Nassau County Interscholastic Mathematics League

Contest # 5

Answers must be in simplest exact form, unless otherwise noted.

2003-2004

No Calculators

Problems 19-20 Time limit: 10 minutes.

- 19) A truth table is made for the logic expression $[p \land (q \lor r)] \rightarrow q$, where symbols \land represents "and", \lor represents "exclusive or", and \rightarrow represents "implies". Of the eight cases for p, q, and r, for how many of them will the statement $[p \land (q \lor r)] \rightarrow q$ be true? (Note, "exclusive or" $p \lor q$ is true when either p or q is true but not both.)
- 20) Point S is in the interior of triangle *POR* such that \overline{RS} bisects $\angle PRS$ and \overline{OS} bisects $\angle POS$. If $m\angle OPR = 88^{\circ}$, find the degree-measure of $\angle OSR$.

Problems 21-22 Time limit: 10 minutes.

- 21) (a classic) The radii of two concentric circles are 10 and 26. Find the length of a chord of the larger circle which is tangent to the smaller circle.
- 22) Find all ordered pairs of integers (x, y) such that 1 + 2x + 3y = xy.

Problems 23-24 Time limit: 10 minutes.

- 23) A function f is even if f(-x) = f(x) for all x. A function f is odd if f(-x) = -f(x) for all x. Suppose that function g is odd and function h is even. Completely simplify the expression $\frac{g(-4) + h(-2) + h(2) + g(4)}{h(2) + g(0)}$.
- 24) Find all exact real numbers x, with $0 \le x < 2\pi$, such that $2\cos(2x) 1 = 0$.

Answers:	19)	7	20)	134	
	21)	48	22)	(4,9),(10,3),(2,-5),(-4,1)	
	23)	2	24)	$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	