Nassau County Interscholastic Mathematics League

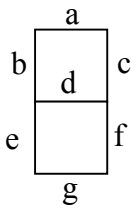
Contest # 1 **2001-2002**

No Calculators

Problems 1-2. Time limit: 10 minutes.

1. On the Planet Skyron, there are eleven cities. Each pair of different cities is connected by exactly one road. There are no other roads on Skyron. How many roads are there on Skyron?

2. On a calculator screen, each digit is formed by lighting some combination of the seven segments labeled a-e in the figure at right. For example, the 2 is formed by lighting the five segments a, c, d, e, and g. How many digits are formed by lighting four or fewer segments?



Problems 3-4. Time limit: 10 minutes.

3. The logical binary operation "or" (denoted by $\vee$) yields true whenever one or both statements are true. The logical operation "exclusive-or" (denoted by $\veebar$) yields true whenever one, but not both, of its statements are true. Consider the two expressions $(p \vee q) \vee r$ and $(p \veebar q) \veebar r$. Of the eight possible cases, how many yield different results for the two expressions?

4. The point $(3,5)$ is on the graph of $y = f(x)$. What point must be on the graph of $y = 2f(x-1)$?

Problems 5-6. Time limit: 10 minutes.

5. After a test, each of the 30 students in the class peeked at the teacher's gradebook. No student saw all the grades, and no student saw his/her own grade. Each student saw exactly ten failures among the grades. Find the least possible number of failures the class could have.

6. Find the largest integer x for which another integer n exists, with $nx = n + 12x$.

Answers.

1. 55
2. 3
3. 3
4. $(4,10)$
5. 11
6. 13