Nassau County Interscholastic Mathematics League

Contest #4 Solutions 2005-2006

T1. Let x = the length of a side of the square.

Then $(x+6)(x-3)=\frac{9}{8}x^2; x^2+3x-18=\frac{9}{8}x^2; 8x^2+24x-144=9x^2;\ x^2-24x+144=0; x=12.$

T2. The radius of the dodecagon is equal to the radius of the hexagon which is equal to the length of a side of the hexagon. The central angle of the dodecagon measures $30°$. The area of the dodecagon is twelve times the area of each of the triangles formed by drawing the radii of the dodecagon.

$$A=12\cdot\frac{1}{2}(9\sqrt{2})^2\sin 30°=486.$$

T3. $3\cdot 9^x+12^x=2\cdot 16^x, x=\log_{\left(\frac{3}{4}\right)} y \Rightarrow y=\left(\frac{3}{4}\right)^x.$

$$3\left(\frac{9}{16}\right)^x+\left(\frac{12}{16}\right)^x=2; 3\left(\left(\frac{3}{4}\right)^x\right)^2+\left(\frac{3}{4}\right)^x=2\Rightarrow 3y^2+y-2=0.$$

$(3y-2)(y+1)=0$; reject $y=-1$; accept $y=\frac{2}{3}$.

T4. A total of seven questions can be answered correctly by choosing 5 MC and 2 TF, 4 MC and 3 TF, 3 MC and 4 TF, or 2 MC and 5 TF. The probabilities of each outcome are as follows:

$$ {}_5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0\cdot{}_5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3+{}_5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1\cdot{}_5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2+$$

$$ {}_5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2\cdot{}_5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1+{}_5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3\cdot{}_5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0=$$

$$\frac{1}{32768}(10+150+450+270)=\frac{55}{2048}.$$

T5. Let t be the time for the faster elevator and $t+14$ be the time for the slower elevator. The rate for the faster elevator is $\frac{600}{t}$ and the rate for the slower one is $\frac{600}{t+14}$.

$$\frac{27.6}{\left(\frac{600}{t}+\frac{600}{t+14}\right)}=0.48; 2760=48\left(\frac{600}{t}+\frac{600}{t+14}\right)\Rightarrow 23=\frac{240}{t}+\frac{240}{t+14}\Rightarrow 23t^2+322t=480t+3360\Rightarrow$$

$$23t^2-158t-3360=0; (23t+210)(t-16)=0\Rightarrow t=16; \frac{600}{t}=37.5$$

T6. The radius of circle P is $6\sqrt{2}$. $OA=OB=3\sqrt{6}; \therefore OP=3\sqrt{2}$ and $m\angle APB=120°$.
$OC=3\sqrt{2}+6\sqrt{2}=9\sqrt{2}$. $\text{Area}_{\odot O}=\pi(9\sqrt{2})^2=162\pi$. The area of the shaded region is

$$81\pi-\frac{2}{3}(72\pi)-\frac{1}{2}(3\sqrt{2})(6\sqrt{6})=33\pi-18\sqrt{3}.$$