Nassau County Interscholastic Mathematics League

Solutions
Contest # 1

2000-2001

1. Getting a common denominator, we have $\frac{9x+2}{x^2+x}$, so the reciprocal is $\frac{x^2+x}{9x+2}$

2. The original price is multiplies by .8 three times, hence $.8^3 = .512$, so the drop is .488

3. $2*(1*((3*1)*2)) = 2*(1*(7*2)) = 2*(1*16) = 2*18 = 22$

4. Compare pair wise by getting common denominator in exponent; e.g. $2^{1/2} = 2^{3/6} = 8^{1/6}$ and $3^{1/3} = 3^{2/6} = 9^{1/6}$, so $3^{1/3} > 2^{1/2}$. Similarly, $5^{1/3} = 125^{1/15}$, $3^{1/5} = 243^{\frac{1}{15}}$, so $5^{1/3} < 3^{1/5}$. Also, $6^{1/6} < 5^{1/3}$

An idea: study the graph of $y = x^{1/x}$ *and see what happens.*

5. Let the legs be a and b. Then $a+b=20$ *and* $ab=64$. Square the first equation and substitute to get $a^2+b^2 = 272$ = hypotenuse^2, so hyp = $\sqrt{272} = 4\sqrt{17}$

6. If a truthteller is asked what they are, they'll say a truthteller; but a liar will also say a truthteller, since they lie. So #1 must have said a Blanc. Thus #2 lied and is actually a Monge. If #3 is a liar, then #1 is not the same, i.e. a Blanc; but if #3 tells the truth, then #1 is the same, so, either way, #1 is a Blanc and tells the truth. So #3 also tells the truth. So #2 is the only Monge.