Notation

Chapter 1

R_n	nth register 9
r_n	contents of R_n 9
$Z(n)$	zero instruction 10
$S(n)$	successor instruction 10
$T(m, n)$	transfer instruction 10
$J(m, n, q)$	jump instruction 11
$r_n := x$	r_n becomes x 10
$P(a_1, a_2, \ldots)$	computation under program P 16
$P(a_1, a_2, \ldots)\downarrow$	the computation stops 16
$P(a_1, a_2, \ldots)\uparrow$	the computation never stops 16
$P(a_1, a_2, \ldots, a_n)\downarrow b$	the final value in R_1 is b 17
$\mathscr{C}, \mathscr{C}_n$	computable functions 17
$f_P^{(n)}$	n-ary function computed by P 21
c_M	characteristic function of M 22

Chapter 2

U_i^n	projection functions 25
PQ or $\begin{matrix}P\\Q\end{matrix}$	concatenation of programs 27
$\rho(P)$	denotes registers affected by P 2
$P[l_1, \ldots, l_n \to l]$	28
$x \dot{-} y$	cut-off subtraction 36

$\mathrm{sg}(x)$, $\overline{\mathrm{sg}}(x)$
signum functions 36
$\mathrm{rm}(x, y)$, $\mathrm{qt}(x, y)$
remainder and quotient functions 36, 37
$\mu z < y(\ldots)$
least z less than y 39
p_x xth prime number 40
$(x)_y$ power of p_y occurring in x 40
$\pi(x, y)$ a pairing function 41
$\mu y(f(\boldsymbol{x}, y) = 0)$
minimalisation operator 43

Chapter 3

$\mathscr{R}$, $\mathscr{R}_0$ (partial) recursive functions 49
$\mathscr{PR}$ primitive recursive functions 51
$\mathscr{TC}$ Turing-computable functions 56
$\overset{Q}{\Rightarrow}$ obtained by productions in Q 59
$\underset{\mathscr{G}}{\vdash}$ Post-system $\mathscr{G}$ generates 59
$T_{\mathscr{G}}$ strings generated by $\mathscr{G}$ 59
$\hat{\sigma}$ coding of a word σ 61
$\tilde{n}$ word representing n 61
$G(f)$ graph of f 62
$\mathscr{PC}$ Post-computable functions 63

Chapter 4

$\mathscr{I}$ URM instructions 74
$\mathscr{P}$ URM programs 74
γ program coding function 75
P_n nth program $= \gamma^{-1}(n)$ 75
$\phi_a^{(n)}$, ϕ_a functions computed by P_a 76–77
$W_a^{(n)}$, W_a
domain of $\phi_a^{(n)}$, ϕ_a 77
$E_a^{(n)}$, E_a range of $\phi_a^{(n)}$, ϕ_a 77

Chapter 5

ψ_{U}, $\psi_{\mathrm{U}}^{(n)}$
universal functions 86
$c_n(e, \boldsymbol{x}, t)$
configuration code 87

$j_n(e, \boldsymbol{x}, t)$
: next instruction 87

$\sigma_n(e, \boldsymbol{x}, t)$
: state function 87

$T_n(e, \boldsymbol{x}, t)$
: Kleene T-predicate 89

$\mathrm{Rec}(f, g)$
: function obtained by recursion from f, g 91

$\mathrm{Sub}(f, g_1, \ldots, g_m)$
: function obtained by substitution from $f, g_1, \ldots, g_m$ 91

Chapter 6

$\mathbb{Q}$ rational numbers 108

$\wedge, \rightarrow$ logical symbols for 'and', 'implies' 111

$0, 1, \ldots$
R
$\mathsf{x}, \mathsf{y}, \ldots$
} symbols in a logical language 110

Chapter 7

$A \oplus B$ $\{2x : x \in A\} \cup \{2x+1 : x \in B\}$ 122

$A \otimes B$ $\{\pi(x, y) : x \in A \text{ and } y \in B\}$ 122

K $\{x : x \in W_x\}$ 123

Chapter 8

$\neg, \vee$ logical symbols for 'not', 'or' 143

$\mathscr{S}$ statements of language L 144

$\mathscr{T}, \mathscr{F}$ true, false statements of L 144

θ_n $(n+1)$th statement of $\mathscr{S}$ 144

$\mathsf{n} \in \mathsf{K}$ formal counterpart of $n \in K$ 145

$\mathscr{P}_{\mathscr{L}}$ provable statements 147

Pr* $\{n : \mathsf{n} \in \mathsf{K} \text{ is provable}\}$ 148

Ref* $\{n : \mathsf{n} \notin \mathsf{K} \text{ is provable}\}$ 148

Chapter 9

$A \leq_{\mathrm{m}} B$ A is many–one reducible to B 158

$\equiv_{\mathrm{m}}$ many–one equivalent 161

$\mathrm{d}_{\mathrm{m}}(A)$ the m-degree of A 161

$\boldsymbol{a} \leq_{\mathrm{m}} \boldsymbol{b}$ partial order on m-degrees 162

$\mathbf{0}_{\mathrm{m}}$ m-degree of recursive sets 163

$\mathbf{o}, \mathbf{n}$ m-degrees of $\varnothing$ and $\mathbb{N}$ 162

$\mathbf{0}'_{\mathrm{m}}$ m-degree of K 163

$\boldsymbol{a} \cup \boldsymbol{b}$ least upper bound of degrees $\boldsymbol{a}, \boldsymbol{b}$ 165
$\mathrm{O}(n)$ oracle instruction 167
P^{χ} URMO program P with χ in the oracle 168
$\mathscr{C}^{\chi}$ χ-computable functions 169
$\mathscr{R}^{\chi}$ χ-partial recursive functions 170
$\phi_m^{\chi,n}, \phi_m^{\chi}$
functions computed by Q_m^{χ} 170–171
W_m^{χ}, E_m^{χ}
domain and range of ϕ_m^{χ} 171
$\psi_{\mathrm{U}}^{\chi,n}$ universal function for χ-computability 171
K^{χ} $=\{x: x \in W_x^{\chi}\}$ 172
$P^A, \mathscr{C}^A, \phi_m^A, W_m^A, E_m^A, K^A$
relativised notions for A-computability 172
$A \leq_{\mathrm{T}} B$ A is Turing reducible to B 174
$\equiv_{\mathrm{T}}$ Turing equivalent 174
$\mathrm{d}_{\mathrm{T}}(A)$ Turing degree of A 175
$\boldsymbol{a} \leq \boldsymbol{b}$ partial order on T-degrees 176
$\mathbf{0}$ T-degree of recursive sets 176
$\mathbf{0}'$ T-degree of K 176
A' jump of A 177
$\boldsymbol{a}'$ jump of $\boldsymbol{a}$ 177
$\boldsymbol{a} | \boldsymbol{b}$ $\boldsymbol{a}, \boldsymbol{b}$ are incomparable degrees 179

Chapter 10

$\mathscr{F}_n$ n-ary partial functions 182
θ a finite function 183
$\tilde{\theta}$ code for a finite function θ 183
f_{Φ} least fixed point for Φ 192
f_{τ} function defined by program τ 196

Chapter 11

$\boldsymbol{E}_k$ sequence of computable functions enumerated by ϕ_k 208
$\boldsymbol{D}$ diagonal enumeration 208

Chapter 12

$t_P^{(n)}(\boldsymbol{x})$ number of steps taken by P to compute $f_P(\boldsymbol{x})$ 213
$t_e^{(n)}(\boldsymbol{x})$ $t_{P_e}^{(n)}(\boldsymbol{x})$ 213
$\mathfrak{C}_b, \mathfrak{C}_b^*$
complexity classes of b 223, 233