QF602 - Assignment

Question

- Let's assume $r = q = 0$, $S_0 = 1$ and the implied volatility for $T = 4$ are given by the following formula:

$$\Sigma(K) = 0.510 - 0.591K + 0.376K^2 - 0.105K^3 + 0.011K^4 \qquad (1)$$

  with an upper limit, which is given by $\Sigma(K) = \Sigma(3)$ for $K > 3$.

- Any payoff that only depends on S_T can be priced with the following formula,

$$\begin{aligned} V_0 &= e^{-rT} V_T(F_0(T)) + \int_0^{F_0(T)} Put(K,T) \frac{\partial^2 V_T(K)}{\partial K^2} dK \\ &\quad + \int_{F_0(T)}^{\infty} Call(K,T) \frac{\partial^2 V_T(K)}{\partial K^2} dK \end{aligned}$$

  where $Call(K,T)$ and $Put(K,T)$ is computed by using the Black Scholes formula using the volatility $\Sigma(K)$ obtained in (1).

- Using the B-L formula, compute numerically the option prices at time 0, for the following payoffs:

  1. $V_T(S_T) = \sqrt{S_T}$
  2. $V_T(S_T) = S_T^3$

  You need to submit a Jupyter Note Book which contains executable Python code. You can modify the Python code for static replication for the square payoff in the lecture note 6.