

1) min  $200x_1 + 180x_2 + 230x_3$   
 s.t.  $5A \geq 3$   $C_A \leq 15$   
 $100 \leq 3s_A + c_A + 4s_B + 2c_B \leq 3x_1 + 6x_2 + 2x_3$   
 $200 \leq 5s_A + 7c_A + 6s_B + 3c_B \leq 2x_1 + 4x_2 + 4x_3$   
 $150 \leq 3s_A + 2c_A + 2s_B + 3c_B \leq 2x_1 + x_3$   
 $y_1 + y_2 + y_3 = 2$   $y_i \leq x_i \leq My_i$   $i=1,2,3$   
 $x_2 = 3w_2$   
 $x_i, s_j, c_j, w_2 \in \mathbb{Z}_+$   $y_i \in \{0,1\}$   $i=1,2,3, j=A,B$

4) (enunciato)  
 $-2 \geq -3$  (verifica Ann. Primal)  
 $1 = 1$   
 $1 \leq 1$   
 $-2 \leq -2$ , dominio ok  
 DUACE  
 $\max -3u_1 + u_2 + u_3 - 2u_4$   
 s.t.  $u_1 + u_2 - u_3 = 1$   
 $2u_1 - 2u_2 + u_4 \geq -2$   
 $-u_1 - 2u_2 + 2u_3 - u_4 \leq 0$   
 $u_1 \geq 0$   $u_2$  lib  $u_3, u_4 \leq 0$

2) min  $-x_2$   
 s.t.  $2x_1 + x_2 + x_4 = 2$   
 $-x_1 + x_2 + x_5 = 0$   
 $x_2 - x_3 = 2$   
 $x_1, x_2, x_3, x_4, x_5 \geq 0$

|       |    |    |   |   |   |    |   |
|-------|----|----|---|---|---|----|---|
|       | 0  | -1 | 0 | 0 | 0 | -1 | 0 |
| $x_1$ | 2  | 1  | 0 | 1 | 0 | 0  | 2 |
| $x_5$ | -1 | 1  | 0 | 0 | 1 | 0  | 0 |
| $x_3$ | 0  | 1  | 1 | 0 | 0 | 0  | 2 |

↓

|       |    |   |   |   |    |    |   |
|-------|----|---|---|---|----|----|---|
|       | -1 | 0 | 0 | 0 | 1  | -1 | 0 |
| $x_4$ | 3  | 0 | 0 | 1 | -1 | 0  | 2 |
| $x_2$ | -1 | 1 | 0 | 0 | 1  | 0  | 0 |
| $x_3$ | 1  | 0 | 1 | 0 | -1 | 0  | 2 |

↓

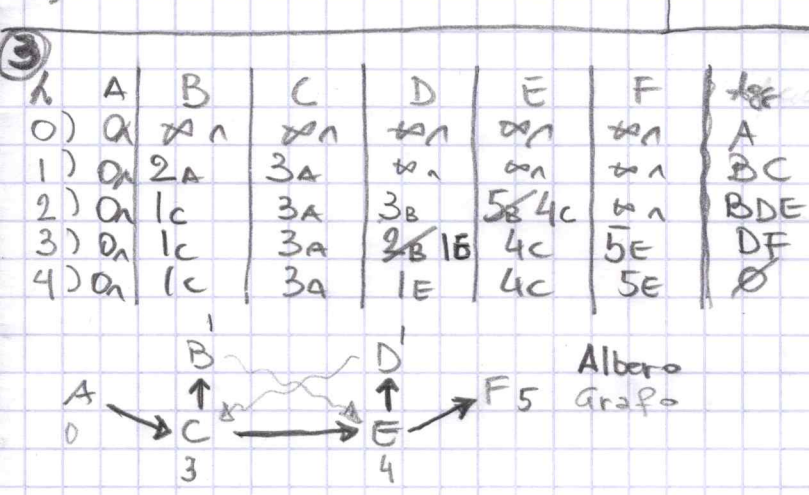
|       |   |   |   |                |                |    |               |
|-------|---|---|---|----------------|----------------|----|---------------|
|       | 0 | 0 | 0 | $\frac{1}{3}$  | $\frac{2}{3}$  | -1 | $\frac{2}{3}$ |
| $x_1$ | 1 | 0 | 0 | $\frac{1}{3}$  | $-\frac{1}{3}$ | 0  | $\frac{2}{3}$ |
| $x_2$ | 0 | 1 | 0 | $\frac{1}{3}$  | $\frac{2}{3}$  | 0  | $\frac{2}{3}$ |
| $x_3$ | 0 | 0 | 1 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | 0  | $\frac{4}{3}$ |

$\frac{1}{3}$  max  $-2x_2 = -(-\frac{1}{3}) = \frac{2}{3}$   
 $x_1 = \frac{2}{3}$   $x_2 = \frac{2}{3}$   $x_3 = -\frac{4}{3}$   $x_4 = x_5 = 0$

CCPD  
 $u_1 \cdot 1 = 0 \Rightarrow u_1 = 0$   
 $u_2$  lib: no capd  
 $u_3 \cdot 0 = 0 \Rightarrow //$   
 $u_4 \cdot 0 = 0 \Rightarrow //$   
 SISTEMA CCPD e A.D.  
 $\begin{cases} u_1 = 0 \\ 2u_1 + u_2 + u_4 = -2 \\ -u_1 - 2u_2 + 2u_3 - u_4 = 0 \\ u_1 + u_2 - u_3 = 1 \end{cases}$  CCPD  
 $\begin{cases} u_1 = 0 \\ u_2 = 0 \\ u_4 = -2 \\ u_3 = -1 \end{cases}$  A.D.  
 VERIFICA ATEN. DUACE  
 $u_1, u_2, u_3, u_4 \forall$ ;  $valore = \text{test} (p.c.)$   
 $\rightarrow \text{CONCLUS.} = \text{verificato!}$   
 $[ver. 1 - 2(-1) = 3 = -3 \cdot 0 + 0 + (-1) - 2(-2) = 3]$

5) a)  $x_6$   $x_1$   $x_3$  (f.c., ann., no ottima trovata)  
 b)  $10(x_2, x_3)$   $5(x_2, x_3)$   $30(x_4, x_5)$   $15(x_4, x_5)$   
 c)  $(x_2, x_3)$  5  
 d)  $0 - 2 \cdot 3 = -6$   
 e) si.

6) max  $2x_2 + 4x_3 + x_4 + x_5$   
 s.t.  $x_2 + 3x_3 + x_4 + 2x_5 \leq 3$   
 $x_2, x_3, x_4, x_5 \in \mathbb{Z}_+$



$P_0$   $x_2 = 1$   $W = 2$   
 $x_1 = \frac{2}{3}$   $x_5$   
 $UB = 2 \cdot 1 + 4 \cdot \frac{2}{3} = 4$   
 $LB = 2 + 1 = 3$

$P_1$   $x_1 = 0$   
 $x_2 = 1$   $W = 2$   
 $x_3 = 1$   $W = 1$   
 $x_4 = \frac{1}{2}$   $x_5 = \frac{1}{2}$   
 $UB = 1 = 2 + 1 + \frac{1}{2} \cdot 1 = 3$   
 $LB = -$

$P_2$   $x_1 = 1 \Rightarrow W = 0$   $UB = LB = 4$

OTTIMO  $x_1 = 1, x_2 = x_3 = x_4 = 0$

\*  $x_i$ : # pacchetti tipo  $i \in \{1,2,3\}$   
 $s_j$ : # server con macchina  $j \in \{A,B\}$   
 $c_j$ : # client  
 $y_i$ : 1 se acquisto pacchetto  $i \in \{1,2,3\}$ , 0 altrimenti  
 $w_2$ : contatore  $x_2$  moltiplo di 3