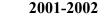
## Nassau County Interscholastic Mathematics League

## Solutions Contest # 4 – TEAM ROUND





T-1. Method 1:  $f(x) = ax^2 + bx + c$ , so  $f(x+1) = a(x+1)^2 + b(x+1) + c$ .

Multiply out and get  $f(x+1) = ax^2 + (2a+b)x + (a+b+c) = 1x^2 + 7x + 4$ .

So a = 1, 2a + b = 7 and a + b + c = 4 So (a, b, c) = (1, 5, -2).

Method 2: Substitute x-1 for x in f(x+1) to get  $f(x-1+1) = (x-1)^2 + 7(x-1) + 4$ .

Simplifying,  $f(x) = 1x^2 + 5x - 2$ 

- T-2. In 1 Down, the sine part yields 200 solutions, the cosine part 32, for a total of 232. There are only two 4-digit perfect cubes starting with 2, 2197 and 2744. Checking, 2704 is a perfect square, 2157 isn't. So 1 Across is 2744. The only 4-digit Fibonacci is 2584, so that's 6 Across. The only three-digit perfect square starting and ending with 4's is 484, so that's 4 Down. Now 5 Across is a 1-digit Fibonacci, followed by a 3-digit, 2-digit then 2-digit, or a 3-digit followed by a 1-digit. But no 2-digit or 3-digit Fibonaccis end in 8, so it must be a 3-digit followed by a 1-digit. The only 3-digit Fibonacci starting with 3 is 377, so 5 Across is 3778. Everything else quickly follows.
- T-3. Method 1:  $x = \frac{-b \pm \sqrt{b^2 4ac}}{2a} = \frac{-(a+c) \pm \sqrt{(a+c)^2 4ac}}{2a} = \frac{-(a+c) \pm \sqrt{a^2 + 2ac + c^2 4ac}}{2a}$ =  $\frac{-(a+c) \pm \sqrt{a^2 - 2ac + c^2}}{2a} = \frac{-(a+c) \pm (a-c)}{2a} = \frac{-c}{a}$  or -1. But -1 is greater.

Method 2:  $ax^2 + (a+c)x + c = 0$ ,  $\rightarrow ax(x+1) + c(x+1) = 0 \rightarrow (ax+c)(x+1) = 0$ , x = -1 or  $x = -\frac{c}{a} < -1$ .

- T-4. Call the edges a, b, and c. So ab = 12, bc = 16, and ac = 8. Multiply and get  $(abc)^2 = 1536$ . Volume  $= abc = \sqrt{1536} = 16\sqrt{6}$ .
- T-5. Method 1:In  $\triangle$ ACD, EG = 6, and in  $\triangle$ ABD, EH = 10 (segment connecting the midpoints of two sides of a triangle is half the third side and parallel to it), so subtract to get GH = 4.

Method 2: Use the theorem that the median of a trapezoid is the average of the bases to get FE = 16.

Since  $FH = GE = \frac{1}{2}CD = 6$ , then HG = 16 - 12 = 4.

- T-6. It could be red, red, with probability  $\frac{4}{9} \cdot \frac{3}{9} = \frac{12}{81}$ , or green, red with probability  $= \frac{5}{9} \cdot \frac{2}{9} = \frac{10}{81}$ , so the total is
- $\frac{22}{81}$  .