

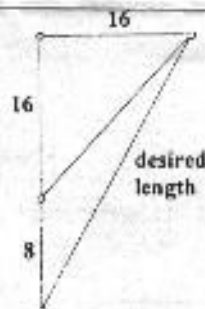
Solutions
Contest # 6

25. $2001 = (3)(23)(29)$ and $2002 = (2)(7)(11)(13)$, so $sp(2001) = 55$ and $sp(2002) = 33$, and the difference is 22.

26. $IL = 12$ and is the altitude to the hypotenuse of right triangle CIM . Let $CI = x$ and $LM = 4x$. So $(4x)(x) = 12^2$ and thus $x = 6$ and $CM = 30$. The area is half the product of the diagonals, $area = \frac{(24)(30)}{2} = 360$

27. The third to last factor is $(x-x) = 0$, so the product is 0.

28. One method is to use Law of Cosines. Another way is to extend the 8 as shown by 16 each way. So the length desired is the hypotenuse of a right triangle with legs 16 and 24, or $8\sqrt{13}$



29. The line has slope $3/2$ and equation $\log_8 y - 6 = \frac{3}{2}(x - 2)$, which becomes $\log_8 y = \frac{3}{2}x + 3$, so $y = 8^{1.5x+3} = 2^{3(1.5x+3)} = 2^{4.5x+9} = 2^9 \cdot 2^{4.5x}$

30. $3^4 = 81 = 1 + 80$. So $3^{2000} = (3^4)^{500} = (1 + 80)^{500}$. Now apply the Binomial Theorem to get $1^{500} + 500 \cdot 1^{499} \cdot 80 + \frac{500 \cdot 499}{2} \cdot 1^{498} \cdot 80^2 + \text{stuff} \cdot 80^3 + \dots$, with every term after that having a higher power of 80,

So the last three digits are $1 + 000 + 000 + 000 + 000$, etc. or 001. Finally to get one extra factor (exponent 2001) multiply by 3 to get 003