DISEQUAZIONI

1) $x < x^2$

R. $x(x-1) > 0 \Rightarrow x < 0$ o $x > 1$

2) $3x^2 + 2x - x^2 < 1 + 2x^2$

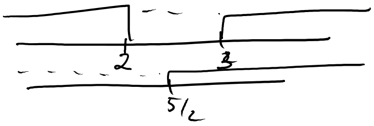
R. $2x - 1 < 0 \Rightarrow x < 1/2$

3) $(x^2-5x+6)(2x-5) \geq 0$

R. $x^2-5x+6 \geq 0$ $\quad x \geq 3$ o $x \leq 2$

$2x-5 \geq 0$ $\quad x \geq 5/2$

$\Rightarrow x \geq 3$ o $2 \leq x \leq 5/2$

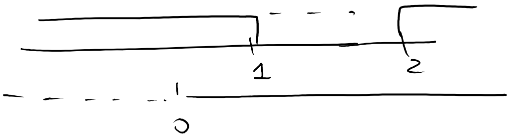


4) $\dfrac{x-2}{x} \leq x-2$

R. $x \neq 0$ $\quad \dfrac{x-2}{x} - x + 2 \leq 0$ $\quad \dfrac{x-2-x^2+2x}{x} \leq 0$

$\dfrac{-x^2+3x-2}{x} \leq 0$ $\quad \dfrac{x^2-3x+2}{x} \geq 0 \longrightarrow x \leq 1$ o $x \geq 2$

$\Rightarrow 0 < x \leq 1, \; x \geq 2$



5) $\dfrac{1}{\alpha x} - 2x > 0, \quad \alpha \in \mathbb{R}, \alpha \neq 0$

$\dfrac{1-2\alpha x^2}{\alpha x} > 0$ $\quad 1 - 2\alpha x^2 = 0$ $\quad x^2 = \dfrac{1}{2\alpha}$ $\quad$ se $\alpha < 0$ $\; 1-2\alpha x^2 > 0$

Quindi $\alpha < 0$ $\quad \dfrac{(\;\;)}{(\;\;)} > 0 \Leftrightarrow x < 0$

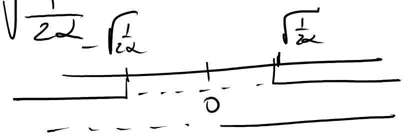
Se $\alpha > 0$ $\quad 1 - 2\alpha x^2 = 0 \Leftrightarrow x = \pm\sqrt{\dfrac{1}{2\alpha}}$

$1 - 2\alpha x^2 > 0 \Leftrightarrow x < -\sqrt{\dfrac{1}{2\alpha}}$ o $x > \sqrt{\dfrac{1}{2\alpha}}$

$\alpha x > 0 \Leftrightarrow x > 0$ perché $\alpha > 0$

Quindi $\alpha > 0$ $\; -\sqrt{\dfrac{1}{2\alpha}} < x < 0, \; x > \sqrt{\dfrac{1}{2\alpha}}$

se $\alpha < 0$ per $x < 0$.



6) $|x-1| < |2x-3|$

a) $x \geq \frac{3}{2}$ $\quad |x-1| < 2x-3$ cioè

$-(2x-3) \underset{①}{\leq} x-1 \underset{②}{\leq} 2x-3$

① $x - 1 > -2x + 3 \Rightarrow 3x > 4 \quad x > 4/3$

② $x - 1 < 2x - 3 \Rightarrow x > 2$

$\Big\} \Rightarrow \{x > 2\} \cap \{x \geq \frac{3}{2}\}$

$\Rightarrow x > 2$

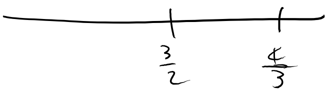
b) $x < \frac{3}{2}$ $\quad |x-1| < 3-2x$ cioè

$2x-3 \underset{①}{\leq} x-1 \underset{②}{\leq} 3-2x$

① $2x - 3 < x - 1 \Rightarrow x < 2$

② $x - 1 < 3 - 2x \Rightarrow 3x < 4 \Rightarrow x < 4/3$ $\quad \Rightarrow \{x < 4/3\} \cap \{x < \frac{3}{2}\}$

$\Rightarrow x < \frac{3}{2}$



Quindi $x < \frac{3}{2}$ o $x > 2$.

7) $|x|x + 5x - 6 > 0$

- $x > 0$ $\quad x^2 + 5x - 6 > 0$ $\quad x = \frac{-5 \pm \sqrt{25+24}}{2} = \frac{-5 \pm 7}{2} = \begin{cases} -6 \\ 1 \end{cases}$

$x > 1$ o $x < -6$ e poiché $x > 0$

$\Rightarrow x > 1$

- $x < 0$ $\quad -x^2 + 5x - 6 > 0$

$x^2 - 5x + 6 < 0$ $\quad x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2} = \begin{cases} 3 \\ 2 \end{cases}$

$2 < x < 3$ ma poiché $x < 0$ mai!

Quindi $x > 1$.

8) $|5x - 1| \geq x$ $\quad \Rightarrow \quad 5x - 1 \geq x$ o $5x - 1 \leq -x$

$\Downarrow \qquad\qquad \Downarrow$

$4x \geq 1 \qquad 6x \leq 1$

$x \geq 1/4$ o $x \leq 1/6$

9) $x^2 + |x+1| \leq 4$

$x \geq -1 \quad \Rightarrow \quad x^2 + x - 3 \leq 0$ $\quad x = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$

$\frac{-1-\sqrt{13}}{2} \leq x \leq \frac{-1+\sqrt{13}}{2}$ ma poiché $x \geq -1$

$\Downarrow$

$-1 \leq x \leq \frac{-1+\sqrt{13}}{2}$

$x < -1$ $\quad x^2 - x - 5 \leq 0$ ..... le soluzioni vanno $\cap$ con $x < -1$

10) $\left|\frac{x-3}{x-5}\right| > 2x$

R. $x \neq 5$ $\qquad \frac{x-3}{x-5} > 2x$ o $\frac{x-3}{x-5} < -2x$

$\Downarrow \qquad\qquad \Downarrow$

$\frac{x-3}{x-5} - 2x > 0 \qquad \frac{x-3}{x-5} + 2x < 0$

$\frac{x-3-2x(x-5)}{x-5} > 0 \qquad \frac{x-3+2x(x-5)}{x-5} < 0$

$\frac{-2x^2+11x-3}{x-5} > 0 \qquad \frac{2x^2-9x-3}{x-5} < 0$

! $\qquad$!

e si fa l'$\cup$ delle soluzioni.

Disequazioni

1) $\boxed{x+|x|>1}$

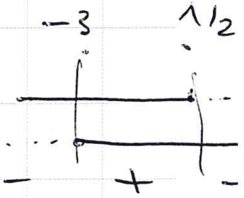
R. $x>0 \Rightarrow \quad 2x>1 \Rightarrow \quad x>\frac{1}{2}$

$x<0 \Rightarrow \quad x-x>1$ nessuna sol. $\Big\}\ x>\frac{1}{2}$

2) $\boxed{\left|\frac{6-5x}{3+x}\right| \leq 1}$

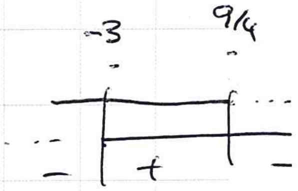
R. $-1 \leq \frac{6-5x}{3+x} \leq 1 \quad \Rightarrow \quad \begin{cases} \frac{6-5x}{3+x} \leq 1 \\ \frac{6-5x}{3+x} \geq -1 \end{cases}$

$\frac{6-5x}{3+x} \leq 1 \Rightarrow \quad \frac{6-5x-3-x}{3+x} \leq 0 \Rightarrow \frac{3-6x}{3+x} \leq 0$



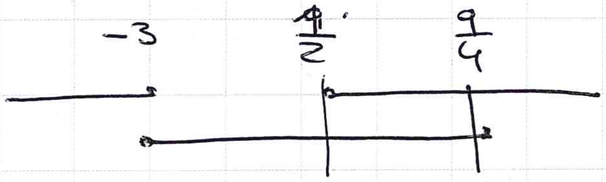
$\left[\begin{array}{l} x<-3 \\ x \geq \frac{1}{2} \end{array}\right.$

$\frac{6-5x}{3+x} \geq -1 \quad \Rightarrow \quad \frac{6-5x+3+x}{3+x} \geq 0 \quad \Rightarrow \quad \frac{9-4x}{3+x} \geq 0$



$-3 < x \leq \frac{9}{4}$

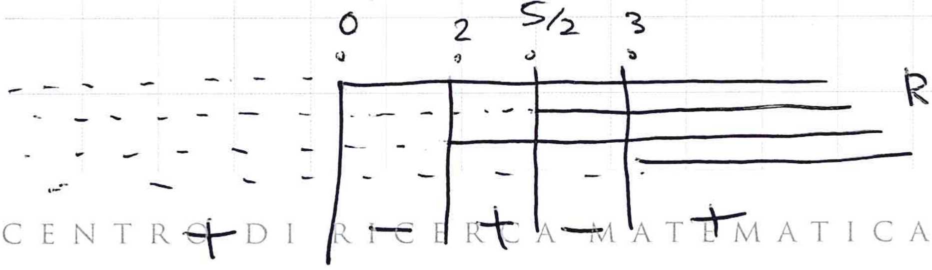
A sistema



R: $\quad \frac{1}{2} \leq x \leq \frac{9}{4}$

3) $\boxed{\frac{x}{x-3} + \frac{x}{x-2} \leq 0}$

$\frac{x[x-2+x-3]}{(x-3)(x-2)} \leq 0 \qquad \frac{x(2x-5)}{(x-3)(x-2)} \leq 0$



R $\boxed{\begin{array}{l} 0 \leq x < 2 \\ \frac{5}{2} \leq x < 3 \end{array}}$

4) $\boxed{\sqrt{x+1} \le x-1}$

$x+1 \ge 0$ condizioni esistenza $\boxed{x \ge -1}$

inoltre $\sqrt{x+1} \ge 0 \Rightarrow x-1 \ge 0$ altrimenti non ho soluzioni

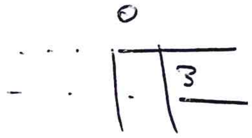
• se $-1 \le x < 1$ nessuna soluzione

$x \ge 1$ $\sqrt{x+1} \le x-1$ elevo al quadrato

$$x+1 \le (x-1)^2.$$

$$x+1 \le x^2-2x+1$$

$$x^2-3x \ge 0 \Rightarrow x(x-3) \ge 0$$



$\boxed{x \ge 3}$

non accettab. ← $x \le 0$

$\Rightarrow$ R $\boxed{x \ge 3}$

5) $\boxed{x+1 < \sqrt{x^2-3x+2}}$

$x^2-3x+2=0$

$x=1$,
$x=2$

$x^2-3x+2 \ge 0$ condizioni esistenza $\Rightarrow$

$\Rightarrow \boxed{x \ge 2,\ x \le +1}$

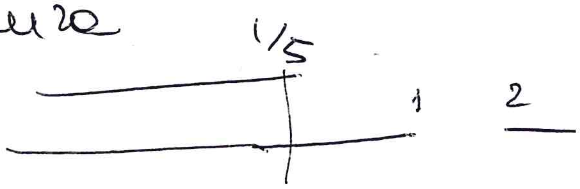
Elevo al quadrato

$$\cancel{x^2+2x+1}\ (x+1)^2 < \left(\sqrt{x^2-3x+2}\right)^2$$

$$x^2+2x+1 < x^2-3x+2$$

$$\Rightarrow 5x-1<0 \Rightarrow \boxed{x<\frac{1}{5}}$$

A sistema con condiz. esistenza



R $\boxed{x<\frac{1}{5}}$