Nassau County Interscholastic Mathematics League

Contest # 2

Answers must be exact or must have 4 (or more) significant digits, correctly rounded, unless otherwise noted

1999-2000

Problems 7-8. Time limit 10 minutes.

7. Let A = {nonnegative integers < 10}, B = {even integers}, and C = {integral multiples of 3 which are less than 10}, and let K' denote the complement of set K in the set of all integers. Find $A \cap (B \cap C)'$.

8. Two students made truth tables with four columns each. The headings were p, q, $p \wedge q$, and $p \vee q$, respectively. One student did the table correctly. The other student mixed up the meanings of $\wedge$ and $\vee$, but otherwise did it correctly

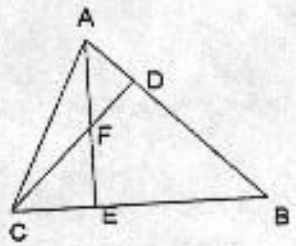
Of the 16 entries in each table, how many are the same?

Problems 9-10. 10 minutes.

9. Find the sum of the infinite geometric series

$$36 + 24 + 16 + \frac{32}{3} + \cdots$$

10. Point F is the orthocenter of ΔABC. If AD = 3, BD = 6, and BE = 5. Find length CE. (The *orthocenter* of a triangle is the point of concurrency of its' altitudes.)



Problems 11-12. 11 minutes.

11. [a classic] Factor the polynomial $81x^4 + 64$ into two factors with integer coefficients, each factor of degree 2.

12. How many distinct isosceles triangles are there with integer sides and perimeter less than 15?