Nassau County Interscholastic Mathematics League

Solutions | **1998-1999**
Contest # 4

T1. There are only four 4-digit Fibonacci numbers: 1597, 2584, 4181, and 6765. So 5-Across must be 4181 or 6765, the only two with repeated digits. The only 3-digit Fibonacci number with a 1 for its second digit is 610, so that could be 2-Down. The only 3-digit Fibonacci number with 7 for its second digit is 377, so it too is a candidate for 2-Down which must be rejected due to 6 across. The only 3 digit cubes are 125, 216, 343, 512, and 729. 343 fits with the first case but there is not cube with 6 as its second digit so case two is impossible. Thus 1-Down is 343. The only 3-digit perfect squares with second digit 8 are 289, 484, and 784. For 3-Down the digits increase so it's 289. Thus the last digit of 1-Across is now a 2 also. The only 3-digit cube starting with 2 is 216, so that's 4-Down. So 6-Across must be 3096

T2. $\Delta ABD \sim \Delta DCB$ by SAS similarity, so $\dfrac{DB}{BC}=\dfrac{AD}{BD}$. Now, $\dfrac{15}{BC}=\dfrac{18}{15}$, so BC = 12.5

T3. Currently she has .2(140) = 28 hits. Let x = number of additional hits needed for average to EQUAL .300. So $\dfrac{28+x}{140+x}=.3$. Thus x = 20. But in order for the average to be OVER .300, there would have to be 21 hits.

T4. This is a conditional probability problem. The condition is that they said the same thing, so the events are not independent. Thus the sample space is where either they both told the truth or both lied. The event is that they both told the truth. So the probability is given by $\dfrac{\frac{2}{3}\cdot\frac{3}{4}}{\frac{2}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{4}}$, which equals $\dfrac{6}{7}$

T5. Since the angles add to $\dfrac{\pi}{2}$, the cosine of one equals the sine of the other. Say that

$\cos\left(Arc\sin\sqrt{\dfrac{3x-1}{25}}\right)=\sin\left(Arc\sin\sqrt{\dfrac{3x-1}{25}}\right)$, and square both sides. Remember on the left side that

$\cos^2\theta = 1-\sin^2\theta$ *and* $\sin(Arc\sin y)=y$, so $1-\dfrac{3x-1}{25}=\dfrac{3x+1}{25}$, which yields $x=\dfrac{25}{6}$

T6. The sum of three absolute values is 0 only if each is 0. So $x+y-1=0$, $|x|-x=0$ and $|x-1|=-(x-1)$. If |x| = x, then $x \geq 0$. If $|x-1|=1-x$, then $x-1\leq 0$, so $x \leq 1$. So the graph is the part of the line x + y = 1 in the first quadrant, and its length is $\sqrt{2}$.