Nassau County Interscholastic Mathematics League

Solutions 1999-2000
Contest #6

MATHLETES

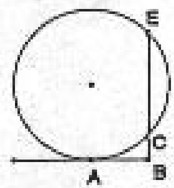
25. $(3a)^2 + (4b)^2 = 5^2$ which only works for integers where $3a = 3, 3a = -3, 4b = 4, 4b = -4$
$a = 1$ *or* $a = -1$ *and* $b = 1$ *or* $b = -1$ Whichever way, $a^2 + b^2 = 2$

26. Let x = m∠ABD = m∠DBC and let y = m∠ACD = m∠BCD. In ΔABC, $102 + 2x + 2y = 180$. Ift follows that $x + y = 39$. Now in ΔDBC, x + y + m∠BDC = 180. Since $x + y = 39$, we can substitute to get 39 + m∠DBC = 180, m∠DBC=141

27. To get $2x^2$ the variable terms must be 2x and x. To get -3, the numeric factors 1, -3 or -1,3 or 3,-1 or -3,1. Try all four and get $a = -5, -1, 1,$ *or* 5

28. Start at top right, go left and down to get a string of 4 > so five numbers must be 5, 4, 3, 2, 1. The third row is now either 2 + 1 = 3 or 2 + 2 = 4. If 2 + 1 =3 is right, then the middle of the fifth row can't be < 1, so third row is 2 + 2 = 4. Now the fifth row must be 1 + 1 = 2. Now the last row must end in 1 and have 1 in the middle, so the bottom left is 2.

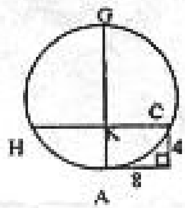
29. Use the Power theorem to get $(BA)^2 = (BC)(BE)$, so CE = 12.
Now draw a radius from the center to the middle of chord $\overline{CE}$.
Draw a radius down to A make a rectangle. Its length is 4 + 6 = 10.



Alternate solution: Let r represent the radius. Draw diameter $\overline{CB}$ and chord $\overline{CH} \perp \overline{AG}$ (with intersection at K)

$$(HK)(KC) = (KA)(KG)$$

Now $$(8)(8) = (4)(2\pi - 4)$$
$$r = 10$$



30. Picture a circle with $\overline{AB}$ as diameter. The center is (2,4) and the radius is 5. The only way to get integers to work is by going from the center 5 vertically or horizontally, or else to go 3 and 4 in perpendicular directions from the center. There are 12 such points but two are A and B. So there are 10 spots left for C.