Cálculo
Lista 4 — Integrais

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Capítulo 1

Integrais imediatas / quase imediatas

Resolva as integrais imediatas ou quase imediatas:

1) $\displaystyle\int (3x^2 - 2x + 4)\mathrm{dx}$

2) $\displaystyle\int \left(\frac{x^3}{2} - 1\right)\mathrm{dx}$

3) $\displaystyle\int \frac{1-x}{2}\mathrm{dx}$

4) $\displaystyle\int \left(\frac{1}{3}x^2 - \frac{1}{2}x - 3\right)\mathrm{dx}$

1)

$$\int (3x^2 - 2x + 4\mathrm{dx}) \implies \overset{\frac{3x^3}{3}}{\cancel{\int 3x^2}} - \overset{\frac{2x^2}{2x}}{\cancel{\int 2}} + \overset{4x}{\cancel{\int 4}} = \frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}} + 4x =$$

$$\boxed{\int (3x^2 - 2x + 4\mathrm{dx}) = x^3 - x^2 + 4x + C}$$

2)

$$\int \left(\frac{x^3}{2} - 1\right)\mathrm{dx} \implies \frac{1}{2}\overset{\frac{x^4}{4}}{\cancel{\int x^3}}\mathrm{dx} - \overset{x}{\cancel{\int 1}}\mathrm{dx} = \frac{1}{2}\frac{x^4}{4} - x + C$$

$$\boxed{\int \left(\frac{x^3}{2} - 1\right) = \frac{x^4}{8} - x + C}$$

3)

$$\int \frac{1-x}{2}\mathrm{dx} \implies \int \frac{1}{2}\mathrm{dx} - \overset{\frac{1}{2}x}{\cancel{\int \frac{x}{2}}}\mathrm{dx} = \frac{x}{2} - \frac{1}{2}\overset{\frac{x^2}{2}}{\cancel{\int x}}\mathrm{dx} = \frac{x}{2} - \frac{1}{2}\frac{x^2}{2} + C$$

$$\boxed{\int \frac{1-x}{2}\mathrm{dx} = \frac{x}{2} - \frac{x^2}{4} + C}$$

4)

$$\begin{aligned}
\int \left(\frac{1}{3}x^2 - \frac{1}{2}x - 3\right) \mathrm{dx} \implies \frac{1}{3}\int x^2 \mathrm{dx} - \frac{1}{2}\int x\mathrm{dx} - \int 3\mathrm{dx} \\
= \frac{1}{3}\frac{x^3}{3} - \frac{1}{2}\frac{x^2}{2} - 3x + C \\
\boxed{\int \left(\frac{1}{3}x^2 - \frac{1}{2}x - 3\right) \mathrm{dx} = \frac{x^3}{9} - \frac{x^2}{4} - 3x + C}
\end{aligned}$$

Capítulo 2

Integral por substituição

Resolva as integrais por substituição:

1) $\displaystyle\int \frac{-6x-5}{-3x^2-5x-2}\mathrm{dx}$

2) $\displaystyle\int \frac{3x-1}{3x^2-2x}\mathrm{dx}$

3) $\displaystyle\int (x^2-5)^3 x\mathrm{dx}$

4) $\displaystyle\int \frac{\mathrm{dx}}{(5-3x)^2}$

1)

$$\int \frac{-6x-5}{-3x^2-5x-2}\mathrm{dx} \implies \int \underbrace{\frac{1}{-3x^2-5x-2}}_{u}\underbrace{(-6x-5)\mathrm{dx}}_{\mathrm{du}}$$

$$u = -3x^2-5x-2$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = -6x-5 \implies \mathrm{du} = (-6x-5)\mathrm{dx}$$

$$\int \frac{1}{u}\mathrm{du} \implies \ln u + C \implies \boxed{\ln|-3x^2-5x-2|+C}$$

2)

$$\int \frac{3x-1}{3x^2-2x}\mathrm{dx} \implies \int \frac{1}{\underbrace{3x^2-2x}_{u}}\underbrace{(3x-1)\mathrm{dx}}_{\mathrm{du}}$$

$$u = 3x^2-2x$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = 6x-2 \implies$$

$$\mathrm{du} = (6x-2)\mathrm{dx} =$$
$$\mathrm{du} = 2(3x-1)\mathrm{dx}$$

$$\boxed{\frac{\mathrm{du}}{2} = (3x-1)\mathrm{dx}}$$

$$\int \frac{1}{u}\frac{\mathrm{du}}{2} =$$

$$\int \frac{1}{u}\frac{\mathrm{du}}{2} = \int \frac{1}{u}\frac{1}{2}\mathrm{du} \implies \frac{1}{2}\int \frac{1}{u}\mathrm{du}$$

$$\frac{1}{2}\ln|u| + C \implies$$

$$\boxed{\int \frac{3x-1}{3x^2-2x}\mathrm{dx} = \frac{1}{2}\ln|3x^2-2x| + C}$$

3)

$$\int \underbrace{(x^2-5)^3}_{u}\underbrace{x\mathrm{dx}}_{\mathrm{du}}$$

$$u = x^2 - 5 \implies u' = 2x$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = 2x \implies \mathrm{du} = 2x\mathrm{dx} \implies \frac{\mathrm{du}}{2} = x\mathrm{dx}$$

$$\int (x^2-5)^3 x\mathrm{dx} \implies$$

$$\int (u)^3\frac{\mathrm{du}}{2} \implies \int (u)^3\mathrm{du} \implies \frac{1}{2}\int u^3\mathrm{du} \implies \frac{1}{2}\frac{u^4}{4} + C = \frac{u^4}{8} + C \implies$$

$$\boxed{\int (x^2-5)^3 x\mathrm{dx} = \frac{(x^2-5)^4}{8} + C}$$

4)

$$\int \frac{\mathrm{dx}}{(5-3x)^2} = \int \frac{1}{\underbrace{(5-3x)^2}_{u}}\underbrace{\mathrm{dx}}_{\mathrm{du}}$$

$$u = -3x + 5 \implies u' = -3$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = -3 \implies \mathrm{du} = -3\mathrm{dx} \implies -\frac{\mathrm{du}}{3} = \mathrm{dx}$$

$$\int \frac{1}{u^2}\left(-\frac{du}{3}\right) \implies$$

$$\int u^{-2}\left(-\frac{1}{3}\mathrm{du}\right) = -\frac{1}{3}\int u^{-2}\mathrm{du} \implies -\frac{1}{3}\frac{u^{-1}}{-1} + C = \frac{1}{3}\frac{1}{-3x+5} + C = \frac{1}{3(-3x+5)} + C$$

$$\boxed{\int \frac{\mathrm{dx}}{(5-3x)^2} = \frac{1}{-9x+15} + C}$$

Capítulo 3

Integrais definidas — aplicações

1) Calcule a área entre os gráficos de $y = x + 2$ e $y = x^2$

2) Calcule a área limitada pela curva $y = -x^2 + 5x$ e pelo *eixo* x.

3) Calcula a área sob o (abaixo do) gráfico da função $y = x$, de $x = 0$ a $x = 3$.

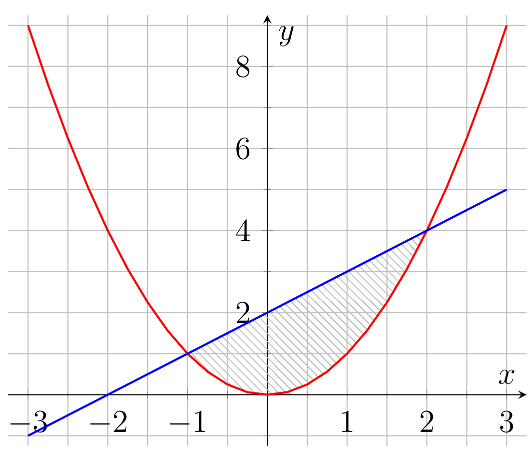
4) Determine o volume do sólido de revolução gerado pela rotação *em torno do eixo* x, da região limitada por $y = 3x + 1$, $x = 0$, $x = 3$ e $y = 0$

5) Determine o volume do sólido de revolução gerado pela rotação *em torno do eixo* x, da região limitada por $y = x^2 + 1$, $x = 1$, $x = 3$ e $y = 0$.

1) Calcule a área entre os gráficos de $y = x + 2$ e $y = x^2$

Parte (a)

Gráfico:



Parte (b)

Área

$$f(x) = x^2 \implies F(x) = \int f(x)\mathrm{dx} = \int x^2\mathrm{dx} = \boxed{\frac{x^3}{3} + C}$$

$$g(x) = x + 2 \implies G(X) = \int g(x)\mathrm{dx} = \int x + 2\mathrm{dx} = \boxed{\frac{x^2}{2} + 2x + C}$$

Pontos de interseção:

$$f(x) = g(x) \implies x^2 = x + 2 \implies \underbrace{x^2}_{a}\underbrace{-x}_{b}\underbrace{-2}_{c} = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \implies \frac{1 \pm \sqrt{-1^2 - 4(1)(-2)}}{2} =$$

$$\frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2} =$$

$$\frac{1 \pm 3}{2} = -\frac{2}{2}, \frac{4}{2}$$

$$\boxed{\{a, b\} = -1, 2}$$

$$A = \int_a^b [f(x) - g(x)]\mathrm{dx}$$

$$A = [F(x) - G(x)]_a^b$$

$$A = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{-1}^{2}$$

Como $G(X) \geq F(X)$ em $[-1, 2]$

$$A = [G(2) - F(2)] - [G(-1) - F(-1)]$$