11/09/2021 (Parte 2 ejercicio)

Thursday, November 11, 2021 8:49 AM

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

$$m^2 + 3m + 2 = 0$$

$$(m+2)(m+1)$$

$$m_1 = -1$$

$$m_2 = -2$$

$$y_1 = c_1 e^{-x}$$

$$y_2 = c_2 e^{-2x}$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & e^{-2x} \\ (1+e^x)^{-1} & -2e^{-2x} \end{vmatrix} = \underline{\underline{-(1+e^x)^{-1}e^{-2x}}}$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & (1+e^x)^{-1} \end{vmatrix} = \underline{\underline{(1+e^x)^{-1}e^{-x}}}$$

$$U_1' = \frac{W_1}{W}$$

$$\boxed{U_1' = \frac{-(1+e^x)^{-1}e^{-2x}}{-e^{-3x}} = \frac{e^x}{1+e^x}}$$

$$U_2' = \frac{W_2}{W}$$

$$\boxed{U_2' = \frac{(1+e^x)^{-1}e^{-x}}{-e^{-3x}} = -\frac{e^{2x}}{1+e^x}}$$

$$U_1 = \int \frac{e^x}{1+e^x}\,dx \Rightarrow \quad u = 1+e^x \qquad \frac{du}{e^x} = dx$$

$$\frac{du}{dx} = e^x$$

$$\boxed{U_1 = \int \frac{du}{u} = \ln u = \ln(1+e^x)}$$

$$u_2 = -\int \frac{e^{2x}}{1+e^x}\,dx$$

$$u = e^x \quad \frac{du}{dx} = e^x \to dx = \frac{du}{e^x}$$

Si $\boxed{u = e^x}$ $\boxed{u^2 = e^{2x}}$

$$u_2 = -\int \frac{u^2}{1+u}\frac{du}{u} \to -\int \frac{u}{1+u}\,du$$

$$-\int \frac{u+1-1}{1+u}\,du \to -\int\left(1 - \frac{1}{1+u}\right)du$$

$$\boxed{u_2 = -u + \ln(1+u) = -e^x + \ln(1+e^x)}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \frac{c_1 \operatorname{Ln}(1+e^x)}{e^x} + \frac{c_2(\operatorname{Ln}(1+e^x) - e^{x})}{e^{2x}}$$

$$= \operatorname{Ln}(1+e^x)\left[\frac{c_1}{e^x} + \frac{c_2}{e^{2x}}\right] - e^{-x}$$