

Contest # 6

Nassau County Interscholastic Mathematics League

Answers must be in simplest exact form, unless otherwise noted.

Calculators

2003-2004

Problems 25-26 Time limit: 10 minutes.

25. Change the base 7 number 63k to base 9 where k is an arbitrary digit.
26. The lengths of the sides of a triangle are a, b, and c, and A is the angle opposite side a.
- If $b^2 + c^2 = a^2 + 4$ and $bc = \frac{a^3}{\cos A}$, find value of $b^2 + c^2$ to the nearest thousandth.
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Problems 27-28 Time limit: 10 minutes.

27. Two circles have radii 6 and 16, and the distance between the centers of the circles is k. Find all values of k so that the circles will intersect in two points.
28. Let f be a function with domain $x > 1$, such that $f(x) + 2f\left(\frac{x + 2002}{x - 1}\right) = 4009 - x$. Find $f(2004)$.
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Problems 29-30 Time limit: 10 minutes.

29. Find all ordered triples (x, y, z) of integers which solve the system:
- $$x + y + z = 2, \quad x^2 - y^2 - z^2 = 2, \text{ and } x - 3y^2 + z = 0.$$
30. Two men, Al and Bill, are married to two women, Clara and Doris, but not necessarily respectively. One of the couples has two children and the other couple has one child. The children are Ed, Fred, and Gilda. If one father, one mother, and two children are randomly selected, find the probability that they are all from one family.
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Answers:	25)	38k	26)	5.587
	27)	$10 < k < 22$	28)	2003
	29)	$(2, -1, 1)$	30)	$\frac{1}{12}$