1.31 For any string $w = w_1 w_2 \cdots w_n$, the ***reverse*** of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}} |\ w \in A\}$.
Show that if A is regular, so is $A^{\mathcal{R}}$.

1.38 An ***all***-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if *every* possible state that M could be in after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

1.40 Recall that string x is a ***prefix*** of string y if a string z exists where $xz = y$, and that x is a ***proper prefix*** of y if in addition $x \neq y$. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

^A**a.** $NOPREFIX(A) = \{w \in A |$ no proper prefix of w is a member of $A\}$.

b. $NOEXTEND(A) = \{w \in A |\ w$ is not the proper prefix of any string in $A\}$.

1.41 For languages A and B, let the ***perfect shuffle*** of A and B be the language

$$\{w |\ w = a_1 b_1 \cdots a_k b_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}.$$

Show that the class of regular languages is closed under perfect shuffle.

1.42 For languages A and B, let the ***shuffle*** of A and B be the language

$$\{w |\ w = a_1 b_1 \cdots a_k b_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}.$$

Show that the class of regular languages is closed under shuffle.

1.70 We define the ***avoids*** operation for languages A and B to be

$$A \textit{ avoids } B = \{w |\ w \in A \text{ and } w \text{ doesn't contain any string in } B \text{ as a substring}\}.$$

Prove that the class of regular languages is closed under the *avoids* operation.

2.43 For strings w and t, write $w \triangleq t$ if the symbols of w are a permutation of the symbols of t. In other words, $w \triangleq t$ if t and w have the same symbols in the same quantities, but possibly in a different order.
For any string w, define $SCRAMBLE(w) = \{t |\ t \triangleq w\}$. For any language A, let $SCRAMBLE(A) = \{t |\ t \in SCRAMBLE(w) \text{ for some } w \in A\}$.

a. Show that if $\Sigma = \{0,1\}$, then the *SCRAMBLE* of a regular language is context free.

b. What happens in part (a) if Σ contains three or more symbols? Prove your answer.

2.44 If A and B are languages, define $A \diamond B = \{xy|\ x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that if A and B are regular languages, then $A \diamond B$ is a CFL.

***2.49** We defined the rotational closure of language A to be $RC(A) = \{yx|\ xy \in A\}$. Show that the class of CFLs is closed under rotational closure.