

Notation

Chapter 1

R_n	n th register	9
r_n	contents of R_n	9
$Z(n)$	zero instruction	10
$S(n)$	successor instruction	10
$T(m, n)$	transfer instruction	10
$J(m, n, q)$	jump instruction	11
$r_n := x$	r_n becomes x	10
$P(a_1, a_2, \dots)$	computation under program P	16
$P(a_1, a_2, \dots) \downarrow$	the computation stops	16
$P(a_1, a_2, \dots) \uparrow$	the computation never stops	16
$P(a_1, a_2, \dots, a_n) \downarrow b$	the final value in R_1 is b	17
$\mathcal{C}, \mathcal{C}_n$	computable functions	17
$f_P^{(n)}$	n -ary function computed by P	21
c_M	characteristic function of M	22

Chapter 2

U_i^n	projection functions	25
PQ or $\begin{smallmatrix} P \\ Q \end{smallmatrix}$	concatenation of programs	27
$\rho(P)$	denotes registers affected by P	27
$P[l_1, \dots, l_n \rightarrow l]$		28
$x \dot{-} y$	cut-off subtraction	36

$\text{sg}(x), \overline{\text{sg}}(x)$	signum functions	36
$\text{rm}(x, y), \text{qt}(x, y)$	remainder and quotient functions	36, 37
$\mu z < y(\dots)$	least z less than y	39
p_x	x th prime number	40
$(x)_y$	power of p_y occurring in x	40
$\pi(x, y)$	a pairing function	41
$\mu y(f(x, y) = 0)$	minimalisation operator	43

Chapter 3

$\mathcal{R}, \mathcal{R}_0$	(partial) recursive functions	49
\mathcal{PR}	primitive recursive functions	51
\mathcal{TC}	Turing-computable functions	56
\Rightarrow_Q	obtained by productions in Q	59
$\vdash_{\mathcal{G}}$	Post-system \mathcal{G} generates	59
$T_{\mathcal{G}}$	strings generated by \mathcal{G}	59
$\hat{\sigma}$	coding of a word σ	61
\tilde{n}	word representing n	61
$G(f)$	graph of f	62
\mathcal{PC}	Post-computable functions	63

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\mathcal{P}	URM programs	74
γ	program coding function	75
P_n	n th program = $\gamma^{-1}(n)$	75
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$W_a^{(n)}, W_a$	domain of $\phi_a^{(n)}, \phi_a$	77
$E_a^{(n)}, E_a$	range of $\phi_a^{(n)}, \phi_a$	77

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$j_n(e, \mathbf{x}, t)$	next instruction	87
$\sigma_n(e, \mathbf{x}, t)$	state function	87
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$\text{Sub}(f, g_1, \dots, g_m)$	function obtained by substitution from f, g_1, \dots, g_m	91

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\mathbb{Q}	rational numbers	108
\wedge, \rightarrow	logical symbols for 'and', 'implies'	111
$0, 1, \dots$	symbols in a logical language	110
R		
x, y, \dots		

Chapter 7

$A \oplus B$	$\{2x : x \in A\} \cup \{2x + 1 : x \in B\}$	122
$A \otimes B$	$\{\pi(x, y) : x \in A \text{ and } y \in B\}$	122
K	$\{x : x \in W_x\}$	123

Chapter 8

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\mathcal{S}	statements of language L	144
\mathcal{T}, \mathcal{F}	true, false statements of L	144
θ_n	$(n + 1)$ th statement of \mathcal{S}	144
$n \in K$	formal counterpart of $n \in K$	145
\mathcal{P}_i	provable statements	147
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Ref^*	$\{n : n \notin K \text{ is provable}\}$	148

Chapter 9

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\equiv_m	many-one equivalent	161
$d_m(A)$	the m -degree of A	161
$a \leq_m b$	partial order on m -degrees	162
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\mathbf{o}, \mathbf{n}	m -degrees of \emptyset and \mathbb{N}	162
$\mathbf{0}'_m$	m -degree of K	163

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a'	jump of a	177
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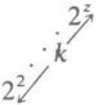
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Chapter 12

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