Algebra e matematica discreta, a.a. 2021/2022,

Scuola di Scienze - Corso di laurea: Informatica

ESERCIZIO TIPO 9

Sia $\mathbf{A}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 1 & \alpha+2i & 0 \\ 2 & 2i & \alpha^2+1 \end{pmatrix}$, dove $\alpha \in \mathbb{C}$.

Per ogni $\alpha \in \mathbb{C}$ si dica qual è $rk(\mathbf{A}_{\boldsymbol{\alpha}})$ e si trovi una base $\boldsymbol{\mathcal{B}}_{\alpha}$ di $C(\mathbf{A}_{\boldsymbol{\alpha}})$.

$$\mathbf{A}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 1 & \alpha+2i & 0 \\ 2 & 2i & \alpha^2+1 \end{pmatrix} \xrightarrow{E_{31}(-2)E_{21}(-1)} \begin{pmatrix} 1 & i & 0 \\ 0 & \alpha+i & 0 \\ 0 & 0 & \alpha^2+1 \end{pmatrix} = \mathbf{B}_{\boldsymbol{\alpha}}$$

$\boxed{1^o CASO}$ $\alpha = -i:$ $\mathbf{B}_{-\boldsymbol{i}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathbf{U}_{-\boldsymbol{i}}$, quindi

$$rk(\mathbf{A}_{-\boldsymbol{i}}) = 1 \text{ e } \boldsymbol{\mathcal{B}}_{-i} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$$

$\boxed{2^o CASO}$ $\alpha \neq -i$

$$\mathbf{B}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 0 & \alpha+i & 0 \\ 0 & 0 & \alpha^2+1 \end{pmatrix} \xrightarrow{E_3(\frac{1}{\alpha+i})E_2(\frac{1}{\alpha+i})} \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha-i \end{pmatrix} = \mathbf{C}_{\boldsymbol{\alpha}}$$

$\boxed{1^o Sottocaso}$ $\alpha = i:$ $\mathbf{C}_{\boldsymbol{i}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathbf{U}_{\boldsymbol{i}}$

$$rk(\mathbf{A}_{\boldsymbol{i}}) = 2 \text{ e } \boldsymbol{\mathcal{B}}_{i} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}; \begin{pmatrix} i \\ 3i \\ 2i \end{pmatrix} \right\}$$

$\boxed{2^o Sottocaso}$ $\alpha \neq -i, i:$

$$\mathbf{C}_{\boldsymbol{\alpha}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha - i \end{pmatrix} \xrightarrow{E_3(\frac{1}{\alpha - i})} \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{U}_{\boldsymbol{\alpha}}$$

$$rk(\mathbf{A}_{\boldsymbol{\alpha}}) = 3 \text{ e } \boldsymbol{\mathcal{B}}_{\alpha} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}; \begin{pmatrix} i \\ \alpha + 2i \\ 2i \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ \alpha^2 + 1 \end{pmatrix} \right\}$$

N.B.: Essendo in questo caso $C(\mathbf{A}_{\boldsymbol{\alpha}}) \leq \mathbb{C}^3$ e $\dim(C(\mathbf{A}_{\boldsymbol{\alpha}})) = 3 = \dim(\mathbb{C}^3)$, allora $C(\mathbf{A}_{\boldsymbol{\alpha}}) = \mathbb{C}^3$ e si sarebbe potuto prendere $\boldsymbol{\mathcal{B}}_{\alpha} = \{\mathbf{e_1}; \mathbf{e_2}; \mathbf{e_3}\}$.