



25. $f(x) = x$, so $|3x - 1| = x$, so $3x - 1 = x$ for $x \geq \frac{1}{3}$ and $3x - 1 = -x$ for $x < \frac{1}{3}$. In the first case $x = \frac{1}{2}$ and in the second, $x = \frac{1}{4}$. Both are non-negative.

26. If the house is blue, then all three statements are true. If the house is red, then Andy's statement is true, Bill's is true, and Colleen's is false. So it's red.

27. Look at $\triangle ABD$. It contains a 40° angle and a right angle, so $m\angle BAE = 50^\circ$.
 $m\angle BE = 2(\text{measure of inscribed } \angle) = 100^\circ$.

28. Let $x = y = 0$ and get $f(0) = 2f(0)g(0)$ and $g(0) = (g(0))^2 - (f(0))^2$. Now either $f(0) = 0$ or $g(0) = \frac{1}{2}$. If $f(0) = 0$, then $g(0) = (g(0))^2$, so $g(0) = 0$ or $g(0) = 1$. If $g(0) = \frac{1}{2}$, then $f(0)$ is not real.

29. $(x + y)(x - y) = x - y$ so $x + y = 1$ or $x - y = 0$. So $x = y$ or $x = 1 - y$. If $x = y$, then $x = 0$. So $x = 1 - y$.
 Substituting, $(1 - y)y = 1 - y - y \rightarrow y - y^2 = 1 - 2y \rightarrow y^2 - 3y + 1 = 0$. Solving, $y = \frac{3 - \sqrt{5}}{2}$ is the only positive solution and, substituting, $x = \frac{-1 + \sqrt{5}}{2}$.

30. Method 1: $\triangle ABC$ has area $\frac{\sqrt{3}}{4}$. So $\triangle BEC$ has half that area, $\frac{\sqrt{3}}{8}$. $BG:BE = 2:3$, so area $\triangle BDC$:area $\triangle BEC = 4:9$. So $\triangle DGC$ is $5/9$ of $\triangle BEC$. The area is $\frac{5}{9} \cdot \frac{\sqrt{3}}{8} = \frac{5\sqrt{3}}{72}$.

Method 2: Draw the median from A, intersecting \overline{BC} at Q. The area of the equilateral triangle is $\frac{\sqrt{3}}{4}$, so the area of $\triangle EGC = \frac{1}{6} \cdot \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{24}$, using the theorem that the 6 triangles formed by the medians have equal area in any \triangle . $BG:GE = 2:1$ and $\overline{GD} \parallel \overline{EC}$, so $BD:CD = 2:1$ by the side splitter theorem. $DC = \frac{1}{3}$ and $QD = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$. That means $DC:QD = 2:1$. So the area of $\triangle DGC = \frac{2}{3} \cdot \frac{\sqrt{3}}{24} = \frac{\sqrt{3}}{36}$. Adding $\frac{\sqrt{3}}{36} + \frac{\sqrt{3}}{24} = \frac{5\sqrt{3}}{72}$.