$\boxed{9}$ Si dica se

$$\mathcal{S} = \left\{ \mathbf{v_1} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}; \mathbf{v_2} = \begin{pmatrix} 3 \\ -4 \\ 8 \end{pmatrix}; \mathbf{v_3} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \right\}$$

è un insieme di generatori di $\mathbb{R}^3$.

TROVARE $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a_1 v_1 + a_2 v_2 + a_3 v_3$

$$= a_1 \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + a_2 \begin{pmatrix} 3 \\ -4 \\ 8 \end{pmatrix} + a_3 \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + 3a_2 + a_3 = a \\ -a_1 - 4a_2 + 2a_3 = b \\ 2a_1 + 8a_2 - 4a_3 = c \end{pmatrix}$$

$\downarrow$ SCRIVO COME MATRICE

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & a \\ -1 & -4 & 2 & b \\ 2 & 8 & -4 & c \end{array}\right) \xrightarrow{E_{31}(-2)\ E_{21}(1)}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & a \\ 0 & -1 & 3 & a+b \\ 0 & 2 & -6 & c-2a \end{array}\right) \xrightarrow{E_{32}(-2)\ E_3(-1)}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 3 & 1 & a \\ 0 & \boxed{1} & -3 & -a-b \\ 0 & 0 & 0 & 2b+c \end{array}\right) \to rk(A)$$

→ RIUSCIAMO AD OTTENERE QUESTO INSIEME DI GENERATORI

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11 Si dica quale dei seguenti sottoinsiemi di $\mathbb{R}^3$ è linearmente indipendente:

$$\left\{ \mathbf{v_1} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}; \mathbf{v_2} = \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}; \mathbf{v_3} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\},$$

$$[a_1 v_1 + a_2 v_2 + a_3 v_3 = \underline{0}]$$

$$a_1 v_1 + a_2 v_2 + a_3 v_3$$

$$\| = a_1 \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix} + a_3 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$= \begin{cases} 4a_2 + 2a_3 = 0 \\ 4a_1 + 6a_2 - a_3 = 0 \\ 2a_2 + a_3 = 0 \end{cases}$$

$$\to \left(\begin{array}{ccc|c} 0 & 4 & 2 & 0 \\ 4 & 6 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right)$$

$$\to \dots \left(\begin{array}{ccc|c} 1 & 3/2 & -1/4 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$rk(A) < rk(B) \to \infty$ SOLUZ.

LINEARMENTE DIPENDENTE

$$\begin{cases} a_1 + \frac{3}{2} a_2 - \frac{1}{4} a_3 = 0 \\ a_2 + \frac{1}{2} a_3 = 0 \end{cases}$$

.... $a_2 = -\frac{1}{2}$

$a_1 = 1$ $\qquad$ $a_3 = ?$

SOL. NON NULLA

$\begin{pmatrix} -1/2 \\ 1 \\ a_3 \end{pmatrix} \to$ SOL. $a_3 \neq 0$

$\boxed{1}$ Sia $\quad \mathbf{A}_\alpha = \begin{pmatrix} 2 & 0 & 0 & 2i \\ 0 & \alpha & 0 & 2i \\ 4 & \alpha-1 & 0 & 4i \\ 0 & 2 & 4\alpha-6 & 0 \end{pmatrix}$, $\quad$ dove $\alpha \in \mathbb{C}$.

$rk(A)$

Per ogni $\alpha \in \mathbb{C}$ si dica qual è $rk(\mathbf{A}_\alpha)$ e si trovi una base $\mathcal{B}_\alpha$ di $C(\mathbf{A}_\alpha)$.

$A \to \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} \xrightarrow{\text{RIDOTTA}} \begin{pmatrix} 1 & -5 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

PRENDO LE COL. DOMINANTI RISPETTO ALLA MATRICE DI PARTENZA

$$\mathbf{A}_\alpha = \begin{pmatrix} 2 & 0 & 0 & 2i \\ 0 & \alpha & 0 & 2i \\ 4 & \alpha-1 & 0 & 4i \\ 0 & 2 & 4\alpha-6 & 0 \end{pmatrix} \xrightarrow{E_{31}(-4)E_1(\frac{1}{2})} \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & \alpha & 0 & 2i \\ 0 & \alpha-1 & 0 & 0 \\ 0 & 2 & 4\alpha-6 & 0 \end{pmatrix} \to$$

$$\xrightarrow{E_{24}} \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 2 & 4\alpha-6 & 0 \\ 0 & \alpha-1 & 0 & 0 \\ 0 & \alpha & 0 & 2i \end{pmatrix} \xrightarrow{E_{42}(-\alpha)E_{32}(-\alpha+1)E_2(\frac{1}{2})}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & 2\alpha-3 & 0 \\ 0 & 0 & -(2\alpha-3)(\alpha-1) & 0 \\ 0 & 0 & -(2\alpha-3)\alpha & 2i \end{pmatrix} = \mathbf{B}_\alpha \qquad \to \alpha = ?$$

1° CASO $\to \underline{\alpha = 1}$

$$B_1 \to \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2i \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2i \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A}_\alpha = \begin{pmatrix} 2 & 0 & 0 & 2i \\ 0 & \alpha & 0 & 2i \\ 4 & \alpha-1 & 0 & 4i \\ 0 & 2 & 4\alpha-6 & 0 \end{pmatrix}, \to \underbrace{\mathcal{B} = \left\{ \begin{pmatrix} 2 \\ 0 \\ 4 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 0 \\ -2 \end{pmatrix} \right\}}$$

$\underline{2^\circ \text{ CASO}}$ $\alpha = 3/2$

$$B_{3/2} \to \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A}_\alpha = \begin{pmatrix} 2 & 0 & 0 & 2i \\ 0 & \alpha & 0 & 2i \\ 4 & \alpha-1 & 0 & 4i \\ 0 & 2 & 4\alpha-6 & 0 \end{pmatrix}, \to \mathcal{B}_{\frac{3}{2}} = \left\{ \begin{pmatrix} 2 \\ 0 \\ 4 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ \frac{3}{2} \\ \frac{1}{2} \\ 2 \end{pmatrix}; \begin{pmatrix} 2i \\ 2i \\ 4i \\ 0 \end{pmatrix} \right\}.$$

3° CASO $\to \alpha \neq 1 \vee \alpha \neq \frac{3}{2}$

$$\mathbf{B}_\alpha = \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & 2\alpha-3 & 0 \\ 0 & 0 & -(2\alpha-3)(\alpha-1) & 0 \\ 0 & 0 & -(2\alpha-3)\alpha & 2i \end{pmatrix} \xrightarrow{E_{43}((2\alpha-3)\alpha)E_3(\frac{1}{-(2\alpha-3)(\alpha-1)})}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & 2\alpha-3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2i \end{pmatrix} \xrightarrow{E_4(\frac{1}{2i})} \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & 2\alpha-3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{U}_\alpha$$

$[rk(A) = 4]$

Una base $\mathcal{B}_\alpha$ di $C(\mathbf{A}_\alpha)$ è $\mathcal{B}_\alpha = \left\{ \begin{pmatrix} 2 \\ 0 \\ 4 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 2 \\ \alpha-1 \\ 2 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4\alpha-6 \end{pmatrix}; \begin{pmatrix} 2i \\ 2i \\ 4i \\ 0 \end{pmatrix} \right\}$.

APPL. LINEARI

$f : V \to W$

$\boxed{0}$ V e W sono spazi vettoriali

$\boxed{1}$ $f(v_1 + v_2) = f(v_1) + f(v_2)$, $v_1, v_2 \in V, W$

$\boxed{2}$ $f(\alpha v) = \alpha \cdot f(v)$

$\boxed{5}$ **Si dica quale delle due seguenti posizioni definisce un'applicazione lineare:**

(a) $f_1 : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ **definita da** $f_1(\mathbf{A}) = \mathbf{A}^T$ **per ogni** $\mathbf{A} \in M_n(\mathbb{C})$;

(b) $f_2 : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ **definita da** $f_2(\mathbf{A}) = \mathbf{A}^2$ **per ogni** $\mathbf{A} \in M_n(\mathbb{C})$.

(1) $f(A+B) = f(A) + f(B)$

(2) $f(\alpha A) = \alpha \cdot f(A)$

In (a) $\to$ VERIFICA (1)?

USANDO QUANTO VALE $f_1 \dots = A^T$

$f_1(A+B)$

$= A^T + B^T \quad \forall A, B \in V, W$

VERIFICATA 1

→ VERIFICA (2) ?

$f_1(\alpha A) = \alpha \cdot f_1(A)$ ⇒ VERIFICATA 2

$= \alpha \cdot A^T$

è appl. lineare

$f_2 = A^2$

① $f_2(A+B)$

$= (A^2+B^2) = \underbrace{(A+B)}^2 = A^2 + B^2 + 2AB$

$\boxed{2}$ $f_2(\alpha A) = \alpha \cdot A^2$ → VERIFICATA

è appl. lineare

1. Si dica se sono lineari le seguenti funzioni:

(a) $f: \mathbb{R}^3 \to \mathbb{R}^2$ dove $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x-4z \\ x+y+z \end{pmatrix}$ per ogni $x, y, z \in \mathbb{R}$.

① SOMMA

② PRODOTTO PER SCALARE α

Prendiamo due vettori

$u = (x_1, y_1, z_1)$, $v = (x_2, y_2, z_2)$

$f(u+v) = f(x_1+x_2, y_1+y_2, z_1+z_2)$

$x - 4z$ $\quad$ $x+y+z$

$= ((x_1+x_2) - 4(z_1+z_2), (x_1+x_2) + (y_1+y_2) + (z_1+z_2))$

$= (x_1 + x_2 - 4z_1 - 4z_2, x_1 + x_2 + y_1 + y_2 + z_1 + z_2$

$= (x_1 - 4z_1, x_1 + y_1 + z_1) + (x_2 - 4z_2, x_2 + y_2 + z_2)$

$= f(u) + f(v) \rightarrow$ RISPETTA (1)

(2) $f(\alpha v) = f((\alpha x, \alpha y, \alpha z))$

$= \alpha(x - 4z) + \alpha(x + y + z)$

$= \alpha(x - 4z, x + y + z)$

$\rightarrow$ RISPETTA (2)

f è un'appl. lineare

3.[2] Si determinino le dimensioni dello spazio nullo $N(A) \subseteq \mathbb{R}^4$ e del sottospazio $C(A) \subseteq \mathbb{R}^3$ e delle basi di tali sottospazi per le seguenti matrici:

(a) $A = \begin{pmatrix} 2 & 1 & 4 & 8 \\ 0 & 6 & -2 & 1 \\ 2 & 7 & 2 & 9 \end{pmatrix}$

(b) $A = \begin{pmatrix} -2 & 0 & 0 & 1 \\ 1 & 0 & 7 & 1 \\ 0 & 1 & 4 & 1 \end{pmatrix}$

$A = \begin{pmatrix} 2 & 1 & 4 & 8 \\ 0 & 6 & -2 & 1 \\ 2 & 7 & 2 & 9 \end{pmatrix} \longrightarrow \dots$

$\begin{pmatrix} 1 & 0 & 4/3 & 7/3 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

TEOREMA NULLITÀ - RANGO

$= \dim(A) - rk(A) = 4 - 2 = 2$

$B_{sp.\,nullo} = \left\{ \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 8 \\ 1 \\ 9 \end{pmatrix} \right\} \rightarrow$ COLONNE LIBERE (PER SP. NULLO)

→ PER DIMENSIONE
SOTTOSPAZIO COLONNA (A)

CONTO LE COLONNE DOMINANTI

→ $\dim C(A) = 2$

② B → $A = \begin{pmatrix} -2 & 0 & 0 & 1 \\ 1 & 0 & 7 & 1 \\ 0 & 1 & 4 & 1 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 15/2 \\ 0 & 0 & 1 & -4 \end{pmatrix}$

dim SP. COLONNA = COL. DOMINANTI = 3

dim. sp. nullo → $\dim(A) - rk(A) = 4 - 3 = 1$

BASI ORTOGONALI E ORTONORMALI

ORTOGONALI → $B = \{v_1, v_2, \dots v_n\}$

→ $\langle v_i, v_j \rangle = 0 \quad \forall i \neq j$

ORTONORMALI
$$\begin{cases} B = \{v_1, v_2, \dots v_n\} \\ = 0 \quad \forall i \neq j \\ \|v_i\| = \sqrt{\langle v_i, v_i \rangle} = 1 \end{cases}$$

NORMALIZZO UN VETTORE

$\{v_1, v_2, \dots v_n\}$ $\qquad u_i$ = VETTORE NORMALIZZATO

$$u_i = \frac{v_i}{\|v_i\|}$$

ALGORITMO DI GRAM-SCHMIDT

$u_1 = v_1$

$u_2 = v_2 - \alpha_{12} u_1$

$u_3 = \underbrace{v_3 - \alpha_{13} u_1 - \alpha_{23} u_2}$

$e_1 = \frac{u_1}{\|u_1\|} = \begin{bmatrix} \vdots \end{bmatrix}$

$e_2 = \frac{u_2}{\|u_2\|} = \begin{bmatrix} \vdots \end{bmatrix}$

$e_3 = \frac{u_3}{\|u_3\|} = \begin{bmatrix} \vdots \end{bmatrix}$

$\rightarrow B = \{e_1, e_2, e_3\}$

$\alpha_{12} \rightarrow \frac{(u_1|v_2)}{(u_1|u_1)} \rightarrow (u_1|v_2)$

$\alpha_{13} \rightarrow \frac{(u_1|v_3)}{(u_1|u_1)} \rightarrow$

$|$ = PIPE = PROD. RIGA COLONNA

H-TRASPOSTA (DA RIGA PASSO A COLONNA O VICEVERSA CAMBIANDO DI SEGNO)

$\boxed{1}$ Si trovi una base ortonormale del sottospazio

$$V = \left\langle \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix}; \begin{pmatrix} -1 \\ -i \\ -1 \\ -i \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 2i \\ 0 \end{pmatrix} \right\rangle$$

di $\mathbb{C}^4$.

1. COSTRUISCO UNA BASE DI A $\underline{\text{RIDOTTA}}$ → PRENDO LE COL. DOMINANTI

2. BASE ORTOGONALE USANDO GRAM-SCHMIDT

3. BASE ORTONORMALE USANDO LA NORMALIZZAZIONE $\|u_1\| = \sqrt{(u_1|u_1)}$

1

$$\mathbf{w_1} = \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix}; \quad \mathbf{w_2} = \begin{pmatrix} -1 \\ -i \\ -1 \\ -i \end{pmatrix}; \quad \mathbf{w_3} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad \mathbf{w_4} = \begin{pmatrix} 0 \\ 0 \\ 2i \\ 0 \end{pmatrix}$$

$$\mathbf{A} = (\mathbf{w_1} \quad \mathbf{w_2} \quad \mathbf{w_3} \quad \mathbf{w_4}) = \begin{pmatrix} i & -1 & 1 & 0 \\ -1 & -i & 0 & 0 \\ i & -1 & 1 & 2i \\ -1 & -i & 0 & 0 \end{pmatrix} \xrightarrow{E_{41}(1)E_{31}(-i)E_{21}(1)E_1(-i)}$$

$$\to \begin{pmatrix} 1 & i & -i & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 2i \\ 0 & 0 & -i & 0 \end{pmatrix} \xrightarrow{E_3(-\frac{1}{2}i)E_{42}(i)E_2(i)} \begin{pmatrix} 1 & i & -i & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \mathbf{U}$$

$$\mathcal{B} = \{w_1, w_3, w_4\}$$

2

$$\left\{ \mathbf{v_1} = \mathbf{w_1} = \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix}; \mathbf{v_2} = \mathbf{w_3} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \mathbf{v_3} = \mathbf{w_4} = \begin{pmatrix} 0 \\ 0 \\ 2i \\ 0 \end{pmatrix} \right\}.$$

$$u_1 = v_1 = \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix}$$

$$u_2 = v_2 - \alpha_{12} u_1$$

$$\alpha_{12} = \frac{\langle u_1 | v_2 \rangle}{\langle u_1 | u_1 \rangle}$$

$$\langle u_1 | v_2 \rangle = u_1^{+} v_2 = (-i \quad -1 \quad -i \quad -1) \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= (-i \cdot 1) - (1 \cdot 0) + (-i \cdot 1) - (1 \cdot 0)$$

$$= -2i$$

$$\langle u_1 | u_1 \rangle = u_1^{+} u_1 = (-i \quad -1 \quad -i \quad -1) \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix} = 4$$

$$\alpha_{12} = -\frac{2i}{4} = -\frac{1}{2} i$$

$$u_2 = v_2 - \alpha_{12} u_1 = v_2 + \frac{1}{2} i u_1$$

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \frac{1}{2} i \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix} = 1/2 \begin{pmatrix} 1 \\ -i \\ 1 \\ -i \end{pmatrix}$$

$$u_3 = v_3 - \alpha_{13} u_1 - \alpha_{23} u_2$$

$$\alpha_{13} = \frac{(u_1 | v_3)}{(u_1 | u_1)}$$

$$(u_1 | v_3) = u_1^H v_3 = (-i \; -1 \; -i \; -1) \begin{pmatrix} 0 \\ 0 \\ 2i \\ 0 \end{pmatrix} = 2$$

$$(u_1 | u_1) = u_1^H u_1 = 4$$

$$\alpha_{13} = \frac{2}{4} = \frac{1}{2}$$

$$\alpha_{23} = \frac{(u_2 | v_3)}{(u_2 | u_2)}$$

$$(u_2 | v_3) = u_2^H v_3 = \frac{1}{2} (1 \; i \; 1 \; i) \begin{pmatrix} 0 \\ 0 \\ 2i \\ 0 \end{pmatrix} = i$$

$$(u_2 | u_2) = u_2^H u_2 = \frac{1}{2} (1 \; i \; \; 1 \; i) \; \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ 1 \\ -i \end{pmatrix} = 1$$

$$\alpha_{23} = \frac{i}{1} = i$$

$$u_3 = v_3 - \alpha_{13} u_1 - \alpha_{23} u_3$$

$$= \begin{pmatrix} 0 \\ 0 \\ 2i \\ 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix} - \frac{1}{2} i \begin{pmatrix} 1 \\ -i \\ 1 \\ -i \end{pmatrix} = \begin{pmatrix} -i \\ 0 \\ i \\ 0 \end{pmatrix}$$

BASE ORTOGONALE

$$\left\{ u_1 = \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix} ; u_2 = \frac{1}{2}\begin{pmatrix} 1 \\ -i \\ 1 \\ -i \end{pmatrix} ; u_3 = \begin{pmatrix} -i \\ 0 \\ i \\ 0 \end{pmatrix} \right\}$$

(3) BASE ORTONORMALE

TUTTE LE NORME DEI VETTORI
DELLA BASE ORTOGONALE (u_1, u_2, u_3)

$$\|u_1\|_2 = \sqrt{\overrightarrow{(i \ -1 \ i \ -1)} \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix}} = \sqrt{4} = 2$$

$$\|u_2\| = \sqrt{\overrightarrow{(u_2 | u_2)}} = \sqrt{1} = 1$$

$$\|u_3\| = \sqrt{\overrightarrow{(u_3 | u_3)}} = \sqrt{(i \ 0 \ -i \ 0) \begin{pmatrix} -i \\ 0 \\ i \\ 0 \end{pmatrix}}$$

$$= \sqrt{1+1} = \sqrt{2}$$

BASE ORTONORMALE

$$B = \left\{ \frac{u_1}{\|u_1\|_2} , \frac{u_2}{\|u_2\|_2} , \frac{u_3}{\|u_3\|_2} \right\}$$

$$\left\{ \frac{1}{2}\begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix} ; \frac{1}{2}\begin{pmatrix} 1 \\ -i \\ 1 \\ -i \end{pmatrix} ; \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 0 \\ i \\ 0 \end{pmatrix} \right\}$$