**Solutions**
**Contest # 5**

19. Let t=the tens digit and u = the units digit. Then the number is 10t + u = 2tu. Rewrite as $2t = \frac{u}{u-5}$. The right side must be an even integer. Only u = 6 works. Then solve for t. t = 3 and. the number is 36.

20. Bob said Dave lied, so if Bob is telling the truth, Dave is the liar. Explore that. Al and Carl must both be truthful. The contrapositive of Carl's statement says if Bob did not do it, then Al did. The contrapositive of Al's statement says if Carl is truthful, then Al didn't do it. These are contradictory statements, unless Bob did it. Further, if Dave is telling the truth, Al and Carl's statements can both also be true leaving Bob as the only liar. Bob did it.
*Alternate solution* If Al is guilty, then Dave's statement is false and so is Al's, which can't be. If Carl or Dave were guilty, then Dave's statement and Carl's statement would both be false, so that can't be. If Bob is guilty, then Bob's statement is false, but all the others are true.

21. $x^2(x-1)-3x^2-7x+10 = x^3-4x^2-7x+10$. At this point either try grouping or finding a root and thus a factor, to get $(x-1)(x+2)(x-5)$.
Alternatively $x^2(x-1)-3x^2-7x+10 = x^2(x-1)-3x(x-1)-10x+10$

$$= x^2(x-1)-3x(x-1)-10(x-1)$$
$$= (x-1)(x^2-3x-10)$$
$$= (x-1)(x+2)(x-5)$$

22. Let a = half major axis, b = half minor axis, and c = distance from center to focus. Using the locus definition of an ellipse, $c^2+b^2=a^2$ and that the sum of the distances to the foci from a point on the ellipse is 2a. So a = 10 and $c = 3\sqrt{2}$. Solving gives $b = \sqrt{82}$ and the minor axis = 2b = $2\sqrt{82}$

23. Find probability it happens on first roll, plus probability not first, but second, plus probability not first 2, but third roll, plus not first three, but fourth roll

$$\frac{1}{6}+\frac{5}{6}\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}=\frac{671}{1296}\approx 0.5177$$

24. The diagonals of a rectangle are congruent, so OQ = PR = 10, and the diameter = 20