Low-level Description of a Turing Machine

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A low-level (or formal) description of a Turing Machine (TM) to recognize a language involves defining all the elements in the 7-tuple

$$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$$

Consider the following language

$$L = \{w\#w | w \in \{0,1\}^*\}$$

Idea

Here is the idea behind building a TM to recognize the above language.

1. Scan the input tape to be sure that it contains a single $\#$. If not, *reject*.
2. Zig-zag across the tape to corresponding positions on either side of $\#$to check whether these positions contain the same symbol. If they do not, *reject*. Cross off the symbols as they are checked.
3. When all symbols to the left of $\#$have been crossed off, check for the remaining symbols to the right of $\#$. If any symbol remain, *reject*; otherwise *accept*.

When the idea explains in detail how the tape is modified, it is called a High-Level Description. This High-Level description can be converted into a formal description.

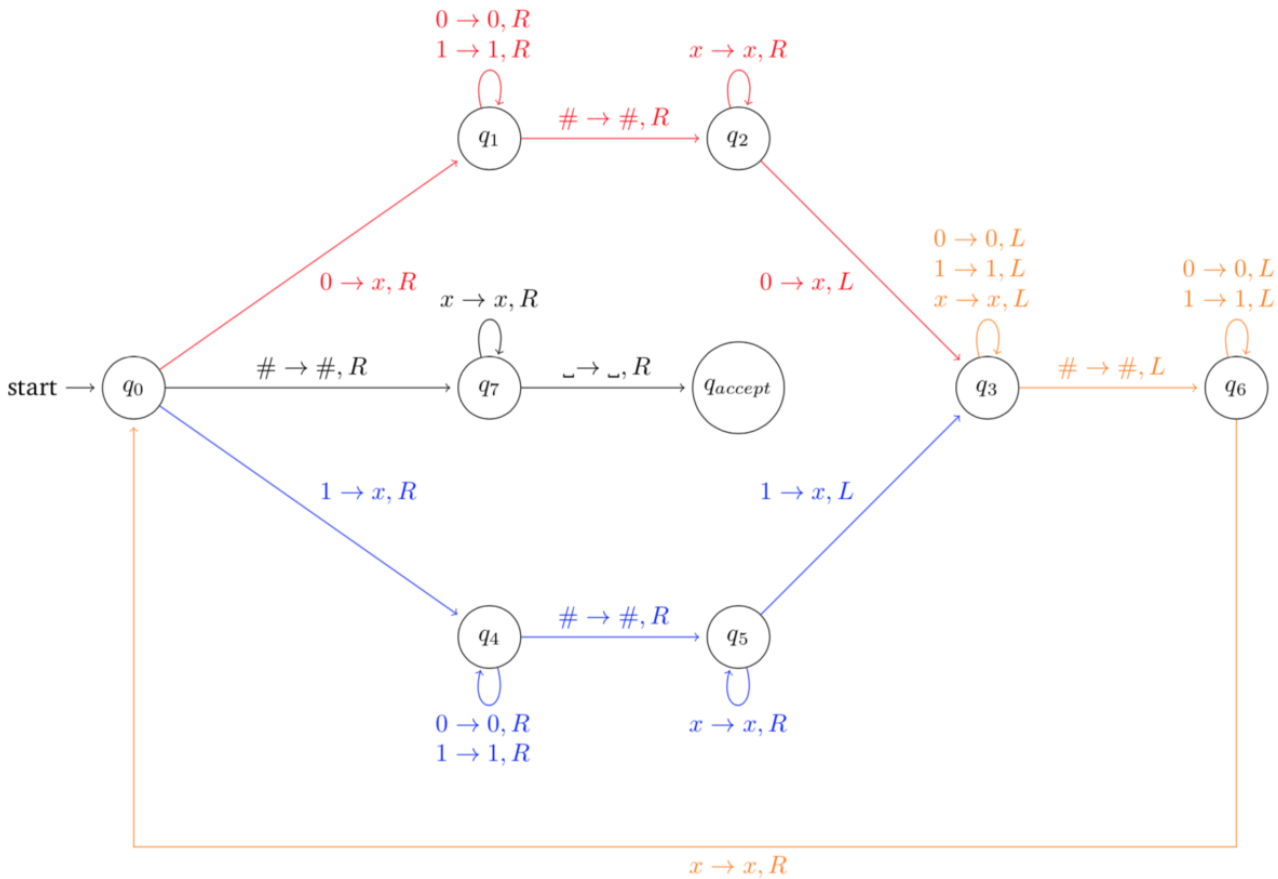
Alphabet

The input alphabet Σ will be $\{0, 1\#\}$ as defined in the language.

A new symbol (x) is needed to keep track of the crossed off symbols. Thus, the tape alphabet Γ will be $\{0, 1\#, \textvisiblespace, x\}$.

Transition Function

The transition state diagram for the above implementation is shown below. The red path matches the 0s on either side of the $\#$.. The blue path matches the 1s on either side of the $\#$.. The orange path brings the tape-head back to the first uncrossed symbol. The black path checks if the count of the crossed out symbols match.



A note about the reject state

The state q_{reject} is not shown here. It is assumed that all undefined transitions (to ensure that the transition function is a total function) go to q_{reject}. For example q_7 has no transition on the symbol 0 in the diagram above. It goes to q_{reject}.

In essence, we only care about ensuring that the strings that are indeed accepted end up in q_{accept}. All other strings that do not match the pattern get rejected. There is no case where the machine will run forever.

Formal Description

Putting it all together, the low-level (formal) description of the TM to recognize the language L will be