

Nassau County Interscholastic Mathematics League

Contest # 5

1998-99

Answers must be exact or must have 4 (or more) significant digits, correctly rounded, unless otherwise noted

Problems 19-20. Time limit 10 minutes.

19. A line parallel to $2x + 5y - 8 = 0$ contains the points $(3, 7)$ and $(a, 3)$. Find a .

20. Ten slips of paper, numbered 1 through 10, are placed in a box. Three slips are drawn without replacement. Find the probability that the largest number drawn is a 6.

Problems 21-22. 10 minutes.

21. Evaluate: $(\log_{16} \sqrt{3})(\log_{27} \sqrt[3]{7})(\log_8 8)(\log_{40} 25)$

22. The midpoints of the sides of regular hexagon ABCDEF are G, H, I, J, K, and L respectively. Find the ratio of the area of ABCDEF to the area of GHIJKL.

Problems 23-24. 11 minutes.

23. If $0 \leq \theta \leq \frac{\pi}{2}$ and $\sin \theta + \cos \theta = \frac{5}{4}$, find the value of $\tan \theta + \cot \theta$.

24. In acute triangle ABC, E is on \overline{AB} and D is on \overline{AC} so that \overline{CE} and \overline{BD} are altitudes. Point N is the midpoint of \overline{ED} and point M is the midpoint of \overline{BC} . If $ED = 20$ and $BC = 30$, find the length of \overline{MN} .