Nassau County Interscholastic Mathematics League

Contest #4 **Answers must be in simplest exact form, unless otherwise noted.** **2003-2004**

Team Problems 35 minutes Calculators allowed

T1) Solve for y: $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$.

T2) Refer to the figure at right. $\overline{AB}$ is a diameter. $\overline{AD}, \overline{CD}, \overline{CB}$ are tangent to the circle at points A, E and B respectively. The radius of the circle is 4 and CD = 12. Find the area of the region inside quadrilateral ABCD which is outside the circle.

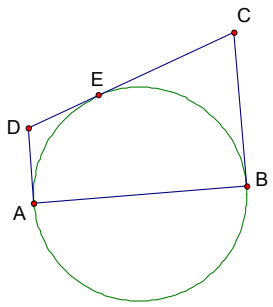

Figure for T2

T3) (a classic) In triangle ABC, AB = 13, AC =15, and median AM = 7.
Find the area of triangle ABC.

T4) In this cross-number puzzle, shape indicated at right, fill in the correct answers. On your answer sheet, write the solution as a 3 by 4 matrix as it appears in the grid.

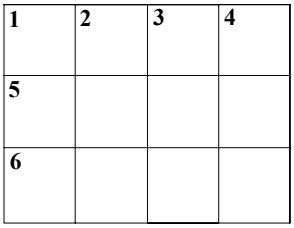

Figure for T4

Across

1. A perfect cube
5. All distinct even digits
6. No repeated digits

Down

1. A perfect square
2. A perfect square
3. A perfect square which is two less than the sum of two 2-digit Fibonacci numbers
4. A palindrome which is divisible by three

T5. Let $D(n) =$ the largest odd divisor of n, where n is a positive integer. For how many n, with $n < 1000$, will $D(n) = 3$?

T6. In the World Series, the winner is the first team to win four games. Assume the probability that each team wins any given game is one-half. Find the probability that the World Series goes seven games.

Answers: T1) 9 T2) $48 - 8\pi$ T3) 84 T4) $\begin{bmatrix} 4 & 9 & 1 & 3 \\ 8 & 0 & 2 & 6 \\ 4 & 0 & 1 & 3 \end{bmatrix}$ T5) 9 T6) $\frac{5}{16}$