

CALCOLO DEGLI INTEGRALI

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Parte 1. INTEGRALI DEFINITI

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Esercizio 1. $\int_0^1 \frac{dx}{1+x} = \ln|1+x|_0^1 = \ln 2 - \ln 1 = \ln 2$

Svolgimento
USANDO $\int_0^1 \frac{dx}{1+x} = \int_0^1 1 + \frac{1}{x} dx = \int_0^1 dx + \int_0^1 \frac{1}{x} dx = \ln|1+x|_0^1 = \ln(2) - \ln(1) = \ln 2$

Esercizio 2. $\int_{-2}^{-1} \frac{dx}{x^3} = \int_{-2}^{-1} x^{-3} dx = -\frac{1}{2x^2} \Big|_{-2}^{-1} = -\frac{1}{2} + \frac{1}{8} = -\frac{3}{8}$

Svolgimento
USANDO $\int_{-2}^{-1} \frac{1}{x^3} = \int_{-2}^{-1} x^{-3} = \left[\frac{x^{-3+1}}{-3+1} \right]_{-2}^{-1} = \left[-\frac{1}{2x^2} \right]_{-2}^{-1} = \underbrace{\left[-\frac{1}{2(-1)^2} \right]}_{f(b)} - \underbrace{\left[-\frac{1}{2(-2)^2} \right]}_{f(a)} = -\frac{1}{2} + \frac{1}{8} = -\frac{3}{8}$

Esercizio 3. $\int_0^x \cos t dt = \sin t \Big|_0^x = \sin x$

Svolgimento
USANDO $\int_0^x \cos(t) dt = [\sin(t)]_0^x = \sin(x) - \sin(0) = \sin(x)$

Esercizio 4. $\int_1^2 (x^2 - 2x + 3) dx = \frac{x^3}{3} - x^2 + 3x \Big|_1^2 = \frac{8}{3} - 4 + 6 - \frac{1}{3} + 1 - 3 = \frac{7}{3}$

Svolgi
risultato
USANDO $\int_1^2 (x^2 - 2x + 3) dx = \int_1^2 x^2 - \int_1^2 2x + \int_1^2 3 = \frac{8}{3} - 4 + 6 - \frac{1}{3} + 1 - 3 = \frac{7}{3}$ *Svolgi*
corris sopra

Esercizio 5. $\int_0^8 (\sqrt{2x} + \sqrt[3]{x}) dx = \int_0^8 (2x)^{\frac{1}{2}} + \int_0^8 x^{\frac{1}{3}} = \frac{(2x)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} \Big|_0^8 = \frac{2^6}{\frac{3}{2}} + \frac{2^4}{\frac{4}{3}} = \frac{100}{3}$

Calcolo $\int_0^8 \sqrt{2x} + \sqrt[3]{x} = \int_0^8 (2x)^{1/2} + \int_0^8 (x)^{1/3}$ *Svolgi*
corris sopra

Esercizio 6. $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx = \int_1^4 x^{-2} dx + \int_1^4 x^{-3/2} dx = -\frac{1}{x} - \frac{2}{\sqrt{x}} \Big|_1^4 = -\frac{1}{4} - 1 + 1 + 2 = \frac{7}{4}$

Calcolo $\int_1^4 \frac{1+\sqrt{x}}{x^2} = \int_1^4 \frac{1}{x^2} + \int_1^4 \frac{x^{1/2}}{x^2} = \int_1^4 x^{-2} + \int_1^4 x^{-3/2}$ *Svolgi*
corris sopra

$\int_2^6 \sqrt{x-2} dx = \int_2^6 (x-2)^{\frac{1}{2}} d(x-2) = \frac{x-2}{\frac{3}{2}} \Big|_2^6 = \frac{2^3}{\frac{3}{2}} - 0 = \frac{16}{3}$

$\int_2^6 (x-2)^{1/2} =$ *Svolgi* *corris sopra*

Esercizio 7.

$$\begin{aligned} \int_0^{-3} \frac{dx}{\sqrt{25+3x}} x &= - \int_{-3}^0 (25+3x)^{-\frac{1}{2}} dx = -\frac{1}{3} \int_{-3}^0 (25+3x)^{-\frac{1}{2}} d(3x+25) = \\ &= -\frac{2}{3} \sqrt{3x+25} \Big|_{-3}^0 = -\frac{10}{3} + \frac{8}{3} = -\frac{2}{3} \end{aligned}$$