

23/04 → Derivate

CAPITOLO 18 | DERIVATE

253 LEGGI IL GRAFICO Trova le equazioni delle rette $y = f(x)$ e $y = g(x)$ rappresentate in figura. Calcola la derivata di:

a. $y = \frac{f(x)}{g(x)}$; b. $y = \frac{g(x)}{f(x)}$.

Quanto valgono le due derivate nel punto A?

[a] $y' = \frac{3}{(3-x)^2}$; b) $y' = -\frac{12}{(x+3)^2}; \frac{3}{4}, -\frac{3}{4}$

Derivata di $y = \tan x$ e di $y = \cot x$ Attività interattiva

Calcola la derivata delle seguenti funzioni.

254 $y = \tan x \cot x$ [y' = 0]

255 $y = \tan x - \cot x$ [y' = $\frac{1}{\cos^2 x \sin^2 x}$]

256 $y = \frac{2}{\cot x} - \tan x$ [y' = $\frac{1}{\cos^2 x}$]

$$\begin{cases} y = mx + q \\ y = q_1 x + b \\ y = q_2 x + c \end{cases}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$q = \frac{x_1 y_2 - x_2 y_1}{x_1 - x_2}$$

PASSANTE PER DUE PUNTI

$$\left[\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \right]$$

$$A(3, 0) \rightarrow y = g(x)$$



$$\frac{x - 3}{0 - 3} = \frac{y - 0}{y - 0} =$$

$$\frac{x - 3}{-3} = \frac{y}{0}$$

$$C(-3, 0) \rightarrow y = f(x)$$

$$\frac{(x-3)}{3} - \frac{y}{4} = 0$$

$$y = q_1 x + b$$

(ESERCIZIO)

$$\frac{4(x-3) - 3y}{42} = 0 - 12$$

$$-4x + 12 + 3y = 0$$

$$\frac{3y}{3} = \frac{4x}{3} - \frac{12}{3}$$

$$(DOSSENTO) y(x) \rightarrow y = \frac{4}{3}x - 4$$

PASSANTE POR DOS PUNTOS

$$\left[\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \right]$$

$$\begin{array}{l} C(-3; 0) \\ D(0; 2) \end{array} \rightarrow y = f(x)$$

$$\frac{x + 3}{0 + 3} = \frac{y - 0}{2 - 0}$$

$$\frac{x + 3}{3} = \frac{y}{2} \rightarrow \frac{x + 3}{3} - \frac{y}{2} = 0 \rightarrow \frac{2(x + 3) - 3y}{6}$$

$$\rightarrow \frac{2x + 6 - 3y}{6} = 0 \rightarrow 0 = 0$$

$$\frac{2x + 6}{3} = \frac{3y}{3}$$

$$y = \frac{2}{3}x + 2 \rightarrow f(x)$$

$$\boxed{0.} \quad g = \frac{f(x)}{g(x)} = \frac{\frac{2}{3}x + 2}{\frac{4}{3}x - 4} \rightarrow f'(x) = ?$$

$$y = \frac{f}{g} = \frac{f'g - fg'}{g^2} = \frac{\left(\frac{2}{3}\right)\left(\frac{4}{3}x - 4\right) - \left(\frac{2}{3}x + 2\right)\left(\frac{4}{3}\right)}{\left(\frac{4}{3}x - 4\right)^2}$$

$$= \frac{\frac{8}{3}x - \frac{8}{3} - \left(\frac{8}{3}x + \frac{8}{3}\right)}{\frac{16}{9}x^2 + 16 - \frac{32}{3}x}$$

$$= \frac{\cancel{\frac{8}{3}x} - \cancel{\frac{8}{3}} - \cancel{\frac{8}{3}x} - \cancel{\frac{8}{3}}}{\frac{16}{9}x^2 + 16 - \frac{32}{3}x}$$

$$\approx -\frac{16}{3}$$

$$\frac{16}{9}x^2 + 16 - \frac{32}{3}x \times \frac{4}{3}x - 4)^2$$

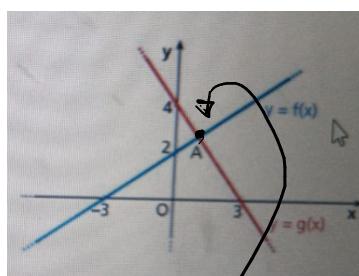
$$-\frac{16}{3} \left(\frac{4}{3}x - 4 \right)^2$$

b) $y = \frac{g(x)}{f(x)} = \frac{\frac{4}{3}x - 4}{\frac{2}{3}x + 2} \Rightarrow \frac{g'f - f'g}{f^2}$

$$= \frac{\frac{4}{3} \left(\frac{2}{3}x + 2 \right) - \left[\frac{2}{3} \left(\frac{4}{3}x - 4 \right) \right]}{\left(\frac{2}{3}x + 2 \right)^2} \rightarrow \frac{\frac{8}{9}x - \frac{8}{3}}{\text{Squando}}$$

$$\frac{\left(\frac{2}{3}x + 2 \right) \left[\frac{4}{3} - \frac{4}{3} + 2 \right]}{\left(\frac{2}{3}x + 2 \right)^2} = \frac{2}{\left(\frac{2}{3}x + 2 \right)}$$

QUANDO IN DERIVARE IN PUNTO A



SISTEMA = INTERSEZIONE

$$y = f(x) = \frac{2}{3}x + 2$$

$$y = g(x) = \frac{4}{3}x - 4$$

$$\begin{cases} \frac{2}{3}x + 2 = y \\ \frac{4}{3}x - 4 = y \end{cases} \Rightarrow \begin{cases} \frac{2}{3}x + 2 - y = 0 \\ \frac{4}{3}x - 4 - y = 0 \end{cases}$$

$$(-2) \quad \begin{cases} \frac{2}{3}x + 2 = y \\ \frac{4}{3}x - 4 - \frac{2}{3}x - 2 = 0 \end{cases} \rightarrow \frac{12x - 12 - 2x - 6}{3} = 0$$

$$12x - 12 - 2x - 6 = 0$$

$$\frac{10x}{10} = \frac{18}{10} \Rightarrow \frac{9}{5} \checkmark$$

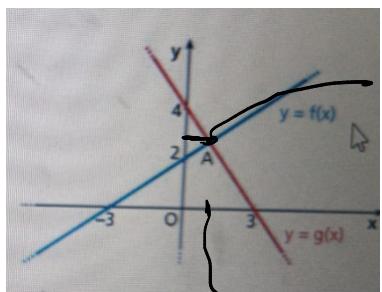
$$\begin{array}{r} 9 \\ \times 5 \\ \hline 45 \end{array}$$

$$\frac{2}{3}\left(\frac{9}{5}\right) + 2 = y$$

$$\frac{18}{15} + 2 = y$$

$$\begin{array}{r} 48 \\ \times 5 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$$

$$\frac{18+30}{15} = y \Rightarrow y = \frac{48}{15} (3, 2)$$



$$\frac{48}{15} = 3 - 2 \quad (y)$$

$$P\left(\frac{9}{5} : \frac{48}{15}\right)$$

$$x = \frac{9}{5} = 1.8 \quad (x)$$

$$y' = \frac{3}{(3-x)^2}$$

sosannenreis

$$\frac{48}{15} = \frac{3}{(3-\frac{9}{5})^2}$$

$$\left[a) y' = \frac{3}{(3-x)^2}; b) y' = -\frac{12}{(x+3)^2}; \frac{3}{4}; -\frac{3}{4} \right]$$

$$\frac{48}{15} = \frac{3}{\cancel{36}} \quad \cancel{36} \downarrow$$

$$\frac{48}{15} = 3 - \frac{27}{\cancel{36}} \quad \cancel{36} \downarrow$$

$$\frac{48}{15} = \frac{25}{12}$$

$$3 - \frac{3}{\cancel{5}} = \frac{15-3}{\cancel{5}} = \frac{6}{\cancel{5}}$$

$$\frac{48}{15} - \frac{25}{12} = \frac{576-375}{180}$$

$$= \frac{201}{180} = \frac{67}{60}$$