Nassau County Interscholastic Mathematics League

Solutions
Contest 2 **1999-2000**

7. We want elements in A, but not in both B and C. The elements in both B and C are 6, 0, -6, -12, ...
The elements in A, not among those, are **1,2,3,4,5,7,8,9.**

8. The differences are when p and q differ in truth value in the "and" and "or" columns, so there are four differences. Consequently **12** are the same.

9. Let $S = 36 + 24 + 16 + \frac{32}{3} + \cdots$ Multiply both sides by $\frac{2}{3}$ and get

$\frac{2}{3}S = 24 + 16 + \frac{32}{3} + \cdots$ Subtract. On the right, all terms are the same except the first

So we get $\frac{1}{3}S = 36$, so $S = 108$. Note: if you know the formula for a geometric series, and when it works, that would be O.K., too, but not nearly as pretty.

10. Let x = length of $\overline{CE}$. ΔABE ~ ΔCBD, so $\frac{BE}{DB} = \frac{AB}{CB}$, or $\frac{5}{6} = \frac{9}{x+5}$. Solve to get **x = 5.8**

11. (Put & Take)

$$\begin{aligned} 81x^4 + 64 &= 81x^4 + 144x^2 + 64 - 144x^2 \\ &= (9x^2 + 8)^2 - (12x)^2 \\ &= (9x^2 + 8 + 12x)(9x^2 + 8 - 12x) \end{aligned}$$

12. To organize, consider possible base lengths (b) and see what leg lengths (L) work.
If b =1, then L = 1,2,3,4,5,or 6.
If b = 2, then L = 2,3,4,5, or 6.
If b =3, then L = 2,3,4, or 5.
If b = 4, then L = 3, 4, or 5.
If b = 5, then L = 3 or 4.
If b = 6, then L = 4..
So there are **21** possibilities.