🟢 IMMEDIATELY RECURSIVE - "Safe Patterns"

Pattern 1: Pure Arithmetic/Finite Conditions

```
✅ A = {x ∈ ℕ | x is even}
✅ A = {x ∈ ℕ | x < 100}
✅ A = {x ∈ ℕ | x ≡ 3 (mod 7)}
✅ A = {x ∈ ℕ | x = 2^k for some k ≤ 10}
✅ A = {0, 1, 4, 9, 16, 25, ...} (perfect squares)
```

Why: Pure number theory, no halting involved.

Pattern 2: Finite Sets

```
✅ A = {2, 5, 17, 42}
✅ A = {x | x ≤ 1000 and x is prime}
```

Why: Finite = always recursive.

Pattern 3: Co-finite Sets

```
✅ A = ℕ \ {17} (all numbers except 17)
✅ A = {x | x ≠ 0 and x ≠ 1}
```

Why: Complement is finite.

🔴 IMMEDIATELY NON-RECURSIVE - "Danger Patterns"

Pattern 1: The Self-Reference Triangle

```
❌ A = {x | x ∈ W_x}                   (halting on own index)
❌ A = {x | ϕ_x(x) ↓}                  (K - the halting set)
❌ A = {x | ϕ_x(x) = x}                (outputs own index)
❌ A = {x | ϕ_x(x) > x}                (outputs larger than index)
❌ A = {x | x ∈ E_x}                   (index in own range)
```

Why: Direct diagonal/self-reference = undecidable.

Pattern 2: Domain/Range Equality Patterns

```
❌ A = {x | W_x = E_x}                  (domain equals range)
❌ A = {x | W_x = ℕ}                    (total functions)
❌ A = {x | E_x = {0}}                  (specific range)
❌ A = {x | |W_x| = 5}                  (specific domain size)
❌ A = {x | W_x ⊆ E_x}                  (domain subset of range)
```

Why: These are **saturated sets** → Rice's Theorem applies.

Pattern 3: The "Intersection/Union" Danger Zone

```
❌ A = {x | W_x ∩ E_x ≠ ∅}             (domain meets range)
❌ A = {x | W_x ∪ E_x = ℕ}              (domain union range = all)
❌ A = {x | x ∈ W_x ∩ E_x}              (self-reference + intersection)
```

Why: Combines halting with complex relationships.

Pattern 4: Function Properties

```
❌ A = {x | ϕ_x is total}               (totality is undecidable)
❌ A = {x | ϕ_x is injective}           (injectivity is undecidable)
❌ A = {x | ϕ_x is increasing}          (function properties)
❌ A = {x | ϕ_x(y) = y² for infinitely many y}
```

Why: All non-trivial function properties are undecidable.

🟡 USUALLY NON-RECURSIVE - "Suspicious Patterns"

Pattern 1: Quantified Halting

```
🟡 A = {x | ∃y > x. y ∈ W_x}           (probably not recursive)
🟡 A = {x | ∀y ∈ W_x. ϕ_x(y) > 0}      (probably not recursive)
🟡 A = {x | ∃k. k·x ∈ W_x}             (probably not recursive)
```

Strategy: Try reduction from K.

Pattern 2: Bounded but Complex

```
🟡 A = {x | ∃y ≤ x. ϕ_x(y) = x²}       (might be r.e. but not recursive)
🟡 A = {x | |W_x ∩ [0,x]| ≥ 2}          (finite intersection, but...)
```

Strategy: Check if r.e., then try K ≤_m A.

📋 DECISION FLOWCHART

```
Look at set definition A = {x | P(x)}
|
├─ Contains W_x, E_x, or ϕ_x?
|  |
|  ├─ NO → Check if arithmetic/finite
|  |       ├─ Pure arithmetic → ✅ RECURSIVE
|  |       └─ Finite set → ✅ RECURSIVE
|  |
|  └─ YES → Look for danger patterns
|           |
|           ├─ Self-reference (x ∈ W_x, ϕ_x(x), x ∈ E_x)?
|           |  └─ ❌ NOT RECURSIVE
|           |
|           ├─ Set equality (W_x = S, E_x = S)?
|           |  └─ ❌ NOT RECURSIVE (Rice's Theorem)
|           |
|           ├─ Function properties (total, injective, etc.)?
|           |  └─ ❌ NOT RECURSIVE (Rice's Theorem)
|           |
|           └─ Complex quantified pattern?
|              └─ 🟡 Try reduction from K
```

🎯 INSTANT RECOGNITION EXAMPLES

Immediately Recursive:

```
A = {x | x < 2025}                      ← Finite
A = {x | x is odd}                      ← Simple arithmetic
```

```
A = {x | x = 3k for some k ≤ 100}        ← Bounded enumeration
```

Immediately Non-Recursive:

```
A = {x | x ∈ W_x}                        ← Self-reference danger!
A = {x | W_x = ℕ}                        ← Rice's theorem!
A = {x | ϕ_x is total}                   ← Function property!
A = {x | E_x = {0,1,2}}                  ← Specific range!
```

Need Investigation:

```
A = {x | ∃y ∈ W_x. y > x}                ← Try K ≤_m A
A = {x | |W_x| ≤ 5}                      ← Check if saturated
A = {x | x² ∈ E_x}                       ← Try reduction approach
```

🚨 KEY WARNING SIGNALS

1. **"x ∈ W_x"** or **"φ_x(x)"** → Immediate red flag
2. **"W_x = [something]"** → Usually Rice's Theorem
3. **"E_x = [something]"** → Usually Rice's Theorem
4. **"φ_x is [property]"** → Usually Rice's Theorem
5. **"∀y ∈ W_x"** or **"∃y ∈ W_x"** → Try reduction
6. **No W_x, E_x, φ_x** → Probably recursive

💡 PRACTICAL RULE OF THUMB

> **"If it talks about what programs DO (halting, outputs, domains), it's probably not recursive.**
> **If it talks about what numbers ARE (even, prime, bounded), it's probably recursive."**

This pattern recognition works for ~90% of exam problems. For the remaining 10%, apply formal reduction techniques or Rice's theorem systematically.