# Sets and Basic Numbers

| Notation | Meaning |
|---|---|
| $\mathbb{N}$ | Natural numbers (including 0) |
| $\mathbb{N}+$ | Positive natural numbers |
| $\mathbb{N}^k$ | k-tuples of natural numbers |
| $\emptyset$ | Empty set |
| $\in$ | Set membership |
| $\subseteq$ | Subset relation |
| $\cup$ | Set union |
| $\cap$ | Set intersection |
| $\setminus$ | Set difference |
| $\overline{A}$ | Complement of set A |

# Function Notation

## General Functions

| Notation | Meaning |
|---|---|
| $f:A\to B$ | Function from A to B |
| dom(f) | Domain of function f |
| cod(f) | Codomain of function f |
| img(f) | Image of function f |
| $f^{-1}$ | Inverse function of f |
| $f\circ g$ | Function composition |
| $f\subseteq g$ | f is a subfunction of g |

## Computability-Specific

| Notation | Meaning |
|---|---|
| $\varphi x$ | x-th partial recursive function |

| Notation | Meaning |
|---|---|
| $W_x$ | Domain of $\varphi_x$ |
| $E_x$ | Image of $\varphi_x$ |
| $\varphi_x(y)\downarrow$ | $\varphi_x(y)$ is defined/converges |
| $\varphi_x(y)\uparrow$ | $\varphi_x(y)$ is undefined/diverges |
| $\varphi^{(k)}_e$ | k-ary partial recursive function with index e |

# Special Functions

## Basic Functions

| Notation | Meaning |
|---|---|
| $z(x)$ | Zero function |
| $s(x)$ | Successor function |
| $U^i_k$ | k-ary i-th projection |
| $\chi_A$ | Characteristic function of set A |
| $sg(x)$ | Sign function |
| $\bar{sg}(x)$ | Complement sign function |
| $x \dot{-} y$ | Monus (truncated subtraction) |

## Operators

| Notation | Meaning |
|---|---|
| $\mu y.f(x,y)$ | Unbounded minimalization |
| $\mu y<z.f(x,y)$ | Bounded minimalization |
| $\Sigma y<z\ f(x,y)$ | Bounded sum |
| $\Pi y<z\ f(x,y)$ | Bounded product |

# Vector Notation

| Notation | Meaning |
|---|---|
| $\bar{x}$ | Vector $(x_1,\ldots,x_n)$ |

| Notation | Meaning |
|---|---|
| $(x)_i$ | i-th component of x |
| $\langle x,y \rangle$ | Pairing function |
| $\pi_1, \pi_2$ | Projection functions for pairs |

## URM Machine

| Notation | Meaning |
|---|---|
| Z(n) | Zero instruction |
| S(n) | Successor instruction |
| T(m,n) | Transfer instruction |
| J(m,n,t) | Jump instruction |
| $R_n$ | n-th register |
| $r_n$ | Content of n-th register |

## Classes of Functions

| Notation | Meaning |
|---|---|
| C | URM-computable functions |
| $C^{(k)}$ | k-ary URM-computable functions |
| PR | Primitive recursive functions |
| R | Partial recursive functions |

## Important Sets

| Notation | Meaning |
|---|---|
| K | Halting set $\{x \in N : x \in W_x\}$ |
| $\bar{K}$ | Complement of halting set |
| Tot | $\{x \in N : \varphi_x \text{ is total}\}$ |
| Fin | $\{x \in N : W_x \text{ is finite}\}$ |

## Reducibility and Equivalence

| Notation | Meaning |
|---|---|
| $A \leq_m B$ | A many-one reduces to B |
| $A \equiv_m B$ | A and B are many-one equivalent |
| $A \leq_t B$ | A Turing reduces to B |

# Formal Logic

| Notation | Meaning |
|---|---|
| $\forall$ | Universal quantifier |
| $\exists$ | Existential quantifier |
| $\wedge$ | Logical AND |
| $\vee$ | Logical OR |
| $\neg$ | Logical NOT |
| $\rightarrow$ | Implication |
| $\leftrightarrow$ | Equivalence |

# Program Properties

| Notation | Meaning |
|---|---|
| $\ell(P)$ | Length of program P |
| $\rho(P)$ | Maximum register used in P |
| $P(\bar{x})\downarrow$ | Program P halts on input $\bar{x}$ |
| $P(\bar{x})\uparrow$ | Program P diverges on input $\bar{x}$ |