Nassau County Interscholastic Mathematics League

Solutions **2001-2002**

Contest # 4 – TEAM ROUND

T-1. Method 1: $f(x) = ax^2 + bx + c,\ so\ f(x+1) = a(x+1)^2 + b(x+1) + c$.

Multiply out and get $f(x+1) = ax^2 + (2a+b)x + (a+b+c) = 1x^2 + 7x + 4$.

So $a = 1,\ 2a + b = 7$ and $a + b + c = 4$ So $(a,b,c) = (1,5,-2)$.

Method 2: Substitute $x-1$ for x in $f(x+1)$ to get $f(x-1+1) = (x-1)^2 + 7(x-1) + 4$.

Simplifying, $f(x) = 1x^2 + 5x - 2$

T-2. In 1 Down, the sine part yields 200 solutions, the cosine part 32, for a total of 232. There are only two 4-digit perfect cubes starting with 2, 2197 and 2744. Checking, 2704 is a perfect square, 2157 isn't. So 1 Across is 2744. The only 4-digit Fibonacci is 2584, so that's 6 Across. The only three-digit perfect square starting and ending with 4's is 484, so that's 4 Down. Now 5 Across is a 1-digit Fibonacci, followed by a 3-digit, 2-digit then 2-digit, or a 3-digit followed by a 1-digit. But no 2-digit or 3-digit Fibonaccis end in 8, so it must be a 3-digit followed by a 1-digit. The only 3-digit Fibonacci starting with 3 is 377, so 5 Across is 3778. Everything else quickly follows.

T-3. Method 1: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(a+c) \pm \sqrt{(a+c)^2 - 4ac}}{2a} = \dfrac{-(a+c) \pm \sqrt{a^2 + 2ac + c^2 - 4ac}}{2a}$

$= \dfrac{-(a+c) \pm \sqrt{a^2 - 2ac + c^2}}{2a} = \dfrac{-(a+c) \pm (a-c)}{2a} = \dfrac{-c}{a}\ or\ -1$. But -1 is greater.

Method 2 : $ax^2 + (a+c)x + c = 0, \rightarrow\ ax(x+1) + c(x+1) = 0 \rightarrow (ax+c)(x+1) = 0$, $x = -1\ or\ x = -\dfrac{c}{a} < -1$.

T-4. Call the edges a, b, and c. So $ab = 12$, $bc = 16$, and $ac = 8$. Multiply and get $(abc)^2 = 1536$.

Volume = $abc = \sqrt{1536} = 16\sqrt{6}$.

T-5. Method 1:In ΔACD, EG = 6, and in ΔABD, EH = 10 (segment connecting the midpoints of two sides of a triangle is half the third side and parallel to it), so subtract to get GH = 4.

Method 2: Use the theorem that the median of a trapezoid is the average of the bases to get $FE = 16$.

Since $FH = GE = \frac{1}{2}CD = 6$, then $HG = 16 - 12 = 4$.

T-6. It could be red, red, with probability $\frac{4}{9} \cdot \frac{3}{9} = \frac{12}{81}$, or green, red with probability = $\frac{5}{9} \cdot \frac{2}{9} = \frac{10}{81}$, so the total is $\frac{22}{81}$.