

# ACM ICPC Reference

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## Contents

## 1 vimrc

```

syntax on
colors evening
set ai si noet ts=4 sw=4 sta sm nu so=7 t_Co=8
imap {<CR> {<CR><Esc>0

```

## 2 hashify.sh

```

#!/bin/bash
while IFS=$'\n' read -r line; do
    trim=$(echo "$line" | tr -d "[:space:]")
    md5=$(echo -n "${trim%\/*}" | md5sum)
    md5=${md5:0:4}
    [ "${trim:~0}" == "$" ] && md5="@${md5}"
    echo "$md5 $line"
done

```

## 3 STL

?

## 4 Geometry

### 4.1 Base

```

d41d // typedef double cood; cood eps = 1e-8; // risky: XXX, untested: TODO
00a0 const double pi = acos(-1.);
ccb5 template<typename T> inline T sq(T x) { return x*x; }
87bc struct vec {
b86a > cood x, y;
6e4f > vec () : x(0), y(0) {} vec (cood a, cood b) : x(a), y(b) {}
741a > inline vec operator - (vec o) { return {x - o.x, y - o.y}; }
ff7e > inline vec operator + (vec o) { return {x + o.x, y + o.y}; }
b6dd > inline vec operator * (cood o) { return {x * o, y * o}; }
2711 > inline vec operator / (cood o) { return {x / o, y / o}; }
6ac9 > inline cood operator ^ (vec o) { return x * o.y - y * o.x; }
83dd > inline cood operator * (vec o) { return x * o.x + y * o.y; }
46ef > inline cood cross (vec a, vec b) { return ((*this)-a) ^ ((*this)-b); } // |(this)a||this)b|sen(angle)
cbad > inline cood inner (vec a, vec b) { return ((*this)-a) * ((*this)-b); } // |(this)a||this)b|cos(angle)
cddd > inline double angle (vec a, vec b) { return atan2(cross(a,b),inner(a,b)); } // ccw angle from (this)a to
    (this)b in range [-pi,pi]
e4d3 > inline int ccw (vec a, vec b) { cood o = cross(a,b); return (eps < o) - (o < -eps); } // this is to the
    (1 left, 0 over, -1 right) of ab
2e1f > inline int dir (vec a, vec b) { cood o = inner(a,b); return (eps < o) - (o < -eps); } // a(this) is to
    the (1 same, 0 none, -1 opposite) direction of ab
5d26 > inline cood sq (vec o = vec()) { return inner(o,o); }
e7cf > inline double nr (vec o = vec()) { return sqrt(sq(o)); } //$
4e72 > inline vec operator ~ () { return (*this)/nr(); }
f149 > inline vec proj (vec a, vec b) { return a + (b-a)*(a.inner((*this),b) / a.sq(b)); } // projects this onto
    line ab
1664 > inline vec rotate (double a) { return vec(cos(a) * x - sin(a) * y, sin(a) * x + cos(a) * y); } // ccw by
    a radians
3206 > inline vec rot90 () { return vec(-y,x); } // rotate(pi/2)$
2810 > bool in_seg (vec a, vec b) { return ccw(a,b) == 0 && dir(a,b) <= 0; } // tips included
5e56 > double dist2_lin (vec a, vec b) { return a.sq(b) <= eps ? sq(a) : double(::sq(cross(a,b)))/a.sq(b); } //
    see cir.has_inter_lin
8831 > double dist2_seg (vec a, vec b) { return a.dir((*this),b) == (b.dir((*this),a)) ? dist2_lin(a,b) :
    min(sq(a),sq(b)); }
436b > inline bool operator == (const vec & o) const { return abs(x-o.x) <= eps && abs(y-o.y) <= eps; }
5522 > inline bool operator < (const vec & o) const { return (abs(x-o.x)>eps)?(x < o.x):(y > o.y); } // lex
    compare (inc x, dec y)
d41d > // full ccw angle strict compare beginning upwards (this+(0,1)) around (*this)
d41d > // incresing distance on ties, this is the first

```

```

69ad > bool compare (vec a, vec b) {
a482 > >   if ((*this < a) != (*this < b)) return *this < b;
bdb1 > >   int o = ccw(a,b); return o>0:((a == *this && !(a == b)) || a.dir(*this,b) < 0);
cbb1 > }
2145 > }; // $
bafe struct lin { // line
6143 > vec p; cood c; // p*(x,y) = c
1105 > lin () {} lin (vec a, cood b) : p(a), c(b) {}
d036 > lin (vec s, vec t) : p((s-t).rot90()), c(p*s) {}
5c8b > inline lin parll (vec v) { return lin(p,v*p); }
1263 > inline lin perp () { return lin(p.rot90(),c); }
3838 > vec inter (lin o) { if (vec(0,0).ccw(p,o.p) == 0) throw 1; cood d = (p^o.p); return vec((c*o.p.y -
p.y*o.c)/d,(o.c*p.x - o.p.x*c)/d); }
1375 > bool contains (vec v) { return abs(p*v - c) <= eps; }
eda5 > vec at_x (cood x) { return vec(x,(c-p.x*x)/p.y); }
c0fb > vec at_y (cood y) { return vec((c-y*p.y)/p.x,y); }
elif > double sign_dist (vec v) { return double(p*v - c)/p.nr(); }
2145 > }; // $
3236 struct cir { // circle
b6d3 > vec c; cood r;
126a > cir () {} cir (vec v, cood d) : c(v), r(d) {}
c118 > cir (vec u, vec v, vec w) { // XXX untreated degenerates
0fb6 > >   vec mv = (u+v)/2; lin s(mv, mv+(v-u).rot90());
bf5f > >   vec mw = (u+w)/2; lin t(mw, mw+(w-u).rot90());
a0c4 > >   c = s.inter(t); r = c.nr(u);
cbb1 > } // $
9e54 > inline bool contains (vec w) { return c.sq(w) <= sq(r) + eps; } // border included
0549 > inline bool border (vec w) { return abs(c.sq(w) - sq(r)) <= eps; }
1cd6 > inline bool has_inter (cir o) { return c.sq(o.c) <= sq(r + o.r) + eps; } // borders included
376d > inline bool has_border_inter (cir o) { return has_inter(o) && c.sq(o.c) + eps >= sq(r - o.r); }
8ab4 > inline bool has_inter_lin (vec a, vec b) { return a.sq(b) <= eps ? contains(a) : sq(c.cross(a,b)) <=
sq(r)*a.sq(b) + eps; } // borders included XXX overflow
9bf7 > inline bool has_inter_seg (vec a, vec b) { return has_inter_lin(a,b) && (contains(a) || contains(b) ||
a.dir(c,b)*b.dir(c,a) != -1); } // borders and tips included XXX overflow
7abe > inline double arc_area (vec a, vec b) { return c.angle(a,b)*r*r/2; } // smallest arc, ccw positive
f967 > inline double arc_len (vec a, vec b) { return c.angle(a,b)*r; } // smallest arc, ccw positive $
771f > pair<vec,vec> tan (vec v) { // XXX low precision
84ec > >   if (contains(v) && !border(v)) throw 0;
2894 > >   cood d2 = c.sq(v); double s = sqrt(d2 - r*r); s = (s==s)?s:0;
0f70 > >   double al = atan2(r,s); vec t = ~(c-v);
3a69 > >   return pair<vec,vec>(v + t.rotate(al)*s, v + t.rotate(-al)*s);
cbb1 > } // $
c56f > pair<vec,vec> border_inter (cir o) {
c4d4 > >   if (!has_border_inter(o) || o.c == (*this).c) throw 0;
2b40 > >   double a = (sq(r) + o.c.sq(c) - sq(o.r))/(2*o.c.nr(c));
b647 > >   vec v = (o.c - c)/o.c.nr(c); vec m = c + v * a;
65b9 > >   double h = sqrt(sq(r) - sq(a)); h = h!=h?0:h;
440c > >   return pair<vec,vec>(m + v.rot90()*h, m - v.rot90()*h);
cbb1 > } // $
5182 > pair<vec,vec> border_inter_lin (vec a, vec b) { // first is closest to a than second
c6e7 > >   if (a.sq(b) <= eps) { if (border(a)) return pair<vec,vec>(a,a); throw 0; }
40f6 > >   if (a.dir(b,c) == -1) swap(a,b);
45ab > >   if (!has_inter_lin(a,b)) throw 0;
5cb6 > >   double d2 = c.dist2_lin(a,b); vec p = (b-a)/a.nr(b);
0aca > >   double h = sqrt(r*r - d2); h = h!=h?0:h;
ddf2 > >   double y = sqrt(c.sq(a) - d2); y = y!=y?0:y;
5539 > >   return pair<vec,vec>(a + p*(y-h), a + p*(y+h));
cbb1 > } // $
be35 > double triang_inter (vec a, vec b) { // ccw oriented, this with (c,a,b)
53ba > >   if (c.sq(a) > c.sq(b)) return -triang_inter(b,a);
148a > >   if (contains(b)) return c.cross(a,b)/2;
7434 > >   if (!has_inter_seg(a,b)) return arc_area(a,b);
773a > >   pair<vec,vec> itr = border_inter_lin(b,a); // order important
12a9 > >   if (contains(a)) return c.cross(a,itr.first)/2 + arc_area(itr.first,b);
c2f4 > >   return arc_area(a,itr.second) + c.cross(itr.second,itr.first)/2 + arc_area(itr.first,b);
cbb1 > }
2145 > }; // $
a71b bool inter_seg (vec a, vec b, vec c, vec d) {
2397 > if (a.in_seg(c, d) || b.in_seg(c, d) || c.in_seg(a, b) || d.in_seg(a, b)) return true;

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bbbd ▶ return (c.ccw(a, b) * d.ccw(a, b) == -1 && a.ccw(c, d) * b.ccw(c, d) == -1);
cbb1 }
e0fd double dist2_seg (vec a, vec b, vec c, vec d){return inter_seg(a,b,c,d)?0.:min({ a.dist2_seg(c,d),
b.dist2_seg(c,d), c.dist2_seg(a,b), d.dist2_seg(a,b) });}
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## 4.2 Advanced

```
484c cir min_spanning_circle (vec * v, int n) { // n
flea ▶ srand(time(NULL)); random_shuffle(v, v+n); cir c(vec(), 0); int i,j,k;
b11a ▶ for (i = 0; i < n; i++) if (!c.contains(v[i]))
e5b6 ▶ ▶ for (c = cir(v[i],0), j = 0; j < i; j++) if (!c.contains(v[j]))
a47c ▶ ▶ ▶ for (c = cir((v[i] + v[j])/2,v[i].nr(v[j])/2), k = 0; k < j; k++) if (!c.contains(v[k]))
3dd3 ▶ ▶ ▶ ▶ c = cir(v[i],v[j],v[k]);
807f ▶ return c;
cbb1 }//$
d45c int convex_hull (vec * v, int n, int border_in) { // nlg | border_in (should border points stay?)
4f17 ▶ swap(v[0], *min_element(v,v+n)); int s, i;
f37e ▶ sort(v+1, v+n, [&v] (vec a, vec b) { int o = b.ccw(v[0], a); return (o?o==1:v[0].sq(a)<v[0].sq(b)); });
a69c ▶ if (border_in) {
9492 ▶ ▶ for (s = n-1; s > 1 && v[s].ccw(v[s-1],v[0]) == 0; s--);
0bb0 ▶ ▶ reverse(v+s, v+n);
cbb1 ▶ }
c497 ▶ for (i = s = 0; i < n; i++) if (!s || !(v[s-1] == v[i])) {
cea9 ▶ ▶ for (; s >= 2 && v[s-1].ccw(v[s-2],v[i]) >= border_in; s--);
ceca ▶ ▶ swap(v[s++],v[i]);
cbb1 ▶ }
0478 ▶ return s;
cbb1 }//$
79b9 int monotone_chain (vec * v, int n, int border_in) { // nlg | border_in (should border points stay?)
5031 ▶ vector<vec> r; sort(v, v+n); n = unique(v, v+n) - v;
d885 ▶ for (int i = 0; i < n; r.pb(v[i++])) while (r.size() >= 2 && r[r.size()-2].ccw(r.back(),v[i]) <=
-border_in) r.pop_back();
dd80 ▶ r.pop_back(); unsigned int s = r.size();
c19d ▶ for (int i = n-1; i >= 0; r.pb(v[i--])) while (r.size() >= s+2 && r[r.size()-2].ccw(r.back(),v[i]) <=
-border_in) r.pop_back();
a255 ▶ return copy(r.begin(), r.end() - (r.size() > 1), v) - v;
cbb1 }//$
f80f double polygon_inter (vec * p, int n, cir c) { // signed area
2eae ▶ return inner_product(p, p+n-1, p+1, c.triang_inter(p[n-1],p[0]), std::plus<double>(), [&c] (vec a, vec b)
{ return c.triang_inter(a,b); });
cbb1 }//$
3214 int polygon_pos (vec * p, int n, vec v) { // lg | p should be simple (-1 out, 0 border, 1 in)
6c2a ▶ int in = -1; // it's a good idea to randomly rotate the points in the double case, numerically safer
6033 ▶ for (int i = 0; i < n; i++) {
2bca ▶ ▶ vec a = p[i], b = p[(i-1)>0?i-1:n-1]; if (a.x > b.x) swap(a,b);
c9e9 ▶ ▶ if (a.x + eps <= v.x && v.x < b.x + eps) { in *= v.ccw(a,b); }
c3b1 ▶ ▶ else if (v.in_seg(a,b)) { return 0; }
cbb1 ▶ }
091d ▶ return in;
cbb1 }//$
271f int polygon_pos_convex (vec * p, int n, vec v) { // lg(n) | (-1 out, 0 border, 1 in) TODO
a868 ▶ if (v.sq(p[0]) <= eps) return 0;
088f ▶ if (n <= 1) { return 0; } if (n == 2) { return v.in_seg(p[0],p[1])?0:-1; }
2ceb ▶ if (v.ccw(p[0],p[1]) < 0 || v.ccw(p[0],p[n-1]) > 0) return -1;
fcfd ▶ int di = lower_bound(p+1,p+n-1,v, [&p](vec a,vec v) { return v.ccw(p[0],a) > 0; }) - p;
adf3 ▶ if (di == 1) return v.ccw(p[1],p[2]) >= 0?0:-1;
cfa4 ▶ return v.ccw(p[di-1],p[di]);
cbb1 }//$
d41d // v is the pointset, w is auxiliary with size at least equal to v's
bf98 cood closest_pair (vec * v, vec * w, int l, int r, bool sorted = 0) { // nlg | r is exclusive TODO (AC on
cf, no test)
91d7 ▶ if (l + 1 >= r) return inf;
900b ▶ if (!sorted) sort(v+l,v+r,[](vec a, vec b){ return a.x < b.x; });
89cd ▶ int m = (l+r)/2; cood x = v[m].x;
1a44 ▶ cood res = min(closest_pair(v,w,l,m,1),closest_pair(v,w,m,r,1));
d046 ▶ merge(v+l,v+m,v+m,v+r,w+l,[](vec a, vec b){ return a.y < b.y; });
2dd0 ▶ for (int i = l, s = 1; i < r; i++) if (sq((v[i] = w[i]).x - x) < res) {
ad96 ▶ ▶ for (int j = s-1; j >= 1 && sq(w[i].y - w[j].y) < res; j--)
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c3b1 > > > res = min(res, w[i].sq(w[j]));
1991 > > w[s++] = v[i];
cbb1 > }
b505 > return res;
cbb1 }//$
ac2e double union_area (cir * v, int n) { // n^2lg | XXX joins equal circles TODO (AC on szkopol, no tests)
c765 > struct I { vec v; int i; } c[2*(n+4)];
cf66 > srand(time(NULL)); cood res = 0; vector<bool> usd(n);
dd83 > cood lim = 1./0.; for (int i = 0; i < n; i++) lim = min(lim, v[i].c.y - v[i].r - 1);
0b02 > for (int i = 0, ss = 0; i < n; i++, ss = 0) {
dc37 > > vec fp = v[i].c + vec(0,v[i].r).rotate(rand()); // rotation avoids corner on cnt initialization
6e87 > > int cnt = 0, eq = 0;
578e > > for (int j = 0; j < n; j++) {
df48 > > > cnt += (usd[j] = v[j].contains(fp));
2311 > > > if (!v[i].has_border_inter(v[j])) continue;
8daa > > > if (v[i].c == v[j].c) eq++;
4e6b > > > else {
e59e > > > > pair<vec,vec> r = v[i].border_inter(v[j]);
0782 > > > > c[ss++] = {r.first, j}; c[ss++] = {r.second, j};
cbb1 > > > }
cbb1 > > }
d21b > > vec d = vec(v[i].r,0); for (int k = 0; k < 4; k++, d = d.rot90()) c[ss++] = {v[i].c + d, i};
85d3 > > int md = partition(c,c+ss,[v,i,fp](I a){return a.v.ccw(v[i].c,fp) > 0;}) - c;
19c7 > > sort(c,c+md,[v,i](I a,I b){return a.v.ccw(v[i].c,b.v) < 0;});
7430 > > sort(c+md,c+ss,[v,i](I a,I b){return a.v.ccw(v[i].c,b.v) < 0;});
56cd > > for (int j = 0; j < ss; j++) {
2b5e > > > if (c[j].i != i) { cnt -= usd[c[j].i]; usd[c[j].i] = !usd[c[j].i]; cnt += usd[c[j].i]; }
b115 > > > vec a = c[j].v, b = c[(j+1)%ss].v;
7c4a > > > cood cir = abs(v[i].arc_area(a,b) - v[i].c.cross(a,b)/2), tra = abs((b.x-a.x)*(a.y+b.y-2*lim)/2);
e20b > > > cood loc = (a.x<b.x)?cir-tra:tra+cir; res += (cnt==eq)?loc/eq:0;
cbb1 > > }
cbb1 > }
b505 > return res;
cbb1 }//$
4ede pii antipodal (vec * p, int n, vec v) { // lg(n) | extreme segments relative to direction v TODO
d41d > // po: closest to dir, ne: furthest from dir
3bd9 > bool sw = ((p[1]-p[0])*v < 0);
d189 > if (sw) v = vec(0,0) - v; // lower_bound returns the first such that lambda is false
0303 > int md = lower_bound(p+1, p+n, v, [p] (vec & a, vec v) { return (a-p[0])*v > eps; }) - p; // chain
separation
25f1 > int po = lower_bound(p, p+md-1, v, [p,n] (vec & a, vec v) { return (p[(&a+1-p)%n]-a)*v > eps; }) - p; //
positive
9dc9 > int ne = (lower_bound(p+md, p+n, v, [p,n] (vec & a, vec v) { return (p[(&a+1-p)%n]-a)*v <= eps; }) -
p)%n; // negative
5703 > if (sw) swap(po,ne);
ef0b > return pii(po,ne);
cbb1 }//$
34e2 int mink_sum (vec * a, int n, vec * b, int m, vec * r) { // (n+m) | a[0]+b[0] should belong to sum, doesn't
create new border points TODO
8d81 > if (!n || !m) { return 0; } int i, j, s; r[0] = a[0] + b[0];
de54 > for (i = 0, j = 0, s = 1; i < n || j < m; s++) {
1ab0 > > if (i >= n) j++;
1dc4 > > else if (j >= m) i++;
4e6b > > else {
4f09 > > > int o = (a[(i+1)%n]+b[j%m]).ccw(r[s-1],a[i%n]+b[(j+1)%m]);
e43c > > > j += (o >= 0); i += (o <= 0);
cbb1 > > }
f5b4 > > r[s] = a[i%n] + b[j%m];
cbb1 > }
162b > return s-1;
cbb1 }//$
9e65 int inter_convex (vec * p, int n, vec * q, int m, vec * r) { // (n+m) | XXX
2d76 > int a = 0, b = 0, aa = 0, ba = 0, inflag = 0, s = 0;
2a6c > while ((aa < n || ba < m) && aa < n+n && ba < m+m) {
b977 > > vec p1 = p[a], p2 = p[(a+1)%n], q1 = q[b], q2 = q[(b+1)%m];
35b2 > > vec A = p2 - p1, B = q2 - q1;
1479 > > int cross = vec(0,0).ccw(A,B), ha = p1.ccw(p2,q2), hb = q1.ccw(q2,p2);
c6e0 > > if (cross == 0 && p2.ccw(p1,q1) == 0 && A*B < -eps) {
507b > > > if (q1.in_seg(p1,p2)) r[s++] = q1;

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5e83 > > > if (q2.in_seg(p1,p2)) r[s++] = q2;
ce58 > > > if (p1.in_seg(q1,q2)) r[s++] = p1;
526a > > > if (p2.in_seg(q1,q2)) r[s++] = p2;
7b25 > > > if (s < 2) return s;
e2a8 > > > inflag = 1; break;
5e6d > > > } else if (cross != 0 && inter_seg(p1,p2,q1,q2)) {
f420 > > > if (inflag == 0) aa = ba = 0;
2b81 > > > r[s++] = lin(p1,p2).inter(lin(q1,q2));
37fd > > > inflag = (hb > 0) ? 1 : -1;
cbb1 > > > }
5499 > > > if (cross == 0 && hb < 0 && ha < 0) return s;
0872 > > > bool t = cross == 0 && hb == 0 && ha == 0;
c0ec > > > if (t ? (inflag == 1) : (cross >= 0) ? (ha <= 0) : (hb > 0)) {
9873 > > > if (inflag == -1) r[s++] = q2;
1146 > > > ba++; b++; b %= m;
9d97 > > > } else {
5c98 > > > if (inflag == 1) r[s++] = p2;
5ecb > > > aa++; a++; a %= n;
cbb1 > > > }
cbb1 > > > }
c1b2 > > > if (inflag == 0) {
3880 > > > if (polygon_pos_convex(q,m,p[0]) >= 0) { copy(p, p+n, r); return n; }
115c > > > if (polygon_pos_convex(p,n,q[0]) >= 0) { copy(q, q+m, r); return m; }
cbb1 > > > }
fc37 > > > s = unique(r, r+s) - r;
2629 > > > if (s > 1 && r[0] == r[s-1]) s--;
0478 > > > return s;
cbb1 > > > }//$
03ae bool isear (vec * p, int n, int i, int prev[], int next[]) { // aux to triangulate
7630 > > > vec a = p[prev[i]], b = p[next[i]];
2d9f > > > if (b.ccw(a,p[i]) <= 0) return false;
578e > > > for (int j = 0; j < n; j++) {
97eb > > > if (j == prev[i] || j == next[i]) continue;
0ef9 > > > if (p[j].ccw(a,p[i]) >= 0 && p[j].ccw(p[i],b) >= 0 && p[j].ccw(b,a) >= 0) return false;
0639 > > > int k = (j+1)%n;
2898 > > > if (k == prev[i] || k == next[i]) continue;
a537 > > > if (inter_seg(p[j],p[k],a,b)) return false;
cbb1 > > > }
8a6c > > > return true;
cbb1 > > > }
1851 int triangulate (vec * p, int n, bool ear[], int prev[], int next[], int tri[][3]) { // O(n^2) | n >= 3
d14e > > > int s = 0, i = 0;
78d0 > > > for (int i = 0, prv = n-1; i < n; i++) { prev[i] = prv; prv = i; next[i] = (i+1)%n; ear[i] =
isear(p,n,i,prev,next); }
6b3b > > > for (int lef = n; lef > 3; lef--, i = next[i]) {
ced7 > > > while (!ear[i]) i = next[i];
e7a9 > > > tri[s][0] = prev[i]; tri[s][1] = i; tri[s][2] = next[i]; s++; // tri[i][0],i,tri[i][1] inserted
e0c0 > > > int c_prev = prev[i], c_next = next[i];
c354 > > > next[c_prev] = c_next; prev[c_next] = c_prev;
84b6 > > > ear[c_prev] = isear(p,n,c_prev,prev,next); ear[c_next] = isear(p,n,c_next,prev,next);
cbb1 > > > }
bc1d > > > tri[s][0] = next[next[i]]; tri[s][1] = i; tri[s][2] = next[i]; s++; // tri[i][0],i,tri[i][1] inserted
0478 > > > return s;
cbb1 > > > }

```

### 4.3 3D

```

f61c const double pi = acos(-1);
d41d // typedef double cood; cood eps = 1e-6; // risky: XXX, untested: TODO
3f73 struct pnt { // TODO it's not tested at all :)
5e43 > > > cood x, y, z;
cf2f > > > pnt () : x(0), y(0), z(0) {} pnt (cood a, cood b, cood c) : x(a), y(b), z(c) {}
4e90 > > > inline pnt operator - (pnt o) { return pnt(x - o.x, y - o.y, z - o.z); }
2b18 > > > inline pnt operator + (pnt o) { return pnt(x + o.x, y + o.y, z + o.z); }
7470 > > > inline pnt operator * (cood o) { return pnt(x*o, y*o, z*o); }
8194 > > > inline pnt operator / (cood o) { return pnt(x/o, y/o, z/o); }
a269 > > > inline cood operator * (pnt o) { return x*o.x + y*o.y + z*o.z; } // inner: |this|*|o|*cos(ang)

```

```

079c > inline pnt operator ^ (pnt o) { return pnt(y*o.z - z*o.y, z*o.x - x*o.z, x*o.y - y*o.x); } // cross:
oriented normal to the plane containing the two vectors, has norm |this||o|*sin(ang)
a2ea > inline cood operator () (pnt a, pnt b) { return (*this)*(a^b); } // mixed: positive on the right-hand
rule (thumb=this,index=a,mid=b)
d41d
f500 > inline cood inner (pnt a, pnt b) { return (a-(*this))*(b-(*this)); }
4114 > inline pnt cross (pnt a, pnt b) { return (a-(*this))^(b-(*this)); } // its norm is twice area of triangle
fa90 > inline cood mixed (pnt a, pnt b, pnt c) { return (a-(*this))(b-(*this),c-(*this)); } // 6 times the
oriented area of thetetrahedra
d41d
4f78 > inline cood sq (pnt o = pnt()) { return inner(o,o); }
113b > inline double nr (pnt o = pnt()) { return sqrt(sq(o)); }
6edf > inline pnt operator ~ () { return (*this)/nr(); }
d41d
11c0 > inline bool in_seg (pnt a, pnt b) { return cross(a,b).sq() <= eps && inner(a,b) <= eps; } // tips included
a6b7 > inline bool in_tri (pnt a, pnt b, pnt c) { return abs(mixed(a,b,c)) <= eps && cross(a,b)*cross(b,c) >=
-eps && cross(a,b)*cross(c,a) >= -eps; } // border included$
d41d
7c26 > inline pnt proj (pnt a, pnt b) { return a + (b-a)*a.inner(b,(*this))/a.sq(b); }
3a26 > inline pnt proj (pnt a, pnt b, pnt c) { pnt n = a.cross(b,c); return (*this) - n*(n-(*this)-a)/n.sq(); }
d41d
8fbb > inline double dist2_lin (pnt a, pnt b) { return cross(a,b).sq()/a.sq(b); }
1880 > inline double dist2_seg (pnt a, pnt b) { return a.inner(b,(*this))*b.inner(a,(*this)) <= eps ?
min(sq(a),sq(b)) : dist2_lin(a,b); }
39c1 > inline double dist_pln (pnt a, pnt b, pnt c) { return abs((~a.cross(b,c))*((~this)-a)); }
5bc2 > inline double dist2_tri (pnt a, pnt b, pnt c) { pnt p = proj(a,b,c); return p.in_tri(a,b,c) ? sq(p) :
min({ dist2_seg(a,b), dist2_seg(b,c), dist2_seg(c,a) }); }
2145 };
eb48 inline cood area (pnt a, pnt b, pnt c) { return abs(a.cross(b,c).nr()) / 2; }
a6c7 inline cood vol (pnt a, pnt b, pnt c, pnt d) { return abs(a.mixed(b,c,d)) / 6; } // tetrahedra
084a pnt inter_lin_pln (pnt s, pnt t, pnt a, pnt b, pnt c) { pnt n = a.cross(b,c); return s +
(t-s)*(n*(a-s))/(n*(t-s)); } //$
fabc struct sph { // TODO it's also not tested at all
af42 > pnt c; cood r;
390f > sph () : c(), r(0) {} sph (pnt a, cood b) : c(a), r(b) {}
baaf > inline pnt operator () (cood lat, cood lon) { return c + pnt(cos(lat)*cos(lon), sin(lon), sin(lat))*r; }
// (1,0,0) is (0,0). z is height.
171a > inline double area_hull (double h) { return 2.*pi*r*h; }
60a4 > inline double vol_hull (double h) { return pi*h/6 * (3.*r*r + h*h); }
2145 };

```

## 5 Graphs

### 5.1 Dinic

```

d41d //typedef int num; const int N = ; const int M = * 2; const num eps = 0;
582d struct dinic {
656d > int hd[N], seen[N], qu[N], lv[N], ei[N], to[M], nx[M]; num fl[M], cp[M]; int en = 2; int when = 0;
1233 > bool bfs(int s, int t) {
876c > > seen[t] = ++when; lv[t] = 0; int ql = 0, qr = 0; qu[qr++] = t;
a872 > > while(ql != qr) {
036d > > > t = qu[ql++]; ei[t] = hd[t]; if(s == t) return true;
9a44 > > > for(int e = hd[t]; e; e = nx[e]) if(seen[to[e]] != when && cp[e ^ 1] - fl[e ^ 1] > eps) {
d4fb > > > > seen[to[e]] = when;
de5c > > > > lv[to[e]] = lv[t] + 1;
f0ff > > > > qu[qr++] = to[e];
cbb1 > > > }
cbb1 > > }
d1fe > > return false;
cbb1 > }
a444 > num dfs(int s, int t, num f) {
f449 > > if(s == t) return f;
cebe > > for(int &e = ei[s]; e; e = nx[e]) if(ei[to[e]] && seen[to[e]] == when && cp[e] - fl[e] > eps &&
lv[to[e]] == lv[s] - 1)
7004 > > > if(num rf = dfs(to[e], t, min(f, cp[e] - fl[e]))) {
805c > > > > fl[e] += rf;
5226 > > > > fl[e ^ 1] -= rf;

```



```

2cb7 > > > > return rf;
cbb1 > > > }
bb30 > > return 0;
cbb1 > > }
d41d > // public $
de22 > num max_flow(int s, int t) {
6cb2 > > num fl = 0;
1c5e > > while (bfs(s, t)) for(num f; (f = dfs(s, t, numeric_limits<num>::max())); fl += f);
e508 > > return fl;
cbb1 > > }
5a3f > void add_edge(int a, int b, num c, num rc=0) {
d03a > > to[en] = b; nx[en] = hd[a]; fl[en] = 0; cp[en] = c; hd[a] = en++;
2f94 > > to[en] = a; nx[en] = hd[b]; fl[en] = 0; cp[en] = rc; hd[b] = en++;
cbb1 > > }
7415 > void reset_flow() { memset(fl, 0, sizeof(num) * en); }
ae0a > void init(int n=N) { en = 2; memset(hd, 0, sizeof(int) * n); } // resets all
2145 > };

```

## 5.2 MinCost MaxFlow

```

d41d //typedef int val; // type of flow
d41d //typedef int num; // type of cost
d41d //const int N = , M = * 2; const num eps = 0;
1854 struct mcmf {
7a62 > int es[N], to[M], nx[M], en = 2, pai[N], seen[N], when, qu[N];
ef55 > val fl[M], cp[M], flow; num cs[M], d[N], tot;
d0cc > val spfa(int s, int t) {
104f > > when++; int a = 0, b = 0;
e0c6 > > for(int i = 0; i < N; i++) d[i] = numeric_limits<num>::max();
3518 > > d[s] = 0; qu[b++] = s; seen[s] = when;
9841 > > while(a != b) {
32d9 > > > int u = qu[a++]; if(a == N) a = 0; seen[u] = 0;
a86f > > > for(int e = es[u]; e; e = nx[e]) if(cp[e] - fl[e] > val(0) && d[u] + cs[e] < d[to[e]] - eps) {
a694 > > > > d[to[e]] = d[u] + cs[e]; pai[to[e]] = e ^ 1;
85b7 > > > > if(seen[to[e]] < when) { seen[to[e]] = when; qu[b++] = to[e]; if(b == N) b = 0; }
cbb1 > > > > }
cbb1 > > > }
8e2a > > if(d[t] == numeric_limits<num>::max()) return false;
91fe > > val mx = numeric_limits<val>::max();
285a > > for(int u = t; u != s; u = to[pai[u]])
7039 > > > mx = min(mx, cp[pai[u] ^ 1] - fl[pai[u] ^ 1]);
6de0 > > tot += d[t] * val(mx);
285a > > for(int u = t; u != s; u = to[pai[u]])
4c48 > > > fl[pai[u]] -= mx, fl[pai[u] ^ 1] += mx;
b9aa > > return mx;
cbb1 > > }
d41d > // public $
8662 > num min_cost(int s, int t) {
3b69 > > tot = 0; flow = 0;
e66e > > while(val a = spfa(s, t)) flow += a;
126a > > return tot;
cbb1 > > }
457a > void add_edge(int u, int v, val c, num s) {
1d08 > > fl[en] = 0; cp[en] = c; to[en] = v; nx[en] = es[u]; cs[en] = s; es[u] = en++;
8015 > > fl[en] = 0; cp[en] = 0; to[en] = u; nx[en] = es[v]; cs[en] = -s; es[v] = en++;
cbb1 > > }
8537 > void reset_flow() { memset(fl, 0, sizeof(val) * en); }
451f > void init(int n) { en = 2; memset(es, 0, sizeof(int) * n); } // XXX must be called
2145 > };

```

## 5.3 Cycle Cancellation

```

d41d //typedef int val; // type of flow
d41d //typedef int num; // type of cost
d41d //const int N = ; const int M = * 2; const val eps = 0;
afb2 struct cycle_cancel {

```



```

0f5c > int hd[N], seen[N], qu[N], lv[N], ei[N], to[M], nx[M], ct[N], pai[N]; val fl[M], cp[M], flow; num cs[M],
    d[N], tot; int en = 2, n; int when = 0;
1233 > bool bfs(int s, int t) {
876c > > seen[t] = ++when; lv[t] = 0; int ql = 0, qr = 0; qu[qr++] = t;
a872 > > while(ql != qr) {
036d > > > t = qu[ql++]; ei[t] = hd[t]; if(s == t) return true;
9a44 > > > for(int e = hd[t]; e; e = nx[e]) if(seen[to[e]] != when && cp[e ^ 1] - fl[e ^ 1] > eps) {
d4fb > > > > seen[to[e]] = when;
de5c > > > > lv[to[e]] = lv[t] + 1;
f0ff > > > > qu[qr++] = to[e];
cbb1 > > > }
cbb1 > > }
dife > > return false;
cbb1 > }
e4d9 > val dfs(int s, int t, val f) {
f449 > > if(s == t) return f;
cebe > > for(int &e = ei[s]; e; e = nx[e]) if(ei[to[e]] && seen[to[e]] == when && cp[e] - fl[e] > eps &&
    lv[to[e]] == lv[s] - 1)
9fe1 > > > if(val rf = dfs(to[e], t, min(f, cp[e] - fl[e]))) {
805c > > > > fl[e] += rf;
5226 > > > > fl[e ^ 1] -= rf;
2cb7 > > > > return rf;
cbb1 > > > }
bb30 > > return 0;
cbb1 > }
5cbe > bool spfa() {
e2f3 > > when++; int a = 0, b = 0, u;
91bc > > for(int i = 0; i < n; i++) { d[i] = 0; qu[b++] = i; seen[i] = when; ct[i] = 0; }
9841 > > while(a != b) {
b492 > > > u = qu[a++]; if(a == N) a = 0; seen[u] = 0;
d627 > > > if(ct[u]++ >= n + 1) { a--; break; }
ccce > > > for(int e = hd[u]; e; e = nx[e]) if(cp[e] - fl[e] > val(0) && d[u] + cs[e] < d[to[e]] - eps) {
a694 > > > > d[to[e]] = d[u] + cs[e]; pai[to[e]] = e ^ 1;
85b7 > > > > if(seen[to[e]] < when) { seen[to[e]] = when; qu[b++] = to[e]; if(b == N) b = 0; }
cbb1 > > > }
cbb1 > > }
5c28 > > if(a == b) return false;
02be > > val mn = numeric_limits<val>::max();
be15 > > when++;
e855 > > for(; seen[u] != when; u = to[pai[u]]) seen[u] = when;
0612 > > for(int v = u; seen[v] != when + 1; v = to[pai[v]]) {
6e6b > > > seen[v] = when + 1;
3225 > > > mn = min(mn, cp[pai[v] ^ 1] - fl[pai[v] ^ 1]);
cbb1 > > > }
ea26 > > for(int v = u; seen[v] == when + 1; v = to[pai[v]]) {
7618 > > > seen[v] = 0;
60f1 > > > fl[pai[v]] -= mn;
0329 > > > fl[pai[v] ^ 1] += mn;
cbb1 > > > }
8a6c > > return true;
cbb1 > }
2b0e > val max_flow(int s, int t) {
e7a0 > > val fl = 0;
036d > > while (bfs(s, t)) for(val f; (f = dfs(s, t, numeric_limits<val>::max())); fl += f);
e508 > > return fl;
cbb1 > }
d41d > // public $
8662 > num min_cost(int s, int t) {
94a7 > > flow = max_flow(s, t);
6c9f > > while(spfa());
ed25 > > tot = 0;
112e > > for(int i = 2; i < en; i++)
b951 > > > if(fl[i] > 0)
dae8 > > > > tot += fl[i] * cs[i];
126a > > return tot;
cbb1 > }
8537 > void reset_flow() { memset(fl, 0, sizeof(val) * en); }
457a > void add_edge(int u, int v, val c, num s) {
d321 > > fl[en] = 0; cp[en] = c; to[en] = v; nx[en] = hd[u]; cs[en] = s; hd[u] = en++;

```

```

f081 ▶ ▶ fl[en] = 0; cp[en] = 0; to[en] = u; nx[en] = hd[v]; cs[en] = -s; hd[v] = en++;
cbb1 ▶ }
bfc4 ▶ void init(int n) { this->n = n; en = 2; memset(hd, 0, sizeof(int) * n); } // XXX must be called
2145 };

```

---

## 5.4 DFS

```

5c82 class Graph
f95b {
f4a3     int V;    // No. of vertices
d41d     // Pointer to an array containing
d41d     // adjacency lists
b363     list<int> *adj;
d41d     // A recursive function used by DFS
d389     void DFSUtil(int v, bool visited[]);
6731 public:
c12d     Graph(int V); // Constructor
d41d     // function to add an edge to graph
a162     void addEdge(int v, int w);
d41d     // DFS traversal of the vertices
d41d     // reachable from v
d55f     void DFS(int v);
2145 };
bd81 Graph::Graph(int V)
f95b {
1549     this->V = V;
16bd     adj = new list<int>[V];
cbb1 }
7862 void Graph::addEdge(int v, int w)
f95b {
3136     adj[v].push_back(w); // Add w to vs list.
cbb1 }
8b82 void Graph::DFSUtil(int v, bool visited[])
f95b {
d41d     // Mark the current node as visited and
d41d     // print it
e758     visited[v] = true;
1812     cout << v << " ";
d41d     // Recur for all the vertices adjacent
d41d     // to this vertex
b046     list<int>::iterator i;
8255     for (i = adj[v].begin(); i != adj[v].end(); ++i)
22a9         if (!visited[*i])
c23d             DFSUtil(*i, visited);
cbb1 }
d41d // DFS traversal of the vertices reachable from v.
d41d // It uses recursive DFSUtil()
d53f void Graph::DFS(int v)
f95b {
d41d     // Mark all the vertices as not visited
c179     bool *visited = new bool[V];
c5e1     for (int i = 0; i < V; i++)
88e7         visited[i] = false;
d41d     // Call the recursive helper function
d41d     // to print DFS traversal
bf0b     DFSUtil(v, visited);
cbb1 }

```

---

## 5.5 BFS

```

2b74 #include <bits/stdc++.h>
efe1 #define pb push_back
931d vector<bool> v;
7899 vector<vector<int> > g;
d41d
cdd5 void edge(int a, int b)
f95b {

```

```

07b3    g[a].pb(b);
d41d    // for undirected graph add this line
d41d    // g[b].pb(a);
cbb1 }
651f void bfs(int u)
f95b {
26a5    queue<int> q;
f736    q.push(u);
ef8c    v[u] = true;
14d7    while (!q.empty()) {
3537        int f = q.front();
8332        q.pop();
5d2b        cout << f << " ";
d41d        // Enqueue all adjacent of f and mark them visited
319e        for (auto i = g[f].begin(); i != g[f].end(); i++) {
eb65            if (!v[*i]) {
9d57                q.push(*i);
b34e                v[*i] = true;
cbb1            }
cbb1        }
cbb1    }
cbb1 }

```

---

## 6 Structures

### 6.1 Ordered Set

```

7747 #include <ext/pb_ds/assoc_container.hpp>
30f4 #include <ext/pb_ds/tree_policy.hpp>
0d73 using namespace __gnu_pbds;
4519 template <typename tA, typename tB=null_type> using ord_set = tree<tA, tB, less<tA>, rb_tree_tag,
    tree_order_statistics_node_update>;
d41d // map: tA -> tB with the less<tA> comparison function
d41d // can be used as a normal map
d41d // s.find_by_order(k) :: returns iterator to the k-th element (0-indexed) (or s.end())
d41d // s.order_of_key(x) :: returns how many elements are strictly less than x

```

---

### 6.2 Treap

```

d41d //const int N = ; typedef int num;
5463 num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
8b25 void calc (int u) { // update node given children info
d4c7 > sz[u] = sz[L[u]] + 1 + sz[R[u]];
d41d > // code here, no recursion
cbb1 }
234f void unlaze (int u) {
e39f > if(!u) return;
d41d > // code here, no recursion
cbb1 }
ee5e void split_val(int u, num x, int &l, int &r) { // l gets <= x, r gets > x
754f > unlaze(u); if(!u) return (void) (l = r = 0);
4bc1 > if(X[u] <= x) { split_val(R[u], x, l, r); R[u] = l; l = u; }
81a7 > else { split_val(L[u], x, l, r); L[u] = r; r = u; }
aaa8 > calc(u);
cbb1 }
9374 void split_sz(int u, int s, int &l, int &r) { // l gets first s, r gets remaining
754f > unlaze(u); if(!u) return (void) (l = r = 0);
e06d > if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1, l, r); R[u] = l; l = u; }
f524 > else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
aaa8 > calc(u);
cbb1 }
c870 int merge(int l, int r) { // els on l <= els on r
67f0 > unlaze(l); unlaze(r); if(!l || !r) return l + r; int u;
7801 > if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
ae90 > else { L[r] = merge(l, L[r]); u = r; }
0ffd > calc(u); return u;

```

```

cbb1 }
500b void init(int n=N-1) { // XXX call before using other funcs
7d1c > for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] = 1; L[i] = R[i] = 0; }
8c5a > random_shuffle(Y + 1, Y + n + 1);
cbb1 }

```

## 6.3 Envelope

```

d41d // typedef ll num; const num eps = 0;
d41d // XXX double: indicates operations specific to integers, not precision related
d79f template<typename line> struct envelope {
5e0f > deque<line> q; num lo,hi; envelope (num _lo, num _hi) : lo(_lo), hi(_hi) {}
01ca > void push_front (line l) { // amort. O(inter) | l is best at lo or never
a86b > > if (q.size() && q[0](lo) < l(lo)) return;
89b8 > > for (num x; q.size(); q.pop_front()) {
cc18 > > > x = (q.size()<=1?hi:q[0].inter(q[1],lo,hi)-1); // XXX double (-1)
4202 > > > if (l(x) > q[0](x)) break;
cbb1 > > }
45bc > > q.push_front(l);
cbb1 > }
f644 > void push_back (line l) { // amort. O(inter) | l is best at hi or never
0334 > > if (q.size() && q[q.size()-1](hi) <= l(hi)) return;
b71c > > for (num x; q.size(); q.pop_back()) {
4e80 > > > x = (q.size()<=1?lo:q[q.size()-2].inter(q[q.size()-1],lo,hi));
1747 > > > if (l(x) >= q[q.size()-1](x)) break;
cbb1 > > }
5e56 > > q.push_back(l);
cbb1 > }
e732 > void pop_front (num _lo) { for (lo=_lo; q.size()>1 && q[0](lo) > q[1](lo); q.pop_front()); } // amort.
O(n)
218a > void pop_back (num _hi) { for (hi=_hi; q.size()>1 && q[q.size()-2](hi) <= q[q.size()-1](hi);
q.pop_back()); } // amort. O(n)
7155 > line get (num x) { // O(lg(R))
e32f > > int lo, hi, md; for (lo = 0, hi = q.size()-1, md = (lo+hi)/2; lo < hi; md = (lo+hi)/2)
c1fb > > > if (q[md](x) > q[md+1](x)) { lo = md+1; }
b029 > > > else { hi = md; }
adf9 > > return q[lo];
cbb1 > }
2145 };
b3a6 struct line { // inter = O(1)
7bd4 > num a,b; num operator () (num x) const { return a*x+b; }
2417 > num inter (line o, num lo, num hi) { return
abs(o.a-a)<=eps?((b<o.b)?hi+1:lo):min(hi+1,max(lo,(o.b-b-(o.b-b<0)*(a-o.a-1))/(a-o.a) + 1)); }
2145 };
16ed struct generic_line { // inter = O(lg(R))
7bd4 > num a,b; num operator () (num x) const { return a*x+b; }
3cfe > num inter (generic_line o, num lo, num hi) { // first point where o strictly beats this
ca4f > > for (num md = lo+((+hi)-lo)/2; lo < hi; md = lo+(hi-lo)/2) { // XXX double
760b > > > if ((*this)(md)<=o(md)) { lo = md+1; } // XXX double
b029 > > > else { hi = md; }
cbb1 > > }
2532 > > return lo;
cbb1 > }
2145 };
11a2 template<typename line> struct full_envelope { // XXX ties are broken arbitrarily
85c9 > vector<envelope<line>> v; full_envelope(envelope<line> c) : v({c}) {} // v.reserve(30);
6aed > void add (line l) { // amort. O(lg(n)*inter)
8cca > > envelope<line> cur(v.back().lo,v.back().hi); cur.push_back(l);
bb4a > > while (!v.empty() && v.back().q.size() <= cur.q.size()) {
ce29 > > > deque<line> aux; swap(aux,cur.q); int i = 0, j = 0;
31d2 > > > for (; i < aux.size(); i++) {
542d > > > > for (; j < v.back().q.size() && v.back().q[j](cur.hi) > aux[i](cur.hi); j++)
0015 > > > > cur.push_back(v.back().q[j]);
70a1 > > > > cur.push_back(aux[i]);
cbb1 > > > }
a0e7 > > > for (; j < v.back().q.size(); j++) cur.push_back(v.back().q[j]);
deff > > > v.pop_back();
cbb1 > > }

```

```

026e ▶ ▶ v.push_back(cur);
cbb1 ▶ }
7155 ▶ line get (num x) { // O(lg(n)lg(R)) | pop_back/pop_front can optimize
9351 ▶ ▶ line a = v[0].get(x);
ad67 ▶ ▶ for (int i = 1; i < (int) v.size(); i++) {
bcbe ▶ ▶ ▶ line b = v[i].get(x);
ad0f ▶ ▶ ▶ if (b(x)<a(x)) a = b;
cbb1 ▶ ▶ }
3f53 ▶ ▶ return a;
cbb1 ▶ }
2145 };
```

---

## 6.4 Centroid

```

0eca vector<int> adj[N]; int cn_sz[N], n;
c864 vector<int> cn_chld[N]; int cn_dep[N], cn_dist[20][N]; // removable
ace4 void cn_setdist (int u, int p, int depth, int dist) { // removable
989e ▶ cn_dist[depth][u] = dist;
59dd ▶ for (int v : adj[u]) if (p != v && cn_sz[v] != -1) // sz = -1 marks processed centroid (not dominated)
4ce5 ▶ ▶ cn_setdist(v, u, depth, dist+1);
cbb1 }
e897 int cn_getsz (int u, int p) {
08c9 ▶ cn_sz[u] = 1;
59dd ▶ for (int v : adj[u]) if (p != v && cn_sz[v] != -1)
b2f6 ▶ ▶ cn_sz[u] += cn_getsz(v,u);
37a9 ▶ return cn_sz[u];
cbb1 }
912c int cn_build (int u, int depth) {
28a0 ▶ int siz = cn_getsz(u,u); int w = u;
0168 ▶ do {
9847 ▶ ▶ u = w;
a786 ▶ ▶ for (int v : adj[u]) if (cn_sz[v] != -1 && cn_sz[v] < cn_sz[u] && cn_sz[v] + cn_sz[v] >= siz)
9a13 ▶ ▶ ▶ w = v;
06ba ▶ } while (u != w); // u becomes current centroid root
094e ▶ cn_setdist(u,u,depth,0); // removable, here you can iterate over all dominated tree
32c2 ▶ cn_sz[u] = -1; cn_dep[u] = depth;
5cff ▶ for (int v : adj[u]) if (cn_sz[v] != -1) {
1df5 ▶ ▶ int w = cn_build(v, depth+1);
2e31 ▶ ▶ cn_chld[u].pb(w); // removable
cbb1 ▶ }
03f4 ▶ return u;
cbb1 }
```

---

## 6.5 Splay Tree

```

d41d //const int N = ;
d41d //typedef int num;
d41d
576f int en = 1;
37e4 int p[N], sz[N];
c7d4 int C[N][2]; // {left, right} children
abac num X[N];
d41d
d41d // update values associated to the nodes that can be calculated from child
8b25 void calc(int u) {
5665 ▶ sz[u] = sz[C[u][0]] + 1 + sz[C[u][1]];
cbb1 }
d41d
d41d // pull child dir of u to its position and return
0584 int rotate(int u, int dir) {
05db ▶ int v = C[u][dir];
2116 ▶ C[u][dir] = C[v][!dir];
6c8a ▶ if(C[u][dir]) p[C[u][dir]] = u;
0928 ▶ C[v][!dir] = u;
c0a7 ▶ p[v] = p[u];
b9c1 ▶ if(p[v]) C[p[v]][C[p[v]][1] == u] = v;
136e ▶ p[u] = v;
```

```

aaa8 ▸ calc(u);
b6b0 ▸ calc(v);
6dc7 ▸ return v;
cbb1 }
d41d
d41d // bring node u to root
81a1 void splay(int u) {
bdd0 ▸ while(p[u]) {
2a84 ▸ ▸ int v = p[u], w = p[p[u]];
1a8a ▸ ▸ int du = C[v][1] == u;
e764 ▸ ▸ if(!w)
76c8 ▸ ▸ ▸ rotate(v, du);
4e6b ▸ ▸ else {
d499 ▸ ▸ ▸ int dv = (C[w][1] == v);
9b57 ▸ ▸ ▸ if(du == dv) {
6c72 ▸ ▸ ▸ ▸ rotate(w, dv);
76c8 ▸ ▸ ▸ ▸ rotate(v, du);
9d97 ▸ ▸ ▸ } else {
76c8 ▸ ▸ ▸ ▸ rotate(v, du);
6c72 ▸ ▸ ▸ ▸ rotate(w, dv);
cbb1 ▸ ▸ ▸ }
cbb1 ▸ ▸ }
cbb1 ▸ }
cbb1 }
d41d
d41d // return node with value x or other if node was not found
8975 int find_val(int u, num x) {
93fe ▸ int v = u;
9a3d ▸ while(u && X[u] != x) {
766a ▸ ▸ v = u;
1b5b ▸ ▸ if(x < X[u]) u = C[u][0];
a73d ▸ ▸ else u = C[u][1];
cbb1 ▸ }
3418 ▸ if(!u) u = v;
6d13 ▸ splay(u);
03f4 ▸ return u;
cbb1 }
d41d
d41d // return nth node
a7c2 int find_sz(int u, int s) {
3939 ▸ while(sz[C[u][0]] != s) {
7ef0 ▸ ▸ if(sz[C[u][0]] < s) {
2777 ▸ ▸ ▸ s -= sz[C[u][0]] + 1;
6bdb ▸ ▸ ▸ u = C[u][1];
66d9 ▸ ▸ } else u = C[u][0];
cbb1 ▸ }
6d13 ▸ splay(u);
03f4 ▸ return u;
cbb1 }
d41d
d41d // concatenate two trees assuming #elements l <= #elements r
c870 int merge(int l, int r) {
db1b ▸ if(!l || !r) return l + r;
45ba ▸ while(C[l][1]) l = C[l][1];
bab4 ▸ splay(l);
0258 ▸ assert(!C[l][1]);
e3ec ▸ C[l][1] = r;
924c ▸ p[r] = l;
f046 ▸ calc(l);
792f ▸ return l;
cbb1 }
d41d
d41d // add node x to splay u and return x
684a int add(int u, int x) {
e29c ▸ int v = 0;
9d2d ▸ while(u) v = u, u = C[u][X[x] >= X[u]];
f257 ▸ if(v) { C[v][X[x] >= X[v]] = x; p[x] = v; }
0b6f ▸ splay(x);
ea56 ▸ return x;

```

```

cbb1 }
d41d
d41d // call 1 time at the top
ca2f void init() {
0cee ▸ en = 1;
cbb1 }
d41d
d41d // create a new node
3e8b int new_node(num val) {
cecb ▸ int i = en++;
9c38 ▸ assert(i < N);
9029 ▸ C[i][0] = C[i][1] = p[i] = 0;
02c8 ▸ sz[i] = 1;
4281 ▸ X[i] = val;
d9a5 ▸ return i;
cbb1 }

```

---

## 7 Strings

### 7.1 Z-function

```

2a61 void Z(char s[], int n, int z[]) { // z[i] = |lcp(s,s[i..n])|
fc15 ▸ for(int i = 1, m = -1; i < n; i++) {
d69b ▸ ▸ z[i] = (m != -1 && m + z[m] >= i)?min(m + z[m] - i, z[i - m]):0;
8a63 ▸ ▸ while (i + z[i] < n && s[i + z[i]] == s[z[i]]) z[i]++;
bbe8 ▸ ▸ if (m == -1 || i + z[i] > m + z[m]) m = i;
cbb1 ▸ }
cbb1 }

```

---

## 8 Math

### 8.1 Linear System Solver

```

d41d //const int N = ;
d41d
46cc double a[N][N];
3793 double ans[N];
d41d
d41d // sum(a[i][j] * x_j) = a[i][n] for 0 <= i < n
d41d // stores answer in ans and returns det(a)
c42a double solve(int n) {
f99b ▸ double det = 1;
6033 ▸ for(int i = 0; i < n; i++) {
0268 ▸ ▸ int mx = i;
197a ▸ ▸ for(int j = i + 1; j < n; j++)
b83d ▸ ▸ ▸ if(abs(a[j][i]) > abs(a[mx][i]))
672f ▸ ▸ ▸ mx = j;
28c6 ▸ ▸ if(i != mx) {
e83f ▸ ▸ ▸ swap_ranges(a[i], a[i] + n + 1, a[mx]);
0143 ▸ ▸ ▸ det = -det;
cbb1 ▸ ▸ }
997e ▸ ▸ if(abs(a[i][i]) < 1e-6); // singular matrix
2f40 ▸ ▸ det *= a[i][i];
94fe ▸ ▸ for(int j = i + 1; j < n; j++) {
12fe ▸ ▸ ▸ for(int k = i + 1; k <= n; k++)
ea32 ▸ ▸ ▸ ▸ a[j][k] -= (a[j][i] / a[i][i]) * a[i][k];
efbc ▸ ▸ ▸ a[j][i] = 0;
cbb1 ▸ ▸ }
cbb1 ▸ }
45bd ▸ for(int i = n - 1; i >= 0; i--) {
7634 ▸ ▸ ans[i] = a[i][n];
197a ▸ ▸ for(int j = i + 1; j < n; j++)
9b00 ▸ ▸ ▸ ans[i] -= a[i][j] * ans[j];
35e5 ▸ ▸ ans[i] /= a[i][i];
cbb1 ▸ }
7a32 ▸ return det;

```



---

cbb1 }

## 8.2 Simplex

---

```

d41d //typedef long double dbl;
bec0 const dbl eps = 1e-6;
d41d //const int N = , M = ;
d41d
79ee struct simplex {
0643 > int X[N], Y[M];
6b50 > dbl A[M][N], b[M], c[N];
e268 > dbl ans;
14e0 > int n, m;
a00d > dbl sol[N];
d41d
c511 > void pivot(int x,int y){
eb91 > > swap(X[y], Y[x]);
c057 > > b[x] /= A[x][y];
8300 > > for(int i = 0; i < n; i++)
7f61 > > > if(i != y)
d311 > > > > A[x][i] /= A[x][y];
3fa2 > > > A[x][y] = 1. / A[x][y];
94f7 > > > for(int i = 0; i < m; i++)
a325 > > > > if(i != x && abs(A[i][y]) > eps) {
6856 > > > > > b[i] -= A[i][y] * b[x];
f90a > > > > > for(int j = 0; j < n; j++)
6739 > > > > > > if(j != y)
8c78 > > > > > > > A[i][j] -= A[i][y] * A[x][j];
e112 > > > > > > A[i][y] = -A[i][y] * A[x][y];
cbb1 > > > > }
8c7e > > > ans += c[y] * b[x];
8300 > > > for(int i = 0; i < n; i++)
7f61 > > > > if(i != y)
bec1 > > > > > c[i] -= c[y] * A[x][i];
0997 > > > > > c[y] = -c[y] * A[x][y];
cbb1 > > > }
d41d
d41d > // maximize sum(x[i] * c[i])
d41d > // element a
d41d > // sum(a[i][j] * x[j]) <= b[i] for 0 <= i < m (Ax <= b)
d41d > // x[i] >= 0 for 0 <= i < n (x >= 0)
d41d > // (n variables, m constraints)
d41d > // stores the answer in ans and returns optimal value
59d9 > dbl solve(int n, int m) {
1f59 > > this->n = n; this->m = m;
f1bf > > ans = 0.;
b1c6 > > for(int i = 0; i < n; i++) X[i] = i;
3e36 > > for(int i = 0; i < m; i++) Y[i] = i + n;
6679 > > while(true) {
ee39 > > > int x = min_element(b, b + m) - b;
988b > > > if(b[x] >= -eps)
c2be > > > > break;
49a2 > > > > int y = find_if(A[x], A[x] + n, [](dbl d) { return d < -eps; }) - A[x];
6f8c > > > > if(y == n) throw 1; // no solution
7fb4 > > > > pivot(x, y);
cbb1 > > > }
6679 > > > while(true) {
f802 > > > > int y = max_element(c, c + n) - c;
b7b6 > > > > if(c[y] <= eps) break;
d6b5 > > > > int x = -1;
06d7 > > > > dbl mn = 1. / 0.;
94f7 > > > > for(int i = 0; i < m; i++)
5877 > > > > > if(A[i][y] > eps && b[i] / A[i][y] < mn)
832b > > > > > > mn = b[i] / A[i][y], x = i;
ff22 > > > > > if(x == -1) throw 2; // unbounded
7fb4 > > > > > pivot(x, y);
cbb1 > > > > }
d094 > > > memset(sol, 0, sizeof(dbl) * n);

```

```

94f7 ▸ ▸   for(int i = 0; i < m; i++)
cff4 ▸ ▸   ▸   if(Y[i] < n)
09d7 ▸ ▸   ▸   ▸   sol[Y[i]] = b[i];
ba75 ▸ ▸   return ans;
cbb1 ▸ ▸   }
2145 };

```

---

## 9 Number Theory

### 9.1 Extended Euclidean Algorithm

```

c25f int egcd(int a, int b, int& x, int& y) { // a*x + b*y = gcd(a, b) [Bezout's Theorem]
8273 ▸ if (b == 0) return x = 1, y = 0, a;
98d1 ▸ int xx, yy;
0c0d ▸ int g = egcd(b, a % b, xx, yy);
512d ▸ x = yy;
a9d0 ▸ y = xx - (a / b) * yy;
96b5 ▸ return g;
cbb1 }

```

---

### 9.2 Miller-Rabin

```

a288 llu llrand() { llu a = rand(); a<= 32; a+= rand(); return a;}
0a9c int is_probably_prime(llu n) {
8dbf   if (n <= 1) return 0;
2373   if (n <= 3) return 1;
7de1   llu s = 0, d = n - 1;
66b4   while (d % 2 == 0) {
90f4     d/= 2; s++;
cbb1   }
6b3a   for (int k = 0; k < 64; k++) {
12c0     llu a = (llrand() % (n - 3)) + 2;
dc17     llu x = exp_mod(a, d, n);
1181     if (x != 1 && x != n-1) {
f0ea       for (int r = 1; r < s; r++) {
708d         x = mul_mod(x, x, n);
61d9         if (x == 1)
bb30           return 0;
68b2         if (x == n-1)
c2be           break;
cbb1       }
34bc       if (x != n-1)
bb30         return 0;
cbb1     }
cbb1   }
6a55   return 1;
cbb1 }

```

---

### 9.3 Diofantine

```

d41d // find all solutions in the form ax + by = c
d41d
080f void shift_solution(int & x, int & y, int a, int b, int cnt) {
526a   x += cnt * b;
fdfb   y -= cnt * a;
cbb1 }
d41d
f0f5 int find_all_solutions(int a, int b, int c, int minx, int maxx, int miny, int maxy) {
ede5   int x, y, g;
7968   if (!find_any_solution(a, b, c, x, y, g))
bb30     return 0;
fe72   a /= g;
ee2d   b /= g;
d41d
0750   int sign_a = a > 0 ? +1 : -1;

```

```

f8be    int sign_b = b > 0 ? +1 : -1;
d41d
ab53    shift_solution(x, y, a, b, (minx - x) / b);
5969    if (x < minx)
8a96        shift_solution(x, y, a, b, sign_b);
6bcc    if (x > maxx)
bb30        return 0;
57f8    int lx1 = x;
d41d
9870    shift_solution(x, y, a, b, (maxx - x) / b);
6bcc    if (x > maxx)
f6f8        shift_solution(x, y, a, b, -sign_b);
eb5e    int rx1 = x;
d41d
7672    shift_solution(x, y, a, b, -(miny - y) / a);
a697    if (y < miny)
bf53        shift_solution(x, y, a, b, -sign_a);
alde    if (y > maxy)
bb30        return 0;
8e42    int lx2 = x;
d41d
e322    shift_solution(x, y, a, b, -(maxy - y) / a);
alde    if (y > maxy)
b156        shift_solution(x, y, a, b, sign_a);
481c    int rx2 = x;
d41d
473e    if (lx2 > rx2)
e723        swap(lx2, rx2);
2b9f    int lx = max(lx1, lx2);
037c    int rx = min(rx1, rx2);
d41d
f0c5    if (lx > rx)
bb30        return 0;
ebb8    return (rx - lx) / abs(b) + 1;
cbb1 }

```

---

## 10 Notes

### 10.1 Modular Multiplicative Inverse

- If  $\gcd(a, m) = 1$ , then let  $ax + my = \gcd(a, m) = 1$  (Bezout's Theorem). Then  $ax \equiv 1 \pmod{m}$ .
- If  $\gcd(a, m) = 1$ , then  $a \cdot a^{\phi(m)-1} \equiv 1 \pmod{m}$  (Euler's Theorem).
- If  $m$  is prime, then  $\phi(m) = m - 1$ , so  $a * a^{m-2} \equiv 1 \pmod{m}$ .

### 10.2 Chinese Remainder Theorem

We are given  $N = n_1 n_2 \cdots n_k$  where  $n_i$  are pairwise coprime. We are also given  $x_1 \cdots x_k$  such that  $x \equiv x_i \pmod{n_i}$ . Let  $N_i = N/n_i$ . There exists  $M_i$  and  $m_i$  such that  $M_i N_i + m_i n_i = 1$  (Bezout). Then, there is only one solution  $x$ , given by:

$$x = \sum_{i=1}^k a_i M_i N_i$$

### 10.3 Euler's Totient Function

Positive integers up to a given integer  $n$  that are relatively prime to  $n$ .  $\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$  where the product is over the distinct prime numbers dividing  $n$ .

### 10.4 Möebius

If  $F(n) = \sum_{d|n} f(d)$ , then  $f(n) = \sum_{d|n} \mu(d) F(n/d)$ .

## 10.5 Burnside

Let  $A: GX \rightarrow X$  be an action. Define:

- $w :=$  number of orbits in  $X$ .
- $S_x := \{g \in G \mid g \cdot x = x\}$
- $F_g := \{x \in X \mid g \cdot x = x\}$

Then  $w = \frac{1}{|G|} \sum_{x \in X} |S_x| = \frac{1}{|G|} \sum_{g \in G} |F_g|$ .

## 10.6 Catalan Number

$C_n$  is solution for:

- Number of correct bracket sequence consisting of  $n$  opening and  $n$  closing brackets.
- The number of rooted full binary trees with  $n + 1$  leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.
- The number of ways to completely parenthesize  $n + 1$  factors.
- The number of triangulations of a convex polygon with  $n + 2$  sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).
- The number of ways to connect the  $2n$  points on a circle to form  $n$  disjoint chords.
- The number of non-isomorphic full binary trees with  $n$  internal nodes (i.e. nodes having at least one son).
- The number of monotonic lattice paths from point  $(0, 0)$  to point  $(n, n)$  in a square lattice of size  $nm$ , which do not pass above the main diagonal (i.e. connecting  $(0, 0)$  to  $(n, n)$ ).
- Number of permutations of length  $n$  that can be stack sorted (i.e. it can be shown that the rearrangement is stack sorted if and only if there is no such index  $i < j < k$ , such that  $a_k < a_i < a_j$ ).
- The number of non-crossing partitions of a set of  $n$  elements.
- The number of ways to cover the ladder  $1 \dots n$  using  $n$  rectangles (The ladder consists of  $n$  columns, where  $i^{th}$  column has a height  $i$ ).

Recursive:

$$C_0 = C_1 = 1$$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, n \geq 2$$

Analytical:

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

## 10.7 Landau

There is a tournament with outdegrees  $d_1 \leq d_2 \leq \dots \leq d_n$  iff:

- $d_1 + d_2 + \dots + d_n = \binom{n}{2}$
- $d_1 + d_2 + \dots + d_k \geq \binom{k}{2} \quad \forall 1 \leq k \leq n$ .

In order to build it, let 1 point to  $2, 3, \dots, d_1 + 1$  and repeat recursively.

## 10.8 Erdős-Gallai

There is a simple graph with degrees  $d_1 \geq d_2 \geq \dots \geq d_n$  iff:

- $d_1 + d_2 + \dots + d_n$  is even
- $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \quad \forall 1 \leq k \leq n$ .

In order to build it, connect 1 with  $2, 3, \dots, d_1 + 1$  and repeat recursively.

## 10.9 Gambler's Ruin

In a game in which we win a coin with probability  $p$  and lose a coin with probability  $q := 1 - p$ , the game stops when we win  $B$  or lose  $A$  coins. Then  $\text{Prob}(\text{win B}) = \frac{1-(p/q)^B}{1-(p/q)^{A+B}}$ .

### 10.10 Extra

- $\text{Fib}(x + y) = \text{Fib}(x + 1)\text{Fib}(y) + \text{Fib}(x)\text{Fib}(y - 1)$