Nassau County Interscholastic Mathematics League

Solutions **2003-2004**

Contest # 2

7. Solve the equation $2x^2-7x-15=0, (2x+3)(x-5)=0, x=-\frac{3}{2}$ or x=5 The integers which satisfy the inequality are the ones between $-\frac{3}{2}$ and 5, including 5. There are **7** of them.

Alternate solution: graph the parabola $y=2x^2-7x-15$ and find the integers for which the graph is below or on the x-axis. They are –1, 0, 1, 2, 3, 4, or 5. So there are **7**.

8. The radius of the circle is 8, so each side is $8\sqrt{2}$ and the area is the square of the side, which is **128**.
Alternate solution. The diameter is 16. The area of a quadrilateral with perpendicular diagonals is half the product of the diagonals. $\frac{1}{2} \bullet 16 \bullet 16 = \mathbf{128.}$

9. One way to tell if a function is even is to look at the graph and see if it's symmetric over the y-axis. Another test would be to try some opposite inputs and see if the same result occurs. For example, $\log_2(-5)$ is not the same as $\log_2(5)$ so $\log_2$ is not even. Of the functions given, the even ones are $\cos(x), 5, x^2+3,$ and $|x|$. There are **4**.

10. Let the base be b. So $(2b^2+5b+7)(9)=b^3+6b^2+4b+3$. The only real solution is b = **15**.

Alternate solution: multiplying the units digits (9)(7) ends in 3, so the remainder when 63 is divided by the base is 3. So the base is a multiple of 5. Also it must be greater than 9, since 9 is one digit. Try 10, 15, etc. until one works. Here **15** works.

11. The probability the first box is acceptable is 1 since it can be anything. The next must be different, so the probability is $\frac{5}{6}$. The third must be again different, probability $\frac{4}{6}$, etc. So the probability they are all different is $\left(\frac{6}{6}\right)\left(\frac{5}{6}\right)\left(\frac{4}{6}\right)\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{6}\right)=\mathbf{\frac{5}{324}} \approx \mathbf{0.0154}$.

12. First note that the triangle is a right triangle since $18^2+80^2=82^2$. Compute the area two ways; either half the product of the legs or half the product of the hypotenuse and the altitude to the hypotenuse (the shortest). So (18)(80) = (82)(alt), and the altitude = $\mathbf{\frac{720}{41}} \approx \mathbf{17.561}$.