Nassau County Interscholastic Mathematics League

Solutions **2003-2004**

Contest # 6

25. $6\cdot 7^2+3\cdot 7+k=315+k=243+72+k=3\cdot 81+8\cdot 9+k$. So it's **38k** in base 9.

26. By the Law of Cosines, $a^2=b^2+c^2-2bc\cos A$. Substituting, $a^2=a^2+4-2bc\cos A$, *so* $bc\cos A=2$.

Now, substitute for bc, and $a^3=2$. So $a^2=2^{\frac{2}{3}}$. Finally, $b^2+c^2=a^2+4=2^{2/3}+4\approx \mathbf{5.587}$.

27. Since the circles intersect in two points, the two radii and the distance between the centers form a triangle. So the triangle inequality holds and $16-6<k<16+6$ or **10 < k < 22**.

28. Substitute 2004 for x and get $f(2004)+2f(2)=2005$. Now substitute 2 for x and get $f(2)+2f(2004)=4007$. Add the two equations and divide by 3 to get $f(2)+f(2004)=2004$.
Now subtract this from the second equation to get $f(2004)=\mathbf{2003}$.

29. Subtract the third equation from the first to get $y+3y^2=2$. The quadratic has two solutions, but only one is an integer, $y=-1$. Now substitute –1 for y to get the system $x+z=3$ *and* $x^2-z^2=3$. Factoring, $(x+z)(x-z)=3$ *and* $x+z=3$, so $x-z=1$. This system yields $x=2, z=1$. So $(x,y,z)=(\mathbf{2,-1,1})$.

30. Refer to everyone by his or her initial. The possible family groupings where A and C are married are ACE & BDFG, ACF & BDEG, ACG & BDEF, ACEF & BDG, ACEG & BDF, or ACFG & BDE. There are also six possibilities for A and D being married. There are 12 possibilities, all equally likely. So the probability that the chosen group is a family is $\mathbf{\frac{1}{12}}$.

Alternate solution: Since you are selecting two children, you need to pick the father and mother and both of the children in the two child family, so there is only one possible way to succeed. You have to select one male of 2, one female of 2 and 2 children of 3, so there are ${}_2C_1\bullet{}_2C_1\bullet{}_3C_2=12$ ways to do the selecting. $\mathbf{\frac{1}{12}}$.