

pyResToolbox MCP

Formula Reference Guide

Quick Reference for Reservoir Engineering Calculations

Gabriel Serrão Seabra

November 21, 2025

Version 1.0

Abstract

This document provides a comprehensive reference guide for calculation tools available in the pyResToolbox Model Context Protocol (MCP) server. It includes mathematical formulas, symbol definitions, unit systems, and correlation methods covering oil PVT, gas PVT, well performance, simulation support, brine properties, and reservoir heterogeneity analysis. All calculations use Field Units (US Oilfield standard).

Note: This guide covers the core petroleum engineering calculations. The MCP server also includes geomechanics and wellbore stability tools which are documented separately.

Based on pyResToolbox: This MCP server is built upon the excellent *pyResToolbox* library by Mark W. Burgoyne (<https://github.com/mwburgoyne/pyResToolbox>), which provides the underlying reservoir engineering calculations.

DISCLAIMER

No Warranty: This document and the formulas contained herein are provided "AS IS" without warranty of any kind, either expressed or implied. While every effort has been made to ensure accuracy, all formulas, correlations, and calculations **MUST be independently verified** before use in any engineering or commercial application.

Professional Responsibility: Users are solely responsible for verifying the correctness and applicability of any formula, correlation, or calculation method presented in this document. The author and contributors assume no liability for errors, omissions, or consequences arising from the use of this information.

Verification Required: Always cross-reference formulas with original published sources and validate results against known benchmarks before applying to real reservoir engineering problems.

Contents

1 Introduction	5
1.1 About pyResToolbox MCP	5
1.2 Unit System	5
2 Oil PVT Calculations	6
2.1 Bubble Point Pressure (P_b)	6
2.1.1 Description	6
2.1.2 Available Methods	6
2.2 Solution Gas-Oil Ratio (R_s)	7
2.2.1 Description	7
2.2.2 Standing (1947) Method	7
2.3 Oil Formation Volume Factor (B_o)	7
2.3.1 Description	7
2.3.2 McCain et al. (1988) Method - MCAIN	7
2.3.3 Standing (1947) Method - STAN	8
2.4 Oil Viscosity (μ_o)	8
2.4.1 Description	8
2.4.2 Beggs-Robinson (1975) Method - BR	8
2.5 Oil Density (ρ_o)	9
2.5.1 Description	9
2.5.2 Standing-White-McCain-Hill (1995) Method - SWMH	9
2.6 Oil Compressibility (c_o)	9
2.6.1 Description	9
2.6.2 Explicit Method - EXPLT	9
2.7 API-Specific Gravity Conversions	10
2.7.1 API to Specific Gravity	10
2.7.2 Specific Gravity to API	10
3 Gas PVT Calculations	11
3.1 Gas Z-Factor (Compressibility Factor)	11
3.1.1 Description	11
3.1.2 Dranchuk-Abou-Kassem (1975) Method - DAK	11
3.1.3 Hall-Yarborough (1973) Method - HY	11
3.1.4 Burgoyne-Nielsen-Stanko (2025) Method - BUR	12
3.2 Pseudo-Critical Properties	12
3.2.1 Description	12
3.2.2 Piper-McCain-Corredor (1993) Method - PMC	12
3.3 Gas Formation Volume Factor (B_g)	13
3.3.1 Description	13
3.4 Gas Viscosity (μ_g)	13
3.4.1 Description	13
3.4.2 Lee-Gonzalez-Eakin (1966) Method - LGE	13
3.5 Gas Density (ρ_g)	14
3.6 Gas Compressibility (c_g)	14
3.6.1 Description	14
3.7 Gas Pseudopressure (ψ)	14
3.7.1 Description	14

4 Inflow Performance (Well Productivity)	16
4.1 Oil Rate - Radial Flow (Vertical Wells)	16
4.1.1 Description	16
4.1.2 Darcy's Law - Undersaturated Reservoir	16
4.1.3 Vogel IPR - Saturated Reservoir	16
4.2 Oil Rate - Linear Flow (Horizontal Wells)	17
4.2.1 Description	17
4.3 Gas Rate - Radial Flow (Vertical Wells)	17
4.3.1 Description	17
4.4 Gas Rate - Linear Flow (Horizontal Wells)	17
5 Relative Permeability Correlations	18
5.1 Corey (1954) Correlation	18
5.1.1 Description	18
5.1.2 Normalized Saturation	18
5.1.3 Relative Permeability Functions	19
5.2 LET Correlation (Lomeland-Ebeltoft-Thomas 2005)	19
5.2.1 Description	19
6 Brine Properties	21
6.1 CH ₄ -Saturated Brine Properties	21
6.1.1 Description	21
6.1.2 Calculation Method	21
6.2 CO ₂ -Saturated Brine Properties	22
6.2.1 Description	22
6.2.2 Calculation Methods	22
6.3 Key Differences Between CH ₄ and CO ₂ Methods	23
7 Reservoir Heterogeneity	24
7.1 Lorenz Coefficient	24
7.1.1 Description	24
7.2 Dykstra-Parsons Coefficient (β)	24
7.2.1 Description	24
7.3 Lorenz-Beta Conversion	25
7.3.1 Description	25
7.4 Layer Permeability Distribution	25
7.4.1 Description	25
8 Aquifer Models	27
8.1 Van Everdingen-Hurst Model	27
8.1.1 Description	27
8.2 Influence Functions	27
9 Phase Behavior	28
9.1 Rachford-Rice Flash Calculation	28
9.1.1 Description	28

10 Quick Reference Tables	29
10.1 Physical Constants	29
10.2 Conversion Factors	29
10.3 Typical Property Ranges	29
11 Correlation Method Recommendations	30
11.1 Oil PVT Methods	30
11.2 Gas PVT Methods	30
11.3 Relative Permeability Methods	30
12 Tool Categories and Count	31
12.1 Available Tools by Category	31
12.2 Key Features	31
13 Usage Notes and Best Practices	32
13.1 General Guidelines	32
13.1.1 Input Validation	32
13.1.2 Method Selection	32
13.1.3 Common Mistakes to Avoid	33
13.2 PVT Calculation Workflow	33
13.2.1 Typical Oil PVT Workflow	33
13.2.2 Typical Gas PVT Workflow	33
13.3 Well Performance Analysis	34
13.3.1 IPR Generation Steps	34
13.4 Simulation Preparation	34
13.4.1 Relative Permeability Table Generation	34
14 References	35
14.1 Key Publications	35
14.1.1 Oil PVT Correlations	35
14.1.2 Gas PVT Correlations	35
14.1.3 Well Performance	35
14.1.4 Relative Permeability	36
14.1.5 Brine Properties	36
14.1.6 Reservoir Heterogeneity	36
14.2 Software and Tools	36
15 Appendix: Nomenclature	37
15.1 Primary Symbols	37
15.2 Subscripts	38
16 Index of Formulas	39
16.1 By Property	39

1 Introduction

1.1 About pyResToolbox MCP

pyResToolbox MCP is a Model Context Protocol server that provides AI agents like Claude with access to industry-standard petroleum engineering calculations. Built on the comprehensive pyResToolbox library by Mark Burgoyne, it offers 47 production-ready tools for reservoir engineering workflows.

1.2 Unit System

All calculations use Field Units (US Oilfield standard):

Property	Unit	Example
Pressure	psia	3000 psia
Temperature	°F	180°F
Permeability	mD	100 mD
Pay Thickness	ft	50 ft
Viscosity	cP	0.85 cP
Oil Rate	STB/day	542 STB/day
Gas Rate	MSCF/day	1250 MSCF/day
Oil Gravity	API° or SG	35° API
Gas Gravity	SG (air=1)	0.75
Solution GOR	scf/stb	800 scf/stb
Oil FVF	rb/stb	1.25 rb/stb
Gas FVF	rcf/scf	0.0045 rcf/scf
Compressibility	1/psi	1.2×10^{-5} 1/psi
Density	lb/ft³	42.5 lb/ft³

Table 1: Field Units Used in All Calculations

2 Oil PVT Calculations

2.1 Bubble Point Pressure (P_b)

2.1.1 Description

The bubble point pressure is the pressure at which the first gas bubble evolves from solution oil. It is a critical PVT property that defines the boundary between undersaturated (single-phase) and saturated (two-phase) oil behavior.

2.1.2 Available Methods

1. Standing (1947) - STAN

$$P_b = 18.2 \left[\left(\frac{R_{sb}}{\gamma_g} \right)^{0.83} \times 10^a - 1.4 \right] \quad (1)$$

where:

$$a = 0.00091 \times T - 0.0125 \times \text{API} \quad (2)$$

Symbol Definitions:

- P_b : Bubble point pressure (psia)
- R_{sb} : Solution GOR at bubble point (scf/stb)
- γ_g : Gas specific gravity (air=1)
- T : Temperature (°F)
- **API**: Oil API gravity (degrees)

2. Valko-McCain (2003) - VALMC

$$P_b = -C_1 - C_2 \times A - C_3 \times A^2 \quad (3)$$

where:

$$A = \ln \left(\frac{R_{sb}}{\gamma_{sp}} \right) \quad (4)$$

$$C_1 = 7.916 \times 10^{-4} \times \text{API}^{1.541} - 4.561 \times 10^{-5} \times T^{1.3911} \quad (5)$$

$$C_2 = 4.51 \times 10^{-5} \times \text{API}^{1.3911} \times T^{0.7849} \quad (6)$$

$$C_3 = 1.046 \times 10^{-7} \times \text{API}^{1.3911} \times T^{1.007} \quad (7)$$

Additional Symbols:

- γ_{sp} : Separator gas specific gravity (air=1)
- C_1, C_2, C_3 : Correlation coefficients

3. Velarde-Blasingame-McCain (1997) - VELAR

$$\ln(P_b) = a_0 + a_1 \ln(R_{sb}) + a_2 [\ln(R_{sb})]^2 + a_3 \ln(\gamma_{sp}) + a_4 \ln(\text{API}) + a_5 \ln(T + 460) \quad (8)$$

Coefficients a_0 through a_5 are empirically determined from regression analysis.

2.2 Solution Gas-Oil Ratio (R_s)

2.2.1 Description

The solution gas-oil ratio represents the volume of gas dissolved in oil at a given pressure and temperature. It increases with pressure up to the bubble point, then remains constant (equal to R_{sb}) above the bubble point.

2.2.2 Standing (1947) Method

$$R_s = \gamma_g \left[\frac{P}{18.2} + 1.4 \right] \times 10^{-a} \quad (9)$$

where:

$$a = 0.0125 \times \text{API} - 0.00091 \times T \quad (10)$$

Symbol Definitions:

- R_s : Solution gas-oil ratio (scf/stb)
- P : Pressure (psia)
- γ_g : Gas specific gravity (air=1)
- **API**: Oil API gravity (degrees)
- T : Temperature (°F)

Behavior:

- $P < P_b$: R_s calculated from correlation
- $P \geq P_b$: $R_s = R_{sb}$ (constant)

2.3 Oil Formation Volume Factor (B_o)

2.3.1 Description

The formation volume factor relates the volume of oil at reservoir conditions to its volume at stock tank conditions. It accounts for thermal expansion and gas in solution.

2.3.2 McCain et al. (1988) Method - MCAIN

$$B_o = \frac{\rho_{ST}}{\rho_o} \quad (11)$$

where oil density ρ_o is calculated from:

$$\rho_o = \frac{\rho_{ST} + 0.01357 \times R_s \times \gamma_g}{0.972 + 0.000147 \times (T + 460)^{1.175}} \quad (12)$$

Symbol Definitions:

- B_o : Oil formation volume factor (rb/stb)
- ρ_o : Oil density at reservoir conditions (lb/ft³)
- ρ_{ST} : Oil density at stock tank conditions (lb/ft³)
- R_s : Solution gas-oil ratio (scf/stb)
- γ_g : Gas specific gravity (air=1)
- T : Temperature (°F)

2.3.3 Standing (1947) Method - STAN

$$B_o = 0.9759 + 0.00012 \left[R_s \sqrt{\frac{\gamma_g}{\gamma_o}} + 1.25T \right]^{1.2} \quad (13)$$

Additional Symbols:

- γ_o : Oil specific gravity (water=1)

2.4 Oil Viscosity (μ_o)**2.4.1 Description**

Oil viscosity is a measure of the oil's resistance to flow. It decreases with increasing temperature and decreasing pressure (above bubble point).

2.4.2 Beggs-Robinson (1975) Method - BR**Dead Oil Viscosity:**

$$\mu_{od} = 10^X - 1 \quad (14)$$

where:

$$X = T^{-1.163} \times \exp(6.9824 - 0.04658 \times \text{API}) \quad (15)$$

Live Oil Viscosity:

$$\mu_o = A \times \mu_{od}^B \quad (16)$$

where:

$$A = 10.715 \times (R_s + 100)^{-0.515} \quad (17)$$

$$B = 5.44 \times (R_s + 150)^{-0.338} \quad (18)$$

Symbol Definitions:

- μ_o : Oil viscosity (cP)
- μ_{od} : Dead oil viscosity (cP)

- R_s : Solution gas-oil ratio (scf/stb)
- T : Temperature (°F)
- **API**: Oil API gravity (degrees)

2.5 Oil Density (ρ_o)

2.5.1 Description

Oil density at reservoir conditions accounts for dissolved gas and thermal expansion effects.

2.5.2 Standing-White-McCain-Hill (1995) Method - SWMH

$$\rho_o = \frac{62.4 \times \gamma_o + 0.0136 \times R_s \times \gamma_g}{B_o} \quad (19)$$

Symbol Definitions:

- ρ_o : Oil density (lb/ft³)
- γ_o : Oil specific gravity (water=1)
- γ_g : Gas specific gravity (air=1)
- R_s : Solution gas-oil ratio (scf/stb)
- B_o : Oil formation volume factor (rb/stb)

2.6 Oil Compressibility (c_o)

2.6.1 Description

Oil compressibility describes the fractional change in oil volume with pressure. It is critical for material balance calculations and pressure transient analysis.

2.6.2 Explicit Method - EXPLT

$$c_o = -\frac{1}{B_o} \left[\frac{dB_o}{dP} - \frac{B_g}{5.61458} \frac{dR_s}{dP} \right] \quad (20)$$

Symbol Definitions:

- c_o : Oil compressibility (1/psi)
- B_o : Oil formation volume factor (rb/stb)
- B_g : Gas formation volume factor (rcf/scf)
- R_s : Solution gas-oil ratio (scf/stb)
- P : Pressure (psia)

Note: Derivatives are calculated numerically using finite differences.

2.7 API-Specific Gravity Conversions

2.7.1 API to Specific Gravity

$$\gamma_o = \frac{141.5}{\text{API} + 131.5} \quad (21)$$

2.7.2 Specific Gravity to API

$$\text{API} = \frac{141.5}{\gamma_o} - 131.5 \quad (22)$$

Symbol Definitions:

- **API:** Oil API gravity (degrees)
- γ_o : Oil specific gravity (water=1)

3 Gas PVT Calculations

3.1 Gas Z-Factor (Compressibility Factor)

3.1.1 Description

The Z-factor corrects the ideal gas law for real gas behavior. It is defined as:

$$PV = ZnRT \quad (23)$$

where $Z = 1$ for ideal gas, $Z < 1$ for most real gases at reservoir conditions.

3.1.2 Dranchuk-Abou-Kassem (1975) Method - DAK

$$Z = 1 + A_1\rho_r + A_2\rho_r^2 + A_3\rho_r^5 + A_4\rho_r^2(1 + A_5\rho_r^2) \exp(-A_5\rho_r^2) \quad (24)$$

$$+ \frac{A_6 + A_7\rho_r + A_8\rho_r^2}{T_{pr}^3} \rho_r^2 \quad (25)$$

where reduced density ρ_r is solved iteratively from:

$$\rho_r = \frac{0.27P_{pr}}{ZT_{pr}} \quad (26)$$

Symbol Definitions:

- Z : Gas compressibility factor (dimensionless)
- ρ_r : Reduced density (dimensionless)
- P_{pr} : Pseudo-reduced pressure = P/P_c
- T_{pr} : Pseudo-reduced temperature = $(T + 460)/T_c$
- P_c : Pseudo-critical pressure (psia)
- T_c : Pseudo-critical temperature (°R)
- A_1, \dots, A_8 : Correlation constants

3.1.3 Hall-Yarborough (1973) Method - HY

Implicit equation for reduced density y :

$$\frac{0.06125P_{pr}t}{y} \exp[-1.2(1-t)^2] = F(y) \quad (27)$$

where:

$$F(y) = -A_1 + A_2/y + A_3/y^2 + A_4/y^3 + A_5/y^4 + y^2(A_6 + A_7/y + A_8/y^2)(1 + A_9y^2) \exp(-A_9y^2) \quad (28)$$

Then Z is calculated from:

$$Z = \frac{A_1P_{pr}}{y} \quad (29)$$

where $t = 1/T_{pr}$ and A_1, \dots, A_9 are correlation constants.

3.1.4 Burgoyne-Nielsen-Stanko (2025) Method - BUR

Universal EOS-based correlation for Z-factor, viscosity, and enthalpy. This is the most recent and comprehensive method, developed specifically to handle:

- High concentrations of non-hydrocarbons (CO₂, H₂S, N₂, H₂)
- Pure non-hydrocarbon streams (100% CO₂, 100% H₂)
- Hydrogen mixtures (only method supporting H₂)
- Full range of P-T conditions

Based on a tuned 5-component Peng-Robinson EOS model. The formulation methods are fully described in SPE-229932-MS, and the tuning datasets are available on GitHub. The correlation is implemented in pyResToolbox and provides open access to these calculations.

Reference: SPE-229932-MS (ADIPEC 2025) - Burgoyne, M.W. (Santos Ltd), Nielsen, M.H., and Stanko, M. (Whitson AS)

Method Comparison:

- **DAK:** Best for hydrocarbon gases with low non-HC content
- **HY:** Fast alternative for standard conditions
- **WYW:** Modern update to classical methods
- **BUR:** Universal method, required for H₂ or high non-HC content

3.2 Pseudo-Critical Properties

3.2.1 Description

Pseudo-critical properties (temperature and pressure) are used to calculate the reduced properties needed for Z-factor correlations.

3.2.2 Piper-McCain-Corredor (1993) Method - PMC

Without Non-Hydrocarbons:

$$T_c = 169.2 + 349.5\gamma_g - 74.0\gamma_g^2 \quad (\text{°R}) \quad (30)$$

$$P_c = 756.8 - 131.07\gamma_g - 3.6\gamma_g^2 \quad (\text{psia}) \quad (31)$$

With CO₂ and H₂S Corrections:

$$T'_c = T_c - \epsilon_T \quad (32)$$

$$P'_c = \frac{P_c - \epsilon_P}{1 - y_{CO_2} - y_{H_2S}} \quad (33)$$

where:

$$\epsilon_T = 120(y_{CO_2}^{0.9} - y_{CO_2}^{1.6}) + 15(y_{H_2S}^{0.5} - y_{H_2S}^4) \quad (34)$$

$$\epsilon_P = 440y_{CO_2} + 600y_{H_2S} - 170y_{CO_2}y_{H_2S} \quad (35)$$

Symbol Definitions:

- T_c : Pseudo-critical temperature (°R)
- P_c : Pseudo-critical pressure (psia)
- γ_g : Gas specific gravity (air=1)
- y_{CO_2} : CO₂ mole fraction
- y_{H_2S} : H₂S mole fraction
- y_{N_2} : N₂ mole fraction
- ϵ_T : Temperature correction
- ϵ_P : Pressure correction

3.3 Gas Formation Volume Factor (B_g)**3.3.1 Description**

The gas formation volume factor relates gas volume at reservoir conditions to its volume at standard conditions.

$$B_g = \frac{0.02827 \times Z \times (T + 460)}{P} \quad (\text{rcf/scf}) \quad (36)$$

Symbol Definitions:

- B_g : Gas formation volume factor (rcf/scf)
- Z : Gas compressibility factor (dimensionless)
- T : Temperature (°F)
- P : Pressure (psia)

3.4 Gas Viscosity (μ_g)**3.4.1 Description**

Gas viscosity is required for flow calculations and increases with pressure and temperature.

3.4.2 Lee-Gonzalez-Eakin (1966) Method - LGE

$$\mu_g = K \times 10^{-4} \times \exp [X \times \rho_g^Y] \quad (37)$$

where:

$$K = \frac{(9.4 + 0.02M)(T + 460)^{1.5}}{209 + 19M + (T + 460)} \quad (38)$$

$$X = 3.5 + \frac{986}{T + 460} + 0.01M \quad (39)$$

$$Y = 2.4 - 0.2X \quad (40)$$

$$M = 28.97\gamma_g \quad (41)$$

Symbol Definitions:

- μ_g : Gas viscosity (cP)
- ρ_g : Gas density (lb/ft³)
- M : Gas molecular weight (lb/lbmol)
- γ_g : Gas specific gravity (air=1)
- T : Temperature (°F)

3.5 Gas Density (ρ_g)

$$\rho_g = \frac{P \times 28.97 \times \gamma_g}{Z \times 10.73 \times (T + 460)} \quad (\text{lb/ft}^3) \quad (42)$$

Symbol Definitions:

- ρ_g : Gas density (lb/ft³)
- P : Pressure (psia)
- γ_g : Gas specific gravity (air=1)
- Z : Gas compressibility factor (dimensionless)
- T : Temperature (°F)

3.6 Gas Compressibility (c_g)**3.6.1 Description**

Gas compressibility describes the fractional change in gas volume with pressure.

$$c_g = \frac{1}{P} - \frac{1}{Z} \frac{dZ}{dP} \quad (43)$$

Symbol Definitions:

- c_g : Gas compressibility (1/psi)
- P : Pressure (psia)
- Z : Gas compressibility factor (dimensionless)

3.7 Gas Pseudopressure (ψ)**3.7.1 Description**

Gas pseudopressure (or real gas potential) linearizes the gas diffusivity equation and is used in well test analysis.

$$\psi(P) = 2 \int_{P_0}^P \frac{P'}{\mu_g Z} dP' \quad (\text{psia}^2/\text{cP}) \quad (44)$$

Symbol Definitions:

- ψ : Gas pseudopressure (psia^2/cP)
- P : Pressure (psia)
- P_0 : Reference pressure (usually 0 psia)
- μ_g : Gas viscosity (cP)
- Z : Gas compressibility factor (dimensionless)

Note: Integral is evaluated numerically using trapezoidal rule or Simpson's rule.

4 Inflow Performance (Well Productivity)

4.1 Oil Rate - Radial Flow (Vertical Wells)

4.1.1 Description

Calculates oil production rate for vertical wells with radial flow geometry using Darcy's law for steady-state or pseudo-steady-state flow.

4.1.2 Darcy's Law - Undersaturated Reservoir

$$q_o = \frac{0.00708 \times k \times h \times (P_i - P_{wf})}{\mu_o \times B_o \times \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + S \right]} \quad (\text{STB/day}) \quad (45)$$

Symbol Definitions:

- q_o : Oil production rate (STB/day)
- k : Permeability (mD)
- h : Net pay thickness (ft)
- P_i : Initial/reservoir pressure (psia)
- P_{wf} : Wellbore flowing pressure (psia)
- μ_o : Oil viscosity (cP)
- B_o : Oil formation volume factor (rb/stb)
- r_e : Drainage radius (ft)
- r_w : Wellbore radius (ft)
- S : Skin factor (dimensionless)

4.1.3 Vogel IPR - Saturated Reservoir

For saturated reservoirs ($P_i < P_b$), use Vogel's correlation:

$$\frac{q_o}{q_{o,max}} = 1 - 0.2 \left(\frac{P_{wf}}{P_i} \right) - 0.8 \left(\frac{P_{wf}}{P_i} \right)^2 \quad (46)$$

where maximum rate is:

$$q_{o,max} = q_o(P_{wf} = 0) \quad (47)$$

Additional Symbols:

- $q_{o,max}$: Maximum oil rate at zero bottomhole pressure (STB/day)
- P_b : Bubble point pressure (psia)

4.2 Oil Rate - Linear Flow (Horizontal Wells)

4.2.1 Description

Calculates oil production rate for horizontal wells or linear flow geometry.

$$q_o = \frac{0.00708 \times k \times h \times L \times (P_i - P_{wf})}{\mu_o \times B_o \times \left[\frac{r_e}{2} + S \right]} \quad (\text{STB/day}) \quad (48)$$

Additional Symbols:

- L : Horizontal well length (ft)

4.3 Gas Rate - Radial Flow (Vertical Wells)

4.3.1 Description

Calculates gas production rate for vertical wells using pseudopressure formulation.

$$q_g = \frac{0.00708 \times k \times h \times [\psi(P_i) - \psi(P_{wf})]}{\left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + S \right]} \quad (\text{MSCF/day}) \quad (49)$$

Symbol Definitions:

- q_g : Gas production rate (MSCF/day)
- $\psi(P)$: Gas pseudopressure (psia²/cP)
- k : Permeability (mD)
- h : Net pay thickness (ft)
- P_i : Reservoir pressure (psia)
- P_{wf} : Wellbore flowing pressure (psia)
- r_e : Drainage radius (ft)
- r_w : Wellbore radius (ft)
- S : Skin factor (dimensionless)

4.4 Gas Rate - Linear Flow (Horizontal Wells)

$$q_g = \frac{0.00708 \times k \times h \times L \times [\psi(P_i) - \psi(P_{wf})]}{\left[\frac{r_e}{2} + S \right]} \quad (\text{MSCF/day}) \quad (50)$$

Additional Symbols:

- L : Horizontal well length (ft)

5 Relative Permeability Correlations

5.1 Corey (1954) Correlation

5.1.1 Description

Simple power-law model for relative permeability as a function of normalized saturation.

5.1.2 Normalized Saturation

Water Phase:

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \quad (51)$$

Oil Phase (Water-Oil):

$$S_o^* = \frac{S_o - S_{or}}{1 - S_{wc} - S_{or}} \quad (52)$$

Gas Phase:

$$S_g^* = \frac{S_g - S_{gc}}{1 - S_{wc} - S_{org}} \quad (53)$$

Symbol Definitions:

- S_w^* : Normalized water saturation
- S_o^* : Normalized oil saturation
- S_g^* : Normalized gas saturation
- S_w : Water saturation
- S_o : Oil saturation
- S_g : Gas saturation
- S_{wc} : Connate water saturation
- S_{or} : Residual oil saturation (to water)
- S_{org} : Residual oil saturation (to gas)
- S_{gc} : Critical gas saturation

5.1.3 Relative Permeability Functions

Water Relative Permeability:

$$k_{rw} = k_{rw}^{max} \times (S_w^*)^{n_w} \quad (54)$$

Oil Relative Permeability (to water):

$$k_{row} = k_{row}^{max} \times (1 - S_w^*)^{n_o} \quad (55)$$

Gas Relative Permeability:

$$k_{rg} = k_{rg}^{max} \times (S_g^*)^{n_g} \quad (56)$$

Oil Relative Permeability (to gas):

$$k_{rog} = k_{rog}^{max} \times (1 - S_g^*)^{n_o} \quad (57)$$

Additional Symbols:

- k_{rw} : Water relative permeability
- k_{row} : Oil relative permeability (to water)
- k_{rg} : Gas relative permeability
- k_{rog} : Oil relative permeability (to gas)
- k_{rw}^{max} : Maximum water relative permeability
- k_{row}^{max} : Maximum oil relative permeability
- k_{rg}^{max} : Maximum gas relative permeability
- n_w : Corey exponent for water
- n_o : Corey exponent for oil
- n_g : Corey exponent for gas

Typical Values:

- n_w : 1.5 - 3.0
- n_o : 2.0 - 4.0
- n_g : 2.0 - 3.5

5.2 LET Correlation (Lomeland-Ebeltoft-Thomas 2005)

5.2.1 Description

Flexible three-parameter model (L, E, T) that can fit complex relative permeability curves.

Water Relative Permeability:

$$k_{rw} = k_{rw}^{max} \times \frac{(S_w^*)^{L_w}}{(S_w^*)^{L_w} + E_w(1 - S_w^*)^{T_w}} \quad (58)$$

Oil Relative Permeability:

$$k_{ro} = k_{ro}^{max} \times \frac{(1 - S_w^*)^{L_o}}{(1 - S_w^*)^{L_o} + E_o(S_w^*)^{T_o}} \quad (59)$$

Gas Relative Permeability:

$$k_{rg} = k_{rg}^{max} \times \frac{(S_g^*)^{L_g}}{(S_g^*)^{L_g} + E_g(1 - S_g^*)^{T_g}} \quad (60)$$

Symbol Definitions:

- L_w, L_o, L_g : Low saturation parameters (controls curve near $S^* = 0$)
- E_w, E_o, E_g : Elevation parameters (controls height/position)
- T_w, T_o, T_g : Top saturation parameters (controls curve near $S^* = 1$)

Typical Ranges:

- L : 1.0 - 3.0
- E : 1.0 - 2.0
- T : 1.0 - 3.0

Advantages over Corey:

- More flexible curve shapes
- Better fitting to experimental data
- Independent control of endpoint behavior
- Smooth transitions

6 Brine Properties

Important Note: pyResToolbox uses **different calculation methods** for CH₄-saturated versus CO₂-saturated brines. The methods are described separately below.

6.1 CH₄-Saturated Brine Properties

6.1.1 Description

For methane-saturated brines, pyResToolbox uses the Modified Spivey Correlation per McCain *Petroleum Reservoir Fluid Properties*, Chapter 4, page 160.

6.1.2 Calculation Method

General Approach:

Properties are calculated using empirical correlations that account for:

- Temperature and pressure effects
- Salinity (weight% NaCl, range 0-100%)
- Methane saturation fraction (0 = no methane, 1 = fully saturated)

Key Features:

- Density calculated from pure water density with salt and methane corrections
- Viscosity uses temperature-dependent base correlation with salinity adjustments
- Compressibility derived from density derivatives with respect to pressure
- Methane solubility from modified Henry's Law with salt correction (lambda and eta coefficients)
- Gas molar volume calculated using Z-factor correlation for methane
- Formation volume factor (B_w) accounts for molar volume changes due to dissolved methane

Properties Calculated:

- B_w : Water formation volume factor (rb/stb)
- ρ_w : Brine density (specific gravity, dimensionless)
- μ_w : Brine viscosity (cP)
- c_w : Brine compressibility (1/psi)
- R_w : Methane solubility (scf/stb)

Reference: McCain, W.D. (2011). *Petroleum Reservoir Fluid Property Correlations*, Chapter 4.

6.2 CO₂-Saturated Brine Properties

6.2.1 Description

For CO₂-saturated brines, pyResToolbox uses a thermodynamic approach with multiple specialized correlations that differ significantly from the methane case.

6.2.2 Calculation Methods

Phase Equilibrium:

- **Spycher & Pruess (2010)**: Modified Redlich-Kwong (SRK) Cubic Equation of State
- Provides CO₂-H₂O mutual solubility via activity coefficients
- Temperature-dependent EOS coefficients (different for T < 99°C vs T > 109°C)
- Blended calculations for 99-109°C transition range

Pure Brine Density:

- **Spivey et al.** correlation (modified)
- Reference: McCain *Petroleum Reservoir Fluid Property Correlations*, Chapter 4

CO₂-Corrected Brine Density:

- **Garcia (2001)** equation for CO₂ molar volume
- Combined with Spicher & Pruess x_{CO_2} and Spivey base density

Pure Brine Viscosity:

- **Mao-Duan (2009)** correlation

CO₂-Corrected Brine Viscosity:

- **Islam-Carlson (2012)** model
- Adjusts base viscosity for dissolved CO₂ mole fraction

Properties Calculated:

- x_{CO_2} : CO₂ mole fraction in brine
- y_{H_2O} : H₂O mole fraction in CO₂-rich phase
- ρ_{CO_2-gas} : CO₂-rich gas density (lb/ft³)
- $\rho_{brine-sat}$: CO₂-saturated brine density (lb/ft³)
- $\rho_{brine-pure}$: Pure brine density (lb/ft³)
- $\mu_{brine-sat}$: CO₂-saturated brine viscosity (cP)
- $\mu_{brine-pure}$: Pure brine viscosity (cP)

- B_{w-sat} : CO₂-saturated brine FVF (rb/stb)
- R_w : CO₂ solubility (scf/stb)
- $c_{w-unsat}$: Undersaturated brine compressibility (1/psi)
- c_{w-sat} : Saturated brine compressibility (1/psi)

Salinity Input: Parts per million (ppm) NaCl

Key References:

- Spycher, N. and Pruess, K. (2010). Geochimica et Cosmochimica Acta
- Garcia, J.E. (2001). Fluid Phase Equilibria
- Mao, S. and Duan, Z. (2009). Chemical Geology
- Islam, A.W. and Carlson, E.S. (2012). Geothermics

6.3 Key Differences Between CH₄ and CO₂ Methods

Aspect	CH ₄ -Saturated	CO ₂ -Saturated
Phase Equilibrium	Henry's Law-based empirical	Cubic EOS (Spycher & Pruess)
Density Correction	Dissolved CH ₄ molar volume	Garcia (2001) CO ₂ molar volume
Viscosity Adjustment	Salinity effect factors	Islam-Carlson (2012) model
Salinity Units	Weight% NaCl (0-100)	ppm NaCl
Compressibility	From density derivatives	Numerical differentiation (sat/unsat)

Table 2: Comparison of CH₄ vs CO₂ Brine Calculation Methods

7 Reservoir Heterogeneity

7.1 Lorenz Coefficient

7.1.1 Description

The Lorenz coefficient quantifies reservoir heterogeneity based on the cumulative flow capacity vs cumulative storage capacity curve. It ranges from 0 (homogeneous) to 1 (completely heterogeneous).

$$L = 2 \times \int_0^1 [F(\phi) - \phi] d\phi \quad (61)$$

where $F(\phi)$ is the Lorenz curve:

$$F(\phi) = \frac{\text{Cumulative Flow Capacity}}{\text{Total Flow Capacity}} \quad (62)$$

Symbol Definitions:

- L : Lorenz coefficient (0-1)
- $F(\phi)$: Lorenz curve function
- ϕ : Cumulative storage fraction

Interpretation:

- $L = 0$: Perfectly homogeneous (all layers have same kh)
- $L = 0.3$: Low heterogeneity
- $L = 0.5$: Moderate heterogeneity
- $L = 0.7$: High heterogeneity
- $L \rightarrow 1$: Extreme heterogeneity

7.2 Dykstra-Parsons Coefficient (β)

7.2.1 Description

The Dykstra-Parsons coefficient measures permeability variation assuming a log-normal distribution.

$$\beta = \frac{k_{50} - k_{84.1}}{k_{50}} \quad (63)$$

where:

- k_{50} is the median permeability (50th percentile)
- $k_{84.1}$ is the 84.1th percentile permeability (1 standard deviation below mean in log space)

Symbol Definitions:

- β : Dykstra-Parsons coefficient (0-1)
- k_{50} : Median permeability (mD)
- $k_{84.1}$: 84.1th percentile permeability (mD)

Interpretation:

- $\beta < 0.5$: Low variation (homogeneous)
- $\beta = 0.5 - 0.7$: Moderate variation
- $\beta > 0.7$: High variation (heterogeneous)

7.3 Lorenz-Beta Conversion

7.3.1 Description

Conversion between Lorenz coefficient and Dykstra-Parsons beta parameter, assuming log-normal permeability distribution.

Beta to Lorenz:

$$L \approx 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) \times \exp(x\sigma) dx \quad (64)$$

where σ is related to β through:

$$\beta = 1 - \exp(-\sigma) \quad (65)$$

Approximate Conversion:

$$L \approx 0.632 \times \beta^{0.9} \quad (66)$$

Symbol Definitions:

- L : Lorenz coefficient
- β : Dykstra-Parsons coefficient
- σ : Standard deviation of $\ln(k)$

7.4 Layer Permeability Distribution

7.4.1 Description

Generates permeability distribution for layered reservoirs based on Lorenz coefficient.

Log-Normal Distribution:

$$k_i = k_{avg} \times \exp \left[\sigma \times \Phi^{-1}(p_i) - \frac{\sigma^2}{2} \right] \quad (67)$$

where:

- k_i is permeability of layer i
- k_{avg} is average permeability
- Φ^{-1} is inverse standard normal CDF
- $p_i = (i - 0.5)/N$ is cumulative probability
- N is number of layers

Symbol Definitions:

- k_i : Layer permeability (mD)
- k_{avg} : Average permeability (mD)
- σ : Standard deviation of $\ln(k)$
- Φ^{-1} : Inverse cumulative normal distribution
- N : Number of layers

8 Aquifer Models

8.1 Van Everdingen-Hurst Model

8.1.1 Description

The Van Everdingen-Hurst model calculates water influx from an aquifer into a reservoir. It uses dimensionless influence functions based on the radial diffusivity equation.

Water Influx:

$$W_e(t) = B \times \sum_{j=1}^n \Delta P_j \times W_D(t_D - t_{D,j}) \quad (68)$$

where:

$$B = 1.119 \times \phi \times c_t \times h \times r_e^2 \quad (\text{rb/psi}) \quad (69)$$

Dimensionless Time:

$$t_D = \frac{0.000264 \times k \times t}{\phi \times \mu \times c_t \times r_e^2} \quad (70)$$

Dimensionless Radius:

$$r_D = \frac{r_{aq}}{r_e} \quad (71)$$

Symbol Definitions:

- W_e : Cumulative water influx (rb)
- B : Aquifer constant (rb/psi)
- ΔP_j : Pressure drop at time step j (psia)
- W_D : Dimensionless water influx function
- t_D : Dimensionless time
- ϕ : Aquifer porosity (fraction)
- c_t : Total compressibility (1/psi)
- h : Aquifer thickness (ft)
- r_e : Reservoir radius (ft)
- r_{aq} : Aquifer outer radius (ft)
- k : Aquifer permeability (mD)
- μ : Water viscosity (cP)
- t : Time (days)

8.2 Influence Functions

The dimensionless water influx $W_D(t_D)$ is tabulated for various dimensionless radius ratios r_D . For:

- Infinite aquifer: $r_D = \infty$
- Finite aquifer: $r_D = 2, 3, 5, 10, \text{etc.}$

9 Phase Behavior

9.1 Rachford-Rice Flash Calculation

9.1.1 Description

The Rachford-Rice equation is used to calculate vapor and liquid phase compositions and amounts for a multi-component mixture at specified temperature and pressure.

Rachford-Rice Equation:

$$\sum_{i=1}^n \frac{z_i(K_i - 1)}{1 + V(K_i - 1)} = 0 \quad (72)$$

where:

- V is vapor mole fraction (solve iteratively)
- $L = 1 - V$ is liquid mole fraction

Component Mole Fractions:

$$x_i = \frac{z_i}{1 + V(K_i - 1)} \quad (\text{liquid}) \quad (73)$$

$$y_i = \frac{K_i \times z_i}{1 + V(K_i - 1)} \quad (\text{vapor}) \quad (74)$$

Symbol Definitions:

- z_i : Overall mole fraction of component i
- x_i : Liquid phase mole fraction of component i
- y_i : Vapor phase mole fraction of component i
- K_i : Equilibrium K-value for component i ($K_i = y_i/x_i$)
- V : Vapor phase mole fraction
- L : Liquid phase mole fraction
- n : Number of components

Solution Method:

Newton-Raphson iteration:

$$V^{n+1} = V^n - \frac{f(V^n)}{f'(V^n)} \quad (75)$$

where:

$$f(V) = \sum_{i=1}^n \frac{z_i(K_i - 1)}{1 + V(K_i - 1)} \quad (76)$$

$$f'(V) = - \sum_{i=1}^n \frac{z_i(K_i - 1)^2}{[1 + V(K_i - 1)]^2} \quad (77)$$

10 Quick Reference Tables

10.1 Physical Constants

Constant	Symbol	Value
Universal gas constant	R	10.73 psia·ft ³ /(lbmol·°R)
Standard pressure	P_{sc}	14.7 psia
Standard temperature	T_{sc}	60°F (520°R)
Air molecular weight	M_{air}	28.97 lb/lbmol
Water density at SC	ρ_w^{SC}	62.4 lb/ft ³
Conversion °F to °R	T_R	$T_F + 459.67$

Table 3: Physical Constants Used in Calculations

10.2 Conversion Factors

From	To	Multiply By
psia	bara	0.06895
°F	°C	($T_F - 32$) × 5/9
ft	m	0.3048
mD	m ²	9.869 × 10 ⁻¹⁶
cP	Pa·s	0.001
STB	m ³	0.1590
MSCF	m ³	28.32
lb/ft ³	kg/m ³	16.02

Table 4: Common Unit Conversions

10.3 Typical Property Ranges

Property	Typical Range	Comments
API gravity	10 - 50°	Light crude: 30-50°, Heavy: 10-25°
Gas gravity	0.6 - 1.2	Dry gas: 0.6-0.7, Wet gas: 0.7-0.9
R_s at P_b	100 - 3000 scf/stb	Depends on oil type and pressure
B_o	1.01 - 2.0 rb/stb	Increases with R_s
μ_o	0.5 - 100 cP	Light oil: 0.5-5, Heavy: 10-100+
Z-factor	0.7 - 1.2	Lower at medium pressures
B_g	0.003 - 0.010 rcf/scf	Increases with T, decreases with P
μ_g	0.01 - 0.03 cP	Increases with P and T
Permeability	0.1 - 1000 mD	Tight: <1, Good: 100-1000
Porosity	0.05 - 0.35	Low: 0.05-0.15, Good: 0.20-0.30

Table 5: Typical Property Ranges for Petroleum Reservoirs

11 Correlation Method Recommendations

11.1 Oil PVT Methods

Property	Recommended	When to Use
Bubble Point	VALMC	Wide range of conditions, most accurate
	STAN	Classic correlation, quick estimates
	VELAR	Regional correlations, comparison
Solution GOR	VELAR	Default choice, good accuracy
	STAN	Compatibility with older studies
	VALMC	Alternative method
FVF	MCAIN	Most accurate, density-based
	STAN	Simple correlation, fast
Viscosity	BR	Industry standard, widely validated

Table 6: Recommended Correlations for Oil PVT

11.2 Gas PVT Methods

Property	Recommended	When to Use
Z-Factor	DAK	Most accurate, industry standard
	HY	Fast, good for most conditions
	WYW	Newer correlation, comparison
	BUR	High non-hydrocarbons, 100% CO ₂ , H ₂
Critical Props	PMC	Best accuracy, accounts for non-HCs
	SUT	Classic method, compatibility
	BUR	High non-hydrocarbon content
Viscosity	LGE	Industry standard

Table 7: Recommended Correlations for Gas PVT

11.3 Relative Permeability Methods

Method	Use For	Advantages
Corey	Quick estimates	Simple, fast, widely used
	Standard cases	Few parameters, easy to understand
	Initial models	Good starting point
LET	History matching	Flexible curve shapes
	Complex systems	Better fit to lab data
	Final models	Independent endpoint control

Table 8: Relative Permeability Method Selection

12 Tool Categories and Count

12.1 Available Tools by Category

12.2 Key Features

- **Production Ready:** All tools tested and validated
- **Field Units:** Consistent use of US oilfield units
- **Array Support:** Calculate properties at multiple pressures simultaneously
- **Multiple Methods:** Choose from various industry-standard correlations
- **Type Safety:** Pydantic models ensure input validation
- **Zero Configuration:** Works out of the box with Claude Desktop
- **GPL-3.0 Licensed:** Free and open source

Category	Description	Tools
Oil PVT	Oil property calculations Bubble point, Rs, Bo, viscosity, density, etc.	17
Gas PVT	Gas property calculations Z-factor, critical props, Bg, viscosity, etc.	11
Inflow	Well performance Oil/gas rates for radial/linear flow	4
Simulation	Reservoir simulation support Rel perm tables, aquifer functions, flash	3
Brine	Brine properties CH ₄ and CO ₂ saturated brine	2
Heterogeneity	Layer analysis Lorenz, beta, layer distributions	5
Library	Component properties Critical properties for 100+ components	1
Config	Configuration resources Units, methods, constants, help	4
Geomechanics	Wellbore stability & stress <i>(Documented separately)</i>	15
Total (Covered)		47
Total (All Tools)		62

Table 9: pyResToolbox MCP Tool Categories (This guide covers 47 core petroleum engineering tools)

13 Usage Notes and Best Practices

13.1 General Guidelines

13.1.1 Input Validation

- Always use absolute pressures (psia, not psig or barg)
- Temperature must be in Fahrenheit, not Celsius
- Check that input ranges are physically realistic
- Verify that saturation endpoints sum correctly (e.g., $S_{wc} + S_{or} < 1.0$)

13.1.2 Method Selection

- Use recommended methods unless specific requirements exist
- For bubble point: VALMC is most accurate for wide ranges
- For Z-factor: DAK is industry standard, BUR for high non-HCs
- For relative permeability: Corey for quick estimates, LET for accuracy

13.1.3 Common Mistakes to Avoid

- Using separator temperature instead of reservoir temperature
- Confusing R_{sb} (solution GOR at bubble point) with separator GOR
- Not setting Vogel=True when $P_i < P_b$ (saturated reservoir)
- Using wrong gas gravity (separator vs weighted average)
- Not accounting for non-hydrocarbon components (H_2S , CO_2 , N_2)
- Forgetting to convert units from SI to Field Units

13.2 PVT Calculation Workflow

13.2.1 Typical Oil PVT Workflow

1. Calculate bubble point pressure (P_b) from R_{sb} , API, T , γ_g
2. Check if reservoir is undersaturated ($P_i > P_b$) or saturated ($P_i < P_b$)
3. Calculate $R_s(P)$ at desired pressures
4. Calculate $B_o(P, R_s)$ using McCain or Standing method
5. Calculate $\mu_o(P, R_s)$ using Beggs-Robinson
6. Calculate $\rho_o(B_o, R_s)$ using SWMH method
7. Calculate c_o from numerical derivatives of B_o
8. Generate complete PVT table for simulation

13.2.2 Typical Gas PVT Workflow

1. Calculate pseudo-critical properties (T_c , P_c) from γ_g and impurities
2. Calculate Z-factor at desired pressures using DAK or BUR
3. Calculate $B_g(P, T, Z)$ from real gas equation
4. Calculate gas density $\rho_g(P, T, Z)$
5. Calculate gas viscosity $\mu_g(\rho_g, T)$ using LGE method
6. Calculate gas compressibility c_g from Z-factor derivatives
7. Calculate pseudopressure $\psi(P)$ by integrating $P/(\mu_g Z)$

13.3 Well Performance Analysis

13.3.1 IPR Generation Steps

1. Gather reservoir data: P_i , P_b , API, T , h , k , S , r_e , r_w
2. Calculate PVT properties at average pressure
3. Choose flow geometry: radial (vertical well) or linear (horizontal well)
4. For saturated reservoir ($P_i < P_b$), set Vogel=True
5. Calculate rates at multiple flowing pressures: $P_{wf} = 0$ to P_i
6. Plot q vs P_{wf} to generate IPR curve
7. Identify AOF (absolute open flow = rate at $P_{wf} = 0$)

13.4 Simulation Preparation

13.4.1 Relative Permeability Table Generation

1. Choose table type: SWOF (water-oil), SGOF (gas-oil), or SGWFN (3-phase)
2. Choose correlation: Corey (simple) or LET (flexible)
3. Specify saturation endpoints: S_{wc} , S_{or} , S_{org} , S_{gc}
4. Set maximum relative permeabilities: k_{rw}^{max} , k_{ro}^{max} , k_{rg}^{max}
5. Define correlation parameters (Corey exponents or LET parameters)
6. Generate table with 20-50 saturation points
7. Validate endpoints and curves against core data
8. Export to ECLIPSE/CMG format for simulation

14 References

14.1 Key Publications

14.1.1 Oil PVT Correlations

- Standing, M.B. (1947). "A Pressure-Volume-Temperature Correlation for Mixtures of California Oils and Gases." *Drilling and Production Practices*, API.
- Valko, P.P. and McCain, W.D. (2003). "Reservoir Oil Bubblepoint Pressures Revisited; Solution Gas-Oil Ratios and Surface Gas Specific Gravities." *Journal of Petroleum Science and Engineering*, Vol. 37, Issues 3-4, pp. 153-169. DOI: 10.1016/S0920-4105(02)00319-4.
- Velarde, J., Blasingame, T.A., and McCain, W.D. (1997). "Correlation of Black Oil Properties at Pressures Below Bubble Point Pressure." *SPE Annual Technical Conference and Exhibition*. (Note: Based on Velarde's 1996 Master's thesis at Texas A&M University).
- McCain, W.D., Spivey, J.P., and Lenn, C.P. (2011). *Petroleum Reservoir Fluid Property Correlations*. PennWell Books.

14.1.2 Gas PVT Correlations

- Dranchuk, P.M. and Abou-Kassem, J.H. (1975). "Calculation of Z Factors for Natural Gases Using Equations of State." *Journal of Canadian Petroleum Technology*.
- Hall, K.R. and Yarborough, L. (1973). "A New Equation of State for Z-Factor Calculations." *Oil and Gas Journal*.
- Wang, X., Ye, F., and Wu, K. (2021). "A New Correlation for Calculating Gas Compressibility Factor." *Energy Reports*, Vol. 8, Supplement 2, pp. 130-137. DOI: 10.1016/j.egyr.2021.11.029.
- Burgoyne, M.W. (Santos Ltd), Nielsen, M.H., and Stanko, M. (Whitson AS) (2025). "A Universal, EOS-Based Correlation for Z-Factor, Viscosity and Enthalpy for Hydrocarbon and H₂, N₂, CO₂, H₂S Gas Mixtures." *SPE Paper 229932-MS*. Presented at ADIPEC, Abu Dhabi, UAE, 3-6 November 2025. Supports high concentrations of non-hydrocarbons including 100% CO₂ and H₂.
- Lee, A.L., Gonzalez, M.H., and Eakin, B.E. (1966). "The Viscosity of Natural Gases." *Journal of Petroleum Technology*.
- Piper, L.D., McCain, W.D., and Corredor, J.H. (1993). "Compressibility Factors for Naturally Occurring Petroleum Gases." *SPE Annual Technical Conference and Exhibition*, SPE-26668-MS, Houston, Texas, October 1993.
- Sutton, R.P. (1985). "Compressibility Factors for High-Molecular-Weight Reservoir Gases." *SPE Annual Technical Conference*.

14.1.3 Well Performance

- Darcy, H. (1856). *Les Fontaines Publiques de la Ville de Dijon*. Victor Dalmont.
- Vogel, J.V. (1968). "Inflow Performance Relationships for Solution-Gas Drive Wells." *Journal of Petroleum Technology*.
- Standing, M.B. (1971). "Concerning the Calculation of Inflow Performance of Wells Producing from Solution Gas Drive Reservoirs." *Journal of Petroleum Technology*.

14.1.4 Relative Permeability

- Corey, A.T. (1954). "The Interrelation Between Gas and Oil Relative Permeabilities." *Production Monthly*, Vol. 19, pp. 38-41.
- Lomeland, F., Ebeltoft, E., and Thomas, W.H. (2005). "A New Versatile Relative Permeability Correlation." *International Symposium of the Society of Core Analysts*.

14.1.5 Brine Properties

- McCain, W.D. (2011). *Petroleum Reservoir Fluid Property Correlations*. PennWell Books, Chapter 4.
- Spycher, N. and Pruess, K. (2010). "A Phase-Partitioning Model for CO₂-Brine Mixtures at Elevated Temperatures and Pressures: Application to CO₂-Enhanced Geothermal Systems." *Transport in Porous Media*, Vol. 82, pp. 173-196. DOI: 10.1007/s11242-009-9425-y.
- Garcia, J.E. (2001). "Density of Aqueous Solutions of CO₂." LBNL Report #49023, October 2001. Also published in *Fluid Phase Equilibria*.
- Mao, S. and Duan, Z. (2009). "The Viscosity of Aqueous Alkali-Chloride Solutions up to 623 K, 1,000 bar, and High Ionic Strength." *Chemical Geology*.
- Islam, A.W. and Carlson, E.S. (2012). "Viscosity Models and Effects of Dissolved CO₂." *Energy & Fuels*, Vol. 26, No. 8, pp. 5330-5336. DOI: 10.1021/ef3006228.

14.1.6 Reservoir Heterogeneity

- Dykstra, H. and Parsons, R.L. (1950). "The Prediction of Oil Recovery by Waterflood." *Secondary Recovery of Oil in the United States*. API.
- Schmalz, J.P. and Rahme, H.D. (1950). "The Variation of Waterflood Performance with Variation in Permeability Profile." *Producers Monthly*.
- Lake, L.W. (1989). *Enhanced Oil Recovery*. Prentice Hall.

14.2 Software and Tools

- Burgoyne, M.W. (2024). *pyResToolbox: A Collection of Reservoir Engineering Utilities*. GitHub: <https://github.com/mwburgoyne/pyResToolbox>
- Anthropic (2024). *Model Context Protocol*. <https://modelcontextprotocol.io/>
- Serrao, G. (2024). *pyResToolbox MCP Server*. GitHub: <https://github.com/gabrielsserrao/pyrestoolbox-mcp>

15 Appendix: Nomenclature

15.1 Primary Symbols

Symbol	Units	Description
API	degrees	Oil API gravity
B_o	rb/stb	Oil formation volume factor
c_o	1/psi	Oil compressibility
μ_o	cP	Oil viscosity
ρ_o	lb/ft ³	Oil density
R_s	scf/stb	Solution gas-oil ratio
R_{sb}	scf/stb	Solution GOR at bubble point
P_b	psia	Bubble point pressure
γ_o	-	Oil specific gravity (water=1)
B_g	rcf/scf	Gas formation volume factor
c_g	1/psi	Gas compressibility
μ_g	cP	Gas viscosity
ρ_g	lb/ft ³	Gas density
Z	-	Gas compressibility factor (Z-factor)
γ_g	-	Gas specific gravity (air=1)
P_c	psia	Pseudo-critical pressure
T_c	°R	Pseudo-critical temperature
P_{pr}	-	Pseudo-reduced pressure
T_{pr}	-	Pseudo-reduced temperature
ψ	psia ² /cP	Gas pseudopressure
B_w	rb/stb	Water/brine formation volume factor
c_w	1/psi	Brine compressibility
μ_w	cP	Brine viscosity
ρ_w	lb/ft ³	Brine density
R_w	scf/stb	Gas solubility in brine
w_t	wt%	Salinity (weight percent NaCl)
k	mD	Permeability
h	ft	Pay thickness (net)
ϕ	fraction	Porosity
S_w	fraction	Water saturation
S_o	fraction	Oil saturation
S_g	fraction	Gas saturation
S_{wc}	fraction	Connate water saturation
S_{or}	fraction	Residual oil saturation
P	psia	Pressure
T	°F	Temperature
P_i	psia	Initial/reservoir pressure
P_{wf}	psia	Wellbore flowing pressure
q_o	STB/day	Oil production rate
q_g	MSCF/day	Gas production rate
r_e	ft	Drainage radius
r_w	ft	Wellbore radius
S	-	Skin factor

Symbol	Units	Description
L	ft	Horizontal well length
k_{rw}	-	Water relative permeability
k_{ro}	-	Oil relative permeability
k_{rg}	-	Gas relative permeability
S^*	-	Normalized saturation
L	-	Lorenz coefficient
β	-	Dykstra-Parsons coefficient
y_{CO_2}	fraction	CO ₂ mole fraction
y_{H_2S}	fraction	H ₂ S mole fraction
y_{N_2}	fraction	N ₂ mole fraction

Table 10: Nomenclature - Primary Symbols

15.2 Subscripts

Subscript	Meaning
o	Oil phase
g	Gas phase
w	Water/brine phase
b	At bubble point
i	Initial conditions
wf	Wellbore flowing
c	Critical or connate
r	Relative or residual
pr	Pseudo-reduced
D	Dimensionless
sc	Standard conditions
ST	Stock tank conditions
sp	Separator conditions

Table 11: Common Subscripts

16 Index of Formulas

16.1 By Property

Oil Properties

- Bubble Point Pressure (P_b): Standing (p.4), Valko-McCain (p.4), Velarde (p.4)
- Solution GOR (R_s): Standing (p.5), Valko-McCain, Velarde
- Formation Volume Factor (B_o): McCain (p.6), Standing (p.6)
- Viscosity (μ_o): Beggs-Robinson (p.7)
- Density (ρ_o): SWMH (p.7)
- Compressibility (c_o): Explicit method (p.8)
- API-SG Conversions (p.8)

Gas Properties

- Z-Factor: DAK (p.9), Hall-Yarborough (p.10)
- Critical Properties: PMC (p.10), Sutton, BUR
- Formation Volume Factor (B_g): Real gas equation (p.11)
- Viscosity (μ_g): Lee-Gonzalez-Eakin (p.11)
- Density (ρ_g): Ideal gas law corrected (p.11)
- Compressibility (c_g): From Z-factor (p.12)
- Pseudopressure (ψ): Integral formulation (p.12)

Well Performance

- Oil Rate - Radial: Darcy's law (p.13), Vogel IPR (p.13)
- Oil Rate - Linear: Darcy's law (p.14)
- Gas Rate - Radial: Pseudopressure formulation (p.14)
- Gas Rate - Linear: Pseudopressure formulation (p.14)

Relative Permeability

- Corey Correlation: Normalized saturation (p.15), Kr functions (p.15)
- LET Correlation: Three-parameter model (p.16)

Brine Properties

- Density (ρ_w): Pressure-temperature-salinity (p.17)
- Viscosity (μ_w): Salinity correction (p.17)
- Compressibility (c_w): From FVF derivative (p.18)

- Formation Volume Factor (B_w): Three-effect model (p.18)
- Gas Solubility (R_w): Henry's Law (p.18)

Heterogeneity

- Lorenz Coefficient: Integral formulation (p.19)
- Dykstra-Parsons Coefficient: Percentile definition (p.19)
- Lorenz-Beta Conversion: Approximate relations (p.20)
- Layer Distribution: Log-normal model (p.20)

Other

- Van Everdingen-Hurst Aquifer: Water influx (p.21)
 - Rachford-Rice Flash: Phase equilibrium (p.22)
-

End of Formula Reference Guide

Compiled by: Gabriel Serrão Seabra

For the latest version and updates, visit:

<https://github.com/gabrielserrao/pyrestoolbox-mcp>

Built on pyResToolbox:

Original library by Mark W. Burgoyne

<https://github.com/mwburgoyne/pyResToolbox>

Remember: Always verify formulas with original sources before professional use.
