Nassau County Interscholastic Mathematics League

Solutions
Contest #6

1998-1999

25. Draw $\overline{BC}$. The diagonals of the faces of a cube are all the same, so the triangle is equilateral. So each angle is 60°.

26. Factor using sum of cubes and get $(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)=\frac{2}{3}$. The first factor is 1 so can be ignored. Now we will "complete the square" for the remaining equation:
$\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x=\frac{2}{3}+3\sin^2 x\cos^2 x$. The left side is $(\sin^2 x+\cos^2 x)^2=1^2=1$. So $\frac{1}{3}=3\sin^2 x\cos^2 x$, $\sin x\cos x=\pm\frac{1}{3}$ and $\sin(2x)=\pm\frac{2}{3}$

27. Team A wins 3 of the first four and loses one, then wins the fifth, so the probability is given by $C(4,3)\cdot(.6)^3(.4)^1\cdot(.6)\approx 0.2074$

28. Note x can't be 2, 3, 5, or 7 since then the data would be bimodal. The mode is 8. The arithmetic mean is $\frac{33+x}{7}$. The median is 5 if $x<5$, 7 if $x>7$, or x if $5<x<7$. If x is the median, then $8+x+\frac{33+x}{7}=21$, which doesn't have an integer solution. If 5 is the median, same thing. If 7 is the median, we get x = 9.

29. The graph is sort of like a big W. The points of equality are $\pm\sqrt{3}$ *and* $\pm\sqrt{7}$. So the solution for > is on the outside ($x>\sqrt{7}$ or $x<-\sqrt{7}$)or in the middle, $-\sqrt{3}<x<\sqrt{3}$.

30. Through the centers of the two smaller circles, draw lines parallel to the common tangent line. These meet the radii of the two larger circles in right angles, creating two similar triangles. In fact, the radii form a geometric progression, so $\frac{6}{8}=\frac{8}{r}$ and $r=\frac{32}{3}$