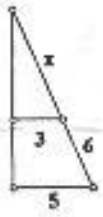
**Nassau County Interscholastic Mathematics League**

**Solutions**
**Contest # 4**

**1999-2000**

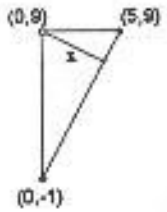
T1. Since all sides are integers and not equal, no side can be 1 because of the triangle inequality. The smallest a side can be is 2. No side can be 6, since the others would then add to at least 7, and the perimeter is too big. So only 2, 3, 4, and 5 are possible. 2,3,4 works, 2,3,5 does not, 2,4,5 works, and 3,4,5 works. There are **3** triangles possible.

T2. Set up similar triangles, using the radii, the slant height, and altitude. So $\frac{3}{5}=\frac{x}{x+6}\rightarrow x=9$.




Now the altitude of the small cone is $6\sqrt{2}$ and the altitude of the large cone is $10\sqrt{2}$. The volume of the frustum is the volume of the large cone minus the volume of the small cone, which is $\frac{1}{3}\cdot 25\pi\cdot 10\sqrt{2}-\frac{1}{3}\cdot 9\pi\cdot 6\sqrt{2}=\frac{196\pi\sqrt{2}}{3}\approx 290.3$

T3. One method is to create a right triangle, as shown. The legs have lengths 10 and 5 and the hypotenuse is $5\sqrt{5}$. Now compute the area two ways and set them equal, $\frac{1}{2}\cdot 10\cdot 5=\frac{1}{2}\cdot 5\sqrt{5}\cdot x$, so $x=2\sqrt{5}$, exact value required.




T4. You start in chair #1 and do an odd number of moves, so you must end in an even number, #2 or #4. Then chairs 1 and 5 are removed. Starting in an even number and doing an odd number of moves, the result will be an odd number. the only odd numbered chair left is #3.

T5. P(first roll a 5) = $\frac{1}{6}$.

P(first not a 5, second a five) = $\frac{5}{6}\cdot\frac{1}{6}$.

P(first two not 5, third is 5) = $\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}$.

P(first 5 on roll 4) = $\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}$ and so on.

The expected value is given by the expression: $1\cdot\frac{1}{6}+2\cdot\frac{5}{6}\cdot\frac{1}{6}+3\cdot\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+4\cdot\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\cdots$, which is an infinite telescoping geometric series like problem #17 on contest 3. Let S = sum, multiply by $\frac{5}{6}$ and subtract. Then use the infinite geometric series formula and compute **S=6**

T6. First, googol$^3$ = $10^{300}$, which is a 1 followed by 300 zeros, so 2 Down is 301. Next look for a four digit cube of a prime. Only 2197 ($13^3$) fits as 6 Across. Note that the upper left digit must be less than 3 (from 1 Across), so look for a 3-digit multiple of 11 that starts with either 1 and ends with 2 or starts with 2 and ends with 2. Only 132 fits for 1 Down. Also, there are two 3-digit Fibonacci numbers ending in 7, 377 and 987. But 4 Down can't start with a 3 (from1 Across) so 4 Down is 987. Since 1 Across is a multiple of 9, it must be 1359. So 3 Down must be 579. Check that 5 Across is 3078.