Nassau County Interscholastic Mathematics League

Contest # 1 Solutions 2005-2006

1. Let $b =$ the number of boys, and $g =$ the number of girls.

$$\frac{b-7}{g}=\frac{5}{4}\Rightarrow 5g=4b-28;\ \frac{b-7}{g-6}=\frac{3}{2}\Rightarrow 3g=2b+4.$$

Solving the system of equations, $g=36$ and $b=52$.

2. Solve by graphing and determine the abscissas of the points of intersection, or solve and check the 4 cases needed for an absolute value equation:

$$8+\frac{1}{2}x=x-2,\ 8+\frac{1}{2}x=2-x,\ 8-\frac{1}{2}x=x-2,\ 8-\frac{1}{2}x=2-x.\ \ x=\left\{-4,\tfrac{20}{3}\right\}$$

3. Choose any convenient number (e.g. 300 miles) for the distance between the cities. This would make the total distance traveled 600 miles. The total time for the trip is $\frac{600}{60}=10$ hours. The time for the first part of the trip is $\frac{300}{50}=6$ hours. The last part of the trip would take 4 hours for a rate of $\frac{300}{4}$ or 75mph.

4. $x+\frac{6}{x}=11.\ \left(x+\frac{6}{x}\right)^3=x^3+3x^2\bullet\frac{6}{x}+3x\bullet\frac{36}{x^2}+\frac{216}{x^3}=1331$

$$x^3+\frac{216}{x^3}+18\left(x+\frac{6}{x}\right)=1331;\ x^3+\frac{216}{x^3}=1331-18\times 11=1133$$

5. Let $n =$ the first consecutive integer, etc.

$$13\times 21=273;\ \frac{273+4n+6}{17}=27;\ 279+4n=459;\ n=45;\ n+3=48.$$

6. $2^{37}\cdot 4^{18}\cdot 5^{63}=2^{37}\cdot 2^{36}\cdot 5^{63}=2^{73}\cdot 5^{63}=2^{10}\cdot 10^{63}=1024\cdot 10^{63}$;

67 digits.