Nassau County Interscholastic Mathematics League

Solutions
Contest # 1

1998-1999

1. Let x = the number. So $x = 1 + \frac{1}{x}$. Solving, $x = \frac{1 \pm \sqrt{5}}{2}$. But only the + solution is positive.

2. $\angle ACD$ is an exterior angle of ΔABC, so $x > 35$. In ΔACD, one angle is 40, so $x < 140$.

3. $\Delta ABE \sim \Delta CDE$, so $\frac{14}{26} = \frac{CD}{13}$ and thus $CD = 7$

4. Case I: $x < 2$. So $2 - x + 5 - x = 4 \Rightarrow x = 1.5$. Case II: $2 \le x \le 5$: So $x - 2 + 5 - x = 4 \Rightarrow 3 = 4$ so no solutions in this case. Case III: $x > 5$. So $x - 2 + x - 5 = 4 \Rightarrow x = 5.5$

5. The midpoints are (1,12), (4,5), (1,-2), and (-2,5). The quadrilateral they form has diagonals which are perpendicular, so the area is half their product. So Area = .5(14)(6) = 42
Alternate solution,: Using encasement, the area of the original quadrilateral is 84, so the midpoint quadrilateral has area $\frac{1}{2}84 = 42$

6. Any combination with only $\wedge$, $\vee$ is false if both p and q are false. If both are $\rightarrow$, then false when p false and q true. So exactly one must be $\rightarrow$. Try the four cases. Only $(p \wedge q) \rightarrow p$ and $(p \rightarrow q) \vee p$ are always true.