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Lista de C III:

① $\mathcal{L}\{te^{10t}\} = \mathcal{L}\{t\}\big|_{s \to s-10} = \frac{1}{s^2}\Big|_{s \to s-10} = \frac{1}{(s-10)^2}$

⑤ $\mathcal{L}\{t(e^t + e^{2t})^2\}$

$= \mathcal{L}\{t(e^{2t} + 2e^{3t} + e^{4t})\} = \mathcal{L}\{t.e^{2t} + 2t.e^{3t} + te^{4t}\}$

$= \mathcal{L}\{t\}\big|_{s \to s-2} + \mathcal{L}\{2t\}\big|_{s \to s-3} + \mathcal{L}\{t\}\big|_{s \to s-4}$

$= \frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^2}$

⑥ $\mathcal{L}\{t^{10}e^{-7t}\} = \mathcal{L}\{t^{10}\}\big|_{s \to s+7} = \frac{10!}{s^{10+1}} = \frac{10!}{(s+7)^{11}}$

⑨ $\mathcal{L}\{(1 - e^t + 3e^{-4t})\cos 5t\}$

$\mathcal{L}\{\cos 5t - e^t.\cos 5t + 3e^{-4t}\cos 5t\}$

$= \mathcal{L}\{\cos 5t\} - \mathcal{L}\{\cos 5t\}\big|_{s \to s-1} + 3\mathcal{L}\{\cos 5t\}\big|_{s \to s+4}$

$\frac{s}{s^2 + 25} - \frac{(s-1)}{(s-1)^2 + 25} + \frac{3(s+4)}{(s+4)^2 + 25}$

12) $\mathcal{L}^{-1}\left\{\frac{(s+1)^2}{(s+2)^4}\right\}$:

- $s+2=u$
- $s=u-2$

$$\mathcal{L}^{-1}\left\{\frac{(u-1)^2}{u^4}\right\} = \mathcal{L}^{-1}\left\{\frac{u^2-2u+1}{u^4}\right\} = \mathcal{L}^{-1}\left\{\frac{u^2}{u^4} - \frac{2u}{u^4} + \frac{1}{u^4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{u^2} - 2\frac{1}{u^3} + \frac{1}{u^4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2} - \frac{2}{(s+2)^3} + \frac{1}{(s+2)^4}\right\}$$

$$f(t) = t - \cancel{2}\frac{t^2}{\cancel{2}} + \frac{t^3}{6}$$

$$\mathcal{L}^{-1}\left\{\frac{(s+1)^2}{(s+2)^4}\right\} = e^{-2t}\left(t - t^2 + \frac{t^3}{6}\right)$$

13) $\mathcal{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$; $s^2-6s+10$; $r = \frac{6 \pm \sqrt{36-40}}{2.1}$ não tem raizes reais.

$s^2-6s+10 = (s^2-6s+9)+1$

s^2-6s+9

$s = \frac{6 \pm \sqrt{36-36}^{\,0}}{2.1} = \frac{6}{2} = 3$

$(s^2-6s+9)+1 = (s-3)^2+1$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+1}\right\} = e^{3t}\,\text{Sen}(t)$$

(14) $\mathcal{L}^{-1}\left\{\dfrac{5s}{(s-2)^2}\right\} =$

$$\frac{5s}{(s-2)^2} = \frac{A}{(s-2)} + \frac{B}{(s-2)^2} = \frac{A(s-2)+B}{(s-2)^2}$$

$5s = A(s-2) + B$

$s(A) - 2A + B = 5s$

$\underline{A = 5}$

$-2A + B = 0$

$-2.5 + B = 0$

$\underline{B = 10}$

$$= \frac{5}{(s-2)} + \frac{10}{(s-2)^2}$$

$$\mathcal{L}^{-1}\left\{\frac{5}{s-2} + \frac{10}{(s-2)^2}\right\} = \underline{e^{2t}(5 + 10t)}$$

(17) $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\}$

$$\frac{s}{(s+1)^2} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} = \frac{A(s+1)+B}{(s+1)^2}$$

$A(s+1) + B = s$

$s(A) + A + B = s$

$A = 1$

$1 + B = 0$

$B = -1$

$$\frac{1}{(s+1)} + \frac{-1}{(s+1)^2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)} - \frac{1}{(s+1)^2}\right\} = e^{-t}(1 - t)$$

(18) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+2s+5}\right\} =$

$$\frac{1}{(s^2+2s+1)+4} = \frac{1}{(s+1)^2+4} = \frac{1}{(s+1)^2+2^2}$$

$$s = \frac{-2 \pm \sqrt{\cancel{4-4}}^{\,0}}{2.1} = \frac{-2}{2} = -1$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\} = \underline{(\text{sen } 2t)\, e^{-t}}$$

(20) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)^4}\right\} = \dfrac{1}{3!}\cdot t^3 . e^t = \underline{\dfrac{t^3}{6} e^t}$

(26) $y'' - 4y' + 4y = t^3 e^{2t}$, $y(0) = 0$, $y'(0) = 0$

$$\left(\mathcal{L}\{y\}s^2 - \cancel{y(0)}^{\,0} . s - \cancel{y'(0)}^{\,0}\right) - 4\left(\mathcal{L}\{y\}s - \cancel{y(0)}^{\,0}\right) + 4\left(\mathcal{L}\{y\}\right) = \frac{3!}{(s-2)^4}$$

$$\mathcal{L}\{y\}(s^2-4s+4) = \frac{3!}{(s-2)^4} \Rightarrow \mathcal{L}\{y\} = \frac{6}{(s-2)^4 . (s-2)} = \frac{6}{(s-2)^6}$$

$s^2 - 4s + 4$

$$s = \frac{4 \pm \sqrt{\cancel{16-16}}^{\,0}}{2.1} = 2$$

$$\frac{6}{(s-2)^6} = \frac{6}{5!}\,\frac{5!}{(s-2)^6} = \frac{1}{20} . \mathcal{L}\{t^5 e^{2t}\}$$

$$\underline{y(t) = \frac{1}{20}\, t^5 e^{2t}}$$

(28)

$y'' - 4y' + 4y = t^3 \ , \ y(0) = 1 \ , \ y'(0) = 0$

$\left(\mathcal{L}\{y\}s^2 - \cancel{y(0)s} - \cancel{y'(0)}\right) - 4\left(\mathcal{L}\{y\}s - \cancel{y(0)}\right) + 4\left(\mathcal{L}\{y\}\right) = \dfrac{3!}{s^4}$

$\mathcal{L}\{y\}(s^2 - 4s + 4) - s + 4 = \dfrac{6}{s^4}$

$\mathcal{L}\{y\}(s^2 - 4s + 4) = \dfrac{6}{s^4} + s - 4$

$s = \dfrac{4 \pm \sqrt{\cancel{16 - 16}}}{2.2.1} = \dfrac{4}{2} = 2$

$(s-2)^2 = (s^2 - 4s + 4)$

$\mathcal{L}\{y\} = \dfrac{6}{s^4(s-2)^2} + \dfrac{s-4}{(s-2)^2}$

$\dfrac{6}{s^4(s-2)^2} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s^3} + \dfrac{D}{s^4} + \dfrac{E}{s-2} + \dfrac{F}{(s-2)^2}$

$6 = As^3(s-2)^2 + Bs^2(s-2)^2 + Cs(s-2)^2 + D(s-2)^2 + Es^4(s-2) + Fs^4$

$\dfrac{6}{s^4(s-2)^2} = \dfrac{1}{s} - \dfrac{4}{s^2} + \dfrac{12}{s^3} - \dfrac{24}{s^4} + \dfrac{1}{s-2} - \dfrac{1}{(s-2)^2}$

$y(t) = 1 - 4t + 6t^2 - 4t^3 + e^{2t} - te^{2t}$

$$y_2 = \frac{s-4}{(s-2)^2} = \frac{(s-2)-2}{(s-2)^2} = \frac{1}{s-2} - \frac{2}{(s-2)^2}$$

$$\mathcal{L}^{-1}\{y_2\} = e^{2t} - 2te^{2t}$$

$$\mathcal{L}^{-1}\{y\} = \mathcal{L}^{-1}\{y_1\} + \mathcal{L}^{-1}\{y_2\}$$

$$y(t) = \left(1 - 4t + 6t^2 - 4t^3 + e^{2t} - te^{2t} + e^{2t} - 2te^{2t}\right)$$

(32) $y'' + 8y' + 20y = 0 \qquad y(0) = 0, \quad y'(\pi) = 0$

$$\left(\mathcal{L}\{y\}s^2 - \cancel{y(0)}^{0}.s - y'(0)\right) + 8\left(\mathcal{L}\{y\}.s - \cancel{y(0)}^{0}\right) + 20\left(\mathcal{L}\{y\}\right) = 0$$

$$\mathcal{L}\{y\}(s^2 + 8s + 20) - y'(0) = 0$$

$$\mathcal{L}^{-1}\{y\} = \frac{y'(0)}{(s^2+8s+20)} = \frac{y'(0)}{(s^2+8s+16)+4} = \frac{y'(0)}{(s+4)^2+4} = y'(0).\left(\frac{1}{(s+4)^4 - 2^2}\right) =$$

$$s = \frac{-8 \pm \sqrt{64-64}}{2.1} = \frac{-8}{2} = -4$$

$$\mathcal{L}^{-1}\{y\} = y'(0).\frac{1}{((s+4)^2)^2 - 2^2} = \mathcal{L}\left\{\frac{1}{2}t\,\text{sen}(2t).e^{-4t}\right\} =$$

$$y(t) = y'(0).\frac{1}{2}t\,\text{sen}(2t).e^{-4t} \Rightarrow y'(0) = 0 \rightarrow y(t) = 0$$

$$y'(t) = \frac{1}{2}e^{-4t}\left[\text{sen}(2t) + 2t\cos(2t) - 4t\,\text{sen}(2t)\right].y'(0)$$

$$y'(\pi) = \pi e^{-4\pi} \qquad 0 = \pi e^{-4\pi}.y'(0)$$

$$y'(0) = 0$$

(37) $\mathcal{L}\{(t-1)\mathcal{U}(t-1)\}$, $e^{-as}F(s)$

$$\mathcal{U}(t-1) = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$$

$f(t) = t \Rightarrow f(t-1) = t-1$

$f(t) = t$

$\mathcal{L}\{t\} = \frac{1}{s^2}$

$$\boxed{\mathcal{L}\{(t-1)\mathcal{U}(t-1)\} = \frac{e^{-s}}{s^2}}$$

(39) $\mathcal{L}\{t\,\mathcal{U}(t-2)\}$

$t = (t-2) + 2$

$t \cdot \mathcal{U}(t-2) = [(t-2)+2]\,\mathcal{U}(t-2) = (t-2)\,\mathcal{U}(t-2) + 2\,\mathcal{U}(t-2)$

. $\mathcal{L}\{(t-2)\mathcal{U}(t-2)\} = e^{-2s} \cdot \frac{1}{s^2}$

. $\mathcal{L}\{\mathcal{U}(t-2)\} = \frac{e^{-2s}}{s}$

$$\boxed{\mathcal{L}\{t \cdot \mathcal{U}(t-2)\} = \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}}$$

(47)

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$$

$$\frac{1}{s(s+1)} \Rightarrow f(t) = 1 - e^{-t}$$

$$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\} = f(t-a)u(t-a)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \left(1 - e^{-(t-1)}\right)u(t-1)$$