Nassau County Interscholastic Mathematics League

Solutions
Contest # 5

1998-1999

19. The line has slope $\frac{-2}{5}$. So a parallel has the same slope. Thus $\frac{3-7}{a-3}=\frac{-2}{5}$ and a = 13

20. The number of possible choices is C(10,3), the number of combinations of 10 things taken three at a time. Think of the numbers as being in three categories: 1-5, 6 alone, and 7-10. The event would be to have 2 out of the first 5, 1 of the one 6, and none of the last four. So the probability is $\frac{C(5,2)\cdot C(1,1)\cdot C(4,0)}{C(10,3)}=\frac{1}{12}$

21. Use the change of base formula to get $\frac{\log\sqrt{3}}{\log 16}\cdot\frac{\log\sqrt[3]{7}}{\log 27}\cdot\frac{\log 8}{\log 5}\cdot\frac{\log 25}{\log 49}$ and rearrange to get

$$\frac{\log\sqrt{3}}{\log 27}\cdot\frac{\log\sqrt[3]{7}}{\log 49}\cdot\frac{\log 8}{\log 16}\cdot\frac{\log 25}{\log 5}=\frac{\left(\frac{1}{2}\right)}{3}\cdot\frac{\left(\frac{1}{3}\right)}{2}\cdot\frac{3}{4}\cdot\frac{2}{1}=\frac{1}{24}$$

22. Say, without loss of generality, that each side of ABCDEF is 2. In Δ GBH, for example, we have a 120° angle included between two sides of length 1, so $GH=\sqrt{3}$. Since the hexagons are similar, the ratio of the areas is the square of the ratio of the, corresponding sides, which is $\frac{4}{3}$.

23. $\sin\theta+\cos\theta=\frac{5}{4}$. Square both sides to get $1+2\sin\theta\cos\theta=\frac{25}{16}$ and $2\sin\theta\cos\theta=\frac{9}{16}$

Playing with $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}=\frac{2}{2\sin\theta\cos\theta}=\frac{2}{9/16}=\frac{32}{9}$

24. Since ΔBEC and ΔDBC are right triangles with the same hypotenuse, draw a circle with $\overline{BC}$ as its diameter. This circle contains points E and D (since an angle inscribed in a semicircle is a right angle). Now $\overline{MB}$ and $\overline{MC}$ are radii each equal to 15 and $\overline{ED}$ is a chord of length 20. Draw in $\overline{MN}$ and $\overline{ME}$ to get a right triangle with $EN=10, ME=15$, so $MN=5\sqrt{5}$.