Basic Operators

Symbol	Meaning	Example
sg(x)	Sign function	sg(0) = 1, sg(x) = 0 for x > 0
$\overline{sg}(x)$	Complemented sign	$\overline{sg}(0) = 0$, $\overline{sg}(x) = 1$ for x > 0
μy	Minimization operator	μy.P(y) = least y where P(y) holds
|x-y|	Absolute difference	|5-3| = 2
rm(x,y)	Remainder	rm(7,3) = 1
qt(x,y)	Quotient	qt(7,3) = 2
$\dot{-}$	Monus (truncated subtraction)	$x \dot{-} y = \max(0, x-y)$

Case Function Notation

Standard Form

```
f(x) = {
    value1  if condition1
    value2  if condition2
    ...     otherwise
}
```

Minimization Form

For converting conditions:

- x = y becomes sg(|x-y|)
- x > y becomes $\overline{sg}(y-x)$
- x ≥ y becomes $\overline{sg}(y-x-1)$
- x divides y becomes sg(rm(y,x))

Boolean Operations in Minimization

Operation	Minimization Form
AND	Multiplication (*)

Operation	Minimization Form
OR	Addition with $s\bar{g}$
NOT	Complement using $s\bar{g}$
x = y	sg(|x-y|)
x > y	$s\bar{g}$(y-x)
x ≥ y	$s\bar{g}$(y-x-1)

Common Patterns

Equality Test

```
f(x) = {
    1 if x = y
    0 otherwise
}
```

Minimization: sg(|x-y|)

Greater Than

```
f(x) = {
    1 if x > y
    0 otherwise
}
```

Minimization: $s\bar{g}$(y-x)

Divisibility

```
f(x) = {
    1 if x divides y
    0 otherwise
}
```

Minimization: sg(rm(y,x))

Function Composition

When combining multiple cases:

```
f(x) * condition1 + g(x) * condition2 + h(x) * condition3
```

where conditions are expressed using sg and $\overline{sg}$

Special Cases

Finite Domain

When function is defined only for certain values:

```
f(x) = value * sg(|x-target|)
```

Threshold Functions

For functions with different behavior above/below threshold:

```
f(x) = value1 * sg(threshold-x) + value2 * sg(threshold-x)
```