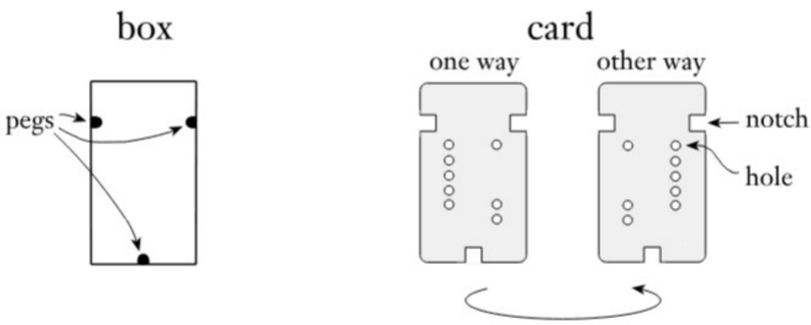
7.28 You are given a box and a collection of cards as indicated in the following figure. Because of the pegs in the box and the notches in the cards, each card will fit in the box in either of two ways. Each card contains two columns of holes, some of which may not be punched out. The puzzle is solved by placing all the cards in the box so as to completely cover the bottom of the box (i.e., every hole position is blocked by at least one card that has no hole there). Let *PUZZLE* $= \{\langle c_1, \ldots, c_k \rangle |$ each c_i represents a card and this collection of cards has a solution$\}$. Show that *PUZZLE* is NP-complete.



7.35 A subset of the nodes of a graph G is a ***dominating set*** if every other node of G is adjacent to some node in the subset. Let

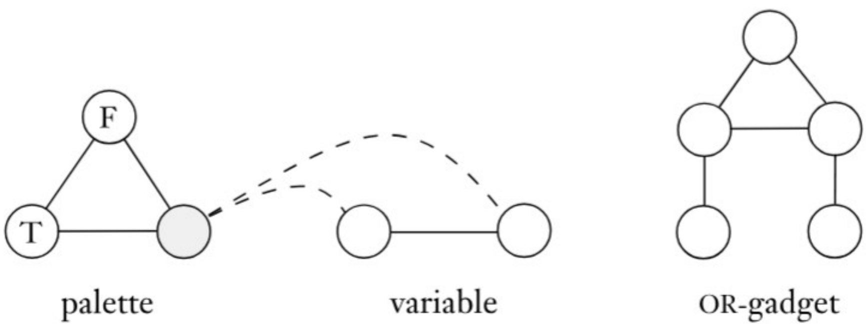
$$\textit{DOMINATING-SET} = \{\langle G, k\rangle |\ G \text{ has a dominating set with } k \text{ nodes}\}.$$

Show that it is NP-complete by giving a reduction from *VERTEX-COVER*.

7.29 A ***coloring*** of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let

$$\textit{3COLOR} = \{\langle G\rangle |\ G \text{ is colorable with 3 colors}\}.$$

Show that *3COLOR* is NP-complete. (Hint: Use the following three subgraphs.)



7.21 Let G represent an undirected graph. Also let

$$SPATH = \{\langle G, a, b, k\rangle \mid G \text{ contains a simple path of length at most } k \text{ from } a \text{ to } b\},$$

and

$$LPATH = \{\langle G, a, b, k\rangle \mid G \text{ contains a simple path of length at least } k \text{ from } a \text{ to } b\}.$$

a. Show that $SPATH \in \mathrm{P}$.

b. Show that $LPATH$ is NP-complete.

5.35 Say that a variable A in CFG G is ***necessary*** if it appears in every derivation of some string $w \in G$. Let $NECESSARY_{\mathsf{CFG}} = \{\langle G, A\rangle \mid A \text{ is a necessary variable in } G\}$.

a. Show that $NECESSARY_{\mathsf{CFG}}$ is Turing-recognizable.

b. Show that $NECESSARY_{\mathsf{CFG}}$ is undecidable.

*__5.36__ Say that a CFG is ***minimal*** if none of its rules can be removed without changing the language generated. Let $MIN_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a minimal CFG}\}$.

a. Show that MIN_{CFG} is T-recognizable.

b. Show that MIN_{CFG} is undecidable.

4.4 Let $A\varepsilon_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG that generates } \varepsilon\}$. Show that $A\varepsilon_{\mathsf{CFG}}$ is decidable.

4.16 Let $A = \{\langle R\rangle \mid R$ is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., $w = x\mathtt{111}y$ for some x and y)$\}$. Show that A is decidable.

3.11 A ***Turing machine with doubly infinite tape*** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.