Nassau County Interscholastic Mathematics League

Solutions **2001-2002**
Contest # 1

1. *Method 1*: City #1 has 10 roads; city #2 adds 9 more roads, etc. So the number of roads is
$10+9+8+7+6+5+4+3+2+1 = 55$
Method 2: Each pair of cities has a road. The number of pairs is ${}_{11}C_2 = 55$.

2. The 1 requires two segments; the 4 requires four segments; and the 7 requires three segments. So there are 3 such digits.

3. They differ when one is false, the other two true. So there are 3 cases.

4. Method 1: The given says that $f(3)=5$. Now let $x=4$. On the new graph,
$y = 2f(x\text{-}1) = 2f(4 - 1) = 2f(3) = 2(5) = 10$.
Method 2: The point (3,5) is on f. $f(x-1)$ moves it one to the right, to $(4,5)$. Then the $2f$ multiplies the height by 2, so the result is $(4,10)$.

5. There are certainly at least 10, but when one of those who failed looked, s/he also saw 10 failures, so there must be at least 11.

6. Solve for x, to get $x=\dfrac{n}{n-12}=\dfrac{n-12+12}{n-12}=1+\dfrac{12}{n-12}$, which will be an integer when $n-12$ divides 12. The largest such is when $n-12=1$, so $x=13$. Coincidentally $n=13$ also, but they need not be equal.