KNF → KLEENE NORMAL FORM

THEORY OF THIS PART...

FUNZIONE (INPUT , OUTPUT , N. PASSI)

$C_K(e, \vec{x}, t)$

STAI CERCANDO DI CODIFICARE

$\varphi_e^{(K)} = (\mu z. \mid T_{S_K}(e, \vec{x}, \underbrace{(z)_1, (z)_2}) \mid$

$\pi = z(z_1, z_2)$ TUPLE

FORMA COMPLETA $(\mu z. \mid S_K(e, \vec{x}, y, t) \mid$

$\underbrace{\quad -1 \quad}$

$(\mu z \mid f(x) - y \mid$

$\in y$ (APPARTENENTE ALL'INSIEME)

$$
\begin{aligned}
sc_A(x) &= \mathbf{1}(\mu w.(S(x,(w)_1,(w)_2,(w)_3) \wedge (w)_2 \in Y)) \\
&= \mathbf{1}(\mu w.(|\chi_S(x,(w)_1,(w)_2,(w)_3) * \chi_Y((w)_2) - 1|))
\end{aligned}
$$

COROLLARY 12.3. *The following predicates are decidable:*

(a) $H_k(e,\vec{x},t) \equiv$ *"$P_e(\vec{x}) \downarrow$ in t or less steps"*

(b) $S_k(e,\vec{x},y,t) \equiv$ *"$P_e(\vec{x}) \downarrow y$ in t or less steps"*

USEFUL EXAMPLES: $\Rightarrow$ HOW TO WRITE FUNCTIONS

READ THE SECTION 12.1

We observed that if $f : \mathbb{N} \to \mathbb{N}$ is a total computable injective function, then

$$
f^{-1}(y) = \begin{cases} x & \text{if exists } y \text{ s.t. } f(x) = y \\ \uparrow & \text{otherwise} \end{cases}
$$

is computable since $f^{-1} = \mu x \,.\, |f(x) - y|$. The hypothesis of *totality* can be omitted.

EXERCISE 12.6. Let $f : \mathbb{N} \to \mathbb{N}$ computable and injective. Then $f^{-1} : \mathbb{N} \to \mathbb{N}$ is computable.

PROOF. Since f is computable, there exists $e \in \mathbb{N}$ such that $\varphi_e = f$. Now it is sufficient to observe that

$$
f^{-1}(y) = (\mu w \,.\, |\chi_S(e,(w)_1,x,(w)_2) - 1|)_1
$$

$\square$