Nassau County Interscholastic Mathematics League

Contest # 6 **2000-2001**

Answers must be exact or must have 4 (or more) significant digits, correctly rounded, unless otherwise noted.

Calculators allowed

Problems 25-26. Time limit: 10 minutes.

25. Find all fixed points (i.e., output = input) of the function $f(x) = |3x - 1|$.

26. Three siblings were asked about the color of their house which is either red or blue. They each made a statement. Andy: If our house is red, then Bill told the truth; Bill: Our house is red or our house is blue; and Colleen: If Andy told the truth, then our home is blue. Not all of the three siblings told the truth. What color is their house?

Problems 26-27 Time limit: 10 minutes.

27. Acute ∠ABC is inscribed in a circle, with A, C on the circle. D is on $\overline{BC}$ and F on $\overline{AB}$ so that $\overline{AD} \perp \overline{BC}$ and $\overline{CF} \perp \overline{AB}$. $\overline{CF}$ extended meets the circle at G and $\overline{AD}$ extended meets the circle at E. If the measure of ∠ABC = 40°, find the measure of minor arc $\overset{\frown}{BE}$.

28. f and g are real, nonconstant functions such that for all x, y:

(1) $f(x+y) = f(x)g(y) + g(x)f(y)$ and

(2) $g(x+y) = g(x)g(y) - f(x)f(y)$.

Find all possible ordered pairs $(f(0), g(0))$.

Problems 29-30 Time limit: 10 minutes.

29. Find the ordered pair of positive numbers (x, y) satisfying $x^2 - y^2 = x - y$ and $xy = x - y$.

30. Triangle ABC is equilateral with side 1. G is the centroid of ΔABC. Point D is on side $\overline{BC}$ so that ∠BGD is a right angle. Ray $\overrightarrow{BG}$ intersects side $\overline{AC}$ at E. Find the area of quadrilateral GDCE.

Answers.

25. $\frac{1}{4}, \frac{1}{2}$ *or* $0.25, 0.5$

26. red

27. 100°

28. $(0,0)(0,1)$

29. $\left(\frac{-1+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right)$ or $(0.6180, 0.3820)$

30. $\frac{5\sqrt{3}}{72} \approx .1203$