Cálculo
Lista 4 — Integrais

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Capítulo 1

Integrais imediatas / quase imediatas

Resolva as integrais imediatas ou quase imediatas:

1) $\displaystyle\int (3x^2 - 2x + 4)\mathrm{dx}$

2) $\displaystyle\int \left(\frac{x^3}{2} - 1\right)\mathrm{dx}$

3) $\displaystyle\int \frac{1-x}{2}\mathrm{dx}$

4) $\displaystyle\int \left(\frac{1}{3}x^2 - \frac{1}{2}x - 3\right)\mathrm{dx}$

1)

$$\int (3x^2 - 2x + 4\mathrm{dx}) \implies \cancelto{\frac{3x^3}{3}}{\int 3x^2} - \cancelto{\frac{2x^2}{2x}}{\int 2} + \cancelto{4x}{\int 4} = \frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}} + 4x =$$

$$\boxed{\int (3x^2 - 2x + 4\mathrm{dx}) = x^3 - x^2 + 4x + C}$$

2)

$$\int \left(\frac{x^3}{2} - 1\right)\mathrm{dx} \implies \frac{1}{2}\cancelto{\frac{x^4}{4}}{\int x^3}\mathrm{dx} - \cancelto{x}{\int 1}\mathrm{dx} = \frac{1}{2}\frac{x^4}{4} - x + C$$

$$\boxed{\int \left(\frac{x^3}{2} - 1\right) = \frac{x^4}{8} - x + C}$$

3)

$$\int \frac{1-x}{2}\mathrm{dx} \implies \int \frac{1}{2}\mathrm{dx} - \cancelto{\frac{1}{2}x}{\int \frac{x}{2}}\mathrm{dx} = \frac{x}{2} - \frac{1}{2}\cancelto{\frac{x^2}{2}}{\int x}\mathrm{dx} = \frac{x}{2} - \frac{1}{2}\frac{x^2}{2} + C$$

$$\boxed{\int \frac{1-x}{2}\mathrm{dx} = \frac{x}{2} - \frac{x^2}{4} + C}$$

4)

$$\int \left(\frac{1}{3}x^2 - \frac{1}{2}x - 3\right)\mathrm{dx} \implies \frac{1}{3}\int x^2\mathrm{dx} - \frac{1}{2}\int x\mathrm{dx} - \int 3\mathrm{dx} = \frac{1}{3}\frac{x^3}{3} - \frac{1}{2}\frac{x^2}{2} - 3x + C$$

$$\boxed{\int \left(\frac{1}{3}x^2 - \frac{1}{2}x - 3\right)\mathrm{dx} = \frac{x^3}{9} - \frac{x^2}{4} - 3x + C}$$

Capítulo 2

Integral por substituição

Resolva as integrais por substituição:

1) $\displaystyle\int \frac{-6x-5}{-3x^2-5x-2}\mathrm{dx}$

2) $\displaystyle\int \frac{3x-1}{3x^2-2x}\mathrm{dx}$

3) $\displaystyle\int (x^2-5)^3 x\mathrm{dx}$

4) $\displaystyle\int \frac{\mathrm{dx}}{(5-3x)^2}$

1)

$$\int \frac{-6x-5}{-3x^2-5x-2}\mathrm{dx} \implies \int \underbrace{\frac{1}{-3x^2-5x-2}}_{u}\underbrace{(-6x-5)\mathrm{dx}}_{\mathrm{du}}$$

$$u = -3x^2-5x-2$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = -6x-5 \implies \mathrm{du} = (-6x-5)\mathrm{dx}$$

$$\int \frac{1}{u}\mathrm{du} \implies \ln u + C \implies \boxed{\ln|-3x^2-5x-2|+C}$$

2)

$$\int \frac{3x-1}{3x^2-2x}\mathrm{dx} \implies \int \underbrace{\frac{1}{3x^2-2x}}_{u}\underbrace{(3x-1)\mathrm{dx}}_{\mathrm{du}}$$

$$u = 3x^2-2x$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = 6x-2 \implies$$

$$\mathrm{du} = (6x-2)\mathrm{dx} =$$
$$\mathrm{du} = 2(3x-1)\mathrm{dx}$$

$$\boxed{\frac{\mathrm{du}}{2} = (3x-1)\mathrm{dx}}$$

$$\int \frac{1}{u}\frac{\mathrm{du}}{2} =$$

$$\int \frac{1}{u}\frac{\mathrm{du}}{2} = \int \frac{1}{u}\frac{1}{2}\mathrm{du} \implies \frac{1}{2}\int \frac{1}{u}\mathrm{du}$$
$$\frac{1}{2}\ln|u|+C \implies$$

$$\boxed{\int \frac{3x-1}{3x^2-2x}\mathrm{dx} = \frac{1}{2}\ln|3x^2-2x|+C}$$

3)

$$\int \underbrace{(x^2-5)^3}_{u}\underbrace{x\mathrm{dx}}_{\mathrm{du}}$$

$$u = x^2-5 \implies u' = 2x$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = 2x \implies \mathrm{du} = 2x\mathrm{dx} \implies \frac{\mathrm{du}}{2} = x\mathrm{dx}$$

$$\int (x^2-5)^3 x\mathrm{dx} \implies$$

$$\int (u)^3 \frac{\mathrm{du}}{2} \implies \int (u)^3 \mathrm{du} \implies \frac{1}{2}\int u^3 \mathrm{du} \implies \frac{1}{2}\frac{u^4}{4} + C = \frac{u^4}{8} + C \implies$$

$$\boxed{\int (x^2-5)^3 x\mathrm{dx} = \frac{(x^2-5)^4}{8} + C}$$

4)

$$\int \frac{\mathrm{dx}}{(5-3x)^2} = \int \frac{1}{\underbrace{(5-3x)}_{u}{}^2} \underbrace{\mathrm{dx}}_{\mathrm{du}}$$

$$u = -3x + 5 \implies u' = -3$$

$$\frac{\mathrm{du}}{\mathrm{dx}} = -3 \implies \mathrm{du} = -3\mathrm{dx} \implies -\frac{\mathrm{du}}{3} = \mathrm{dx}$$

$$\int \frac{1}{u^2}\left(-\frac{du}{3}\right) \implies$$

$$\int u^{-2}\left(-\frac{1}{3}\mathrm{du}\right) = -\frac{1}{3}\int u^{-2}\mathrm{du} \implies -\frac{1}{3}\frac{u^{-1}}{-1} + C = \frac{1}{3}\frac{1}{-3x+5} + C = \frac{1}{3(-3x+5)} + C$$

$$\boxed{\int \frac{\mathrm{dx}}{(5-3x)^2} = \frac{1}{-9x+15} + C}$$

Capítulo 3

Integrais definidas — aplicações

1) Calcule a área entre os gráficos de $y = x + 2$ **e** $y = x^2$

2) Calcule a área limitada pela curva $y = -x^2 + 5x$ **e pelo *eixo*** x**.**

3) Calcula a área sob o (abaixo do) gráfico da função $y = x$**, de** $x = 0$ **a** $x = 3$**.**

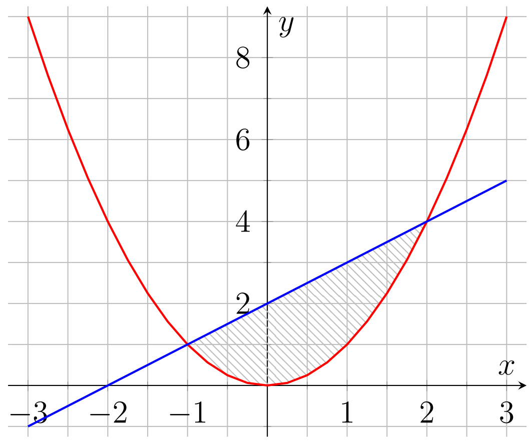
4) Determine o volume do sólido de revolução gerado pela rotação *em torno do eixo* x**, da região limitada por** $y = 3x + 1$**,** $x = 0$**,** $x = 3$ **e** $y = 0$

5) Determine o volume do sólido de revolução gerado pela rotação *em torno do eixo* x**, da região limitada por** $y = x^2 + 1$**,** $x = 1$**,** $x = 3$ **e** $y = 0$**.**

1) Calcule a área entre os gráficos de $y = x + 2$ e $y = x^2$

Parte (a)

Gráfico:



Parte (b)

Área

$$f(x) = x^2 \implies F(x) = \int f(x)\mathrm{dx} = \int x^2\mathrm{dx} = \boxed{\frac{x^3}{3} + C}$$

$$g(x) = x + 2 \implies G(X) = \int g(x)\mathrm{dx} = \int x + 2\mathrm{dx} = \boxed{\frac{x^2}{2} + 2x + C}$$

Pontos de interseção:

$$f(x) = g(x) \implies x^2 = x + 2 \implies \underbrace{x^2}_{a}\underbrace{-x}_{b}\underbrace{-2}_{c} = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \implies \frac{1 \pm \sqrt{-1^2 - 4(1)(-2)}}{2} =$$

$$\frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2} =$$

$$\frac{1 \pm 3}{2} = -\frac{2}{2}, \frac{4}{2}$$

$$\boxed{\{a, b\} = -1, 2}$$

$$A = \int_a^b [f(x) - g(x)]\mathrm{dx}$$

$$A = [F(x) - G(x)]_a^b$$

$$A = \left[\frac{x^3}{3} - \left(\frac{x^2}{2} + 2x\right)\right]_{-1}^{2}$$

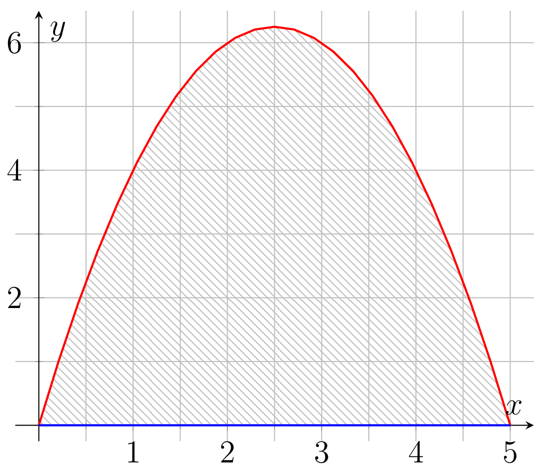
Como $G(X) \geq F(X)$ em $[-1, 2]$,

$$\begin{aligned}
A &= [G(2) - F(2)] - [G(-1) - F(-1)] \\
&= \left[\left(\frac{2^2}{2} + 2(2)\right) - \frac{2^3}{3}\right] - \left[\left(\frac{-1^2}{2} + 2(-1)\right) - \frac{-1^3}{3}\right] \\
&= \left[\left(\frac{4}{2} + 4\right) - \frac{8}{3}\right] - \left[\left(\frac{-1}{2} + -2\right) - \frac{-1}{3}\right] \\
&= \left[2 + 4 - \frac{8}{3}\right] - \left[\left(\frac{-1}{2} - 2\right) + \frac{1}{3}\right] \\
&= 2 + 4 - \frac{8}{3} + \frac{1}{2} - 2 - \frac{1}{3} \\
&= \frac{12 + 24 - 16 + 3 - 12 - 2}{6} \\
A &= \boxed{\frac{9}{6}u^2}
\end{aligned}$$

2) Calcule a área limitada pela curva $y = -x^2 + 5x$ e pelo *eixo* x.

Parte (a)

Gráfico



Parte (b)

Área

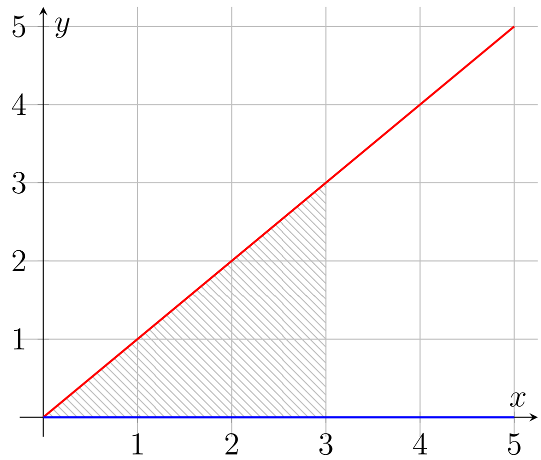
$$f(x) = -x^2 + 5x \implies F(x) = \int f(x)\mathrm{dx} = \int -x^2 + 5x\mathrm{dx} = \frac{-x^3}{3} + \frac{5x^2}{2}$$

$$\begin{aligned}
A &= \int_0^5 f(x)\mathrm{dx} \\
&= \frac{-x^3}{3} + \frac{5x^2}{2}\bigg|_0^5 \\
&= \frac{-5^3}{3} + \frac{5(5)^2}{2} \cancel{- \frac{-0^3}{3} + \frac{5(0)^2}{2}} \\
&= -\frac{125}{3} + \frac{125}{2} \\
&= \frac{-125(2) + 125(3)}{6} \\
&= \frac{-250 + 375}{6} \\
A &= \boxed{\frac{125}{6}u^2}
\end{aligned}$$

3) Calcula a área sob o (abaixo do) gráfico da função $y = x$, de $x = 0$ a $x = 3$.

Parte (a)

Gráfico



Parte (b)

Área

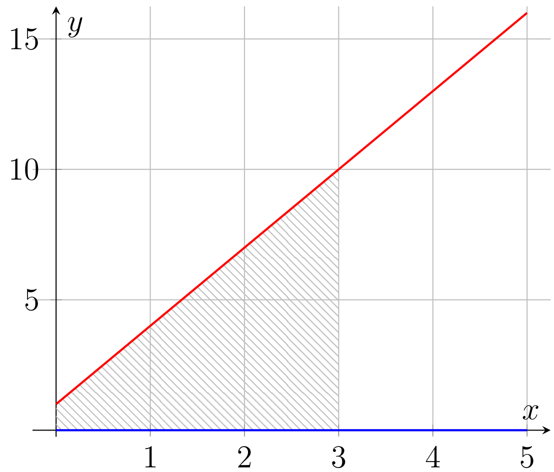
$$f(x) = x \implies F(x) = \int f(x)\mathrm{dx} = \int x\mathrm{dx} = \frac{x^2}{2}$$

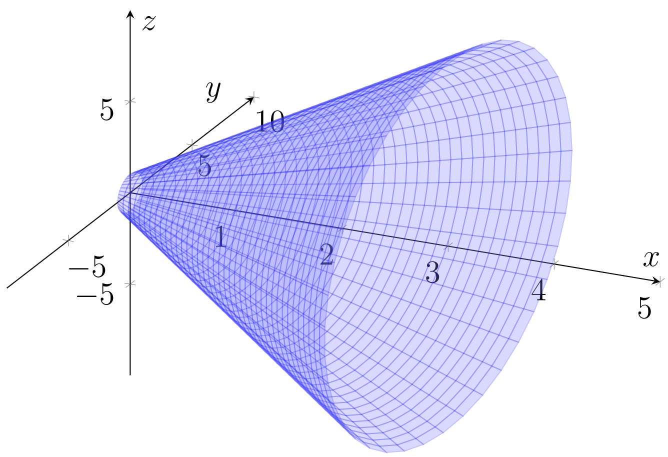
$$\begin{aligned} A &= \int_0^3 f(x)\mathrm{dx} \\ &= \left.\frac{x^2}{2}\right|_0^3 \\ &= \frac{3^2}{2} - \cancel{\frac{0^2}{2}} \\ &= \frac{9}{2} \\ A &= \boxed{\frac{9}{2}u^2} \end{aligned}$$

4) Determine o volume do sólido de revolução gerado pela rotação *em torno do eixo* x, da região limitada por $y = 3x + 1$, $x = 0$, $x = 3$ e $y = 0$

Parte (a)

Gráficos





Parte (b)

Área

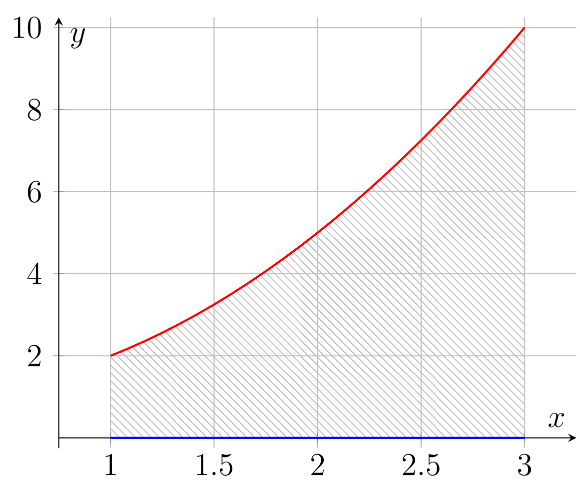
$$f(x) = 2x + 1 \implies F(x) = \int f(x)\mathrm{dx} = \int 2x + 1\mathrm{dx} = \frac{3x^2}{2} + x$$

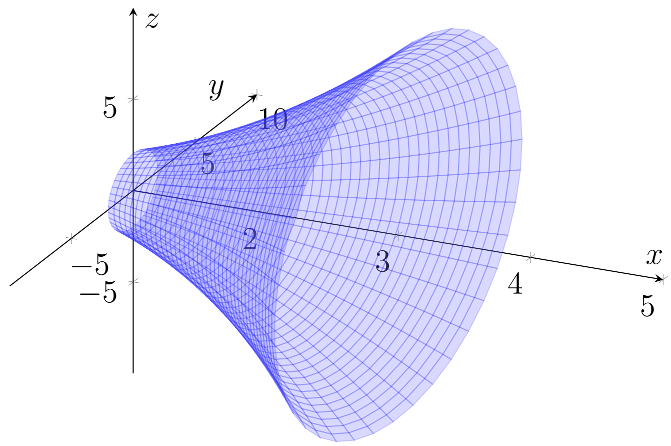
$$\begin{aligned}
V &= \pi \int_0^3 f(x)^2 \mathrm{dx} \\
&= \pi \int_0^3 (2x+1)^2 \mathrm{dx} \\
&= \pi \int_0^3 4x^2 + 4x + 1 \mathrm{dx} \\
&= \pi \left[\frac{4x^3}{3} + \frac{4x^2}{2} + x \right]_0^3 \\
&= \pi \left(\frac{4(3)^3}{3} + \frac{4(3)^2}{2} + (3) \right) \\
&= \pi \left(\frac{108}{3} + \frac{36}{2} + 3 \right) \\
V &= \boxed{57\pi u^3}
\end{aligned}$$

5) Determine o volume do sólido de revolução gerado pela rotação *em torno do eixo* x, da região limitada por $y = x^2 + 1$, $x = 1$, $x = 3$ e $y = 0$

Parte (a)

Gráficos





Parte (b)

Área

$$f(x) = x^2 + 1 \implies F(x) = \int f(x)\mathrm{dx} = \int x^2 + 1\mathrm{dx} = \frac{x^3}{3} + x$$

$$\begin{aligned}
V &= \pi \int_1^3 f(x)^2 \mathrm{dx} \\
&= \pi \int_1^3 (x^2+1)^2 \mathrm{dx} \\
&= \pi \int_1^3 x^4 + 2x^2 + 1\mathrm{dx} \\
&= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_1^3 \\
&= \pi \left(\left[\frac{(3)^5}{5} + \frac{2(3)^3}{3} + (3) \right] - \left[\frac{(1)^5}{5} + \frac{2(1)^3}{3} + (1) \right] \right) \\
&= \pi \left(\left[\frac{253}{5} + \frac{54}{3} + 3 \right] - \left[\frac{1}{5} + \frac{8}{3} + 1 \right] \right) \\
&= \pi \left(\frac{759 + 270 + 45}{15} - \frac{3 + 40 + 15}{15} \right) \\
V &= \boxed{\frac{1016}{15}\pi u^3}
\end{aligned}$$