Nassau County Interscholastic Mathematics League

Contest # 4 **2001-2002**

Answers must be exact or must have 4 (or more) significant digits, correctly rounded, unless otherwise noted .

Team Problems 35 minutes Calculators allowed

T-1. If $f(x) = ax^2 + bx + c$ and $f(x+1) = x^2 + 7x + 4$, find the ordered triple (a, b, c).

T-2. In this cross-number puzzle, shape indicated at right, fill in the correct answers. On your answer sheet, write the solution as a 3 by 4 matrix as it appears in the grid.

1	2	3	4
5			
6			

Across

1. A perfect cube that is 40 more than a perfect square
5. One Fibonacci number followed by another Fibonacci number
6. A Fibonacci number

Down

1. Number of real solutions between 0 and 2π to the equation $2(\sin 100x)(\cos 16x) = \cos 16x$
2. If p > q are odd primes, this number is of the form pq^2
3. The digits are in strictly increasing order
4. A perfect square

T-3. Suppose $b > c > a > 0$ and $b = a + c$. Find the numerical value of the larger solution to the equation $ax^2 + bx + c = 0$.

T-4. A box (rectangular prism) has two faces each of area 12, two faces each of area 16, and two faces each of area 8. Find the exact volume of the box. Write your answer in simplified radical form.

T-5. ABCD is a trapezoid with bases AB = 20 and CD = 12. E is the midpoint of leg $\overline{AD}$ and F is the midpoint of leg $\overline{BC}$. Diagonal $\overline{AC}$ meets $\overline{EF}$ at G and diagonal $\overline{BD}$ meets $\overline{EF}$ at H. Find length of $\overline{GH}$.

T-6. Box A contains 4 red marbles and 5 green marbles. Box B contains 2 red marbles and 6 green marbles. One marble is randomly chosen from Box A and put into Box B. Then one marble is randomly chosen from Box B. Find the probability that this marble is red.

Answers: T-1. (1, 5, -2) T-2. $\begin{bmatrix} 2 & 7 & 4 & 4 \\ 3 & 7 & 7 & 8 \\ 2 & 5 & 8 & 4 \end{bmatrix}$ T-3. –1 T-4. $16\sqrt{6}$ T-5. 4 T-6. $\frac{22}{81} \approx 0.2716$