Solutions Contest # 2



- 7. Method 1:There are 29 roads to Monty, plus $_{29}C_2$ = other pairs, which gives a total of 435 Method 2: $_{30}C_2$
- 8. If the anti-calculator requires 2 or fewer, then on a regular calculator it would use 5 or more segments. The number 2, 3, and 5 each require five segments, 6 and 9 require five or six depending on the calculator, 0 requires six, and 8 needs all seven.
- 9. If Al were guilty, then both of his statements are true. If Carl is guilty, then Al's statements are still both true. If Dave did it, then both of Bob's statements are true. If Bob is guilty, than Al (2), Carl (2), and Dave (2) are true, the others false.

$$100^{2} - 99^{2} + 98^{2} - 97^{2} + \dots + 4^{2} - 3^{2} + 2^{2} - 1^{2}$$

$$= (100 + 99)(100 - 99) + (98 + 97)(98 - 97) + \dots + (4 + 3)(4 - 3) + (2 + 1)(2 - 1)$$

$$= (100 + 99)(1) + (98 + 97)(1) + \dots + (2 + 1)(1)$$

$$= 100 + 99 + 98 + \dots + 3 + 2 + 1$$

$$= \frac{100(101)}{2}$$

$$= 5050$$

- 11. $m \angle A = 68^{\circ}$ and $m \angle BDC = 80^{\circ}$. In each triangle, apply the inequality about smallest side opposite smallest angle. In $\triangle ABD$, AB < AD < BD. In $\triangle BCD$, BD < CD < CB.
- 12. Algebraic Method: We know f(7) = 5. So now let x = 4. y = 3f(x) = 3f(5) = 15. So the point is (4,15). Geometric Transformation Method: (7,5) shifts one right to (8,5). Then x is cut in half, to (4,5). Finally, y triples to (4,15).