

$x_i^j = 1$  se gruppo  $g \in \{1,2\}$  usa l'impronta  $i \in \{A,B,C,D,E\}$ , 0 altrimenti  
 $y_i$ : numero di sciatori individuali che usa l'impronta  $i \in \{A..E\}$   
 $t_g$ : tempo di risalita del gruppo  $g \in \{1,2\}$ ;  $w = |t_1 - t_2|$   
 $z = 1$  se sciatori individuali usano impronta A, 0 altrimenti

min  $w$   
 n.l.  $w \geq t_1 - t_2 \quad w \geq t_2 - t_1$   
 tempi  $t_1 = 15x_A^1 + 10x_B^1 + 20x_C^1 + 15x_D^1 + 10x_E^1 \leq 30$   
 $t_2 = 15x_A^2 + 10x_B^2 + 20x_C^2 + 15x_D^2 + 10x_E^2 \leq 30$

ASSEGNA  $y_D \geq 10$   
 $y_A + y_B + y_C = 30$   
 $x_A^1 + x_B^1 + x_C^1 = 1$   
 $x_A^2 + x_B^2 + x_C^2 = 1$   
 $y_D + y_E = 30$   
 $x_D^1 + x_E^1 = 1$   
 $x_D^2 + x_E^2 = 1$

capacità  $\begin{cases} 30x_A^1 + 40x_B^1 + y_A \leq 50 \\ 30x_B^1 + 40x_C^1 + y_B \leq 40 \\ 30x_C^1 + 40x_D^1 + y_C \leq 30 \end{cases}$   
 $\begin{cases} 30x_D^2 + 40x_E^2 + y_D \leq 60 \\ 30x_E^2 + 40x_A^2 + y_E \leq 40 \end{cases}$

Budget  $\begin{cases} 16 \cdot 30 x_A^1 + 16 \cdot 40 x_B^1 + 20 y_A + 12 \cdot 30 x_D^2 + 12 \cdot 40 x_E^2 + 15 y_D + \\ + 8 \cdot 30 x_C^1 + 8 \cdot 40 x_D^1 + 10 y_C + 12 \cdot 30 x_A^2 + 12 \cdot 40 x_B^2 + 15 y_B + \\ + 20 \cdot 30 x_E^2 + 20 \cdot 40 x_A^2 + 25 y_E \leq 2750 \end{cases}$

logico  $y_A \leq Mz \quad x_A^1 + z \leq 1 \quad (y_A \geq z, \text{max score})$   
 $x_i^j \in \{0,1\} \quad y_i \in \mathbb{Z}_+ = \mathbb{Z} \cap \{0,1\} \quad w, t_1, t_2 \in \mathbb{R}$

2) min  $-x_1 - \hat{x}_2 - 3x_3$   
 n.l.  $x_1 - \hat{x}_2 - 2x_3 + x_4 = 1$   
 $2x_1 - \hat{x}_2 + x_3 + x_5 = 2$   
 $-x_1 + 2x_3 + x_6 = 4$   
 $x_1, \hat{x}_2, x_3, x_4, x_5, x_6 \geq 0$   
 Tableau:  
 $\begin{array}{ccccccc|c} -1 & -1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 1 & -1 & -2 & 1 & 0 & 0 & 0 & 1 \\ 2 & -1 & 1 & 0 & 1 & 0 & 0 & 2 \\ -1 & 0 & 2 & 0 & 0 & 1 & 0 & 4 \end{array}$   
 ILLIM. !!!  
 $\begin{array}{ccccccc|c} 0 & -2 & -5 & 1 & 0 & 0 & -1 & 1 + R_1 \\ 1 & -1 & -2 & 1 & 0 & 0 & 0 & 1 = \\ 0 & 1 & 5 & -2 & 1 & 0 & 0 & 0 - 2R_1 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 5 + R_1 \end{array}$   
 $\begin{array}{ccccccc|c} 0 & 0 & 5 & -3 & 2 & 0 & -1 & 1 + 2R_2 \\ 1 & -0 & 3 & -1 & 1 & 0 & 0 & 1 + R_2 \\ 0 & 1 & 5 & -2 & 1 & 0 & 0 & 0 = \\ 0 & 0 & 5 & -1 & 1 & 1 & 0 & 5 + R_2 \end{array}$

4) max  $-u_1 + u_2 + 3u_3$   
 n.l.  $u_1 + u_2 \leq 1$   
 $2u_2 + u_3 \geq -2$   
 $u_1 - 2u_3 \leq -1$   
 $-u_1 + u_3 = 2$   
 $u_1 \leq 0, u_2 \geq 0, u_3$  libera  
 $u_1 \cdot (1 + 0 - 3 + 1) = 0 \Rightarrow u_1 \cdot (-1) = 0 \Rightarrow u_1 = 0$   
 $u_2 \cdot (1 - 1) = 0 \Rightarrow 0 = 0$   
 $u_3$  libera  $\Rightarrow$  NO CCPD (ATTN. PRER.)  
 $x_1 (u_1 + u_2 - 1) = 0 \Rightarrow u_1 + u_2 = 1$   
 $x_2 = 0 \Rightarrow 0$   
 $x_3 = 0 \Rightarrow 0$   
 $x_4$  libera  $\Rightarrow$  NO CCPD (ATTN. PRER.)  
 $\begin{cases} u_1 = 0 \text{ CCPD} \\ u_1 + u_2 = 1 \text{ CCPD} \\ -u_1 + u_3 = 2 \text{ A.D.} \end{cases} \Rightarrow \begin{cases} u_1 = 0 \\ u_2 = 1 \\ u_3 = 2 \end{cases}$   
 $u_1 + u_2 = 1 \vee \quad 2u_2 + u_3 = 2 > -2 \vee \Rightarrow$  OTTIMA  
 $u_1 - 2u_3 = -4 < -1 \vee$   
 (VERIF.  $1 + 6 = 7 = 1 + 6 \checkmark$ )

b) PRIMA IL LIM  $\Rightarrow$  DUALE INAMMISSIB. (Corollario dualita' debole)

	A	B	C	D	E	F	G	Age
0	0A	10A	10A	10A	10A	10A	10A	A
1	0A	5A	5A	1A	10A	10A	10A	B
2	0A	3D	4B 3D	1A	8B	9C 3D	10A	BEEF
3	0A	3D	2B	1A	6B 5F	3D	1F	CEA
4	0A	3D	2B	1A	2A	3D	1F	E
5	0A	3D	2B	1A	2A	3D	1F	

d)

- 5) a)  $(x_2, x_1) = (21, 67, 0)$  b)  $(x_2, x_1)$   
 c)  $(73, x_1)$  d)  $(x_4, x_2)$  e)  $(x_4, x_3)$   
 f)  $-5 = -5 - 14 \cdot 0$   
 6) a)  $\in [16, 9; 17, 1]$  b)  $[16, 0; 16, 9]$   
 c)  $P_5$  e  $P_6$  d)  $P_4$  e)  $SA = UB \in [16, 7; 16, 9]$