

ESERCIZIO TIPO 4

Sia $\mathbf{A}(\alpha) = \begin{pmatrix} \alpha-1 & 1 & \alpha-1 \\ \alpha-1 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix}$, dove $\alpha \in \mathbb{R}$. Per quegli $\alpha \in \mathbb{R}$ per cui $\mathbf{A}(\alpha)$ è non singolare, si calcoli $\mathbf{A}(\alpha)^{-1}$.

$$\begin{aligned} (\mathbf{A}(\alpha) \mid \mathbf{I}_3) &= \left(\begin{array}{ccc|ccc} \alpha-1 & 1 & \alpha-1 & 1 & 0 & 0 \\ \alpha-1 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\boxed{\alpha \neq 1 : \mathbf{A}(1) \text{ non ha inversa}}} \\ &\xrightarrow{E_{21}(-\alpha+1)E_1(\frac{1}{\alpha-1})} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 1 & \frac{1}{\alpha-1} & 0 & 0 \\ 0 & 0 & -\alpha & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{E_{23}} \\ &\rightarrow \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 1 & \frac{1}{\alpha-1} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\alpha & -1 & 1 & 0 \end{array} \right) \xrightarrow{\boxed{\alpha \neq 0 : \mathbf{A}(0) \text{ non ha inversa}} E_3(-\frac{1}{\alpha})} \\ &\rightarrow \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 1 & \frac{1}{\alpha-1} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{array} \right) \xrightarrow{E_{13}(-1)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 0 & \frac{1}{\alpha(\alpha-1)} & \frac{1}{\alpha} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{array} \right) \rightarrow \\ &\xrightarrow{E_{12}(-\frac{1}{\alpha-1})} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{\alpha(\alpha-1)} & \frac{1}{\alpha} & -\frac{1}{\alpha-1} \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{array} \right) = (\mathbf{I}_3 \mid \mathbf{A}(\alpha)^{-1}). \end{aligned}$$

$$\text{Se } \boxed{\alpha \notin \{0, 1\}} \quad \mathbf{A}(\alpha)^{-1} = \begin{pmatrix} \frac{1}{\alpha(\alpha-1)} & \frac{1}{\alpha} & -\frac{1}{\alpha-1} \\ 0 & 0 & 1 \\ \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{pmatrix}.$$