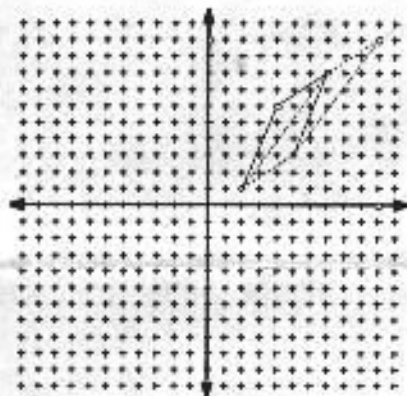


Solutions
Contest # 3

13. Slope $\overline{AB} = \frac{k-1}{3}$ and slope $\overline{CB} = \frac{-3}{5}$. The lines are perpendicular so $\frac{k-1}{3} = \frac{5}{3} \rightarrow k = 6$



14. There are actually three ways to make a parallelogram but only two put the fourth vertex in the first quadrant. Start at a vertex and move horizontally and vertically by the same numbers the opposite segment determines. this makes opposite sides \parallel and \cong . See the figure.

15. There are 49 elements in A, all of which are included. The elements in B but not C are 3, 9, 15, ... except 30, 60, 90. Those are in A. The number of elements 3, 9, 15, ..., 99 is 17, making a total of 66 elements.

16. $(a+b)(a-b) = 143 = (143)(1) = (13)(11)$ Since $a > b > 0$, $a+b > a-b$.

Then $a+b=143$ and $a-b=1$ or $a+b=13$ and $a-b=11$. Solving the systems, $(a,b) = (72,71)$ or $(12,1)$

17. Let $S = 1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)^3 + \dots$ and multiply by $\frac{1}{3}$ to get

$$\frac{1}{3}S = 1\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right)^3 + \dots \text{ and when we subtract, we get}$$

$$\frac{2}{3}S = \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots, \text{ which is infinite geometric.}$$

$$\text{So } \frac{2}{3}S = \frac{1}{2} \text{ and then } S = \frac{3}{4}$$

18. We check the possibilities. Any thing with "and" can't work since p can be false or $\sim p$ can be false. $\vee \sim$ fails when p True, q False. The \rightarrow fails when p false. The $\rightarrow \sim$ fails when p False, q True. Only "or" works.