z)$

NOT R.E

$\wedge (\varphi_x(y) > \varphi_x(z))\}$

$B = \{x : \varphi_x \in \mathcal{B}\}$ where $f = ID$, $\theta = \emptyset$

$f \in \mathcal{B}$, $\theta \notin \mathcal{B}$

$B = \{ f \in \mathcal{C} : \forall y \in dom(f), \exists z \in dom(f) . (y < z) \wedge (f_x(y) > f_x(z)) \}$

SATURATED (B)

$\bar{B} = \{ x \in \mathbb{N} \mid \underline{\exists y \in W_x, \forall z \in W_x . (y > z)} \vee \underline{[(\varphi_x(y) \le \varphi_x(z))]} \}$

NOT R.E

$(w)_1 \; (w)_2$

$SC_{\bar{B}} = [\cancel{1}(\mu(x, \underset{w}{y, A}) \cdot H(x, y, A)]$

$\vee \; S(x, y, y, A) \le S(x, y, z, A)$

$\bar{B}$ IS R.E, / B IS NOT R.E

NOT RECURSIVE — NOT RECURSIVE

[SMN - THEOREM]

$x \to y$

① STATE SMN - THEOREM

EXP / LOG

② $s : \mathbb{N} \to \mathbb{N} \mid \forall x \in \mathbb{N}, |W_x| = 2^x, |E_x| = x+1$

$g(x, y) = \begin{cases} \lfloor \log_2(y) + 1 \rfloor, & [y < 2^x] \to [0, 2^x - 1] \\ \uparrow, & \text{otherwise} \end{cases}$

$\varphi_{s(x)}(y)$

$\Rightarrow \exists s : \mathbb{N} \to \mathbb{N}, \forall x, y \in \mathbb{N}$

$\varphi_{s(x)}(y) = g(x, y)$

$|W_x| = \{y \mid g(x, y)\downarrow\} = [0, 2^x - 1]$

$|E_x| = \{g(x, y) \mid 0 \le y \le 2^x\}$

$\{\log_2(y) + 1 \mid 0 \le y \le 2^x\} = [0, x]$

$\boxed{8.4}$ ① → STATE 2R.T.

② → USE IT TO PROVE

$\exists x \in \mathbb{N}$ s.t. $W_n = E_n = \{x \cdot n : x \in \mathbb{N}\}$

① $\exists h : \mathbb{N} \to \mathbb{N}$, $\exists e \in \mathbb{N}$ s.t. $\varphi_{h(e)} = \varphi_{(e)}$

$(n \in \mathbb{N})$

② $h(x, n) = \begin{cases} x \cdot n & x \in W_n \quad (x \in \mathbb{N}) \\ \uparrow & \text{otherwise} \end{cases}$

$\downarrow$

$\varphi_n = \varphi_{h(n)} = \varphi_{s(n)}$

$h(e, n) = \varphi_e(n) = \begin{cases} e \cdot n & , x \in W_e \\ \uparrow & , \text{otherwise} \end{cases}$

2015 – PARTIAL EXAM

(2) $CPR : \mathbb{N}^2 \to \mathbb{N}$ is PR defined as

$CPR(x,y) = |\{p \mid x \leq p < y \wedge p \text{ prime}\}|$

(1) GIVE PR DEFINITION

→ 0, S, U_s^k DEFINED ∧ CLOSED W.R.T COMPOSITION AND PR

(2)

$$\begin{cases} CPR(x,0) = 0 & (0 \leq 1 \leq y \wedge 1 \text{ PRIME}) \\ CPR(x,k+1) = CPR(x,k) + \chi_{PR}(x+k) \end{cases}$$

PR → SET → COMP?

χ_{PR} (EXISTS, ETC.)

– 2022 EXAM

→ $ISQRT(x) = \lfloor \sqrt{x} \rfloor \in PR$

→ LP(X) → LARGEST PRIME DIVISOR ∈ PR

(EXAMPLE TO READ / STUDY)