RAPPORTO INCREMENTALE

$$\left[\frac{f(x_0+h)-f(x_0)}{x_0+}\right]$$

$$\left\{ \underset{(SX)}{\longleftarrow} \quad (x_0) \quad \underset{(DX)}{\longrightarrow} \right\} \text{INTORNO}$$

FUNZIONI COMPOSTE

└ FONDAMENTALI — COS — SIN

7. $f(x) = \sqrt{x + 2x^2}$ $\rightarrow f'(x)$

$f(x)$

$\rightarrow (x+2x^2)^{1/2}$ — POTENZA

$g(x)$

$$= \left(\frac{1}{2}-1\right)(x+2x^2)^{1/2-1} \cdot (1+4x)$$

$x^1 = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1$

$$\underbrace{m \cdot x^{m-1}}_{\text{POTENZA}} \qquad \underbrace{}_{\text{PARTE INTERNA}}$$

$$= -\frac{1}{2}(x+2x^2)^{-1/2}(1+4x)$$

$$\frac{d}{dx}(2x^2) = 2\cdot(2x^{2-1}) = 4x$$

14. $f(x) = \log(\cos x)$

$f(x)$

$$f'(x) = -\sin(x) \cdot \frac{1}{\cos(x)}$$

$$= -\frac{\sin(x)}{\cos(x)}$$

9. $f(x) = \dfrac{x^2}{\sqrt{x-3}}$

$$f'(x) = x^2 \cdot (x-3)^{-1/2}$$

$$= \underbrace{2x^{2-1}}_{\text{POTENZA}} \cdot \left(-\frac{1}{2} \underline{(x-3)}^{-3/2} \cdot 1 \right)$$

$$= 2x \left(-\frac{1}{2} (x-3)^{-3/2} \right)$$

$$f(x) = \frac{e^x}{\sqrt{e^x - 1}} = e^x \cdot (e^x - 1)^{-\frac{1}{2}}$$

$$= e^x \cdot \left(-\frac{1}{2}\right)(e^x - 1)^{-\frac{1}{2}-1} \cdot e^x$$

POTENZA: $n \cdot \underbrace{x}^{n-1}$

$$f(x) = (x+1)^2 \cdot e^{x^2+2x} \quad \frac{d}{dx} = e^{x^2+2x} \cdot (2x+2)$$

PRODOTTO:

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$f'(x) = \underbrace{2 \cdot (x+1)}_{f'} \cdot \underbrace{e^{x^2+2x}}_{g} +$$

$$\underbrace{(x+1)^2}_{f} \cdot \underbrace{e^{x^2+2x} \cdot (2x+2)}_{g'}$$

$$f'(x) = (2x+2) \cdot e^{x^2+2x} + (x+1)^2 \cdot e^{x^2+2x} \cdot (2x+2)$$

$$= (2x+2)(e^{x^2+2x}) \left[(x+1)^2 + 1 \right]$$

QUOZIENTE:

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$\frac{d}{dx} \sqrt{x} = x^{1/2} = \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2\sqrt{x}}$$

4) $y = \frac{\overset{f}{1+\sqrt{x}}}{\underset{g}{1-\sqrt{x}}}$

$$f' = \frac{d}{dx}\left(\underbrace{1}_{0} + \underbrace{\sqrt{x}}_{\frac{1}{2\sqrt{x}}}\right) = \frac{1}{2\sqrt{x}}$$

$$g' = \frac{d}{dx}\left(\underbrace{1}_{0} - \underbrace{\sqrt{x}}_{-\frac{1}{2\sqrt{x}}}\right) = -\frac{1}{2\sqrt{x}}$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2} = \frac{\underbrace{\frac{1}{2\sqrt{x}}}_{f'} \cdot \underbrace{(1-\sqrt{x})}_{g} - \underbrace{(1+\sqrt{x})}_{f} \cdot \underbrace{-\frac{1}{2\sqrt{x}}}_{g'}}{\underbrace{(1-\sqrt{x})^2}_{g^2}}$$

$$\Rightarrow (a-b)^2 = a^2 + b^2 - 2ab$$

$$= \frac{\frac{(1-\sqrt{x})}{2\sqrt{x}} + \frac{(1+\sqrt{x})}{2\sqrt{x}}}{(1 + x - 2\sqrt{x})}$$

6) $y = \frac{\log(x)}{\cos(x)}$

$$\frac{d}{dx} \log(x) = \frac{1}{x}$$

$$\frac{d}{dx} = \sin(x)$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$\Rightarrow \frac{\underbrace{\frac{1}{x}}_{f'} \cdot \underbrace{\cos x}_{g} - \underbrace{\log(x)}_{f} \cdot \underbrace{-\sin(x)}_{g'}}{[\cos^2(x)] \to g^2}$$

$$\frac{\underbrace{\frac{1}{x}}_{f'} \cdot \underbrace{\cos x}_{g} - \underbrace{\log(x)}_{f} \cdot \underbrace{-\sin x}_{g'}}{[\cos^2(x)] \to 2} = \frac{\frac{\cos(x)}{x} + \sin(x)\log(x)}{\cos^2(x)}$$

6) $y = xe^x \sin(x)$ — f', g

$$\underbrace{xe^x}_{f \cdot g} = 1 \cdot e^x + e^x \cdot x = e^x(1+x)$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$$= \underbrace{e^x(1+x)}_{f'} \cdot \underbrace{\sin(x)}_{g} + \underbrace{\cos(x)}_{g'} \cdot \underbrace{(xe^x)}_{f}$$

$$= e^x(1+x)\sin(x) + xe^x\cos(x)$$

9) $y = \underbrace{x \log_{\frac{1}{2}}(x)}_{1^o} + \dfrac{x}{4\log(x)} + \dfrac{5x}{3}$

$$x^1 \cdot x^{-1/2} \Rightarrow x^{1/2}$$

$$= f' \cdot g + f \cdot g' = 1 \cdot \log_{\frac{1}{2}}(x) + x \cdot \frac{1}{x^{1/2}} \to i^o$$

$$= \frac{x}{4\log(x)} = \frac{f' \cdot g - f \cdot g'}{(g)^2}$$

$$\frac{d}{dx}[4\log(x)] = 0 \cdot \log(x) + 4 \cdot \frac{1}{x} = \frac{4}{x}$$

$$\frac{d}{dx}(x) = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1$$

$$= \frac{\left[\underbrace{1}_{f'} \cdot \underbrace{4\log(x)}_{g}\right] - \left[\underbrace{\cancel{x}}_{f} \cdot \underbrace{\frac{4}{\cancel{x}}}_{g'}\right]}{[4\log(x)]^2}$$

$$\frac{4\log(x) - 4}{16\log^2(x)} = \frac{\cancel{4}(\log(x) - 1)}{4\,\cancel{16}\log^2(x)} = \frac{\log(x) - 1}{4\log^2(x)}$$

$$\underbrace{\frac{5}{3}x} = f' \cdot g + f \cdot g' = 0 \cdot x + 1 \cdot \frac{5}{3} = \frac{5}{3}$$

$$\underbrace{\text{SOMMA}} \rightarrow \log_{\frac{1}{2}}(x) \cdot x^{1/2} + \frac{\log(x) - 1}{4\log^2(x)} + \frac{5}{3}$$

DI TUTTE
LE DERIVATE

$D\,k = 0$	dove k è una costante
$D\,x^n = n\,x^{n-1}$	

$$D \ln x = \frac{1}{x}$$

$$D\,e^x = e^x$$

$$D\,senx = cosx$$

$$D\,cosx = -\,senx$$

$$(f+g)' = f' + g'$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$\underbrace{\frac{d}{dx}[g(f(x))] = g'(f(x)) \cdot f'(x)}$$

COMPOSTA = DERIVA
TUTTO