Team Problems 35 minutes Calculators allowed

T1) . If $a, b,$ and c are arbitrary complex numbers, HOW MANY of the following statements are necessarily true?

(i) $a + b = b + a$ (ii) $|a + b| = |a| + |b|$ (iii) $a(bc) = (ab)c$ (iv) $|abc| = |a| + |b| + |c|$ (v) $a(b + c) = ab + ac$

(vi) If $ab = 0,$ then $a = 0$ or $b = 0$ (vii) If a and b are conjugates, then ab is real

T2) . Each of five slips of paper in a box contains exactly one of the following equations:

$$x^2 - x = 0 \qquad x^2 + 2x + 1 = 0 \qquad x^2 - 5x + 6 = 0 \qquad x^2 + 4 = 0 \qquad x^2 - 10x + 20 = 0$$

Five times, Pandora randomly selects one slip of paper, with replacement, from the box.
Find, to the nearest thousandth, the probability that she will remove an equation with
only positive, real solutions at least twice.

T3) Solve for x: $\log_2(x + 2) + 5 = 8 + \log_2(x)$.

T4) Al, Bob, and Carl are brothers. Each does exactly two sports (out of baseball, football, soccer, tennis, gymnastics, and basketball). No two do the same sport. Gymnastics and basketball are the only indoor sports.
We know the following:
   (1) Al doesn't do indoor sports; (2) No one does both indoor sports; (3) Bob and the football player went to watch the gymnast; (4) Football and soccer are played during the same season so no one can do both; (5) Baseball and tennis are played during the same season, so no one can do both; (6) Carl and the basketball player like to watch the baseball player; (7) No one does both soccer and gymnastics.
         What are Bob's two sports?

T5) The vertices of a triangle in 3-D are $A(1,3,2), B(4,-2,1)$ and $C(6,2,5).$ Find (to the nearest tenth of a degree) the measure of $\angle BAC$.

T6) Consider a parabola whose focus is the point $(0,4)$ and whose directrix is the line $y = -2$. Write a polynomial $d(x)$, in terms of the single variable x, that gives the distance from any point (x, y) on the parabola to the focus.

Answers: T1) 5   T2)   0.663   T3) $\dfrac{2}{7}$ T4) soccer, basketball   T5)   60.9°   T6) $d(x) = \dfrac{x^2}{12} + 3$