

13.  $\frac{527}{62} = 8.5$ , which is midway between the first integer and the last.  $8 - 30 = -22$ .

14.  $7^1 = 7$ ,  $7^2 = 49$ ,  $7^3 = 343$ ,  $7^4 = 2401$ , etc.  $\Rightarrow 17^{325}$  has a units' digit of 7.  
 $3^1 = 3$ ,  $3^2 = 9$ ,  $3^3 = 27$ ,  $3^4 = 81$ , etc.  $\Rightarrow 23^{108}$  has a units' digit of 1.  $1 + 7 = 8$ .

$$\begin{array}{r} 15. \quad 7x + 62y = -2 \\ + (23x - 27y = 16) \\ \hline 30x + 35y = 14 \Rightarrow 6x + 7y = \frac{14}{5} = 2.8 \end{array}$$

16. A sum of 8 can be rolled with three dice if the first two dice have a sum of 2, 3, 4, 5, 6, or 7. The probability that this will occur is  $\frac{21}{36}$ . A sum of 10 can be rolled with three dice if the first two dice have a sum of 4, 5, 6, 7, 8, or 9. The probability that this will occur is  $\frac{27}{36}$ . The probability of rolling 8 or 10 with three dice is  $\frac{1}{6}(\frac{21}{36} + \frac{27}{36}) = \frac{2}{9}$ .

17. The area of a square is one half the square of the length of the diagonal. Let  $x$  = the length of the diagonal of the original square.

$$\frac{(x+8)^2}{2} = \frac{x^2}{2} + 144; x^2 + 16x + 64 = x^2 + 288 \Rightarrow 16x = 224 \Rightarrow x = 14.$$

$$\text{The side of the original square} = \frac{14}{\sqrt{2}} = 7\sqrt{2}.$$

18. Let  $x$  = the radius of the circumscribed circle and  $y$  = the radius of the inscribed circle. The ratio of the area of the circumscribed circle to the area of the inscribed circle is  $\left(\frac{x}{y}\right)^2$ .

$$x = y + y\sqrt{2} \Rightarrow y = \frac{x}{1+\sqrt{2}} \Rightarrow \left(\frac{x}{y}\right)^2 = (1+\sqrt{2})^2 = 3+2\sqrt{2}.$$

