$\boxed{S_{m,n}}$ → KLEENE'S SMN THEOREM

$\boxed{\text{PARAMETRIZATION}}$ ALSO CALLED... (MORE IMPORTANTLY!)

$f(x) \to y$

$e \to \boxed{\varphi_{(x)}(y)}$

$g(x,y) \to$ COMPUTABLE

$\varphi_{s(x)}(y)$

S^{n}_{m}

$\boxed{\varphi_{s(x)}(y)}$

$$A = \{ x \in \mathbb{N} \mid x \in W_x \ \wedge \ \varphi_x(x) = x^2 \}$$

SATURATED $\Rightarrow$ SMN

NOT RECURSIVE

K

HALTING SET

NOT R.E

$$\Rightarrow \underbrace{g(x,y)}_{SMN} = \begin{cases} y & x \in K \\ \uparrow & \text{otherwise} \end{cases}$$

K

1) $$g(x,y) = \begin{cases} y^2 & x \in K \\ \uparrow & \text{otherwise} \end{cases}$$

↓ CLASSIC K DEFINITION

$$\mathcal{I}_K \to \left[g(x,y) = \begin{cases} y & x \in W_x \\ \uparrow & \text{otherwise} \end{cases} \right]$$

$$g(x,y) = \begin{cases} y^2 & x \in K \\ \uparrow & \text{otherwise} \\ & (x \notin K) \end{cases}$$

$$g(x,y) = \mu w \,.\, |y^2 - x^2|$$

$$\simeq y^2 \cdot SC_K$$

↓ R.G

$$SC_K = \begin{cases} 1 & x \in K \\ \uparrow & \text{otherwise} \end{cases}$$

↓ REC. / B

$$\mathcal{I}_K = \begin{cases} 1 & x \in K \\ 0 & \text{otherwise} \end{cases}$$

SMN-THEOREM

$\exists s : \mathbb{N} \to \mathbb{N}$ s.t. PARAM.

$\exists x, y \in \mathbb{N} \mid \varphi_{s(x)}(y) = g(x,y)$

$$g(x,y) = \begin{cases} y^2 & x \in K \\ \uparrow & \text{otherwise} \end{cases}$$

We use parametrisation to make the function do whatever we want!

$\varphi_{s(x)}(y) = g(x,y)$

s is a reduction function "for something"

CORE IDEA...

$$K \leq A$$

$$\begin{bmatrix} -\text{ if } x \in K & \to y^2 \\ -\text{ if } x \notin K & \to \uparrow \end{bmatrix}$$

$-$ if $x \in K$

$$\varphi_{S(x)}(y) = g(x, y) = y^2 \quad \forall y \in \mathbb{N}$$

$\to$ DOM / CODOM TO COMPUTE

$$\underbrace{W_{S(x)}}_{\text{DOMAIN OF } \varphi_{S(x)} = y^2} = y^2 = \mathbb{N}$$

$$\left[\varphi_{S(x)}(S(x))\right] \Longleftrightarrow x^2$$

SUBSTITUTE...

$$= S(x)^2$$

if $x \in K$ | $\underbrace{s(x)}_{y} \in A$

$\uparrow$

s is a reduction function of K to A.

- if $x \notin K$ =

$\varphi_{s(x)}(y) = g(x, y) = \uparrow \quad \forall y \in \mathbb{N}$

$s(x) \notin W_x = \emptyset$

$\varphi_{s(x)}(s(x)) = \uparrow$ | $k \notin W_x$

Thus, $s(x) \notin A$.

$A / \bar{A}$

$$A = \{x \in \mathbb{N} : x \in W_x \wedge \varphi_x(x) = x^2\}$$

$$\underbrace{\bar{A} = \{x \in \mathbb{N} : x \notin W_x \vee \varphi_x(x) \neq x^2\}}$$

WRITE THE NEGATED VERSION...

UNIVERSAL URM-MACHINE

||

UNIVERSAL TM

Ψ] PSI → (UPPERCASE)

PARAMETRIZES ...

<u>Theorem</u> (Universal Program)

Let $k \geq 1$ then the universal function

$$\Psi_U^k : \mathbb{N}^{k+1} \to \mathbb{N}$$

$$\underbrace{\Psi_U(e, \vec{x}) = \varphi_e^{(k)}(\vec{x})}_{}$$

is computable

SMN!

$$\varphi_x(x) = \text{FIX } x \text{ ON } x$$

$$\Psi_U(x, x)$$

A is r.e

WE CAN WRITE WITH μ-OPERATOR... SOMETHING!

$$SC_A(x) =$$

$$1(\mu w . \; |x^2 - y^2| = |x^2 - \varphi_x(x)|$$

1/0

FAKE VAR.

$$\varphi_{s(x)}(s(x))$$

$$= \mathbb{1}\,(\mu w.\, |x^2 - \Phi_0(x,x)|\,|$$

$$= \left[\mathbb{1}\,(\mu w.\, |x^2 - \Phi_0(x,x)|\,|\right._{1}$$

$N^{k \neq 1} \to \mathbb{N}$

1 - SUBSCRIPT

$$(\pi) \to (w_1), (w_3)$$

COMPLETE FORM

$$= (\mathbb{1}\,(\mu\,(y,z,A)\,\underbrace{\Sigma}_{A}\,(x,y,A) \wedge$$

$$\underbrace{\Sigma_{\xi}}\,(x,z,y,A)\,)_{1}$$

$$\left[= \mathbb{1}\,(\mu w.\, H(x, (w)_1, (w)_3) \wedge \right.$$

$$\left. S(x, (w)_2, (w)_1, (w)_3) \right]$$

COMPACT FORM!

The smn-theorem states that given a function of two arguments $g(x, y)$ which is computable, there exists a total and computable function such that $\phi_s(x)(y) = g(x, y)$ basically "fixing" the first argument of g. It's like partially applying an argument to a function. This is generalized over m, n tuples for x, y. In other words, it addresses the idea of "parametrization" or "indexing" of computable functions. It's like creating a simplified version of a function that takes an additional parameter (index) to mimic the behavior of a more complex function.

The function s_m^n is designed to mimic the behavior of $\phi(x, y)$ when given the appropriate parameters. Essentially, by selecting the right values for m and n, you can make s behave like for a specific computation. Instead of dealing with the complexity of $\phi(x, y)$, we can work with a simpler s_m^n that captures the essence of the computation.

(THINK ON THAT – IF YOU WILL!)

FINISHING...