

General Pattern

1. Original Problem Pattern

Find $s: \mathbb{N}^2 \rightarrow \mathbb{N}$ where $Ws(x,y)$ has some property $P(x,y)$

2. Solution Strategy

- Define a helper function g that takes additional arguments
- Use smn theorem to transform g into the desired s

Steps to Determine Domains

1. For the Helper Function g

If trying to control $|Ws(x,y)|$:

- Input: typically needs (x, y, z) where z is the "counter" or "tester"
- Output: usually \mathbb{N} (typically $\{0,1\}$ or $\{0\}$ to control the domain)

Example from the image:

$g: \mathbb{N}^3 \rightarrow \mathbb{N}$

$g(x,y,z) = \begin{cases} 0, & \text{if } z < x*y \\ 1, & \text{otherwise} \end{cases}$

2. For the Final Function s

- Domain: Always matches what you need to "fix" (typically \mathbb{N}^2 for these exercises)
- Codomain: Always \mathbb{N} (as s produces program indices)

Example from the image:

$s: \mathbb{N}^2 \rightarrow \mathbb{N}$

where $|Ws(x,y)| = x*y$

Key Points to Remember

1. The helper function g typically needs:

- All parameters from s (x, y in these examples)
 - An extra parameter z for testing/counting
 - Boolean output (0/1) to control domain size
2. The final function s always:
- Takes the parameters you want to "fix"
 - Returns a program index (so codomain is \mathbb{N})
 - Controls $Ws(x, y)$ through the helper function
3. Look at what you're trying to control:
- If controlling size of $Ws(x, y)$, you need a way to count elements
 - If controlling content of $Ws(x, y)$, you need a way to test membership

This structured approach helps ensure you get the domains right every time.