

R Primer

Getting Started with R

- ▶ You can download R from <https://www.r-project.org/>
- ▶ R is an open-source language mostly used for Statistics and data analysis.
- ▶ It is based on a core build with basic functions plus packages that can be installed.
- ▶ A package is a set of functions/data.
- ▶ R community is huge and you should use it! Most of your questions were already answered on Stack Overflow (<https://stackoverflow.com>).
- ▶ Stack Overflow is a forum where people post questions about programming.

RStudio

- ▶ R alone is just a console that looks like a notepad. It is not very friendly.
- ▶ RStudio is an Integrated development environment (IDE) made for R.
- ▶ You can download RStudio from <https://rstudio.com/products/rstudio/download/>
- ▶ Make sure to get the free desktop version.
- ▶ RStudio will automatically find your R once installed.

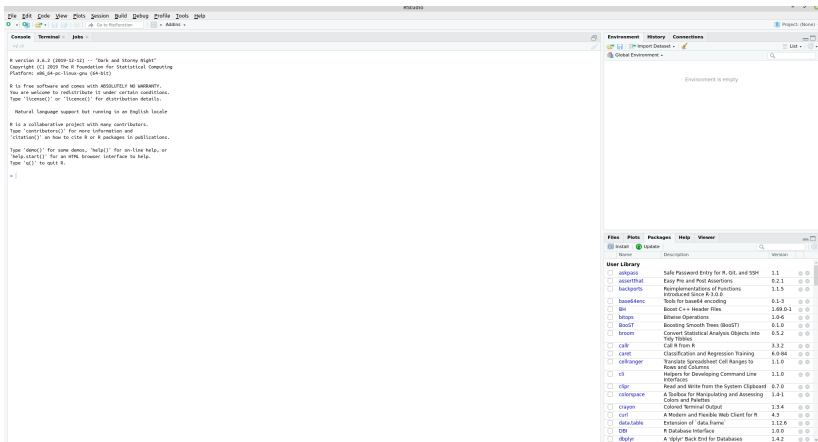
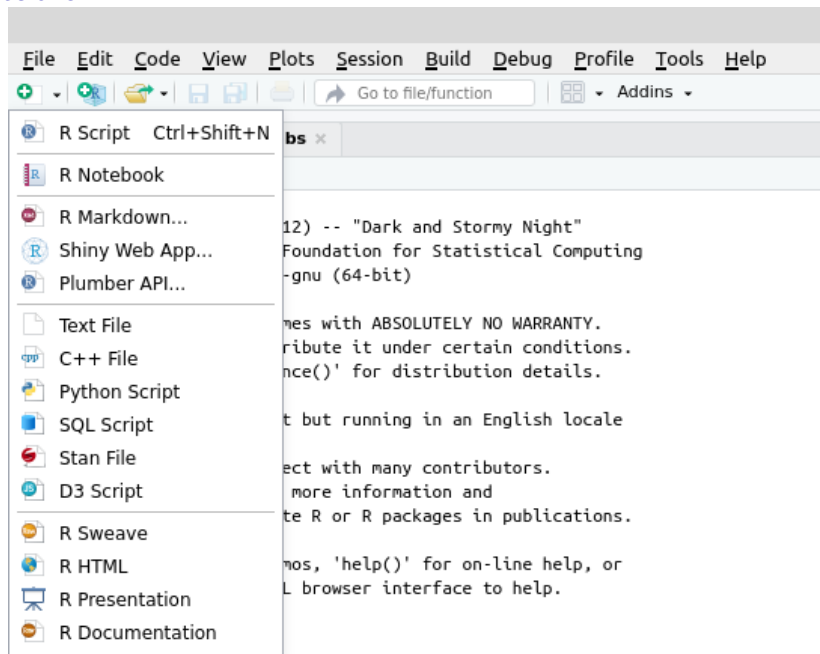


Figure 1: RStudio

RStudio



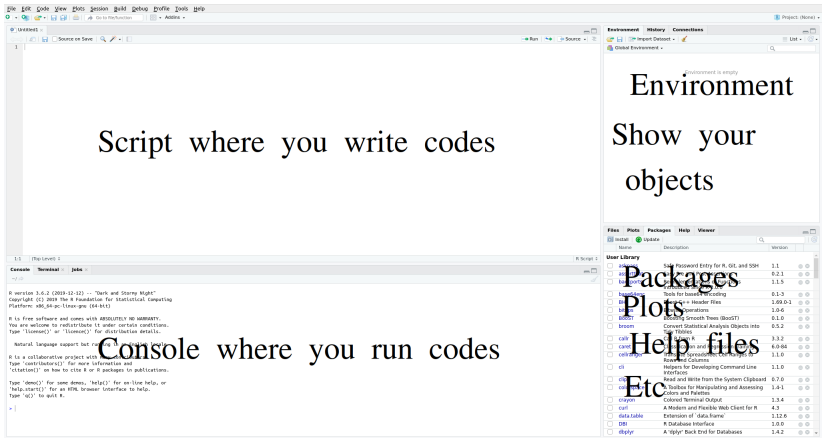


Figure 3: RStudio

RStudio

- ▶ You can run commands on the console. For example, try `1+1` and hit enter.
- ▶ You can write codes on the script and run in the console by selecting them and pressing `Ctrl + Enter` (`Cmd+Enter` for MAC). You can also run the selected codes by pressing the Run button on the top right corner of the script.
- ▶ Type `1+1` on the script and run it.

Creating Objects

- ▶ Objects are created with the symbols `=` or `<-`.
- ▶ creating an object is equivalent to storing the result.

```
x = 1
```

```
y <- 1
```

```
z = x + y
```


Creating Objects

- The objects you create will show in your environment.

Global Environment ▾		<input type="text"/>
Values		
x	1	
y	1	
z	2	

Objects

- ▶ You can see all your objects with:

```
ls()
```

```
## [1] "x" "y" "z"
```

- ▶ You can delete an object with:

```
rm(list = c("x", "y"))
```

- ▶ You can clear your environment with:

```
rm(list = ls())
```

Working Directory

- ▶ The working directory (WD) is the folder where R is working.
- ▶ If you read or write a file, the default location will be your WD.
- ▶ If you want to read/write from somewhere else you will have to tell R the path.
- ▶ Type the following command to find your WD:

```
getwd()
```

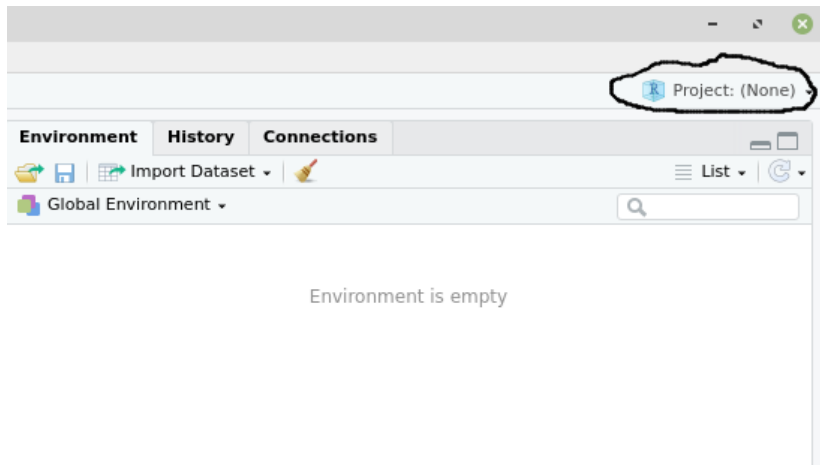
- ▶ You can change with the following command:

```
setwd("path")
```

- ▶ The path must be between quotation marks.

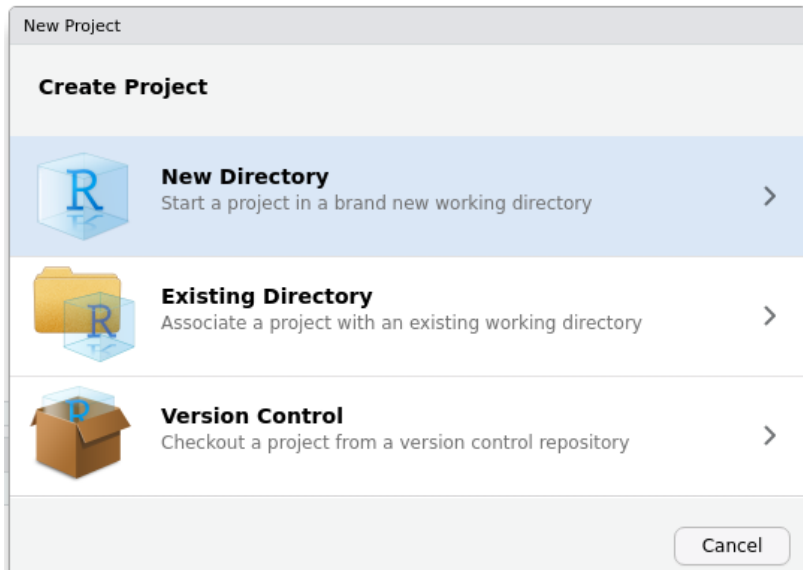
Projects

- ▶ Projects are a very good way to keep your work organized.
- ▶ Once you setup a project it will be linked to a folder, which will be the project WD.
- ▶ You can resume your work by just opening the project and everything will be ready for you.

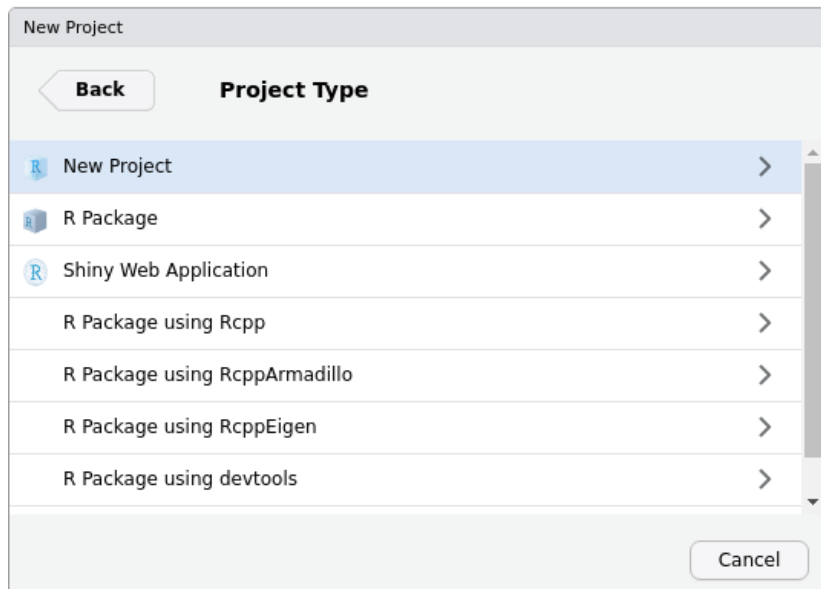


Projects

To start a project on a new folder open file > new project and follow these steps:



Projects




Projects

New Project

Back

Create New Project



Directory name:
project1

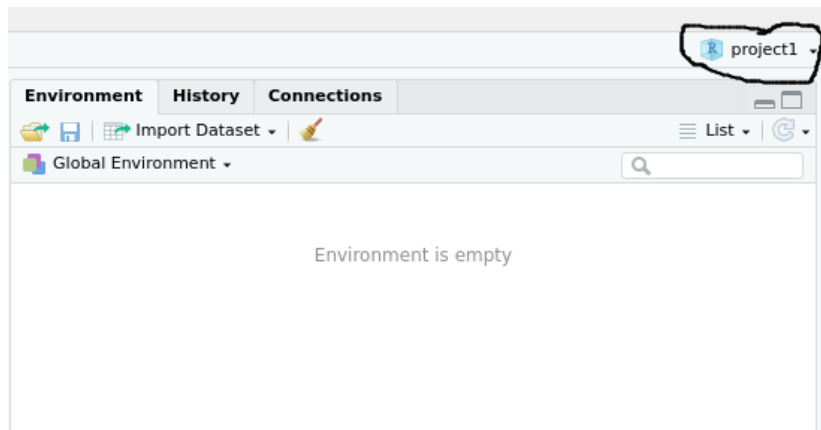
Create project as subdirectory of:
~ Browse...

☐ Open in new session

Create Project

Cancel

Projects



Projects

- ▶ The project will create a file `project1.Rproj` in the selected WD. You can use it to open your project.
- ▶ You can also switch between projects by clicking on `project1` like the last slide.

Basic Operations

```
# Addition
```

```
3 + 3
```

```
## [1] 6
```

```
# Subtraction
```

```
5 - 4
```

```
## [1] 1
```

```
# Multiplication
```

```
5 * 6
```

```
## [1] 30
```

```
# Division
```

```
10 / 5
```

```
## [1] 2
```

```
# Exponent
```

```
6 ^ 2
```

```
## [1] 36
```

Basic Operations

- ▶ The order of priority is Exponent > Division > Multiplication > Addition = Subtraction.
- ▶ This is the order R will use in an expression like this:

```
2 + 2 * 3 + 5 / 2 ^ 2
```

```
## [1] 9.25
```

- ▶ You can change the priority using parentheses.

```
# The first addition should go first and  
# the division should go before the exponent  
(2 + 2) * 3 + (5 / 2) ^ 2
```

```
## [1] 18.25
```

Logical/Misc Operators

Logical/Misc Operators	
==	equal to
!=	not equal equal to
<	less than
<=	less than or equal to
>	greater than
>=	greater than or equal to
	OR
&	AND
a%in%b	is 'a' contained in 'b'
%*%	matrix multiplication

Logical/Misc Operators

Some examples:

```
1 == 2
```

```
## [1] FALSE
```

```
1 != 2
```

```
## [1] TRUE
```

```
3<4
```

```
## [1] TRUE
```

We will deal more with this operators latter.

Data Types, Classes, and Structures

Up to this point, we have been working with numbers. There are actually six data types in R:

- ▶ Double: Numeric, real.
- ▶ Integer: Numeric, integer.
- ▶ Character: Name.
- ▶ logical: TRUE, FALSE
- ▶ complex: $a + bi$ - A complex number.
- ▶ raw: a byte.

We will work only with the first four types.

Data Types, Classes, and Structures

- ▶ You can ask R the type of the object:

```
typeof(2)
```

```
## [1] "double"
```

```
typeof(2L)
```

```
## [1] "integer"
```

```
typeof("a")
```

```
## [1] "character"
```

```
typeof(TRUE)
```

```
## [1] "logical"
```

Data Types, Classes, and Structures

- You can also ask R if an object is of a particular type:

```
is.double(2)
```

```
## [1] TRUE
```

```
is.integer(2L)
```

```
## [1] TRUE
```

```
is.character("a")
```

```
## [1] TRUE
```

```
is.logical(TRUE)
```

```
## [1] TRUE
```

```
is.character(2)
```

```
## [1] FALSE
```


Data Types, Classes, and Structures

- ▶ You can also for the class of the object.
- ▶ Class is not the same as type. For example, a matrix is a class and its type can be double, integer, character, etc. . .

```
# creates a 2x2 matrix of zeros.
```

```
A = matrix(0, 2, 2)
```

```
class(A)
```

```
## [1] "matrix"
```

```
typeof(A)
```

```
## [1] "double"
```

Data Types, Classes, and Structures

- ▶ A particular class we will be using is called factor.
- ▶ They are used for categorical variables.
- ▶ They only accept a particular set of values called levels.

```
fact = c("a", "b", "b")  
fact = as.factor(fact)
```

```
levels(fact)
```

```
## [1] "a" "b"
```

```
class(fact)
```

```
## [1] "factor"
```

Data Types, Classes, and Structures

- ▶ So far we mostly worked with objects of a single element.
- ▶ R have several data structures:

	Homogeneous	Heterogeneous
1-dimensional	vector	list
2-dimensional	matrix	data frame
more than 2 dimensions	array	

Data Types, Classes, and Structures

- Vectors are created with the following command:

```
vec1 = c(2,4,6,8,10)
```

```
is.vector(vec1) # is it a vector?
```

```
## [1] TRUE
```

```
is.double(vec1) # is it type double
```

```
## [1] TRUE
```

```
is.numeric(vec1) # is it class numeric?
```

```
## [1] TRUE
```

```
is.atomic(vec1) # is it homogeneous?
```

```
## [1] TRUE
```

Data Types, Classes, and Structures

- There are other ways to create vectors:

```
vec2 = seq(2,10,2) #seq(from,to,by)  
vec2
```

```
## [1] 2 4 6 8 10
```

```
vec3 = 1:10 #seq with by = 1  
vec3
```

```
## [1] 1 2 3 4 5 6 7 8 9 10
```

```
length(vec3)
```

```
## [1] 10
```

Data Types, Classes, and Structures

► Vector operations

```
vec4 = 1:5
```

```
vec5 = 6:10
```

```
vec4 + vec5
```

```
## [1] 7 9 11 13 15
```

```
vec4 - vec5
```

```
## [1] -5 -5 -5 -5 -5
```

```
vec4 * vec5 #inner product
```

```
## [1] 6 14 24 36 50
```

Data Types, Classes, and Structures

- ▶ We can transpose vectors with the `t` function.

```
t(vec4)
```

```
##      [,1] [,2] [,3] [,4] [,5]  
## [1,]    1    2    3    4    5
```

```
vec4 %*% t(vec5) #5X1 times 1X5
```

```
##      [,1] [,2] [,3] [,4] [,5]  
## [1,]    6    7    8    9   10  
## [2,]   12   14   16   18   20  
## [3,]   18   21   24   27   30  
## [4,]   24   28   32   36   40  
## [5,]   30   35   40   45   50
```

```
t(vec4) %*% vec5 #1X5 times 5X1
```

```
##      [,1]  
## [1,]  130
```

Data Types, Classes, and Structures

We can also do logical operations with vectors:

```
vec4 == vec5
```

```
## [1] FALSE FALSE FALSE FALSE FALSE
```

```
1 %in% vec4 #ask if 1 and 2 is in vec4
```

```
## [1] TRUE
```

```
vec5 > vec4
```

```
## [1] TRUE TRUE TRUE TRUE TRUE
```


Data Types, Classes, and Structures

```
A = matrix(c(1,2,3,4,5,6),nrow=2,byrow=TRUE)
```

```
A
```

```
##      [,1] [,2] [,3]
```

```
## [1,]    1    2    3
```

```
## [2,]    4    5    6
```

```
is.atomic(A)
```

```
## [1] TRUE
```

```
is.matrix(A)
```

```
## [1] TRUE
```

```
B = matrix(c(1,6,3,5,7,2),nrow=3)
```

```
B
```

```
##      [,1] [,2]
```

```
## [1,]    1    5
```

```
## [2,]    6    7
```

```
## [3,]    3    2
```

Data Types, Classes, and Structures

- Here are some other matrix operations:

```
dim(A) # Dimension
```

```
## [1] 2 3
```

```
nrow(A) # Number of rows
```

```
## [1] 2
```

```
ncol(A) # Number of columns
```

```
## [1] 3
```

```
C = A%*%B # Multiplication (vectors also work)
```

```
C
```

```
##      [,1] [,2]
```

```
## [1,]   22   25
```

```
## [2,]   52   67
```

```
A*A # Inner product
```

```
##      [,1] [,2] [,3]
```

```
## [1,]    1    4    9
```

```
## [2,]   16   25   36
```

Data Types, Classes, and Structures

► You can also try:

```
det(C) # Determinant
```

```
## [1] 174
```

```
eigen(C) # Eigenvalues/ vectors
```

```
## eigen() decomposition
```

```
## $values
```

```
## [1] 87 2
```

```
##
```

```
## $vectors
```

```
##           [,1]      [,2]
```

```
## [1,] -0.3589791 -0.7808688
```

```
## [2,] -0.9333456  0.6246950
```

```
solve(C) # Inverse
```

```
##           [,1]      [,2]
```

```
## [1,]  0.3850575 -0.1436782
```

```
## [2,] -0.2988506  0.1264368
```

Other Built-in Functions

- R has several built-in functions:

```
mean(vec1)
```

```
## [1] 6
```

```
sd(vec1) # standard deviation
```

```
## [1] 3.162278
```

```
sum(vec1)
```

```
## [1] 30
```

```
prod(vec1)
```

```
## [1] 3840
```

```
log(vec1) #ln
```

```
## [1] 0.6931472 1.3862944 1.7917595 2.0794415 2.3025851
```

```
exp(vec1) # Exponential
```

```
## [1] 7.389056 54.598150 403.428793 2980.957987 22026.465795
```

```
summary(log(vec1))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
```

```
## 0.6931  1.3863  1.7918  1.6506  2.0794  2.3026
```

Indexing

- ▶ You can access a particular element of a vector/matrix using indexes:

```
vec1[2] # second element of vec1
```

```
## [1] 4
```

```
vec1[1:3] # elements 1 2 and 3 of vec1
```

```
## [1] 2 4 6
```

```
C[2,1] # row 2 column 1 of matrix C
```

```
## [1] 52
```

```
which(vec1 > 3) #position of elements in vec1 bigger than 3
```

```
## [1] 2 3 4 5
```

```
vec1[which(vec1 > 3)] # elements in vec1 bigger than 3
```

```
## [1] 4 6 8 10
```

Creating and combining strings

The command **paste** is used to combine strings

```
names<-paste("samp",1:4,sep="")  
names
```

```
## [1] "samp1" "samp2" "samp3" "samp4"
```

```
namesWithSp<-paste("samp",1:4,sep=" ")  
namesWithSp
```

```
## [1] "samp 1" "samp 2" "samp 3" "samp 4"
```

```
namesByMach<-paste("samp",1:4,sep="Mach")  
namesByMach
```

```
## [1] "sampMach1" "sampMach2" "sampMach3" "sampMach4"
```

Getting Help

- ▶ You can see how a function works with `?functionName`.
- ▶ This will give you access to the function documentation, which is standardized for all R functions.
- ▶ If you get an error or warning message, take a deep breath, read the error or warning, and try to figure out your error.
- ▶ If all else fails, Google the error.

Working with data

- ▶ The most usual way to read data in R is from csv files with **read.csv**.
- ▶ However, R has tools to read many types of data like xlsx, dta, etc.
- ▶ R also has its own way of storing data (.rda and .RData files).
- ▶ This is how you can read a csv file:

```
data = read.csv("cars.csv", row.names = 1)
head(data) # shows the first 6 entries.
```

##	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
## Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
## Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
## Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
## Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
## Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
## Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

- ▶ The **rownames = 1** is to set the first column in the csv file as the names of the rows in the data. In this case we have the names of the cars.
- ▶ You can also write csv files.

```
write.csv(data, file = "path/name.csv")
```


Working with data

- ▶ The **summary** function gives descriptive information on the data. The statistics will be calculated for each individual column.

```
summary(data)
```

- ▶ The **View** function opens the data on a sheet similar to excel, but in read only mode.

```
View(data)
```

- ▶ You can open the same sheet by clicking at the data object in the environment (top right part of RStudio).

Data Frames

- ▶ When we loaded the data a new class of object was introduced:

```
class(data)
```

```
## [1] "data.frame"
```

- ▶ Data Frames are a special class made to deal with data. Each column is a variable and each row is an observation.
- ▶ It falls in the heterogeneous classes. Each column can have a different type.

Data Frames

- ▶ Variable names play an important role in Data Frames. You can get a particular variable with the `\textbf{$}` element.

```
head(data$cyl)
```

```
## [1] 6 6 4 6 8 6
```

```
mean(data$cyl)
```

```
## [1] 6.1875
```

- ▶ You can also use indexes like in a vector/matrix:

```
data[1,2]
```

```
## [1] 6
```

```
data$disp[3]
```

```
## [1] 108
```

Lists

- ▶ Lists are the most heterogeneous data in R. They can store anything.
- ▶ For example, you can have a list with a matrix, a vector, a string and even a function.

```
l1 = list(matrix = C, vector = vec1, string = "hello")
```

- ▶ Note that the elements in the list were named. This is a good practice that allow us to access the elements using their names.

```
l1$vector
```

```
## [1] 2 4 6 8 10
```

- ▶ You can also use numbers to index elements:

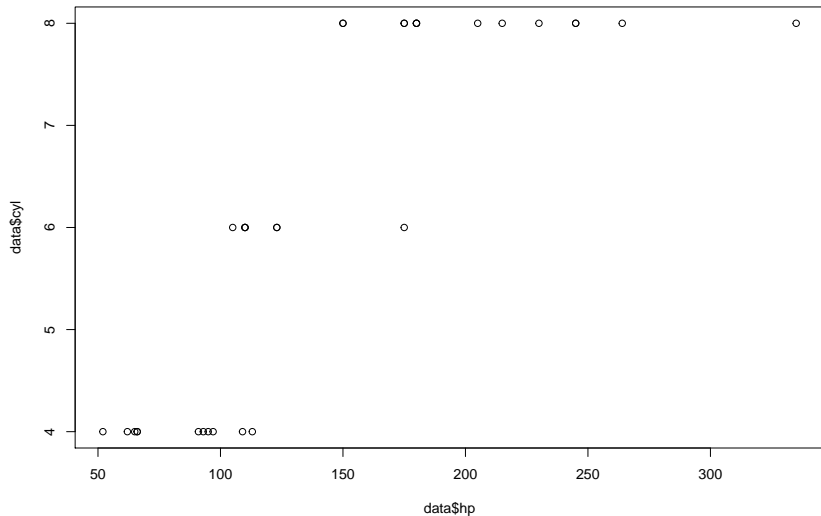
```
l1[[3]]
```

```
## [1] "hello"
```

- ▶ Data Frames can be seen as a special case of lists where each element is a vector of the same size.
- ▶ Lists may seem weird now, but later you will see that the output of many functions are lists.

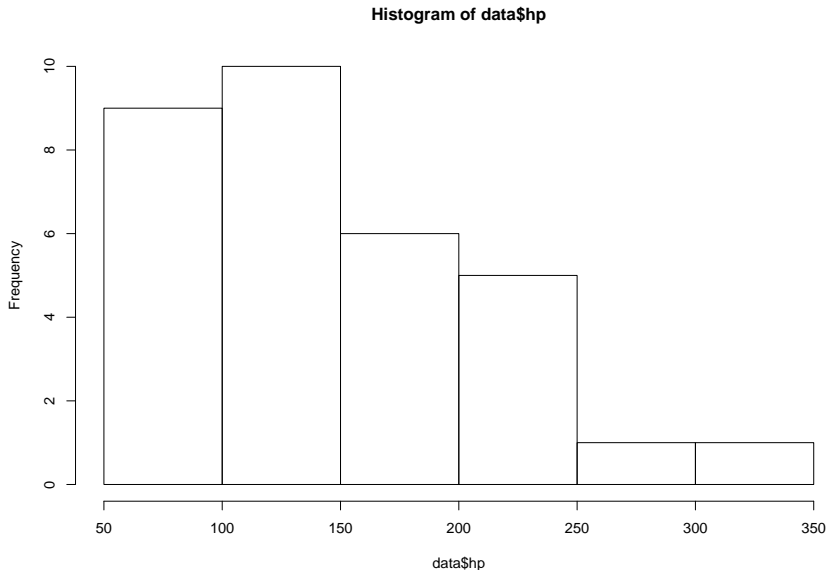
Basic Graphical Tools

```
plot(data$hp, data$cyl)
```



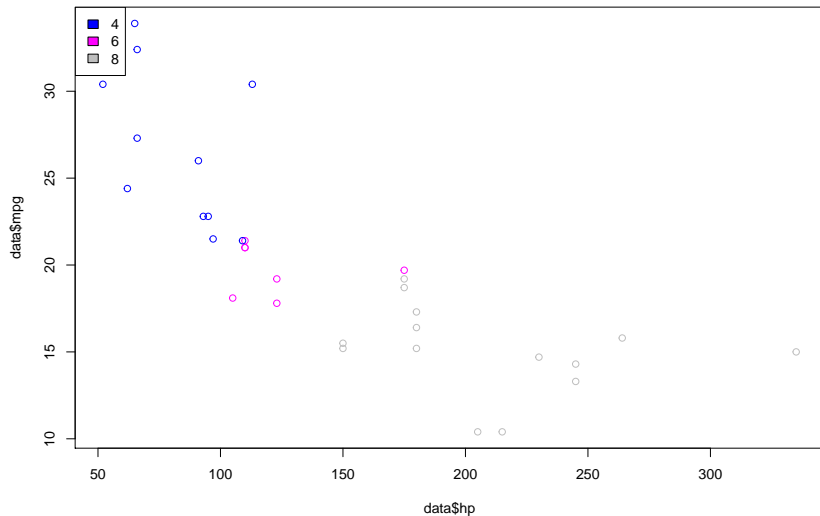
Basic Graphical Tools

```
hist(data$hp)
```



Basic Graphical Tools

```
plot(data$hp,data$mpg, col=data$cyl)  
legend("topleft",fill=c(4,6,8),legend=levels(as.factor(data$cyl)))
```



Loop, If and Else

- ▶ Loops are codes for iterative processes. The most basic way to do is using the **for** interface.
- ▶ A good example of something that can only be made in a loop is to create data that follow a random walk.
- ▶ A random walk is when we have

$$y_{t+1} = y_t + \varepsilon_{t+1}$$

where ε_t is an error term that we will sample from a normal distribution with mean 0 and variance 1.

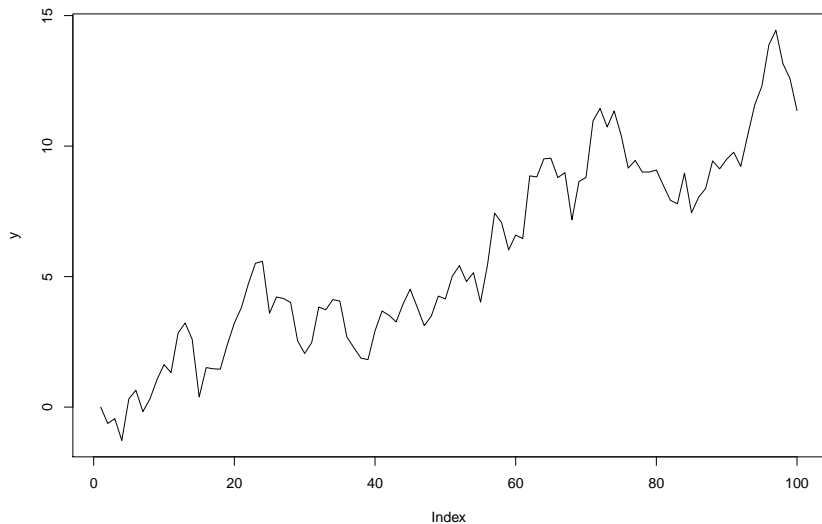
```
N = 100
y = rep(NA, N)
y[1] = 0

set.seed(1)
for(i in 1:(N-1)){
  y[i+1] = y[i] + rnorm(1)
}
head(y)
```

```
## [1] 0.0000000 -0.6264538 -0.4428105 -1.2784391 0.3168417 0.6463495
```


Loop, If and Else

```
plot(y,type = "l")
```



Loop, If and Else

- ▶ You can also write loops that will only stop when a criterion is met with the **while** interface.
- ▶ In the example below we will keep adding a random number between 0 and 1 to x until x becomes bigger or equal to 100.
- ▶ Then we will evaluate the variable i , which tells us how many iterations were needed.

```
x = 0
i = 0
set.seed(1)
while(x < 100){
  x = x + runif(1)
  i = i+1
}
i
```

```
## [1] 194
```

Loop, If and Else

- ▶ When one iteration depends on the previous it is hard to escape from the **for** loop.
- ▶ However, when the iterations are independent, we can use a set of functions from the **apply** family.
- ▶ For example, suppose we want to calculate the standard deviation of all columns in the car dataset;

```
# apply(date, dim, function, ... extra arguments)  
apply(data,2,sd)
```

##	mpg	cyl	disp	hp	drat	wt
##	6.0269481	1.7859216	123.9386938	68.5628685	0.5346787	0.9784574
##	qsec	vs	am	gear	carb	
##	1.7869432	0.5040161	0.4989909	0.7378041	1.6152000	

Loop, If and Else

- ▶ If we are dealing with lists we can use the **lapply** function:

```
# A list with the tree matrices we created  
matlist = list(A = A, B = B, C = C)  
# This will get the first line of all matrices  
lapply(matlist, function(x) x[1,] )
```

```
## $A  
## [1] 1 2 3  
##  
## $B  
## [1] 1 5  
##  
## $C  
## [1] 22 25
```

- ▶ The function **sapply** is for the cases where the input is a list and the output is a single element:

```
sapply(matlist, sum)
```

```
##   A   B   C  
## 21  24 166
```

- ▶ **sapply** also works for vectors.

Loop, If and Else

- ▶ **if** statements are used when we want R to do something once a certain condition is met.
- ▶ The syntax is **if (condition) {what to do}**
- ▶ We can use **if** to do the something we did in the **while** code, but with a for:

```
x = 0
set.seed(1)
for(i in 1:100000){
  x = x + runif(1)
  if(x>100){
    break
  }
}
```

```
## [1] 194
```

User Defined Functions (UDF)

A UDF is a function created by the user. It follows the structure

functionName = function(arguments)calculation

For Example:

```
subtract<-function(x,y){  
  result = x-y  
  return(result)  
}
```

- ▶ A function always stops when it reaches the **return** command.

User Defined Functions (UDF)

- ▶ Now we can use our function:

```
subtract(10,5)
```

```
## [1] 5
```

```
subtract(x = 20, y = 10)
```

```
## [1] 10
```

User Defined Functions (UDF)

- ▶ Now let's write a more interesting function.
- ▶ This function will create data from autoregressive models like:

$$y_{t+1} = \rho y_t + \varepsilon_{t+1}$$

- where $0 \leq \rho \leq 1$

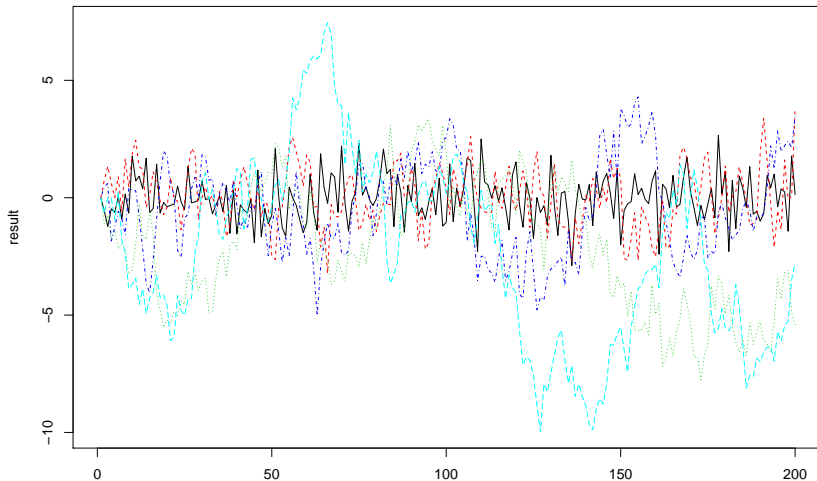
```
AR = function(N, rho){  
  y = rep(NA, N)  
  y[1] = 0  
  for(i in 1:(N-1)){  
    y[i+1] = rho*y[i] + rnorm(1)  
  }  
  return(y)  
}
```

- ▶ We can use our UDF in an lapply with several values of ρ

```
rho_vector = c(0, 0.5, 0.95, 0.99, 1)  
result = sapply(rho_vector, function(x) AR(200,x))
```


User Defined Functions (UDF)

```
matplot(result, type = "l")
```



Generation of Random Numbers

- ▶ Several codes so far had a **set.seed** before the functions we were running.
- ▶ This seed is used when we are dealing with random numbers. It allows some other user to replicate the exact same experiment.
- ▶ R has build in functions to generate data from several distributions like normal, uniform, Student t, etc.

```
# 10 numbers from a normal distribution with mean 0 and sd 1  
rnorm(10,0,1)
```

```
## [1] -0.337691156 -0.009148952 -0.125309208 -2.090846097 1.697393895  
## [6] 1.063881154 -0.766616636 0.382007559 0.241895904 -1.132759411
```

```
# 10 numbers from a uniform distribution with min 0 max 1  
runif(10,-1,1)
```

```
## [1] 0.863751416 -0.116035867 -0.196056773 -0.268954999 0.145659955  
## [6] -0.787665198 0.314427664 0.600958316 -0.679837722 -0.001700387
```

```
# 10 numbers from a t distribution 5 degrees of freedom  
rt(10,5)
```

```
## [1] -1.2347379 -0.4441516 -2.9575715 -1.2655547 -1.0570831 -0.6317558  
## [7] -2.1978297 -0.4304928 -1.5235338 -2.8844815
```

Generation of Random Numbers

- ▶ Note that all functions that we just used started with a **r**, which comes from random.
- ▶ If you use a **d** you will calculate the density, **p** for the distribution and **q** for the quantile.

Packages

- ▶ Registered packages are stored in the Comprehensive R Archive Network (CRAN).
- ▶ These packages can be installed with `install.packages("pkg name")`
- ▶ And they can be loaded with **`library(pkg name)`**.
- ▶ Once you load the package you have access to all its functions, data and documentation.

```
install.packages("glmnet")  
library(glmnet)  
?glmnet
```

The Tidyverse

- ▶ The Tidyverse is a set of dozens of packages, all compatible with each other, made for data treatment and analysis.
- ▶ It is the state of the art for data treatment. You can install and load it with:

```
install.packages("tidyverse")  
library(tidyverse)
```

The Tidyverse

- ▶ One of the most important features of Tidyverse is the pipe operator `%>%`.
- ▶ Consider the code:

```
log(exp(sin(2^2)))
```

```
## [1] -0.7568025
```

- ▶ Once we keep using functions inside function the code may look very confusing and it is very easy to get lost in the parentheses. The same results can be obtained with the pipe operator:

```
2^2 %>% sin() %>% exp() %>% log()
```

```
## [1] -0.7568025
```

The Tidyverse

- ▶ The pipe operator can be used in the same way with data frames. It is very fast and you can perform a lot of operations in a single step.
- ▶ For example, we can select columns:

```
df1 = data %>% select(cyl,mpg,hp)  
head(df1)
```

##	cyl	mpg	hp
## Mazda RX4	6	21.0	110
## Mazda RX4 Wag	6	21.0	110
## Datsun 710	4	22.8	93
## Hornet 4 Drive	6	21.4	110
## Hornet Sportabout	8	18.7	175
## Valiant	6	18.1	105

The Tidyverse

- ▶ We can filter the data with some criterion:

```
df2 = df1 %>% filter(cyl == 4)
head(df2)
```

```
##   cyl  mpg  hp
## 1   4 22.8  93
## 2   4 24.4  62
## 3   4 22.8  95
## 4   4 32.4  66
## 5   4 30.4  52
## 6   4 33.9  65
```


The Tidyverse

- It is also possible to create new variables:

```
df3 = df2 %>% mutate(newvar = hp/mpg)
head(df3)
```

```
##   cyl  mpg  hp  newvar
## 1    4 22.8 93 4.078947
## 2    4 24.4 62 2.540984
## 3    4 22.8 95 4.166667
## 4    4 32.4 66 2.037037
## 5    4 30.4 52 1.710526
## 6    4 33.9 65 1.917404
```

Tidyverse

- ▶ The `group_by` and the **summarise** functions together are used to group the observations by some variable.

```
df4 = df1 %>% group_by(cyl) %>%  
  summarise(hp = mean(hp), mpg = mean(mpg))  
head(df4)
```

```
## # A tibble: 3 x 3  
##   cyl    hp  mpg  
##   <int> <dbl> <dbl>  
## 1     4  82.6  26.7  
## 2     6 122.   19.7  
## 3     8 209.   15.1
```

Tidyverse

- ▶ Finally, we can combine as many operations as we want in one chain of pipe codes.

```
df5 = data %>% select(mpg, hp, cyl) %>%  
  filter(cyl > 4) %>%  
  group_by(cyl) %>%  
  summarise(hp = mean(hp), mpg = mean(mpg)) %>%  
  mutate(newvar = hp/mpg)
```

df5

```
## # A tibble: 2 x 4  
##   cyl    hp    mpg newvar  
##   <int> <dbl> <dbl> <dbl>  
## 1     6  122.  19.7   6.19  
## 2     8  209.  15.1  13.9
```