

Answers must be exact or must have 4 (or more) significant digits, correctly rounded, unless otherwise noted

Problems 13-14. Time limit 10 minutes.

13. Three vertices of a parallelogram are $(2,1)$, $(5,3)$, and $(7,8)$. If the fourth vertex is in the first quadrant, what point or points could it be?

14. Points $A(7,k)$, $B(4,1)$, and $C(9,-2)$ are given. Find k so that $\overline{AB} \perp \overline{BC}$

Problems 15-16. 10 minutes.

15. Assume the universal set is the set of positive integers less than 100. $A = \{\text{integers divisible by 2}\}$, $B = \{\text{integers divisible by 3}\}$, and $C = \{\text{integers divisible by 10}\}$. How many elements are in the set $A \cup (B \cap C)$?

16. Find all ordered pairs of positive integers (a,b) , with $a > b$, for which $a^2 - b^2 = 143$

Problems 17-18. 11 minutes.

17. [a classic] Find the sum of the infinite series $1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)^3 + \cdots + n\left(\frac{1}{3}\right)^n + \cdots$

18. Tilly wrote a tautology [statement that is always true], but it got smudged. It looks like $(\sim p \vee q) \blacksquare p$. Which of the following could be under the \blacksquare ? The choices are $\{ \wedge, \vee, \neg, \rightarrow, \leftrightarrow \}$