Computability
July 4, 2023

Exercise 1

a. Provide the definition of a recursive set.

b. Provide the definition of a recursively enumerable (r.e.) set.

c. Show that if $A, B \subseteq \mathbb{N}$ are recursive then also $A \setminus B = \{x \in \mathbb{N} \mid x \in A \ \wedge \ x \notin B\}$ is recursive. Does this extend to r.e. sets, i.e., is it the case that if A and B are r.e. then also $A \setminus B$ is r.e.? Provide a proof or a counterexample.

Exercise 2

State the s-m-n theorem and use it to prove that there exists a total computable function $s : \mathbb{N} \to \mathbb{N}$ such that $|W_{s(x)} \cap E_{s(x)}| = 2x$.

Exercise 3

Classify the following set from the point of view of recursiveness

$$A = \{x \mid W_x = E_x \cup \{0\}\},$$

i.e., establish if A and $\bar{A}$ are recursive/recursively enumerable.

Exercise 4

Classify the following set from the point of view of recursiveness

$$B = \{x \in \mathbb{N} \mid 4x + 1 \in E_x\},$$

i.e., establish if A and $\bar{A}$ are recursive/recursively enumerable.

Note: Each exercise contributes with the same number of points (8) to the final grade.