Assignment 1: Efficient Frontier

Industry_Portfolios.xlsx contains monthly nominal (net) returns for ten industry portfolios, expressed in percent. These returns cover the ten-year period from Jan 2004 through Dec 2013.

→ Estimate the vector of mean returns and the covariance matrix of returns for the ten industry portfolios.

→ Create a table showing the mean returns and standard deviation of returns for the ten industry portfolios.

→ Plot the minimum-variance frontier (without the riskless asset) generated by the ten industry portfolios, with expected (monthly) return on the vertical axis and standard deviation of (monthly) return on the horizontal axis. This plot should cover the range from 0% to 2% on the vertical axis, in increments of 0.1% (or less).

→ Briefly explain the economic significance and relevance of the minimum-variance frontier to an investor.

Now suppose that the risk-free rate is 0.13% per month.

→ Plot the efficient frontier (with the riskless asset) on the same plot as the minimum-variance frontier generated by the ten industry portfolios.

→ Briefly explain the economic significance and relevance of the efficient frontier to an investor.

The two frontiers will intersect at single point: the tangency portfolio.

→ Calculate the Sharpe ratio for the tangency portfolio, and also the tangency portfolio weights for the ten industry portfolios.

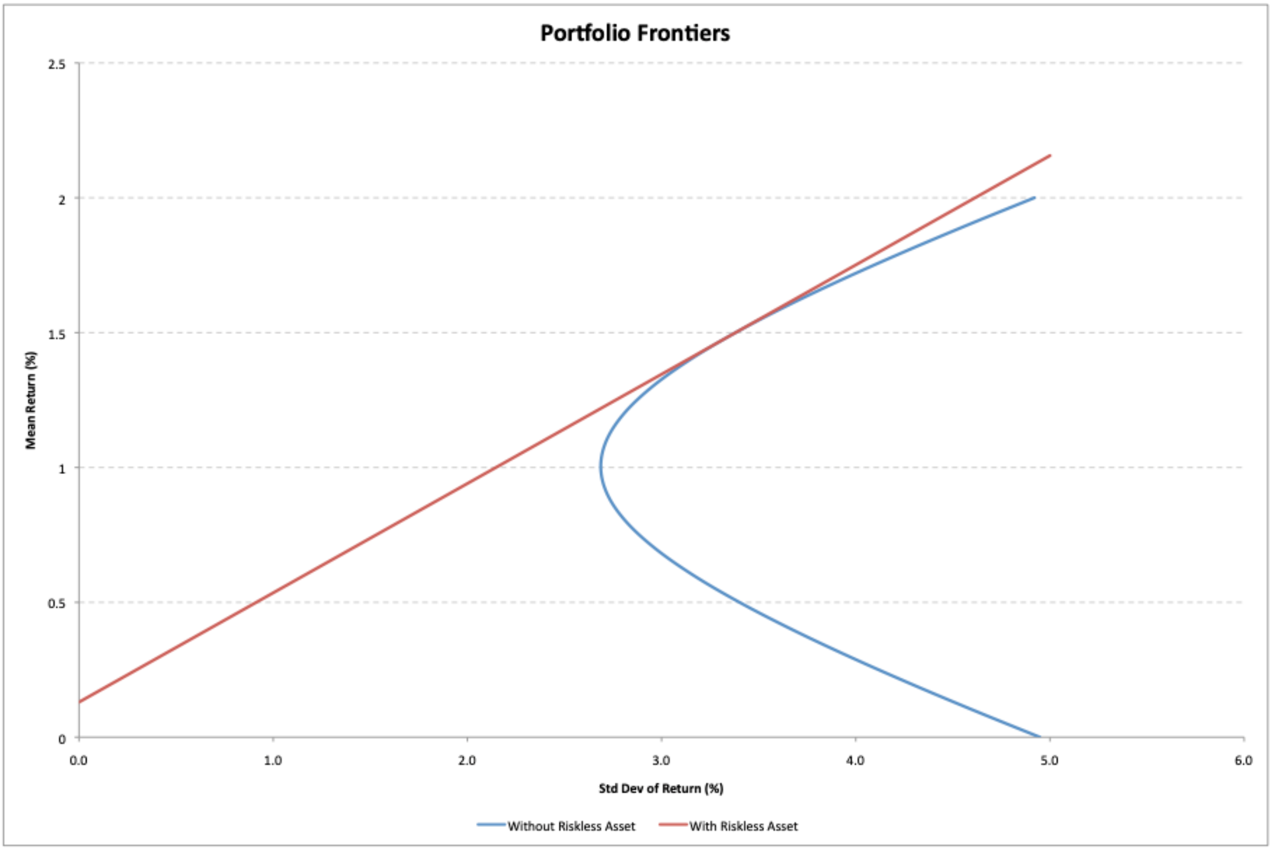
→ Briefly explain the economic significance and relevance of the tangency portfolio to an investor.

Economic significance:

- Minimum-variance frontier consists of portfolios with least risk for specified mean return, so no portfolios exist to the left of this frontier.

- Efficient frontier consists of portfolios with highest mean return for specified level of risk, so risk-averse investor must invest in portfolio on this frontier in order to maximise expected utility (of wealth).

- Tangency portfolio has the highest possible Sharpe ratio.



Assignment 2: Capital Asset Pricing Model (CAPM) & Linear Factor Model

Market_Portfolio.xlsx contains monthly nominal (net) returns for the market portfolio, expressed as a percentage. These returns cover the ten-year period from Jan 2004 through Dec 2013. Assume that the risk-free rate is 0.13% per month.

Market Model

→ Regress the monthly *excess* returns for each of the ten industry portfolios on the monthly *excess* returns for the market portfolio, so as to estimate the intercept coefficient (α) and slope coefficient (β) for each of the ten industry portfolios.

→ Create a table showing the intercept and slope coefficients for the ten industry portfolios.

→ Briefly explain the economic significance of the intercept and slope coefficients.

Security Market Line (SML)

→ Calculate the mean monthly return for each of the ten industry portfolios, as well as the market portfolio.

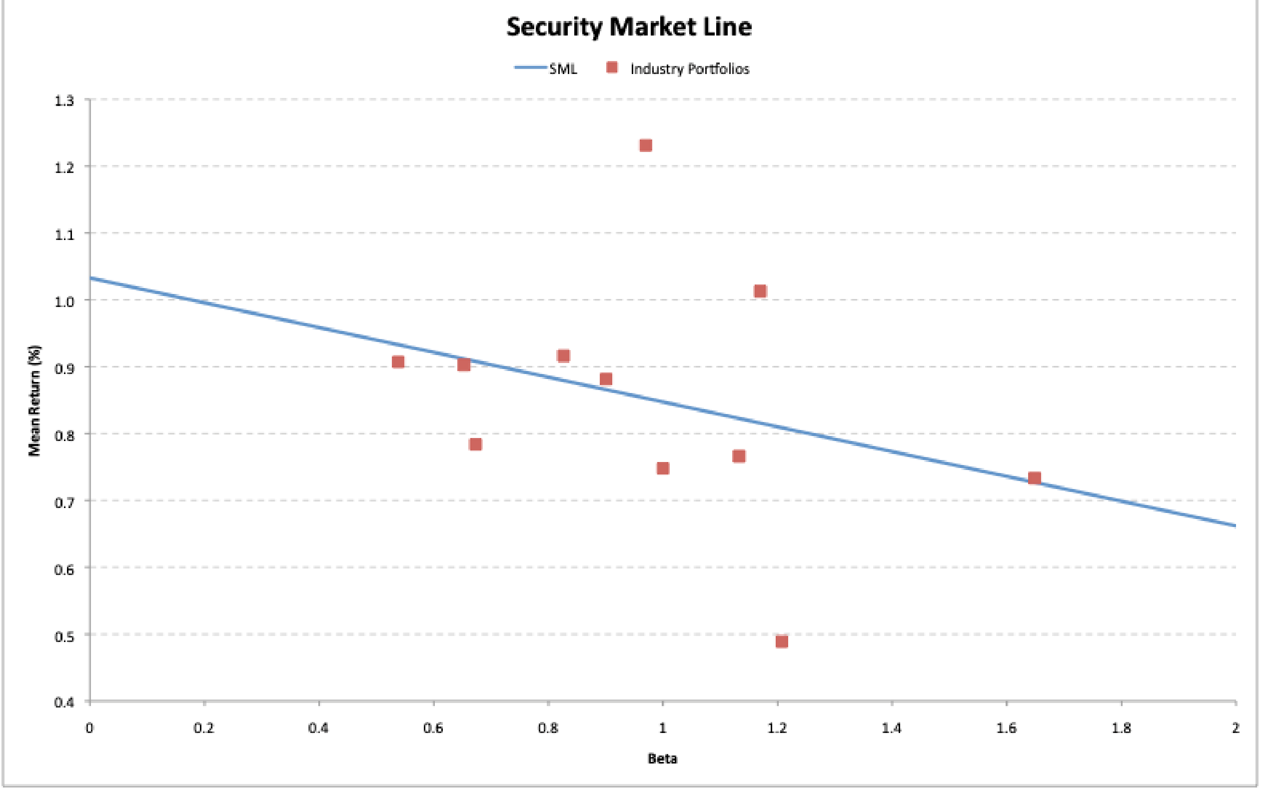
→ Regress the mean monthly returns of the ten industry portfolios and the market portfolio on the corresponding β (by construction, the market portfolio has β of one). This will give you the intercept and slope coefficients for the SML. (Warning: the results may be very different from what you would expect!)

→ Using the estimated intercept and slope coefficients for the SML, plot the SML in the range of β from zero to two on the horizontal axis. Also plot the positions of the ten industry portfolios and the market portfolio. (You are NOT required to label the individual portfolios.)

→ Briefly explain the economic significance of the SML.

Economic significance:

- Intercept coefficient from market model regression represents CAPM pricing error.
- Slope coefficient from market model regression represents amount of exposure to market risk.
- SML shows that all appropriately-priced assets should have same Treynor ratio. Assets that lie above SML are underpriced, while assets that lie below SML are overpriced.



Assignment 3: Performance Measurement

Risk_Factors.xlsx contains monthly observations of the risk-free rate and the three Fama–French risk factors, all expressed as a percentage. These observations cover the ten-year period from Jan 2004 through Dec 2013.

→ Using excess returns for the ten industry portfolios, calculate the following performance metrics:

- Sharpe ratio
- Sortino ratio (using risk-free rate as target)
- Treynor ratio (using CAPM β)
- Jensen's α
- Three-factor α

The sample semi-variance can be estimated as:

$$\frac{1}{T}\sum_{t=1}^{T}\min\{R_{it}-R_{ft},0\}^{2}$$

where Ri is return on industry portfolio and Rf is risk-free rate.

→ Create a table showing the performance metrics for the ten industry portfolios.

→ Plot your results as a bar chart for each performance metric.

→ Briefly explain the economic significance of each performance metric.

Economic significance:

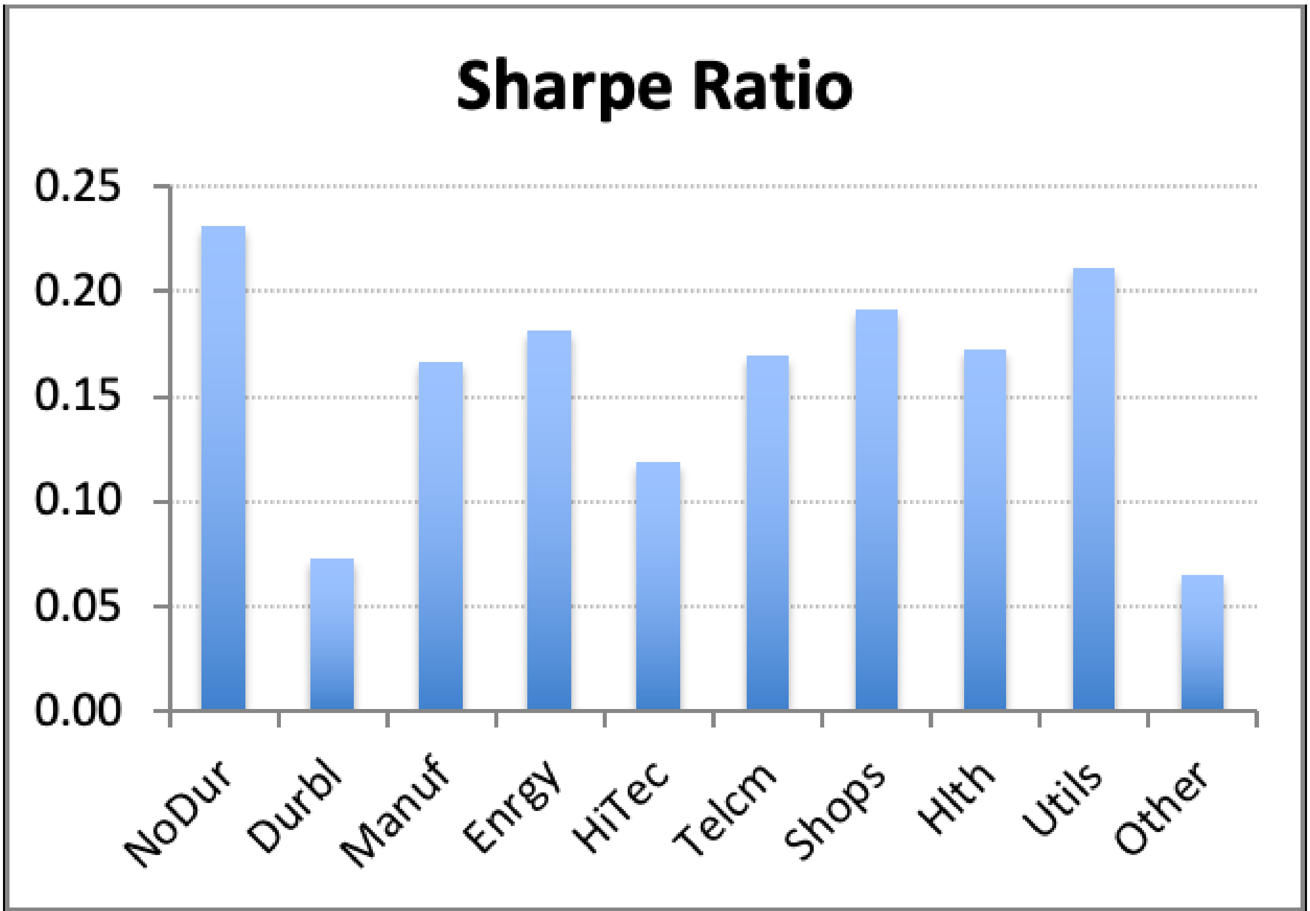
- Sharpe ratio represents risk premium per unit of total risk: o Includes all types of systematic risk and also idiosyncratic risk, which penalises individual investments and non-diversified portfolios.
    - Implicitly assumes normal returns, so cannot distinguish between return distributions with same variance but different skewness or kurtosis.

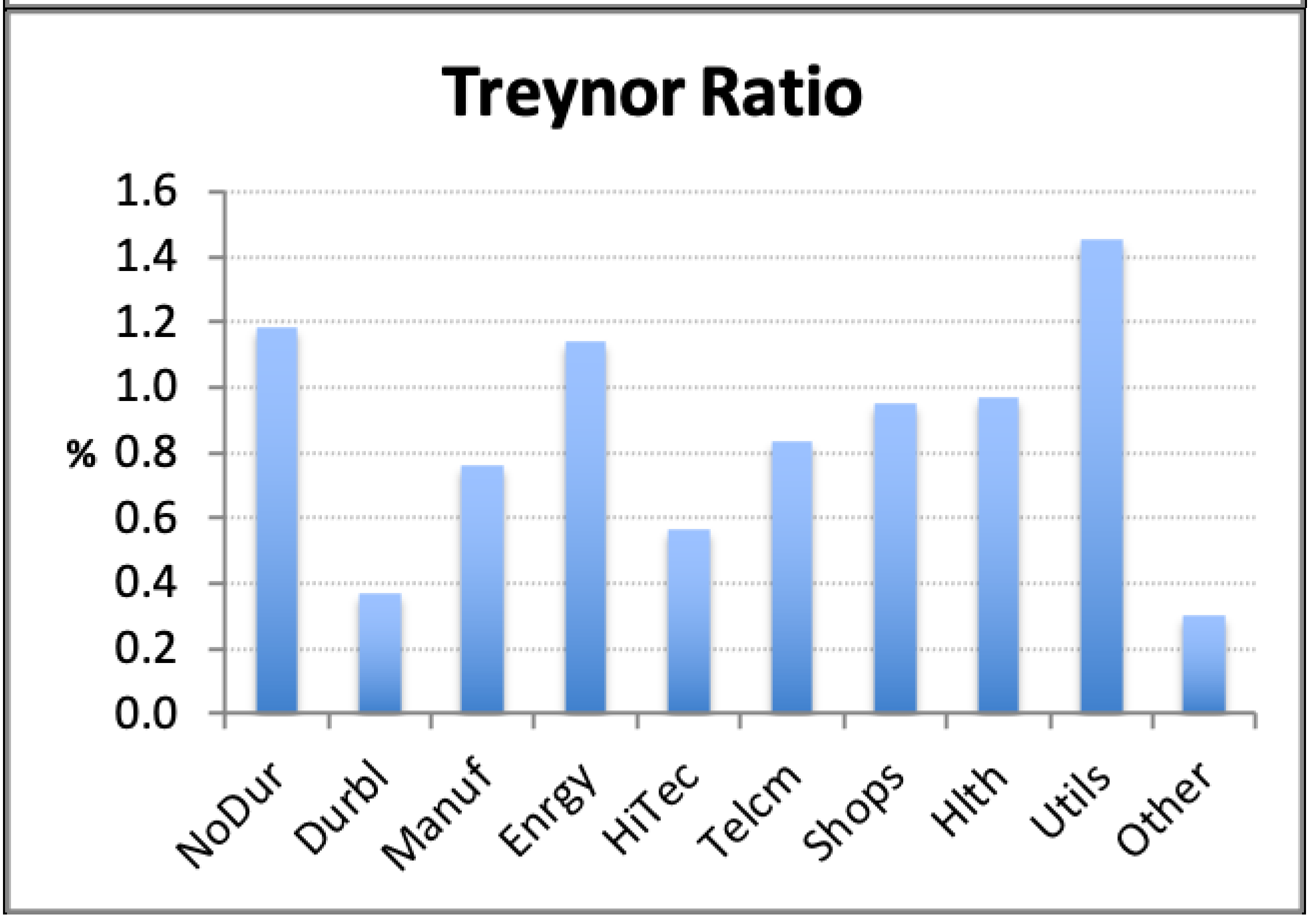
- Treynor ratio represents risk premium per unit of market risk, which ignores other types of systematic risk.

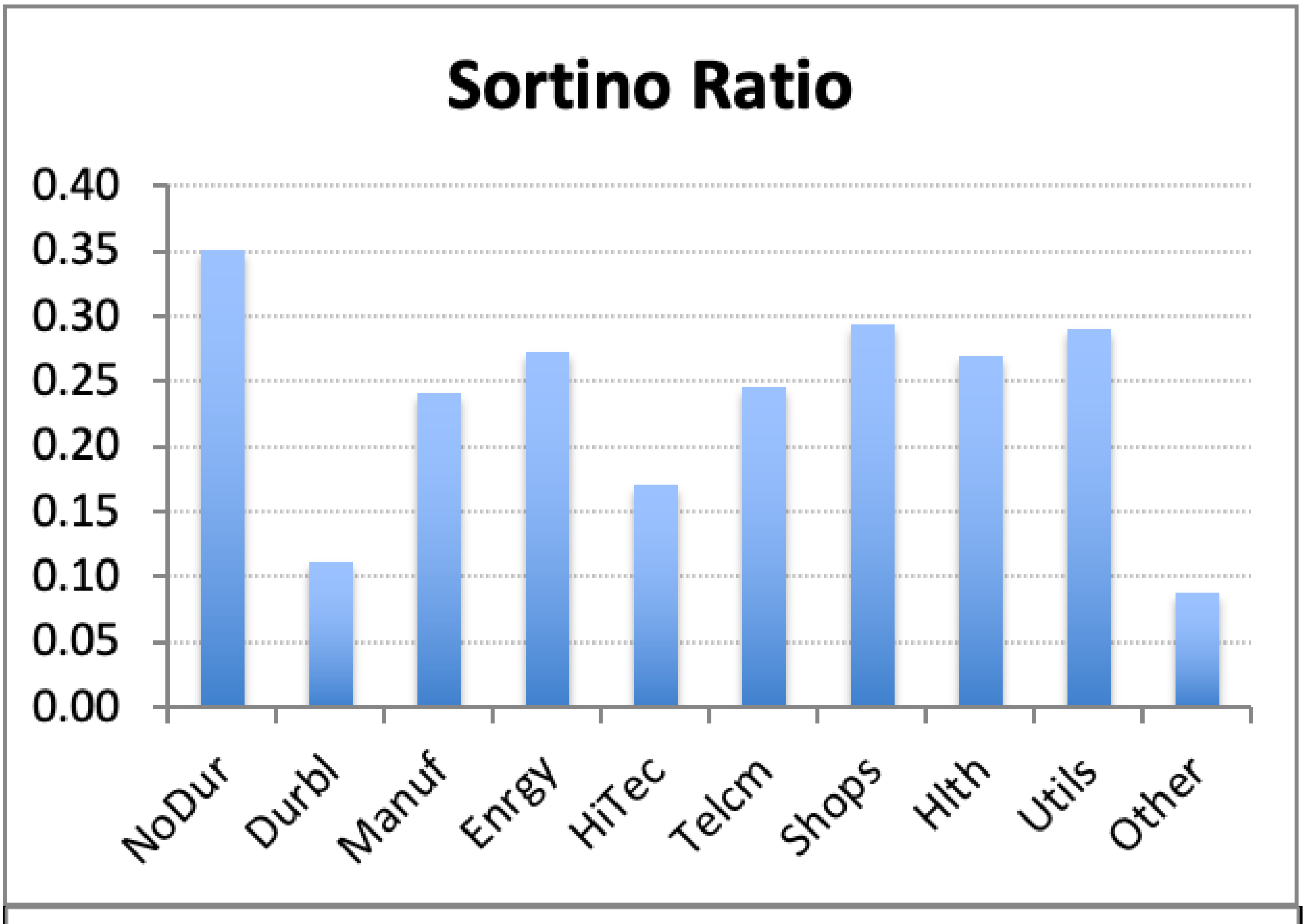
- Sortino ratio represents risk premium per unit of downside risk, so can distinguish between asymmetric return distributions with same variance but different skewness.

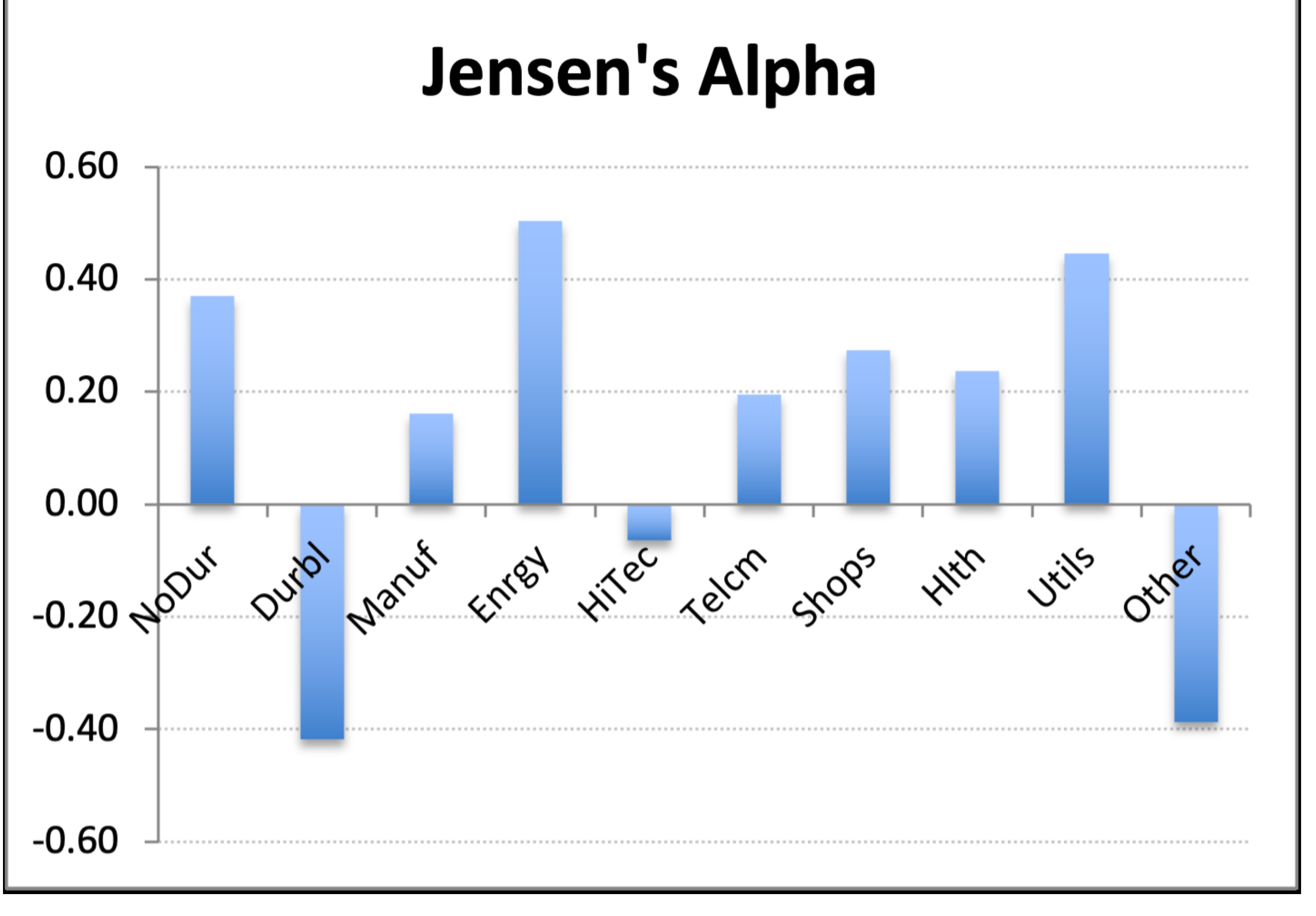
- Jensen's α represents pricing error (or abnormal mean return) after adjusting for exposure to market risk.

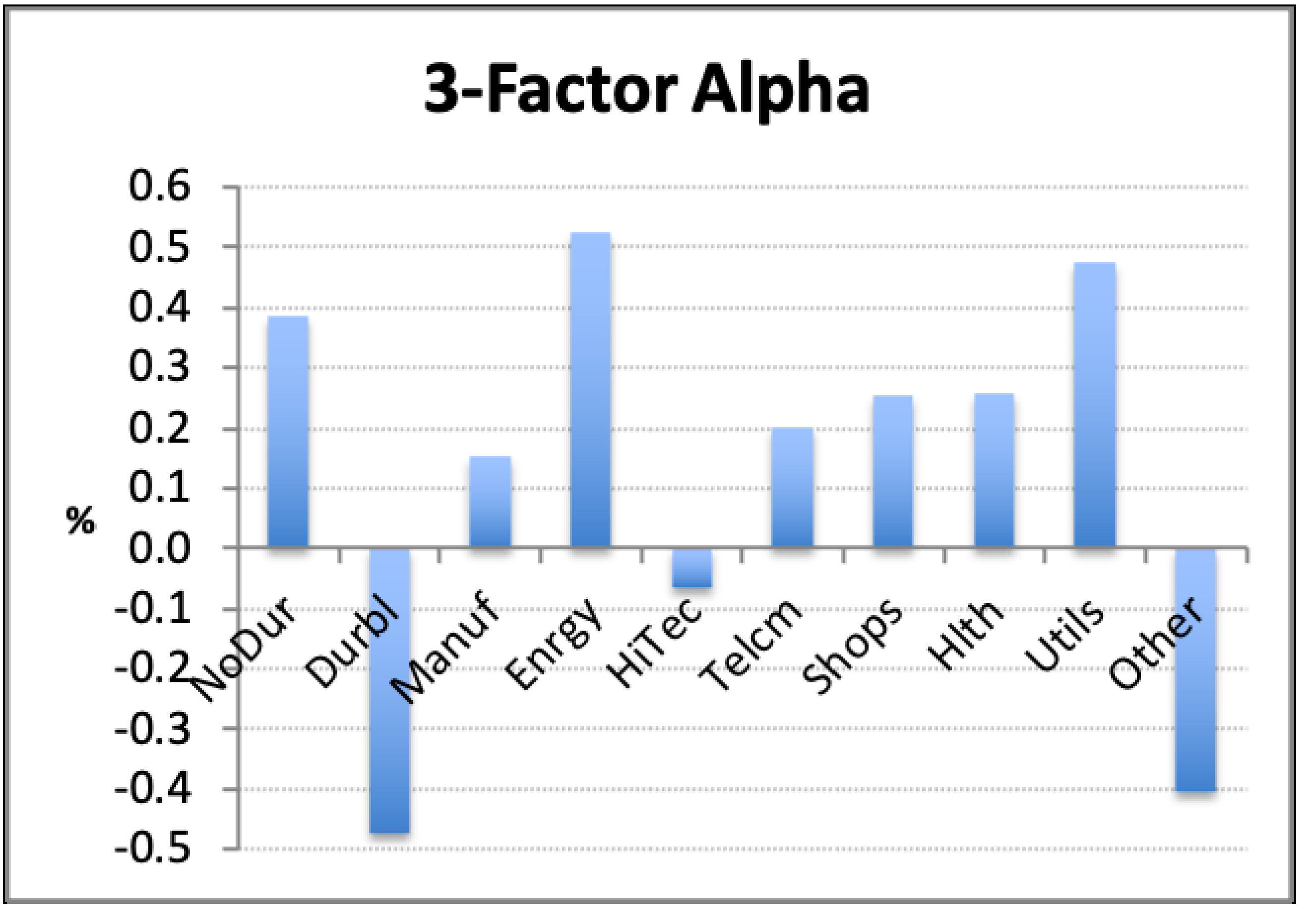
- Three-factor α represents pricing error (or abnormal mean return) after adjusting for exposure to market risk, size risk, and value risk.


Sharpe Ratio
0.25
0.20
0.15
0.10
0.05
0.00
NoDur
Durbl
Manuf
Enrgy
HiTec
Telcm
Shops
Hlth
Utils
Other


Treynor Ratio
%
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
NoDur
Durbl
Manuf
Enrgy
HiTec
Telcm
Shops
Hlth
Utils
Other


Sortino Ratio
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
NoDur
Durbl
Manuf
Enrgy
HiTec
Telcm
Shops
Hlth
Utils
Other


Jensen's Alpha
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
NoDur
Durbl
Manuf
Enrgy
HiTec
Telcm
Shops
Hlth
Utils
Other


3-Factor Alpha
0.6
0.5
0.4
0.3
0.2
0.1
%
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
NoDur
Durbl
Manuf
Enrgy
HiTec
Telcm
Shops
Hlth
Utils
Other

Assignment 4: Efficient Frontier Revisited

Part 1: Minimum-Tracking-Error Frontier

Let the market return be the target return. Estimate the expected deviation from market return, for the ten industry portfolios:

$$R_i = E(\tilde{R}_i - \tilde{R}_m)$$

Also estimate the covariance matrix of return deviations, for the ten industry portfolios:

$$V_{ij} = Cov[(\tilde{R}_i - \tilde{R}_m), (\tilde{R}_j - \tilde{R}_m)]$$

→ Plot the minimum-tracking-error frontier generated by the ten industry portfolios, with expected (monthly) return deviation on the vertical axis and (monthly) tracking error on the horizontal axis. This plot should cover the range from 0% to 0.1% on the vertical axis, in increments of 0.005% (or less).

→ Also plot the line starting from the origin that is tangent to the upper half of the minimum-tracking-error frontier, and calculate the information ratio and portfolio weights for the "tangency" portfolio.

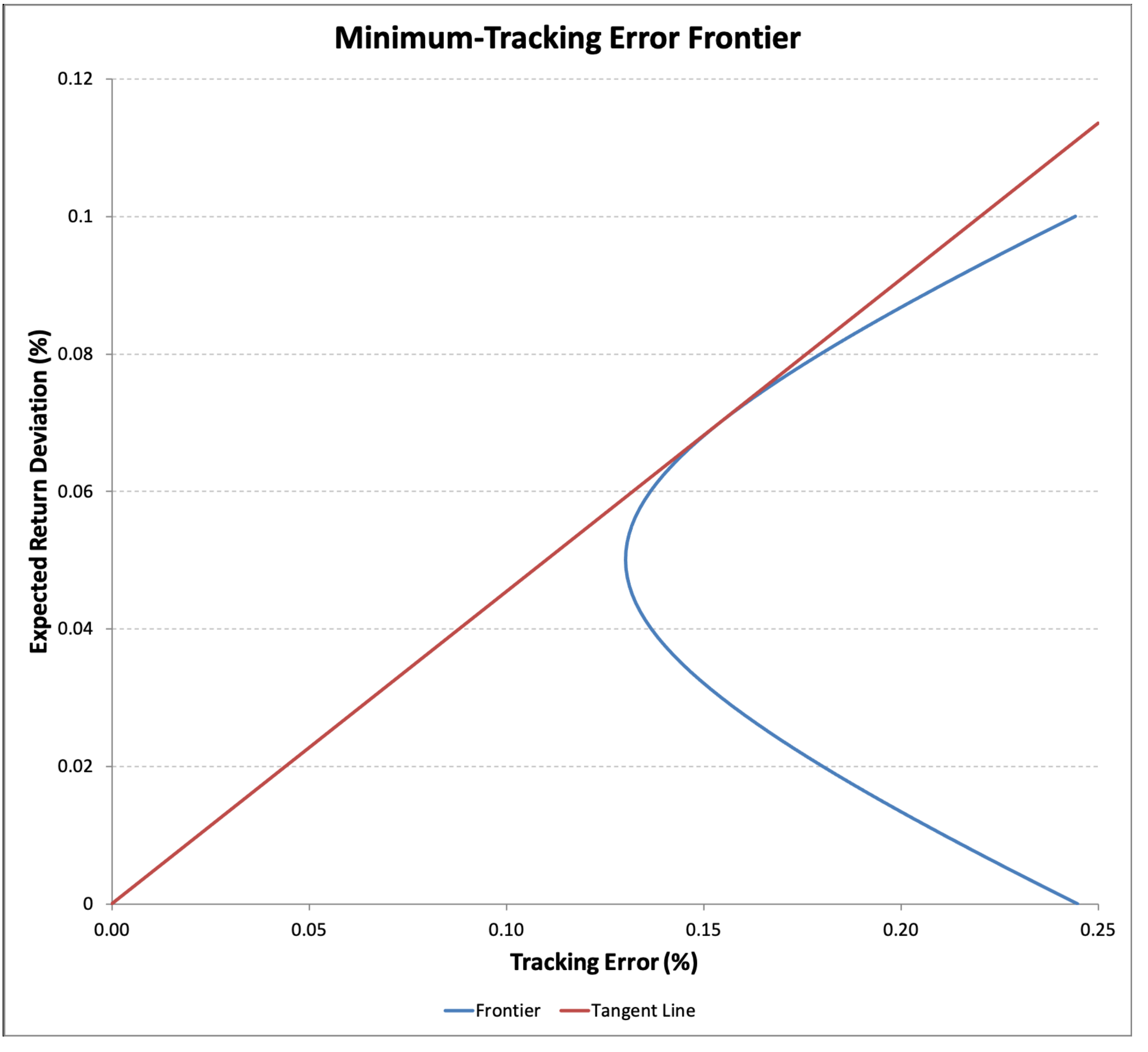
Part 2: Minimum-Variance Frontier w/o Short Sales

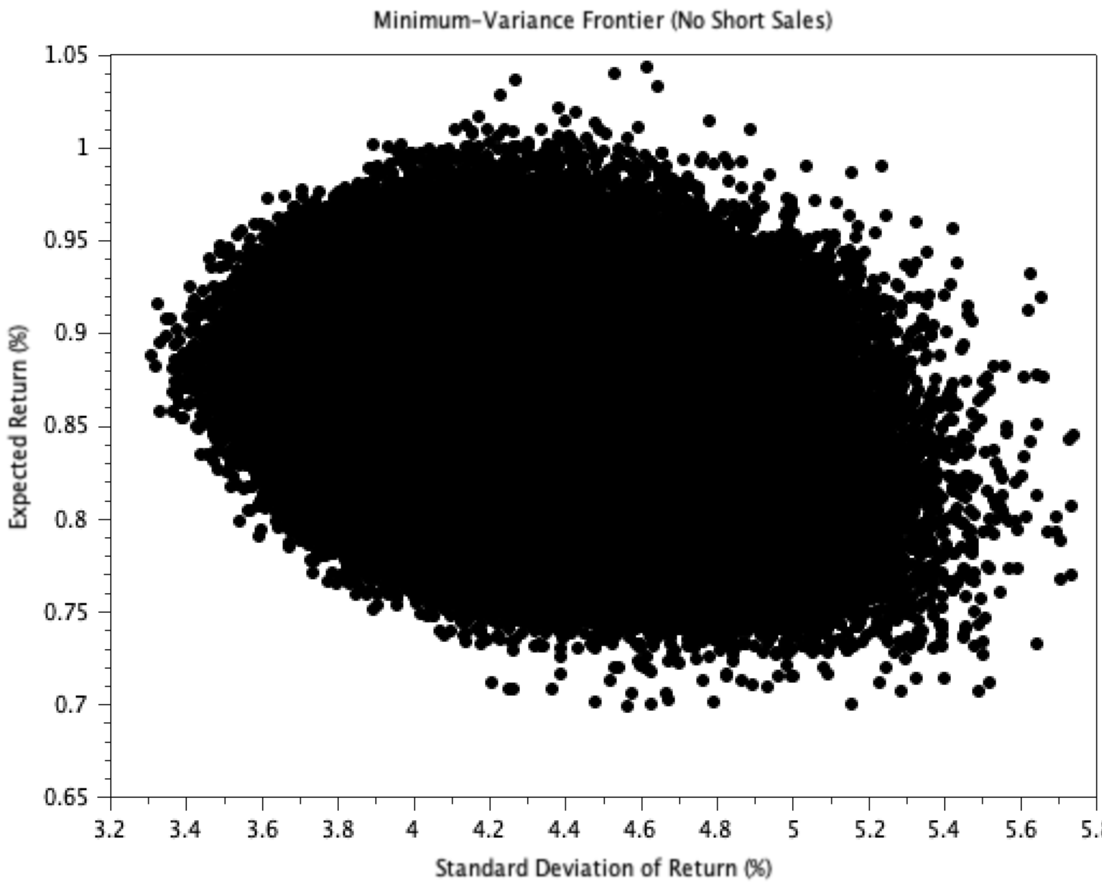
Use the monthly returns of the ten industry portfolios to generate the minimum-variance frontier without short sales, using Monte Carlo simulation. Portfolio weights will be limited to the range [0, 1].

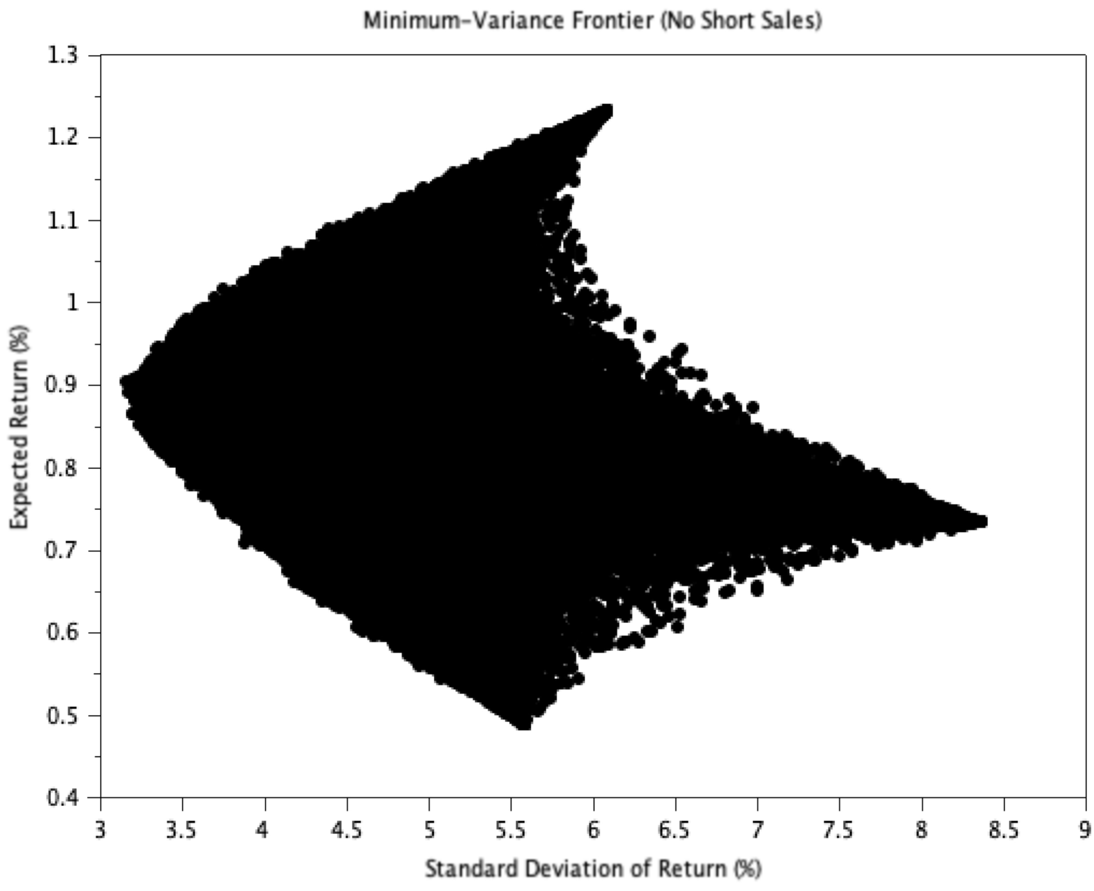
Randomly draw each element of **w**, the vector of portfolio weights, from the (standard) uniform distribution in the range [0, 1]. Divide **w** by the sum of the portfolio weights, to ensure that the portfolio weights sum to one. Use the normalised **w** to calculate the mean return and standard deviation of return for the simulated portfolio. Repeat this process until you have (at least) 105 data points.

→ Plot the data points with mean return on the vertical axis and standard deviation of return on the horizontal axis to show the minimum-variance frontier.
Repeat this entire process by simulating 1/w using the standard uniform distribution: i.e., take the reciprocal of the random draw from the standard uniform distribution as the portfolio weight.

→ Plot the new data points to show the minimum-variance frontier on a separate graph.


Minimum-Tracking Error Frontier
0.12
0.1
0.08
0.06
0.04
0.02
0
0.00
0.05
0.10
0.15
0.20
0.25
Expected Return Deviation (%)
Tracking Error (%)
Frontier
Tangent Line


Minimum-Variance Frontier (No Short Sales)
Expected Return (%)
Standard Deviation of Return (%)
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
4.8
5
5.2
5.4
5.6
5.8


Minimum-Variance Frontier (No Short Sales)
Expected Return (%)
Standard Deviation of Return (%)
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9

Assignment 5: Stochastic Discount Factor

Suppose that consumption growth has a lognormal distribution with the possibility of rare disasters:

$$\ln \tilde{g} = 0.02 + 0.02\tilde{\epsilon} + \tilde{\nu}$$

Here ε is a standard normal random variable, while ν is an independent random variable that has value of either zero (with probability of 98.3%) or ln(0.65) (with probability of 1.7%).

→ Simulate ε with (at least) 104 random draws from a standard normal distribution, and simulate ν with (at least) 104 random draws from a standard uniform distribution.

→ Use the simulated distribution of consumption growth to calculate the pricing kernel for power utility:

$$\tilde{M} = 0.99\tilde{g}^{-\gamma}$$

for γ in the range [1,4], in increments of 0.1 (or less).

→ Calculate the mean and standard deviation of the pricing kernel for each values of γ, and plot the ratio σm/μm (on the vertical axis) vs γ (on the horizontal axis).

→ Take note of the smallest value of γ for which σm/μm > 0.4 (i.e., for which the Hansen–Jagannathan bound is satisfied).

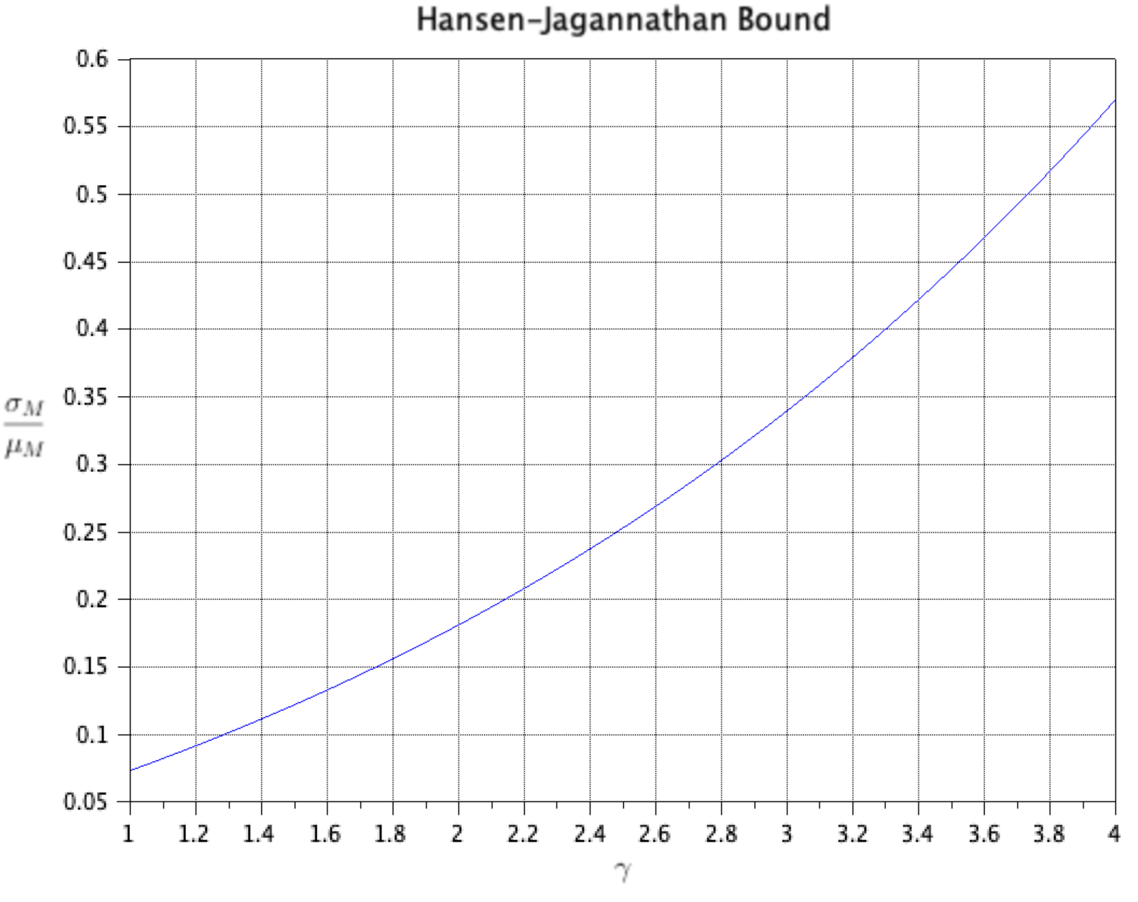
→ Briefly explain the economic significance of this result.

Economic Significance:

- US stock market has Sharpe ratio of 0.4, which can be used to derive lower bound on (constant) coefficient of relative risk aversion for investor with power utility (of consumption).

- Distribution of consumption growth becomes highly skewed (to the left) when rare disasters are included.

- H–J bound is satisfied for reasonably low level of relative risk aversion, so no equity premium puzzle.



Assignment 6: Behavioural Finance

Consider a Barberis, Huang and Santos (2001) economy with the following parameter choices for the investor's utility function:

$$\delta=0.99, \gamma=1, \lambda=2$$

Consumption growth has a lognormal distribution:

$$\ln g\sim=0.02+0.02\epsilon\sim$$

where ε is a standard normal random variable. With these parameter choices, the risk-free rate is constant at 1.0303 per year.

→ Simulate the distribution for consumption growth with (at least) 104 random draws for ε. Define x as one plus the dividend yield for the market portfolio:

$$x=(1+PD)DP=1+DP$$

and define the error term:

$$e(x)=0.99b_0E[\nu\hat{}(xg\sim)]+0.99x-1$$

where utility from financial gain or loss is given by:

$$\nu\hat{}(R)=R-1.0303 \text{ for } R\geq1.0303$$

$$\nu\hat{}(R)=2(R-1.0303) \text{ for } R<1.0303$$

→ Calculate the equilibrium values of x for b0 in the range [0, 10], in increments of 0.1, using an iterative procedure known as bisection search:

1. Set x− = 1 and x+ = 1.1. Use the simulated distribution of consumption growth to confirm that e(x−) < 0 and e(x+) > 0. Hence solution for x must lie between x− and x+.

2. Set x = 0.5*(x− + x+), and use the simulated distribution of consumption growth to calculate e(x).

3. If |e(x)| < 10−5, then x is (close enough to) the solution.

4. Otherwise, if e(x) < 0, then the solution lies between x and x+, so repeat the procedure from step 2 with x− = x.

5. Otherwise, if e(x) > 0, then the solution lies between x− and x, so repeat the procedure from step 2 with x+ = x.

Use x to calculate the price-dividend ratio for the market portfolio:

$$PD=1x-1$$

→ Plot the price-dividend ratio (on the vertical axis) vs b0.

Also, calculate the expected market return:

$$E[R_m]=E[x\tilde{g}]$$

→ Plot the equity premium (on the vertical axis) vs b0.

→ Briefly explain the economic significance of the investor's utility function for financial gain or loss [i.e., ν(R)], as well as the economic significance of the parameters b0 and λ.

Economic significance:

- Utility function for recent financial gain or loss is based on prospect theory, where financial gain or loss is measured relative to reference level based on risk-free rate, and investor is more sensitive to financial loss.

- λ determines degree of loss aversion.

- b0 determines balance between utility from consumption vs utility from recent financial gain or loss.