Contest # 6

Answers must be exact or must have 4 (or more) significant digits, correctly rounded, unless otherwise noted

1999-2000

Problems25-26. Time limit 10 minutes.

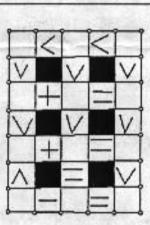
25. If (a,b) is an ordered pair of integers that solves $9a^2 + 16b^2 = 25$, find all possible values of $a^2 + b^2$

26. [a classic] In the figure, \overline{BD} bisects $\angle ABC$ and \overline{CD} bisects $\angle ACB$. If the measure of $\angle A=102^{\circ}$, find the measure of $\angle BDC$

Problems 27-28 10 minutes.

27. For what values of α can the polynomial $2x^2 + \alpha x - 3$ be factored into two polynomials with integer coefficients?

28. Fill in each blank square in the grid with a digit from the set {1,2,3,4,5} in such a way that the equations and inequalities are all true. Give your answer as a 4x3 array of digits, as they appear in the figure. Note that the third, fifth, and seventh rows are simple arithmetic equations that must be true.



Problems 29-30 11 minutes.

29. A ball sits on a staircase as shown. If AB = 8 inches and BC = 4 inches, find (in inches) the radius of the ball. Note that the ball is tangent to the step at A and $\overline{AB} \perp \overline{BC}$

30. Points A(-1,0), B(5,8), and C(h,k) are given, with h and k being integers.

For how many points C will ∠ACB be a right angle?

