Nassau County Interscholastic Mathematics League

Solutions
Contest # 5

2000-2001

19. $1\cdot2\cdot3\cdot2\cdot3\cdot4\cdot5\cdot2\cdots7 = 1\cdot2\cdots7\cdot(2\cdot4)\cdot(3\cdot3)\cdot(2\cdot5) = 10!$, so n = 10

20. Draw the circle, and extend $\overline{DC}$ past C to hit the circle again at E. $\overline{DE}$ is the diameter. Let CE = x and by the Power Theorems, 4x=100, so x = 25, and diameter = 29

21. The Arithmetic Mean of any arithmetic progression is the average of the first and last terms, hence $\frac{101}{2} = 50.5$

22. $(2x-1)(4x+1) = 90$, so $x = 3.5$ or -4.25, but x can't be negative, so BC = 6 is the altitude of the triangle whose base is 4, which makes the area 12.

23. Note that $f^{-1}(x) = \sqrt[3]{x-5}$. From the second equation, $g(x) = f^{-1}(\sqrt{x-2})$ So $g(11) = f^{-1}(3) = 2$

24. Either the first card is a non-heart king and the second is any heart, or else the first card is the king of hearts and the second card is one of the other hearts, so the probability will be $\frac{3}{52}\cdot\frac{13}{51}+\frac{1}{52}\cdot\frac{12}{51} = \frac{1}{52}$