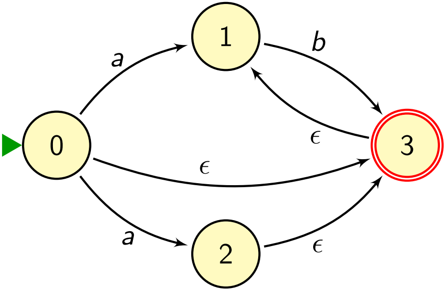
Convert an NFA to DFA
An Example
Rabin-Scott subset construction
Lounden, Exercise 2.14, page 92

Convert an NFA to DFA

Convert the NFA below into a DFA using the subset construction.



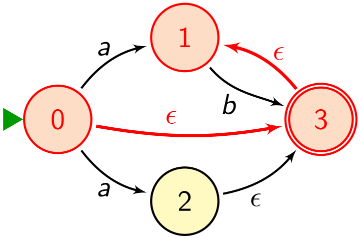
Q	Σ_ϵ	Q
0	ϵ	3
0	a	1
0	a	2
1	b	3
2	ϵ	3
3	ϵ	1

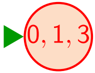
An example NFA. Lounden, Example 2.10, page 58.

Convert an NFA to DFA

Convert the NFA into a DFA using the subset construction. Each state of the DFA is a set of states of the NFA. The initial state of the DFA is the ϵ-closure of the initial state of the NFA.

$$\epsilon\text{-}\textsc{close}\{0\} = \{0, 1, 3\} = S_0$$

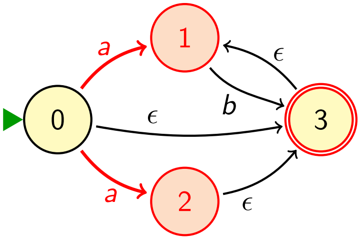


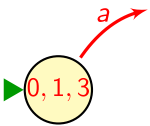


Convert an NFA to DFA

Determine the transition function of the DFA on all inputs $\sigma \in \Sigma$. Begin with the initial state S_0, and determine the transition on input a.

$$\begin{aligned} \epsilon\text{-CLOSE}\{0\} &= \{0,1,3\} = S_0 \\ \delta(S_0, a) &= \epsilon\text{-CLOSE}\{1,2\} \\ \delta(S_0, b) &= \end{aligned}$$

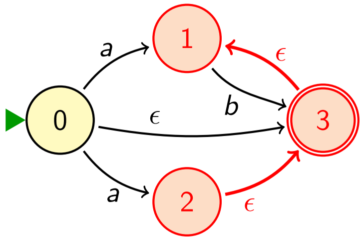


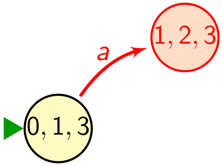


Convert an NFA to DFA

With the initial state $S_0 = \{0, 1, 3\}$, determine the transition on input a. The ϵ-closure of the set $\{1, 2\}$ is $\{1, 2, 3\}$. This is a new state in the DFA, call it S_1.

$$\begin{aligned} \epsilon\text{-}\mathrm{CLOSE}\{0\} &= \{0, 1, 3\} = S_0 \\ \delta(S_0, a) = \epsilon\text{-}\mathrm{CLOSE}\{1, 2\} &= \{1, 2, 3\} = S_1 \\ \delta(S_0, b) = & \end{aligned}$$

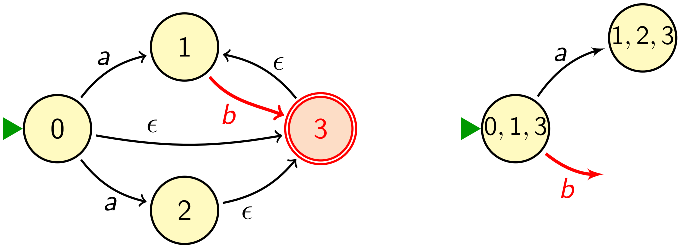




Convert an NFA to DFA

With the initial state $S_0 = \{0, 1, 3\}$, determine the transition on input b.

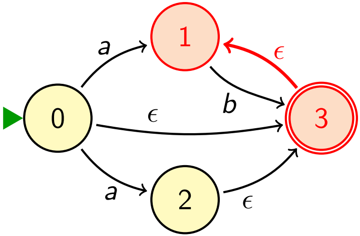
$$\begin{aligned}
\epsilon\text{-CLOSE}\{0\} &= \{0,1,3\} = S_0 \\
\delta(S_0, a) = \epsilon\text{-CLOSE}\{1,2\} &= \{1,2,3\} = S_1 \\
\delta(S_0, b) = \epsilon\text{-CLOSE}\{3\} &
\end{aligned}$$

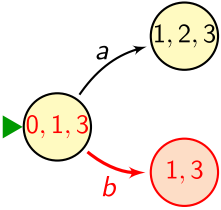


Convert an NFA to DFA

The ϵ-closure of the set $\{3\}$ is $\{1, 3\}$. This is a new state in the DFA, call it S_2.

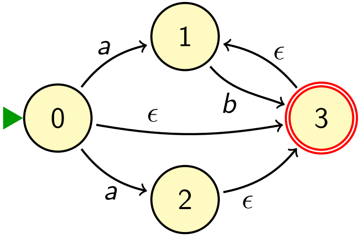
$$\begin{aligned} \epsilon\text{-CLOSE}\{0\} &= \{0,1,3\} = S_0 \\ \delta(S_0, a) = \epsilon\text{-CLOSE}\{1,2\} &= \{1,2,3\} = S_1 \\ \delta(S_0, b) = \epsilon\text{-CLOSE}\{3\} &= \{1,3\} = S_2 \end{aligned}$$

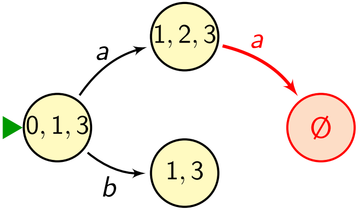




Determine the transition function of the DFA from state S_1 on inputs a and b. On a there is nowhere to go in the NFA, so we create a "sink" state for the DFA.

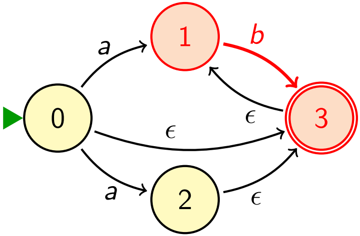
$$\begin{array}{lcccccc}
\delta(S_0, a) & = & \epsilon\text{-CLOSE}\{1,2\} & = & \{1,2,3\} & = & S_1 \\
\delta(S_0, b) & = & \epsilon\text{-CLOSE}\{3\} & = & \{1,3\} & = & S_2 \\
\delta(S_1, a) & = & \epsilon\text{-CLOSE}\{\} & = & \emptyset & = & S_3 \\
\delta(S_1, b) & = & & & & &
\end{array}$$

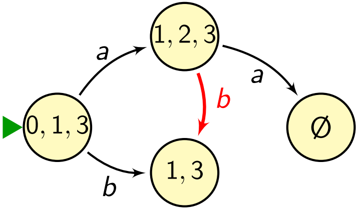




Determine the transition function of the DFA from state S_1 on inputs a and b. On b we happen to transition to an existing state S_2.

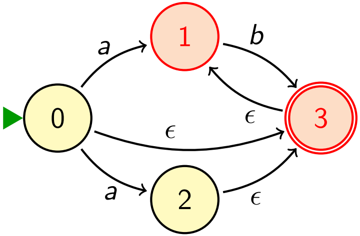
$$\begin{aligned}
\delta(S_0, a) &= \epsilon\text{-CLOSE}\{1, 2\} = \{1, 2, 3\} = S_1 \\
\delta(S_0, b) &= \epsilon\text{-CLOSE}\{3\} = \{1, 3\} = S_2 \\
\delta(S_1, a) &= \epsilon\text{-CLOSE}\{\} = \emptyset = S_3 \\
\delta(S_1, b) &= \epsilon\text{-CLOSE}\{3\} = \{1, 3\} = S_2
\end{aligned}$$

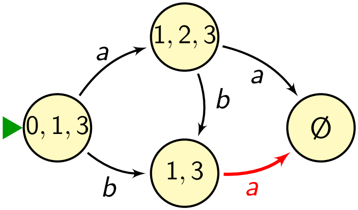




Determine the transition from state S_2 on inputs a and b.

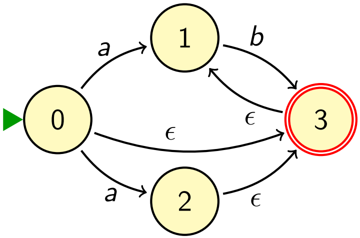
$$
\begin{aligned}
\delta(S_0, a) &= \epsilon\text{-CLOSE}\{1,2\} &&= \{1,2,3\} &&= S_1 \\
\delta(S_0, b) &= \epsilon\text{-CLOSE}\{3\} &&= \{1,3\} &&= S_2 \\
\delta(S_1, a) &= \epsilon\text{-CLOSE}\{\} &&= \emptyset &&= S_3 \\
\delta(S_1, b) &= \epsilon\text{-CLOSE}\{3\} &&= \{1,3\} &&= S_2 \\
\delta(S_2, a) &= \epsilon\text{-CLOSE}\{\} &&= \emptyset &&= S_3 \\
\delta(S_2, b) &=
\end{aligned}
$$

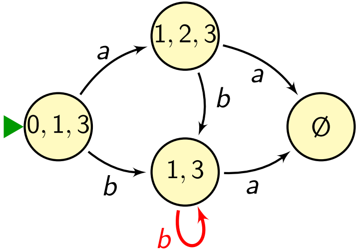




Determine the transition from state S_2 on inputs a and b.

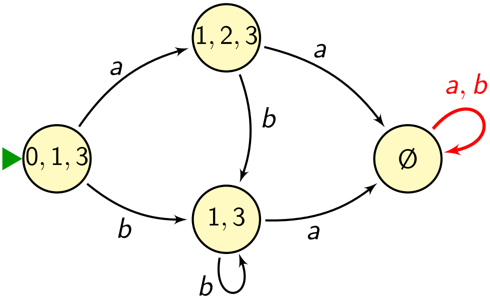
$$\begin{aligned}
\delta(S_0, a) &= \epsilon\text{-CLOSE}\{1,2\} = \{1,2,3\} = S_1 \\
\delta(S_0, b) &= \epsilon\text{-CLOSE}\{3\} = \{1,3\} = S_2 \\
\delta(S_1, a) &= \epsilon\text{-CLOSE}\{\} = \emptyset = S_3 \\
\delta(S_1, b) &= \epsilon\text{-CLOSE}\{3\} = \{1,3\} = S_2 \\
\delta(S_2, a) &= \epsilon\text{-CLOSE}\{\} = \emptyset = S_3 \\
\delta(S_2, b) &= \epsilon\text{-CLOSE}\{3\} = \{1,3\} = S_2
\end{aligned}$$





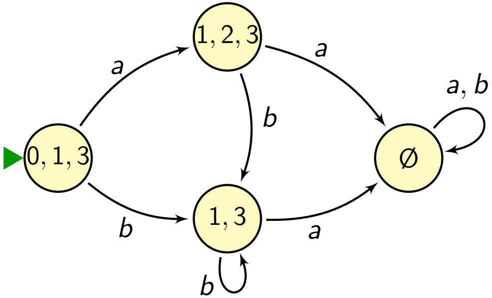
Convert an NFA to DFA

Determining the transition from state S_2 on inputs a and b is easy; from the empty set of states there are no transitions in the NFA. In the DFA this is represented by a transition from the empty set back to itself.



Convert an NFA to DFA

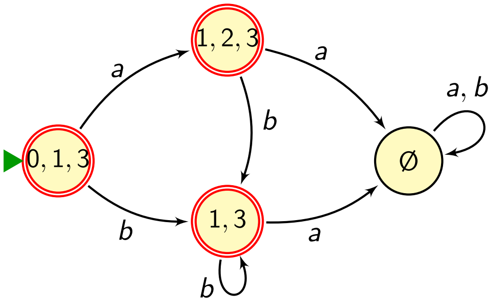
All the edges of the DFA have been discovered.



Q	Σ	Q
$\{0,1,3\}$	a	$\{1,2,3\}$
$\{0,1,3\}$	b	$\{1,3\}$
$\{1,2,3\}$	a	$\emptyset$
$\{1,2,3\}$	b	$\{1,3\}$
$\{1,3\}$	a	$\emptyset$
$\{1,3\}$	b	$\{1,3\}$
$\emptyset$	a	$\emptyset$
$\emptyset$	b	$\emptyset$

Convert an NFA to DFA

The final states of the DFA are determined from the final states of the NFA. State 3 was the only final state in the NFA. Any set of NFA states containing a final state is a final state in the DFA.



Q	Σ	Q
$\{0,1,3\}$	a	$\{1,2,3\}$
$\{0,1,3\}$	b	$\{1,3\}$
$\{1,2,3\}$	a	$\emptyset$
$\{1,2,3\}$	b	$\{1,3\}$
$\{1,3\}$	a	$\emptyset$
$\{1,3\}$	b	$\{1,3\}$
$\emptyset$	a	$\emptyset$
$\emptyset$	b	$\emptyset$

Convert an NFA to DFA (Solution)

Lounden, Exercise 2.14, page 92. Convert the NFA of Example 2.10 into a DFA using the subset construction. The resulting DFA is shown below on the right.