Nassau County Interscholastic Mathematics League

Solutions Contest # 1

2002-2003



- 1. Being a multiple of 3 and of 4 means multiples of 12; so 12, 24, 36, 48, 60, 72, 84, 96. The only ones that reverse are 48, 84.
- 2. Let r be the radius of the inner circle. So $\pi (2r)^2 \pi r^2 = 75\pi$ which simplifies to $3r^2 = 75$, and $r^2 = 25$, so r = 5, and the radius of the LARGE circle is 10.
- 3. Call the points A(4,2), B(10,4), and C(6,8). Method 1: If \overline{AB} is a diagonal, then its midpoint is (7,3), and the fourth vertex is the reflection of C over (7,3), which is (8,-2). If \overline{AC} is a diagonal, its midpoint is (5,5) and the fourth vertex is the reflection of B over (5,5), which is (0,6). If \overline{CB} is a diagonal, its midpoint is (8,6) and the fourth vertex would be (12,10). Only (0,6) is on an axis.

Method 2: Going from A to B is (+6,+2), so from C to fourth vertex would also be (+6,+2), so the fourth vertex would be (12,10). From B to A is (-6,-2) so from B that would lead to (0,6). From C to A is (-2,-6) so from C that leads to (8,-2). Again, only (0,6) is on an axis.

- 4. If Dewey is guilty, then his second statement is true, but the guilty party's statements are both false. If Cheatem is guilty, then his second statement is true. If Howe is guilty, then Dewey's are False, True; Cheatem's are False, True; and Howe's statements are both false. So Howe is the guilty party.
- 5. m(p-q) = pqr, so mp mq = pqr, then mp = mq + pqr, and now factor the right side to get mp = q(m+pr), and so $q = \frac{mp}{m+pr}$
- 6. Substitute each pair in for (x, y). Using (0,0) we get c = 0. Then we get the system of equations 16a + 4b = 0 Solving, we get a = -2 and b = 8. So a = -2