

343

$$f(x) = (x^2 - 1)^2 (x + 1)^3$$

$f \quad g \quad (f'g') = f'g + f \cdot g'$

$$f'(x) = \frac{[(x^2 - 1) \cdot 2x]}{f'} \cdot \frac{[(x + 1)^3]}{g}$$

$$+ \frac{(x^2 - 1)^2 \cdot 3(x + 1)^2 \cdot 1}{g'}$$

$$= 2x(x^2 - 1) \cdot (x + 1)^3 + (x^2 - 1)^2 \cdot 3(x + 1)^2$$

$$= (x + 1)^2 (x^2 - 1) [2x(x + 1) + 3(x^2 - 1)]$$

366

$$f(x) = 2x \cos x + (x^2 - 2) \sin x$$

$f \quad g \quad f' \quad g'$

$$\frac{d}{dx} (2x \cos x) = \frac{2}{f'} \cdot \frac{\cos x}{g} +$$

$$\frac{2x(-\sin x)}{f' \cdot g'}$$

$$\frac{d}{dx} (x^2 - 2) \sin x = \frac{2x}{f'} \cdot \frac{\sin x}{g}$$

$$+ \frac{(x^2 - 2) \cdot \cos x}{f' \cdot g'}$$

$$= 2 \cos x - 2x \sin x + 2x \sin x + (x^2 - 2) \cos x$$

$$= \cos x [2 + (x^2 - 2)]$$

389 $f(x) = \frac{(x^2-1)^4}{(x+1)^3}$

$$= \frac{\frac{2x}{f'} \cdot \frac{(x+1)^3}{g} - \frac{(x^2-1)}{f} \cdot \frac{3(x+1)^2}{g'}}{(x+1)^6}$$

$$= \frac{(x+1)^2 [2x - 3(x^2-1)]}{(x+1)^6}$$

$$= \frac{[2x - 3(x^2-1)]}{(x+1)^4}$$

$$\frac{(x+1)^1 (3-x)}{(x+1)^4 \cdot 3} = \frac{(3-x)}{(x+1)^3}$$

351 $f(x) = e^{2x} + \sqrt[3]{e^x}$

$$f'(x) = e^{2x} \cdot 2 + \frac{(e^x)^{1/3}}{3}$$

POSSIBLE

$$= e^{2x} \cdot 2 + \frac{1}{3} (e^x)^{-2/3} \cdot e^x$$

$$= 2e^{2x} + \frac{1}{3} e^{1/3 x}$$

$$= e^x (2e^x + \frac{1}{3} e^{-2/3 x})$$

371

$$f(x) = \ln \left(\frac{x^2 - 2x}{x+1} \right)$$

LOGARITHMO

QUOTIENT

$$\frac{d}{dx} \left(\frac{x^2 - 2x}{x+1} \right) = \frac{(2x-2)(x+1) - \frac{x^2-2x}{f} \cdot \frac{1}{g'}}{(x+1)^2}$$

$$\frac{2x^2 + 2x - 2x - 2 - x^2 + 2x}{(x+1)^2} =$$

$$= \frac{x^2 - 2x - 2}{(x+1)^2} \cdot \frac{x^2 - 2x}{x(x-2)}$$

$$371 \quad f(x) = \ln \left(\frac{x^2 - 2x}{x+1} \right)$$

$$\ln(f(x)) = \frac{1}{f(x)}$$

$$= \frac{x^2 - 2x - 2}{(x+1)x(x-2)} \quad (\checkmark)$$

$$384 \quad f(x) = \frac{(x \ln x + x^2 \ln^3 x)}{(x^2 \ln x)} f'$$

$$\frac{d}{dx} [x \ln(x)] = \frac{1}{f'} \cdot \frac{\ln(x)}{g} + \frac{x}{f} \cdot \frac{1}{g'} = \ln(x) + 1$$

$$\frac{d}{dx} [x^2 \ln^3(x)] = \frac{2x}{f'} \frac{\ln^3(x)}{g} + \frac{x^2}{f} \cdot \frac{3 \ln^2(x)}{g'}$$

$$\ln^3(x) \text{ COMPOSTA}$$

$$/ 3 \ln^2(x) \cdot \frac{1}{x}$$

$$\frac{d}{dx} [x^2 \ln(x)]$$

$$= \frac{2x}{f'} \frac{\ln(x)}{g} + \frac{x^2}{f} \frac{1}{g'}$$

$$= 2x \ln(x) + x$$

$$\text{384 } f(x) = \frac{(x \ln x + x^2 \ln^3 x)}{(x^2 \ln x) g} = \frac{f' g - f g'}{(g)^2}$$

$$= \frac{2x \ln^3(x) + 3x \ln^2(x) + (2x \ln(x) + x) \cdot x^2 \ln(x)}{f'}$$

$$- \frac{[2x \ln(x) + x] \cdot (x \ln(x) + x^2 \ln^3(x))}{g' f}$$

$$\frac{x^4 \ln^2(x)}{g^2}$$

$$= [2x \ln(x) + x] [2x \ln^3(x) + 3x \ln^2(x)] \cdot x^2 \ln(x) + [x \ln(x) + x^2 \ln^3(x)]$$

$$\frac{x^4 \ln^2(x)}{g^2}$$

395 $f(x) = \sin(\sqrt{1+\ln x})$ — $0 + \frac{1}{x} = f'(x)$ $[1+\ln(x)]^{1/2}$

$$f'(x) = \cos \sqrt{1+\ln(x)} \cdot \frac{1}{2} [1+\ln(x)]^{-1/2} \cdot \frac{1}{x}$$

397 $f(x) = \underbrace{(x^2+1)}_{f'} \underbrace{(x^2+2)^2}_{g} \underbrace{(x^2+3)}_{g'}$

$$\frac{d}{dx} [(x^2+1)(x^2+2)^2] = f' \cdot g + f \cdot g'$$

$$= \frac{2x}{f'} \cdot \frac{(x^2+2)^2}{g} + \frac{(x^2+1) \cdot 4x}{g'} \cdot \frac{(x^2+2)}{f}$$

$$= 2x(x^2+2)^2 + (x^2+1)[4x+1]$$

$$\frac{d}{dx} [(x^2+2)^2(x^2+3)] = \frac{(x^2+2) \cdot 4x}{f'} \cdot \frac{(x^2+3)}{g}$$

$$+ \frac{(x^2+2)^2}{f} \cdot \frac{2x}{g'} = (x^2+2)2x [2(x^2+3) + (x^2+2)]$$

$$2x(x^2+2)^2 + (x^2+1)[4x+1] \rightarrow f$$

$$\frac{(x^2+2)2x}{f} [2(x^2+3) + (x^2+2)] \rightarrow g$$

$$f' = 2(x^2+2) \cdot 2x + \underbrace{[2 \times [4x+1] + 4(x^2+1)]}_{\text{prod}}$$

$$g' = \underbrace{6x^2 + 2(x^2+2)}_{f'} \underbrace{[2(x^2+3) + (x^2+2)]}_g$$

$$+ \underbrace{6x}_{g'} + \underbrace{(x^2+2x)}_f$$

$$\{ 2(x^2+2) \cdot 2x + [2 \times [4x+1] + 4(x^2+1)] \}$$

f'

$$\cdot (x^2+2)2x [2(x^2+3) + (x^2+2)] +$$

g

$$\underbrace{2x(x^2+2)^2 + (x^2+1)[4x+1]}_f +$$

$$\underbrace{6x^2 + 2(x^2+2)}_{f'} \underbrace{[2(x^2+3) + (x^2+2)]}_g$$

$$+ \underbrace{6x}_{g'} + \underbrace{(x^2+2x)}_f$$

$$f(x) = \sqrt[3]{x^2(x-1)} \rightarrow [x^2(x-1)]^{1/3}$$

$$\begin{aligned} \frac{d}{dx} \left[\frac{x^2}{t} \cdot \frac{(x-1)}{g} \right] &= \frac{2x}{t^1} \cdot \frac{(x-1)}{g} + \frac{x^2}{t} \cdot \frac{1}{g'} \\ &= 2x^2 - 2x + x^2 = 3x^2 - 2x \\ &= x(3x - 2) \end{aligned}$$

$$\frac{1}{3} [x^2(x-1)]^{-2/3} \cdot x(3x-2)$$

$$\frac{1}{3} \sqrt[3]{\frac{1}{[x^2(x-1)]^2}} [x(3x-2)]$$

Trova l'equazione della retta tangente al grafico della funzione $f(x) = \frac{x^2-4}{x-2}$ nel punto di ascissa $x=3$.

$$f(3)$$

$$f(x) = \frac{x^2-4}{x-2} \rightarrow f(3) = \frac{3^2-4}{3-2} = \frac{9-4}{1} = 5$$

$$f'(x) = \frac{2x(x-2) - (x^2-4)}{(x-2)^2} =$$

$$\frac{2x^2 - 4x - x^2 + 4}{(x-2)^2} = \frac{x^2 - 4x + 4}{(x-2)^2} = \frac{(x-2)^2}{(x-2)^2}$$

$$= 1$$

$$(x - x_0)$$

RETTA TANGENTE $\rightarrow y - 5 = 1 \cdot (x - 3) \rightarrow y = x + 2$

A $x_0 = 3$