(1) $X \doteq Y \mid Y \sim \text{Unif}[-1, 1]$.

$$\underbrace{\frac{1}{b-a}}_{\text{DENSITÀ} \sim f} \sim \frac{1}{1-(-1)} \sim \frac{1}{2} \cdot \mathbb{1}_{[-1,1]}$$

$\int$

RIPARTIZIONE $\sim F$

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{2} \int_{-1}^{1} x\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{1} = \frac{1}{2} \cdot \left(\frac{1}{2} - \frac{1}{2}\right) = 0.$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\,dx = \frac{1}{2} \int_{-1}^{1} x^2\,dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{3}\right) = \frac{1}{3}.$$

$\rightsquigarrow \quad var(X) = E[X^2] - E[X]^2 = \frac{1}{3}.$

(ii) X con funzione di ripartizione F_X data da $F_X(x) \doteq (x^2/4) \cdot \mathbf{1}_{(0,2)}(x) + \mathbf{1}_{[2,\infty)}(x)$, $x \in \mathbb{R}$;

$$F(x) = \frac{x^2}{4} \cdot \mathbb{1}_{[0,2)} + \mathbb{1}_{[2,\infty)}, \quad x \in \mathbb{R}$$

$\updownarrow$

DENSITÀ = DERIVATA $f(x)$

$$f(x) = \frac{x}{2} + c$$

$$E(x) = \frac{1}{2} \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$$

$$= \frac{1}{2} \int_0^2 x^2 = \frac{1}{2} \cdot \left[\frac{x^3}{3} \right]_0^2$$

$$E(x^2) = \frac{1}{2} \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\, dx$$

$$var(x) = E(x^2) - E(x^2)$$

$$\rightsquigarrow \quad E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx = \int_0^2 \frac{x^2}{2}\,dx$$

$$= \left[\frac{x^3}{6}\right]_0^2 = \frac{8}{6} = \frac{4}{3}.$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(dx) = \int_0^2 \frac{x^3}{2}\,dx$$

$$= \left[\frac{x^4}{8}\right]_0^2 = \frac{16}{8} = 2$$

$$\rightsquigarrow \quad var(X) = E[X^2] - E[X]^2 = 2 - \frac{16}{9} = \frac{2}{9}.$$

(iii) $X \doteq e^Y$ per una variabile aleatoria Y esponenziale di parametro quattro.

$$X = e^Y \iff Y = Exp(4) \qquad Y \to X$$

$$F(x) = e^Y \sim f(x) = \lambda \cdot e^{-\lambda}$$

$$= 4 \cdot e^{-4x}$$

$$E[X] = \int_{-\infty}^{\infty} e^x \cdot f_Y(x)\,dx = \int_0^{\infty} e^x (4 \cdot e^{-4x})$$

$$E[X^2] = \int e^{2x} \cdot f_Y(x)\,dx = \int e^{2x} \cdot (4 \cdot e^{-4x})$$

$$\rightsquigarrow \quad E[X] = \int_{-\infty}^{\infty} e^{x} \cdot f_Y(x)\,dx = \int_0^{\infty} e^{x} \cdot (4 \cdot e^{-4x})\,dx$$

$$= 4 \int_0^{\infty} e^{-3x}\,dx = 4 \left[-\frac{1}{3} \cdot e^{-3x}\right]_0^{\infty}$$

$$= 0 + \frac{4}{3} = \underline{\underline{\frac{4}{3}}}.$$

$$E[X^2] = \int_{-\infty}^{\infty} e^{2x} \cdot f_Y(x)\,dx = 4 \cdot \int_0^{\infty} e^{-2x}\,dx$$

$$= 4 \cdot \left[-\frac{1}{2} e^{-2x}\right]_0^{\infty} = \frac{4}{2} = 2.$$

$$\rightsquigarrow \quad var(X) = E[X^2] - E[X]^2 = 2 - \frac{16}{9} = \underline{\underline{\frac{2}{9}}}.$$

SCRITTO Ø_SOL

Esercizio 3. Siano $X_1, X_2, \ldots, X_{1000}$ variabili aleatorie indipendenti ed identicamente distribuite su $(\Omega, \mathcal{F}, \mathbf{P})$ con comune distribuzione di Bernoulli di parametro $1/400$. Poniamo

$$S(\omega) \doteq \sum_{i=1}^{1000} X_i(\omega), \ \omega \in \Omega, \qquad M \doteq \min\{m \in \mathbb{N} : \mathbf{P}(S \leq m) \geq 0.99\}.$$

Sia dia una stima per M in tre modi diversi, usando

a) la disuguaglianza di Chebyshev;

b) l'approssimazione di Poisson (legge dei piccoli numeri);

c) l'approssimazione normale.

→ TAVOLE

$$\underbrace{X_i \sim Ber\left(\frac{1}{400}\right)}_{\text{SINGOLO}} \quad \rightarrow \quad \mathbb{E}[S] = 1000 \cdot \frac{1}{400} = \frac{5}{2}$$

$$\sigma[S] = \frac{5}{2}$$

$$E[X] = p \qquad var(X) = p(1-p)$$

$$= \frac{1}{400} \qquad var(X) = \frac{1}{400}\left(\frac{399}{400}\right)$$

$$E[S] = \frac{5}{2} \qquad var(S) = 1000 \cdot var(X)$$

$$= \underbrace{\cancel{1000}^{5} \cdot \frac{1}{\cancel{400}_{2}} \cdot \frac{399}{400}}_{\frac{1995}{800}}$$

CHEBYSHEV

$\downarrow$

CONSEGNA $\rightarrow P(S > K) = P(S \geq K+1)$

$$= P(S - E[S] \geq K+1 - E[S])$$

$$= P\left(S - E[S] \geq K+1 - \frac{5}{2}\right)$$

$$\underset{\text{CHEB}}{\leq} \frac{var(S)}{(K-1)^2} \;\underline{\vee}\; \underbrace{1 - \frac{1995}{800} \cdot \frac{1}{(K-1)^2}}_{1 - P(S > K)}$$

POISSON $\quad Bin\begin{pmatrix} n \sim 1000 \\ K \sim \frac{1}{400} \end{pmatrix}$

$$S \sim Bin\left(1000, \frac{1}{400}\right)$$

$$\lambda = 1000 \cdot \frac{1}{400} = \frac{5}{2}$$

$$F_{POISS(5/2)}(K) \geq 0.89$$

5 FILO

↘ LEGGIAMO LE TAVOLE

NORMALE

$$E[S] = \frac{5}{2}, \quad var(S) = \frac{1998}{800}$$

$$\bar{S} = \frac{1}{\sqrt{var(S)}}[S - E(S)]$$

$$= \frac{1}{\sqrt{1000 \cdot var(S)}} \sum_{i=1}^{1000} (X_i - E[X_i])$$

CW5BY5H97

Per $K \in \mathbb{N}$:

$$P(S \leq K) = P(S - E[S] \leq K - E[S])$$

$$= P\Big(\underbrace{\frac{1}{\sqrt{Var(S)}}(S - E[S])}_{=\bar{S}} \leq \frac{K - E[S]}{\sqrt{Var(S)}}\Big)$$

$$\approx \Phi\Big(\frac{K - E[S]}{\sqrt{Var(S)}}\Big)$$

$\uparrow$ funzione di ripartizione della normale standard

$$y \in \mathbb{R} \mid \Phi(y) \geq 0.99$$

§ Rws

$$\exists K \in \mathbb{N} \mid \frac{K - E(S)}{\sqrt{Var(S)}} \geq 2.33$$

PRIMO $\geq$ 0.99

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916