Computability
Jan 19 2022

Exercise 1

a. Provide the definition of reducibility, i.e., given sets $A, B \subseteq \mathbb{N}$ define what it means that $A \leqslant_m B$.

b. Show that if A is not recursive and $A \leqslant_m B$ then B is not recursive.

c. Show that if A is recursive then $A \leqslant_m \{1\}$.

Exercise 2

Is there a non-computable total function $f : \mathbb{N} \to \mathbb{N}$ such that $f(x) = f(x+1)$ on infinitely many inputs x, i.e., such that the set $\{x \in \mathbb{N} \mid f(x) = f(x+1)\}$ is infinite? Provide an example or show that such a function cannot exist.

Exercise 3

Say that a function $f : \mathbb{N} \to \mathbb{N}$ is quasi-total if it is undefined on a finite number of inputs, i.e., $\overline{dom(f)}$ is finite. Classify the set $A = \{x \in \mathbb{N} \mid \varphi_x \text{ quasi-total}\}$ from the point of view of recursiveness, i.e., establish whether A and $\bar{A}$ are recursive/recursively enumerable. .

Exercise 4

Classify the set $B = \{x \in \mathbb{N} \mid \exists y > 2x.\ y \in E_x\}$ from the point of view of recursiveness, i.e., establish whether B and $\bar{B}$ are recursive/recursively enumerable.

Note: Each exercise contributes with the same number of points (8) to the final grade.