Nassau County Interscholastic Mathematics League

Contest # 3 **2001-2002**

No Calculators

Problems 13-14. Time limit: 10 minutes.

13. On the planet Triad there are twelve cities. Every set of three cities is connected by one triangular road. How many triangular roads are there on Triad?

14. The symbol $\vee$ means "or", and the symbol $\veebar$ means "exclusive or"(true if one or the other is true, but not both). Consider the two expressions $(p \vee q) \veebar r$ and $(p \veebar q) \vee r$. Of the eight cases, how many have different truth values?

Problems 15-16. Time limit: 10 minutes.

15. For all real numbers x, $2f(x) + f(6-x) = x^2$. Find the value of $f(3)$.

16. Find the exact volume of a cube if the space diagonal (longest diagonal; connecting opposite vertices of a cube) has length 18.

Problems 17-18. Time limit: 10 minutes.

17. Find the equation (in slope-intercept form) of the line tangent to the circle with equation $x^2 + (y-2)^2 = 25$ at the point $(3,6)$.

18. How many five-digit numbers can be made from the set of digits $\{2,3,4,5,6,7,8,9\}$ with the following rules:

(a) no digit can be repeated;
(b) the digits must be in decreasing order?

Answers.

13. 220 14. 4

15. 3 16. $648\sqrt{3}$

17. $y = \frac{-3}{4}x + \frac{33}{4}$ 18. 56