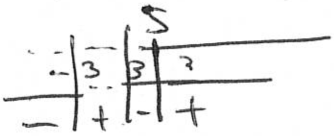
Soluzioni esercizi su asintoti

1) $f(x) = \dfrac{x-5}{x^2-9}$

Dominio $x^2-9 \neq 0 \Rightarrow x \neq \pm 3$ $\quad (-\infty,-3)\cup(-3,3)\cup(3,+\infty)$

Segno $\dfrac{x-5}{x^2-9} \geq 0$ $\quad x \geq 5$ $\quad x<-3 \quad x>3$



$f(x) \geq 0 \iff x \geq 5$
$\quad -3<x<3$

Simmetrie $f(-x) = \dfrac{-x-5}{x^2-9} \neq f(x)$, $\neq -f(x)$ non ci sono simmetrie

Asintoti $\lim_{x\to+\infty} \dfrac{x-5}{x^2-9} = \lim_{x\to+\infty} \dfrac{x\,(1-5/x)}{x^2(1-9/x^2)} = 0$ $\quad (1-5/x)\to 1,\ (1-9/x^2)\to 1,\ \dfrac{x}{x^2}\to 0$

$\lim_{x\to-\infty} \dfrac{x-5}{x^2-9} = \lim_{x\to-\infty} \dfrac{x\,(1-5/x)}{x^2(1-9/x^2)} = 0$

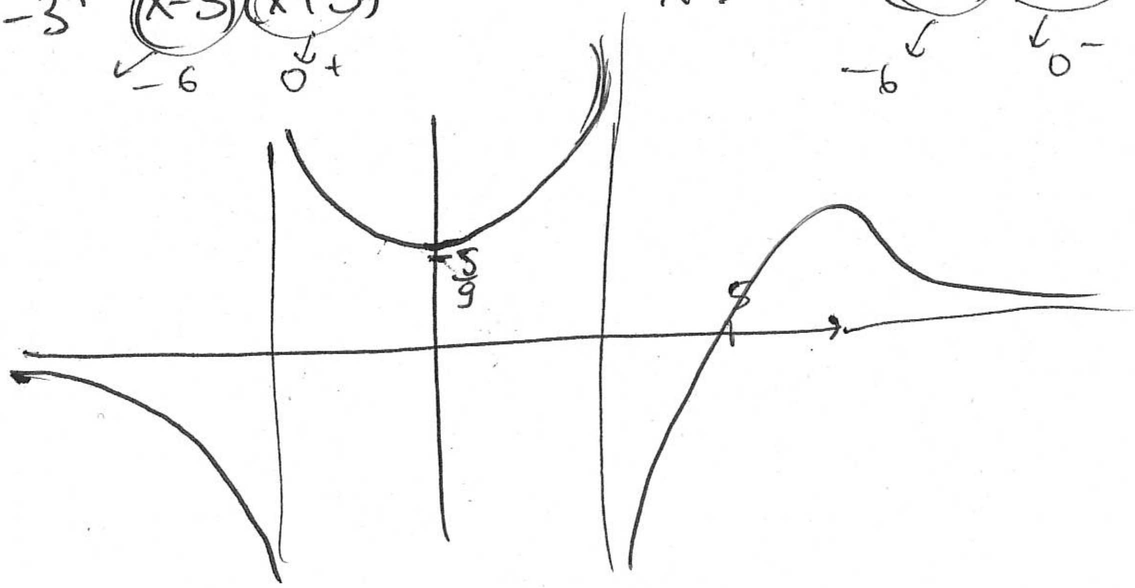
$y=0$ asintoto orizzontale a $\pm\infty$.

$\lim_{x\to 3^+} \dfrac{(x-5)}{(x-3)(x+3)} = -\infty$ $\quad (x-5)\to -2,\ (x-3)\to 0^+,\ (x+3)\to 6$

$\lim_{x\to 3^-} \dfrac{(x-5)}{(x-3)(x+3)} = +\infty$ $\quad (x-5)\to -2,\ (x-3)\to 0^-,\ (x+3)\to 6$

$\lim_{x\to -3^+} \dfrac{(x-5)}{(x-3)(x+3)} = +\infty$ $\quad (x-5)\to -8,\ (x-3)\to -6,\ (x+3)\to 0^+$

$\lim_{x\to -3^-} \dfrac{(x-5)}{(x-3)(x+3)} = -\infty$ $\quad (x-5)\to -8,\ (x-3)\to -6,\ (x+3)\to 0^-$



2) $f(x) = \dfrac{x^3}{x^2-1}$

<u>dominio</u> $x^2-1 \neq 0 \Rightarrow x \neq \pm 1$ $\quad (-\infty,-1) \cup (-1,1) \cup (1,+\infty)$

<u>segno</u> $f(x) \geq 0$ $\quad x^3 \geq 0 \quad x \geq 0$

$x^2-1>0 \quad x<-1 \ x>1$

$f(x) \geq 0 \iff x > 1, \ -1 < x \leq 0$

<u>simmetrie</u> $f(-x) = \dfrac{(-x)^3}{(-x)^2-1} = -\dfrac{x^3}{x^2-1} = -f(x)$ $\quad$ <u>f dispari</u>

<u>asintoti</u> $\lim_{x\to+\infty} \dfrac{\cancel{x^3}}{\cancel{x^2}(1-\frac{1}{x^2})} = +\infty$ $\quad \lim_{x\to-\infty} \dfrac{x^3}{x^2(1-\frac{1}{x^2})} = -\infty$

non ci sono asintoti orizzontali.
cerco gli obliqui

$\lim_{x\to+\infty\,(-\infty)} \dfrac{f(x)}{x} = \lim_{x\to+\infty} \dfrac{x^3}{x(x^2-1)} = \lim_{x\to+\infty} \dfrac{\cancel{x^3}}{\cancel{x}\cdot\cancel{x^2}(1-\frac{1}{x^2})} = 1 = a$

$\lim_{x\to+\infty} f(x)-x = \lim_{x\to+\infty} \dfrac{\cancel{x^3}-\cancel{x^3}+x}{x^2-1} = \lim_{x\to+\infty\,(-\infty)} \dfrac{x}{x^2(1-\frac{1}{x^2})} = 0 = b$

$y=x$ asintoto obliquo a $+\infty$ e a $-\infty$.

$\lim_{x\to1^+} \dfrac{x^3}{(x-1)(x+1)} = +\infty$ $\quad$ ($x^3 \to 1$, $x-1 \to 0^+$, $x+1 \to 2$)

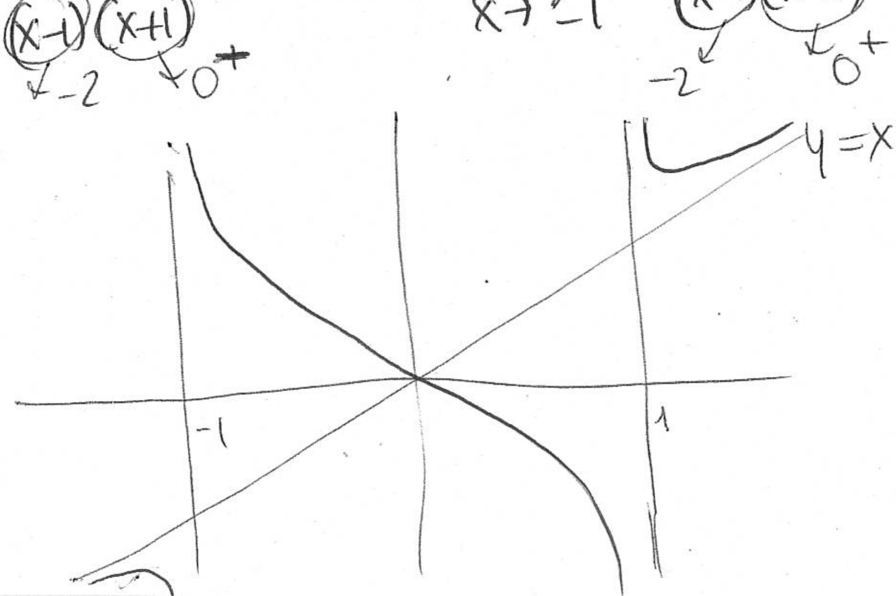
$\lim_{x\to1^-} \dfrac{x^3}{(x-1)(x+1)} = -\infty$ $\quad$ ($x^3 \to 1$, $x-1 \to 0^-$, $x+1 \to 2$)

$x=1$ asintoto verticale

$\lim_{x\to-1^+} \dfrac{x^3}{(x-1)(x+1)} = +\infty$ $\quad$ ($x^3 \to -1$, $x-1 \to -2$, $x+1 \to 0^+$)

$\lim_{x\to-1^-} \dfrac{x^3}{(x-1)(x+1)} = +\infty$ $\quad$ ($x^3 \to -1$, $x-1 \to -2$, $x+1 \to 0^+$)

$x=-1$ as. verticale



3) $f(x) = \lg(\cosh x)$ NB $\cosh x = \dfrac{e^x + e^{-x}}{2} =$ ③

$= e^x \dfrac{(1+e^{-2x})}{2} = e^{-x}\dfrac{(e^{2x}+1)}{2}$

dominio $\cosh x > 0 \ \forall x$ quindi
$D = \mathbb{R}$

segno $\cosh x \geq 1 \ \forall x$ quindi $f(x) \geq 0$ e $f(x) = 0$
$\iff x = 0$.

simmetrie $\cosh(-x) = \cosh x \implies f(x) = f(-x)$ f pari

asintoti $\lim_{x \to +\infty} f(x) = +\infty$.

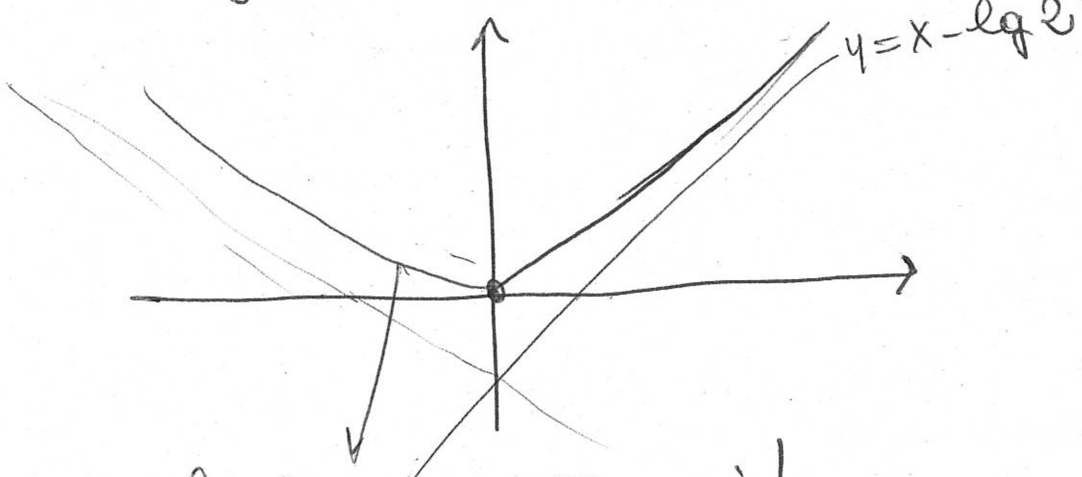
cerco as. obliqui $\lim_{x \to +\infty} \dfrac{\lg(\cosh x)}{x} = \lim_{x \to +\infty} \dfrac{1}{x} \lg\left(\dfrac{e^{x}(1+e^{-2x})}{2}\right)$

$= \lim_{x \to +\infty} \dfrac{x + \lg\left(\frac{1+e^{-2x}}{2}\right)}{x} = \lim_{x \to +\infty} \dfrac{\cancel{x}\left(1 + \frac{\lg\left(\frac{1+e^{-2x}}{2}\right)}{x}\right)}{\cancel{x}} = 1$
$\downarrow 0$ $\quad \| \ a$

$\lim_{x \to +\infty} f(x) - x = \lim_{x \to +\infty} \lg\left(e^{x}\left(\frac{1+e^{-2x}}{2}\right)\right) - x =$

$= \lim_{x \to +\infty} \cancel{x} + \lg\left(\frac{1+e^{-2x}}{2}\right) \cancel{-x} = \lg\left(\frac{1}{2}\right) = -\lg 2 = b$ $\quad (e^{-2x} \to 0)$

$y = x - \lg 2$ è asintoto obliquo (a $+\infty$)



La funzione è pari!
$y = -x - \lg 2$ è asintoto obliquo a $-\infty$.

$$\lim_{x\to-\infty} \lg \underbrace{(e^{2x}-1)^2}_{\downarrow\, 1} = 0 \qquad y=0$$

ASINTOTO ORIZZONTALE a $-\infty$

$$\lim_{x\to+\infty} \lg \underbrace{(e^{2x}-1)^2}_{\downarrow\, +\infty} = +\infty$$

cerco as. obliquo

$$\lim_{x\to+\infty} \frac{\lg (e^{2x}-1)^2}{x} = \lim_{x\to+\infty} \frac{2 \lg (e^{2x}(1-e^{-2x}))}{x}$$

$$= \lim_{x\to+\infty} \frac{2\left[2x+\lg(1-e^{-2x})\right]}{x} =$$

$$= \lim_{x\to+\infty} \frac{2\cancel{x}\left[2+\underbrace{\frac{\lg(1\pm e^{-2x})}{x}}_{\downarrow\, 0}\right]}{\cancel{x}} = 4 = a$$

$$\lim_{x\to+\infty} f(x)-4x = \lim_{x\to+\infty} 2(2x+\lg(1-e^{-2x}))-4x=$$

$$= \lim_{x\to+\infty} 2\lg\underbrace{(\underbrace{1-e^{-2x}}_{\downarrow\, 1})}_{\downarrow\, 0} = 0 .$$

$y=4x$ asintoto obliquo a $+\infty$

$$\lim_{x\to 0^{\pm}} \lg \underbrace{(e^{2x}-1)^2}_{\downarrow\, 0} = -\infty \qquad x=0 \text{ AS. VERTICALE}$$

f non ha simmetrie

5) $f(x) = \lg\left(\frac{x+1}{x-1}\right)$

Dominio $\frac{x+1}{x-1} > 0$ $\quad x > -1$ $\quad x > 1$ $\quad$ $x > 1$ e $x < -1$

$(-\infty, -1) \cup (1, +\infty)$.

segno $f(x) \geq 0$ $\quad \frac{x+1}{x-1} \geq 1 \Longleftrightarrow \frac{x+1-(x-1)}{x-1} \geq 0 \quad \frac{2}{x-1} \geq 0 \Rightarrow \boxed{x>1}$

simmetrie $f(-x) = \lg\left(\frac{-x+1}{-x-1}\right) = \lg\left(\frac{x-1}{x+1}\right) = -\lg\left(\frac{x+1}{x-1}\right) = -f(x)$

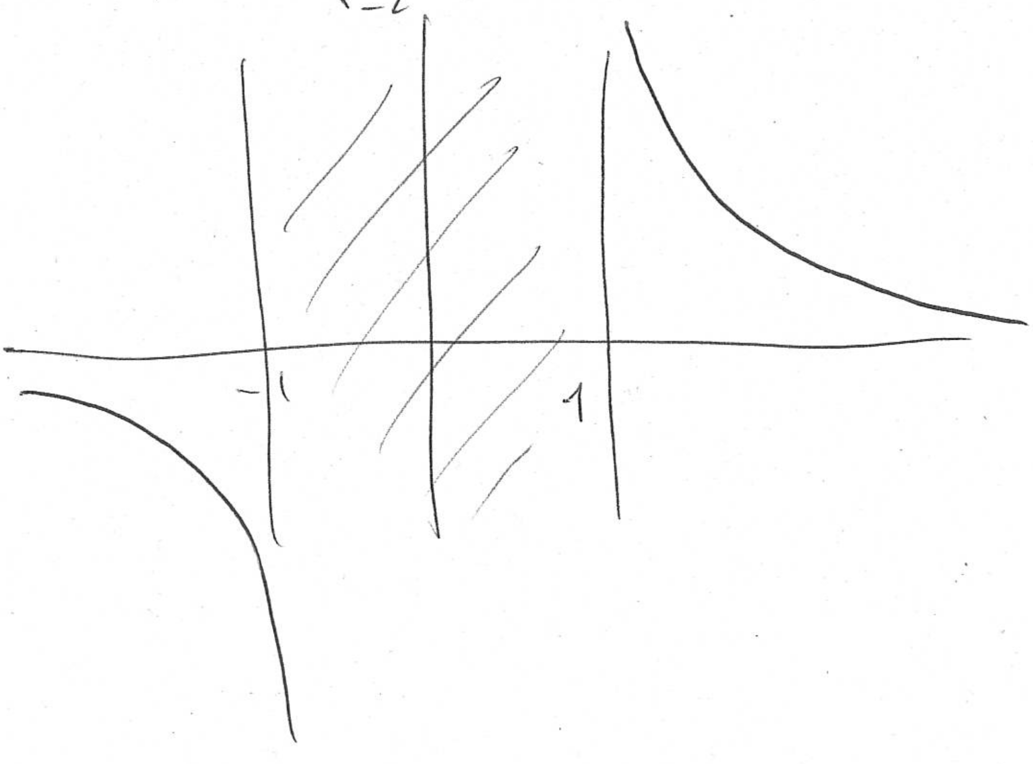
f è dispari.

asintoti $\lim_{\substack{x \to +\infty \\ -\infty}} \lg\left(\frac{x+1}{x-1}\right) = \lim_{\substack{x \to +\infty \\ -\infty}} \lg\left(\frac{\cancel{x}\overbrace{\left(1+\frac{1}{x}\right)}^{\to 1}}{\cancel{x}\underbrace{\left(1-\frac{1}{x}\right)}_{1}}\right) = \lg 1 = 0$

$y = 0$ è asintoto orizzontale a $+\infty$ e $-\infty$.

$\lim_{x \to 1^+} \lg\left(\frac{\overbrace{x+1}^{\to 2}}{\underbrace{x-1}_{\to 0^+}}\right) = +\infty$ $\quad x = 1$ asintoto verticale destro

$\lim_{x \to -1^-} \lg\left(\frac{\overbrace{x+1}^{\to 0^-}}{\underbrace{x-1}_{\to -2}}\right) = -\infty$ $\quad x = -1$ asintoto verticale sinistro



6) $f(x) = x\, e^{\frac{10x}{x^2+9}}$

dominio $D = \mathbb{R}$ $\quad (x^2+9>0 \quad \forall x)$

segno $f(x) \geq 0 \iff x \geq 0$ $\quad \left(\text{perché } e^{\frac{10x}{x^2+9}} > 0\right)$

~~asintoti~~ simmetrie $f(-x) = -x\, e^{\frac{-10x}{x^2+9}} \neq -f(x)$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad f(x)$

né pari né dispari

asintoti

$$\lim_{\substack{x\to+\infty\\ -\infty}} x\, e^{\frac{10x}{x^2+9}} = \lim_{\substack{x\to+\infty\\ -\infty}} x\, e^{\overbrace{\frac{10}{x\left(1+\frac{9}{x^2}\right)}}^{\to 0}} = \begin{matrix}+\infty\\ -\infty\end{matrix}$$ no as. orizzontali

cerco asintoti obliqui

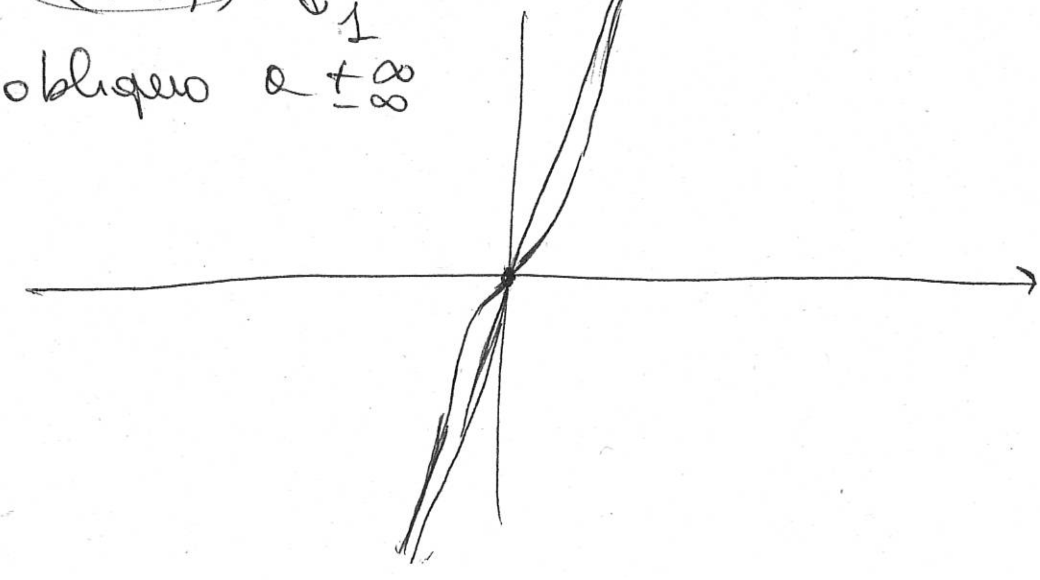
$$\lim_{\substack{x\to+\infty\\ -\infty}} \frac{x\, e^{\frac{10x}{x^2+9}}}{x} = \lim_{\substack{x\to+\infty\\ -\infty}} e^{\overbrace{\frac{10}{x\left(1+\frac{9}{x^2}\right)}}^{\to 0}} = 1 = a$$

$$\lim_{\substack{x\to+\infty\\ -\infty}} f(x) - x = \lim_{\substack{x\to+\infty\\ -\infty}} x\left[e^{\frac{10}{x\left(1+\frac{9}{x^2}\right)}} - 1\right] =$$

$$y = \frac{1}{x} \quad = \lim_{\substack{y\to 0^+\\ (y\to 0^-)}} \frac{1}{y}\left(e^{\frac{10y}{1+9y^2}} - 1\right) =$$

$$= \lim_{\substack{y\to 0^+\\ y\to 0^-}} \underbrace{\frac{e^{10y/1+9y^2} - 1}{\left(\frac{10y}{1+9y^2}\right)}}_{\downarrow\, 1} \frac{10\cancel{y}}{1+9y^2}\frac{1}{\cancel{y}} = 10 = b$$

$y = x + 10$ as. obliquo a $\pm\infty$



7) $f(x) = e^{\frac{x^2-4}{x-1}}$

Dominio $x \neq 1$ $(-\infty, 1) \cup (1, +\infty)$

segno $f(x) > 0$ $\forall x$

simmetrie $f(-x) = e^{\frac{x^2-4}{-x-1}} \neq f(x)$, $\neq -f(x)$ no simmetrie

asintoti $\lim_{x\to+\infty} e^{\frac{x^2-4}{x-1}} = \lim_{x\to+\infty} e^{\frac{x^{\cancel{2}}(1-4/x^2)}{\cancel{x}(1-x)}} = +\infty$

$\lim_{x\to-\infty} e^{\frac{x^{\cancel{2}}(1-4/x^2)}{\cancel{x}(1-1/x)}} = 0$ $y=0$ asintoto orizzontale a $-\infty$.

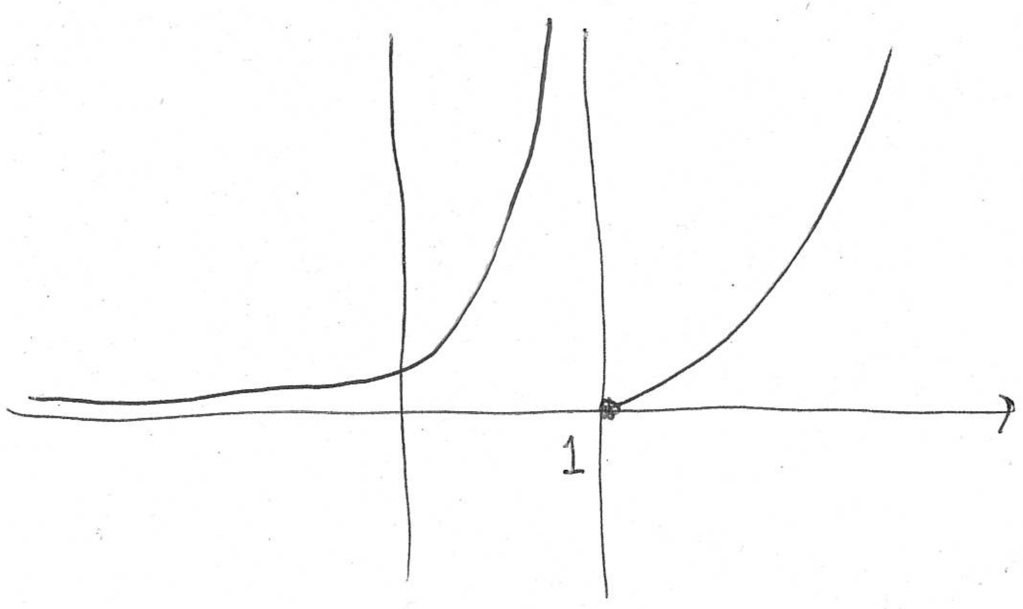
cerco as. obliquo a $+\infty$

$\lim_{x\to+\infty} \frac{e^{x\frac{(1-4/x^2)}{(1-1/x)}}}{x} = +\infty$ (per confronto infiniti)

non ci sono asintoti obliqui.

$\lim_{x\to1^+} e^{\frac{(x^2-4)\to-3}{(x-1)\to0^+}} \Big]\to-\infty = 0$

$\lim_{x\to1^-} e^{\frac{(x^2-4)\to-3}{(x-1)\to0^-}} \Big]\to+\infty = +\infty$ $x=1$ è asintoto verticale sinistro



8) $f(x) = 5x+2-\sqrt{25x^2+12x}$

dominio $\quad 25x^2+12x \geq 0 \iff x(25x+12) \geq 0$ $\quad x \geq 0$, $x \geq -\frac{12}{25}$ $\quad + \mid - \mid +$

$x \geq 0 \quad e \quad x \leq -\frac{12}{25}$

$(-\infty, -\frac{12}{25}] \cup [0, +\infty)$

segno $\quad 5x+2 \geq \sqrt{25x^2+12x}$

se $5x+2<0$ non è mai verificata.

se $5x+2 \geq 0$ elevo entrambi al quadrato

$$\begin{cases}(5x+2)^2 \geq 25x^2+12x \\ 5x+2 \geq 0\end{cases} \qquad \begin{cases}\cancel{25x^2}+20x+4 \geq \cancel{25x^2}+12x \\ x \geq -\frac{2}{5}\end{cases}$$

$$\begin{cases}x \geq -\frac{4}{8} = -\frac{1}{2} \\ x \geq -\frac{2}{5}\end{cases} \qquad -1/2 \quad -2/5 \qquad \Rightarrow x \geq -\frac{2}{5}$$

$x \in D \Rightarrow \boxed{x \geq 0}$

$-2/5$ $\quad -12/5$ $\quad 0$

Dunque $f(x) \geq 0 \Rightarrow x \geq 0$.

simmetrie $\quad$ non ci sono simmetrie.

$f(0) = 2 \qquad f(-\frac{12}{25}) = -\frac{12}{5}+2 = -\frac{2}{5}$.

$$\lim_{x\to+\infty} \underset{+\infty \quad -\infty}{\left(5x+2-\sqrt{25x^2+12x}\right)} \cdot \frac{\left(5x+2+\sqrt{25x^2+12x}\right)}{\left(5x+2+\sqrt{25x^2+12x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{(5x+2)^2-(25x^2+12x)}{5x+2+\sqrt{x^2}\sqrt{25+\frac{12}{x}}} = \lim_{x\to+\infty} \frac{8x+4}{x\left[5+\frac{2}{x}+\sqrt{25+\frac{12}{x}}\right]}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\overbrace{\left(8+\frac{4}{x}\right)}^{\to 8}}{\cancel{x}\underbrace{\left[5+\frac{2}{x}+\sqrt{25+\frac{12}{x}}\right]}_{\to 10}} = \frac{8}{10} = \frac{4}{5}$$

$y = \frac{4}{5}$ è asintoto orizzontale a $+\infty$.

$$\lim_{x\to-\infty} 5x-2-\sqrt{25x^2+12x} = -\infty$$

cerco asintoto obliquo a $-\infty$

$-x = \sqrt{x^2}$!

$$\lim_{x\to-\infty} \frac{5x+2-\sqrt{25x^2+12x}}{x} = \lim_{x\to-\infty} \frac{5x+2-\sqrt{x^2}\sqrt{25+12/x}}{x} =$$

$$\lim_{x\to-\infty} \frac{\cancel{x}\left(5+\frac{2}{x}+\sqrt{25+\frac{12}{x}}\right)}{\cancel{x}} = 10 = a$$

$$\lim_{x\to-\infty} 5x+2-\sqrt{25x^2+12x}-10x = \lim_{x\to-\infty} \underbrace{-5x+2}_{+\infty}\underbrace{-\sqrt{25x^2-12x}}_{-\infty} = \text{f.i}$$

$$= \lim_{x\to-\infty} \frac{\left(-5x+2-\sqrt{25x^2+12x}\right)\left(-5x+2+\sqrt{25x^2+12x}\right)}{\left(-5x+2+\sqrt{25x^2+12x}\right)} =$$

$$= \lim_{x\to-\infty} \frac{(-5x+2)^2-(25x^2+12x)}{-5x+2+\sqrt{x^2}\sqrt{25+\frac{12}{x}}} = \lim_{x\to+\infty} \frac{\cancel{25x^2}-20x+4-\cancel{25x^2}-12x}{-5x+2-x\sqrt{25+\frac{12}{x}}} =$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(-32+\frac{4}{x}\right)}{\cancel{x}\left(-5+\frac{2}{x}-\sqrt{25+\frac{12}{x}}\right)} = \frac{-32}{-10} = \frac{16}{5}$$

$y = 10x + \frac{16}{5}$ è as. obliquo a $-\infty$.