Cálculo
Lista 2 - Limites

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Capítulo 1

Noção Intuitiva / Limites laterais

Problema 1. Dadas as funções e seus respectivos gráficos:

(a) calcule os limites laterais da $f(x)$

(b) compare os limites laterais e verifique se eles são iguais ou diferentes

(c) conclua se existe o limite da função e se existir, indique qual é o seu valor

1)

$$\lim_{x \to 0} f(x), \text{para } f(x) = \begin{cases} x^2 & \text{, se } x \leq 0 \\ 1 + x^2 & \text{, se } x > 0 \end{cases}$$

2)

$$\lim_{x \to 2} f(x), \text{para } f(x) = \begin{cases} x^2 & \text{, se } x \geq 2 \\ x + 1 & \text{, se } x < 2 \end{cases}$$

3)

$$\lim_{x \to 0} f(x), \text{para } f(x) = \begin{cases} -x^2 + 3 & \text{, se } x < 0 \\ -4x + 3 & \text{, se } x \geq 0 \end{cases}$$

4)

$$\lim_{x \to 1} f(x), \text{para } f(x) = \begin{cases} 3x + 1 & \text{, se } x \neq 1 \\ 0 & \text{, se } x = 1 \end{cases}$$

1)

$$\lim_{x\to 0} f(x), \text{para}\, f(x) = \begin{cases} x^2 & \textbf{, se } x \leq 0 \\ 1 + x^2 & \textbf{, se } x > 0 \end{cases}$$

$$\begin{aligned} \lim_{x\to 0^-} f(x) &= x^2 \\ f(0) &= 0^2 \\ f(0) &= 0 \\ \lim_{x\to 0^-} f(x) &= 0 \end{aligned} \qquad \begin{aligned} \lim_{x\to 0^+} f(x) &= 1 + x^2 \\ f(0) &= 1 + 0^2 \\ f(0) &= 1 \\ \lim_{x\to 0^+} f(x) &= 1 \end{aligned}$$

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

2)

$$\lim_{x\to 2} f(x), \text{para}\, f(x) = \begin{cases} x^2 & \textbf{, se } x \geq 2 \\ x + 1 & \textbf{, se } x < 2 \end{cases}$$

$$\begin{aligned} \lim_{x\to 2^-} f(x) &= x^2 \\ f(2) &= 2^2 \\ f(2) &= 4 \\ \lim_{x\to 2^-} f(x) &= 4 \end{aligned} \qquad \begin{aligned} \lim_{x\to 2^+} f(x) &= x + 1 \\ f(2) &= 2 + 1 \\ f(2) &= 3 \\ \lim_{x\to 2^+} f(x) &= 3 \end{aligned}$$

$$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$$

3)

$$\lim_{x\to 0} f(x), \text{para}\, f(x) = \begin{cases} -x^2 + 3 & \textbf{, se } x < 0 \\ -4x + 3 & \textbf{, se } x \geq 0 \end{cases}$$

$$\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} (-x^2 + 3) \\ &= -1^2 + 3 \\ &= -1 + 3 \\ \lim_{x\to 1^-} f(x) &= 2 \end{aligned} \qquad \begin{aligned} \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} (-4x + 3) \\ &= -4 * 1 + 3 \\ &= -4 + 3 \\ \lim_{x\to 1^+} f(x) &= -1 \end{aligned}$$

$$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

4)

$$\lim_{x \to 1} f(x), \text{para}\, f(x) = \begin{cases} 3x + 1 & \textbf{, se } x \neq 1 \\ 0 & \textbf{, se } x = 1 \end{cases}$$

$$\begin{aligned} \lim_{x \to 1} f(x) &= \lim_{x \to 1} (3x + 1) \\ &= 3 * 1 + 1 \\ \lim_{x \to 1} f(x) &= 4 \end{aligned}$$

Capítulo 2

Limites com indeterminação

Problema 2. **Calcule os limites, indicando o passo a passo.**

1)
$$\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$$

2)
$$\lim_{x\to 10} \frac{x^2-100}{x-10}$$

3)
$$\lim_{x\to 7} \frac{x^2-49}{x-7}$$

4)
$$\lim_{x\to 2} \frac{3x^2-5x-2}{x^2+3x-10}$$

5)
$$\lim_{x\to -1} \frac{x^2+3x+2}{x^2-1}$$

1)

$$\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}$$

Multiplicando divisor e numerador pelo conjugado do numerador:

$$\frac{\sqrt{x+3}-\sqrt{3}}{x}=$$

$$\frac{(\sqrt{x+3}-\sqrt{3})(\sqrt{x+3}+\sqrt{3})}{x(\sqrt{x+3}+\sqrt{3})}=\frac{x}{x(\sqrt{x+3}+\sqrt{3})}=\frac{1}{\sqrt{x+3}+\sqrt{3}}$$

$$\boxed{\lim_{x\to 0}\frac{1}{\sqrt{x+3}+\sqrt{3}}=\frac{1}{\sqrt{0+3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}}$$

Multiplicando numerador e divisor por $\sqrt{3}$:

$$\lim_{x\to 0}\frac{1}{\sqrt{x+3}+\sqrt{3}}=$$

$$\frac{1}{2\sqrt{3}}=\frac{1(\sqrt{3})}{2\sqrt{3}(\sqrt{3})}=\frac{\sqrt{3}}{2*3}$$

$$\boxed{\lim_{x\to 0}\frac{1}{\sqrt{x+3}+\sqrt{3}}=\frac{\sqrt{3}}{6}}$$

2)

Sabendo que $a^2-b^2=(a+b)(a-b)$:

$$\lim_{x\to 10}\frac{x^2-100}{x-10}$$

$$\frac{x^2-10^2}{x-10}=\frac{(x+10)(x-10)}{x-10}=x+10$$

$$\boxed{\lim_{x\to 10}x+10=20}$$

3)

Sabendo que $a^2-b^2=(a+b)(a-b)$:

$$\lim_{x\to 7}\frac{x^2-49}{x-7}$$

$$\frac{x^2-49}{x-7}=\frac{x^2-7^2}{x-7}=\frac{(x-7)(x+7)}{x-7}=x+7$$

$$\boxed{\lim_{x\to 7}x+7=14}$$

4)

$$\lim_{x\to 0} \frac{3x^2-5x-2}{x^2+3x-10}$$

$$\text{Sabendo que } ax^2+bx+c = a(x-x1)(x-x2)$$
$$x1, x2 \equiv \text{Raizes da função}$$

Utilizando a fórmula de báskara para achar as raízes da função:
Numerador $(3x^2-5x-2)$:

$$\{x1, x2\} = \frac{-(-5) \pm \sqrt{(-5)^2-4*3*2}}{2*3}$$
$$= \frac{5 \pm \sqrt{25+24}}{6} = \frac{5 \pm \sqrt{49}}{6} = \frac{5 \pm 7}{6}$$
$$\boxed{\{x1, x2\} = \left\{\frac{12}{6}, \frac{-2}{6}\right\} = \left\{2, -\frac{1}{3}\right\}}$$

Divisor: $(x^2+3x-10)$

$$\{x1, x2\} = \frac{-3\sqrt{3^2-4*1*-10}}{2*1}$$
$$= \frac{-3\sqrt{3^2-4*1*-10}}{2*1} = \frac{-3 \pm \sqrt{9+40}}{2} = \frac{-3 \pm 7}{2}$$
$$\boxed{\{x1, x2\} = \left\{\frac{4}{2}, \frac{-10}{2}\right\} = \left\{2, -5\right\}}$$

Simplificando o divisor e numerador:

$$3x^2-5x-2 = 3(x-2)(x+\frac{1}{3})$$

$$x^2+3x-10 = 1(x-2)(x+5)$$

$$\lim_{x\to 2} \frac{3x^2-5x-2}{x^2+3x-10}$$
$$\frac{3x^2-5x-2}{x^2+3x-10} = \frac{3(x-2)(x+\frac{1}{3})}{(x-2)(x+5)} = \frac{3(x+\frac{1}{3})}{x+5} = \frac{3x+1}{x+5} = \frac{3x+1}{x+5} = \frac{3*2+1}{2+5} = \frac{7}{7}$$
$$\boxed{\lim_{x\to 2} \frac{3x^2-5x-2}{x^2+3x-10} = 1}$$

5)

$$\lim_{x\to -1} \frac{x^2+3x+2}{x^2-1}$$

$$\text{Sabendo que } ax^2+bx+c = a(x-x1)(x-x2)$$

$$x1, x2 \equiv \text{Raizes da função}$$

$$\frac{-3 \pm \sqrt{-3^2-4*1*2}}{2*1} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2}$$

$$\{x1, x2\} = \left\{\frac{-2}{2}, \frac{-4}{2}\right\} = \left\{-1, -2\right\}$$

$$x^2+3x+2 = (x+1)(x+2)$$

Sabendo que $a^2-b^2 = (a+b)(a-b)$

$$x^2-1 = x^2-1^2 = (x-1)(x+1)$$

Então:

$$\lim_{x\to -1} \frac{x^2+3x+2}{x^2-1}$$

$$\frac{x^2+3x+2}{x^2-1} = \frac{(x+1)(x+2)}{(x-1)(x+1)} = \frac{x+2}{x-1} = \frac{x+2}{x-1} = \frac{-1+2}{-1-1} = \frac{1}{-2}$$

$$\boxed{\lim_{x\to -1} \frac{x^2+3x+2}{x^2-1} = -\frac{1}{2}}$$