Nassau County Interscholastic Mathematics League

Contest # 1 **2002-2003**

No Calculators

Problems 1-2. Time limit: 10 minutes.

1. Find all two digit numbers which have all the following properties: (a) the number is a multiple of 3; (b) the number is a multiple of 4; (c) reversing the digits gives another number with properties (a) and (b).

2. Two circles are concentric. The area of the annulus (ring) between them is 75π. The radius of the large circle is twice the radius of the small circle. Find the radius of the large circle.

Problems 3-4. Time limit: 10 minutes.

3. Three of the four vertices of a parallelogram are (not necessarily in order) the points (4,2), (10,4), and (6,8). The fourth vertex is on one of the coordinate axes. Find the coordinates of this fourth vertex.

4. At the law firm of Dewey, Cheatem, and Howe, one of the three partners has committed a crime. When questioned by the police, each gave two statements. The two innocent men each gave one true statement and one false statement. The guilty person gave two false statements. Their statements were:
   - *Dewey*: (1) Cheatem and Howe are both guilty; (2) If Cheatem is guilty, then Howe is innocent.
   - *Cheatem*: (1) Howe is innocent and I'm innocent. (2) If Dewey is guilty, then Howe is guilty.
   - *Howe*: (1) Dewey or Cheatem did it. (2) Dewey's second statement is false.

   Who is the guilty party?

Problems 5-6. Time limit: 10 minutes.

5. If $m = \dfrac{pqr}{p-q}$, solve the equation for q. [Assume that no denominator is zero]

6. A parabola has an equation in the form $y = ax^2 + bx + c$. It passes through the points (0,0), (4,0), and (2,8). Find the value of coefficient a.

Answers.

1. 48, 84 (any order)
2. 10
3. $(0,6)$
4. Howe
5. $q = \dfrac{mp}{m+pr}$
6. –2