A rapid introduction to Complex Networks

UFOP

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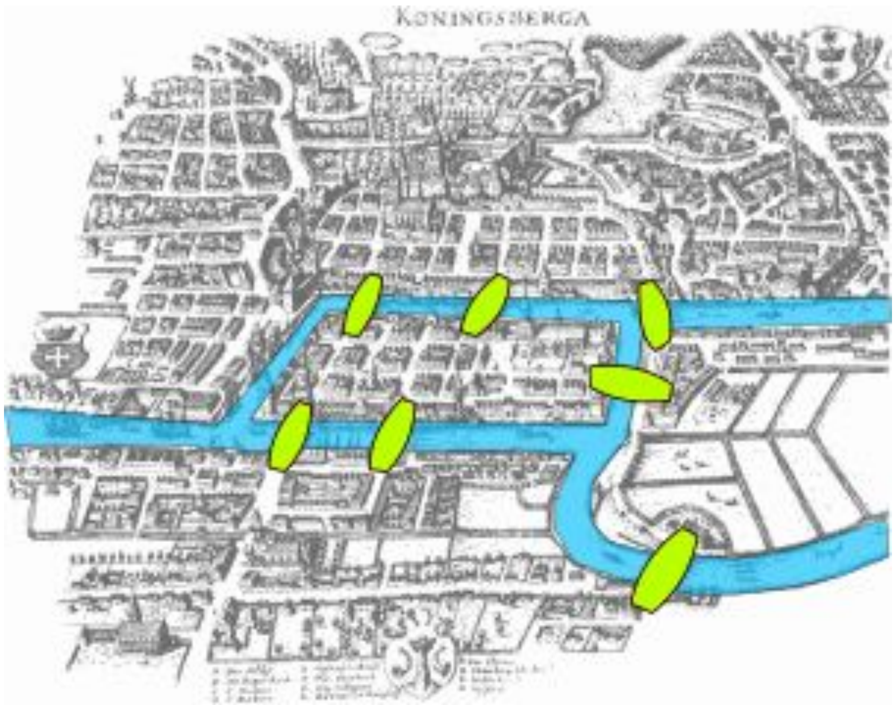
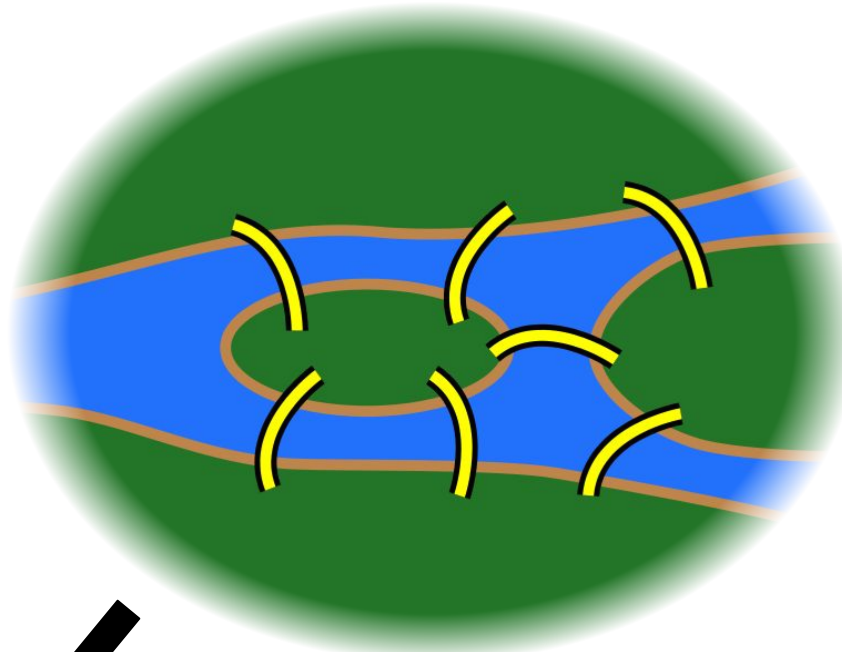
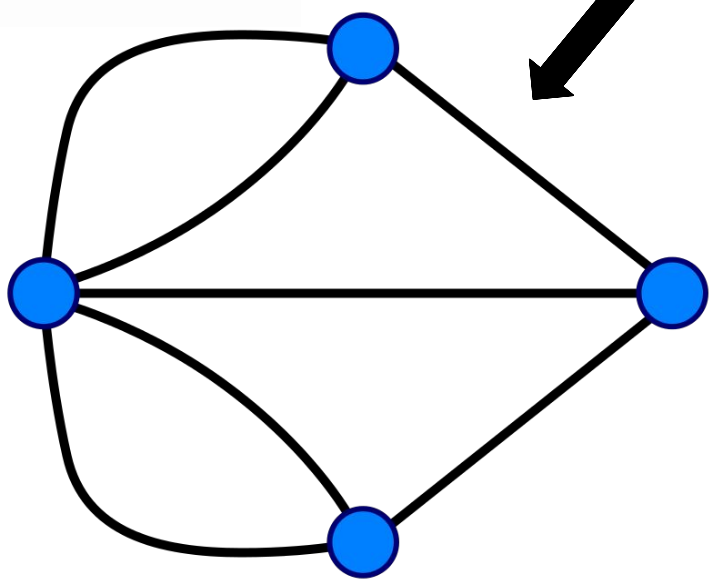
Agenda

- Introduction
- Network Models
- Degree correlations
- Robustness
- Community detection
- Spreading Phenomena
- References

Introduction - Seven Bridges of Königsberg (1736)







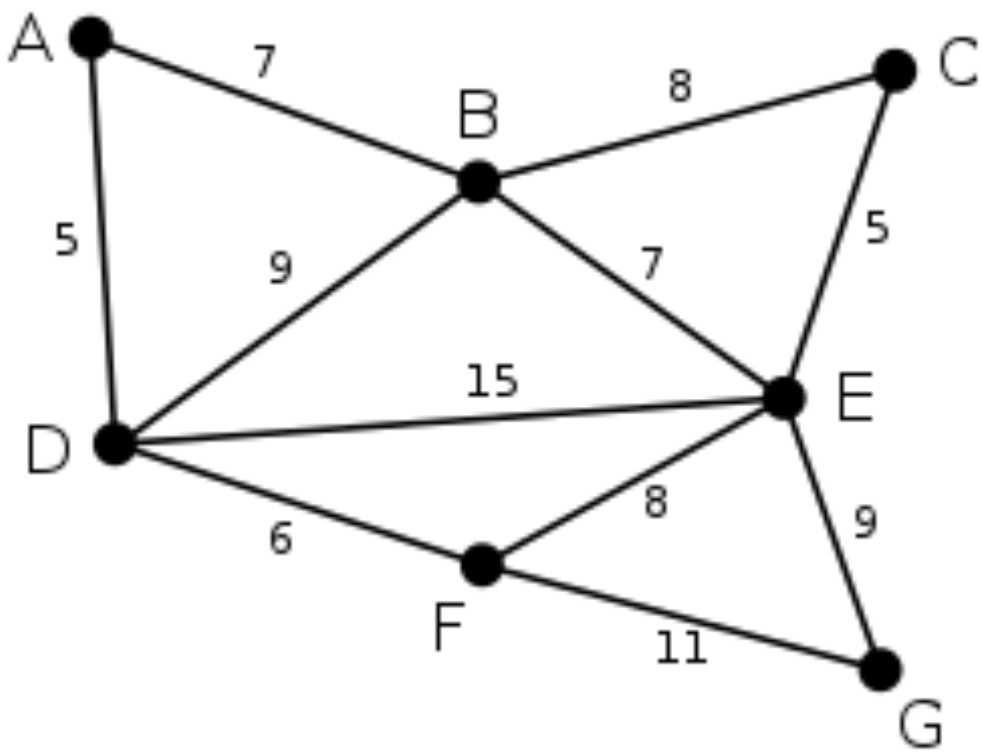
Graph: G(V,E)
Vertex (nodes)
and Edges (links)

Eulerian path:
connected graph
with exactly zero or
two nodes of odd
degree

https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

Introduction

Graph (network): G(V,E)
Vertex (nodes) and Edges (links)
Graphs in Complex Systems: Complex Networks



Complex

[adj., v. kuh m-pleks, kom-pleks; n. kom-pleks]
–adjective
1. composed of many interconnected parts; compound; composite: a complex highway system.
2. characterized by a very complicated or involved arrangement of parts, units, etc.: complex machinery.
3. so complicated or intricate as to be hard to understand or deal with: a complex problem.

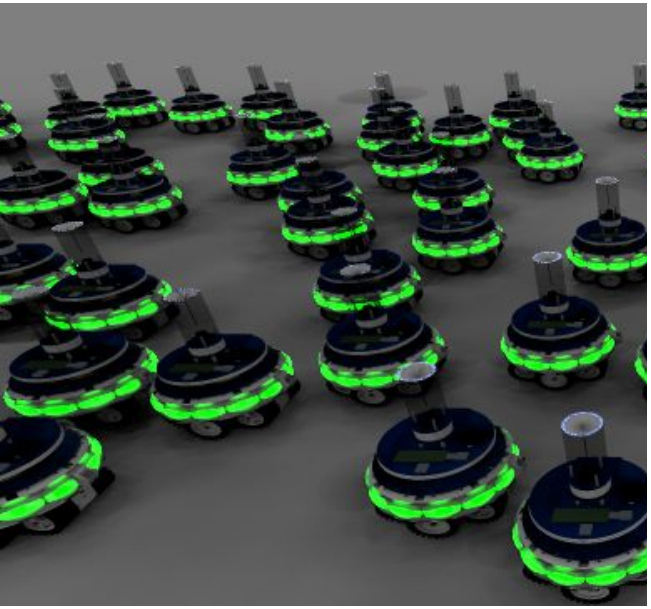
Source: Dictionary.com

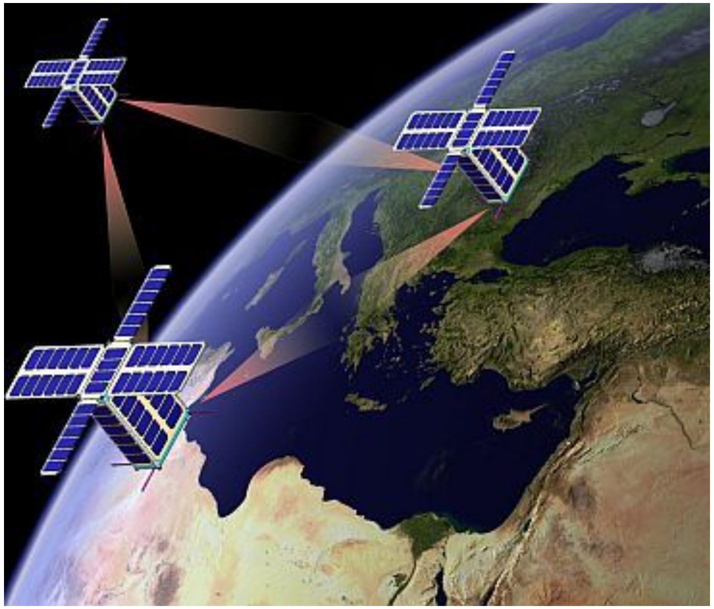
Complexity

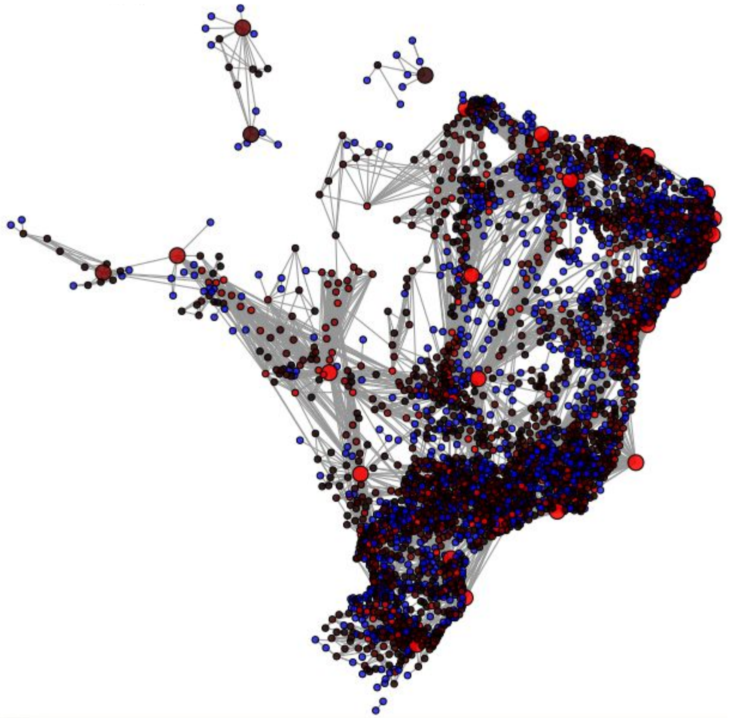
Complexity, a scientific theory which asserts that some systems display behavioral phenomena that are completely inexplicable by any conventional analysis of the systems' constituent parts. These phenomena, commonly referred to as emergent behaviour, seem to occur in many complex systems involving living organisms, such as a stock market or the human brain.

Source: John L. Casti, Encyclopædia Britannica

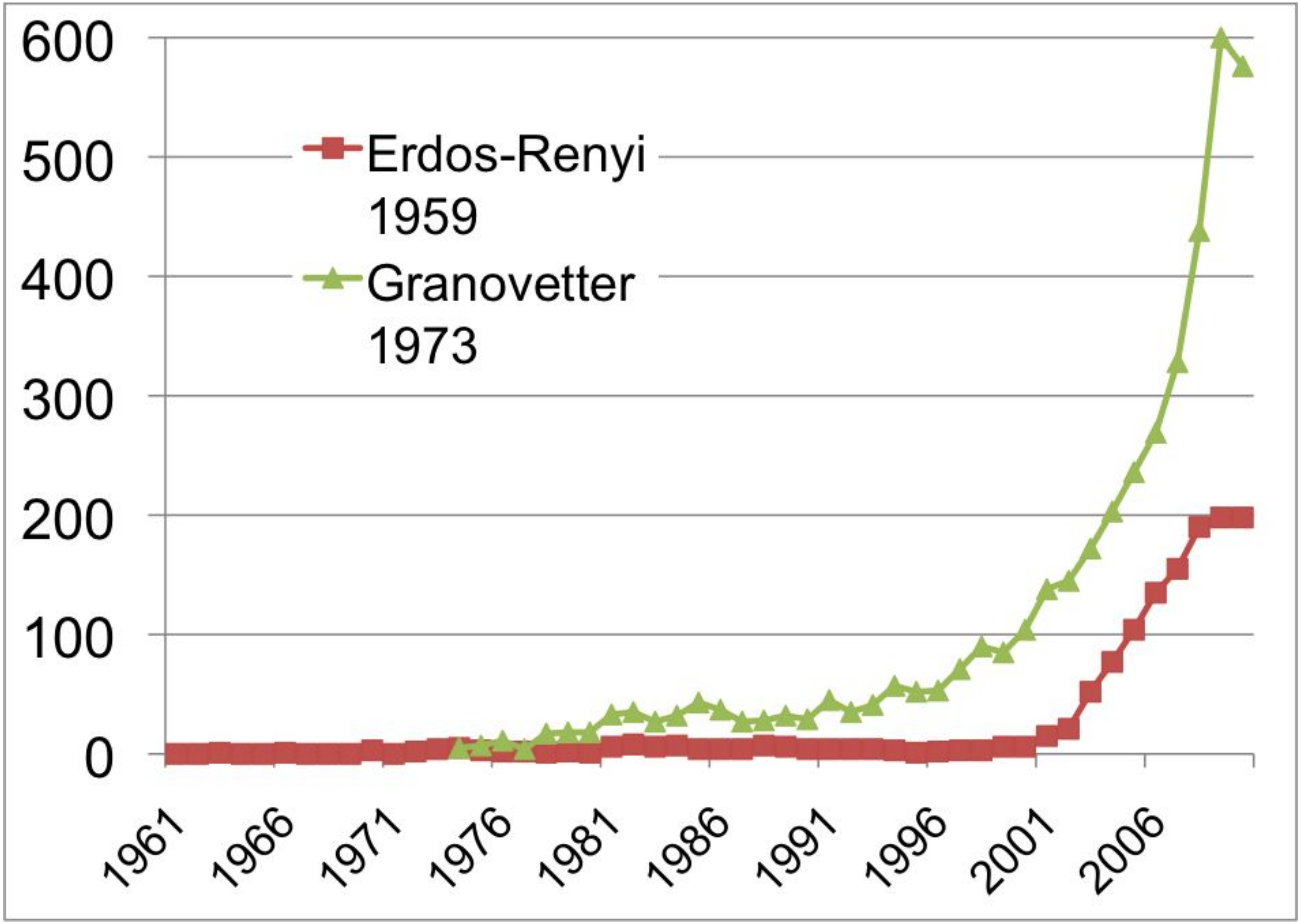
Introduction - Complex systems







Introduction



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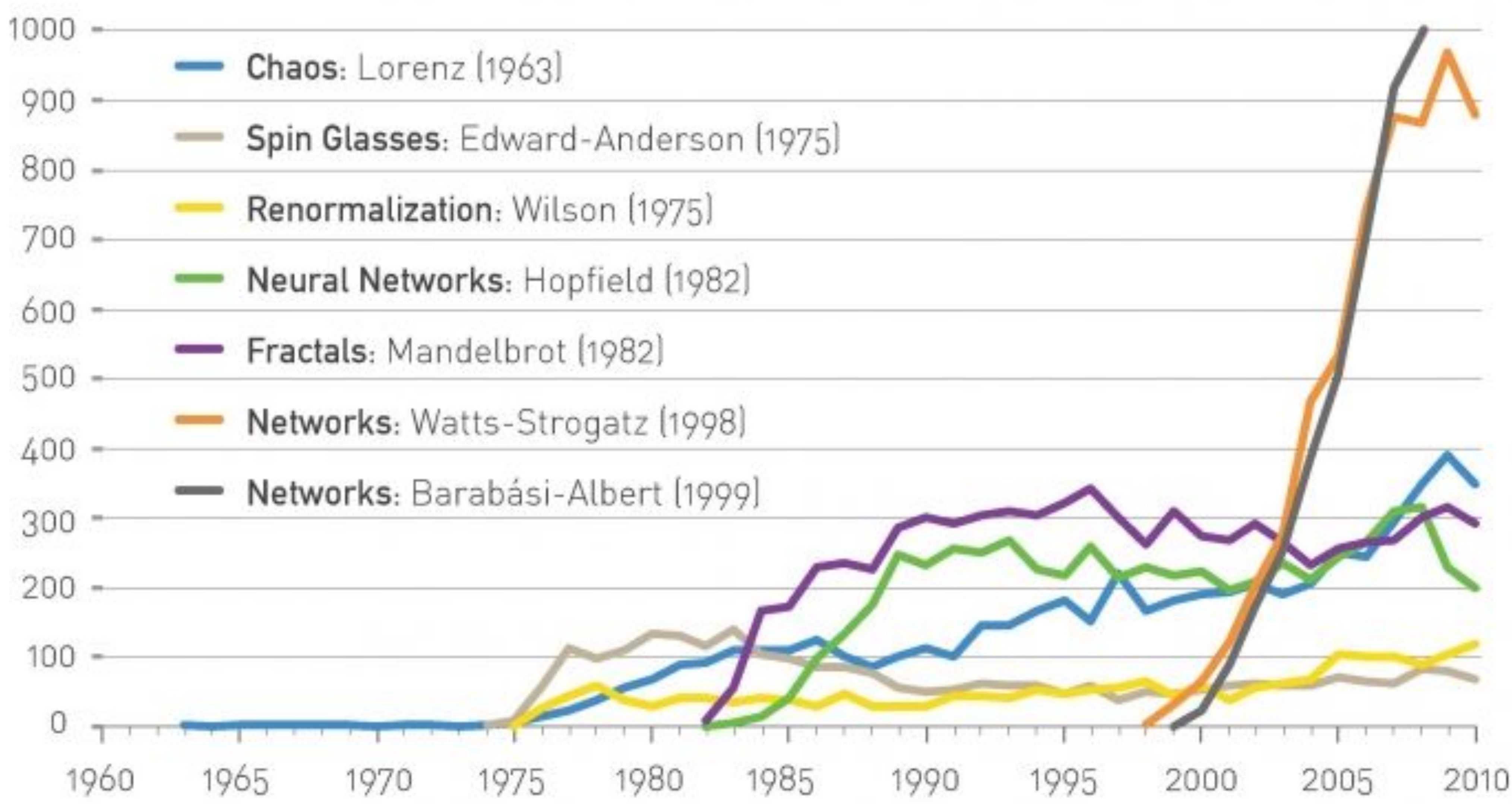
Introduction



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Introduction - Important people in the field

Albert Barabasi (1967)
Romanian Physicist

Réka Albert (1972)
Romanian-Hungarian Physicist

Paul Erdős (1913-1996)
Hungarian Mathematician

Alfred Rényi (1921-1970)
Hungarian Mathematician

Duncan Watts (1971)
American Physicist and Sociologist

Steven Strogatz (1959)
American Physicist

Wikipedia, 2020

Introduction

- The emergence of network maps:
  - Movie Actor Network, 1998;
  - World Wide Web, 1999.
  - C elegans neural wiring diagram 1990
  - Citation Network, 1998
  - Metabolic Network, 2000;
  - PPI network, 2001

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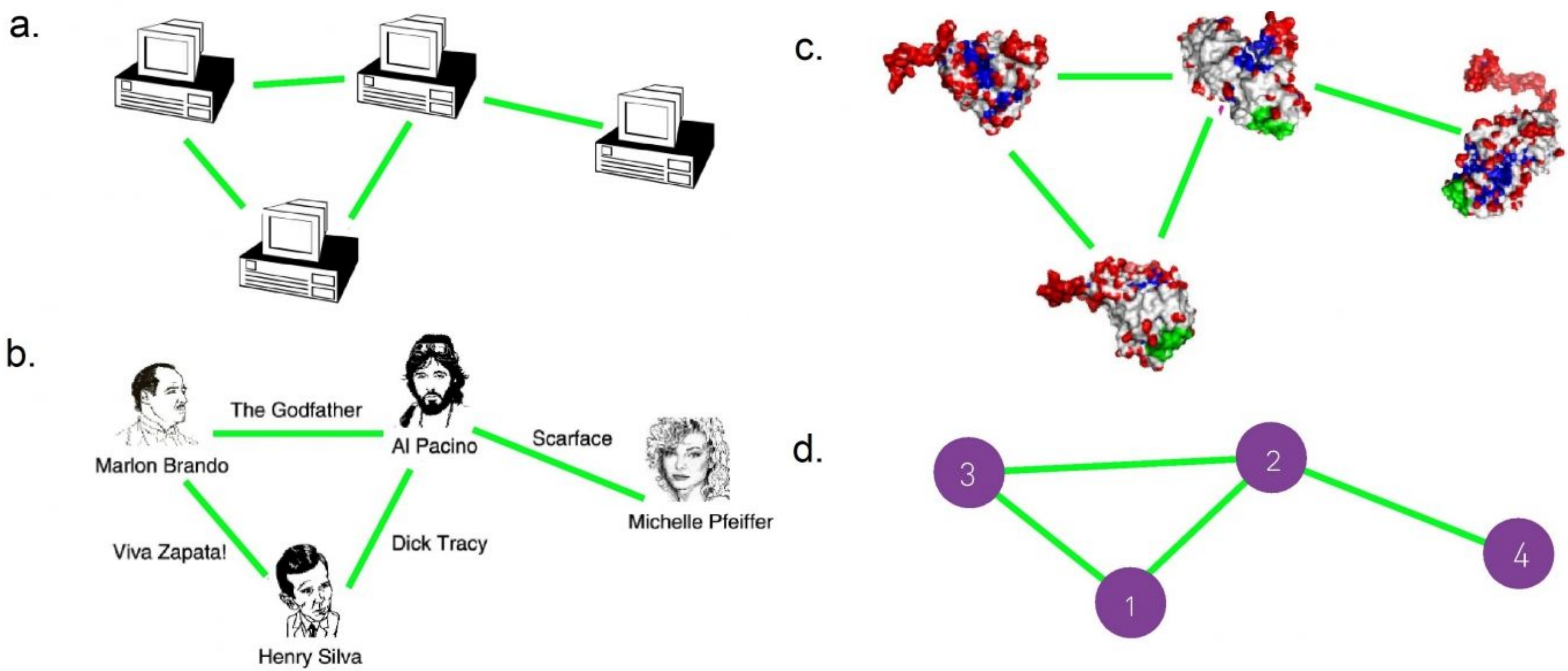
The universality of network characteristics:

The architecture of networks emerging in various domains of science, nature, and technology are more similar to each other than one would have expected.

Introduction - Different Networks, same Graph



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Introduction

Network	Nodes	Links	Directed / Undirected	N	L	‹K›
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.34
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile-Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorships	Undirected	23,133	93,437	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Papers	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

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Introduction

Science (2009):

Special Issue for the 10 year anniversary of Barabasi & Albert 1999 paper.

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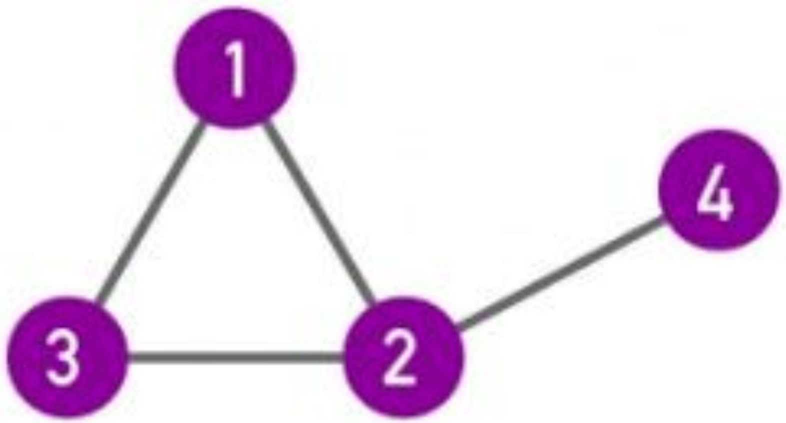
Introduction

2019

https://www.nature.com/immersive/d42859-019-00121-0/index.html?fbclid=IwAR0ien4-IHA29v0konDmB5gyGnGl-klf-b3cv6T94Nv98wH8dDEsQj5b6-k

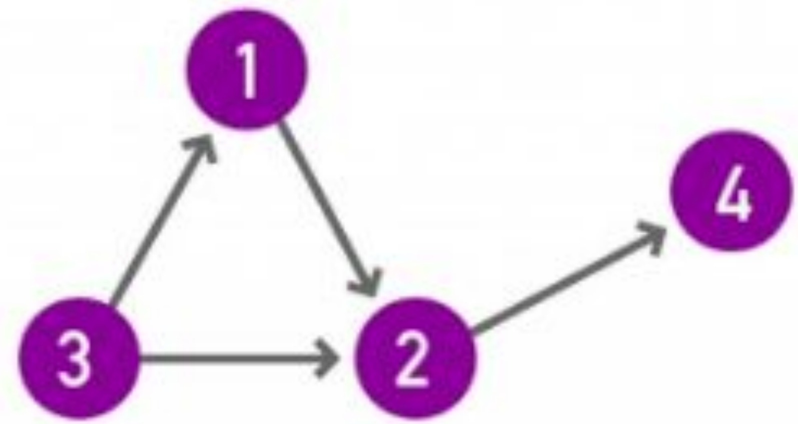
Introduction - Adjacency matrix

b. **Undirected network**



$$A_{ij} = \begin{matrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$$

c. **Directed network**



$$A_{ij} = \begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$$

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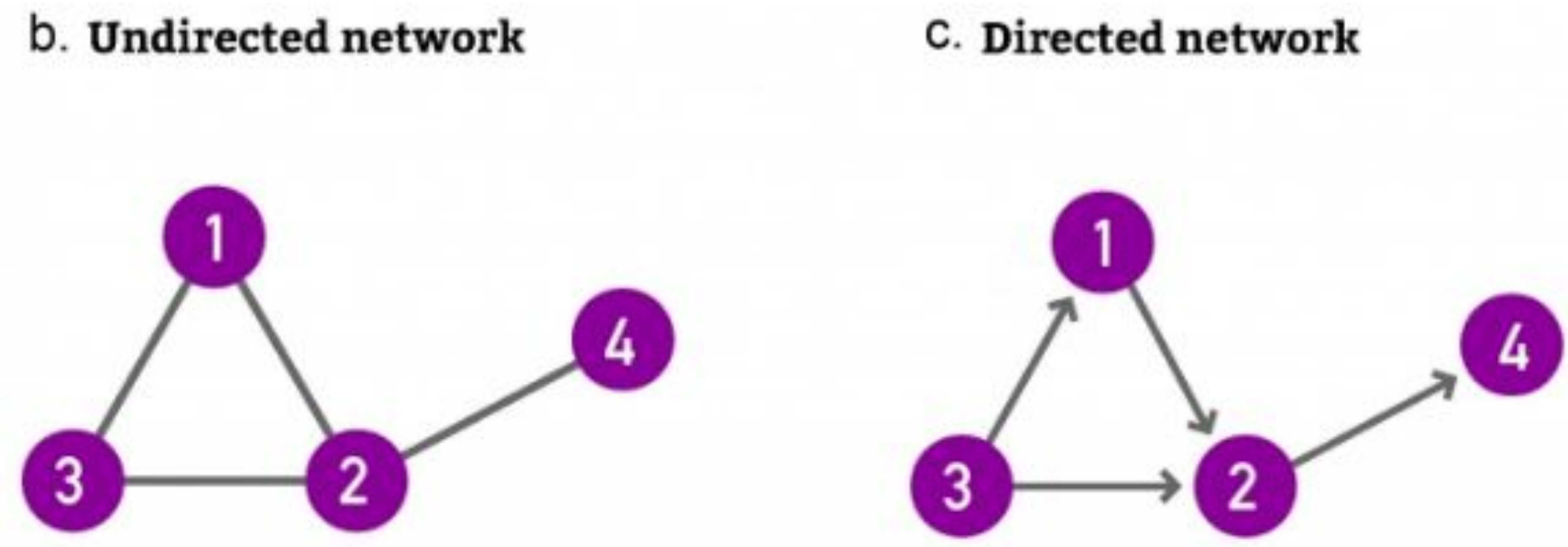
Introduction - Adjacency list



G = ((1,2), (1,3), (2,3), (2,4))

G = ((1,2), (2,4), (3,1), (3,2))

The adjacency matrix shows whether the network is undirected or not. On the other hand, the adjacency list does not.

However, the list needs **less** disk space to be stored.

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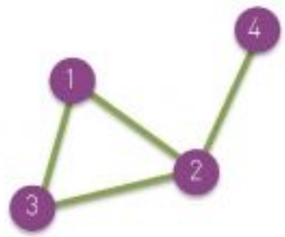
Introduction - Adjacency matrix

a. Undirected

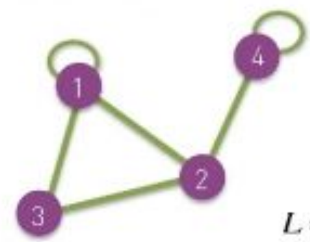


$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad \langle k \rangle = \frac{2L}{N}$$

b. Self-loops

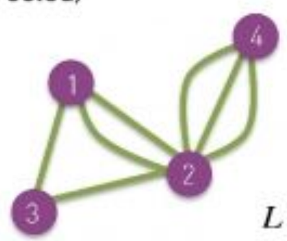


$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\exists i, A_{ii} \neq 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1, i\neq j}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii} \qquad ?$$

c. Multigraph
(undirected)

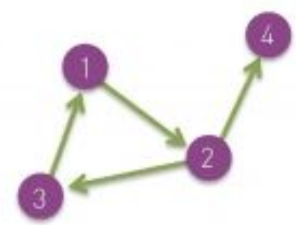


$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad \langle k \rangle = \frac{2L}{N}$$

d. Directed

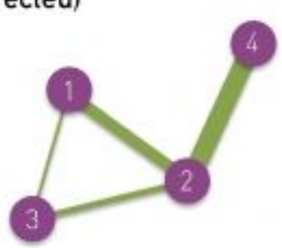


$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{ij} \neq A_{ji}$$

$$L = \sum_{i,j=1}^{N} A_{ij} \qquad \langle k \rangle = \frac{L}{N}$$

e. Weighted
(undirected)

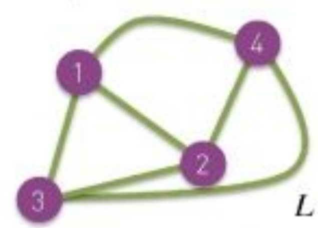


$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$\langle k \rangle = \frac{2L}{N}$$

f. Complete Graph
(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{i\neq j} = 1$$

$$L = L_{\max} = \frac{N(N-1)}{2} \qquad \langle k \rangle = N-1$$

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Introduction - Metrics

Degree *k*: Number of nodes a node is connected to.

A B C D E F G

Ex: [2, 4, 2, 4, 5, 3, 2]

Strength *s*: Accumulated weights from incident edges.

$$s_i = \sum_{j=1}^{N} F_{ij}$$

A B C D E F G

Ex: [12, 31, 13, 35, 44, 25, 20]

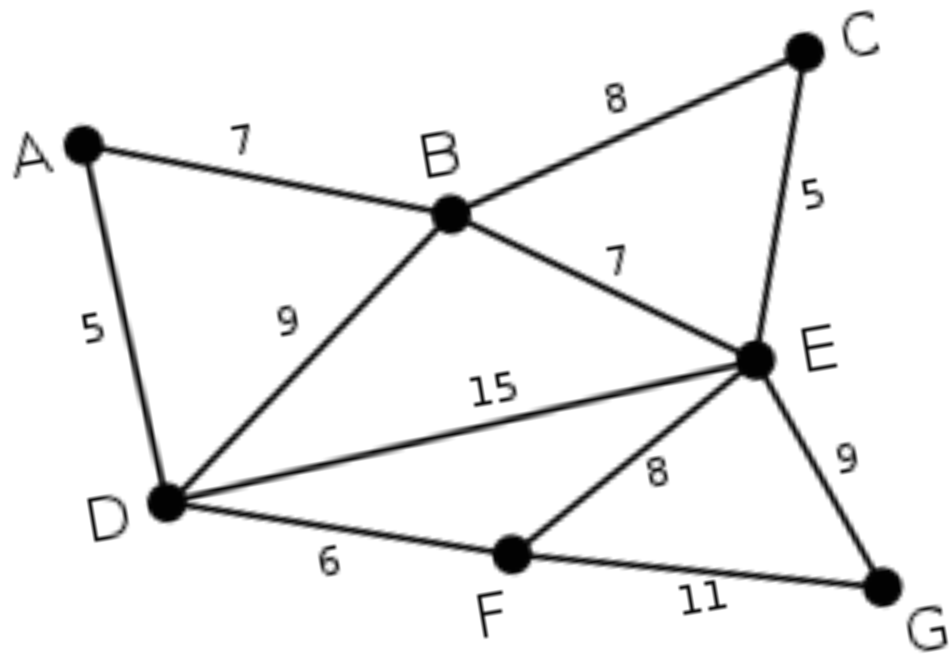
Betweenness *b*: Rate of shortest paths that pass through a node.

A B C D E F G

Ex: [0, 2.33, 0, 2.67, 5.17, 0.83, 0]

$$b_i = \sum_{v \neq w \neq i} \frac{\sigma_{vw}(i)}{\sigma_{vw}}$$

Diameter *D*: Length of the largest shortest path. Ex: [3]



Introduction

The empirical study of networks:

- Technological networks
    - The internet
    - The telephone network
    - Power grids
    - Transportation networks
    - Delivery and distribution networks
- Networks of information
    - The World Wide Web (WWW)
    - Citation networks
    - Peer-to-peer networks
    - Recommender networks
    - Keyword indexes
- Social Networks
    - Affiliation networks
    - Twitter (X), Instagram, Facebook
    - Co-workers
    - Sex
    - Friends
- Biological networks
    - Biochemical networks: metabolic, protein-protein
    - Brain networks
    - Ecological networks
    - Food web

(NEWMAN, 2018)

Introduction

Typical questions:

- What are the most influential nodes?
- What are the shortest paths between nodes?
- Is the network assortative? (i.e. most connected nodes are friends with other influential nodes, and the less connected nodes have links with other of the same kind - degree correlation)
- What is the degree distribution? (Poisson, Power Law, ...)
- Is the network bipartite?
- Is the network connected?
- Are your friends also friends with each other? (Clustering Coefficient, triadic closure)
- Which network model resembles a certain network? (Erdős-Rényi, Small-world, Scale-free)

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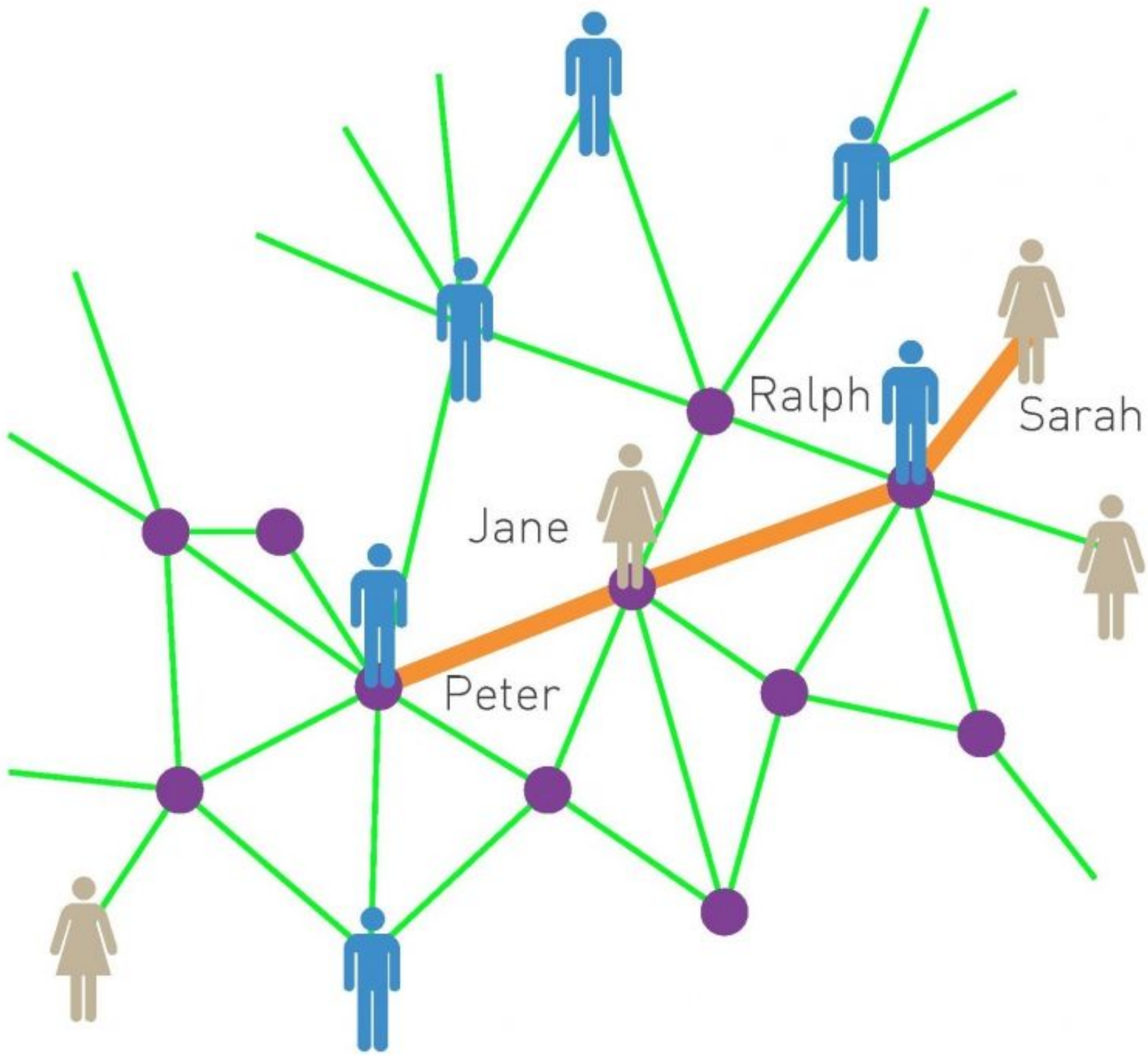
Network topology and models

Network topology - Small-world

Six Degree of Separation



(MILGRAM, 1967)

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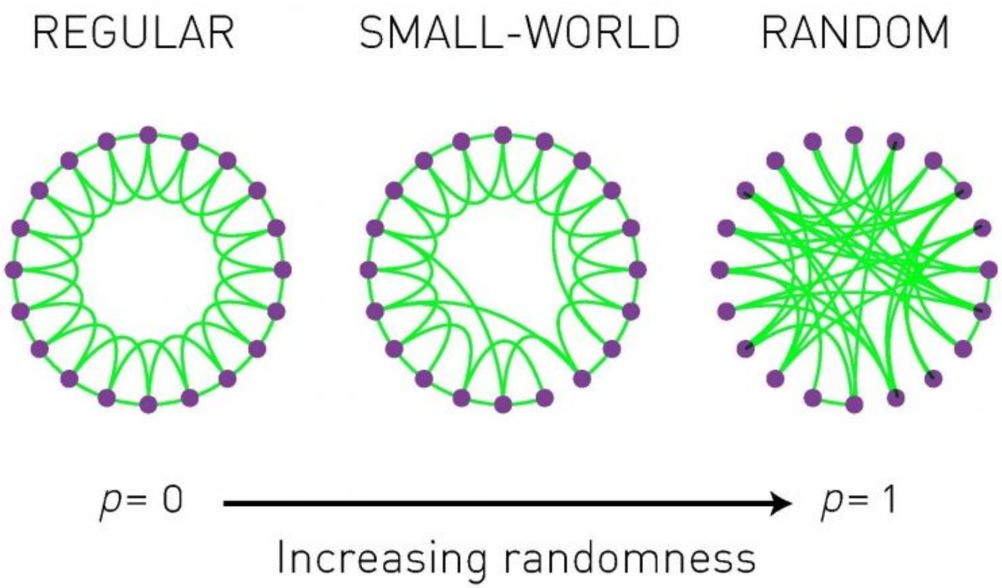
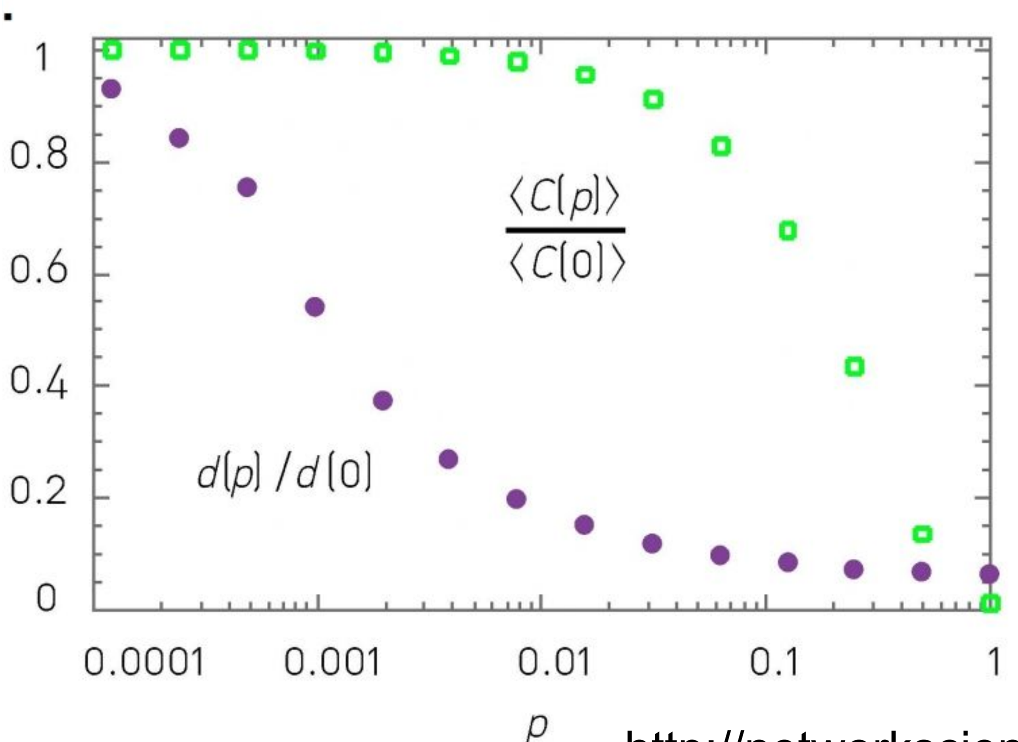
Network models - Small-world





(WATTS and STROGATZ, 1998)

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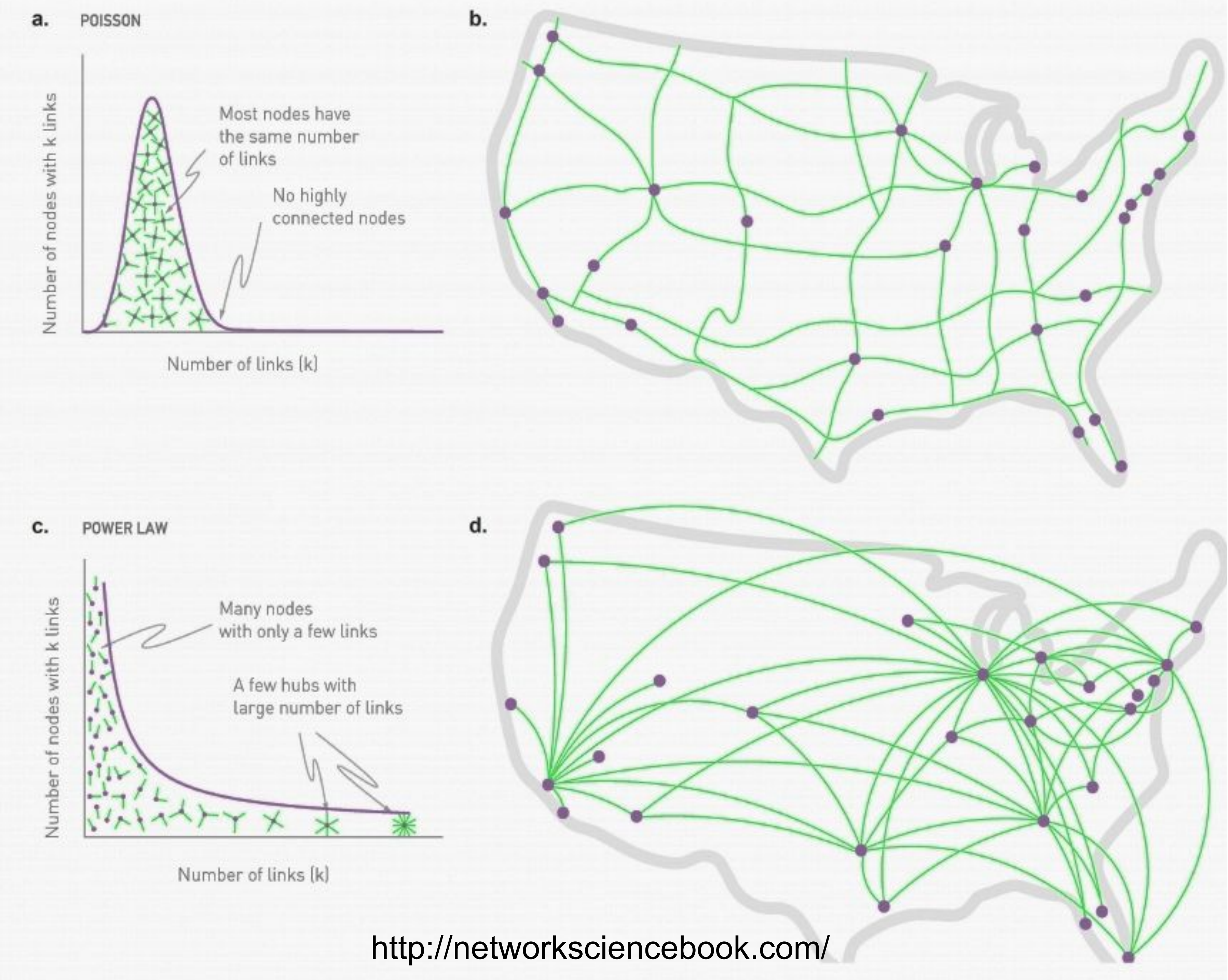
Network topology - Scale-free



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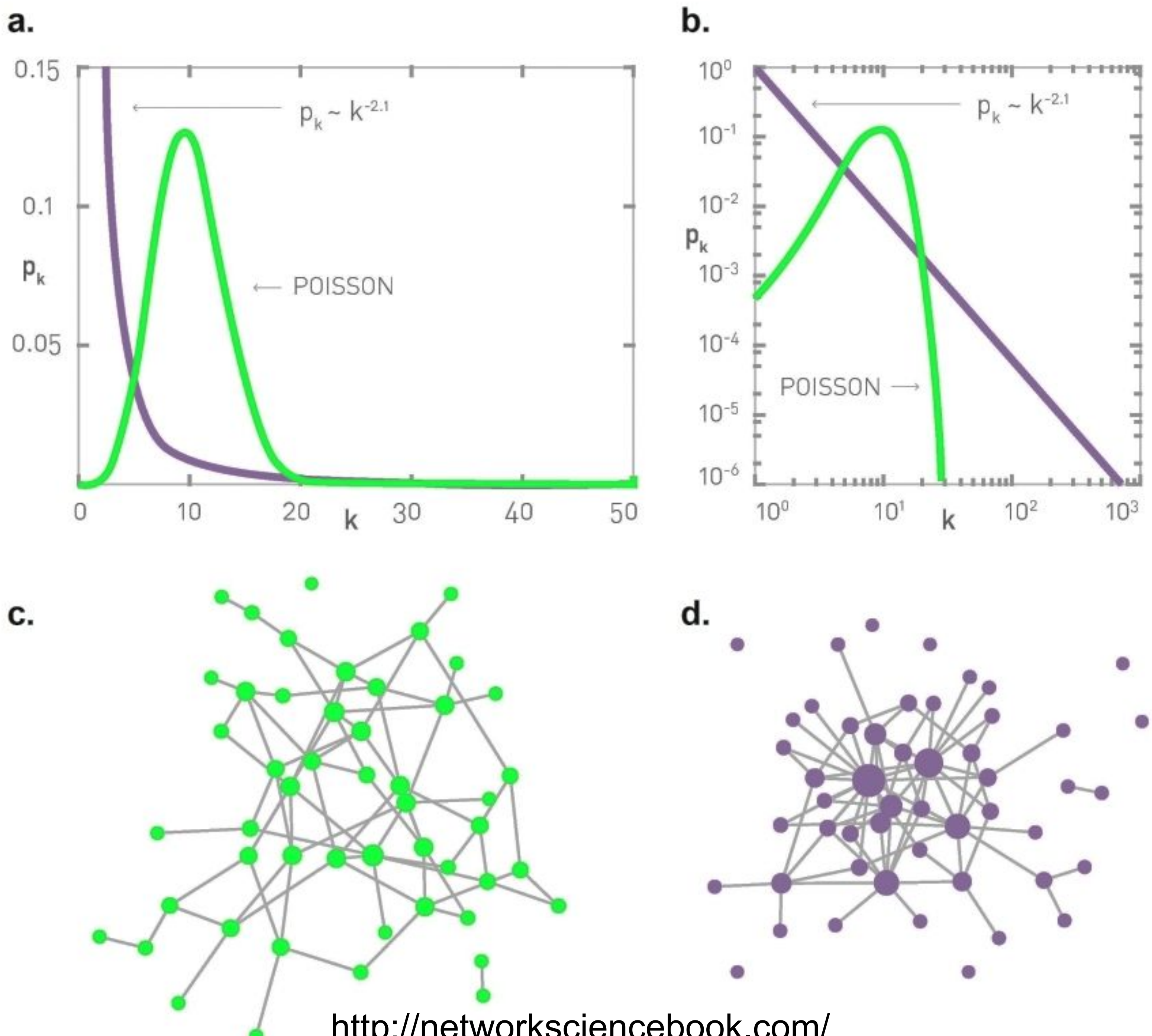
Network topology - Scale-free



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Network models - Scale-free

Barabási-Albert Model

- Growth: at each time step we add a new node with *m* links that connect to *m* nodes added earlier.
- Preferential Attachment (rich-gets-richer phenomenon)

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

Video: http://networksciencebook.com/images/ch-05/video-5-2.webm

http://networksciencebook.com/

Network models - Scale-free

Bianconi-Barabási Model

- Growth: at each time step we add a new node with *m* links and fitness η_j, where η_j is a random number chosen from a fitness distribution $\rho(\eta)$.The fitness of a node does not change.
- Preferential Attachment considers fitness as well

$$\Pi(k_i) = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$$

Video: http://networksciencebook.com/images/ch-06/video-6-1.webm

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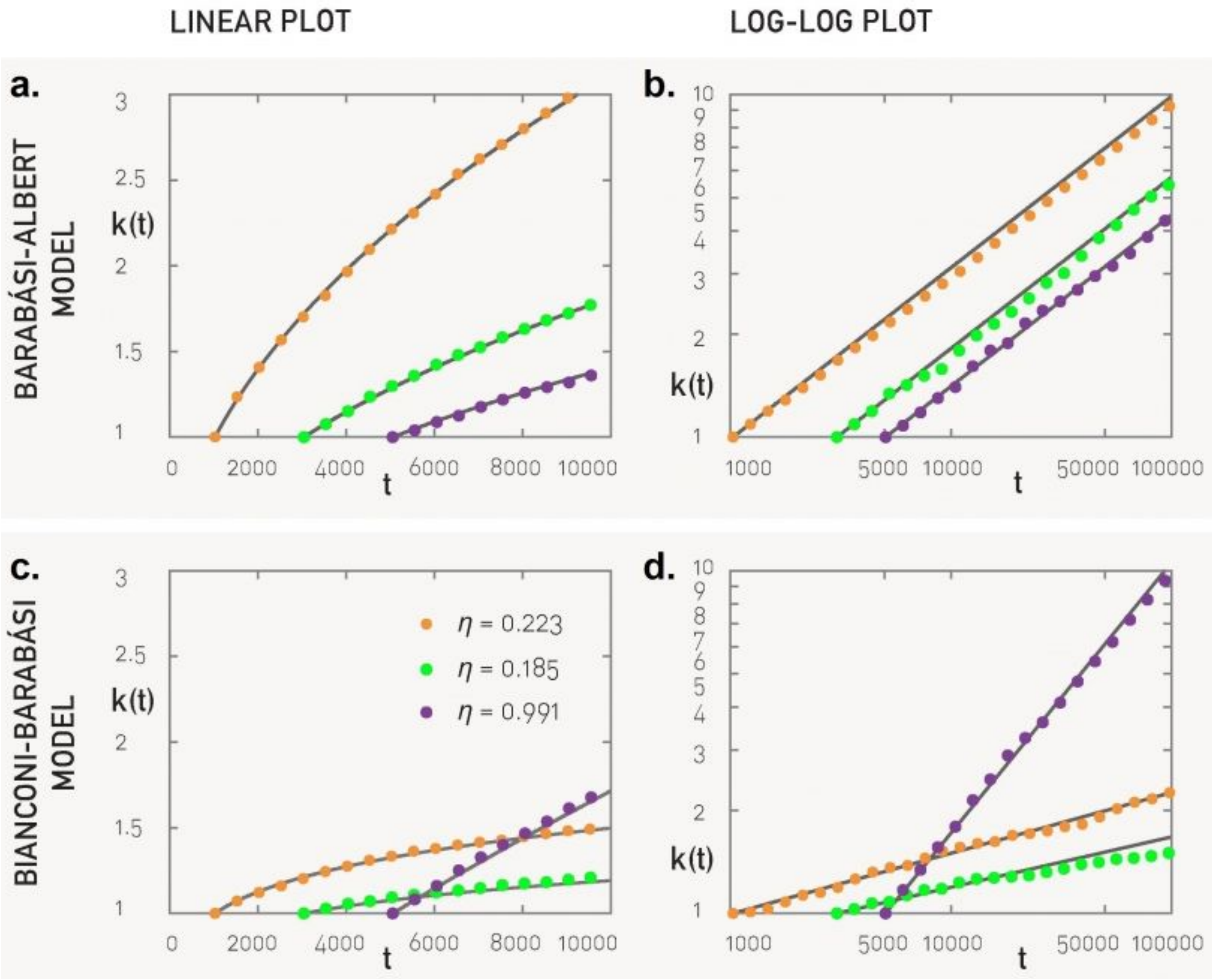
Network models - Scale-free



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Network models - Summary

Model Class	Examples	Characteristics
Static Models	Erdos–Rényi Watts-Strogatz	• N fixed • pk exponentially bounded • Static, time independent topologies
Generative Models	Configuration Model Hidden Parameter Model	• Arbitrary pre-defined p_k • Static, time independent topologies
Evolving Network Models	Barabási–Albert Model Bianconi-Barabási Model Initial Attractiveness Model Internal Links Model Node Deletion Model Accelerated Growth Model Aging Model	• p_k is determined by the processes that contribute to the network's evolution. • Time-varying network topologies

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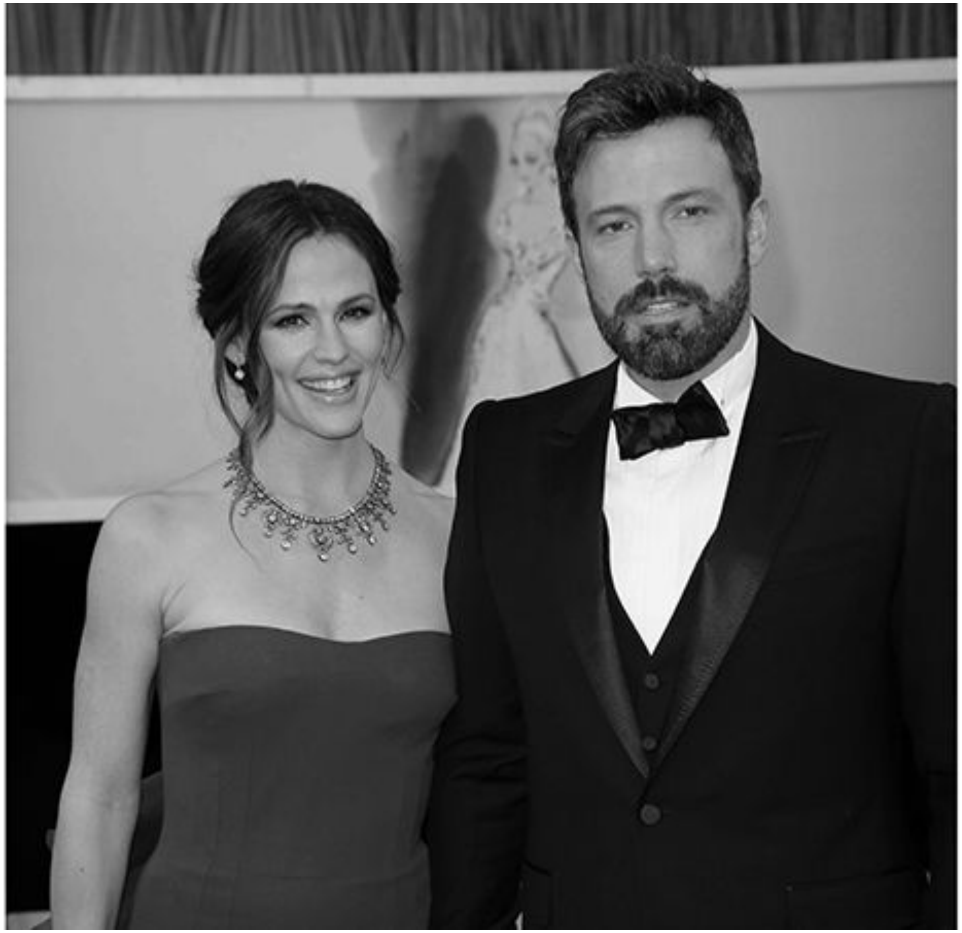
Assortativity and homophily

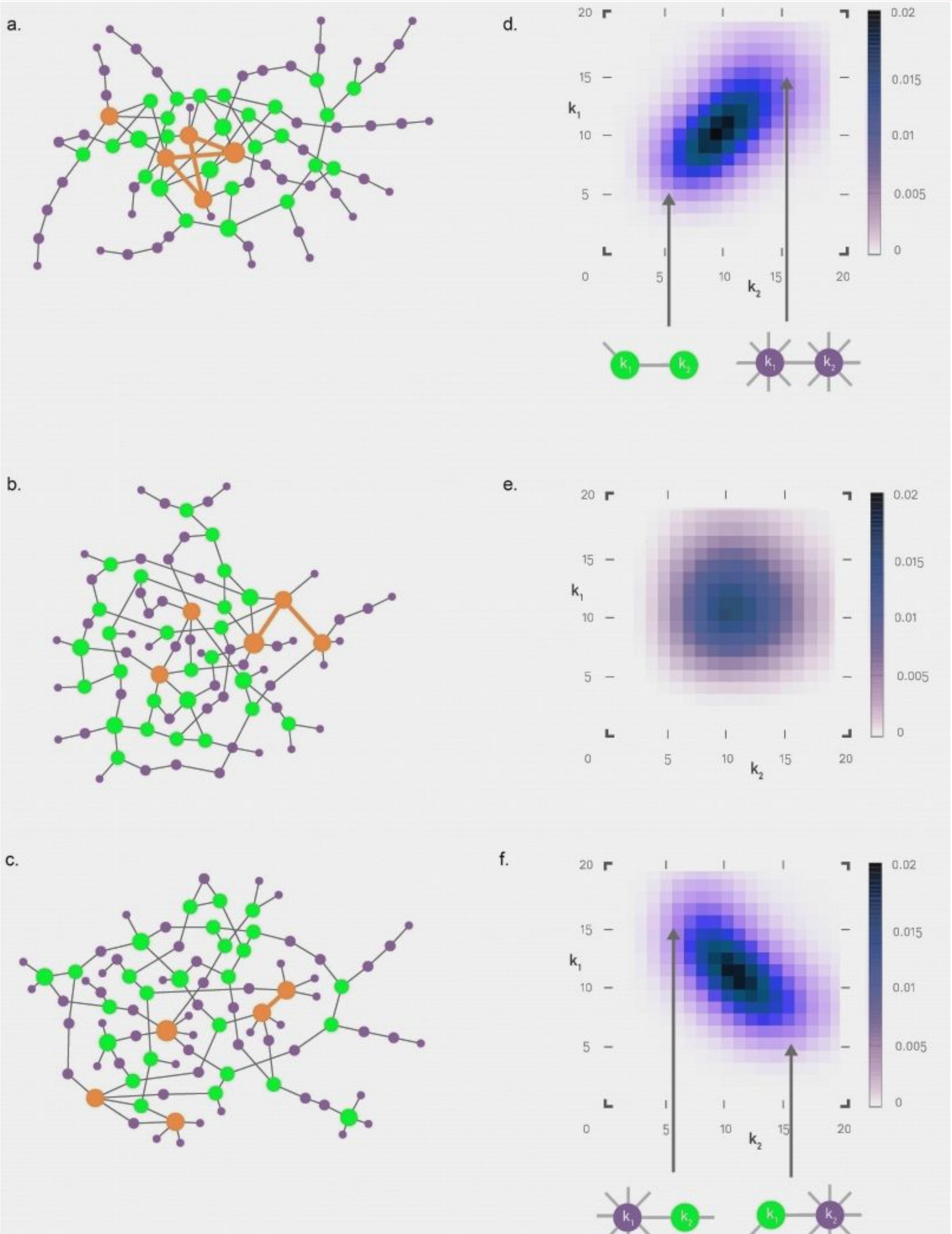
Degree correlations

What is the true chance that a celebrity marries another celebrity?



http://www.whosdatedwho.com
http://networksciencebook.com/

Degree correlations



Assortative Network

Neutral Network

Disassortative Network

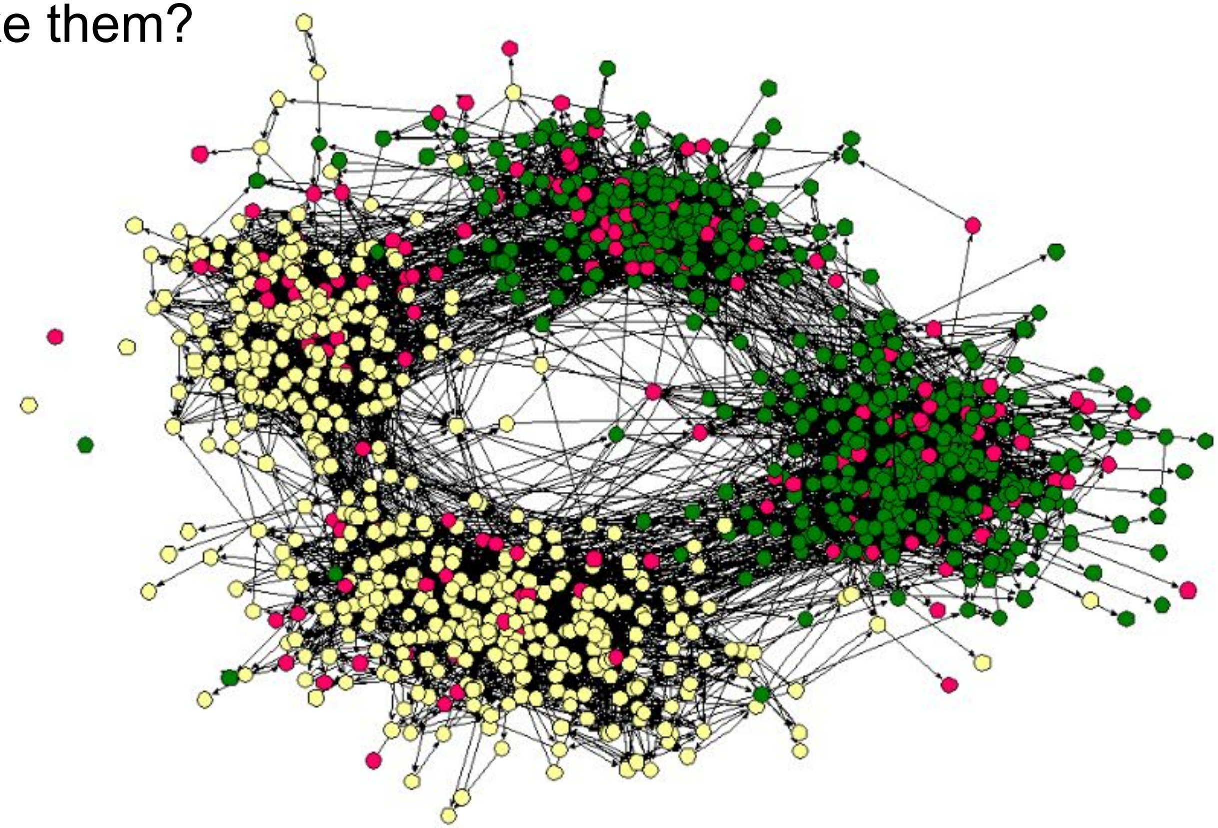
http://networksciencebook.com/

Homophily

Based on ideas such as:

- "Similarity begets friendship" (Plato)
- People "love those who are like themselves" (Aristotle)
- "birds of a feather flock together" (probably William Turner, 1545)

Selection and social influence: Have the people in the network adapted their behaviors to become more like their friends, or have they sought out people who were already like them?



Two divisions in the network are apparent: one based on race (with students of different races drawn as differently colored circles), and the other based on friendships in the middle and high schools respectively (EASLEY and KLEINBERG, 2010)

Homophily

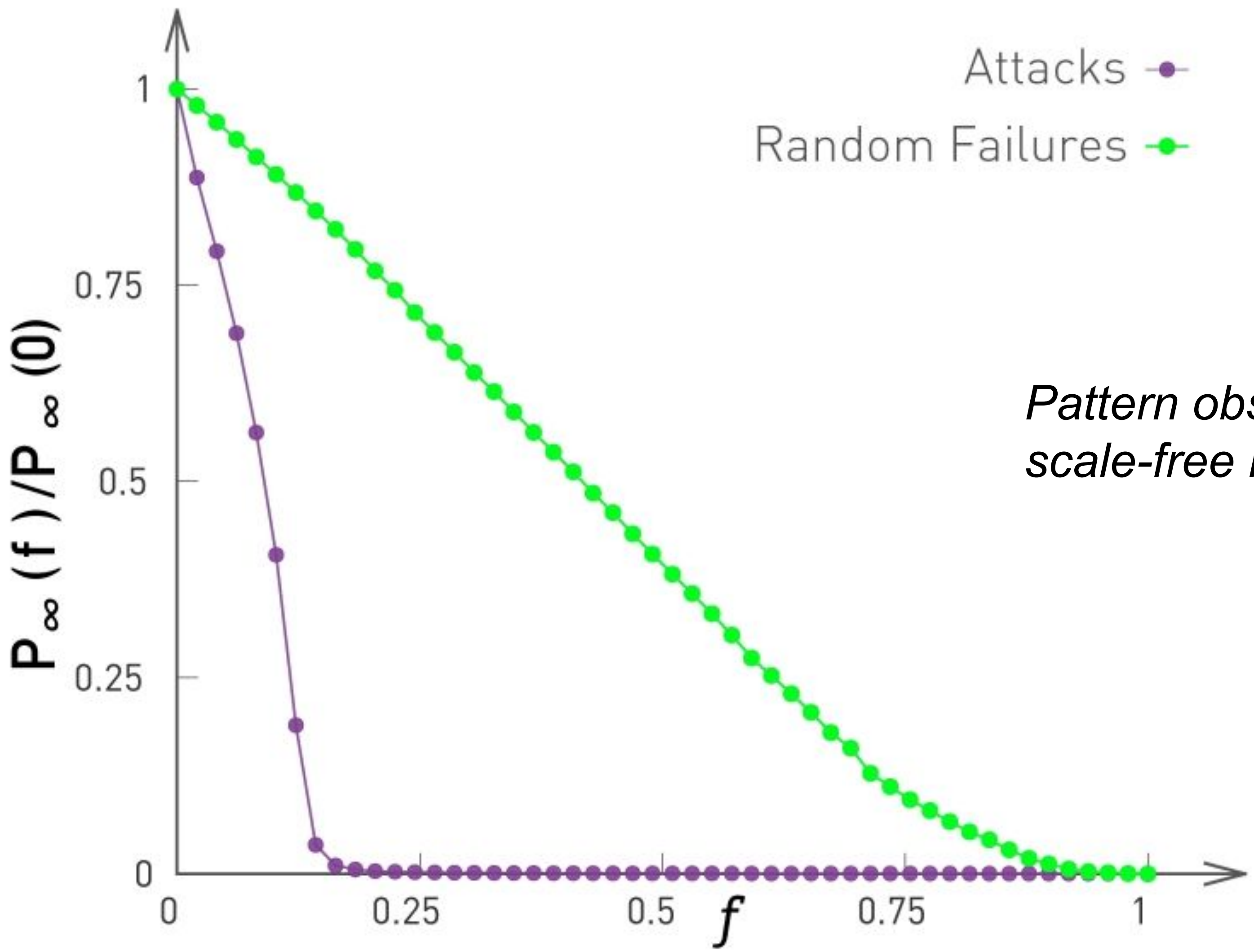
TED Talk by Prof. Nicholas Christakis: https://www.ted.com/talks/nicholas_christakis_the_hidden_influence_of_social_networks

Robustness

Robustness

How may one efficiently fragment a network?

Video: http://networksciencebook.com/images/ch-08/video-8-2.webm



Pattern observed in scale-free networks

Fonte: http://networksciencebook.com/chapter/8#robustness

Robustness

Network	Random Failures (Real Network)	Random Failures (Randomized Network)	Attack (Real Network)
Internet	0.92	0.84	0.16
WWW	0.88	0.85	0.12
Power Grid	0.61	0.63	0.20
Mobile Phone Calls	0.78	0.68	0.20
Email	0.92	0.69	0.04
Science Collaboration	0.92	0.88	0.27
Actor Network	0.98	0.99	0.55
Citation Network	0.96	0.95	0.76
E. Coli Metabolism	0.96	0.90	0.49
Protein Interactions	0.88	0.66	0.06

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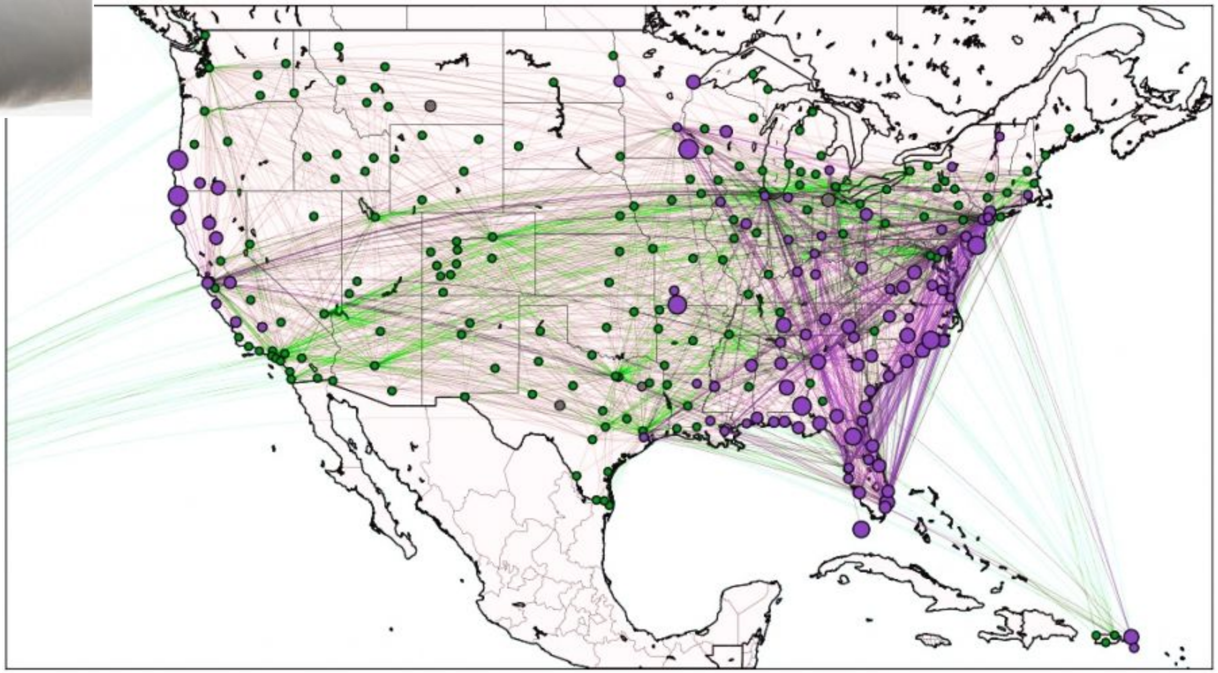
Robustness

Cascading Failures



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Robustness

Failure Propagation Model

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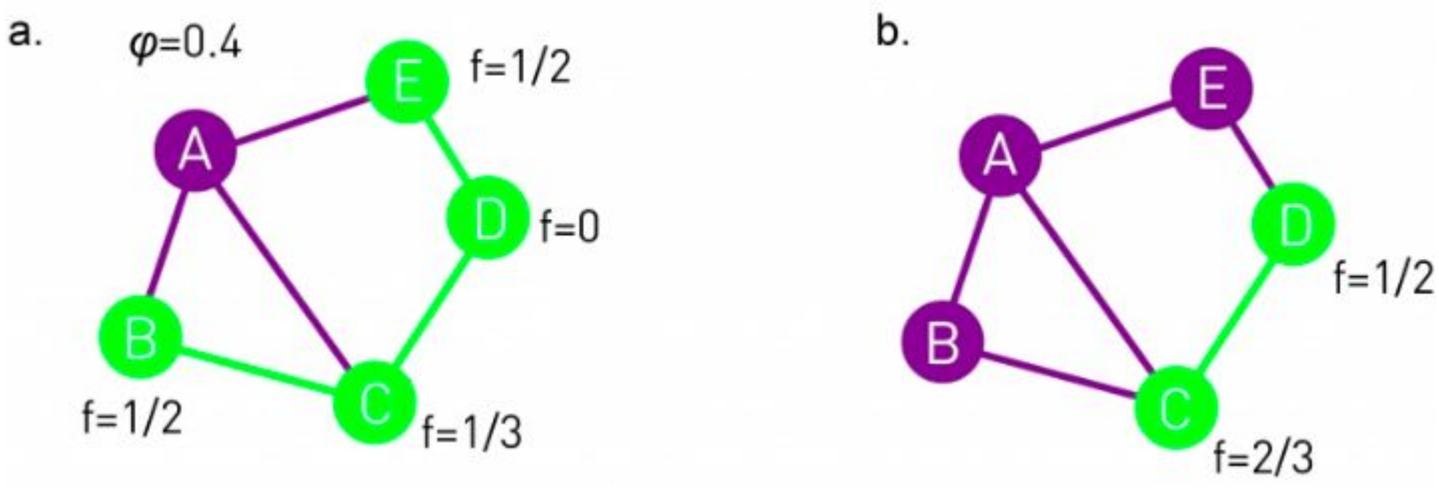
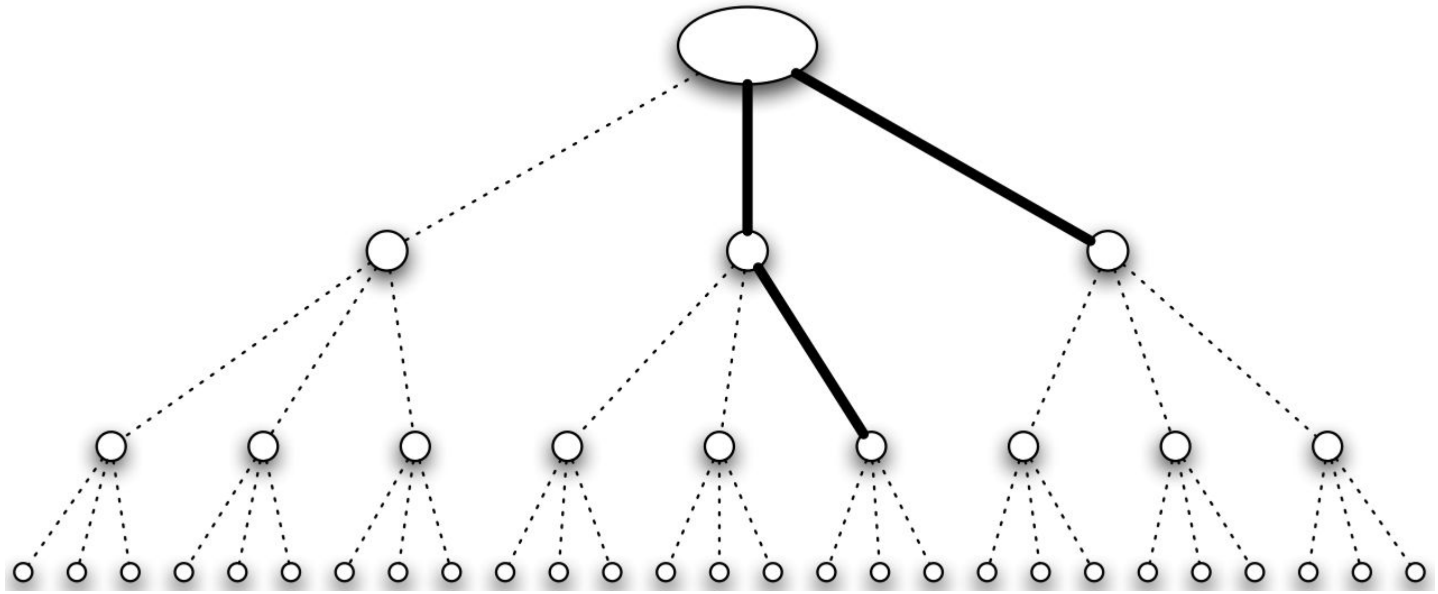


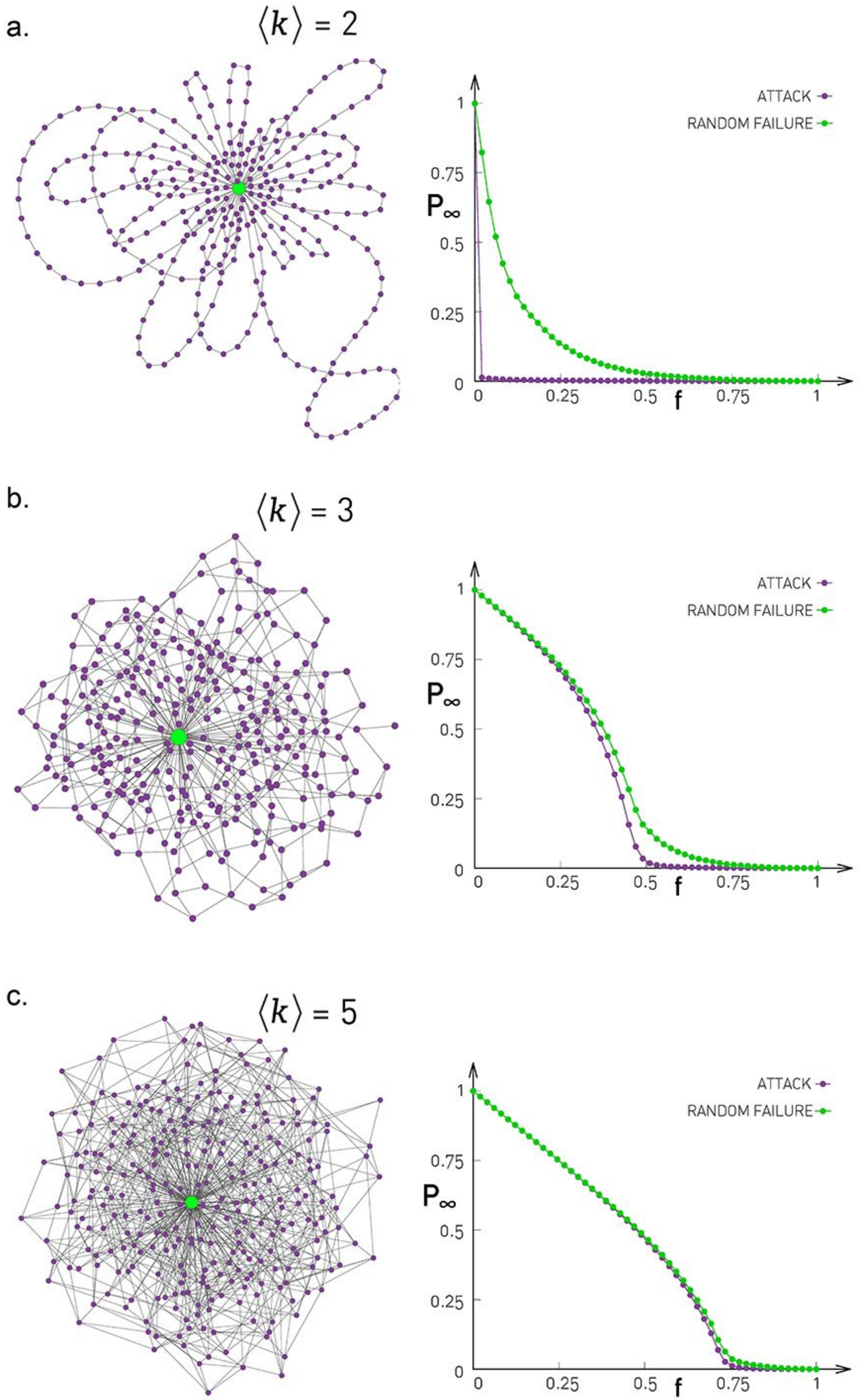
Branching Model



(EASLEY and KLEINBERG, 2010)

Robustness

Building robustness



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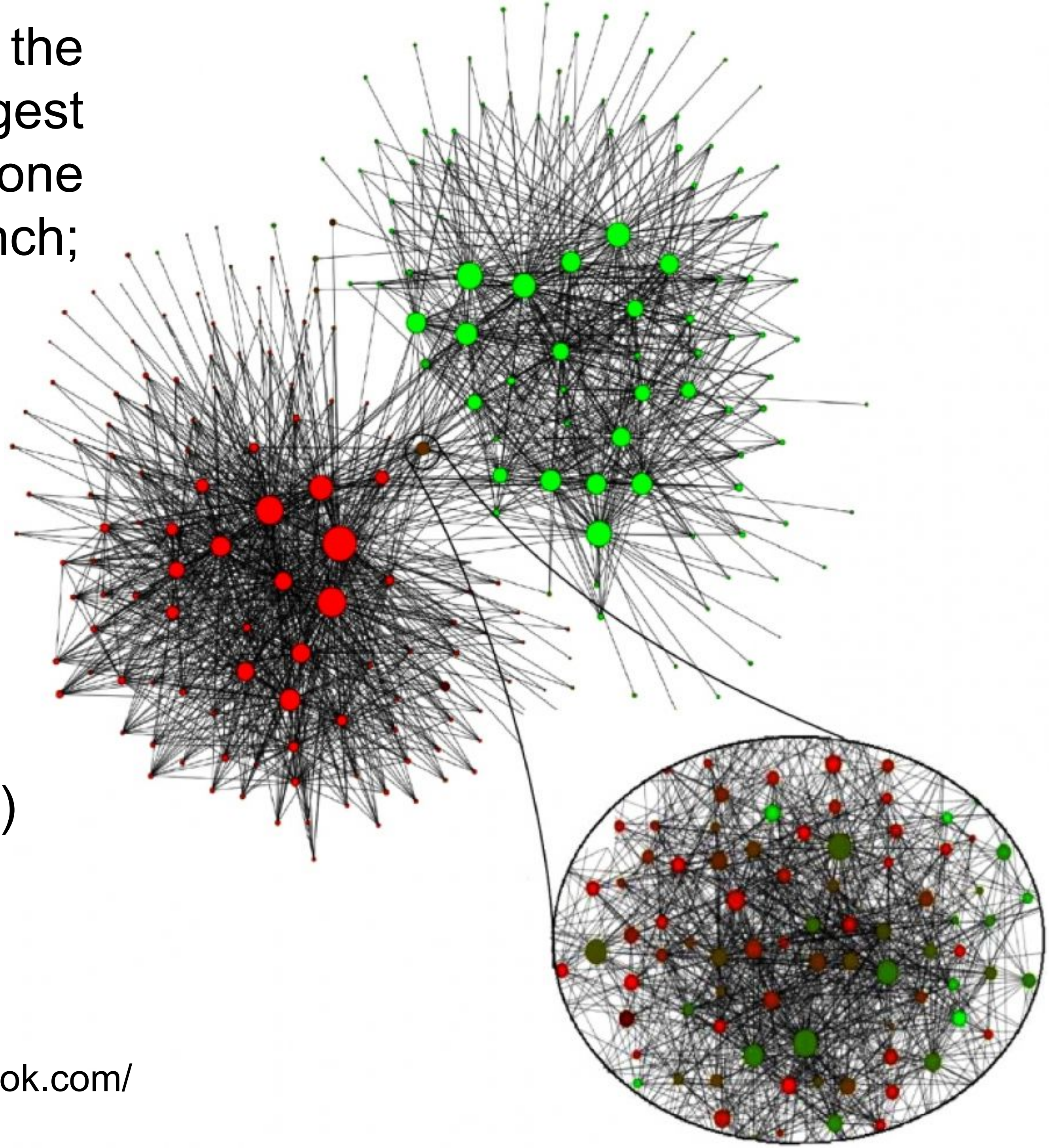
Communities

Communities

Call pattern of the consumers of the largest Belgian mobile phone company. Red: french; Green: Dutch.



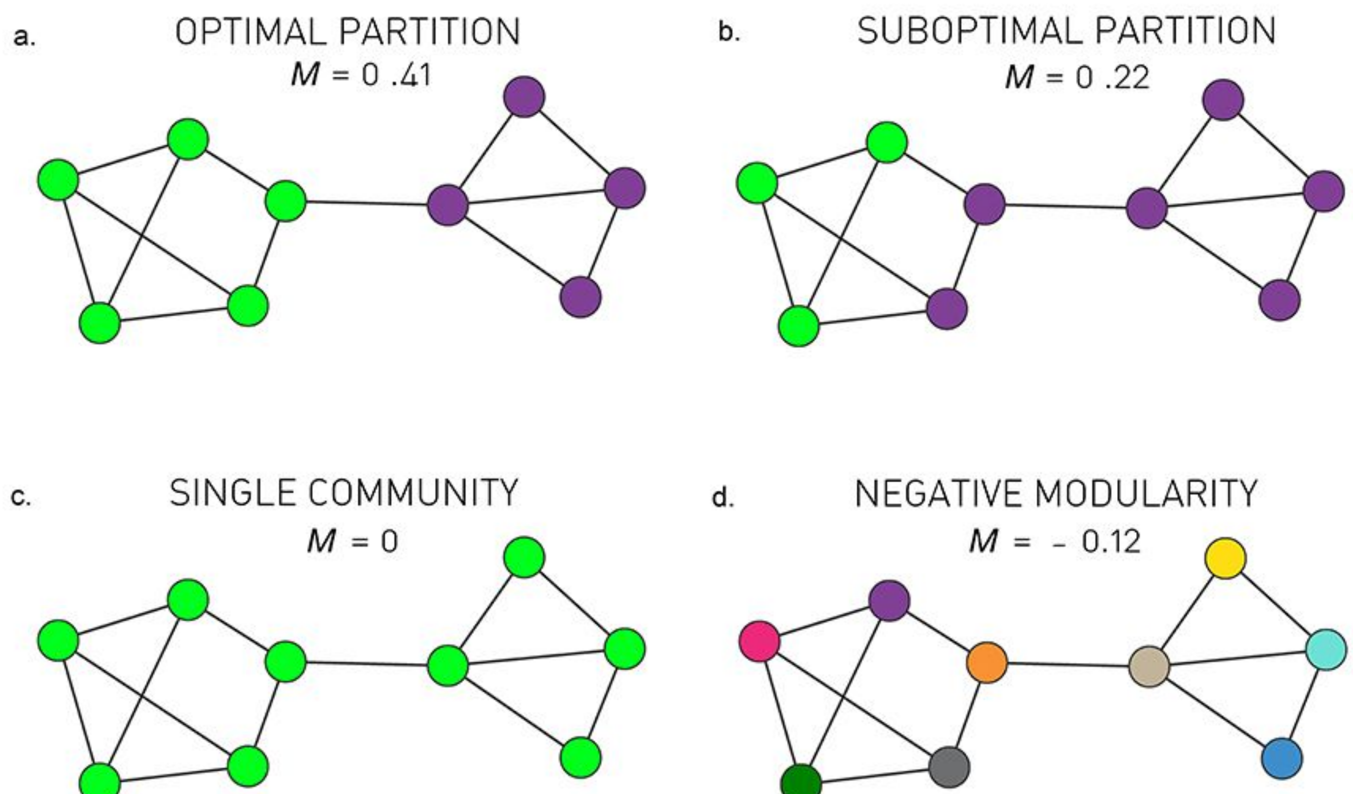
(FORTUNATO, 2010)

http://networksciencebook.com/

Communities

- Graph partitioning vs Community detection
- Agglomerative Procedures: the Ravasz Algorithm
- Divisive Procedures: the Girvan-Newman Algorithm

- Modularity: When the number of links between pairs of nodes is higher than the "expected", the modularity increases, showing possible true communities.



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Communities

Name	Nature	Comp.
Ravasz	Hierarchical Agglomerative	$O(N^2)$
Girvan-Newman	Hierarchical Divisive	$O(N^2)$
Greedy Modularity	Modularity Optimization	$O(N^2)$
Greedy Modularity (Optimized)	Modularity Optimization	$O(N\log^2 N)$
Louvain	Modularity Optimization	$O(L)$
Infomap	Flow Optimization	$O(N\log N)$
Clique Percolation (CFinder)	Overlapping Communities	$Exp(N)$
Link Clustering	Hierarchical Agglomerative; Overlapping Communities	$O(N^2)$

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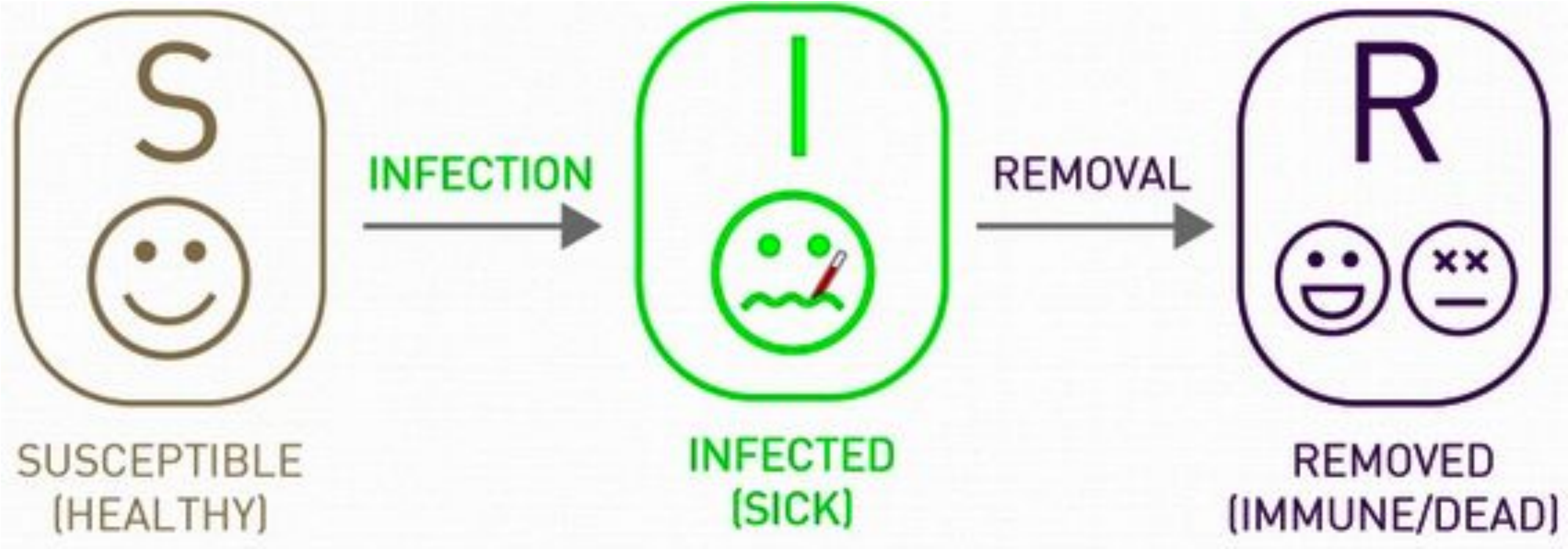
Spreading Phenomena

Spreading Phenomena

Phenomena	Agent	Network
Venereal Disease	Pathogens	Sexual Network
Rumor Spreading	Information, Memes	Communication Network
Diffusion of Innovations	Ideas, Knowledge	Communication Network
Computer Viruses	Malwares, Digital viruses	Internet
Mobile Phone Virus	Mobile Viruses	Social Network/Proximity Network
Bedbugs	Parasitic Insects	Hotel - Traveler Network
Malaria	Plasmodium	Mosquito - Human network

http://networksciencebook.com/

Spreading Phenomena

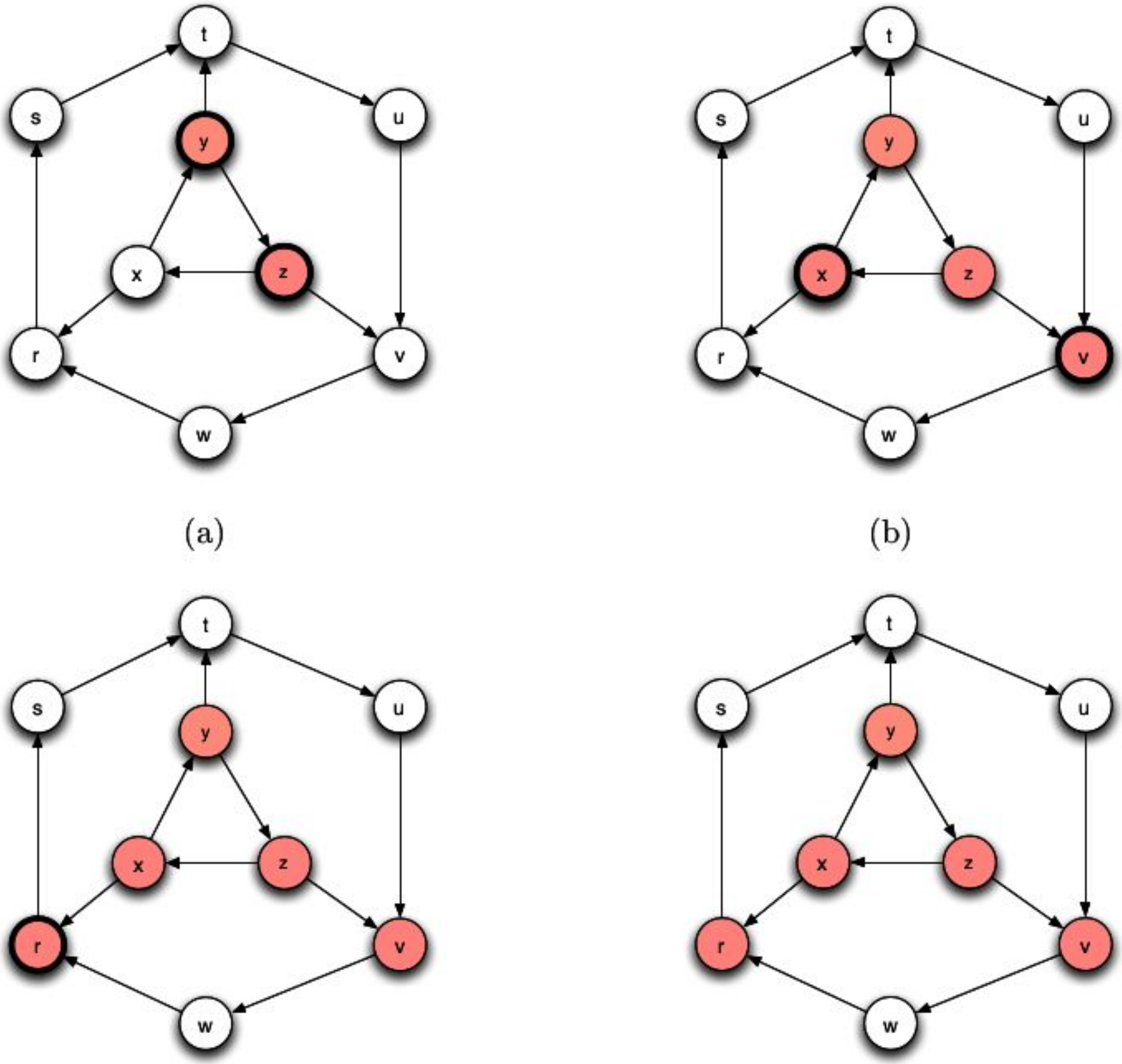


http://networksciencebook.com/

Spreading Phenomena

Simulation:
y and z initially infected

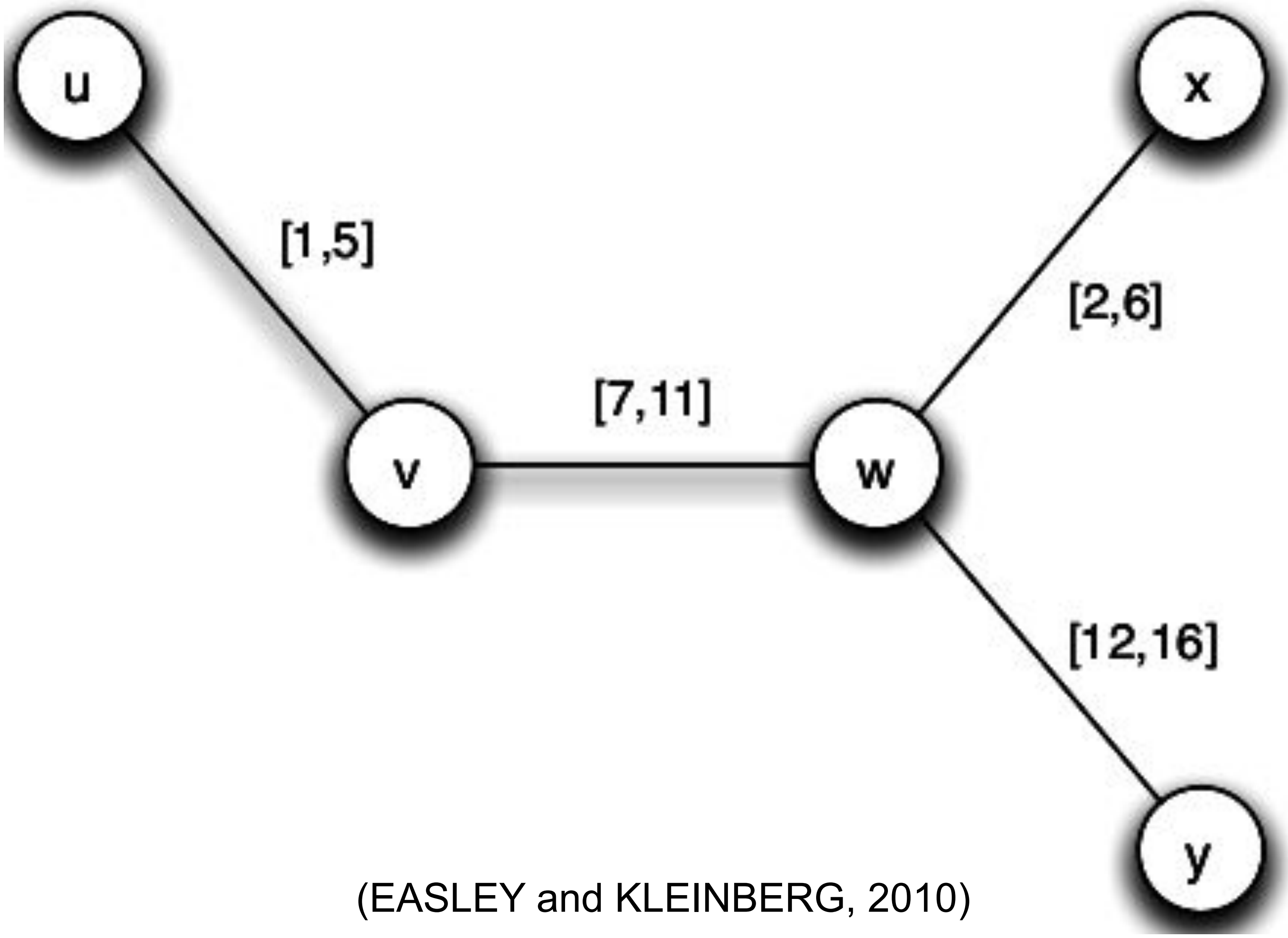
(EASLEY and KLEINBERG, 2010)



(a) (b) (c) (d)

Spreading Phenomena

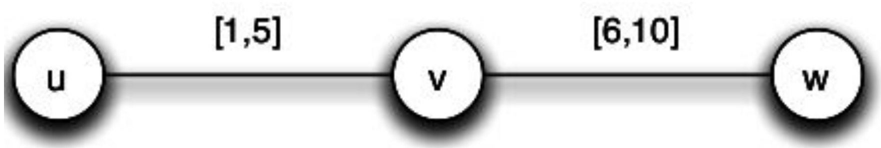
Transient contact networks



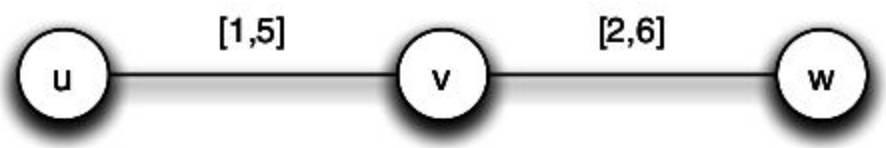
(EASLEY and KLEINBERG, 2010)

Spreading Phenomena

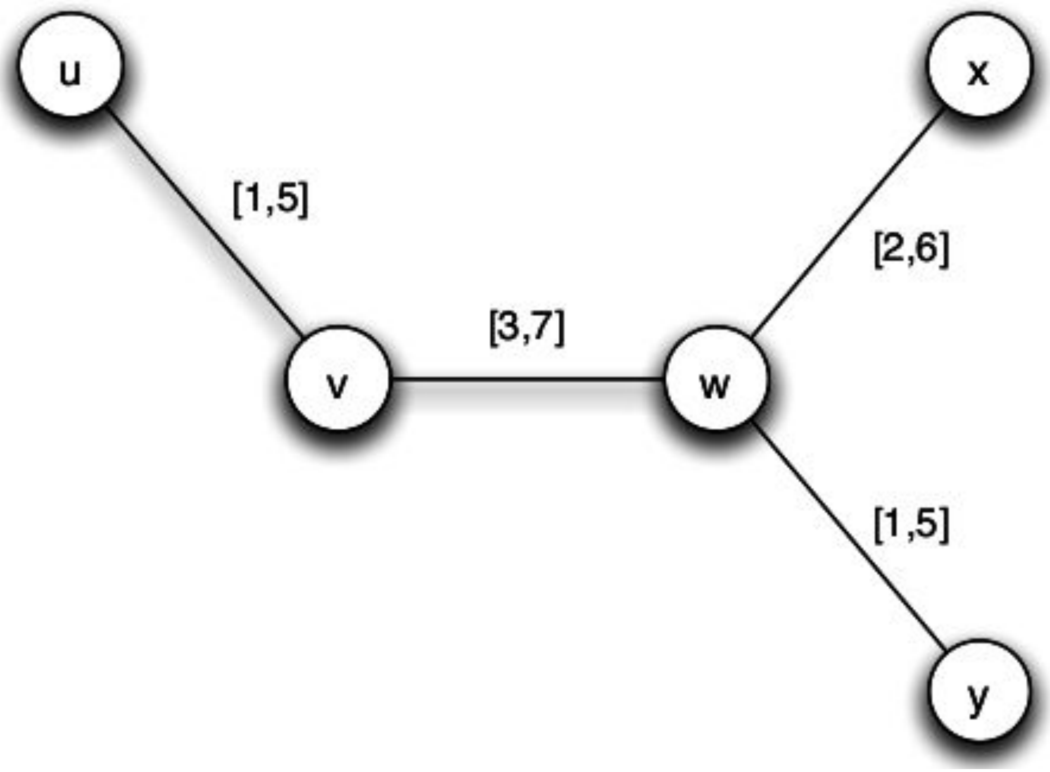
Concurrency:



(a) *v's two partnerships happen serially*



(b) *v's two partnership's happen concurrently*

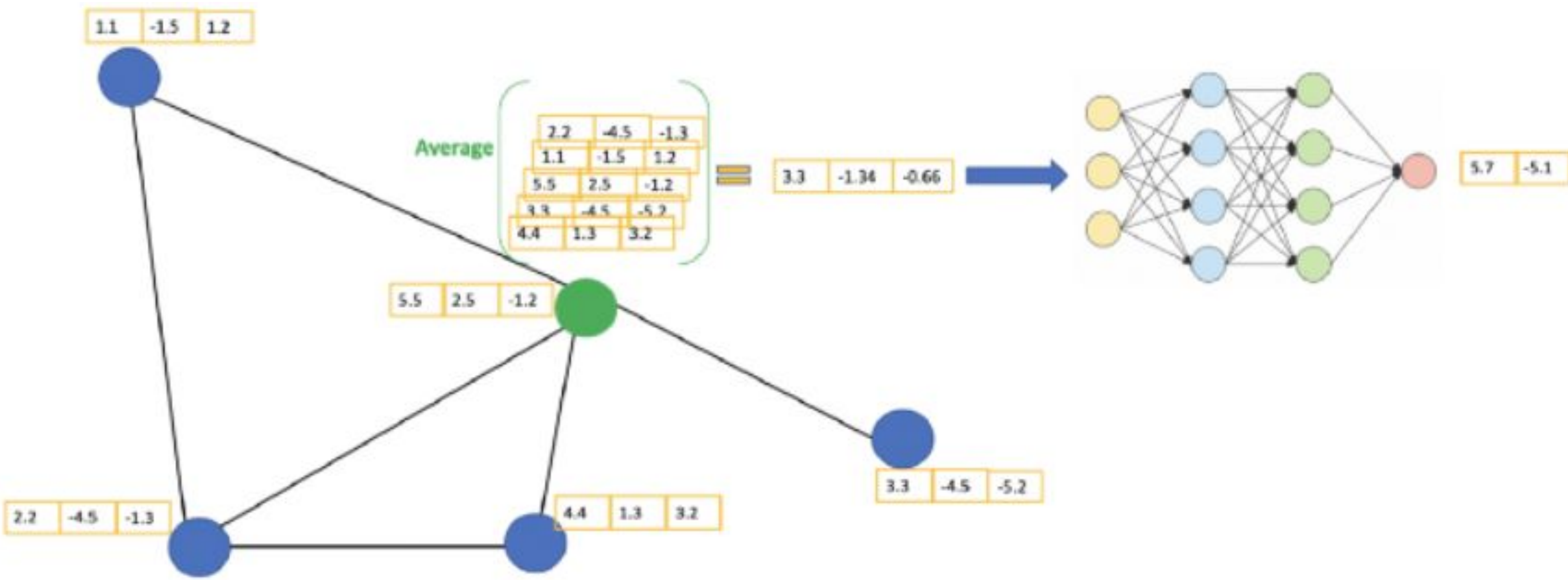


Small concurrency addition implies big propagation addition.

(EASLEY and KLEINBERG, 2010)

GNN

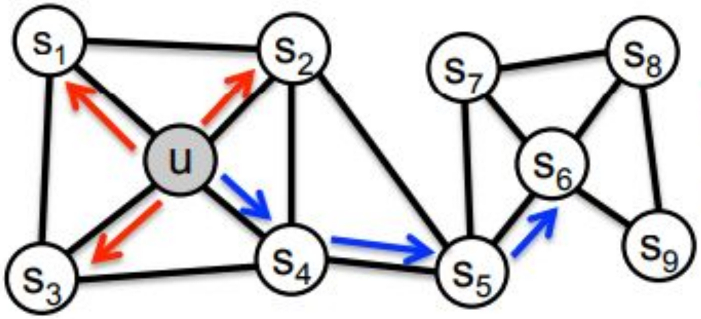
- Basics of Graph Neural Networks: https://www.graphneuralnets.com/p/basics-of-gnns/?src=yt%29
- GCN: https://www.topbots.com/graph-convolutional-networks/

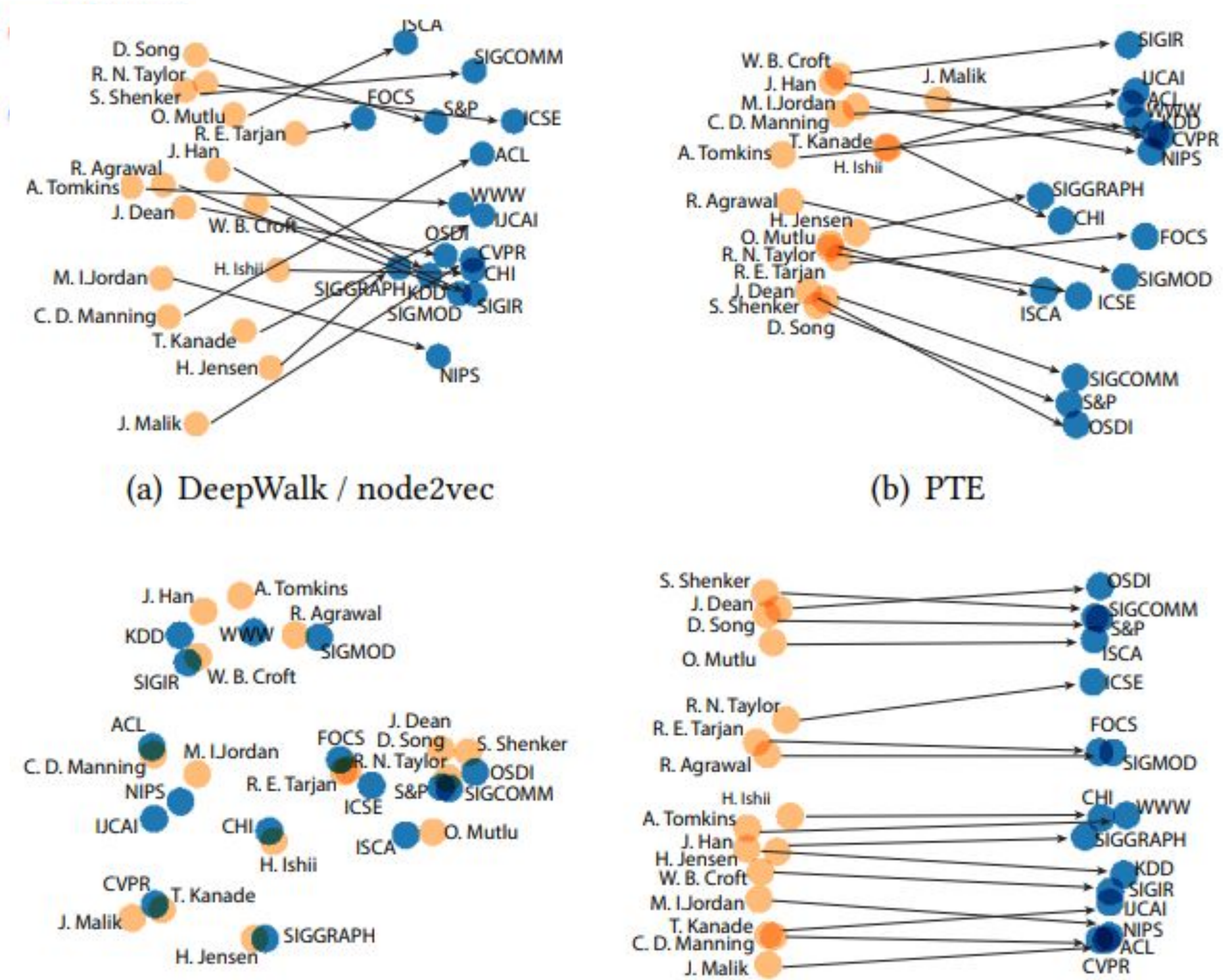


Network Science + Machine Learning

Embeddings:

- node2vec (https://dl.acm.org/doi/10.1145/2939672.2939754)
- metapath2vec (https://dl.acm.org/doi/10.1145/3097983.3098036)





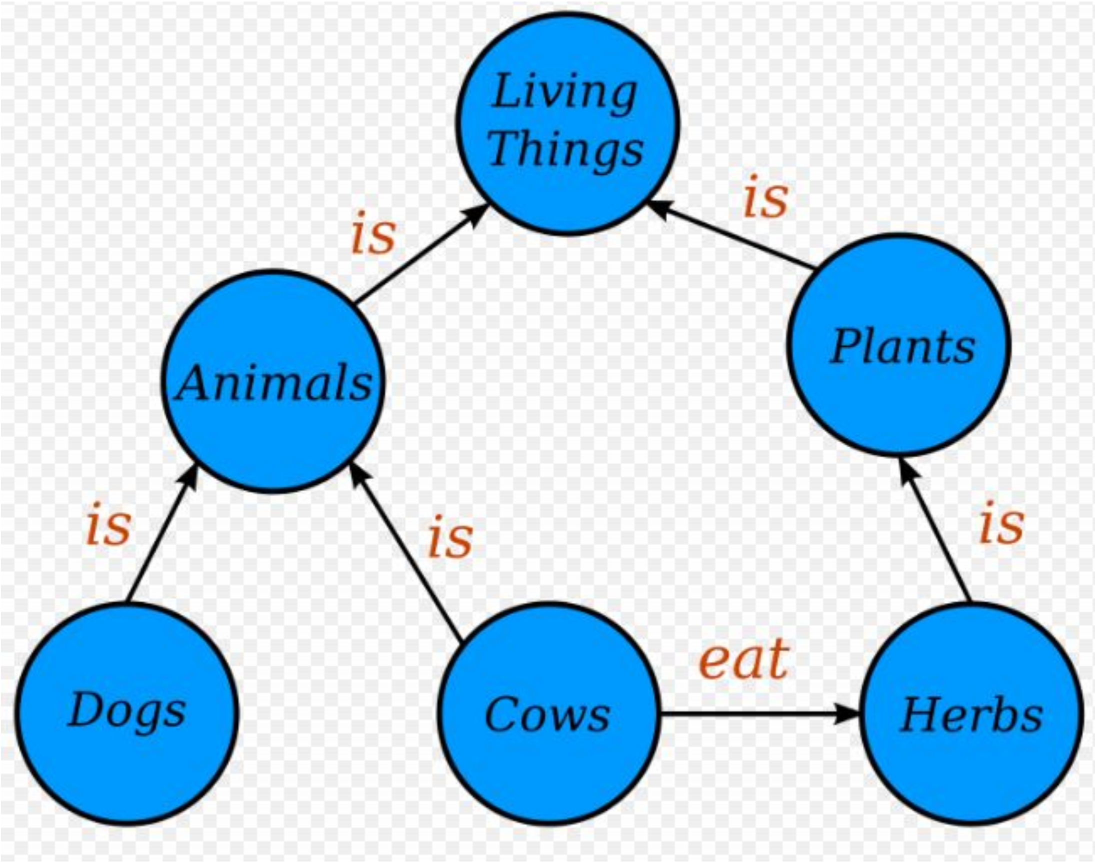
(a) DeepWalk / node2vec

(b) PTE

Network Science + Machine Learning

Knowledge Graphs:

- Natural Language Processing + Graphs (https://doi.org/10.1016/j.eswa.2019.112948)
- GraphRAG (https://www.microsoft.com/en-us/research/project/graphrag/)



Conclusions

- Complex networks are everywhere
  - sociology, engineering, math, physics, computer science, biology, ...
- Highly interdisciplinary

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Questions?

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