ACM ICPC Reference

University of Notre Dame

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Contents

1 vimrc

```
syntax on
colors evening
set ai si noet ts=4 sw=4 sta sm nu so=7 t_Co=8
imap {<CR> {<CR>}<Esc>O
```

2 hashify.sh

```
#!/bin/bash
while IFS=$'\n' read -r line; do
    trim=$(echo "$line" | tr -d "[:space:]")
    md5=$(echo -n "${trim%%\/\/*}" | md5sum)
    md5=${md5:0:4}
    [ "${trim:~0}" == "$" ] && md5="@$md5@"
    echo "$md5 $line"
done
```

3 STL

?

4 Geometry

4.1 Base

```
d41d // typedef double cood; cood eps = 1e-8; // risky: XXX, untested: TODO
00a0 const double pi = acos(-1.);
ccb5 template<typename T> inline T sq(T x) { return x*x; }
87bc struct vec {
b86a ▹ cood x, y;
6e4f ▹ vec () : x(0), y(0) {} vec (cood a, cood b) : x(a), y(b) {}
741a ▹ inline vec operator - (vec o) { return {x - o.x, y - o.y}; }
ff7e ▹ inline vec operator + (vec o) { return {x + o.x, y + o.y}; }
b6dd ▹ inline vec operator * (cood o) { return {x * o, y * o}; }
2711 ▹ inline vec operator / (cood o) { return {x / o, y / o}; }
6ac9 ▹ inline cood operator ^ (vec o) { return x * o.y - y * o.x; }
83dd ▹ inline cood operator * (vec o) { return x * o.x + y * o.y; }
46ef ▹ inline cood cross (vec a, vec b) { return ((*this)-a) ^ ((*this)-b); } // |(this)a||(this)b|sen(angle)
cbad ▹ inline cood inner (vec a, vec b) { return ((*this)-a) * ((*this)-b); } // |(this)a||(this)b|cos(angle)
cddd ▹ inline double angle (vec a, vec b) { return atan2(cross(a,b),inner(a,b)); } // ccw angle from (this)a to (this)b in range [-pi,pi]
e4d3 ▹ inline int ccw (vec a, vec b) { cood o = cross(a,b); return (eps < o) - (o < -eps); } // this is to the (1 left, 0 over, -1 right) of ab
2e1f ▹ inline int dir (vec a, vec b) { cood o = inner(a,b); return (eps < o) - (o < -eps); } // a(this) is to the (1 same, 0 none, -1 opposite) direction of ab
5d26 ▹ inline cood sq (vec o = vec()) { return inner(o,o); }
e7cf ▹ inline double nr (vec o = vec()) { return sqrt(sq(o)); } //$
4e72 ▹ inline vec operator ~ () { return (*this)/nr(); }
f149 ▹ inline vec proj (vec a, vec b) { return a + (b-a)*(a.inner((*this),b) / a.sq(b)); } // projects this onto line ab
1664 ▹ inline vec rotate (double a) { return vec(cos(a) * x - sin(a) * y, sin(a) * x + cos(a) * y); } // ccw by a radians
3206 ▹ inline vec rot90 () { return vec(-y,x); } // rotate(pi/2)$
2810 ▹ bool in_seg (vec a, vec b) { return ccw(a,b) == 0 && dir(a,b) <= 0; } // tips included
5e56 ▹ double dist2_lin (vec a, vec b) { return a.sq(b) <= eps ? sq(a) : double(::sq(cross(a,b)))/a.sq(b); } // see cir.has_inter_lin
8831 ▹ double dist2_seg (vec a, vec b) { return a.dir((*this),b) == (b.dir((*this),a)) ? dist2_lin(a,b) : min(sq(a),sq(b)); }
436b ▹ inline bool operator == (const vec & o) const { return abs(x-o.x) <= eps && abs(y-o.y) <= eps; }
5522 ▹ inline bool operator < (const vec & o) const { return (abs(x-o.x)>eps)?(x < o.x):(y > o.y); } // lex compare (inc x, dec y)
d41d ▹ // full ccw angle strict compare beginning upwards (this+(0,1)) around (*this)
d41d ▹ // incresing distance on ties, this is the first
```

```
69ad 	bool compare (vec a, vec b) {
a482 		if ((*this < a) != (*this < b)) return *this < b;
bdb1 		int o = ccw(a,b); return o?o>0:((a == *this && !(a == b)) || a.dir(*this,b) < 0);
cbb1 	}
2145 }; //$
bafe struct lin { // line
6143 	vec p; cood c; // p*(x,y) = c
1105 	lin () {} lin (vec a, cood b) : p(a), c(b) {}
d036 	lin (vec s, vec t) : p((s-t).rot90()), c(p*s) {}
5c8b 	inline lin parll (vec v) { return lin(p,v*p); }
1263 	inline lin perp () { return lin(p.rot90(),c); }
3838 	vec inter (lin o) { if (vec(0,0).ccw(p,o.p) == 0) throw 1; cood d = (p^o.p); return vec((c*o.p.y -
     p.y*o.c)/d,(o.c*p.x - o.p.x*c)/d); }
1375 	bool contains (vec v) { return abs(p*v - c) <= eps; }
eda5 	vec at_x (cood x) { return vec(x,(c-p.x*x)/p.y); }
c0fb 	vec at_y (cood y) { return vec((c-y*p.y)/p.x,y); }
e1ef 	double sign_dist (vec v) { return double(p*v - c)/p.nr(); }
2145 }; //$
3236 struct cir { // circle
b6d3 	vec c; cood r;
126a 	cir () {} cir (vec v, cood d) : c(v), r(d) {}
c118 	cir (vec u, vec v, vec w) { // XXX untreated degenerates
0fb6 		vec mv = (u+v)/2; lin s(mv, mv+(v-u).rot90());
bf5f 		vec mw = (u+w)/2; lin t(mw, mw+(w-u).rot90());
a0c4 		c = s.inter(t); r = c.nr(u);
cbb1 	}//$
9e54 	inline bool contains (vec w) { return c.sq(w) <= sq(r) + eps; } // border included
0549 	inline bool border (vec w) { return abs(c.sq(w) - sq(r)) <= eps; }
1cd6 	inline bool has_inter (cir o) { return c.sq(o.c) <= sq(r + o.r) + eps; } // borders included
376d 	inline bool has_border_inter (cir o) { return has_inter(o) && c.sq(o.c) + eps >= sq(r - o.r); }
8ab4 	inline bool has_inter_lin (vec a, vec b) { return a.sq(b) <= eps ? contains(a) : sq(c.cross(a,b)) <=
     sq(r)*a.sq(b) + eps; } // borders included XXX overflow
9bf7 	inline bool has_inter_seg (vec a, vec b) { return has_inter_lin(a,b) && (contains(a) || contains(b) ||
     a.dir(c,b)*b.dir(c,a) != -1); } // borders and tips included XXX overflow
7abe 	inline double arc_area (vec a, vec b) { return c.angle(a,b)*r*r/2; } // smallest arc, ccw positive
f967 	inline double arc_len (vec a, vec b) { return c.angle(a,b)*r; } // smallest arc, ccw positive$
771f 	pair<vec,vec> tan (vec v) { // XXX low precision
84ec 		if (contains(v) && !border(v)) throw 0;
2894 		cood d2 = c.sq(v); double s = sqrt(d2 - r*r); s = (s==s)?s:0;
0f70 		double al = atan2(r,s); vec t = (~(c-v));
3a69 		return pair<vec,vec>(v + t.rotate(al)*s, v + t.rotate(-al)*s);
cbb1 	}//$
c56f 	pair<vec,vec> border_inter (cir o) {
c4d4 		if (!has_border_inter(o) || o.c == (*this).c) throw 0;
2b40 		double a = (sq(r) + o.c.sq(c) - sq(o.r))/(2*o.c.nr(c));
b647 		vec v = (o.c - c)/o.c.nr(c); vec m = c + v * a;
65b9 		double h = sqrt(sq(r) - sq(a)); h = h!=h?0:h;
440c 		return pair<vec,vec>(m + v.rot90()*h, m - v.rot90()*h);
cbb1 	}//$
5182 	pair<vec,vec> border_inter_lin (vec a, vec b) { // first is closest to a than second
c6e7 		if (a.sq(b) <= eps) { if (border(a)) return pair<vec,vec>(a,a); throw 0; }
40f6 		if (a.dir(b,c) == -1) swap(a,b);
45ab 		if (!has_inter_lin(a,b)) throw 0;
5cb6 		double d2 = c.dist2_lin(a,b); vec p = (b-a)/a.nr(b);
0aca 		double h = sqrt(r*r - d2); h = h!=h?0:h;
ddf2 		double y = sqrt(c.sq(a) - d2); y = y!=y?0:y;
5539 		return pair<vec,vec>(a + p*(y-h), a + p*(y+h));
cbb1 	}//$
be35 	double triang_inter (vec a, vec b) { // ccw oriented, this with (c,a,b)
53ba 		if (c.sq(a) > c.sq(b)) return -triang_inter(b,a);
148a 		if (contains(b)) return c.cross(a,b)/2;
7434 		if (!has_inter_seg(a,b)) return arc_area(a,b);
773a 		pair<vec,vec> itr = border_inter_lin(b,a); // order important
12a9 		if (contains(a)) return c.cross(a,itr.first)/2 + arc_area(itr.first,b);
c2f4 		return arc_area(a,itr.second) + c.cross(itr.second,itr.first)/2 + arc_area(itr.first,b);
cbb1 	}
2145 }; //$
a71b bool inter_seg (vec a, vec b, vec c, vec d) {
2397 	if (a.in_seg(c, d) || b.in_seg(c, d) || c.in_seg(a, b) || d.in_seg(a, b)) return true;
```

```
bbbd ▹ return (c.ccw(a, b) * d.ccw(a, b) == -1 && a.ccw(c, d) * b.ccw(c, d) == -1);
cbb1 }
e0fd double dist2_seg (vec a, vec b, vec c, vec d){return inter_seg(a,b,c,d)?0.:min({ a.dist2_seg(c,d),
     b.dist2_seg(c,d), c.dist2_seg(a,b), d.dist2_seg(a,b) });}
```

4.2 Advanced

```
484c cir min_spanning_circle (vec * v, int n) { // n
f1ea ▹ srand(time(NULL)); random_shuffle(v, v+n); cir c(vec(), 0); int i,j,k;
b11a ▹ for (i = 0; i < n; i++) if (!c.contains(v[i]))
e5b6 ▹ ▹ for (c = cir(v[i],0), j = 0; j < i; j++) if (!c.contains(v[j]))
a47c ▹ ▹ ▹ for (c = cir((v[i] + v[j])/2,v[i].nr(v[j])/2), k = 0; k < j; k++) if (!c.contains(v[k]))
3dd3 ▹ ▹ ▹ ▹ ▹ c = cir(v[i],v[j],v[k]);
807f ▹ return c;
cbb1 }//$
d45c int convex_hull (vec * v, int n, int border_in) { // nlg | border_in (should border points stay?)
4f17 ▹ swap(v[0], *min_element(v,v+n)); int s, i;
f37e ▹ sort(v+1, v+n, [&v] (vec a, vec b) { int o = b.ccw(v[0], a); return (o?o==1:v[0].sq(a)<v[0].sq(b)); });
a69c ▹ if (border_in) {
9492 ▹ ▹ for (s = n-1; s > 1 && v[s].ccw(v[s-1],v[0]) == 0; s--);
0bb0 ▹ ▹ reverse(v+s, v+n);
cbb1 ▹ }
c497 ▹ for (i = s = 0; i < n; i++) if (!s || !(v[s-1] == v[i])) {
cea9 ▹ ▹ for (; s >= 2 && v[s-1].ccw(v[s-2],v[i]) >= border_in; s--);
ceca ▹ ▹ swap(v[s++],v[i]);
cbb1 ▹ }
0478 ▹ return s;
cbb1 }//$
79b9 int monotone_chain (vec * v, int n, int border_in) { // nlg | border_in (should border points stay?)
5031 ▹ vector<vec> r; sort(v, v+n); n = unique(v, v+n) - v;
d885 ▹ for (int i = 0; i < n; r.pb(v[i++])) while (r.size() >= 2 && r[r.size()-2].ccw(r.back(),v[i]) <=
     -border_in) r.pop_back();
dd80 ▹ r.pop_back(); unsigned int s = r.size();
c19d ▹ for (int i = n-1; i >= 0; r.pb(v[i--])) while (r.size() >= s+2 && r[r.size()-2].ccw(r.back(),v[i]) <=
     -border_in) r.pop_back();
a255 ▹ return copy(r.begin(), r.end() - (r.size() > 1), v) - v;
cbb1 }//$
f80f double polygon_inter (vec * p, int n, cir c) { // signed area
2eae ▹ return inner_product(p, p+n-1, p+1, c.triang_inter(p[n-1],p[0]), std::plus<double>(), [&c] (vec a, vec b)
     { return c.triang_inter(a,b); });
cbb1 }//$
3214 int polygon_pos (vec * p, int n, vec v) { // lg | p should be simple (-1 out, 0 border, 1 in)
6c2a ▹ int in = -1; // it's a good idea to randomly rotate the points in the double case, numerically safer
6033 ▹ for (int i = 0; i < n; i++) {
2bca ▹ ▹ vec a = p[i], b = p[i?i-1:n-1]; if (a.x > b.x) swap(a,b);
c9e9 ▹ ▹ if (a.x + eps <= v.x && v.x < b.x + eps) { in *= v.ccw(a,b); }
c3b1 ▹ ▹ else if (v.in_seg(a,b)) { return 0; }
cbb1 ▹ }
091d ▹ return in;
cbb1 }//$
271f int polygon_pos_convex (vec * p, int n, vec v) { // lg(n) | (-1 out, 0 border, 1 in) TODO
a868 ▹ if (v.sq(p[0]) <= eps) return 0;
088f ▹ if (n <= 1) { return 0; } if (n == 2) { return v.in_seg(p[0],p[1])?0:-1; }
2ceb ▹ if (v.ccw(p[0],p[1]) < 0 || v.ccw(p[0],p[n-1]) > 0) return -1;
fcfd ▹ int di = lower_bound(p+1,p+n-1,v, [&p](vec a,vec v) { return v.ccw(p[0],a) > 0; }) - p;
adf3 ▹ if (di == 1) return v.ccw(p[1],p[2]) >= 0?0:-1;
cfa4 ▹ return v.ccw(p[di-1],p[di]);
cbb1 }//$
d41d // v is the pointset, w is auxiliary with size at least equal to v's
bf98 cood closest_pair (vec * v, vec * w, int l, int r, bool sorted = 0) { // nlg | r is exclusive TODO (AC on
     cf, no test)
91d7 ▹ if (l + 1 >= r) return inf;
900b ▹ if (!sorted) sort(v+l,v+r,[](vec a, vec b){ return a.x < b.x; });
89cd ▹ int m = (l+r)/2; cood x = v[m].x;
1a44 ▹ cood res = min(closest_pair(v,w,l,m,1),closest_pair(v,w,m,r,1));
d046 ▹ merge(v+l,v+m,v+m,v+r,w+l,[](vec a, vec b){ return a.y < b.y; });
2dd0 ▹ for (int i = l, s = l; i < r; i++) if (sq((v[i] = w[i]).x - x) < res) {
ad96 ▹ ▹ for (int j = s-1; j >= l && sq(w[i].y - w[j].y) < res; j--)
```

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c3b1            res = min(res, w[i].sq(w[j]));
1991        w[s++] = v[i];
cbb1    }
b505    return res;
cbb1 }//$
ac2e double union_area (cir * v, int n) { // n^2lg | XXX joins equal circles TODO (AC on szkopul, no tests)
c765    struct I { vec v; int i; } c[2*(n+4)];
cf66    srand(time(NULL)); cood res = 0; vector<bool> usd(n);
dd83    cood lim = 1./0.; for (int i = 0; i < n; i++) lim = min(lim, v[i].c.y - v[i].r - 1);
0b02    for (int i = 0, ss = 0; i < n; i++, ss = 0) {
dc37        vec fp = v[i].c + vec(0,v[i].r).rotate(rand()); // rotation avoids corner on cnt initialization
6e87        int cnt = 0, eq = 0;
578e        for (int j = 0; j < n; j++) {
df48            cnt += (usd[j] = v[j].contains(fp));
2311            if (!v[i].has_border_inter(v[j])) continue;
8daa            if (v[i].c == v[j].c) eq++;
4e6b            else {
e59e                pair<vec,vec> r = v[i].border_inter(v[j]);
0782                c[ss++] = {r.first, j}; c[ss++] = {r.second, j};
cbb1            }
cbb1        }
d21b        vec d = vec(v[i].r,0); for (int k = 0; k < 4; k++, d = d.rot90()) c[ss++] = {v[i].c + d, i};
85d3        int md = partition(c,c+ss,[v,i,fp](I a){return a.v.ccw(v[i].c,fp) > 0;}) - c;
19c7        sort(c,c+md,[v,i](I a,I b){return a.v.ccw(v[i].c,b.v) < 0;});
7430        sort(c+md,c+ss,[v,i](I a,I b){return a.v.ccw(v[i].c,b.v) < 0;});
56cd        for (int j = 0; j < ss; j++) {
2b5e            if (c[j].i != i) { cnt -= usd[c[j].i]; usd[c[j].i] = !usd[c[j].i]; cnt += usd[c[j].i]; }
b115            vec a = c[j].v, b = c[(j+1)%ss].v;
7c4a            cood cir = abs(v[i].arc_area(a,b) - v[i].c.cross(a,b)/2), tra = abs((b.x-a.x)*(a.y+b.y-2*lim)/2);
e20b            cood loc = (a.x<b.x)?cir-tra:tra+cir; res += (cnt==eq)?loc/eq:0;
cbb1        }
cbb1    }
b505    return res;
cbb1 }//$
4ede pii antipodal (vec * p, int n, vec v) { // lg(n) | extreme segments relative to direction v TODO
d41d    // po: closest to dir, ne: furthest from dir
3bd9    bool sw = ((p[1]-p[0])*v < 0);
d189    if (sw) v = vec(0,0) - v; // lower_bound returns the first such that lambda is false
0303    int md = lower_bound(p+1, p+n, v, [p] (vec & a, vec v) { return (a-p[0])*v > eps; }) - p; // chain
     separation
25f1    int po = lower_bound(p, p+md-1, v, [p,n] (vec & a, vec v) { return (p[(&a+1-p)%n]-a)*v > eps; }) - p; //
     positive
9dc9    int ne = (lower_bound(p+md, p+n, v, [p,n] (vec & a, vec v) { return (p[(&a+1-p)%n]-a)*v <= eps; }) -
     p)%n; // negative
5703    if (sw) swap(po,ne);
ef0b    return pii(po,ne);
cbb1 }//$
34e2 int mink_sum (vec * a, int n, vec * b, int m, vec * r) { // (n+m) | a[0]+b[0] should belong to sum, doesn't
     create new border points TODO
8d81    if (!n || !m) { return 0; } int i, j, s; r[0] = a[0] + b[0];
de54    for (i = 0, j = 0, s = 1; i < n || j < m; s++) {
1ab0        if (i >= n) j++;
1dc4        else if (j >= m) i++;
4e6b        else {
4f09            int o = (a[(i+1)%n]+b[j%m]).ccw(r[s-1],a[i%n]+b[(j+1)%m]);
e43c            j += (o >= 0); i += (o <= 0);
cbb1        }
f5b4        r[s] = a[i%n] + b[j%m];
cbb1    }
162b    return s-1;
cbb1 }//$
9e65 int inter_convex (vec * p, int n, vec * q, int m, vec * r) { // (n+m) | XXX
2d76    int a = 0, b = 0, aa = 0, ba = 0, inflag = 0, s = 0;
2a6c    while ((aa < n || ba < m) && aa < n+n && ba < m+m) {
b977        vec p1 = p[a], p2 = p[(a+1)%n], q1 = q[b], q2 = q[(b+1)%m];
35b2        vec A = p2 - p1, B = q2 - q1;
1479        int cross = vec(0,0).ccw(A,B), ha = p1.ccw(p2,q2), hb = q1.ccw(q2,p2);
c6e0        if (cross == 0 && p2.ccw(p1,q1) == 0 && A*B < -eps) {
507b            if (q1.in_seg(p1,p2)) r[s++] = q1;
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5e83 			if (q2.in_seg(p1,p2)) r[s++] = q2;
ce58 			if (p1.in_seg(q1,q2)) r[s++] = p1;
526a 			if (p2.in_seg(q1,q2)) r[s++] = p2;
7b25 			if (s < 2) return s;
e2a8 			inflag = 1; break;
5e6d 		} else if (cross != 0 && inter_seg(p1,p2,q1,q2)) {
f420 			if (inflag == 0) aa = ba = 0;
2b81 			r[s++] = lin(p1,p2).inter(lin(q1,q2));
37fd 			inflag = (hb > 0) ? 1 : -1;
cbb1 		}
5499 		if (cross == 0 && hb < 0 && ha < 0) return s;
0872 		bool t = cross == 0 && hb == 0 && ha == 0;
c0ec 		if (t ? (inflag == 1) : (cross >= 0) ? (ha <= 0) : (hb > 0)) {
9873 			if (inflag == -1) r[s++] = q2;
1146 			ba++; b++; b %= m;
9d97 		} else {
5c98 			if (inflag == 1) r[s++] = p2;
5ecb 			aa++; a++; a %= n;
cbb1 		}
cbb1 	}
c1b2 	if (inflag == 0) {
3880 		if (polygon_pos_convex(q,m,p[0]) >= 0) { copy(p, p+n, r); return n; }
115c 		if (polygon_pos_convex(p,n,q[0]) >= 0) { copy(q, q+m, r); return m; }
cbb1 	}
fc37 	s = unique(r, r+s) - r;
2629 	if (s > 1 && r[0] == r[s-1]) s--;
0478 	return s;
cbb1 }//$
03ae bool isear (vec * p, int n, int i, int prev[], int next[]) { // aux to triangulate
7630 	vec a = p[prev[i]], b = p[next[i]];
2d9f 	if (b.ccw(a,p[i]) <= 0) return false;
578e 	for (int j = 0; j < n; j++) {
97eb 		if (j == prev[i] || j == next[i]) continue;
0ef9 		if (p[j].ccw(a,p[i]) >= 0 && p[j].ccw(p[i],b) >= 0 && p[j].ccw(b,a) >= 0) return false;
0639 		int k = (j+1)%n;
2898 		if (k == prev[i] || k == next[i]) continue;
a537 		if (inter_seg(p[j],p[k],a,b)) return false;
cbb1 	}
8a6c 	return true;
cbb1 }
1851 int triangulate (vec * p, int n, bool ear[], int prev[], int next[], int tri[][3]) { // O(n^2) | n >= 3
d14e 	int s = 0, i = 0;
78d0 	for (int i = 0, prv = n-1; i < n; i++) { prev[i] = prv; prv = i; next[i] = (i+1)%n; ear[i] =
    isear(p,n,i,prev,next); }
6b3b 	for (int lef = n; lef > 3; lef--, i = next[i]) {
ced7 		while (!ear[i]) i = next[i];
e7a9 		tri[s][0] = prev[i]; tri[s][1] = i; tri[s][2] = next[i]; s++; // tri[i][0],i,tri[i][1] inserted
e0c0 		int c_prev = prev[i], c_next = next[i];
c354 		next[c_prev] = c_next; prev[c_next] = c_prev;
84b6 		ear[c_prev] = isear(p,n,c_prev,prev,next); ear[c_next] = isear(p,n,c_next,prev,next);
cbb1 	}
bc1d 	tri[s][0] = next[next[i]]; tri[s][1] = i; tri[s][2] = next[i]; s++; // tri[i][0],i,tri[i][1] inserted
0478 	return s;
cbb1 }
```

4.3 3D

```
f61c const double pi = acos(-1);
d41d // typedef double cood; cood eps = 1e-6; // risky: XXX, untested: TODO
3f73 struct pnt { // TODO it's not tested at all :)
5e43 	cood x, y, z;
cf2f 	pnt () : x(0), y(0), z(0) {} pnt (cood a, cood b, cood c) : x(a), y(b), z(c) {}
4e90 	inline pnt operator - (pnt o) { return pnt(x - o.x, y - o.y, z - o.z); }
2b18 	inline pnt operator + (pnt o) { return pnt(x + o.x, y + o.y, z + o.z); }
7470 	inline pnt operator * (cood o) { return pnt(x*o, y*o, z*o); }
8194 	inline pnt operator / (cood o) { return pnt(x/o, y/o, z/o); }
a269 	inline cood operator * (pnt o) { return x*o.x + y*o.y + z*o.z; } // inner: |this||o|*cos(ang)
```

```
079c    inline pnt operator ^ (pnt o) { return pnt(y*o.z - z*o.y, z*o.x - x*o.z, x*o.y - y*o.x); } // cross: oriented normal to the plane containing the two vectors, has norm |this||o|*sin(ang)
a2ea    inline cood operator () (pnt a, pnt b) { return (*this)*(a^b); } // mixed: positive on the right-hand rule (thumb=this,index=a,mid=b)
d41d
f500    inline cood inner (pnt a, pnt b) { return (a-(*this))*(b-(*this)); }
4114    inline pnt cross (pnt a, pnt b) { return (a-(*this))^(b-(*this)); } // its norm is twice area of triangle
fa90    inline cood mixed (pnt a, pnt b, pnt c) { return (a-(*this))(b-(*this),c-(*this)); } // 6 times the oriented area of thetahedra
d41d
4f78    inline cood sq (pnt o = pnt()) { return inner(o,o); }
113b    inline double nr (pnt o = pnt()) { return sqrt(sq(o)); }
6edf    inline pnt operator ~ () { return (*this)/nr(); }
d41d
11c0    inline bool in_seg (pnt a, pnt b) { return cross(a,b).sq() <= eps && inner(a,b) <= eps; } // tips included
a6b7    inline bool in_tri (pnt a, pnt b, pnt c) { return abs(mixed(a,b,c)) <= eps && cross(a,b)*cross(b,c) >= -eps && cross(a,b)*cross(c,a) >= -eps; } // border included$
d41d
7c26    inline pnt proj (pnt a, pnt b) { return a + (b-a)*a.inner(b,(*this))/a.sq(b); }
3a26    inline pnt proj (pnt a, pnt b, pnt c) { pnt n = a.cross(b,c); return (*this) - n*(n*((*this)-a))/n.sq(); }
d41d
8fbb    inline double dist2_lin (pnt a, pnt b) { return cross(a,b).sq()/a.sq(b); }
1880    inline double dist2_seg (pnt a, pnt b) { return a.inner(b,(*this))*b.inner(a,(*this)) <= eps ? min(sq(a),sq(b)) : dist2_lin(a,b); }
39c1    inline double dist_pln (pnt a, pnt b, pnt c) { return abs((~a.cross(b,c))*((*this)-a)); }
5bc2    inline double dist2_tri (pnt a, pnt b, pnt c) { pnt p = proj(a,b,c); return p.in_tri(a,b,c) ? sq(p) : min({ dist2_seg(a,b), dist2_seg(b,c), dist2_seg(c,a) }); }
2145 };
eb48 inline cood area (pnt a, pnt b, pnt c) { return abs(a.cross(b,c).nr()) / 2; }
a6c7 inline cood vol (pnt a, pnt b, pnt c, pnt d) { return abs(a.mixed(b,c,d)) / 6; } // thetahedra
084a pnt inter_lin_pln (pnt s, pnt t, pnt a, pnt b, pnt c) { pnt n = a.cross(b,c); return s + (t-s)*(n*(a-s))/(n*(t-s)); } //$
fabc struct sph { // TODO it's also not tested at all
af42    pnt c; cood r;
390f    sph () : c(), r(0) {} sph (pnt a, cood b) : c(a), r(b) {}
baaf    inline pnt operator () (cood lat, cood lon) { return c + pnt(cos(lat)*cos(lon), sin(lon), sin(lat))*r; } // (1,0,0) is (0,0). z is height.
171a    inline double area_hull (double h) { return 2.*pi*r*h; }
60a4    inline double vol_hull (double h) { return pi*h/6 * (3.*r*r + h*h); }
2145 };
```

5 Graphs

5.1 Dinic

```
d41d //typedef int num; const int N = ; const int M = * 2; const num eps = 0;
582d struct dinic {
656d    int hd[N], seen[N], qu[N], lv[N], ei[N], to[M], nx[M]; num fl[M], cp[M]; int en = 2; int when = 0;
1233    bool bfs(int s, int t) {
876c        seen[t] = ++when; lv[t] = 0; int ql = 0, qr = 0; qu[qr++] = t;
a872        while(ql != qr) {
036d            t = qu[ql++]; ei[t] = hd[t]; if(s == t) return true;
9a44            for(int e = hd[t]; e; e = nx[e]) if(seen[to[e]] != when && cp[e ^ 1] - fl[e ^ 1] > eps) {
d4fb                seen[to[e]] = when;
de5c                lv[to[e]] = lv[t] + 1;
f0ff                qu[qr++] = to[e];
cbb1            }
cbb1        }
d1fe        return false;
cbb1    }
a444    num dfs(int s, int t, num f) {
f449        if(s == t) return f;
cebe        for(int &e = ei[s]; e; e = nx[e]) if(ei[to[e]] && seen[to[e]] == when && cp[e] - fl[e] > eps && lv[to[e]] == lv[s] - 1)
7004            if(num rf = dfs(to[e], t, min(f, cp[e] - fl[e]))) {
805c                fl[e] += rf;
5226                fl[e ^ 1] -= rf;
```

```
2cb7                return rf;
cbb1            }
bb30        return 0;
cbb1    }
d41d    // public $
de22    num max_flow(int s, int t) {
6cb2        num fl = 0;
1c5e        while (bfs(s, t)) for(num f; (f = dfs(s, t, numeric_limits<num>::max())); fl += f);
e508        return fl;
cbb1    }
5a3f    void add_edge(int a, int b, num c, num rc=0) {
d03a        to[en] = b; nx[en] = hd[a]; fl[en] = 0; cp[en] = c; hd[a] = en++;
2f94        to[en] = a; nx[en] = hd[b]; fl[en] = 0; cp[en] = rc; hd[b] = en++;
cbb1    }
7415    void reset_flow() { memset(fl, 0, sizeof(num) * en); }
ae0a    void init(int n=N) { en = 2; memset(hd, 0, sizeof(int) * n); } // resets all
2145 };
```

5.2 MinCost MaxFlow

```
d41d //typedef int val; // type of flow
d41d //typedef int num; // type of cost
d41d //const int N = , M = * 2; const num eps = 0;
1854 struct mcmf {
7a62    int es[N], to[M], nx[M], en = 2, pai[N], seen[N], when, qu[N];
ef55    val fl[M], cp[M], flow; num cs[M], d[N], tot;
d0cc    val spfa(int s, int t) {
104f        when++; int a = 0, b = 0;
e0c6        for(int i = 0; i < N; i++) d[i] = numeric_limits<num>::max();
3518        d[s] = 0; qu[b++] = s; seen[s] = when;
9841        while(a != b) {
32d9            int u = qu[a++]; if(a == N) a = 0; seen[u] = 0;
a86f            for(int e = es[u]; e; e = nx[e]) if(cp[e] - fl[e] > val(0) && d[u] + cs[e] < d[to[e]] - eps) {
a694                d[to[e]] = d[u] + cs[e]; pai[to[e]] = e ^ 1;
85b7                if(seen[to[e]] < when) { seen[to[e]] = when; qu[b++] = to[e]; if(b == N) b = 0; }
cbb1            }
cbb1        }
8e2a        if(d[t] == numeric_limits<num>::max()) return false;
91fe        val mx = numeric_limits<val>::max();
285a        for(int u = t; u != s; u = to[pai[u]])
7039            mx = min(mx, cp[pai[u] ^ 1] - fl[pai[u] ^ 1]);
6de0        tot += d[t] * val(mx);
285a        for(int u = t; u != s; u = to[pai[u]])
4c48            fl[pai[u]] -= mx, fl[pai[u] ^ 1] += mx;
b9aa        return mx;
cbb1    }
d41d    // public $
8662    num min_cost(int s, int t) {
3b69        tot = 0; flow = 0;
e66e        while(val a = spfa(s, t)) flow += a;
126a        return tot;
cbb1    }
457a    void add_edge(int u, int v, val c, num s) {
1d08        fl[en] = 0; cp[en] = c; to[en] = v; nx[en] = es[u]; cs[en] = s; es[u] = en++;
8015        fl[en] = 0; cp[en] = 0; to[en] = u; nx[en] = es[v]; cs[en] = -s; es[v] = en++;
cbb1    }
8537    void reset_flow() { memset(fl, 0, sizeof(val) * en); }
451f    void init(int n) { en = 2; memset(es, 0, sizeof(int) * n); } // XXX must be called
2145 };
```

5.3 Cycle Cancelling

```
d41d //typedef int val; // type of flow
d41d //typedef int num; // type of cost
d41d //const int N = ; const int M = * 2; const val eps = 0;
afb2 struct cycle_cancel {
```

```
0f5c ▹ int hd[N], seen[N], qu[N], lv[N], ei[N], to[M], nx[M], ct[N], pai[N]; val fl[M], cp[M], flow; num cs[M],
    d[N], tot; int en = 2, n; int when = 0;
1233 ▹ bool bfs(int s, int t) {
876c ▹ ▹ seen[t] = ++when; lv[t] = 0; int ql = 0, qr = 0; qu[qr++] = t;
a872 ▹ ▹ while(ql != qr) {
036d ▹ ▹ ▹ t = qu[ql++]; ei[t] = hd[t]; if(s == t) return true;
9a44 ▹ ▹ ▹ for(int e = hd[t]; e; e = nx[e]) if(seen[to[e]] != when && cp[e ^ 1] - fl[e ^ 1] > eps) {
d4fb ▹ ▹ ▹ ▹ seen[to[e]] = when;
de5c ▹ ▹ ▹ ▹ lv[to[e]] = lv[t] + 1;
f0ff ▹ ▹ ▹ ▹ qu[qr++] = to[e];
cbb1 ▹ ▹ ▹ }
cbb1 ▹ ▹ }
d1fe ▹ ▹ return false;
cbb1 ▹ }
e4d9 ▹ val dfs(int s, int t, val f) {
f449 ▹ ▹ if(s == t) return f;
cebe ▹ ▹ for(int &e = ei[s]; e; e = nx[e]) if(ei[to[e]] && seen[to[e]] == when && cp[e] - fl[e] > eps &&
    lv[to[e]] == lv[s] - 1)
9fe1 ▹ ▹ ▹ if(val rf = dfs(to[e], t, min(f, cp[e] - fl[e]))) {
805c ▹ ▹ ▹ ▹ fl[e] += rf;
5226 ▹ ▹ ▹ ▹ fl[e ^ 1] -= rf;
2cb7 ▹ ▹ ▹ ▹ return rf;
cbb1 ▹ ▹ ▹ }
bb30 ▹ ▹ return 0;
cbb1 ▹ }
5cbe ▹ bool spfa() {
e2f3 ▹ ▹ when++; int a = 0, b = 0, u;
91bc ▹ ▹ for(int i = 0; i < n; i++) { d[i] = 0; qu[b++] = i; seen[i] = when; ct[i] = 0; }
9841 ▹ ▹ while(a != b) {
b492 ▹ ▹ ▹ u = qu[a++]; if(a == N) a = 0; seen[u] = 0;
d627 ▹ ▹ ▹ if(ct[u]++ >= n + 1) { a--; break; }
ccce ▹ ▹ ▹ for(int e = hd[u]; e; e = nx[e]) if(cp[e] - fl[e] > val(0) && d[u] + cs[e] < d[to[e]] - eps) {
a694 ▹ ▹ ▹ ▹ d[to[e]] = d[u] + cs[e]; pai[to[e]] = e ^ 1;
85b7 ▹ ▹ ▹ ▹ if(seen[to[e]] < when) { seen[to[e]] = when; qu[b++] = to[e]; if(b == N) b = 0; }
cbb1 ▹ ▹ ▹ }
cbb1 ▹ ▹ }
5c28 ▹ ▹ if(a == b) return false;
02be ▹ ▹ val mn = numeric_limits<val>::max();
be15 ▹ ▹ when++;
e855 ▹ ▹ for(; seen[u] != when; u = to[pai[u]]) seen[u] = when;
0612 ▹ ▹ for(int v = u; seen[v] != when + 1; v = to[pai[v]]) {
6e6b ▹ ▹ ▹ seen[v] = when + 1;
3225 ▹ ▹ ▹ mn = min(mn, cp[pai[v] ^ 1] - fl[pai[v] ^ 1]);
cbb1 ▹ ▹ }
ea26 ▹ ▹ for(int v = u; seen[v] == when + 1; v = to[pai[v]]) {
7618 ▹ ▹ ▹ seen[v] = 0;
60f1 ▹ ▹ ▹ fl[pai[v]] -= mn;
0329 ▹ ▹ ▹ fl[pai[v] ^ 1] += mn;
cbb1 ▹ ▹ }
8a6c ▹ ▹ return true;
cbb1 ▹ }
2b0e ▹ val max_flow(int s, int t) {
e7a0 ▹ ▹ val fl = 0;
036d ▹ ▹ while (bfs(s, t)) for(val f; (f = dfs(s, t, numeric_limits<val>::max())); fl += f);
e508 ▹ ▹ return fl;
cbb1 ▹ }
d41d ▹ // public $
8662 ▹ num min_cost(int s, int t) {
94a7 ▹ ▹ flow = max_flow(s, t);
6c9f ▹ ▹ while(spfa());
ed25 ▹ ▹ tot = 0;
112e ▹ ▹ for(int i = 2; i < en; i++)
b951 ▹ ▹ ▹ if(fl[i] > 0)
dae8 ▹ ▹ ▹ ▹ tot += fl[i] * cs[i];
126a ▹ ▹ return tot;
cbb1 ▹ }
8537 ▹ void reset_flow() { memset(fl, 0, sizeof(val) * en); }
457a ▹ void add_edge(int u, int v, val c, num s) {
d321 ▹ ▹ fl[en] = 0; cp[en] = c; to[en] = v; nx[en] = hd[u]; cs[en] = s; hd[u] = en++;
```

```
f081 ▹ ▹  fl[en] = 0; cp[en] = 0; to[en] = u; nx[en] = hd[v]; cs[en] = -s; hd[v] = en++;
cbb1 ▹  }
bfc4 ▹  void init(int n) { this->n = n; en = 2; memset(hd, 0, sizeof(int) * n); } // XXX must be called
2145 };
```

6 Structures

6.1 Ordered Set

```
7747 #include <ext/pb_ds/assoc_container.hpp>
30f4 #include <ext/pb_ds/tree_policy.hpp>
0d73 using namespace __gnu_pbds;
4519 template <typename tA, typename tB=null_type> using ord_set = tree<tA, tB, less<tA>, rb_tree_tag,
     tree_order_statistics_node_update>;
d41d // map: tA -> tB with the less<tA> comparison function
d41d // can be used as a normal map
d41d // s.find_by_order(k) :: returns iterator to the k-th element (0-indexed) (or s.end())
d41d // s.order_of_key(x) :: returns how many elements are strictly less than x
```

6.2 Treap

```
d41d //const int N = ; typedef int num;
5463 num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
8b25 void calc (int u) { // update node given children info
d4c7 ▹  sz[u] = sz[L[u]] + 1 + sz[R[u]];
d41d ▹  // code here, no recursion
cbb1 }
234f void unlaze (int u) {
e39f ▹  if(!u) return;
d41d ▹  // code here, no recursion
cbb1 }
ee5e void split_val(int u, num x, int &l, int &r) { // l gets <= x, r gets > x
754f ▹  unlaze(u); if(!u) return (void) (l = r = 0);
4bc1 ▹  if(X[u] <= x) { split_val(R[u], x, l, r); R[u] = l; l = u; }
81a7 ▹  else { split_val(L[u], x, l, r); L[u] = r; r = u; }
aaa8 ▹  calc(u);
cbb1 }
9374 void split_sz(int u, int s, int &l, int &r) { // l gets first s, r gets remaining
754f ▹  unlaze(u); if(!u) return (void) (l = r = 0);
e06d ▹  if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1, l, r); R[u] = l; l = u; }
f524 ▹  else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
aaa8 ▹  calc(u);
cbb1 }
c870 int merge(int l, int r) { // els on l <= els on r
67f0 ▹  unlaze(l); unlaze(r); if(!l || !r) return l + r; int u;
7801 ▹  if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
ae90 ▹  else { L[r] = merge(l, L[r]); u = r; }
0ffd ▹  calc(u); return u;
cbb1 }
500b void init(int n=N-1) { // XXX call before using other funcs
7d1c ▹  for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] = 1; L[i] = R[i] = 0; }
8c5a ▹  random_shuffle(Y + 1, Y + n + 1);
cbb1 }
```

6.3 Envelope

```
d41d // typedef ll num; const num eps = 0;
d41d // XXX double: indicates operations specific to integers, not precision related
d79f template<typename line> struct envelope {
5e0f ▹  deque<line> q; num lo,hi; envelope (num _lo, num _hi) : lo(_lo), hi(_hi) {}
01ca ▹  void push_front (line l) { // amort. O(inter) | l is best at lo or never
a86b ▹ ▹  if (q.size() && q[0](lo) < l(lo)) return;
89b8 ▹ ▹  for (num x; q.size(); q.pop_front()) {
cc18 ▹ ▹ ▹  x = (q.size()<=1?hi:q[0].inter(q[1],lo,hi)-1); // XXX double (-1)
4202 ▹ ▹ ▹  if (l(x) > q[0](x)) break;
```

```
cbb1         }
45bc         q.push_front(l);
cbb1     }
f644     void push_back (line l) { // amort. O(inter) | l is best at hi or never
0334         if (q.size() && q[q.size()-1](hi) <= l(hi)) return;
b71c         for (num x; q.size(); q.pop_back()) {
4e80             x = (q.size()<=1?lo:q[q.size()-2].inter(q[q.size()-1],lo,hi));
1747             if (l(x) >= q[q.size()-1](x)) break;
cbb1         }
5e56         q.push_back(l);
cbb1     }
e732     void pop_front (num _lo) { for (lo=_lo; q.size()>1 && q[0](lo) > q[1](lo); q.pop_front()); } // amort.
    O(n)
218a     void pop_back (num _hi) { for (hi=_hi; q.size()>1 && q[q.size()-2](hi) <= q[q.size()-1](hi);
    q.pop_back()); } // amort. O(n)
7155     line get (num x) { // O(lg(R))
e32f         int lo, hi, md; for (lo = 0, hi = q.size()-1, md = (lo+hi)/2; lo < hi; md = (lo+hi)/2)
c1fb             if (q[md](x) > q[md+1](x)) { lo = md+1; }
b029             else { hi = md; }
adf9         return q[lo];
cbb1     }
2145 };
b3a6 struct line { // inter = O(1)
7bd4     num a,b; num operator () (num x) const { return a*x+b; }
2417     num inter (line o, num lo, num hi) { return
    abs(o.a-a)<=eps?((b<o.b)?hi+1:lo):min(hi+1,max(lo,(o.b-b-(o.b-b<0)*(a-o.a-1))/(a-o.a) + 1)); }
2145 };
16ed struct generic_line { // inter = O(lg(R))
7bd4     num a,b; num operator () (num x) const { return a*x+b; }
3cfe     num inter (generic_line o, num lo, num hi) { // first point where o strictly beats this
ca4f         for (num md = lo+((++hi)-lo)/2; lo < hi; md = lo+(hi-lo)/2) { // XXX double
760b             if ((*this)(md)<=o(md)) { lo = md+1; } // XXX double
b029             else { hi = md; }
cbb1         }
2532         return lo;
cbb1     }
2145 };
11a2 template<typename line> struct full_envelope { // XXX ties are broken arbitrarily
85c9     vector<envelope<line> > v; full_envelope(envelope<line> c) : v({c}) {} // v.reserve(30);
6aed     void add (line l) { // amort. O(lg(n)*inter)
8cca         envelope<line> cur(v.back().lo,v.back().hi); cur.push_back(l);
bb4a         while (!v.empty() && v.back().q.size() <= cur.q.size()) {
ce29             deque<line> aux; swap(aux,cur.q); int i = 0, j = 0;
31d2             for (; i < aux.size(); i++) {
542d                 for (; j < v.back().q.size() && v.back().q[j](cur.hi) > aux[i](cur.hi); j++)
0015                     cur.push_back(v.back().q[j]);
70a1                 cur.push_back(aux[i]);
cbb1             }
a0e7             for (; j < v.back().q.size(); j++) cur.push_back(v.back().q[j]);
deff             v.pop_back();
cbb1         }
026e         v.push_back(cur);
cbb1     }
7155     line get (num x) { // O(lg(n)lg(R)) | pop_back/pop_front can optimize
9351         line a = v[0].get(x);
ad67         for (int i = 1; i < (int) v.size(); i++) {
bcbe             line b = v[i].get(x);
ad0f             if (b(x)<a(x)) a = b;
cbb1         }
3f53         return a;
cbb1     }
2145 };
```

6.4 Centroid

```
0eca vector<int> adj[N]; int cn_sz[N], n;
c864 vector<int> cn_chld[N]; int cn_dep[N], cn_dist[20][N]; // removable
ace4 void cn_setdist (int u, int p, int depth, int dist) { // removable
```

```
989e     cn_dist[depth][u] = dist;
59dd     for (int v : adj[u]) if (p != v && cn_sz[v] != -1) // sz = -1 marks processed centroid (not dominated)
4ce5         cn_setdist(v, u, depth, dist+1);
cbb1 }
e897 int cn_getsz (int u, int p) {
08c9     cn_sz[u] = 1;
59dd     for (int v : adj[u]) if (p != v && cn_sz[v] != -1)
b2f6         cn_sz[u] += cn_getsz(v,u);
37a9     return cn_sz[u];
cbb1 }
912c int cn_build (int u, int depth) {
28a0     int siz = cn_getsz(u,u); int w = u;
0168     do {
9847         u = w;
a786         for (int v : adj[u]) if (cn_sz[v] != -1 && cn_sz[v] < cn_sz[u] && cn_sz[v] + cn_sz[v] >= siz)
9a13             w = v;
06ba     } while (u != w); // u becomes current centroid root
094e     cn_setdist(u,u,depth,0); // removable, here you can iterate over all dominated tree
32c2     cn_sz[u] = -1; cn_dep[u] = depth;
5cff     for (int v : adj[u]) if (cn_sz[v] != -1) {
1df5         int w = cn_build(v, depth+1);
2e31         cn_chld[u].pb(w); // removable
cbb1     }
03f4     return u;
cbb1 }
```

6.5 Splay Tree

```
d41d //const int N = ;
d41d //typedef int num;
d41d
576f int en = 1;
37e4 int p[N], sz[N];
c7d4 int C[N][2]; // {left, right} children
abac num X[N];
d41d
d41d // update values associated to the nodes that can be calculated from child
8b25 void calc(int u) {
5665     sz[u] = sz[C[u][0]] + 1 + sz[C[u][1]];
cbb1 }
d41d
d41d // pull child dir of u to its position and return
0584 int rotate(int u, int dir) {
05db     int v = C[u][dir];
2116     C[u][dir] = C[v][!dir];
6c8a     if(C[u][dir]) p[C[u][dir]] = u;
0928     C[v][!dir] = u;
c0a7     p[v] = p[u];
b9c1     if(p[v]) C[p[v]][C[p[v]][1] == u] = v;
136e     p[u] = v;
aaa8     calc(u);
b6b0     calc(v);
6dc7     return v;
cbb1 }
d41d
d41d // bring node u to root
81a1 void splay(int u) {
bdd0     while(p[u]) {
2a84         int v = p[u], w = p[p[u]];
1a8a         int du = C[v][1] == u;
e764         if(!w)
76c8             rotate(v, du);
4e6b         else {
d499             int dv = (C[w][1] == v);
9b57             if(du == dv) {
6c72                 rotate(w, dv);
76c8                 rotate(v, du);
9d97             } else {
```

```
76c8                rotate(v, du);
6c72                rotate(w, dv);
cbb1            }
cbb1        }
cbb1    }
cbb1 }
d41d
d41d // return node with value x or other if node was not found
8975 int find_val(int u, num x) {
93fe    int v = u;
9a3d    while(u && X[u] != x) {
766a        v = u;
1b5b        if(x < X[u]) u = C[u][0];
a73d        else u = C[u][1];
cbb1    }
3418    if(!u) u = v;
6d13    splay(u);
03f4    return u;
cbb1 }
d41d
d41d // return nth node
a7c2 int find_sz(int u, int s) {
3939    while(sz[C[u][0]] != s) {
7ef0        if(sz[C[u][0]] < s) {
2777            s -= sz[C[u][0]] + 1;
6bdb            u = C[u][1];
66d9        } else u = C[u][0];
cbb1    }
6d13    splay(u);
03f4    return u;
cbb1 }
d41d
d41d // concatenate two trees assuming #elements l <= #elements r
c870 int merge(int l, int r) {
db1b    if(!l || !r) return l + r;
45ba    while(C[l][1]) l = C[l][1];
bab4    splay(l);
0258    assert(!C[l][1]);
e3ec    C[l][1] = r;
924c    p[r] = l;
f046    calc(l);
792f    return l;
cbb1 }
d41d
d41d // add node x to splay u and return x
684a int add(int u, int x) {
e29c    int v = 0;
9d2d    while(u) v = u, u = C[u][X[x] >= X[u]];
f257    if(v) { C[v][X[x] >= X[v]] = x; p[x] = v; }
0b6f    splay(x);
ea56    return x;
cbb1 }
d41d
d41d // call 1 time at the top
ca2f void init() {
0cee    en = 1;
cbb1 }
d41d
d41d // create a new node
3e8b int new_node(num val) {
cecb    int i = en++;
9c38    assert(i < N);
9029    C[i][0] = C[i][1] = p[i] = 0;
02c8    sz[i] = 1;
4281    X[i] = val;
d9a5    return i;
cbb1 }
```

7 Strings

7.1 Z-function

```
2a61 void Z(char s[], int n, int z[]) { // z[i] = |lcp(s,s[i..n])|
fc15     for(int i = 1, m = -1; i < n; i++) {
d69b         z[i] = (m != -1 && m + z[m] >= i)?min(m + z[m] - i, z[i - m]):0;
8a63         while (i + z[i] < n && s[i + z[i]] == s[z[i]]) z[i]++;
bbe8         if (m == -1 || i + z[i] > m + z[m]) m = i;
cbb1     }
cbb1 }
```

8 Math

8.1 Linear System Solver

```
d41d //const int N = ;
d41d
46cc double a[N][N];
3793 double ans[N];
d41d
d41d // sum(a[i][j] * x_j) = a[i][n] for 0 <= i < n
d41d // stores answer in ans and returns det(a)
c42a double solve(int n) {
f99b     double det = 1;
6033     for(int i = 0; i < n; i++) {
0268         int mx = i;
197a         for(int j = i + 1; j < n; j++)
b83d             if(abs(a[j][i]) > abs(a[mx][i]))
672f                 mx = j;
28c6         if(i != mx) {
e83f             swap_ranges(a[i], a[i] + n + 1, a[mx]);
0143             det = -det;
cbb1         }
997e         if(abs(a[i][i]) < 1e-6); // singular matrix
2f40         det *= a[i][i];
94fe         for(int j = i + 1; j < n; j++) {
12fe             for(int k = i + 1; k <= n; k++)
ea32                 a[j][k] -= (a[j][i] / a[i][i]) * a[i][k];
efbc             a[j][i] = 0;
cbb1         }
cbb1     }
45bd     for(int i = n - 1; i >= 0; i--) {
7634         ans[i] = a[i][n];
197a         for(int j = i + 1; j < n; j++)
9b00             ans[i] -= a[i][j] * ans[j];
35e5         ans[i] /= a[i][i];
cbb1     }
7a32     return det;
cbb1 }
```

8.2 Simplex

```
d41d //typedef long double dbl;
bec0 const dbl eps = 1e-6;
d41d //const int N = , M = ;
d41d
79ee struct simplex {
0643     int X[N], Y[M];
6b50     dbl A[M][N], b[M], c[N];
e268     dbl ans;
14e0     int n, m;
a00d     dbl sol[N];
d41d
c511     void pivot(int x,int y){
eb91         swap(X[y], Y[x]);
```

```
c057        b[x] /= A[x][y];
8300        for(int i = 0; i < n; i++)
7f61            if(i != y)
d311                A[x][i] /= A[x][y];
3fa2        A[x][y] = 1. / A[x][y];
94f7        for(int i = 0; i < m; i++)
a325            if(i != x && abs(A[i][y]) > eps) {
6856                b[i] -= A[i][y] * b[x];
f90a                for(int j = 0; j < n; j++)
6739                    if(j != y)
8c78                        A[i][j] -= A[i][y] * A[x][j];
e112                A[i][y] = -A[i][y] * A[x][y];
cbb1            }
8c7e        ans += c[y] * b[x];
8300        for(int i = 0; i < n; i++)
7f61            if(i != y)
bec1                c[i] -= c[y] * A[x][i];
0997        c[y] = -c[y] * A[x][y];
cbb1    }
d41d
d41d    // maximize sum(x[i] * c[i])
d41d    // element a
d41d    //  sum(a[i][j] * x[j]) <= b[i] for 0 <= i < m (Ax <= b)
d41d    //  x[i] >= 0 for 0 <= i < n (x >= 0)
d41d    // (n variables, m constraints)
d41d    // stores the answer in ans and returns optimal value
59d9    dbl solve(int n, int m) {
1f59        this->n = n; this->m = m;
f1bf        ans = 0.;
b1c6        for(int i = 0; i < n; i++) X[i] = i;
3e36        for(int i = 0; i < m; i++) Y[i] = i + n;
6679        while(true) {
ee39            int x = min_element(b, b + m) - b;
988b            if(b[x] >= -eps)
c2be                break;
49a2            int y = find_if(A[x], A[x] + n, [](dbl d) { return d < -eps; }) - A[x];
6f8c            if(y == n) throw 1; // no solution
7fb4            pivot(x, y);
cbb1        }
6679        while(true) {
f802            int y = max_element(c, c + n) - c;
b7b6            if(c[y] <= eps) break;
d6b5            int x = -1;
06d7            dbl mn = 1. / 0.;
94f7            for(int i = 0; i < m; i++)
5877                if(A[i][y] > eps && b[i] / A[i][y] < mn)
832b                    mn = b[i] / A[i][y], x = i;
ff22            if(x == -1) throw 2; // unbounded
7fb4            pivot(x, y);
cbb1        }
d094        memset(sol, 0, sizeof(dbl) * n);
94f7        for(int i = 0; i < m; i++)
cff4            if(Y[i] < n)
09d7                sol[Y[i]] = b[i];
ba75        return ans;
cbb1    }
2145 };
```

9 Number Theory

9.1 Extended Euclidean Algorithm

```
c25f int egcd(int a, int b, int& x, int& y) { // a*x + b*y = gcd(a, b) [Bezout's Theorem]
8273    if (b == 0) return x = 1, y = 0, a;
98d1    int xx, yy;
0c0d    int g = egcd(b, a % b, xx, yy);
512d    x = yy;
```

```
a9d0     y = xx - (a / b) * yy;
96b5     return g;
cbb1 }
```

9.2 Miller-Rabin

```
a288 llu llrand() { llu a = rand(); a<<= 32; a+= rand(); return a;}
0a9c int is_probably_prime(llu n) {
8dbf    if (n <= 1) return 0;
2373    if (n <= 3) return 1;
7de1    llu s = 0, d = n - 1;
66b4    while (d % 2 == 0) {
90f4       d/= 2; s++;
cbb1    }
6b3a    for (int k = 0; k < 64; k++) {
12c0       llu a = (llrand() % (n - 3)) + 2;
dc17       llu x = exp_mod(a, d, n);
1181       if (x != 1 && x != n-1) {
f0ea          for (int r = 1; r < s; r++) {
708d             x = mul_mod(x, x, n);
61d9             if (x == 1)
bb30                return 0;
68b2             if (x == n-1)
c2be                break;
cbb1          }
34bc          if (x != n-1)
bb30             return 0;
cbb1       }
cbb1    }
6a55    return 1;
cbb1 }
```

10 Notes

10.1 Modular Multiplicative Inverse

- If $gcd(a,m) = 1$, then let $ax + my = gcd(a,m) = 1$ (Bezout's Theorem). Then $ax \equiv 1 \pmod{m}$.
- If $gcd(a,m) = 1$, then $a \cdot a^{\phi(m)-1} \equiv 1 \pmod{m}$ (Euler's Theorem).
- If m is prime, then $\phi(m) = m - 1$, so $a * a^{m-2} \equiv 1 \pmod{m}$.

10.2 Chinese Remainder Theorem

We are given $N = n_1 n_2 \cdots n_k$ where n_i are pairwise coprime. We are also given $x_1 \cdots x_k$ such that $x \equiv x_i \pmod{n_i}$. Let $N_i = N/n_i$. There exists M_i and m_i such that $M_i N_i + m_i n_i = 1$ (Bezout). Then, there is only one solution x, given by: $x = \sum_{i=1}^{k} a_i M_i N_i$

10.3 Euler's Totient Function

Positive integers up to a given integer n that are relatively prime to n. $\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$ where the product is over the distinct prime numbers dividing n.

10.4 Möebius

If $F(n) = \sum_{d|n} f(d)$, then $f(n) = \sum_{d|n} \mu(d) F(n/d)$.

10.5 Burnside

Let $A\colon GX \to X$ be an action. Define:

- $w :=$ number of orbits in X.

- $S_x := \{g \in G \mid g \cdot x = x\}$
- $F_g := \{x \in X \mid g \cdot x = x\}$

Then $w = \frac{1}{|G|} \sum\limits_{x \in X} |S_x| = \frac{1}{|G|} \sum\limits_{g \in G} |F_g|$.

10.6 Catalan Number

C_n is solution for:

- Number of correct bracket sequence consisting of n opening and n closing brackets.
- The number of rooted full binary trees with $n+1$ leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.
- The number of ways to completely parenthesize $n+1$ factors.
- The number of triangulations of a convex polygon with $n+2$ sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).
- The number of ways to connect the $2n$ points on a circle to form n disjoint chords.
- The number of non-isomorphic full binary trees with n internal nodes (i.e. nodes having at least one son).
- The number of monotonic lattice paths from point $(0,0)$ to point (n,n) in a square lattice of size nn, which do not pass above the main diagonal (i.e. connecting $(0,0)$ to (n,n)).
- Number of permutations of length n that can be stack sorted (i.e. it can be shown that the rearrangement is stack sorted if and only if there is no such index $i < j < k$, such that $a_k < a_i < a_j$).
- The number of non-crossing partitions of a set of n elements.
- The number of ways to cover the ladder $1 \ldots n$ using n rectangles (The ladder consists of n columns, where i^{th} column has a height i).

Recursive:

$$C_0 = C_1 = 1$$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, n \geq 2$$

Analytical:

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

10.7 Landau

There is a tournament with outdegrees $d_1 \leq d_2 \leq \ldots \leq d_n$ iff:

- $d_1 + d_2 + \ldots + d_n = \binom{n}{2}$
- $d_1 + d_2 + \ldots + d_k \geq \binom{k}{2} \quad \forall 1 \leq k \leq n$.

In order to build it, let 1 point to $2, 3, \ldots, d_1 + 1$ and repeat recursively.

10.8 Erdös-Gallai

There is a simple graph with degrees $d_1 \geq d_2 \geq \ldots \geq d_n$ iff:

- $d_1 + d_2 + \ldots + d_n$ is even
- $\sum\limits_{i=1}^{k} d_i \leq k(k-1) + \sum\limits_{i=k+1}^{n} \min(d_i, k) \quad \forall 1 \leq k \leq n$.

In order to build it, connect 1 with $2, 3, \ldots, d_1 + 1$ and repeat recursively.

10.9 Gambler's Ruin

In a game in which we win a coin with probability p and lose a coin with probability $q := 1 - p$, the game stops when we win B ou lose A coins. Then $Prob(\text{win B}) = \frac{1-(p/q)^B}{1-(p/q)^{A+B}}$.

10.10 Extra

- $Fib(x + y) = Fib(x + 1)Fib(y) + Fib(x)Fib(y - 1)$