Contest # 5

Nassau County Interscholastic Mathematics League

Answers must be exact unless otherwise noted

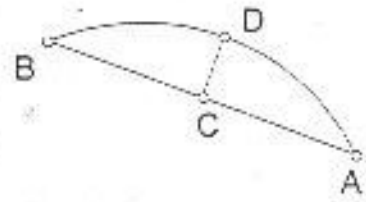
2000-2001

No Calculator

Problems 19-20 Time limit 10 minutes.

19. (a classic) Solve for the positive integer n: $(3!)(5!)(7!) = n!$

20. In the figure at right $\widehat{AB}$ is a minor arc of a circle and $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$. If AB = 20 and CD = 4, find the diameter of the circle.



Problems 21-22 10 minutes.

21. Find the arithmetic mean of the first 100 positive integers.

22. ABCD is a rectangle with area 180 and E is the midpoint of side $\overline{CD}$. If AF = 4, with F on side $\overline{AB}$, $BC = 2x-1$, *and* $CE = 4x+1$, for some x, find the numerical area of ΔAEF.

Problems 23-24. 10 minutes.

23. If $f(x) = x^3 - 5$ and $f \circ g(x) = \sqrt{x-2}$, where f and g are real-valued functions, find the value of $g(11)$

24. In a standard, well-mixed deck of 52 cards, find the probability that the first card is a king and the second card is a heart.

Answers.

19. 10

20. 29

21. 50.5 or equivalent

22. 12

23. 2

24. $\frac{1}{52}$